The Assignment of Workers to Jobs with Endogenous Information Selection

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Abstract

We present a model where information processing constraints on workers and firms lead to an endogenous matching function. We provide conditions under which the matching process has a unique equilibrium computable in closed-form. The main finding is that equilibrium matching is generally inefficient. This result does not depend on the form of heterogeneity, the distribution of surplus or bargaining rules. It is driven by information processing constraints which weaken the strategic complementarities and enhance the negative externalities in search efforts of workers and firms. A closed-form solution of the model provides a bound on the size of this inefficiency.

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1 Introduction

Efficiency of the matching process between workers and jobs and equilibrium unemployment have been a central issue of dispute in economics since Phelps (1967) and Friedman (1968) introduced the notion of the "natural" rate of unemployment. The quest for efficiency has taken the literature to look into the differences between models designed for centralized and decentralized markets and, ultimately, attributed these differences to frictions and externalities in order to account for wage and employment fluctuations. A suitable tool to explore these sources of inefficiency has been the 'matching function', which gives the flow of hires as a function of the stocks of vacancies and unemployed workers. Despite convenience of such a function in accommodating search and matching frictions, the link between its properties and the conditions for efficiency has not been fully investigated.[1]

In this paper, we identify information frictions as the link between the two. We argue that information frictions play a crucial role in understanding matching of large numbers of workers to jobs and the inefficiency of this matching process. In particular, we show that costly information processing by workers and firms leads to an endogenous matching function in which the outcome of the matching process might be inefficient.

We present a novel model of interaction between stocks of unemployed workers and vacant jobs. In our economy, firms are actively searching for workers and workers are actively searching for jobs. Both workers and firms are heterogeneous. We assume that each firm has one vacancy and each worker can apply for one job. A match is formed if it is mutually accepted and the surplus from the match is split between the two parties. Building on rational inattention theory of Sims (2003), we postulate an economy where both job seekers and job creators have a cost of processing information that prevents them from instantaneously finding and matching with each other.

In our decentralized model, workers have limited abilities of processing information about perspective employers. As a result, their optimal search effort balances precision of the information about employers with costs of processing the information acquired. An analogous mechanism is at play on the employers' side. We also study a centralized economy. We postulate that the planner has information-processing constraints similar to the ones of the

[1]Petrongolo and Pissarides (2001) provide a survey of the literature on the matching function. The paper by Stevens (2007) represents one of the more recent advances.

two parties. We study conditions for efficiency of the matching process.

Our analysis consists of three parts. First, we consider a steady-state labor market equilibrium and compare the centralized economy to the decentralized one. It emerges that, in equilibrium, the constraint on information processing leads to (1) lower search effort than is socially optimal; (2) inefficient search outcomes, with a lower number of matches than is socially optimal.

Second, building on the meeting protocol of Stevens (2007), we consider a dynamic model where rationally inattentive workers and firms endogenously choose the number of applications sent and processed, respectively. This framework together with lattice-theoretical methods based on Vives (1990) allow us to derive an endogenous aggregate matching function of the economy based on information-processing frictions. Third, we derive conditions under which an equilibrium of the the matching process exists, is unique and computable in closed-form.

The main finding of the paper is that the planner's solution cannot be decentralized. Albeit in contrast with the literature on random and assortative matching,[2] this result can be explained by carefully analyzing the implications of information-processing constraints in our environment. The technology that limits people's ability to process information at infinite rate captures people's physical constraints to pay attention to all the available information. As a result, it forces people to select and focus on only a narrow partition of the available information. This technology is at the core of individual decision making. Since there is a physical barrier to processing information and information selection is an endogenous process, a planner cannot enforce a socially optimal equilibrium by encompassing the externalities generated by individual processes of information selection. Thus, a social planner fails to exploit the complementarities in search efforts of workers and firms when information-processing constraints are present. Moreover, from the analogy between information-processing limits and human ability of processing information, it follows that it is unlikely that any market mechanism could successfully internalize the inefficiency created by limited processing capacities.

A corollary of our theoretical results is that, in our economy, assignment of workers and jobs does not need to be assortative. The rationale behind this result is that information frictions weaken the perceived complementarities

[2]See, inter alios, Hosios (1990), Shi (2001), Eeckhout and Kircher (2010).

of search efforts of firms and workers alike. Hence, equilibrium mismatch occurs with a positive probability in our framework owing only to information frictions. Because supermodularity of the surplus function is not required for existence or uniqueness of equilibrium, matching does not have to be assortative as derived by Becker (1973).[3]

Endogenous information selection is the driving force of matching patterns in our model. It is the key mechanism responsible for having inefficient matching rates and, as a result, elevated unemployment rates as equilibrium outcomes. Moreover, the technology of information frictions based on information theoretic measures makes our setting general and applicable to any matching market,[4] while the lattice approach allows for a rich theoretical setting with continuous types and type distributions on both sides of the market.

This paper relates to three main strands of literature. First, the paper contributes to the literature on efficient assignment. Several directed and random search models have analyzed the efficiency properties of the equilibrium assignments in an environment with time-consuming search.[5] Shi (2001) shows that a wage posting mechanism can induce efficiency in a decentralized matching framework with heterogeneous types. Assuming exogenous matching functions and Nash bargaining rules, Shimer and Smith (2000), (2001) characterize endogenous evolution of type distributions of heterogeneous workers and firms and show that a decentralized framework with taxes and transfers can implement the socially efficient allocation of resources in this environment. Eeckhout and Kircher (2010) exploit the interaction between complementarities in match values and complementarities in search technologies to augment the standard Hosios necessary condition for efficiency. By contrast, our paper with an endogenous information selection sees the failure of decentralizing assignments as structural. In our model, almost surely there exists no market mechanism that can implement the planner's solution.[6] This outcome occurs because of reduced strategic complementar-

[3] A similar result has been derived by Anderson and Smith (2010) in the context of incomplete information about types and learning. Both our finding and Anderson and Smith (2010)'s rest on the fact that informational frictions undermine match supermodularity.

[4] Inter alia, marriage as in Becker (1973), education and trade.

[5] See, inter alios, Hosios (1990), Moen (1997) and Acemoglu and Shimer (1999).

[6] As we show in the text, only under the two extreme cases extensively used in the literature of zero and infinite cost of processing information, it is possible to recover the efficient equilibrium.

ities in the information acquisition process itself. Hence, traditional market indicators are of little guidance towards efficiency for market participants.

Second, the paper contributes to the literature on assignment and coordination frictions. Shimer (2005) introduces the notion of coordination frictions into the job matching process by postulating a labor market where workers cannot coordinate perfectly on where to queue for jobs. Combined with the assumption that different jobs pay different wages to any equally able workers, this kind of coordination friction places enough restrictions on the joint strategy space of workers and firms to loosen the complementarities between workers and firms' types. As a result, Shimer (2005) obtains that the decentralized market outcome is constrained efficient. By contrast, with finite processing capacity of both workers and firms, our framework accommodates coordination frictions on both sides of the market. With limited attention to process and send applications, firms cannot perfectly coordinate with workers and workers cannot perfectly coordinate with firms. The optimal distribution of attention chosen by each type induces an equilibrium joint distribution of strategies of workers and firms where cross-correlations are as tight as each agent desires within his information-processing limits. Even though information-processing constraints loosen the degree of complementarities between workers and firms types, we show that the outcome of a rational inattention model with two-sided coordination frictions generally is not socially optimal.

Third, the paper contributes to the literature on microfoundations of the aggregate matching function. Petrongolo and Pissarides (2001) survey the state of the empirical and theoretical literature.[7] In a paper most relevant for ours, Stevens (2007) postulates a Poisson queuing process for workers and firms with approximately constant search costs and derives a Cobb-Douglas matching function from the model's primitives. Building on Stevens (2007)'s meeting protocol, we identify information-processing constraints as search costs and let firms and workers endogenously choose the queuing process

[7] Broadly speaking, the two competing theories are the stock-flow matching theory and the random matching theory. A necessarily non-exhaustive list of examples of the stock-flow literature includes, inter alios, Lagos (2000), Coles and Smith (1998) and Coles (1999). Lagos (2000) postulates a framework where stock-flow matching at a disaggregate level provides a microfoundation of the aggregate matching function. Coles and Smith (1998) and Coles (1999) use the stock-flow matching model to explain misspecification of the aggregate matching function. Coles and Petrongolo (2008) test the random matching model against the stock-flow matching model and find evidence consistent with the former.

based on their optimal information structure. This approach allows us to fully characterize the matching technology as an endogenous response to fundamentals. Closed-form solutions obtained under symmetry assumptions illustrate the potential magnitude of matching inefficiency and its relationship to the size of information frictions.

The paper proceeds as follows: Section 2 outlines the one-shot model and provides conditions for existence and uniqueness of the equilibrium. Section 3 introduces the dynamic model and derives the matching function in equilibrium. Section 4 develops a special case which leads to an equilibrium computable in closed-form. Section 5 concludes. Proofs that are not included in the main text can be found in the Appendix.

2 One Shot Model

For expositional purposes, in this section we first restrict our attention to a stylized one-shot model. It fully illustrates the main intuition and derivations of the main theoretical results. In the next section we show that this simple framework naturally extends to repeated interactions, to a continuum of firm and worker types, to multiple simultaneous applications and to alternative meeting protocols.

2.1 Setup

We assume that there are N workers indexed by $x \in \{1, ..., N\}$ and M firms indexed by $y \in \{1, ..., M\}$. Workers and firms search for each other in order to match. A match between worker x and firm y generates a surplus $f(x, y)$. If a firm and a worker match, the surplus is split between the worker and the firm by unilateral bargaining in such a way that the worker gets a wage $w(x, y)$ and the firm gets a profit $\pi(x, y)$. The surplus, wage and profit are known to potential participants of each match.

The way we model endogenous information acquisition builds on elements of information theory and the rational inattention literature. We assume that workers and firms face costs of search, $c_w(x, \kappa_w(x))$ and $c_f(y, \kappa_f(y))$, respectively, which incorporate costs of processing information. These costs are functions of information capacities, denoted $\kappa_w(x)$ and $\kappa_f(y)$, of worker x and firm y respectively. Information capacities represent amounts of information processed by agents measured in bits.

Had workers and firms had infinite information processing capacity, they would be able to perfectly identify mutual best matches. The outcome of the model would be an equilibrium of the classical assignment model. In this case the optimal strategies of firms and workers would be to match with their equilibrium counterparts with probability one, i.e. they would be able to choose infinitely precise strategies.

When information processing is costly, agents are bound to make probabilistic choices. We model this by assuming that agents choose mixed strategies. We denote these strategies $p(x, y)$ and $q(y, x)$. They represent probabilities of worker x applying to firm y and firm y considering application of worker x respectively. We also refer to them as allocations of attention of workers and firms. We assume that agents can rationally choose these strategies in order to maximize their payoff and economize on processing information.

We denote $m_w(x, y)$ the equilibrium matching rate faced by worker x when applying to firm y. Similarly, we denote $m_f(y, x)$ the equilibrium matching rate faced by firm y when considering worker x. These rates are assumed to be common knowledge to participating parties. We model worker applications and firm acceptances as random draws from the chosen distributions $p(x, y)$ and $q(y, x)$. For now we restrict ourselves to a one shot model. That is, each worker sends an application to a single firm, and each firm accepts applications from a single worker. A match is formed between worker x and firm y if and only if, according to the worker's random draw from $p(x, y)$, worker x applies to firm y, according to the firm's random draw from $q(y, x)$, firm y accepts the application from worker x, and their payoffs are non-negative. Since negative payoffs lead to de facto zero payoffs due to absence of a match, without loss of generality, we can assume that all payoffs are non-negative:

$$f(x, y) \geq 0, \qquad \pi(x, y) \geq 0, \qquad w(x, y) \geq 0.$$

2.2 Equilibrium

The probabilistic nature of the model allows us to consider ex ante optimal choices of strategies by firms and workers. Worker x chooses his strategy $p(x, y)$ to maximize his expected income flow:

$$Y_w(x) = \sum_{y=1}^{M} w(x,y)\, m_w(x,y)\, p(x,y) - c_w(x, \kappa_w(x)) \rightarrow \max_{p(x,y)}$$

We normalize the outside options of the worker and the firm to zero. The worker gets his expected wage in a match with firm y conditional on matching with that firm. He also incurs a search cost, which depends on the amount of information processed by the worker, defined as follows:

$$\kappa_w(x) = \sum_{y=1}^{M} p(x,y) \log_2 \frac{p(x,y)}{1/M} \tag{1}$$

where the worker's strategy must satisfy $\sum_{y=1}^{M} p(x,y) = 1$ and $p(x,y) \geq 0$ for all y. Our definition of information, $\kappa_w(x)$, represents the relative entropy between a uniform prior $\{1/M\}$ over firms and the posterior mixed strategy, $p(x,y)$. Shannon's relative entropy can be interpreted as the reduction of uncertainty that the worker can achieve by choosing his mixed strategy. This definition is a special case of Shannon's channel capacity when information structure is the only choice variable. Thus, our assumption is a special case of a uniformly accepted definition of information tailored to our problem.

Similarly, firm y chooses her strategy $q(y,x)$ to maximize her expected income flow:

$$Y_f(y) = \sum_{x=1}^{N} \pi(x,y)\, m_f(y,x)\, q(y,x) - c_f(y, \kappa_f(y)) \rightarrow \max_{q(y,x)}$$

The firm profits from a match with worker x conditional on matching with that worker and pays the cost of search. The search cost on the firm's side also depends on the amount of information processed by the firm:

$$\kappa_f(y) = \sum_{x=1}^{N} q(y,x) \log_2 \frac{q(y,x)}{1/N} \tag{2}$$

where the firm's strategy must satisfy $\sum_{x=1}^{N} q(y,x)\, dx = 1$ and $q(y,x) \geq 0$ for all x.

Definition 1 *An matching* equilibrium *is a set of strategies of workers,* $\{p(x,y)\}_{x=1}^{N}$*, and firms,* $\{q(y,x)\}_{y=1}^{M}$*, and matching rates* $\{m_f(y,x)\}_{x,y=1}^{N,M}$ *and* $\{m_w(x,y)\}_{x,y=1}^{N,M}$ *such that:*

1) strategies solve problems of the workers and firms;

2) matching rates satisfy equilibrium conditions:

$$m_f(y,x) = p(x,y), \qquad m_w(x,y) = q(y,x) \tag{3}$$

2.3 Properties of equilibrium

Using condition (3) of the definition of equilibrium, payoffs of workers and firms can be rewritten as follows:

$$Y_w(x) = \sum_{y=1}^{M} w(x,y)\, q(y,x)\, p(x,y) - c_w(x, \kappa_w(x)),$$

$$Y_f(y) = \sum_{x=1}^{N} \pi(x,y)\, p(x,y)\, q(y,x) - c_f(y, \kappa_f(y)).$$

Theorem 1 *A (Nash) equilibrium of the matching model exists.*

Proof. The equilibrium of the matching model can be interpreted as a standard Nash equilibrium. All the results for lattices described by Vives (1990) apply to it. Since cross-derivatives of objective functions in our case are all non-negative, this game is supermodular. Hence there exists a Nash equilibrium. ■

When cost functions are non-decreasing and convex, it is easy to verify that first-order conditions are necessary and sufficient conditions for equilibrium.[8] Rearranging the first order conditions for the worker, we obtain:

[8] Taking derivatives of the Lagrangian function corresponding to the problem of worker x, we obtain for all y:

$$w(x,y)\, q^*(y,x) - \left.\frac{\partial c_w(\kappa_w)}{\partial \kappa_w}\right|_{p^*} \frac{1}{\ln 2}\left(\ln \frac{p^*(x,y)}{1/M} + 1\right) = \lambda_w(x)$$

We can invert this first-order condition to characterize the optimal strategy:

$$p^*(x,y) = \frac{1}{M}\exp\left(\frac{w(x,y)q^*(y,x)}{\frac{1}{\ln 2}\left.\frac{\partial c_w(\kappa_w)}{\partial \kappa_w}\right|_{p^*}} - \frac{\lambda_w(x)}{\frac{1}{\ln 2}\left.\frac{\partial c_w(\kappa_w)}{\partial \kappa_w}\right|_{p^*}} - 1\right)$$

$$p^*(x,y) = \exp\left(\frac{w(x,y)\, q^*(y,x)}{\frac{1}{\ln 2} \left. \frac{\partial c_w(\kappa_w)}{\partial \kappa_w} \right|_{p^*}} \right) / \sum_{y'=1}^{M} \exp\left(\frac{w(x,y')\, q^*(y',x)}{\frac{1}{\ln 2} \left. \frac{\partial c_w(\kappa_w)}{\partial \kappa_w} \right|_{p^*}} \right) \tag{4}$$

This sufficient condition for equilibrium casts the optimal strategy of worker x in the form of a best response to optimal strategies of firms. Following a similar route we derive the optimal strategy of firm y in the form of a best response to optimal strategies of workers:

$$q^*(y,x) = \exp\left(\frac{\pi(x,y)\, p^*(x,y)}{\frac{1}{\ln 2} \left. \frac{\partial c_f(\kappa_f)}{\partial \kappa_f} \right|_{q^*}} \right) / \sum_{y'=1}^{M} \exp\left(\frac{\pi(x',y)\, p^*(x',y)}{\frac{1}{\ln 2} \left. \frac{\partial c_f(\kappa_f)}{\partial \kappa_f} \right|_{q^*}} \right) \tag{5}$$

Theorem 2 *The equilibrium of the model is unique, if*

a) cost functions are non-decreasing and convex;

b) $\left. \frac{\partial c_w(\kappa_w)}{\partial \kappa_w} \right|_{p^*(x,y)} > w(x,y)\, p^*(x,y);$

c) $\left. \frac{\partial c_f(\kappa_f)}{\partial \kappa_f} \right|_{q^*(y,x)} > \pi(x,y)\, q^*(y,x).$

Proof. The payoffs of all firms and workers are continuous in their strategies. They are also concave in these strategies when cost functions are (weakly) increasing and convex in information capacities. "Diagonal dominance" conditions (b) and (c) guarantee that the Hessian of the game is negative definite along the equilibrium path. Then, by the generalized Poincare-Hopf index theorem of Acemoglu, Simsek and Ozdaglar (2007), the equilibrium is unique. ■

Note that assumptions we use to prove uniqueness are by no means restrictive. The assumption that cost functions are non-decreasing and convex is a natural one. Most of the literature on information processing assumes that either the cost function is linear, or there is a capacity constraint on processing information, which implies a vertical cost function after a certain amount of information has been processed. Our assumption incorporates both of these as special cases. "Diagonal dominance" conditions in our case can be interpreted as implying that the marginal cost of information processing should be sufficiently high for the equilibrium to be unique. If these conditions don't hold, then there can be multiple equilibria. This is a well-known outcome in the assignment model, which is a special case of our model under zero marginal information costs.

Note also, that by the nature of the index theorem used in the proof, it is enough to check diagonal dominance conditions locally in the neighborhood of equilibrium. There is no requirement for them to hold globally. This suggests a simple way of finding equilibria of our model in most interesting cases. We first need to find one solution to the first-order conditions (4-5) and then check that diagonal dominance conditions are satisfied for this solution.

Equilibrium conditions (4) and (5) have an intuitive interpretation. They predict that the higher the worker's private gain from matching with a firm, the higher the probability of applying to that firm. Similarly, the higher the probability that a firm considers a particular worker, the higher the probability that that worker applies to the firm. Overall, workers place higher probabilities on applications to firms which give them higher expected private gains. Thus, firms are naturally ordered in each worker's strategy by probabilities of applying to those firms. The strategies of firms have the same properties due to symmetry of the problem. In equilibrium, firms' strategies are best responses to strategies of workers, and workers' strategies are best responses to strategies of firms.

It is instructive to understand properties of equilibria for two limiting cases. First, consider the case when marginal costs of processing information go to zero. In this case, application and consideration strategies become more and more precise. In the limit, in every equilibrium workers place unit probabilities on particular firms, and those firms respond with unit probabilities of considering these same workers. All such equilibria implement equilibria of the classical assignment problem.

Second, consider the opposite case when marginal costs go to infinity. In this case, the difference between probabilities of applying to different firms shrinks. In the limit, in the unique equilibrium optimal strategies of firms and workers approach a uniform distribution. This unique equilibrium implements the standard uniform random matching assumption extensively used in the literature. Thus, the assignment model and the random matching model are special cases of our model, when costs of information are either tiny or enormous. A characterization of equilibria for intermediate values of costs of information processing is a novel theoretical result.

2.4 Social Planner's problem

It is instructive to compare equilibrium outcomes of our model to a social planner's solution. We assume that the social planner maximizes the total surplus of the economy, which is a utilitarian welfare function. In order to achieve a social optimum, the planner can instruct workers and firms upon which mixed strategy to choose. If no costs of processing information were present, the planner would always choose to implement the efficient solution of the assignment problem and instruct agents to follow infinitely precise strategies. To study the constrained efficient allocation we impose upon the social planner the same information processing constraints that we place on workers and firms. Thus, the social planner maximizes the following weighted sum of surpluses:

$$\sum_{x=1}^{N}\sum_{y=1}^{M} f\left(x,y\right)p\left(x,y\right)q\left(y,x\right)-\sum_{x=1}^{N} c_w\left(x,\kappa_w\left(x\right)\right)-\sum_{y=1}^{M} c_f\left(y,\kappa_f\left(y\right)\right)$$

subject to information constraints (1-2) and to the constraints that $p\left(x,y\right)$ and $q\left(y,x\right)$ are well-defined probability distributions. Under the assumption of increasing convex cost functions, the social welfare function is concave in the strategies of workers and firms. Hence, first-order conditions are sufficient conditions for a maximum. Rearranging and substituting out Lagrange multipliers, we arrive at the following characterization of the social planner's allocation:

$$p^o\left(x,y\right)=\exp\left(\frac{f\left(x,y\right)q^o\left(y,x\right)}{\frac{1}{\ln 2}\left.\frac{\partial c_w(\kappa_w)}{\partial\kappa_w}\right|_{p^o}}\right)/\sum_{y'=1}^{M}\exp\left(\frac{f\left(x,y'\right)q^o\left(y',x\right)}{\frac{1}{\ln 2}\left.\frac{\partial c_w(\kappa_w)}{\partial\kappa_w}\right|_{p^o}}\right),\qquad(6)$$

$$q^o\left(y,x\right)=\exp\left(\frac{f\left(x,y\right)p^o\left(x,y\right)}{\frac{1}{\ln 2}\left.\frac{\partial c_f\left(\kappa_f\right)}{\partial\kappa_f}\right|_{q^o}}\right)/\sum_{y'=1}^{M}\exp\left(\frac{f\left(x',y\right)p^o\left(x',y\right)}{\frac{1}{\ln 2}\left.\frac{\partial c_f\left(\kappa_f\right)}{\partial\kappa_f}\right|_{q^o}}\right).\qquad(7)$$

The first observation to make is that the structure of the social planner's solution is very similar to the structure of decentralized equilibrium. Second, from the workers' perspective, the only difference between the centralized and decentralized equilibrium strategies is that the probability of applying to a firm depends on the social gain from a match rather than on the private gain. Notice the same difference holds from the perspective of firms. Thus, it is socially optimal to consider the whole expected surplus in determining the socially optimal strategies, while in the decentralized equilibrium workers and firms only consider their private gains.

Since the social gain is always the sum of private gains, it is not feasible to split the surplus between the worker and the firm in such a way as to achieve the socially optimal allocation. For any finite positive values of costs of information, for the equilibrium to be socially optimal the following conditions have to hold:

$$\pi(x,y) = f(x,y), \qquad w(x,y) = f(x,y).$$

Recall that private gains have to add up to the whole surplus, $\pi(x,y) + w(x,y) = f(x,y)$. Therefore, these optimality conditions can only hold in equilibrium if the surplus is zero. Thus, we have just proven the following theorem:

Theorem 3 *The equilibrium is socially inefficient if the equilibrium is unique and all of the following hold:*

1) $0 < \left.\frac{\partial c_w(\kappa_w)}{\partial \kappa_w}\right|_{p^*} < \infty$

2) $0 < \left.\frac{\partial c_f(\kappa_f)}{\partial \kappa_f}\right|_{q^*} < \infty$

3) $f(x,y) > 0$ *for some* (x,y).

The only special cases, when inefficiency does not arise, are the limiting cases discussed earlier. When costs of information are absent, the best equilibrium of the assignment model is socially optimal. When costs of information are enormous, the random matching outcome is the best possible outcome. The case when all potential matches yield zero surplus is a trivial case of no gains from matching. For all intermediate values of costs the decentralized equilibrium is socially inefficient.

One notable property of the equilibrium is that, by considering only fractions of the total surplus in choosing their strategies, workers and firms place lower probabilities on applying to their best matches. This implies that in equilibrium attention of workers and firms is more dispersed and the number of matches is lower than is socially optimal. The main reason for this inefficiency is the reduction in strategic complementarities.

To illustrate these complementarities consider the case of a firm, which chooses its strategy under the assumption that all workers implement socially optimal application strategies. Because the firm only considers its private gains from matches with workers, the firm's optimal response would be to pay less attention to the best workers, than is socially optimal. In a second step, taking as given these strategies of firms, workers will be dis-incentivized not only by the fact that they consider fractions of total match gains, but also by the fact that firms pay less attention to them than it is socially optimal. This double dis-incentive will lower the probabilities of applying to their best matches for all workers. Iterating in this way on strategies of workers and firms, at each step we get a reduction in the probability of applying to the best match.

The negative externalities of considering only private gains by workers and firms reinforce each other through strategic best responses of workers to firms and firms to workers. Thus, we have uncovered a major source of inefficiency in the matching process. Information processing constraints weaken strategic complementarities between strategies of workers and firms. By the same token they reduce synergies from cooperation and enhance negative externalities in search efforts of workers and firms. As a consequence of these negative externalities, firms and workers fail to fully internalize the gains from coordination.

In the next section we extend our results to a continuous-tim framework with a continuum of worker and firm types, multiple applications and a more realistic meeting protocol.

3 Extended model

3.1 Primitives

Let worker and firm types be continuously distributed on compact measurable sets X and Y. Let there be a measure $u(x)$ of workers of each type $x \in X$ and a measure $v(y)$ of firms of each type $y \in Y$. Workers and firms search for each other in order to match. Like before, a match between a worker of type x and firm y generates a surplus $f(x, y)$. If a firm and a worker match, the surplus is split between the worker and the firm in a such a way that the worker gets a wage $w(x,y)$ and the firm gets a profit $\pi(x,y)$. The surplus, wage and profit conditional on types are common knowledge.

We assume that the worker and the firm face relatively general search costs of the forms:

$$c_w(\alpha(x), \kappa(x)) = \chi_w(x) \frac{\alpha(x)^{k_w(x)}}{k_w(x)} + \alpha(x)\theta_w(x)\kappa_w(x)$$
$$c_f(\gamma(x), \kappa(x)) = \chi_f(y) \frac{\gamma(y)^{k_f(y)}}{k_f(y)} + \gamma(y)\theta_f(y)\kappa_f(y)$$

Each search cost has two components. The first component represents a convex cost of processing applications, which depends only on the numbers of applications, $\alpha(x)$ and $\gamma(y)$. The second component is the cost of processing information. It is proportional to the number of applications. Following notation of the one-shot model, $\kappa_w(x)$ and $\kappa_f(y)$ are amounts of information per application processed by firms and workers. Both are measured in bits. Agent-specific parameters, denoted $\theta_w(x)$ and $\theta_f(y)$, stand for marginal costs of processing information in dollars per bit.

Denote the equilibrium matching rate faced by the worker of type x when applying to a firm of type y as $m_w(x,y)$. Similarly, we denote the matching rate faced by firm y when considering worker of type x as $m_f(y,x)$. The worker maximizes his expected income flow:

$$Y_w(x) = \int_Y w(x,y)\, m_w(x,y)\, p(x,y)\, \alpha(x)\, dy - c_w(\alpha(x), \kappa_w(x))$$

with respect to his search intensity $\alpha(x)$ and allocation of attention $p(x,y)$. The worker x gets his expected wage conditional on matching with a firm of type y net of the cost of search. The search cost depends on the amount of information processed by the worker, defined as follows:

$$\kappa_w(x) = \int_Y p(x,y) \log_2 \frac{p(x,y)}{v(y) / \int_Y v(y)\, dy} dy \tag{8}$$

where $p(x,y)$ is a probability distribution, which satisfies the usual assumptions:

$$\int_Y p(x,y)\,dy = 1, \qquad p(x,y) \geq 0. \tag{9}$$

The firm also maximizes her expected income flow:

$$Y_f(y) = \int_X \pi(x,y)\, m_f(y,x)\, q(y,x)\, \gamma(y)\, dx - c_f(\gamma(y), \kappa_f(y))$$

with respect to her search intensity $\gamma(y)$ and allocation of attention $q(y,x)$. Firm y gets a profit conditional on matching with a worker of type x net of the cost of search. The search cost depends on the amount of information processed by the worker, defined as follows:

$$\kappa_f(y) = \int_X q(y,x) \log_2 \frac{q(y,x)}{u(x) / \int_X u(x)\,dx} dx \tag{10}$$

where $q(y,x)$ is a probability distribution, which also satisfies the usual assumptions:

$$\int_X q(y,x)\,dx = 1, \qquad q(y,x) \geq 0. \tag{11}$$

3.2 Meeting protocol

We extend the telephone line meeting protocol of Stevens (2007) to allow for two-sided heterogeneity. We assume that out of the stock of $u(x)$ workers of type x, $\alpha(x)\,u(x)$ are sending applications, while the rest are enjoying leisure/waiting. The expected number of applications sent by worker of type x to firm of type y is $p(x,y)\,\alpha(x)\,u(x)$.

Out of the stock of $v(y)$ firms of type y, $v_p(y)$ spend time processing applications. Before knowing the type of worker they are facing, firms choose applications from which types of workers to pay attention to, and how quickly to respond. Upon receiving an application from worker of type x, the firm processes on average $\gamma(y)$ applications and accepts the application with probability $q(y,x)$.

We denote $v_p(x,y)$ the stock of firms of type y processing applications from workers of type x. A fraction $\gamma(y)$ of them transition to the waiting state per period. The total outflow to the waiting state is $\gamma(y)\,v_p(y)$. Those firms that accepted the application hire the worker and are replaced by a copy of them in the waiting pool. Those which rejected the application start waiting for another application to arrive. In a stationary equilibrium, the inflow of firms into the processing pool equals the outflow:

$$u_s(x,y)\frac{v_w(y)}{v(y)} = \gamma(y)\, v_p(x,y).$$

Using the accounting identity for the number of firms of type y, we can solve for the numbers of firms in each state. Then, the equilibrium number of matches for each pair of types equals:

$$m(x,y) = \gamma(y)\, v_p(x,y)\, q(y,x) = \frac{p(x,y)\,\alpha(x)\,u(x)\,q(y,x)\,\gamma(y)\,v(y)}{\int_X \left(v(y)\,\gamma(y)\,q(y,x') + p(x',y)\,\alpha(x')\,u(x')\right)dx'}$$

The personal meeting rates arising from this meeting protocol are computed as follows:

$$\mu_w(x,y) = \frac{m(x,y)}{q(y,x)\,p(x,y)\,\alpha(x)\,\gamma(y)}\frac{1}{u(x)} \qquad \mu_f(y,x) = \frac{m(x,y)}{q(y,x)\,p(x,y)\,\alpha(x)\,\gamma(y)}\frac{1}{v(y)}$$

3.3 Equilibrium

Definition 2 *An equilibrium matching process is a set of strategies of workers $\{p(x,y), \alpha(x)\}$ and firms $\{q(y,x), \gamma(y)\}$, matching rates $m_f(y,x)$ and $m_w(x,y)$ such that:*

1) strategies solve problems of the workers and firms;

2) matching rates satisfy steady-state equilibrium conditions:

$$m_w(x,y) = q(y,x)\,\gamma(y)\,\mu_w(x,y), \qquad m_f(y,x) = p(x,y)\,\alpha(x)\,\mu_f(x,y).$$

We can simplify the definition of equilibrium and cast it into a Bayesian Nash equilibrium by redefining strategies of firms and workers. We introduce the following notation:

$$\hat{p}(x,y) = \alpha(x)\,p(x,y), \qquad \hat{q}(y,x) = \gamma(y)\,q(y,x)$$

Utilizing this notation, the workers' and firms' problems can be rewritten as an unconstrained maximization problems with payoffs:

$$Y_x(\hat{p}_x,\hat{q}) = \left[\begin{array}{c} \int_Y w(x,y)\,\mu_w(x,y)\,\hat{q}(y,x)\,\hat{p}(x,y)\,dy - \chi_w(x)\frac{\left(\int_Y \hat{p}(x,y)dy\right)^{k_w(x)}}{k_w(x)} \\ -\frac{\theta_w(x)}{\ln 2}\int_Y \hat{p}(x,y)\ln\frac{\hat{p}(x,y)/\int_Y \hat{p}(x,y')dy'}{v(y)/\int_Y v(y')dy'}dy \end{array} \right]$$

$$Y_y(\hat{q}_y,\hat{p}) = \left[\begin{array}{c} \int_X \pi(x,y)\,\mu_f(x,y)\,\hat{p}(x,y)\,\hat{q}(y,x)\,dx - \chi_f(y)\frac{\left(\int_X \hat{q}(y,x)dx\right)^{k_f(y)}}{k_f(y)} \\ -\frac{\theta_f(y)}{\ln 2}\int_X \hat{q}(y,x)\ln\frac{\hat{q}(y,x)/\int_X \hat{q}(y,x')dx'}{u(x)/\int_X u(x')dx'}dx \end{array} \right]$$

where equilibrium meeting rates are taken as given. These payoffs can be analyzed and optimized using standard techniques borrowed from the calculus of variations. We refer the reader to the appendix for technical details. We introduce the following assumptions:

A1. Type sets $x \in X$ and $y \in Y$ are compact.

A2. Action sets $\hat{p}_x \in [0, P]$ and $\hat{q}_y \in [0, Q]$ are compact, i.e. P and Q are finite.

A3. $w(x,y)\,\mu_w(x,y) \geq 0$ and $\pi(x,y)\,\mu_f(x,y) \geq 0$ for all x and y.

A4. Costs parameters $\theta_w(x)\,,\ \theta_f(y)\,,\ \chi_w(x)\,,\ \chi_f(x)$ are non-negative. Costs of applications are convex: $k_w(x) \geq 1$, $k_f(y) \geq 1$.

A5. "Diagonal dominance" conditions are satisfied along the equilibrium path:

$$\left|\frac{\partial^2 Y_x(\hat{p}_x,\hat{q}_y)}{\partial\hat{p}_x\partial\hat{p}_x}\right|_{\hat{p}_x^*,\hat{q}_y^*} > \left|\frac{\partial^2 Y_x(\hat{p}_x,\hat{q}_y)}{\partial\hat{p}_x\partial\hat{q}_y}\right|_{\hat{p}_x^*,\hat{q}_y^*}$$

$$\left|\frac{\partial^2 Y_y(\hat{q}_y,\hat{p}_x)}{\partial\hat{q}_y\partial\hat{q}_y}\right|_{\hat{p}_x^*,\hat{q}_y^*} > \left|\frac{\partial^2 Y_y(\hat{q}_y,\hat{p}_x)}{\partial\hat{q}_y\partial\hat{p}_x}\right|_{\hat{p}_x^*,\hat{q}_y^*}$$

Assumptions A1-A2 postulate that types and actions lie on compact domains, while Assumption A3 states that matching is profitable for both parties. Assumption A4 requires information processing costs to be non-negative. This assumption is important for uniqueness of equilibrium since information-processing constraints lower the perceived degree of complementarities between search efforts of workers and firms. Finally, A.5 guarantees that we have a contraction mapping of the best response functions.

Theorem 4 *Under assumptions A1, A2 and A3 Nash Equilibria exist.*

Proof. The proof is achieved in three steps and follows Vives (1990):

(a) The set of all measurable functions mapping a compact set into a compact set is a lattice under the natural ordering.

(b) The game is supermodular since the cross-derivatives of the objective functions are all non-negative.

$$\frac{\partial^2 Y_x\left(\hat{p}_x, \hat{q}_y\right)}{\partial \hat{p}_x \partial \hat{q}_y} = w\left(x, y\right) \mu_w\left(x, y\right) \qquad \frac{\partial^2 Y_y\left(\hat{q}_y, \hat{p}_x\right)}{\partial \hat{q}_y \partial \hat{p}_x} = \pi\left(x, y\right) \mu_f\left(x, y\right)$$

(c) In a supermodular game on a lattice Nash equilibria exist. ■

Lemma 1 *Under A1 and A2 Y_x and Y_y are continuous in $\hat{p}_x$ and $\hat{q}_y$ respectively.*

Proof. All the integrands are continuously differentiable with respect to strategies, and all the integrals are taken over compact sets. ■

Lemma 2 *Under assumptions A1, A2 and A4 Y_x and Y_y are concave in $\hat{p}_x$ and $\hat{q}_y$ respectively.*

Proof. Using the previous lemma, it remains to verify that the second variational derivatives are everywhere non-positive. That is indeed the case under assumption A4. ■

Theorem 5 *Under A.1, A.2, A.3, A.4 the first-order conditions are necessary and sufficient conditions for equilibrium.*

Proof. This theorem is a direct consequence of the previous two lemmas and assumption A3. ■

Theorem 6 *Under assumptions A.1, A.2, A.3, A.4, A.5 the matching process has a unique Nash equilibrium.*

Proof. Diagonal dominance conditions guarantee that the Hessian of the game is negative definite along the equilibrium path. It follows from lemmas 1 and 2 that the payoff functionals are continuous and concave. Then, the generalized Poincare-Hopf index theorem of Acemoglu, Simsek and Ozdaglar (2007) implies that the equilibrium is unique. ■

The first-order conditions can be simplified and rewritten using the original notation to yield distributions of attention and search intensities for both firms and workers. We relegate the derivation to the appendix and only report the necessary and sufficient conditions here:

$$p^*(x,y) = \frac{v(y)\exp\left(\frac{\ln 2}{\theta_w(x)} g_w(x,y)\right)}{\int_Y v(y')\exp\left(\frac{\ln 2}{\theta_w(x)} g_w(x,y')\right) dy'}$$

$$\alpha^*(x) = \left[\frac{1}{\ln 2}\frac{\theta_w(x)}{\chi_w(x)}\ln\frac{\int_Y v(y)\exp\left(\frac{\ln 2}{\theta_w(x)} g_w(x,y)\right) dy}{\int_Y v(y)\,dy}\right]^{\frac{1}{k_w(x)-1}}$$

$$q^*(y,x) = \frac{u(x)\exp\left(\frac{\ln 2}{\theta_f(y)} g_f(x,y)\right)}{\int_X u(x')\exp\left(\frac{\ln 2}{\theta_f(y)} g_f(x',y)\right) dx'}$$

$$\gamma^*(y) = \left[\frac{1}{\ln 2}\frac{\theta_f(y)}{\chi_f(y)}\ln\frac{\int_X u(x)\exp\left(\frac{\ln 2}{\theta_f(y)} g_f(x,y)\right) dx}{\int_X u(x)\,dx}\right]^{\frac{1}{k_f(y)-1}}$$

where private gains of workers and firms are defined as follows:

$$g_w(x,y) = w(x,y)\,\mu_w(x,y)\,q^*(y,x)\,\gamma^*(y)$$

$$g_f(y,x) = \pi(x,y)\,\mu_f(x,y)\,p^*(x,y)\,\alpha^*(x)$$

Like in the one shot model, equilibrium allocations of attention have an intuitive interpretation. The higher agents' expected private gains from matching with each other, the higher the probabilities of applying/processing applications. Firms and workers are naturally ordered in probabilities of allocating attention to each other. In equilibrium, firms' strategies are best responses to strategies of workers, and workers' strategies are best responses to strategies of firms. The strategies of firms and workers have similar properties due to the symmetry of the problem.

The rich structure of heterogeneity in costs, surpluses and types is fully taken into account by all agents in the model. Relatively unrestrictive conditions for uniqueness allow us to accomodate a rich class of matching models with different structures of fundamentals. Each element of this rich structure of fundamentals potentially has an impact on matching rates between all type pairs, which in turn affect the number and quality of matches in equilibrium. Therefore, this model can be extremely useful for understanding the consequences of heterogeneity for the aggregate matching function.

Note, that neither existence nor uniqueness of equilibrium relies on supermodularity of the surplus function. Therefore, assortative matching (in expected terms) needs not be an equilibrium outcome of the model. Thus, our model can generate a rich structure of equilibrium outcomes and has a potential to speak to the rich empirical literature on the determinants of wages.

3.4 The social planner's problem

Similarly to the one-shot model, we assume that the social planner maximizes the total surplus of the economy subject to the the same constraints that we place on workers and firms in equilibrium. We relegate the details of the derivation to the appendix and note that under the aforementioned assumptions the resulting conditions for social optimality are the same as for equilibrium, except social gains are defined as follows:

$$g_w^o(x,y) = f(x,y)\,\mu_w(x,y)\,q^o(y,x)\,\gamma^o(y) - \phi(y),$$

$$g_f^o(y,x) = f(x,y)\,\mu_f(x,y)\,p^o(x,y)\,\alpha^o(x) - \phi(y),$$

where $\phi(y)$ is a constant which only depends on the firm types.

Theorem 7 *The equilibrium is socially inefficient under assumptions A1-A5 and if all of the following hold:*
1) $0 < \theta_w(x) < \infty$
2) $0 < \theta_f(y) < \infty$
3) $f(x,y) > 0$ *for some* (x,y).

See proof in the appendix. The proof has a similar intuition to the one shot model. It is not feasible to achieve the social optimum, because to do that the planner needs to promise private gains which violate the resource constraint. This result is crucial for understanding the magnitude of potential inefficiencies in the matching process. For that it is useful to compute the aggregate number of equilibrium matches. Note that in this framework the matching rate, an analog of the matching function, can be computed as:

$$M = \int_X \int_Y \frac{q(y,x)\,p(x,y)\,\alpha(x)\,u(x)\,\gamma(y)\,v(y)}{v(y)\,\gamma(y) + \int_X p(x',y)\,\alpha(x')\,u(x')\,dx'} dxdy$$

3.5 Simplifying assumptions

To facilitate quantitative explorations of the properties of equilibrium outcomes and the size of inefficiency we make several auxiliary assumptions.

A6 Workers and firms are distributed uniformly: $u(x) = U$, $v(y) = V$.

A7 Workers are identical: $\theta_w(x) = \theta_w,\ \ \chi_w(x) = \chi_w,\ \ k_w(x) = k_w$.
Firms are identical: $\theta_f(x) = \theta_f,\ \chi_f(x) = \chi_f,\ \ k_f(y) = k_f$.

A8 Workers and firms are placed on connected unit intervals:

$$X = [0,1], \qquad Y = [0,1].$$

A9 Match surplus and Nash bargaining weights depend on distance, $d(x,y)$, only:

$$f(x,y) = f(d(x,y)), \qquad w(d) = \beta(d)\,f(d), \qquad \pi(d) = (1-\beta(d))\,f(d).$$

where $d(x,y) = \min\{|x-y|, 1-x+y, 1-y+x\} \in \left[0, \frac{1}{2}\right]$.

Thus, we place workers and firms on connected unit intervals and define the surplus of each match as a function of the distance between types. Firm and worker types are symmetric. Symmetry and uniformity simplify the analysis substantially. Conditional on assumptions A6-A9, all match-specific variables become distance-specific, all firm- or worker-specific variables lose this dependence. Therefore, the solution to the model can be rewritten as follows:

$$\alpha^* = \left[\frac{1}{\ln 2} \frac{\theta_w}{\chi_w} \ln 2 \int_0^{\frac{1}{2}} \exp \left(\frac{\ln 2}{\theta_w} \frac{V\gamma^*}{V\gamma^* + \alpha^* U} w(d) \, q^*(d) \right) dd \right]^{\frac{1}{k_w - 1}}$$

$$\gamma^* = \left[\frac{1}{\ln 2} \frac{\theta_f}{\chi_f} \ln 2 \int_0^{\frac{1}{2}} \exp \left(\frac{\ln 2}{\theta_f} \frac{\alpha^* U}{V\gamma^* + \alpha^* U} \pi(d) \, p^*(d) \right) dd \right]^{\frac{1}{k_f - 1}}$$

$$p^*(d) = \frac{\exp \left(\frac{\ln 2}{\theta_w} \frac{V\gamma^*}{V\gamma^* + \alpha^* U} w(d) \, q^*(d) \right)}{2 \int_0^{\frac{1}{2}} \exp \left(\frac{\ln 2}{\theta_w} \frac{V\gamma^*}{V\gamma^* + \alpha^* U} w(d') \, q(d') \right) dd'}$$

$$q^*(d) = \frac{\exp \left(\frac{\ln 2}{\theta_f} \frac{\alpha^* U}{V\gamma^* + \alpha^* U} \pi(d) \, p^*(d) \right)}{2 \int_0^{\frac{1}{2}} \exp \left(\frac{\ln 2}{\theta_f} \frac{\alpha^* U}{V\gamma^* + \alpha^* U} \pi(d') \, p^*(d') \right) dd'}$$

Socially optimal allocations are similar, with the exception that private gains $w(d)$ and $\pi(d)$ are replaced by social gains, $f(d)$. Therefore, it is straightforward to see that no bargaining weights can help achieve the socially optimal allocation, unless $w(d) = \pi(d) = f(d)$, which is not feasible. The matching rate in this case equals:

$$M = 2 \frac{\alpha U \gamma V}{\alpha U + \gamma V} \int_0^{\frac{1}{2}} p(d) \, q(d) \, dd$$

The matching function takes the form of a constant-returns-to-scale matching function with a constant elasticity of substitution between unemployed workers and vacant firms. In practice, it can be approximated by a CES or Cobb-Douglas function. Parameters of this function are fully endogenous. They are determined exclusively by the distribution of surplus and by costs of search. The solution to this matching process is easily computable using standard optimization algorithms. It also allows for a closed-form solution under additional assumptions, which we describe next.

3.6 Closed-Form Solution

We proceed to a closed-form solution by adding assumptions that cost functions, numbers of workers and bargaining powers are also symmetric:

A10 $\chi_f = \chi_w = \chi, \qquad \theta_w = \theta_f = \theta, \qquad k_w = k_f = k,$
$U = V$ and $\pi(d) = w(d) = \frac{1}{2} f(d)$.

In this case, the solution is symmetric with $p(d) = q(d)$ and $\alpha = \gamma$, and can be solved in closed-form:

$$p^*(d) = \frac{1}{A^*} \exp\left(-W\left(-\frac{1}{A^*}\frac{\ln 2}{4\theta} f(d)\right)\right) \qquad \alpha^* = \left[\frac{\theta}{\ln 2}\frac{1}{\chi}\ln A^*\right]^{\frac{1}{k-1}}$$

where $W(y)$ is the real branch of the Lambert-W function, defined as the solution to $y = We^W$ for $W(y) \geq -1$, and A^* is a normalizing constant, which makes sure that the distribution of attention integrates to one. The planner's allocation has a similar form with both workers and firms assuming they will get the whole surplus instead of a half. Assuming the existence of an upper bound, F, on the surplus function, the equilibrium is unique if:

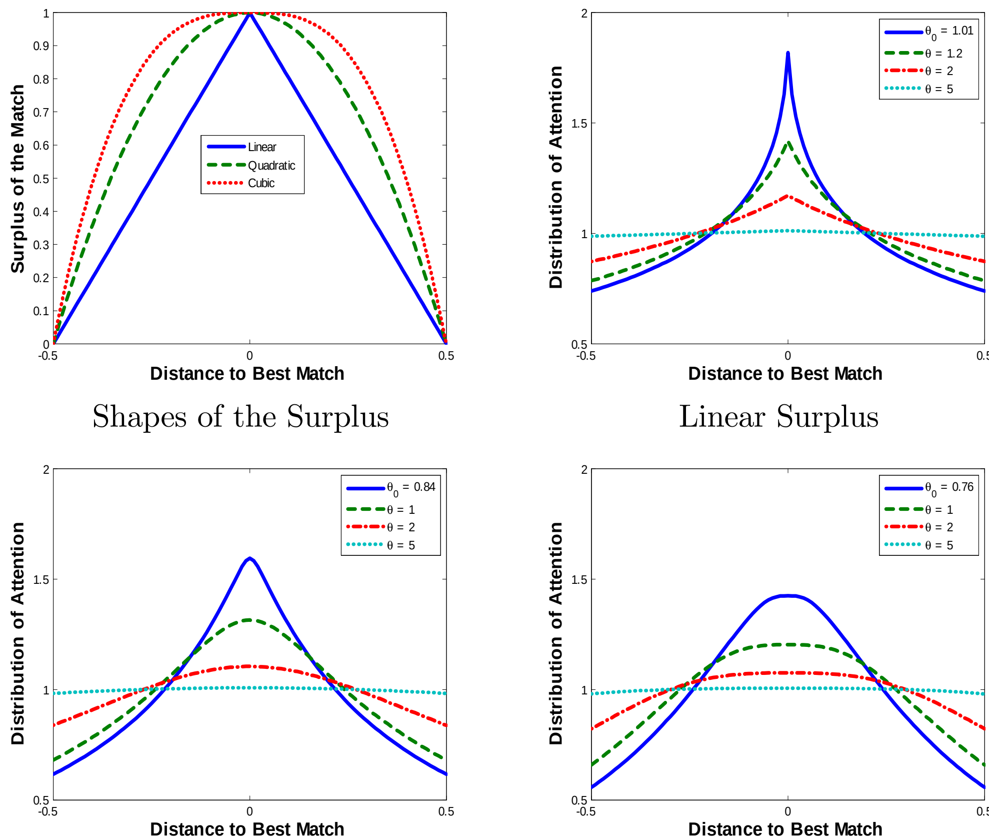
$$\theta \geq \theta_0 = \frac{Fe \ln 2}{8 \int_0^{\frac{1}{2}} \exp\left(-W\left(-\frac{f(d)}{Fe}\right)\right) dd}$$

Constraints on costs of information illustrate that a high enough cost is necessary to weaken the strategic complementarity between strategies of workers and firms. The intuition behind the lower bounds is that, for $\theta < \theta_0$, the marginal cost is smaller than the marginal benefit of information:

$$\frac{F}{A}\frac{\ln 2}{4\theta} > \frac{1}{e} > \frac{\ln p(d)}{p(d)}$$

For lower costs of information, the strategic complementarities dominate. One solution to the problem in this case is the solution to the assignment model, characterized by infinitely precise strategies described by the Dirac-delta function, $p(d) = \delta(d)$. There is a multiplicity of other infinitely precise strategies which are also equilibria.

In Figure 1 we plot distributions of attention for three shapes of the surplus $f(d) = 1 - (2d)^p$ for different values of costs above their limiting values. For different values of curvature, $p = \{1, 2, 3\}$, the limiting values of costs for equilibria to be unique equal $\theta_0 = \{1.00, 0.83, 0.75\} * \ln 2$. The matching rate in these cases also has a closed-form solution:



Shapes of the Surplus | Linear Surplus

Quadratic Surplus | Cubic Surplus

Figure 1. Shapes of the Surplus and Distributions of Attention

$$M = U\alpha^* \int_0^{\frac{1}{2}} (p^* (d))^2 \, dd$$

For each of the aforementioned surplus functions the matching function is strictly decreasing in the cost of information as illustrated in the Figure 2 for the case $k \to \infty$. Figure 2 helps quantify losses in efficiency due to existence of strategic complementarities. For the symmetric economy the efficient outcome is equivalent to the equilibrium outcome under the assumption that cost of information is reduced in half. Figure 2 shows that, for intermediate values of costs, the number of lost matches in equilibrium can reach 50% compared to the social planner's allocation.

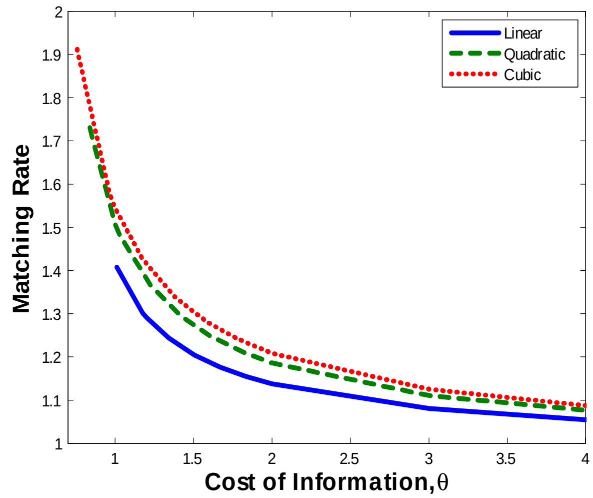


Figure 2. Matching Efficiency

4 Conclusion

An assignment model with both workers and firms having limited information-processing capacities produces a decentralized equilibrium with (1) lower search efforts than it is socially optimal; (2) inefficient search outcomes, with a lower number of matches than it is socially optimal. This property of equilibrium holds in a static as well as a dynamic framework with a continuum of heterogenous workers and firms.

The key mechanism behind this result is that information-processing constraints weaken the perceived complementarities of search efforts of firms and workers in a way that cannot be internalized by a social planner facing similar constraints.

The dynamic version of the model allows us to derive the matching rate in this labor market from the model's fundamentals. Imposing additional discipline on our matching model, we are able to compute the unique equilibrium in closed form and to characterize a bound on the inefficiency of the matching function. We find this inefficiency to be potentialy large. We leave for future research whether and how second-best allocations can be achieved in this framework.

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5 Appendix NOT FOR PUBLICATION

[to be added]