Are State and Time dependent models of price setting really different?

A sufficient statistic approach.

Fernando Alvarez

University of Chicago

2016 SED, Toulouse

Two hypothesis to explain infrequent price changes

(1) Firms monitor "economic conditions" and pay a fixed cost to adjust prices $\rightarrow$ State-Dependent rules

(2) Firms inattentive to "economic conditions" and pay a fixed cost to observe state $\rightarrow$ Time-Dependent rules

Does the rule type matter for the response to an aggregate shock?

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Analytic comparison of (1) and (2).

- If (1) and (2) fit same key moments , then no difference for **small** shocks
- For **large** shocks instead (1) and (2) are different
- (scattered) evidence of models w/both state & time dependent behavior

Based upon joint work with *Francesco Lippi* and others
(Paciello, Le Bihan, Beraja, Passadore, Neumeyer)

- *Optimal Price Setting with Observation and Menu Costs*, QJE, 2011.
- *Price setting with menu cost for multi-product firms*, Etca 2014.
- *The real effects of monetary shocks in sticky price models: a sufficient statistic approach*, AER, 2016.
- *Monetary Shocks in a Model with Inattentive Producers*, RES, 2016.
- *Phillips curve with Observation and menu Costs*, JEEA, 2016.
- *Are State and Time dependent models really different?* NBER Macro Annual 2016.
- *Price plans and the real effects of monetary policy*, under review.
- *From Hyperinflation to Stable Prices: Argentina's Evidence on Menu Cost Models*, under review.

- GE setup: Woodford, Golosov-Lucas model

$$\text{Lifetime Utility}: \int_0^\infty e^{-r\,t}\left(\frac{c(t)^{1-\epsilon}-1}{1-\epsilon}-\alpha\,\ell(t)+\log\frac{M(t)}{P(t)}\right)dt$$

$$\text{CES aggregate}: c(t)=\left(\int_0^1\left[\quad A_k(t)\,c_k(t)\quad\right]^{\left(1-\frac{1}{\eta}\right)}dk\right)^{\frac{\eta}{\eta-1}}$$

- Intra-temp. subst. elasticity η (firms k)
- Linear technology $c_k(t)=\frac{\ell_k(t)}{Z_k(t)}$ and $Z_k(t)=\exp\left(\sigma\,\mathcal{W}_k(t)\right)$.
- Marg Cost shocks: $\mathcal{W}_k\perp$ all k, standard Brownian Motion
- **Equilibrium:** constant nominal interest rate & wages $W(t)=\mathbf{a}\,M(t)$.

Profit maximizing (log) price, case of $n = 1$ per firm

$$p^*(t) \;=\; \underbrace{\Big(\textit{idosyncratic}(t)\Big)}_{\textit{marginal cost}\text{: random walk } \perp \text{ across firms}} \;+\; \underbrace{\log \textit{ of markup}}_{\frac{\eta}{\eta-1}}$$

price $P(t)$, cost $Cost(t)$ & agg. consumption $c(t)$:

$$\textit{Profits}\Big(\; P(t), Cost(t)\;;\; c(t)\Big) \approx \textit{ Maximized Profits}_t$$

$$-\;\frac{\eta(\eta-1)}{2}\Big[\; g^2(t)\;\Big] \;+\; \textit{terms indep. of } g(t)$$

"price gaps": $g(t) \equiv \log P(t) - p^*(t)$

Between price adjustments, each price gaps evolve as:

$$dg(t) = -\pi dt \;+\; \sigma\, d\mathcal{W}(t) \quad \text{(focus on case } \pi/\sigma^2 \approx 0\text{)}$$

- GE setup: Midrigan (**multiproduct**) model

$$\text{Lifetime Utility}: \int_0^\infty e^{-r\,t}\left(\frac{c(t)^{1-\epsilon}-1}{1-\epsilon}-\alpha\,\ell(t)+\log\frac{M(t)}{P(t)}\right)dt$$

$$\text{CES aggregate}: c(t)=\left(\int_0^1\left[\sum_{i=1}^{n}\left(A_{ki}(t)\,c_{ki}(t)\right)^{1-\frac{1}{\varrho}}\right]^{\left(\frac{\varrho}{\varrho-1}\right)\left(1-\frac{1}{\eta}\right)}dk\right)^{\frac{\eta}{\eta-1}}$$

- Intra-temp. subst. elasticity η (firms k) & ϱ (products i); $n \geq 1$ goods
- Linear technology $c_{ki}(t)=\frac{\ell_{ki}(t)}{Z_{ki}(t)}$ and $Z_{ki}(t)=\exp\left(\sigma\,\mathcal{W}_{ki}(t)+\bar{\sigma}\,\bar{\mathcal{W}}_k(t)\right)$.
- Marg Cost shocks: $\mathcal{W}_{ki}\perp\bar{\mathcal{W}}_k$ all i,k, standard Brownian Motion
- **Equilibrium:** constant nominal interest rate & wages $W(t)=\mathbf{a}\,M(t)$.

Profit maximizing (log) price of each product $j = 1, 2, ..., n$

$$p_j^*(t) = \underbrace{\Big(common(t) \quad + \quad idosyncratic_j(t) \Big)}_{marginal\ cost\text{: random walk } \perp \text{ across firms}} + \underbrace{\log\ of\ markup}_{\frac{\eta}{\eta-1}}$$

price vector $P(t)$, cost vector $Cost(t)$ & agg. consumption $c(t)$

$$Profits \Big(P_1(t), ..., P_n(t), Cost_1(t), ..., Cost_n(t)\ ;\ c(t) \Big) \approx Maximized\ Profits_t$$

$$- \frac{\eta(\eta-1)}{2} \Big[\sum_{j=1}^{n} g_j^2(t) \Big] + terms\ indep.\ of\ g_1(t), ..., g_n(t)$$

"price gaps": $g_j(t) \equiv \log P_j(t) - p_j^*(t)$, <u>same elasticity: $\eta = \varrho$</u>

Between price adjustments, each price gaps evolve as:

$$dg_j(t) = -\pi dt + \sigma\, d\mathcal{W}_j(t) + \bar{\sigma}\, d\bar{\mathcal{W}}(t) \quad j = 1, \ldots n$$

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$$- B_1 \Big[\sum_{j=1}^{n} g_j^2(t) \Big] + B_2 \Big[\sum_{j=1}^{n} g_j(t) \Big]^2 + terms\ indep.\ of\ g_1(t), ..., g_n(t)$$

"price gaps": $g_j(t) \equiv \log P_j(t) - p_j^*(t)$, $B_1 = \dfrac{\varrho(\eta - 1)}{2}$, $B_2 = \dfrac{(\varrho - \eta)(\eta - 1)}{2\,n}$

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State-dependent decision rules

- "Menu costs": ψ_m paid to <u>reset the entire state</u>: $g \equiv \{g_1(t), \ldots, g_n(t)\}$
- Optimal policy: inaction region $\mathcal{I} \subset \mathbb{R}^n$ and optimal return point g^*:
  - if $g \in \mathcal{I} \implies$ no action ,
  - if $g \notin \mathcal{I} \implies$ change prices to vector $g^* \in \mathcal{I}$
  - $g^* = (0, ..., 0)$ with no inflation ($\pi/\sigma^2 \approx 0$).
- Inaction set described by a function $b : \mathbb{R}^n \to \mathbb{R}$:

$$g \in \mathcal{I} \implies b(g_1, \ldots, g_n) \leq 0 \text{ and } g \notin \mathcal{I} \implies b(g_1, \ldots, g_n) > 0$$

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- Examples:
  - $n = 1$ Barro, Golosov-Lucas: $b(g_1) = g_1^2 - \bar{g}^2$ so $\mathcal{I} = [-\bar{g}, \bar{g}]$.

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  $n > 1$: Llach-Tsiddon, Midrigan : $b(g_1, \ldots, g_n) = \sum_{i=1}^n g_i^2 - \bar{g}^2$

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  $n > 1$: If $\varrho \neq \eta$, or corr. socks : $b(g_1, \ldots, g_n) = \sum_{i=1}^n g_i^2 - \varphi\left([\sum_{i=1}^n g_i]^2\right)$

Time-dependent decision rules

- Fixed 'observation cost' ψ_o paid to **observe the state**: $g(t)$
- At observation t: signal $\zeta(t)$ on future observation cost $\psi_o(t+s)$, all $s \geq 0$.
- Observation cost and signal (ψ_o, ζ) independent of g.

  Example: Observation cost ψ_o is Markov serves as signal $\implies \zeta = \psi_o$.
- Optimal policy at the time of a review (observation)
  - Change price using information gathered, so that g changes to $g^* = 0$.
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  - **time** $\mathsf{T}(\zeta)$ until the next observation
- Multiple dimensions without loss of generality.
- Generalization of Reis (2006), related to Sim's rational innatention.

  Examples: Taylor, Caballero: $\mathsf{T} \sim$ degenerate, Calvo: $\mathsf{T} \sim$ exponential

Mixed Models: both state and time dependence.

- Multiproduct Calvo$^+$ model (Nakamura-Steinsson):
  - Assume menu cost $\psi_m(t) = 0$ with (Poisson) probability λdt, otherwise $\psi_m(t) = \bar{\psi}_m > 0$ with probability $1 - \lambda dt$.
  - Change every price every at the time at which menu cost ψ_m is zero.
  - Otherwise change when hit border of inaction set $\mathcal{I}$.
- Observation $\psi_o > 0$ and Menu Cost $\psi_m > 0$ model (Carvalho et all):
  - Observe infrequently, but change price only if price gap g is large.
  - Inaction set for price changes, so state dependence.
  - Time to the next observation T>0, so time dependence.

Steady state

- Number of price changes per unit of time $N(\Delta p)$.
- State Dependent Decision rules:
  - cross-section distribution of gaps $f(g_1, ...g_n)$ for $g \in \mathcal{I}$
  - $f(\cdot)$ and $N(\Delta p)$ **only** matters for effect of a small aggregate shock
- Time Dependent decision rules:
  - Cross-section distribution of times until next observation $Q(t)$.
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- Time Dependent decision rules:
  - Cross-section distribution of times until next observation $Q(t)$.
  - $Q(\cdot)$ and $N(\Delta p)$ **only** matters for effect of a small aggregate shock.
- Shape of <u>distribution of price changes</u> and <u>hazard rates</u>:

  depends on $f(\cdot)$ and $N(\Delta p)$ in state dependent and

  depends on on $Q(\cdot)$ and $N(\Delta p)$ in time dependent.
- Mixed models: states that combine both.

Effect of an "MIT" monetary shock

- Start at steady state at $t = 0$, normalizing level of aggregates to zero.
- Increase money, unexpectedly, once and for all by δ log points.
- Nominal wages increase, once and for all by δ log points.
- Outcomes: aggregate price level $\mathcal{P}(\delta, t)$ & output $\mathcal{Y}(\delta, t) = \frac{1}{\epsilon}\left[\delta - \mathcal{P}(\delta, t)\right]$.

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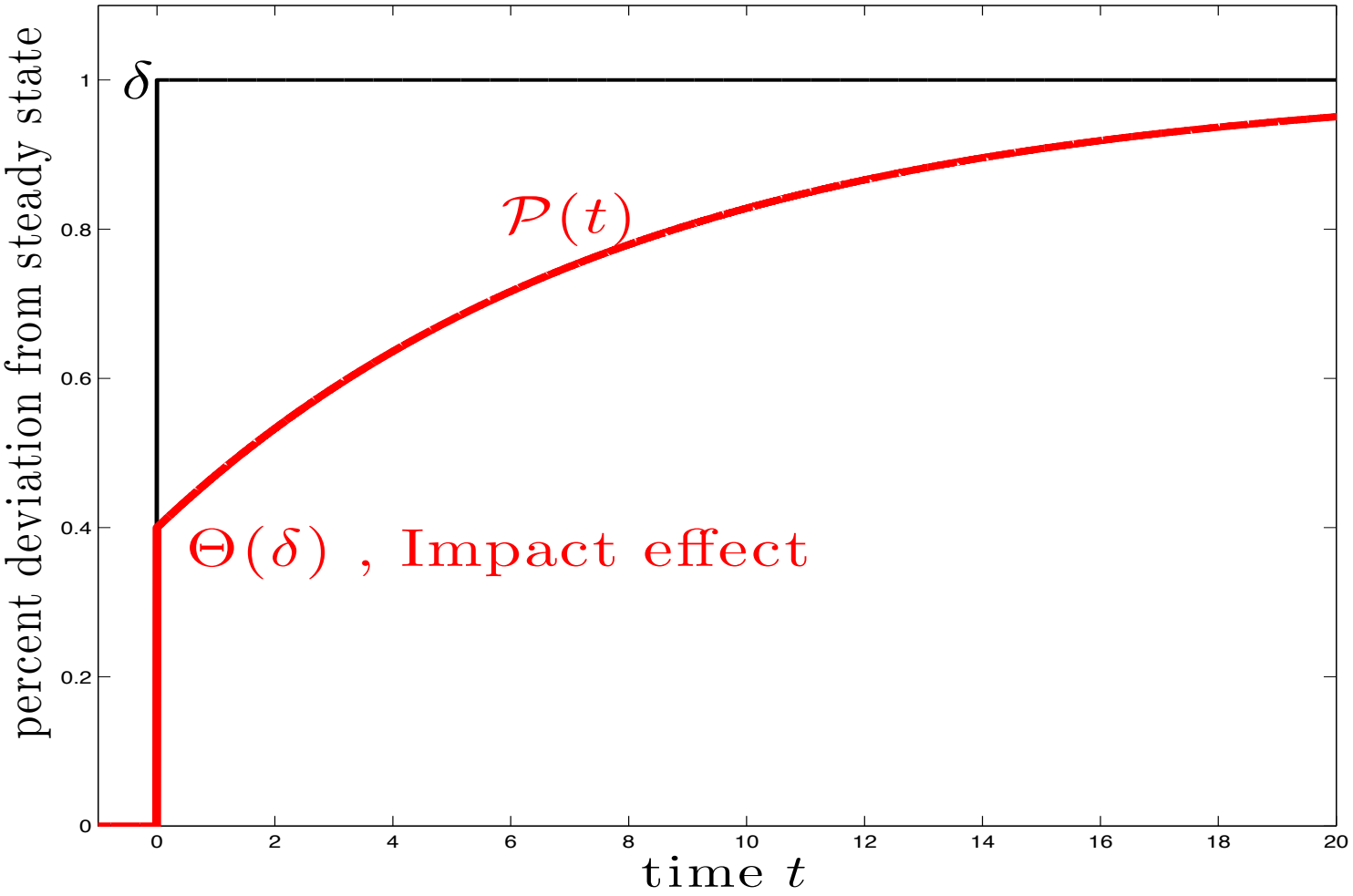
- Outcomes: aggregate price level $\mathcal{P}(\delta, t)$ & output $\mathcal{Y}(\delta, t) = \frac{1}{\epsilon}\left[\delta - \mathcal{P}(\delta, t)\right]$.
- Results for two measures:
  - impact effect on prices $\Theta(\delta) \equiv \mathcal{P}(\delta, 0)$ (Caballero-Engel Price Flex. index)
  - cumulative output effect $\mathcal{M}(\delta) \equiv \int_0^\infty \mathcal{Y}(\delta, t)\, dt = \frac{1}{\epsilon}\int_0^\infty \left[\delta - \mathcal{P}(\delta, t)\right]\, dt$

Impulse response of Price Level to nominal shock: δ



$$\mathcal{P}(\delta, t) = \Theta(\delta) + \int_0^t \theta(\delta, s) ds$$

Impact effect on in **TD** vs SD

$$\mathcal{P}(\delta, t) = \Theta(\delta) + \int_0^t \theta(\delta, s) ds \qquad \text{by definition} \quad \Theta(0) = 0$$

- <u>in TD Models</u>:
  - no impact: $\Theta(\delta) = 0$, since # adjustments proportional to period length "dt"
  - <u>linear</u> in shock size: $\mathcal{P}(\delta, t) = \delta \cdot \mathcal{P}(1, t) = \delta \int_0^t Q(s) ds$

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- Consider firms whose decision of review is of length τ.
  - Among them price change $\Delta p \sim \mathcal{N}\left(\delta, \tau\sigma^2\right)$.
    Only expected value matters for IRF.
  - $Q(s)$: how many will adjust exactly at s periods after the shock.

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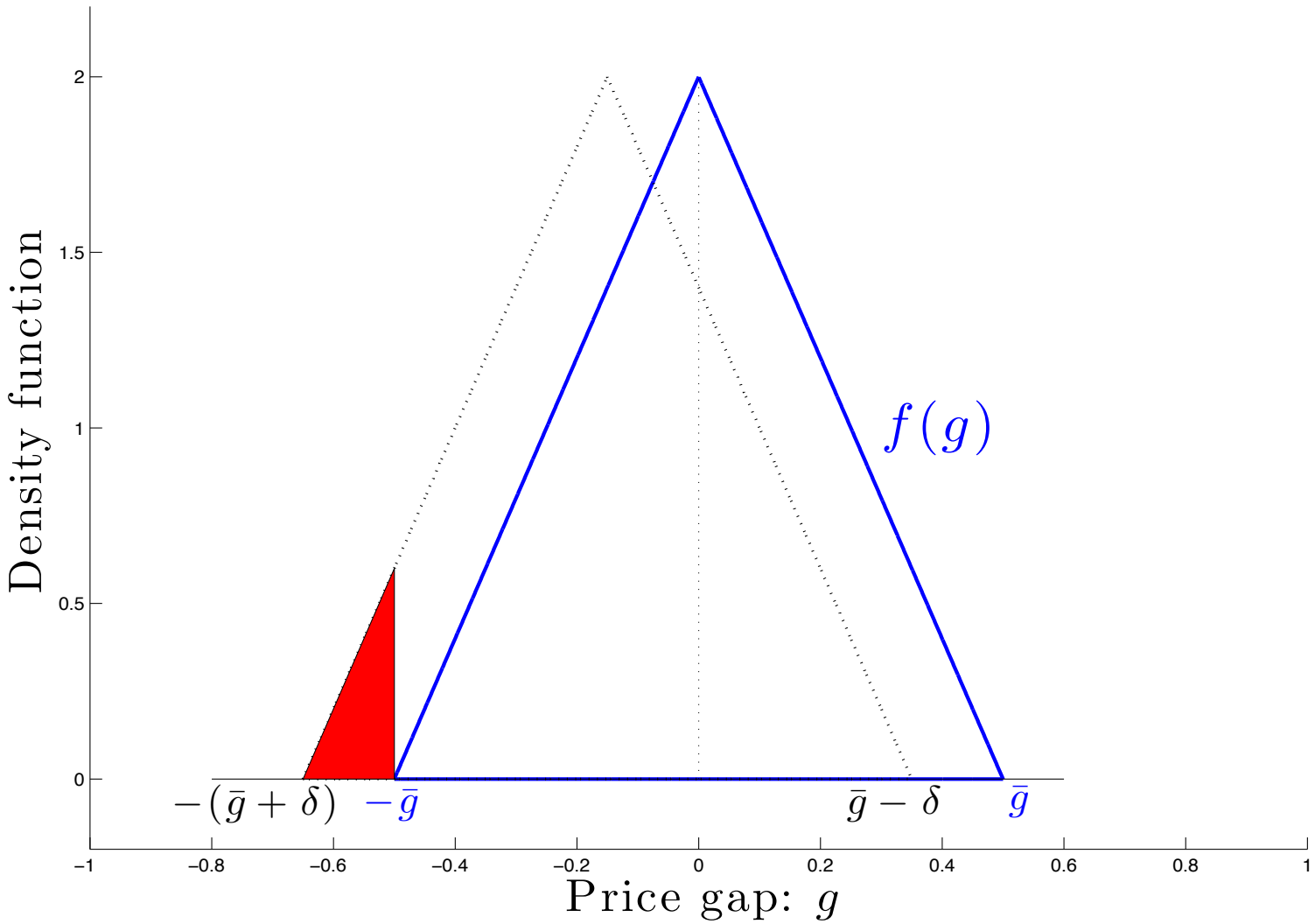
- <u>in SD Models</u>:
  - impact $\Theta(\delta) > 0$, as some firms fall out of the inaction region
  - But no first-order effect if $\sigma > 0 \implies \Theta(\delta) \approx \underbrace{\Theta(0)}_{=0} + \underbrace{\Theta'(0)}_{=0} \delta + \frac{\Theta''(0)}{2} \delta^2$

No first-order response in SD models: $\Theta'(0) = 0$



Proof: if $\sigma > 0$ then $f(\underline{g}) = 0$, so mass of adjusters $\approx \left(f'(\underline{g})\delta\right)\delta/2 \approx \delta^2$

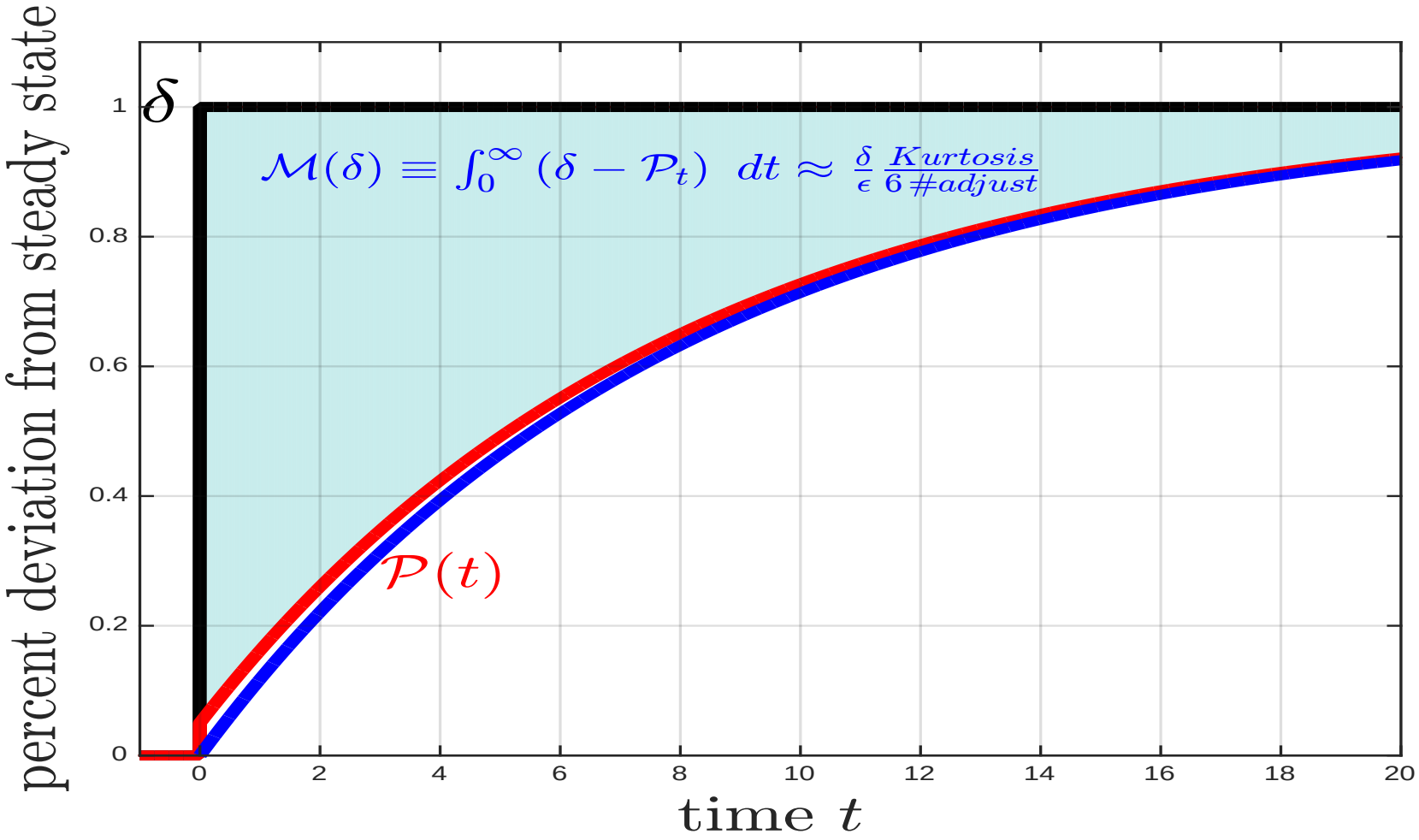
No first-order response in SD models: $\Theta'(0) = 0$ (Similar logic hold for $n > 1$ products)

- $f(\bar{g}) = 0$ because it is an exit point. Requires $\sigma^2 > 0$.
- In $n > 1$ result is similar: $f(g_1, ..., g_n) = 0$ on the boundary of $\mathcal{I}$.

  Using notation above $b(g_1, ..., g_n) = 0 \implies f(g_1, ..., g_n) = 0$.
- Computation of fraction of adjusters more involved, but logic is the same.
- This result only requires $\{g_1, ..., g_n\}$ be regular a diffusion

  and that $b(\cdot)$ defines a closed curve.

Small shocks: similarities beyond impact



Same cumulated output response if **frequency** and **kurtosis** of Δp coincide

Sufficient Statistic for small monetary shock δ

- $\mathcal{M}(\delta) \equiv$ cumulative IRF of output to a (small) monetary shock δ

$$\mathcal{M}(\delta) \approx \frac{\partial \mathcal{M}(0)}{\partial \delta}\,\delta = \frac{\delta}{\epsilon}\,\frac{Kurt(\Delta p)}{6\,N(\Delta p)}$$

  - Frequency of price changes $N(\Delta p)$ has a first order effect.
  - Kurtosis of price changes $Kurt(\Delta p)$ has a first order effect.
  - $Kurt(\Delta p)$ controls for **selection** of time and size of price changes.
  - different distributions of <u>size and time</u> of Δp may lead to same $Kurt(\Delta p)$.

Cum. Output IRF small shock: $\mathcal{M}(\delta) \approx \frac{\delta}{\epsilon} \frac{Kurt(\Delta p)}{6\ N(\Delta p)}$

- Several models can have the same $Kurt(\Delta p)$
- Examples with $Kurt(\Delta p) = 3$:
  - Observation cost model with constant observation cost ($\psi_o > 0, \psi_m = 0$).
  - Multiproduct model with $n \to \infty$.
  - Many Calvo$^+$ models: for any n there is a $\lambda > 0$ for which $Kurt(\Delta p) = 3$.
- $Kurt(\Delta p) = 1$ in Golosov-Lucas vs. $Kurt(\Delta p) = 6$ in Calvo.
- Data: $Kurt(\Delta p) \approx 4$ (measurement error, aggregation)

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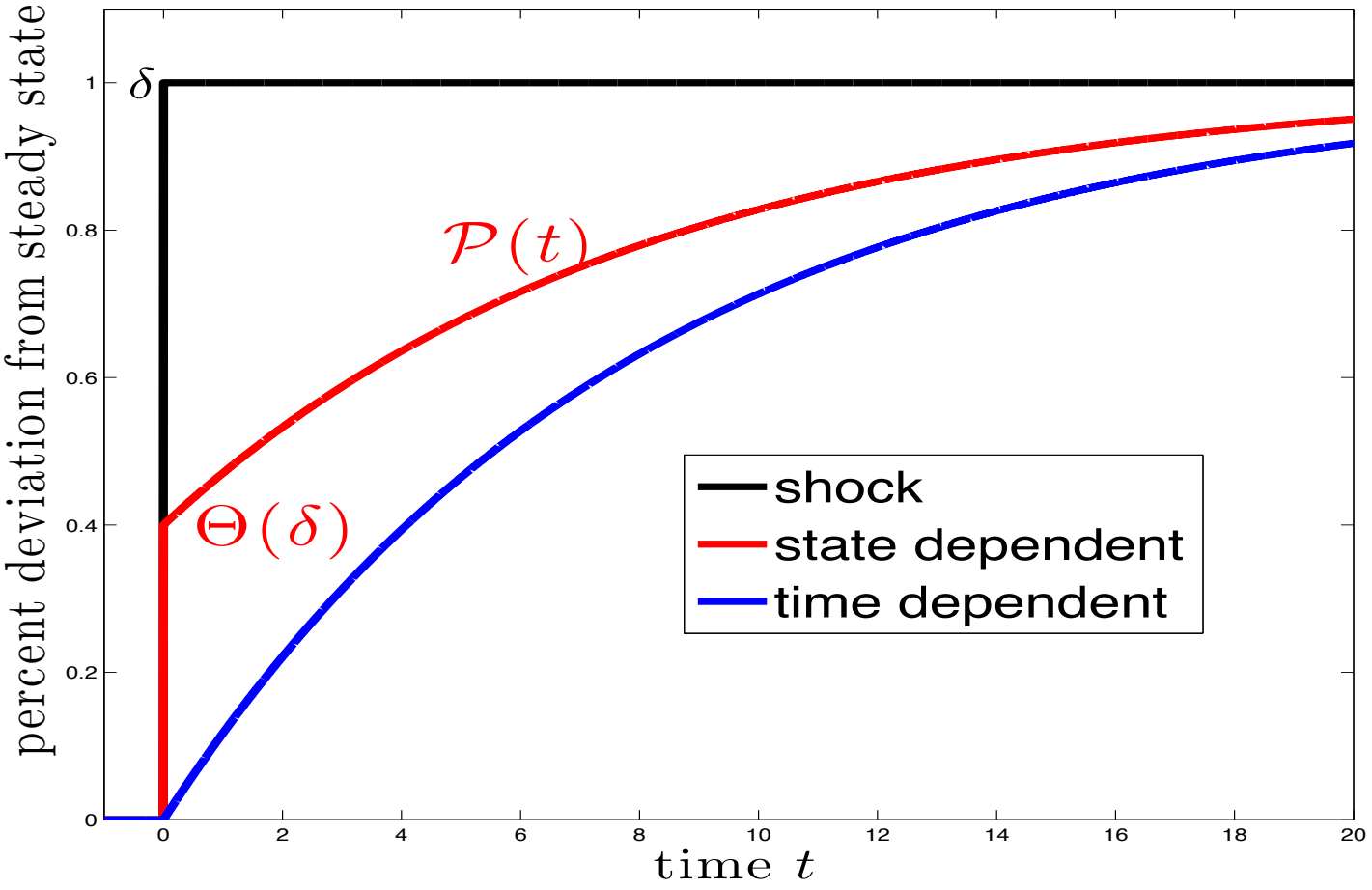
- $Kurt(\Delta p) = 1$ in Golosov-Lucas vs. $Kurt(\Delta p) = 6$ in Calvo.
- Data: $Kurt(\Delta p) \approx 4$ (measurement error, aggregation)
- For all models can choose parameters to match any $N(\Delta p)$.
- In each model: $Kurt(\Delta p)$ and $N(\Delta p) \implies$ structural parameters.

Differential Impact effect of large shocks



$\Theta'(\delta) = 1$ if $\delta > \underline{\delta} \approx 2\,Std(\Delta p_i)$; full flexibility for sufficiently large shocks

Lack of sensitivity to inflation around $\pi = 0$

- Inflation has only second order effect around $\pi = 0$ on
  - entire *hazard rate* $h(t)$ and *frequency* of price changes ($N(\Delta p)$).
  - all centered *even moments* of marginal price changes (e.g. $Kurt(\Delta p)$).
  - results on $\mathcal{M}(\delta; \pi)$ due to symmetry around w.r.t. (π, δ) around (0,0).
- Thus expression for holds for *small* inflation rates π and shocks δ:

$$\mathcal{M}(\delta) \approx \frac{\delta}{\epsilon} \frac{Kurt(\Delta p)}{6\, N(\Delta p)}$$

Limitations of the results

- Cost follow a random walk.
  (Alternatively, large idiosyncratic shocks, regime changes, etc.)
- Aggregate shock is once-and-for all change in level.
- GE setup and firm problem lack strategic complementarity.
- Shock when economy is at steady state.
- Price adjusted Δp firms close gap

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  (Alternatively, change in growth rate, announcement of future change)
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Limitations of the results

- Cost follow a random walk.
- Aggregate shock is once-and-for all change in level.
- GE setup and firm problem lack strategic complementarity.
  (Profit function does not depend on endogenous aggregates variables)
- Shock when economy is at steady state.
- Price adjusted Δp firms close gap (adjust to static maximizing price)

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- Cost follow a random walk.
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- GE setup and firm problem lack strategic complementarity.
- Shock when economy is at steady state.

  (Alternatively, response can depend on distribution across firm, e.g. Caplin-Lehay "overheated" economy)
- Price adjusted Δp firms "close gap"

Limitations of the results

- Cost follow a random walk.
- Aggregate shock is once-and-for all change in level.
- GE setup and firm problem lack strategic complementarity.
- Shock when economy is at steady state.
- Price adjusted Δp firms "close gap", i.e. adjust to static optimal price.

  (Alternatively, price plans as in "sales" in ERJ or AL, where there is a positive impact effect.)

Impact effect on Aggregate Price is:

- **zero** for any $\delta > 0$ in **TD** models
- **zero** for small δ, but positive for large $\delta > 0$ in **SD** models

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- **Exchange rate Innovations** Δe_t are an interesting candidate....
  - Δe_t **close to RW** (theory uses once-and-for-all nominal shock)
  - Δe_t available at **high frequency** (need to estimate impact!)
  - Δe_t available for **many countries** (need many large shocks)

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  - Δe_t **close to RW** (theory uses once-and-for-all nominal shock)
  - Δe_t available at **high frequency** (need to estimate impact!)
  - Δe_t available for **many countries** (need many large shocks)
- Alternatively, cases studies: Mexico (Gagnon), Switzerland (BFS).

Non-linear exchange rate pass-through

- Effect of exchange rate changes Δe_t on change on prices π_{t+1}
- Focus on short term impact (first month), relative to long-run.
- Compare small vs large changes in exchange rates
- Finding:
  large exchange rate changes have larger short term pass-through

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- Finding:
  large exchange rate changes have larger short term pass-through
- Causality?

Analysis of non-linear pass-through in the short-run

IFS **monthly** data on CPI and Bilateral Exchange rate with US

- 64 countries with avg inflation below 8% (MA 5 yrs)
- sample with or without de facto Fixed ER regime

Baseline specification:

- "local projection" for horizon $h = 1, 3, 6, ...$ (in months) :

$$\pi_{i,t,t+h} = \alpha_i + \delta_t + \beta_h \, \Delta e_{i,t} + \gamma_h \, sign(\Delta e_{i,t}) \, (\Delta e_{i,t})^2 + \epsilon_{it}$$

- $\Delta e_{i,t}$: country i nominal exchange rate change from $t-1$ to t
- $\pi_{i,t,t+h}$: country i CPI inflation between t and $t+h$

Pass-through: Baseline regressions (1974-2014)

	exclude Fixed ER country (6,811 obs.)		
horizon h $\rightarrow$	1 month	3 month	6 month
β_h (linear term)	0.009^{**}	0.027^{***}	0.056^{***}
	(0.004)	(0.008)	(0.010)
$\gamma_h \times 100$ (quadratic term)	0.114^{***}	0.152^{***}	0.104
	(0.027)	(0.054)	(0.106)

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$\gamma_h \times 100$ (quadratic term)	0.114^{***}	0.152^{***}	0.104
	(0.027)	(0.054)	(0.106)
	All countries (13,273 obs)		
horizon h:	1 month	3 month	6 month
β_h (linear term)	0.019^{***}	0.039^{***}	0.062^{***}
	(0.004)	(0.010)	(0.013)
$\gamma_h \times 100$ (quadratic term)	0.058^{***}	0.166	0.097
	(0.028)	(0.112)	(0.140)

Robustness checks for regressions

- Piecewise linear specification.
- Classification of ER regime (LS vs RR).
- Other sample selection criteria (unclassified ctrys.).
- Definition of "low inflation" country
- Sample selection criteria: "mean output above 10K threshold".
- Inclusions of time dummies.
- Coefficients robust to exclude outliers (highest two devaluations) but
- Statistical significance **NOT** robust to exclude largest two outliers.

Recent (Jan 2015) change on Swiss Mark policy

- "The Speed of the Exchange Rate Pass-Through"
  by Bonadio, Fischer and Saure.
- Unanticipated exchange rate shock: SNB lifted floor of 1 euro per 1.2 SF.
- Permanent appreciation of the SF $\sim$ 11 % against euro.
- Daily unit value data all Swiss transactions-level trade.
- Speed of price adjustment is much faster than for day-to-day changes.
- Pass-through between 2 and 7 days.

Are State and Time dependent models different?

- for small shocks: nature of friction is irrelevant, “same propagation” ....
  ..... provided **frequency** and **kurtosis** of price changes are the same
- results extend to model that mix SD and TD elements (e.g. Calvo^{+})
- most applications use small shocks...hence distinction is not important

Are State and Time dependent models different?

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  ..... provided **frequency** and **kurtosis** of price changes are the same
- results extend to model that mix SD and TD elements (e.g. Calvo$^+$)
- most applications use small shocks...hence distinction is not important
- for large shocks: propagation to prices quicker in SD models
- some evidence from pass-through regressions consistent w/ SD
- distinguishing between SD & TD relevant for some polices:
  effect of announcement of future actions (aka forward guidence).

Thank you for your (rational) attention !

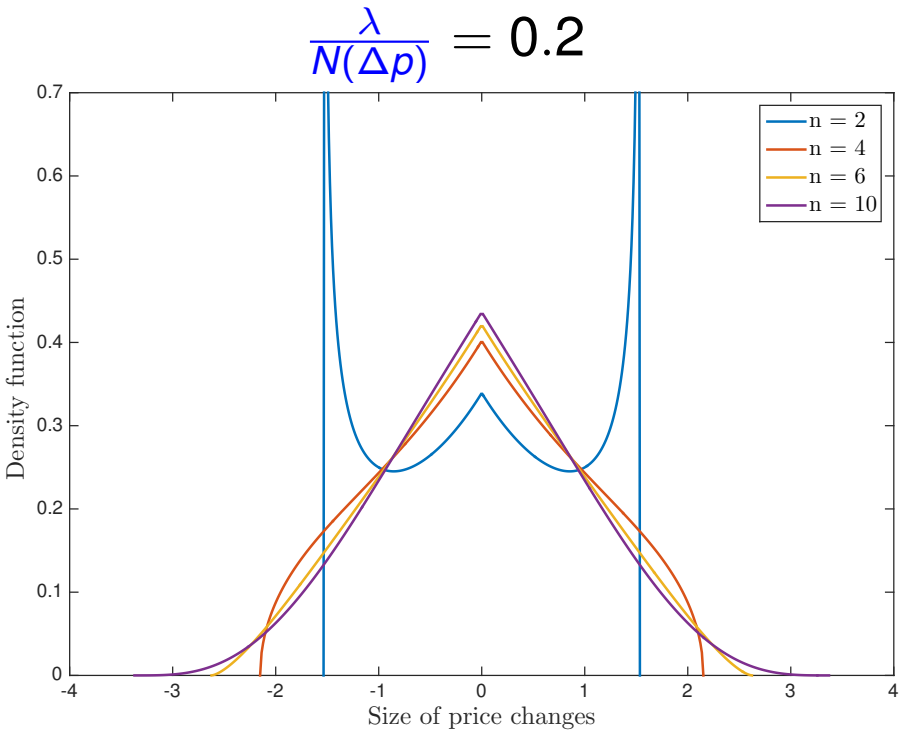
State-dependent decision rules (simpler $n = 1$ case)

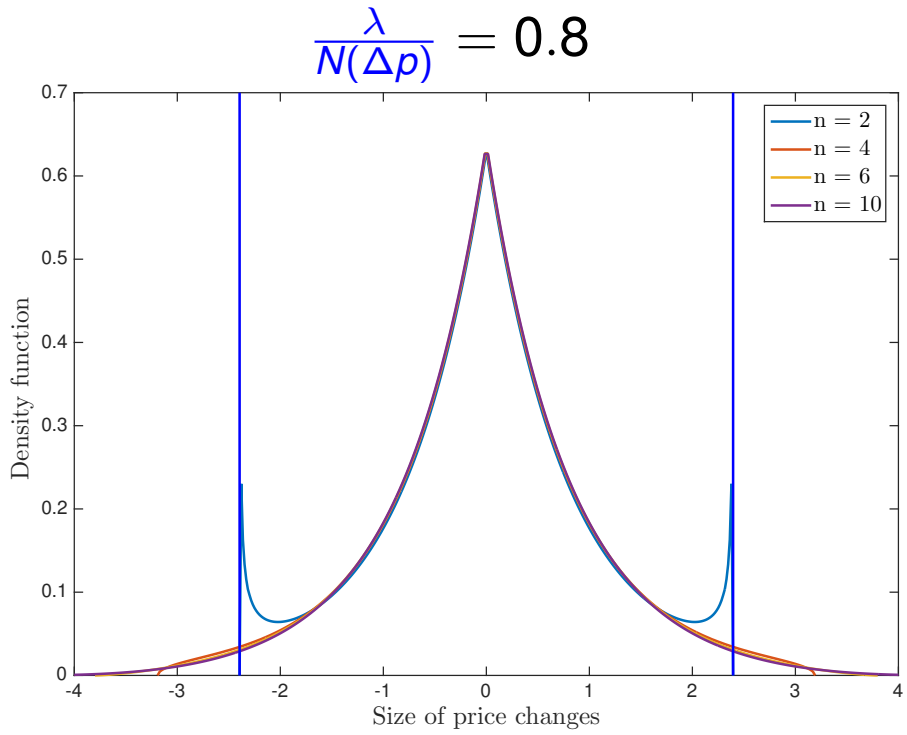
- Fixed 'menu cost' ψ_m paid to **reset the price**
- Price gap $g \equiv p - p^*$ is the state
- Optimal sS rule: three numbers $\{\underline{g},\ g^*,\ \bar{g}\}$. Inaction optimal if $g \in \left(\underline{g}, \bar{g}\right)$
- keep state $g \in \left(\underline{g}, \bar{g}\right)$ and reset it to g^* if g hits a boundary
- No inflation: $\underline{g} = -\bar{g} < 0$ and reset to $g^* = 0$

Extensions & further documentation

- Effects of inflation (on rules and steady state moments)
- Size distribution of *Dp* Models Data
- Robustness of empirical results Robust
- Experimental evidence on dec. rules Models

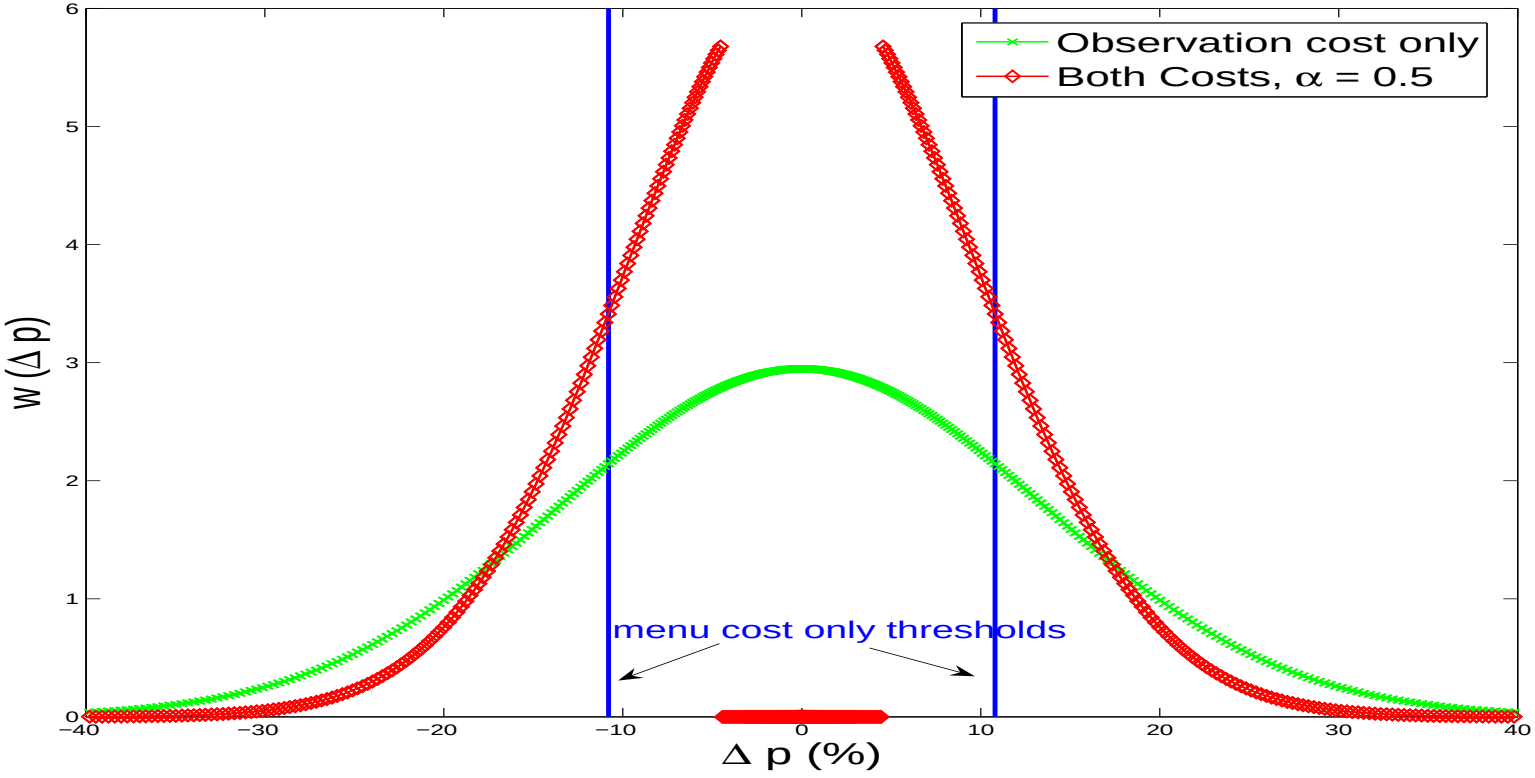
Distribution of price changes: multiproduct Calvo$^+$





- $\frac{\lambda}{N(\Delta p)}$ maps one-to-one with $\lambda \frac{\bar{g}/n}{\sigma^2}$
- each line for different number of products n

Distribution of price changes for different $\alpha = \frac{\psi_o}{\psi_m}$



- No menu cost (time dependent)
- No observation cost (state dependent)
- Mixed case with $\psi_m > 0$ and $\psi_o > 0$.

Size distribution of price and cost changes: data

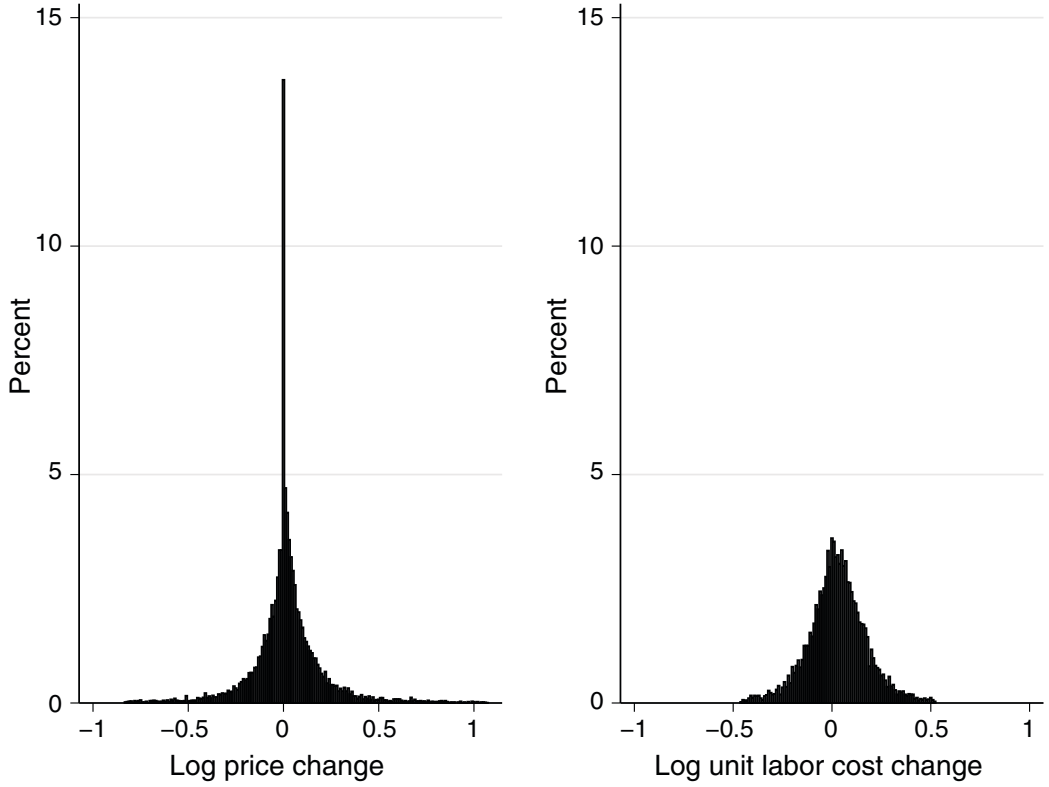


FIGURE 1. HISTOGRAMS OF FINAL DATA

Notes: The left-hand panel describes the distribution of log price changes across 13,772 observations (for 1,610 different products across 702 firms). The right-hand panel describes the distribution of log unit labor cost changes across 8,424 observations (for 702 firms). Bin size 0.01.

kurtosis price changes $\approx$ 8, kurtosis cost changes $\approx$ 3.8

Measurement error & distribution of price changes

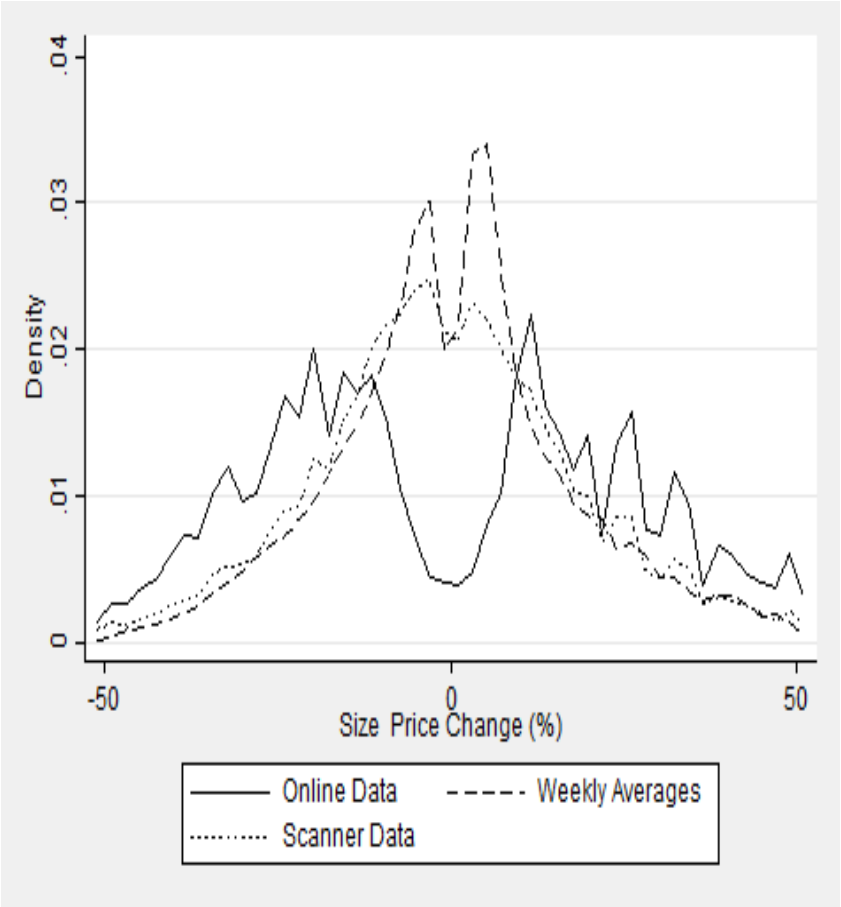


Figure 1: The Distribution of the Size of Price Changes in the US

Notes: The online and scanner data in the US was collected at the same retailer during the same time period. Scanner data was collected by Nielsen and provided by the Kilts Center at Chicago Booth.

Measures of Kurtosis by Cavallo and Rigobon (2016)

Table 2: Stickiness Statistics - Retailer Averages by Country

Country	(1) Duration (months)	(2) Percent Inc.	(3) Percent $< \|1\%\|$	(4) Mean Size PC	(5) Kurtosis PC
ARGENTINA	2	72	3	6	7
AUSTRALIA	3	50	4	5	3
AUSTRIA	22	38	15	-8	5
BELGIUM	41	58	8	2	6
BRAZIL	2	55	3	2	6
CANADA	6	51	2	3	3
CHILE	15	49	3	1	4
CHINA	5	47	6	1	7
COLOMBIA	5	51	2	2	4
FRANCE	5	55	10	2	4
GERMANY	5	48	7	1	8
GREECE	3	48	13	1	5
HUNGARY	5	47	3	4	3
INDIA	5	49	3	2	5
INDONESIA	4	61	5	3	7
IRAN	30	47	6	1	4
IRELAND	4	46	1	2	3
ISRAEL	4	36	2	-8	4
ITALY	6	43	10	-7	4
JAPAN	8	60	2	-4	5
KOREA	6	44	2	0	4
NETHERLANDS	24	41	9	-2	6
NEW ZEALAND	2	48	1	5	2
NORWAY	17	62	8	4	6
POLAND	2	51	16	2	13
PORTUGAL	7	47	6	-2	6
RUSSIA	4	55	16	2	6
SINGAPORE	25	51	1	2	3
SOUTH AFRICA	5	47	5	-3	5
SPAIN	4	46	11	0	5
SWEDEN	3	27	3	-6	6
THAILAND	2	51	1	2	4
TURKEY	3	56	2	4	4
UAE	28	17	3	-9	3
UK	4	44	6	1	5
URUGUAY	6	58	17	1	45
USA	9	45	3	0	4
VENEZUELA	11	59	5	8	4
MEAN	9	49	6	0	6

Note: We use a sample of 200 of the largest multi-channel (online and offline) retailers in the world. Prices were collected between 2007 and 2014, with different starting dates in each country. Each statistics is calculated at the retailer level and then averaged within countries. The average is unweighted, giving the same importance to each retailer within a country. Similarly, no category weights are applied. The unweighted mean over all countries is reported on the last row.

data aim to minimize (1) measur. error and (2) heterogeneity

Conjecture: generality of result for small δ

Result $\mathcal{M}(\delta) \approx \frac{\delta}{\epsilon} \frac{Kurt(\Delta p)}{6\, N(\Delta p)}$ holds in any model where where:

- Price gap vector $\{g_1(t), \ldots, g_n(t)\}$ follows *driftless* BMs.

  w/correlated common component.

- Price changes: each component of price gaps are reversed to *zero*.

- All prices are simultaneously changed according to stopping time: τ

  a function of $\sum g_i^2$, $\sum g_i$, and independent random variable(s).

Evidence for models w/both TD and SD

Constant observation and menu cost ($\psi_o > 0$ and $\psi_m > 0$).

- Experimental (Lab) evidence.

  Subjects in lab solving menu cost model behaves as if they also have observation cost ($\psi_o > 0$).

- Shape of distribution of price changes

- Survey evidence on "review" and "adjustments"

Evidence for models w/both TD and SD

Constant observation and menu cost ($\psi_o > 0$ and $\psi_m > 0$).

- Experimental (Lab) evidence.

- Shape of distribution of price changes

  When price changes Δp are cleaned of measurement error, "dip" on density close to zero, but disperse large changes (bimodal distribution).

- Survey evidence on "review" and "adjustments"

Evidence for models w/both TD and SD

Constant observation and menu cost ($\psi_o > 0$ and $\psi_m > 0$).

- Experimental (Lab) evidence.
- Shape of distribution of price changes
- Survey evidence on "review" and "adjustments"

  In model with $\psi_o > 0$ and $\psi_m > 0$ frequency of observations (reviews) > frequency of price changes.

  Firm's surveys: frequency of reviews $\approx$ 3 frequency of price changes.

Frequency of review vs adjustment

- Survey: firms reviewing adequacy of the price of main product.
- Interpreting reviews as observing states
- Pure rational inattentiveness model:

  every review (observation) leads to an adjustment
- Pure menu cost model :

  constantly reviewing (observing), and infrequent adjustment
- Mixed model :

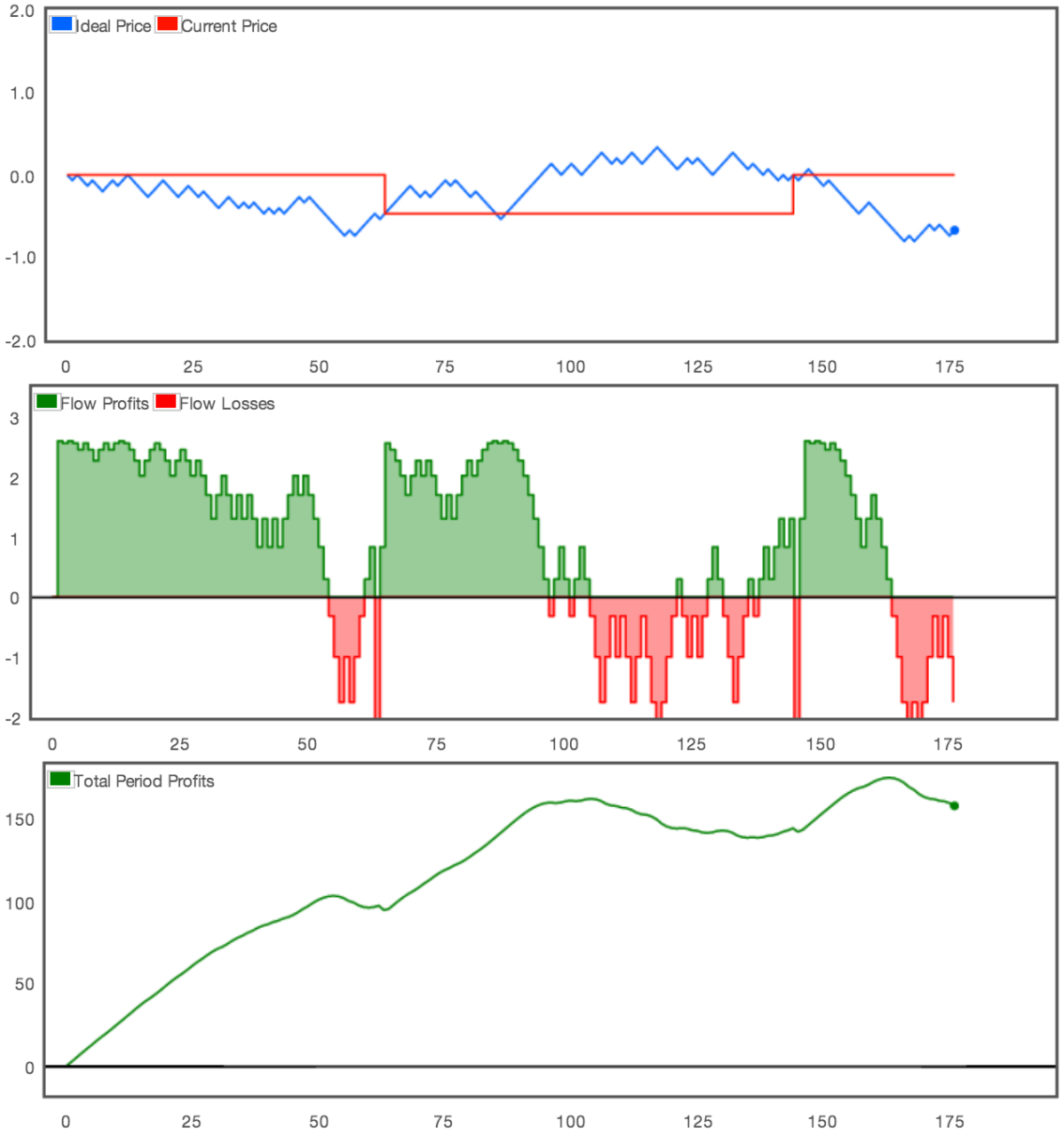
  finite frequency of review and adjustment, but more frequent review.

  ratio of observation/menu cost 1-to-1 to frequency adjustment/review

Experimental Evidence

- Experiment description
  - Subjects are paid to solve a quadratic tracking problem
  - Subject must track a random walk with no drift
  - Subjects paid proportional to square deviation of current tracking point
  - Subjects pay a fixed cost to adjust to current position
  - Experiments imitates problem of a firm adjusting prices subject to fixed cost
- Results
  - Subject act as if they also have an observation cost
  - Implied distribution of adjustment $\approx$ model w/ observation & menu costs.

Description of Experiment



Period:	1
Period Profits:	164.69
All Profits:	164.69

Change Size:	0.067
Prob(up):	50.00%
Prob(end):	0.25%
Adjust Cost:	3

Ready

Distributions of adjustment in the lab

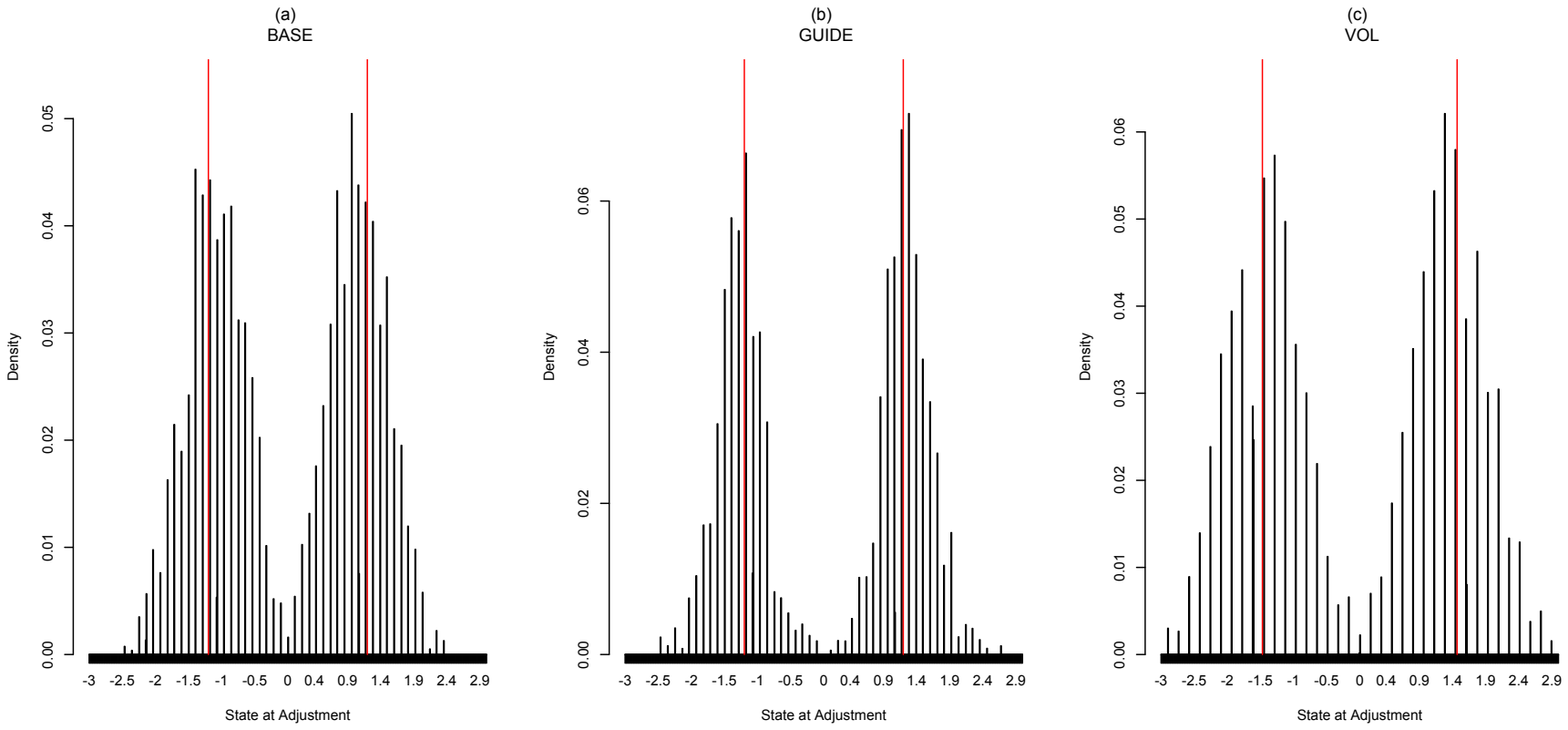


Figure 4: Weighted histograms of adjustment states for each treatment.

Distributions of adjustment in the theory

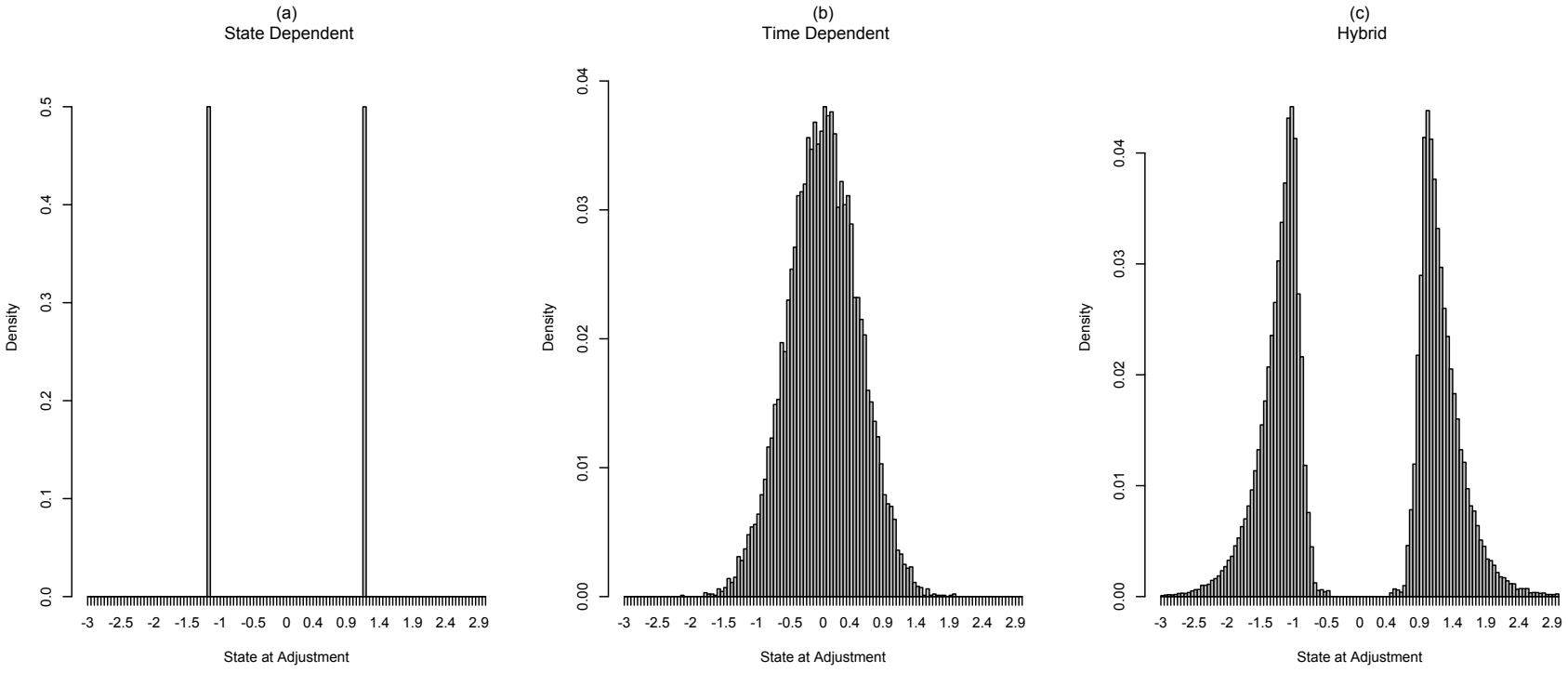
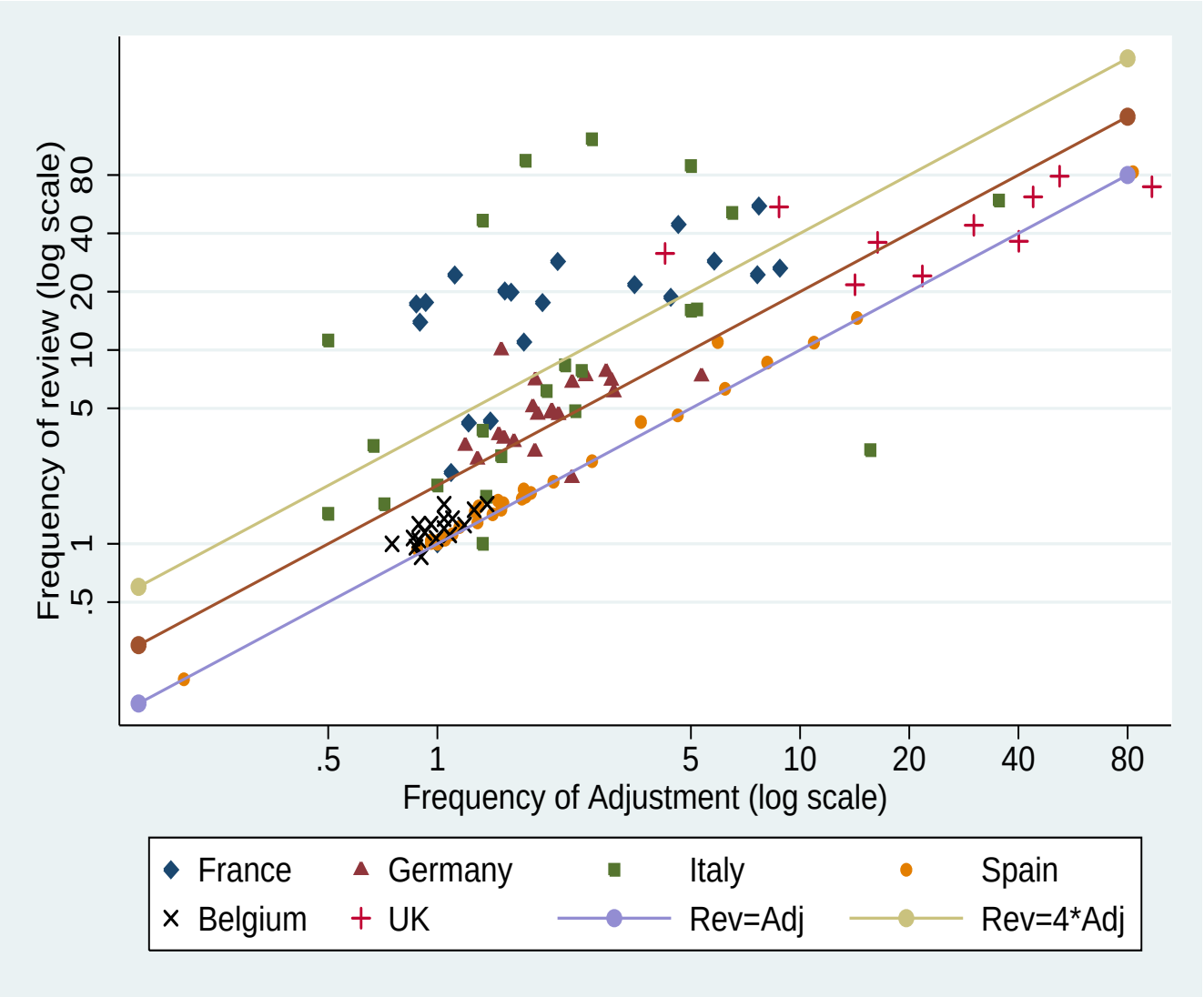


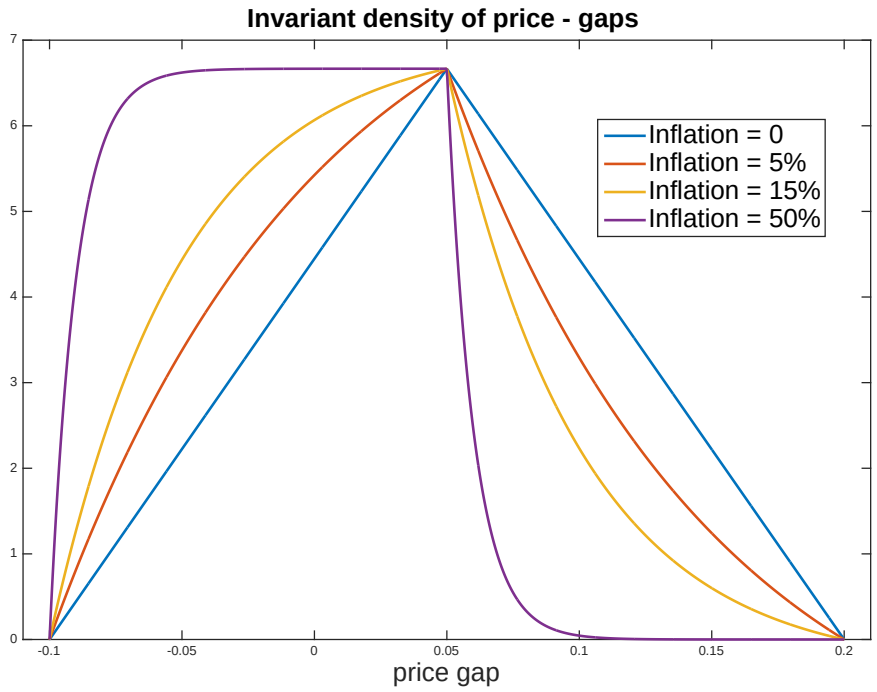
Figure 2: Histogram of simulated adjustments for (a) state dependent agents, (b) time dependent agents and (c) costly attention agents.

Frequency of price reviews and price changes



Industry year averages symbols, country colors (Source ECB)

Impact effect on prices (SD models): non-negligible for high inflation



$$\Theta(\delta;\pi) \approx \frac{1}{2}\left[\frac{1}{\bar{g}^2} + \frac{1}{\sigma^2\bar{g}}\pi\right]\delta^2 = \frac{1}{Std\left[\Delta p_i\right]}\left[\frac{2}{Std\left[\Delta p_i\right]} + \frac{\pi}{\sigma^2}\right]\delta^2$$

As $\frac{\pi}{\sigma^2}$ increases distribution $\rightarrow$ uniform (Shesinsky-Weiss)

TD models and inflation

- "Pure" TD model useful for conceptual purposes.
- Yet "pure" TD model display full indexation.
- Full indexation is highly counterfactual.
- Positive observation ($\psi_o > 0$) and menu cost ($\psi_m > 0$) eliminates this.

TD models and inflation

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- Yet "pure" TD model display full indexation.
- Full indexation is highly counterfactual.
- Positive observation ($\psi_o > 0$) and menu cost ($\psi_m > 0$) eliminates this.
- Interestingly they also eliminate "price plans"

  (price changes between observations, also counterfactual).

Inflation Pass-through: Piecewise Linear

	1974-2014 Sample, excluding Fixed ER countries (6,816 obs.)				
horizon h:	1 month	3 month	6 month	12 month	24 month
β_h (linear term)	0.015***	0.034**	0.058***	0.058***	0.0943***
	(0.004)	(0.007)	(0.009)	(0.014)	(0.020)
$\gamma_h \times 100$ (non-linear term)	0.0174***	0.027**	0.027	0.019	-0.005
	(0.008)	(0.016)	(0.023)	(0.035)	(0.049)
R^2	0.30	0.45	0.41	0.51	0.62

	1974-2014 Sample, All countries (13,733 obs)				
horizon h:	1 month	3 month	6 month	12 month	24 month
β_h (linear term)	0.022***	0.047***	0.064***	0.092***	0.157***
	(0.007)	(0.018)	(0.024)	(0.038)	(0.046)
$\gamma_h \times 100$ (non-linear term)	0.012*	0.027	0.029	0.033	-0.047
	(0.007)	(0.018)	(0.024)	(0.038)	(0.046)
R^2	0.16	0.26	0.28	0.39	0.46

Inflation Pass-through: Classification ER Regime

	1974-2014 Sample, excluding Fixed ER countries (9,570 obs.)				
horizon h:	1 month	3 month	6 month	12 month	24 month
β_h (linear term)	0.011***	0.031***	0.065***	0.088***	0.125***
	(0.004)	(0.009)	(0.012)	(0.018)	(0.027)
$\gamma_h \times 100$ (quadratic term)	0.108***	0.160**	0.082	-0.010	-0.227*
	(0.036)	(0.076)	(0.121)	(0.133)	(0.130)
R^2	0.29	0.44	0.42	0.54	0.61

	1974-2014 Sample, All countries (13,733 obs)				
horizon h:	1 month	3 month	6 month	12 month	24 month
β_h (linear term)	0.020***	0.040***	0.064***	0.092***	0.184***
	(0.004)	(0.010)	(0.013)	(0.021)	(0.024)
$\gamma_h \times 100$ (quadratic term)	0.057**	0.166	0.091	0.097	-0.448***
	(0.028)	(0.112)	(0.138)	(0.255)	(0.149)
R^2	0.16	0.26	0.28	0.39	0.46

Pass-through on Impact: Definition Low Inflation

	1974-2014 sample: All Countries				
inflation threshold below:	4%	5%	6%	8%	10%
Non-linear effect	×	✓✓✓	✓✓	✓✓	×
# obs	8,263	9,774	11,030	13,723	16,157

	1974-2014 sample: excluding Fixed ER countries				
inflation threshold below:	4%	5%	6%	8%	10%
Non-linear effect	×	✓✓✓	✓✓✓	✓✓✓	×
# obs	4,566	5,240	5,795	6,811	7,587

	1990-2014 sample: All Countries				
inflation threshold below:	4%	5%	6%	8%	10%
Non-linear effect	×	✓✓✓	✓✓✓	✓✓✓	✓
# obs	6,678	7,651	8,314	9,179	9,813

Inflation Pass-through: Unclassified Countries Out

	1974-2014 Sample, excluding Fixed ER countries (3,896 obs.)				
horizon h:	1 month	3 month	6 month	12 month	24 month
β_h (linear term)	0.008	0.013	0.029**	0.029*	0.081***
	(0.005)	(0.009)	(0.012)	(0.017)	(0.024)
$\gamma_h \times 100$ (quadratic term)	0.096***	0.149***	0.134	0.138	-0.034
	(0.021)	(0.038)	(0.089)	(0.119)	(0.152)
R^2	0.26	0.41	0.55	0.61	0.69

	1974-2014 Sample, All countries (10,808 obs)				
horizon h:	1 month	3 month	6 month	12 month	24 month
β_h (linear term)	0.024***	0.044***	0.062***	0.097***	0.188***
	(0.005)	(0.010)	(0.014)	(0.023)	(0.026)
$\gamma_h \times 100$ (quadratic term)	0.029	0.133	0.079	0.090	-0.434***
	(0.020)	(0.041)	(0.054)	(0.068)	(0.084)
R^2	0.11	0.19	0.29	0.40	0.47

Descriptive Statistics: samples

	Post-1974 Sample, Inflation Threshold 8%						
Sample	Mean(π)	sd(π)	Mean(Δe)	sd(Δe)	# Large innovations $\lvert\Delta e\rvert$		
					>7%	>10%	>15%
All countries (13,025 obs)	3.51	3.76	0.08	2.81	368	131	22
No Fixed ER (6,137 obs)	3.14	3.38	0.014	3.00	229	88	18
	Post-1990 Sample, Inflation Threshold 8%						
All countries (8,488 obs)	2.94	2.80	0.12	2.86	272	109	18
No Fixed ER (5,010 obs)	2.76	2.75	0.19	3.07	204	82	16