Abstract

In this paper, I explore the asset pricing implications of a general equilibrium model with production, in which investment projects need time-to-build. This investment friction allows me to explain the observed negative correlation of investment growth and stock returns at the aggregate level – an observation that has been interpreted as evidence for irrational markets since it cannot be reconciled with the Q theory of investment. When I further enhance the time-to-build model with adjustment costs, the model is able to explain three empirical facts of aggregate data: (i) the negative comovement of stock returns and investment growth, (ii) the prolonged positive correlation between stock returns and future investment growth, and (iii) the negative correlation between investment growth and future stock returns. Further, since time-to-build implies an investment commitment, firms are levered by the commitment, leading to a risk premium and thereby alleviating the equity premium and excess volatility puzzles.
1 Introduction

The empirical correlation between investment growth and stock returns is negative. This finding contrasts with the prediction of the Q theory of investment that investment growth and stock returns will exhibit a positive comovement. According to the Q theory, when discount rates fall, investment increases in value and thus firms increase investment. At the same time, stock prices appreciate, leading to positive comovement with investment growth. From an economic perspective, this empirical finding suggests that firms irrationally reduce investment when the stock market signals good times. The goal of the paper is to rationalize this finding in a general equilibrium model with production.

To be able to differentiate between the effect of irrational and rational market forces on real investment is an important research question – a point nicely summarizes by Baker, Ruback, and Wurgler (2006):

Of paramount importance are the real consequences of market inefficiency. It is one thing to say that investor irrationality has an impact on capital market prices, or even financing policy, which lead to transfers of wealth among investors. It is another to say that mispricing leads to underinvestment, overinvestment, or the general misallocation of capital and deadweight losses for the economy as a whole.

The failure of the Q theory of investment has far reaching implications for theoretical asset pricing models. It is well known that Q theory implies the state-by-state equivalence of stock and investment returns—returns from investment in capital reflect the firm’s tradeoff between the investment’s marginal costs and marginal benefits whereas stock returns are the consumer’s tradeoff of investing in the stock market. The equivalence of these two returns is a crucial assumption for many asset pricing models, for instance, Cox, Ingersoll, and Ross (1985), Gomes, Kogan, and Zhang (2003), Hall (2001), Jermann (2005), Zhang (2006), and Liu, Whited, and Zhang (2006). I perform an empirical exercise which clearly shows that stock return are not the same as investment returns.

The main insight of this paper is that a model, in which capital accumulation exhibits time-to-build, can generate the empirical correlation pattern between stock returns and investment growth. Further, time-to-build creates a wedge between average and marginal
Q, thereby breaking the link between stock and investment returns. This implication helps to generate a higher excess return and return volatility than in standard production economies and thus alleviates the equity premium and excess volatility puzzles.

Time-to-build captures the idea that investment projects are not instantaneously completed. Instead, firms have to allow for several quarters to expand capacity. Since investment studies have found that the completion of investment takes on average one to one and a half years, time-to-build is an important investment friction.

In a time-to-build model, firms commit to invest in the future but only incur a fraction of the investment costs immediately. The remaining investment costs are spread over future periods. The delay between investment commitment and the payment of investment costs is critical to generating the observed negative correlation between investment growth and stock returns. Fortunately, such a lag is consistent with investment data. If firms incur most of the costs at the time of the investment decision, then investment growth and stock returns covary positively.

How does time-to-build lead to a negative correlation between investment growth and stock returns? To explain the economic intuition, consider a two period time-to-build framework, in which firms do not incur any investment costs at the commitment date. Instead they incur all costs in the next period.

After a positive technology shock, firms commit to invest two periods ahead. Since the investment costs are incurred with a lag, consumption as well as dividends are initially high, but then fall leading to negative consumption and dividend growth. Since negative expected consumption growth necessitates a low risk-free rate, the expected stock return is low and thus the stock price and the realized stock return are high. In the period after the shock, the expected stock return returns to the steady state level. Since firms incur the investment costs now, investment growth and realized stock returns covary negatively as observed in the data.

This general equilibrium mechanism is in stark contrast to partial equilibrium considerations where the pricing kernel is exogenous. Here, the investment commitment makes the firm risky because the assets in place are levered by the investment commitment. Thus, the investment commitment creates operating leverage, driving up the expected return. In general equilibrium, however, expected returns are low because the pricing kernel and
thus the risk-free rate are endogenous. Yet consistent with the idea of operating leverage, I find that the expected excess return is higher the less investment costs are incurred at the commitment date.

In a general equilibrium model with time-to-build, stock returns reflect operating leverage because the value of the firm depends not only on the value of its productive capital but also on the value attributed to unfinished investment projects. As a result, average Q deviates from marginal Q, creating a wedge between investment and stock returns. This fact helps to explain the equity premium and the excess volatility puzzle because stock returns are not tied to the marginal rate of transformation of capital, i.e. investment returns. In standard production economies based on the Q theory, investment and stock returns are identical state by state. I test this implication and find it "rejected".

The standard time-to-build model, in which the investment costs are spread evenly over periods, has the drawback that it implies an oscillating optimal investment policy. After a positive shock, the investment commitment is high, then low, and so forth. Since the investment costs are a weighted average of past investment decisions, the firm thereby smooths the average investment costs.

This implication is counterintuitive because investment decisions are persistent in the data. A remedy to this drawback is to introduce adjustment costs. The standard adjustment cost function, which is defined in terms of the investment rate, the ratio of investment to capital, does not solve the problem. In contrast, I assume that the firm faces adjustment costs in the investment growth rate, the ratio of current investment decision to past investment decision. Thus the firm is penalized when it changes the growth rate.

The time-to-build model with adjustment costs produces persistent investment growth after a positive technology shock, as observed in the data. The model is able to explain three empirical facts of aggregate data: (i) the negative comovement of stock returns and investment growth, (ii) the prolonged positive correlation between stock returns and future investment growth, and (iii) the negative correlation between investment growth and future stock returns.

In the empirical part of the paper, I report findings consistent with time-to-build at the aggregate level. Nonresidential investment growth and stock returns are negatively correlated contemporaneously but positively correlated for six quarters afterwards. Further,
there is a negative correlation between investment growth and future stock returns.

Interestingly, there is already plenty of empirical support for the existence of time-to-build not only in the empirical investment literature but also in the empirical finance literature. The respective authors have, however, interpreted the findings as behavioral and thus not rational.

The first paper which documents findings consistent with time-to-build is McConnell and Muscarella (1985). They conduct an event study around the dates when firms publicly announce future capital expenditure plans and find that the announcement of a budget increase of plant and equipment spending is associated with a significant positive excess return. In the light of time-to-build, at these events firms commit to future investment spending and thus are more risky than a firm without such a commitment. More recently, Blose and Shieh (1997) and Vogt (1997) confirm these findings.

Further evidence for time-to-build is contained in the SEO underperformance literature, starting with Loughran and Ritter (1995) and Spiess and Affleck-Graves (1995). If we assume that the motivation for the majority of SEOs is as increase in upcoming investment expenditures, then SEO announcements can be interpreted as investment commitments. Consequently, these firms earn a positive excess return at the announcement date and underperform in the long-run as they incur the investment costs and thus unlever.

Recent papers by Titman, Wei, and Xie (2004) and Anderson and Garcia-Feijóo (2004) document that firms with high investment expenditure earn low excess returns and vice versa. This phenomenon has been dubbed the investment anomaly. Titman, Wei, and Xie (2004) interpret their empirical evidence as investor underreaction to the empire-building implication. However, this phenomenon is perfectly consistent with time-to-build since firms with high investment expenditure unlever and thus are less risky than firms with high commitment and thus low investment.

Direct evidence of time-to-build is presented by Lamont (2000). He tests a hypothesis suggested by Cochrane (1991) "If there are lags in the investment process, then investment will not rise for a few periods, but orders or investment plans rise immediately". Lamont (2000) finds strong empirical support for this hypothesis because investment plans explain more than three quarters of annual variation of aggregate investment growth.

Concluding, the results of my model imply that time-to-build is a necessary feature to
explain stock market and investment data simultaneously. The existence of time-to-build can explain a failure of the Q theory of investment and helps to explain asset pricing facts such as the equity premium and excess volatility.

The road map of the paper is as follows. In section two, I summarize the related literature. The empirical results for aggregate investment and stock returns are presented in section three. In section four, I present the time-to-build model and derive the optimality conditions. In section five, I explain the asset pricing implication and section six concludes.

2 Related Literature

Investment Literature

The investment literature concerned with investment lags has tried to answer two separate questions: How long is the investment process and when do firms incur most of the costs? The investment process comprises two distinct periods: the planning phase and the construction phase. The planning phase contains the design of the project involving, for instance, the work of engineers. A bidding and contracting phase follows in which the conditions of the project are finalized. The actual implementation of the project happens in the construction period.

Several papers have estimated the total time of the investment process around one and a half years. For instance, Mayer (1960) conducts a survey of 110 companies and finds that the average length of the time between the decision to build a plant and its completion is 21 months. Montgomery (1995a,b) uses survey data collected by the U.S. Department of Commerce to construct the completion pattern for nonresidential structures between 1961 and 1991. He finds that the construction period averages between 5 and 6 quarters. Further evidence is presented in Mayer and Sonenblum (1955), Jorgenson and Stephenson (1967), Ghemawat (1984), and Koeva (2000).

The second important feature of time-to-build is the timing of the costs during the investment process. For many projects it is reasonable to conjecture that the planning phase involves lower costs than the construction period since the latter contains the construction of real goods such as factories and machinery.

Krainer (1968) finds that expenditures on major investment projects exhibit the clas-
sic S-curve shape with very low expenditures initially. Estimating a time-to-build Euler equation with annual Compustat data, Koeva (2001) reports that in the first year roughly 10% and in the second year 90% of the costs are incurred.

**Macroeconomic Literature**

Time-to-build was first introduced into a business cycle model by Kydland and Prescott (1982). Based on investment studies, they assume that it takes four quarters for an investment project to be finished and the investment costs are spread evenly over this period. In contrast, in the standard RBC model investment increases next period’s capital stock.

More recently, Christiano and Todd (1996) argue that most of the investment costs are incurred at the end of the project. Their reasoning is that most investment projects begin with a lengthy planning period which is less resource intensive than the actual construction phase of the project. They call this idea time-to-plan. The only difference compared to the time-to-build specification is that the investment costs are not evenly spread over the investment project horizon but most investment costs are incurred at the end of the project.¹

Christiano and Vígefusson (2003) employ frequency domain tools to estimate and test dynamic stochastic general equilibrium models and confirm the importance of time-to-plan to explain business cycle variations.

**Asset Pricing Literature**

In line with the equity premium puzzle in an endowment economy, Rouwenhorst (1995) demonstrates the failure of the standard RBC model to account for the equity premium. Jermann (1998) and, more recently, Boldrin, Christiano, and Fisher (2001) show that the RBC model generates a reasonable equity premium when the model is enhanced with frictions. The key insight of these papers is that frictions in the capital market as well as modifications of preferences are necessary. Both papers rely on internal habit, yet Jermann (1998) includes capital adjustment costs and Boldrin, Christiano, and Fisher

¹In this paper, I use the words time-to-build and time-to-plan synonymously even though time-to-plan would be more precise.
The only paper that analyzes the asset pricing implication of time-to-build is Boldrin, Christiano, and Fisher (2001). They, however, examine the investment return and not the stock return. As I will show below, these two return deviate from each other.

The cross-sectional book-to-market and size effects have been explained in partial equilibrium models by Berk, Green, and Naik (1999), Carlson, Fisher, and Giammarino (2004), Zhang (2005, 2006), and Cooper (2004). These papers explore the investment decision of dynamic optimizing firms and differ in the investment frictions they consider to generate a realistic cross-section of returns. In a general equilibrium model, Kogan (2001, 2004) analyzes the effects of irreversible investment on stock returns. None of these papers has, however, explored the effects of time-to-build on stock returns.

3 Empirical Findings

The goal of this section is to estimate investment returns based on the widely used capital adjustment cost framework. Capital adjustment costs are the key ingredient of the Q theory of investment. The empirical exercise is therefore also a test of the Q theory. As "testable" implications for the model with time-to-build, I also analyze the correlation between stock returns and investment growth.

In the first section, I derive the investment return in partial equilibrium production-based asset pricing model or, put differently, partial equilibrium Q theory of investment model. In the second section, I estimate the derived investment return for gross investment, nonresidential and residential investment.

3.1 A Neoclassical Production-Based Asset Pricing Model

Production-based asset pricing models are derived from firm’s optimal investment decision. The firm’s problem can be stated as maximizing firm value \( P_t \) by optimally choosing future real investment \( I_t \), i.e.

\[
P_t = \max_{\{I_{t+s}\}_{s=1}^{\infty}} \mathbb{E}_t \sum_{s=1}^{\infty} \frac{\Lambda_{t+s}}{\Lambda_t} D_{t+s} \quad D_t = Y_t - I_t - w_t N_t
\]

where \( \Lambda_t \) denotes the pricing kernel and \( D_t \) the dividend payment to the stock holder. Dividends are defined as the residual payment after subtracting investment \( I_t \), and labor

\(^2\)See also Danthine and Donaldson (2002), Lettau (2003), Christiano and Fisher (2003), and Guvenen (2004).
costs, which are labor $N_t$ times wage rate $w_t$, from output $Y_t$. Output is determined by a constant returns to scale Cobb-Douglas production function $F$

$$Y_t = Z_t F(K_t, N_t) \quad F(K_t, N_t) = K_t^\alpha N_t^{1-\alpha}$$

(2)

where $K_t$ denotes capital, $Z_t$ an exogenous technology shock, and $\alpha$ the capital share of production.

Firms incur adjustment costs when they invest. Adjustment costs reduce tomorrow’s capital and thus the law of motion for capital is

$$K_{t+1} = (1 - \delta)K_t + G(I_t, K_t)$$

(3)

where $\delta$ denotes depreciation and $G$ is a constant returns to scale function given by

$$G(I_t, K_t) = \left( \frac{a_1}{1-1/\xi} \left( \frac{I_t}{K_t} \right)^{1-1/\xi} + a_2 \right) K_t$$

(4)

This function is concave in $I$ and decreasing in $K$ and thus captures the notion that it is more costly to change the capital stock quickly. The parameter $\xi$ is the elasticity of investment-capital ratio with respect to marginal $Q$ and controls the concavity of the function. The concavity of $G$ also implies irreversibility of investment because $G$ is not defined for negative $I_t$. As noted by Hayashi (1982), this feature does not affect the dynamics, since optimal investment is never negative.

As a consequence of the capital adjustment costs, a Tobin’s Q interpretation arises. Marginal $Q$ is the ratio of the marginal value of an additional unit of capital $\Lambda_t^K$ over the price of a unit of capital $\Lambda_t$. I define marginal denote by $q_t$

$$q_t = \frac{\Lambda_t^K}{\Lambda_t}$$

(5)

where $\Lambda_t^K$ is the Lagrange multiplier on (3) and therefore the price of capital.

Investment should take place when $q_t > 1$ and destruction of capital when $q_t < 1$. Adjustment costs prevent the firm from adjusting the capital stock every period to its optimal level and, therefore, marginal $Q$ is time-varying and deviates from unity.

The solution of (1) can be characterized by the Euler equation

$$E_t \frac{\Lambda_{t+1}}{\Lambda_t} R_{t,t+1}^I = 1$$

(6)

\footnote{A derivation of the following results is contained in the appendix.}
where

\[ R_{t,t+1}^I = \frac{MPK_{t+1} + q_{t+1}(G_2(I_{t+1}, K_{t+1}) + (1 - \delta))}{q_t} \]  

(7)

defines the investment return on capital and \( MPK_{t+1} = Z_{t+1}F_1(K_{t+1}, N_{t+1}) \) is the marginal product of capital. The investment return is the ratio of marginal productivity plus capital gains tomorrow divided by marginal costs and therefore reflects the firm’s intertemporal tradeoff of investing.

One of the standard neoclassical assumptions is constant returns to scale (CRS). This innocuous-looking assumption has, however, major asset pricing implications. Hayashi (1982) proves in a non-stochastic setting and Abel and Eberly (1994) in a stochastic setting that when production function and adjustment cost function are both CRS, marginal \( Q \) equals Tobin’s average \( Q \). Thus, firm value is given by

\[ P_t = q_tK_{t+1} \]  

(8)

This equation says that firm value is the value of capital in terms of the numeraire times the amount of capital. Substituting (8) into the investment return (7) and assuming that labor is paid its marginal product, it follows that

\[ R_{t,t+1}^I = \frac{P_{t+1} + D_{t+1}}{P_t} = R_{t,t+1}^E \]

and hence the investment return on capital is equal to the stock return \( R_{t,t+1}^E \), state by state. This equivalence was also noticed by Restoy and Rockinger (1994).

The only paper that tries to test this implication is Cochrane (1991). In contrast to Cochrane (1991), I assume a different functional form for the adjustment cost function. The adjustment cost function assumed here has been employed, for instance, by Jermann (1998). This specification is used more often in theoretical work and, therefore, my empirical analysis can be understood as a direct test of these models. The results in this paper, however, are not driven by the functional form of the adjustment cost. The main driving force is the relation between investment and stock prices.

Different adjustment cost functions, or more generally, different production-based asset pricing models result in different functional forms for the investment return. This fact implies that investment returns are not uniquely identified and, thus, are model dependent whereas stock returns are not. Nevertheless, one can rebut production-based asset pricing models for their implications for investment returns. That is precisely done here.
3.2 Data

To construct an investment return time-series, I use aggregate quarterly US NIPA data covering the period 1947 Q1 until 2004 Q4. Output $Y_t$ is real GDP minus government expenditure. I compute the investment return based on real gross private domestic investment and the two subcategories real private nonresidential and residential fixed investment. I leave out the third subcategory which is changes in private inventories.

Gross private investment comprises nonresidential and residential fixed investment as well as changes in private inventories. Nonresidential fixed investment comprises structures as well as equipment and software. Nonresidential structures are, for instance, new constructions (e.g., hotels or mining explorations), improvements to existing structures, and brokers’ commissions of sales of structures. Nonresidential equipment and software are purchases by private business of new machinery, equipment, furniture, vehicles, and computer software. Residential fixed investment consists, for instance, of new construction of permanent single and multi family units, improvements to housing units, and brokers’ commissions on the sale of residential property.

Cochrane (1991) computes the investment return based on gross investment and Lamont (2000) focuses on nonresidential fixed investment because ”excluding residential investment is arguably more appropriate for relating investment and stock returns, because most of the residential capital stock is not traded on equity markets” (p.2729). Recently, however, the effects of housing has been linked to the stock market, see e.g. or Lustig and Nieuwerburgh (2005).

Further, residential and nonresidential behave very differently over the business cycle. Residential investment is known to lead the cycle whereas nonresidential investment tends to lag the cycle.

The capital stock time-series is constructed by aggregating investment following (3). I set the initial value of the capital stock equal to the BEA value reported for 1946. Since by assumption the production function (2) features constant returns to scale, the marginal product of capital can be estimated from

$$MPK_t = \alpha \frac{Y_t}{K_t}$$  

(9)

Inherent in $MPK_t$ is a technology shock and labor. By making use of the constant return
Table 1: Investment Returns
Annualized mean and standard deviation of real investment returns calculated for real gross private investment, real private nonresidential and residential fixed investment. The last row of each panel is the contemporaneous correlation of investment and stock returns in percent, \( \text{corr}(R_{t-1}^E, R_{t-1}^I) \). The last column is the annualized mean and standard deviation of the quarterly real return of the CRSP value-weighted index. The sample period is 1955-2004.

<table>
<thead>
<tr>
<th>Panel A: Gross private investment, ( \alpha = 0.2 )</th>
<th>Panel B: Gross private investment, ( \alpha = 0.3 )</th>
<th>Panel C: Nonresidential investment, ( \alpha = 0.1 )</th>
<th>Panel D: Residential investment, ( \alpha = 0.1 )</th>
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<td>-4.10</td>
<td>-3.71</td>
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</table>

3.2.1 Investment Returns
The investment return depends on the capital share \( \alpha \), depreciation \( \delta \), and the adjustment cost parameter \( \xi \). It is sensitive to the capital share \( \alpha \) and adjustment cost parameter \( \xi \) but insensitive to the depreciation parameter \( \delta \) which I therefore set to the common value of 0.025.

In table 1, I present the annualized first and second moments of investment returns computed for gross investment, residential and nonresidential investment. I exclude data prior to 1955 since the estimated investment return time series are extremely volatile.
before 1955.

Panel A and B is the gross investment return with a capital share of $\alpha = 0.2$ and $\alpha = 0.3$, respectively. Panel C is the nonresidential investment return and panel D the residential investment return both with a capital of $\alpha = 0.1$. The last column is the annualized real return of the quarterly value-weighted CRSP index.

I firstly discuss the implications for the gross investment return, panel A and B. Two effects are noticeable: Firstly, the capital share $\alpha$ controls the mean level of investment return. A higher capital share implies a higher mean investment return. From the definition of the investment return (7), it follows that the marginal product of capital controls the mean level of investment returns. Further, a higher capital share implies a higher marginal product of capital via (9).

Secondly, the adjustment cost parameter $\xi$ affects mainly the investment return volatility. By increasing the adjustment cost parameter $\xi$ and thereby making investment more and more costly, the investment return standard deviation increases and almost perfectly matches the corresponding moment of the CRSP index. Specifically, the choice of $\alpha = 0.2$ and $\xi = 0.55$ results in an annualized mean investment return of 7.89% and standard deviation of 17.19% compared with the mean CRSP return of 7.86% and standard deviation of 17.14%.

To compute the investment return for residential and nonresidential investment, I have to lower the capital share. Otherwise, the mean investment return would be too large because the capital stock in each subcategory is smaller than the capital stock for gross investment resulting in an increase of the marginal product of capital (9) which controls the mean return.

In panel C and D of table 1, the annualized first and second moments of nonresidential and residential investment returns are depicted. For a capital share $\alpha = 0.1$ and adjustment cost $\xi = 0.45$, the mean nonresidential investment return and stock return are almost identical, however, the standard deviation of the nonresidential investment return is considerably lower than the standard deviation of stock returns. In the case of residential investment with $\alpha = 0.1$ and $\xi = 0.55$, the standard deviations match perfectly. Yet the mean residential investment return is higher than the mean stock return.

Cochrane (1991) is only able to match the mean of gross investment returns and stock
returns. The standard deviation of investment returns is roughly half the standard deviation of stock returns. The fact that I can match first and second moments whereas Cochrane (1991) only matches the first moment is the result of the adjustment cost function. In the specification of Cochrane (1991), the elasticity of the investment-capital ratio is fixed at two whereas here the adjustment cost parameter $\xi$ affects the elasticity and thus the concavity of the function.

In the following, I focus on gross investment returns based on $\alpha = 0.2$ and $\xi = 0.55$, nonresidential investment returns based on $\alpha = 0.1$ and $\xi = 0.45$, and residential investment returns based on $\alpha = 0.1$ and $\xi = 0.55$ because these parameter choices give the best fit in terms of matching the first and second moments of investment and stock returns. Notice, the results are not sensitive to the particular parameter choice.

### 3.2.2 Correlation of Investment and Stock Returns

The last row of each panel of table 1 presents the contemporaneous correlation of quarterly investment and stock returns, $\text{corr}(R_{E_{t-1},t}^{I}, R_{I_{t-1},t}^{I})$. As shown above, the production-based asset pricing model implies that investment returns and stock returns have to be equal state by state. Hence, all moments have to be identical and, more importantly, the two time series have to be perfectly positively correlated.

For gross investment returns, I find that even though the first and second moments of investment and stock returns almost perfectly match, they are contemporaneously uncorrelated invalidating the model, $\text{corr} = -4.1\%$. The result is even stronger for nonresidential investment returns, $\text{corr} = -8.95\%$. Yet residential investment returns are positively correlated with stock returns, $\text{corr} = 18.19$.

In figure 1, the correlation of gross investment returns and stock returns are depicted when shifting investment returns by $k$ leads and lags, $\text{corr}(R_{E_{t-1},t}^{I}, R_{I_{1-k},t+k}^{I})$. Even though the returns are contemporaneously uncorrelated, gross investment and stock returns have a correlation of almost $34\%$ when gross investment returns are shifted by half a year forward, $\text{corr} = 33.96\%$. Thus, gross investment returns lag stock returns.

The lag of nonresidential investment returns is even more pronounced. In figure 2, the correlation of nonresidential investment returns and stock returns at leads and lags are plotted. The correlation pattern is humped shaped indicating that stock returns have
Table 2: Correlation of investment and stock returns

Correlation (in percent) of stock returns with gross investment returns (panel A), nonresidential investment returns (panel B), and residential investment returns (panel C) at $k$ leads and lags, $\rho_k = \text{corr}(R_{t-1,t}^E, R_{t-1+k,t+k}^I)$, $k = -12, -11, ..., 11, 12$. The second column of each panel is t-stat of the null hypothesis of zero correlation, $H_0: \rho_k = 0$ and the third columns is the corresponding p-value. Bold p-values are significant at the 5% level. The sample period is 1955-2004.

<table>
<thead>
<tr>
<th>$k$</th>
<th>Panel A</th>
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<th>Panel B</th>
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<th>Panel C</th>
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a prolonged effect on nonresidential investment. The correlation peaks at 31.08% after shifting investment returns half a year forward.

Residential investment returns behave differently. The correlations at leads and lags of residential investment returns with stock returns is depicted in figure 3. Residential investment returns lag stock returns by only one quarter, corr = 41.09%.

In table 2, I report the correlation of stock returns with gross investment returns (panel A), nonresidential investment returns (panel B), and residential investment returns (panel C) at k leads and lags, as plotted in figure 1-3. The second column of each panel is the t-statistic of the null hypothesis of zero correlation and the third column is the corresponding p-value.

The correlation of real stock returns with gross investment returns is significant at the 5% level at 1 to 3 quarters leads, with nonresidential investment at 2 to 5 quarters leads, and with residential investment at 0 to 2 leads.

In the long run, current stock returns are negatively correlated with investment returns. The correlation of stock returns with gross investment returns is significant at the 5% level at 7 quarters lead, with nonresidential investment returns at the 10% level at 11 quarters lead, and with residential investment at the 5% level at 5 to 7 quarters lead.

Cochrane (1991) reports a contemporaneous correlation of quarterly returns of 0.241. He obtains this positive correlation because he shifts stock returns by a half quarter so that they go from center to center of each quarter. His reasoning for the adjustment is that investment is a quarterly aggregate and stock prices are point to point. I do not make this adjustment because the correlation pattern between investment and stock returns cannot be resolved by this simple shift. Further, Lamont (2000) does not follow the timing convention of Cochrane (1991) as well.

In figure 4-6, I plot the annual gross investment return, nonresidential, and residential investment return, respectively. The solid blue line in each figure is the annual real CRSP return and the dashed black line the investment return. Especially for gross investment returns, it is visible that the volatility prior to 1955 is fairly large. Further, the lag of gross investment returns and nonresidential investment returns is clearly apparent and the positive correlation of residential investment returns and stock returns.
3.2.3 Investment Growth

What drives the correlation pattern between investment returns and stock returns? To this end, I compute the correlation of log changes of quarterly investment and log changes of the real CRSP index, when shifting log investment growth $k$ quarters forward and backward, $corr(\Delta \log P_t, \Delta \log I_{t+k})$, $\Delta \log I_{t+k} = \log I_{t+k} - \log I_{t+k-1}$, and $\Delta \log P_t = \log P_t - \log P_{t-1}$. In figure 7-9, I plot the correlation of stock prices changes with gross investment growth, nonresidential investment growth, and residential investment growth, respectively.

Comparing the correlations of investment returns and stock returns (figure 1-3) with the correlations of investment and stock price changes (figure 7-9), it is striking how similar the graphs are. This fact implies that most of the variation in investment returns is driven by investment growth.

One of the main puzzles in the investment literature is the negative contemporaneous correlation between gross investment growth and stock price changes (Figure 7) because it contradicts the Q theory of investment. A decline in expected returns raises marginal Q and as a result investment should increase. At the same time, stock prices increase due to lower expected returns. Hence, investment and stock prices should be contemporaneously positively correlated. Yet empirically, the contemporaneous correlation is negative, contradicting the Q theory.

The contemporaneous negative comovement is even stronger for firm (nonresidential) investment (Figure 8). Lamont (2000) nicely summarizes the point with "The significant negative contemporaneous covariation is particularly puzzling since it seems to suggest that firms perversely cut investment when stock prices go up".

Even though the contemporaneous correlation is negative, stock returns are positively correlated with future gross as well as nonresidential investment growth. Thus, the impact of high stock returns leads to positive investment growth for a prolonged period. The highest correlation between future gross investment growth and stock returns occurs at half a year lag, $corr = 32.1\%$.

Another important feature of gross as well as nonresidential investment growth is the negative correlation with future stock returns, i.e. $corr(\Delta \log P_t, \Delta \log I_{t-k}) < 0$. Thus,

\[^4\text{Cf. Barro (1990).}\]
high gross and nonresidential investment growth predict low stock returns.

The correlation pattern between residential investment growth and stock returns is very different. The contemporaneous correlation is positive and, therefore, accords with the Q theory. The correlation peaks with one quarter lag at 40.7%.

These findings suggest that it is important to differentiate between nonresidential and residential investment because they behave very differently. Nonresidential investment is mainly firm investment and residential investment is mainly housing expenditures by households. Firms seem react to slowly to good news as conveyed by positive stock returns and thus the decision process lags behind. Residential investment, which is mainly housing expenditures by households, reacts quickly to positive returns and lags by one quarter.

One reason for the slow reactions of firms to good news might be adjustment costs. In good times, firms want to expand their production capacity to meet higher demands. Convex adjustment costs, as assumed in the Q-theory of investments, slow the adjustment process because firms are penalized for high investment rates. However, convex adjustment costs still imply a positive correlation between investment growth and stock returns. Thus, only non-convex adjustment costs might be the cause.

In addition to facing adjustment costs, firms also need time to expand capacity. When the economy enters a boom, firms may decide to build a new factory, for instance. In standard models, this new capacity is available in the next period. Yet as documented above, the investment process takes on average one and a half years. In the following section, I show that time-to-build can cause the negative correlation.

In Figures 10-11, the correlation of the two subcategories of nonresidential investment growth, namely equipment and software as well as structures, with stock returns are depicted. The positive correlation between stock returns and future structures investment growth is even more pronounced.

4 Model

The goal of the model is to explain three facts of stock market and (nonresidential) firm investment data which have been documented above: First, the negative comovement of stock returns and investment growth; second, the prolonged positive correlation between stock returns and future investment growth; and third, the negative correlation between
investment growth and future stock returns.

To this end, I solve a stochastic general equilibrium with production where capital needs time-to-build. The model is similar to Kydland and Prescott (1982) and Christiano and Todd (1996). Since the goal of the model is not to solve the equity premium, I assume power utility.

4.1 Household

The representative household maximizes expected lifetime utility over consumption

\[
\max_{\{C_t, s_t, B_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t) \quad (10)
\]

where \( \beta \in (0, 1) \) denotes the individual discount rate, \( C_t \) consumption, and \( u \) a time-separable utility function. Since the goal of the model is not to solve the equity premium, I assume power utility

\[
u(C_t) = \frac{1}{1 - \gamma} C_t^{1-\gamma}
\]

where \( \gamma \) is the coefficient of relative risk aversion. The household can buy a risky claim on the firm’s dividend stream and a risk-free bond \( B_t \) such that the time \( t \) budget constraint reads

\[
C_t + s_tP_t + B_t \leq s_{t-1}(P_t + D_t) + R_f B_{t-1} + l_t N_t
\]

where \( s_t \) is the number of asset hold, \( P_t \) the stock price, \( D_t \) the dividend payment, \( l_t \) the wage rate, and \( N_t \) the amount of time working.

4.2 Firm

Firms own the economy’s real capital and decide about investments. Their objective is to maximize expected firm value \( P_t \) by making optimal real investment decisions \( X_t \), i.e.

\[
P_t = \max_{\{X_t, s_t\}_{t=1}^{\infty}} \mathbb{E}_t \sum_{s=1}^{\infty} \frac{\Lambda_{t+s}}{\Lambda_t} D_{t+s} \quad D_t = Y_t - I_t - l_t N_t
\]

where \( \Lambda_t \) denotes the pricing kernel and \( D_t \) the dividend payment to the stock holder. Dividends are defined as the residual payment of output after subtracting investment costs \( I_t \) and labor costs \( l_t N_t \). Output is determined by a Cobb-Douglas production function \( F \)

\[
Y_t = Z_t F(K_t, N_t) \quad F(K_t, N_t) = K_t^\alpha N_t^{1-\alpha}
\]
where $K_t$ denotes capital, $Z_t$ an exogenous technology shock, and $\alpha$ the capital share of production.

Contrary to previous papers, I assume that the current investment decision $X_t$ becomes productive 2 periods later. The law of motion for capital is therefore

$$K_{t+2} = (1 - \delta)K_{t+1} + X_t$$

where $X_t$ is the investment decision at time $t$ but not the cost. The investment costs $I_t$ are a weighted average of past investment decisions

$$I_t = wX_t + (1 - w)X_{t-1}$$

where the weight $w$ determines the timing of the costs. When $w = 1$, firms incur the full cost within the same period whereas firms incur the cost a period later when $w = 0$. The specification of the investment costs (15) follows Kydland and Prescott (1982) and Christiano and Todd (1996).

Previous papers have assumed four quarters time-to-build. Since the asset pricing implications are driven by the timing of the costs, I have simplified the standard time-to-build model to 2 quarters. As a result, only a single parameter, namely $w$, determines the timing of the investment costs.

Output is subject to a technology shock $Z_t$ which follows an AR(1) processes

$$\ln Z_t = \rho \ln Z_{t-1} + \varepsilon_t$$

with $\varepsilon_t \sim \mathcal{N}(0, \sigma)$.

### 4.3 Equilibrium

A competitive rational expectations equilibrium is a sequence of allocations $\{C_t, K_t\}_{t=0}^{\infty}$ and a price system $\{\Lambda_t, P_t\}_{t=0}^{\infty}$ such that

1. given the price system, the representative household maximizes (10) s.t. (11)
2. given the price system, the representative firm maximizes (12) s.t. (14)
3. the good market clears
   $$Y_t = C_t + I_t$$
4. the stock market clears $s_t = 1$.

By Walras’ law, the labor market clears as well.
4.4 Returns

The household’s first order condition with respect to $s_t$ gives the standard Lucas Euler equation for stock returns

$$ u'(C_t) = \beta \mathbb{E}_t u'(C_{t+1}) R_{t,t+1}^E $$

where

$$ R_{t,t+1}^E = \frac{P_{t+1} + D_{t+1}}{P_t} $$

denotes the return on equity. Note that in equilibrium $N_t = 1$. The price of the consumption numeraire is $\Lambda_t = u'(C_t)$ and thus the pricing kernel is

$$ M_{t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t} $$ (16)

The risk-free rate is given by

$$ 1/R_{t,t+1}^f = \mathbb{E}_t M_{t+1} $$

Since the firm’s optimality conditions are non-standard, I derive them explicitly. The firm’s Lagrange function is

$$ \max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \{ Z_t K_t^\alpha N_t^{1-\alpha} - w X_t + (1-w) X_{t-1} - \alpha X_t - 1 - w N_t \} + \beta \mathbb{E}_t \Gamma_{t+1} (1-\delta) $$

where $\Gamma_t$ is the Lagrange multiplier on (14) and therefore the price of capital. The first order conditions with respect to $X_t, K_{t+2}, N_t$ are

$$ \Gamma_t = w \Lambda_t + (1-w) \beta \mathbb{E}_t \Lambda_{t+1} $$ (17)

$$ \Gamma_t = \beta^2 \mathbb{E}_t \Lambda_{t+2} Z_{t+2} K_{t+2}^{\alpha-1} N_{t+2}^{1-\alpha} + \beta \mathbb{E}_t \Gamma_{t+1} (1-\delta) $$ (18)

$$ l_t = Z_t (1-\alpha) K_t^\alpha N_t^{1-\alpha} $$ (19)

Since time-to-build is an investment friction, a Tobin’s Q interpretation arises. Marginal Q is the ratio of the marginal value of an additional unit of capital, $\Gamma_t$, divided by the price of a unit of capital, $\Lambda_t$. Dividing the FOC (17) by $\Lambda_t$ yields marginal Q, denoted by $q_t$,

$$ q_t = \frac{\Gamma_t}{\Lambda_t} = w + (1-w) \beta \mathbb{E}_t \Lambda_{t+1} \Lambda_t $$ (20)

Thus, marginal Q is a weighted average of the investment costs where the weight is given by the risk-free rate. Marginal Q is time-varying because parts of the investment costs are incurred in the future and these sure costs have to be discounted at the risk-free rate.
Substituting (17) into (18) results in the firm’s Euler equation

\[ \Lambda_t = \beta \mathbb{E}_t \Lambda_{t+1} R^f_{t,t+1} \]

where the the investment return is defined by

\[ R^f_{t,t+1} = \frac{\beta \Lambda_{t+2} Z_{t+2} F_{1}(K_{t+2}, N_{t+2}) + q_{t+1}(1 - \delta)}{q_t} \]

The investment return \( R^f_{t,t+1} \) defines the firm’s intertemporal tradeoff of investing one more unit of capital. It is the ratio of tomorrow’s marginal benefits of investing one additional unit of capital divided by today’s marginal costs. Since the marginal product of an additional unit of capital occurs in 2 periods, it has to be discounted at the risk-free rate.

Due to constant returns to scale, firm value \( P_t \) can be solved analytically and is given by

\[ P_t = \mathbb{E}_t \sum_{s=1}^{\infty} M_{t+s} D_{t+s} \]

\[ = q_t K_{t+2} - (1 - w) X_t \mathbb{E}_t M_{t+1} + \mathbb{E}_t M_t Z_{t+1} F(K_{t+1}, N_{t+1}) \]  

Under time-to-build, the equivalence of marginal and average \( Q \) breaks down and hence leads to a deviation of investment and stock returns. To see this note that average \( Q \) is defined as

\[ Q_t = \frac{P_t}{K_{t+2}} \]

Substituting (22) into (23) and comparing the resulting expression with (20), it follows that \( Q_t \neq q_t \). Altuğ (1993) first noticed that time-to-build leads to a divide between average and marginal \( Q \).

The difference between marginal and average \( Q \) arises because firm value (22) reflects not only the value of productive capital \( q_t K_{t+2} \) but also the value of unfinished investment projects. More specifically, firm value has to be reduced by future investment costs, which surely occur because of past investment decisions. The second term of (22) captures these costs. The third term is discounted future profits due to past investment decisions.

4.5 Calibration

In table 3, I summarize the parameters’ choice. These values are similar to Cooley and Prescott (1995) and correspond to quarterly frequency.
Table 3: Parameter Choice

\( \beta \) denotes the discount rate, \( \alpha \) the capital share, \( \gamma \) risk aversion, \( \delta \) depreciation, \( \rho \) the autocorrelation and \( \sigma \) the standard deviation of the technology shock.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( \alpha )</th>
<th>( \delta )</th>
<th>( \rho )</th>
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<td>1.03(^{-1/4})</td>
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<td>0.36</td>
<td>0.025</td>
<td>0.95</td>
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The household discounts future consumption at an annual rate of 3 percent implying \( \beta = 1.03^{-1/4} \). I set the coefficient of risk aversion equal to 5 so that the results are not driven by extreme risk aversion.

The capital share of production is \( \alpha = 0.36 \). The quarterly depreciation rate of capital, \( \delta \), is set to 0.025, which implies 10% annual depreciation. The technology shock is mean zero with autocorrelation \( \rho \) of 0.95 and standard deviation \( \sigma \) of 0.007 percent.

I solve the model with a second order perturbation around the simulated mean of the model, following Collard and Juillard (2001).

5 Results

Since the asset pricing implications are driven by the timing of the investment costs, I present three cases with different timing. In the first case, I assume that the total investment costs are incurred in the period after the investment decision, i.e. \( w = 0 \). Thus, the firm commits to invest in the future but does not incur any costs in the commitment period.

Following Kydland and Prescott (1982), in the second case the investment costs are spread evenly over the two periods, i.e. \( w = 0.5 \). The specification of Kydland and Prescott (1982) results in an oscillating investment decision because firms thereby smooth the average investment costs.

In the third case, I enhance the two period time-to-build with convex adjustment costs. Consistent with the empirical investment literature, I assume that some investment costs are incurred in the commitment period, i.e. \( w = 0.2 \). This specification results in a very realistic correlation pattern between investment growth and stock returns.
5.1 Pure Commitment: \( w = 0 \)

To explore the timing of events, I report the impulse response functions generated by the model. In Figure 12, the impulse response functions of output \( Y \), capital \( K \), consumption \( C \), dividends \( D \), the investment decision \( X \), and the investment costs \( I \) are plotted after a one percent technology shock in period one.

The investment decision and output rise on impact of the shock. The investment costs have the same impulse response pattern as the investment decision but lag one period. The response of capital is hump-shaped due to time-to-build of the capital stock.

Consumption and dividends first rise because of the technology shock but then sharply decrease because of the investment costs. After the initial spike, dividends exhibit hump-shaped dynamics which arise because of the delayed dynamics of capital and the decreasing investment costs.

Figure 13 shows the impulse response of the realized and expected stock return, and the risk-free rate. Because of the negative expected consumption growth after the shock, the household wants to buy the bond to smooth consumption. But since the bond is in zero net supply, the risk-free rate has to adjust and therefore be below the steady state.

The effect of the low risk-free rate on the expected stock return is apparent in the impulse response. Since the expected stock return equals the risk-free rate plus risk premium, the low risk-free rate depresses the expected stock return. Moreover, the risk premium component of the expected return is not high enough to offset the negative risk-free rate effect.

The low expected return results in a high stock price and thus a high realized return. The realized stock return falls after the initial peak because the firm now incurs the investment costs. This effect leads to the negative comovement of stock returns and investment growth. Figure 14 shows the correlation of stock returns and investment growth, \( \text{corr}(\Delta \log P_t, \Delta \log I_{t+k}) \), based on a simulation of the model. This graph is computed in the same way as the ones reported above and therefore should look similar. Consistent with the data, stock returns and investment growth are negatively correlated contemporaneously and positively correlated with a lag. Missing, however, is the prolonged positive effect.

Even though the expected return is low on impact of the shock, the expected excess
return is positive because the stock holders receive a risk compensation for operating leverage. Since the firm commits to invest in the next period, the firm is levered by the amount of future investment costs. The amount of tomorrow’s investment cost is fixed, but the future is uncertain due to the technology shock and, thus, the firm is a risky investment. The stock holder therefore receives a compensation for the investment risk in terms of a positive expected excess return, which can be seen in Table 4.

Table 4 summarizes the moments generated by the model. Most interesting are the asset pricing implications. Even though the household has power utility, the model generates almost a one percent excess return. This result is quite surprising and has not been achieved in the previous literature with power utility.

In the standard RBC model, stock and investment returns are identical state by state. To generate a high stock return in such a model, the marginal rate of transformation of capital, i.e. investment return, has to be very volatile as well. This close tie between stock and investment returns is broken in a model with time-to-build because average Q diverge from marginal Q. Here, investment returns can remain smooth but stock returns are volatile. Further, the model generates a high standard deviation of the stock return, alleviating the excess volatility puzzle.

5.2 Time-to-Build: $w = 0.5$

Following Kydland and Prescott (1982), the investment costs are now spread evenly over the two periods, i.e. $w = 0.5$. When firms incur some of the investment cost in the same period as the investment decision, the dynamics are more complex. In Figure 15, the impulse response functions of output $Y$, capital $K$, consumption $C$, dividends $D$, the investment decision $X$, and the investment costs $I$ are depicted after a one percent technology shock in period one.

A striking feature of these impulse response function is the oscillation. This pattern is not unique to two periods of time-to-build, but also apparent in the graphs reported by Christiano and Todd (1996) with 4 periods time-to-build.

The rationale behind the dynamics is the objective of a smooth consumption stream for the household, which necessitates a smooth dividend stream. Since the firm cannot initially smooth output because of time-to-build of the capital stock, it wants to dampen
As the technology shock is highest on impact, the firm wants to incur most of the costs at time 1 and thus the investment decision is above the steady state. In the second period, the firm commits to invest only the steady state level $\bar{X}$. As a result, the investment costs are smooth

$$I_1 = 0.5X_1 + 0.5\bar{X} = I_2 = 0.5\bar{X} + 0.5X_1$$

Hence, oscillating between high and low investment commitment is optimal.

Figure 16 shows the corresponding impulse response functions for the realized and expected stock return, as well as the risk-free rate. Following the same reasoning as above, negative expected consumption growth after the shock results in a low risk-free rate. Since consumption growth oscillates after the shock, the risk-free rate oscillates as well.

The expected stock return is again low on impact of the shock, which is induced by the low risk-free rate. This implies that the risk premium component does not offset the negative risk-free rate effect. Another feature of the expected stock return impulse response is its fluctuation, which is caused by the dividend process. Since the stock price is the present value of an oscillating dividend stream with next period’s dividend having the highest weight, the stock price fluctuates as well.

The low expected return causes a high stock price and high realized return but, in the model specification with $w = 0.5$, the investment costs are high as well. As a result, investment growth and stock return are positively correlated, contradicting empirical facts. Figure 17 presents the correlation between investment growth and stock returns based on a simulation of the model. It is apparent, that the standard time-to-build specification generates an unrealistic correlation pattern between investment growth and stock returns. Since firms incur already half of the investment costs in the commitment period, the investment costs and stock returns are positively correlated.

The second column of Table 4 summarizes the moments of data generated by this specification. Most importantly, the mean excess return with $w = 0.5$ is lower than with $w = 0$. The reason is that with $w = 0.5$ half of the investment costs are already incurred in the period of the investment commitment. Thus, the operating leverage effect is reduced because the assets in place are levered by a smaller amount of investment costs.
Concluding, time-to-build does not necessarily imply a negative correlation between investment growth and stock returns. The timing of the investment costs are the crucial determinant of the asset pricing implications.

5.3 Time-to-Build with Adjustment Costs

As soon as firms incur some of the costs in the commitment period, it is optimal for the firm to oscillate between high and low investment decisions. To remedy this problem, I enhance the time-to-build specification with convex adjustment costs.

Q-theory based investment models assume convex adjustment costs in the investment rate, \( \frac{I_t}{K_t} \), so that firms are penalized for quick capital adjustment. This specification, however, does not solve the oscillation because the investment costs are fairly smooth.

The adjustment costs employed here penalize the firm for switching between high and low investment decisions. The adjustment costs are therefore a function of the investment decision \( \frac{X_t}{X_{t-1}} \). This adjustment cost function induces more realistic firm behavior because, in real life, firms do not shift between high and low investment as shown in the correlation graphs.

Adjustment costs reduce the 2 periods ahead capital stock and thus the new law of motion for capital is

\[
K_{t+2} = (1 - \delta)K_{t+1} + \left( 1 - S \left( \frac{X_t}{X_{t-1}} \right) \right) X_t
\]

where \( S \) is a concave function in \( \frac{X_t}{X_{t-1}} \)

\[
S(x) = \frac{\chi}{2} \left( e^{x-1} + e^{-(x-1)} - 2 \right)
\]

A similar specification has been used by Christiano, Eichenbaum, and Evans (2005).

Since it is unrealistic to assume that no investment costs are due in the commitment period, I assume \( w = 0.2 \). Further, I set the adjustment costs parameter \( \chi = 1 \).

As a result of the adjustment costs, the impulse response functions of the model are smooth; see Figure 18. The impulse responses of the investment decision and investment costs are hump-shaped because the firm would otherwise incur high adjustment costs.

Figure 19 shows the impulse response of the realized and expected stock return, as well as the risk-free rate. As before, the expected stock return and risk-free rate are low on impact of the shock, resulting in a high realized return.
In Figure 20, the correlation between investment growth and stock returns is depicted. This graph shows that the model is able to replicate three features of investment and stock market data: First, investment growth is negatively correlated with future stock returns; second, the contemporaneous correlation between investment growth and stock return is negative; and third, stock returns are positively correlated with future investment growth.

The negative correlation between investment growth and future stock returns arises because the adjustment costs cause the investment costs to be persistent. Further, since most of the investment costs are incurred next period, high investment costs are followed by even lower dividends causing a low realized return. Therefore, high investment growth is negatively correlated with future realized stock returns.

A drawback of this model is that consumption growth is too volatile, causing a high standard deviation of the risk-free rate. The third column of table 4 summarizes the moments of the model. The model further generates a considerable standard deviation of the stock return and a small excess return which is larger than in the standard time-to-build with \( w = 0.5 \).

6 Conclusion

The findings in this paper support the importance of time-to-build in order to explain stock market and investment data. A model with time-to-build rationalizes the negative correlation between investment growth and stock returns at the aggregate level, which contradicts the Q theory of investment.

Time-to-build also induces risk changes due to operating leverage. Since firms commit to invest in the future, they are levered and thus risky. A partial equilibrium model would therefore predict high expected returns at the commitment date. But in general equilibrium model, in which the pricing kernel depends on consumption, expected returns are low at the commitment. The reason is a low risk-free rate because the households expects negative consumption growth after the commitment.

Consistent with the idea of operating leverage, the excess return generated by a time-to-build model is higher the more investment costs are incurred in the future because the assets in place are levered by higher investment costs.

The risk effects of operating leverage arise because the value of the firm depends not
only on the value of its productive capital but also on the value attributed to unfinished investment projects. As a result, there is a wedge between marginal and average Q so that stock returns deviate from investment returns. Since stock returns are not tied to the marginal rate of transformation, i.e. investment return, the model generates an annual excess return of almost one percent with power utility and a coefficient of risk aversion of five.

The simple two period time-to-build specification with adjustment costs is able to explain three phenomena: First, the negative correlation of investment growth and future stock returns; second, the negative contemporaneous comovement of stock returns and investment growth; and third, the prolonged positive correlation of stock returns and future investment growth.
References


Table 4: Moments
This table shows moments of the simulated models. The first column, $w = 0$, are the moments from the pure commitment model, the second column, $w = 0.5$, from the standard time-to-build specification, and the third column from the time-to-build model enhanced with capital adjustment costs. $R^E$ denotes the stock return, $R^f$ the risk-free rate, and SD its standard deviation. All asset pricing moments are annualized. Further, $\sigma(.,.)$ denotes the standard deviation of consumption $C$, output $Y$, and investments $I$, respectively, and $corr(.,.)$ the correlation between two variables.

<table>
<thead>
<tr>
<th></th>
<th>$w = 0$</th>
<th>$w = 0.5$</th>
<th>$w = 0.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean $R^E$</td>
<td>3.242</td>
<td>2.929</td>
<td>3.163</td>
</tr>
<tr>
<td>SD $R^E$</td>
<td>7.593</td>
<td>1.601</td>
<td>5.209</td>
</tr>
<tr>
<td>Mean $R^E - R^f$</td>
<td>0.728</td>
<td>0.007</td>
<td>0.224</td>
</tr>
<tr>
<td>Mean $R^f$</td>
<td>2.514</td>
<td>2.922</td>
<td>2.940</td>
</tr>
<tr>
<td>SD $R^f$</td>
<td>7.610</td>
<td>1.601</td>
<td>8.306</td>
</tr>
<tr>
<td>$\sigma(C)/\sigma(Y)$</td>
<td>0.641</td>
<td>0.593</td>
<td>0.641</td>
</tr>
<tr>
<td>$\sigma(I)/\sigma(Y)$</td>
<td>4.639</td>
<td>2.171</td>
<td>2.091</td>
</tr>
<tr>
<td>$corr(C,Y)$</td>
<td>0.935</td>
<td>0.958</td>
<td>0.949</td>
</tr>
<tr>
<td>$corr(I,Y)$</td>
<td>0.960</td>
<td>0.979</td>
<td>0.968</td>
</tr>
</tbody>
</table>
Figure 1: Cross-correlation of gross private investment returns and stock returns at leads and lags, $\text{corr}(R^{E}_{t-1,t}, R^{I}_{t-1+k,t+k}), \alpha = 0.2, \xi = 0.55, 1955-2004$

Figure 2: Cross-correlation of nonresidential fixed investment returns and stock returns at leads and lags, $\text{corr}(R^{E}_{t-1,t}, R^{I}_{t-1+k,t+k}), \alpha = 0.1, \xi = 0.45, 1955-2004$

Figure 3: Cross-correlation of residential fixed investment returns and stock returns at leads and lags, $\text{corr}(R^{E}_{t-1,t}, R^{I}_{t-1+k,t+k}), \alpha = 0.1, \xi = 0.55, 1955-2004$
Figure 4: Annual gross private investment returns and stock returns based on $\alpha = 0.2, \xi = 0.55$

Figure 5: Annual nonresidential fixed investment returns and stock returns based on $\alpha = 0.1, \xi = 0.45$

Figure 6: Annual residential fixed investment returns and stock returns based on $\alpha = 0.1, \xi = 0.55$
Figure 7: Correlation of real gross private investment growth and real stock price changes, $corr(\Delta \log P_t, \Delta \log I_{t+k})$, 1955-2004

Figure 8: Correlation of nonresidential fixed investment growth and real stock price changes, $corr(\Delta \log P_t, \Delta \log I_{t+k})$, 1955-2004

Figure 9: Correlation of residential fixed investment growth and real stock price changes, $corr(\Delta \log P_t, \Delta \log I_{t+k})$, 1955-2004
Figure 10: Correlation of (nonresidential fixed) equipment and software investment growth and real stock price changes, \( \text{corr}(\Delta \log P_t, \Delta \log I_{t+k}) \), 1955-2004

Figure 11: Correlation of (nonresidential fixed) structures investment growth and real stock price changes, \( \text{corr}(\Delta \log P_t, \Delta \log I_{t+k}) \), 1955-2004
Figure 12: Impulse response functions of output $Y_t$, capital stock $K_t$, consumption $C_t$, dividends $D_t$, investment decision $X_t$, investment costs $I_t$ for $w = 0$
Figure 13: Impulse response function of the stock price $P_t$, stock return $R_t^E$, risk-free rate $R_t^f$, and commitment $X_t - I_t$ for $w = 0$

Figure 14: Correlation of investment growth and stock price changes, $corr(\Delta \log P_t, \Delta \log I_{t+k}), w = 0$
Figure 15: Impulse response functions of output $Y_t$, capital stock $K_t$, consumption $C_t$, dividends $D_t$, investment decision $X_t$, investment costs $I_t$ for $w = 0.5$.
Figure 16: Impulse response function of the stock price $P_t$, stock return $R_t^E$, risk-free rate $R_t^f$, and commitment $X_t - I_t$ for $w = 0.5$.

Figure 17: Correlation of investment growth and stock price changes, $corr(\Delta \log P_t, \Delta \log I_{t+k})$ for $w = 0.5$. 

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Figure 18: Impulse response functions of output $Y_t$, capital stock $K_t$, consumption $C_t$, dividends $D_t$, investment decision $X_t$, investment costs $I_t$ with adjustment costs $\chi = 1$ and $w = 0.2$. 

\[ x \times 10^{-3} \]
Figure 19: Impulse response function of the stock price $P_t$, stock return $R_t^E$, risk-free rate $R_t^f$, and commitment $X_t - I_t$ with adjustment costs $\chi = 1$ for $w = 0.2$.

Figure 20: Correlation of investment growth and stock price changes, $\text{corr}(\Delta \log P_t, \Delta \log I_{t+k})$ with adjustment costs $\chi = 1$ and $w = 0.2$.