Testing for Common Valuation in Treasury Bills Auctions

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Abstract

We develop a test for common values in treasury bill auction. The test is based on different bidding dynamics within an auction under the two competing models. Bidders who obtain information about rivals' bids in the private values model use this information only to update their prior on the distribution of the residual supplies. In the model with common value component, they also update their prior of the valuation itself. We use this different updating effect to construct our test and we apply it to data from Canadian treasury bill market.

Keywords: multiunit auctions, treasury auctions, structural estimation, nonparametric identification and estimation, test for common value

JEL Classification: D44

1 Introduction

Is the private or common valuation component more important in treasury bill auctions? Can we use data to provide an answer? These are two major questions that we attempt to address in this paper. We exploit variation in observed bids by several bidders before the deadline for bid submission to develop an econometric test for presence and importance of the common valuation.

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component in treasury bill markets. The basic idea underlying our test is that as new information about bidding behavior of her rivals becomes available a bidder should change her bid in a different way when her valuation is private and when the valuation has an important common component.

Most governments sell their short term debt via auctions. The economic theory does not have a definitive answer as to what the optimal selling mechanism would be, and it is perhaps not surprising that the actual auction mechanisms differ substantially across countries. In previous empirical work discussed below, researchers tried to compare discriminatory and uniform price auction formats, yet most of the structural work discussed below restricts attention to the private values paradigm. While many economists agree that for the short term debt the private valuation component is probably more important, because most investors hold these papers in their portfolios until maturity so that there is almost no resale, there is still some controversy in modelling auctions of government debt using private valuation models. In particular, for example due to different expectations of some global risk, say of interest rate fluctuations, there might still be an important common valuation component involved. It therefore remains a matter of taste as to which model to apply. So far, there is relatively thin literature on testing for common value component, moreover, it deals solely with a setting where a single unit of a good is being auctioned. The auctions of government debt clearly do not fall into this category. In particular, in these multiunit auctions bidders submit whole demand curves as their bids rather than just a simple real-valued bid signalling their willingness to pay. It turns out that using the two-dimensionality of bidders demands will help us develop our test. The proposed test is quite different from those employed previously in the literature and as such is less susceptible to unobserved heterogeneity across auctions. In particular, we will make use of dynamics in bidding behavior within a particular auction, where the common and private value paradigm would predict different bidding patterns.

As an example, consider a situation in which bidder $i$ is about to submit her bid (demand) function $y_i$, but before submitting $y_i$ she observes a bid actually submitted by bidder $j$. With private valuations bidder $i$ obtains better information about the location and shape of residual supply she will be facing in the upcoming auction. Using this additional information, she revises her initial bid $y_i$ and submits an alternative bid $y'_i$. In an auction with a common value component,
on top of the additional information about the location and shape of the residual supply curve, she also obtains new important information about the common value. Therefore she submits a new bid $y''_i$ taking into account both of these two pieces of new information. In general, the way she will revise her bid $y_i$ will differ under the two scenarios and this distinction motivates our test.

The question of finding a way to distinguish between the common and private valuation paradigms is not new to economics literature. The theory of equilibrium bidding in different auction environment which was spelled out in the seminal paper of Milgrom and Weber (1982) motivated empirical researchers to develop formal techniques that would help them decide which theoretical model would seem more appropriate in a given setting. In a single unit setting, in which a single object is auctioned, researchers proposed a reduced form testing approach based on examining how bids vary with the number of participants (e.g., Gilley and Karels (1981)). For second-price sealed-bid and English auctions, Paarsch (1991) and Bajari and Hortacsu (2003) suggest testing for CV using standard regression techniques. Pinkse and Tan (2002) establish, however, that such a reduced form test cannot distinguish unambiguously a CV from PV model in first price auctions. Therefore, structural modelling seems necessary in order to achieve the goal of distinguishing CV and PV. Paarsch’s (1992) seminal paper was indeed motivated by this question. His method, however, relies on parametric assumptions about the distribution of bidder’s private information, and hence it is hard to disentangle the influence of the parametric assumptions on the actual outcomes of the testing procedure. Our approach, instead, will be non-parametric. Haile, Hong and Shum (2003) (henceforth HHS) is the most closely related paper. They propose a non-parametric test for common value making use of variation in the number of bidders across auctions. They use non-parametric techniques developed in empirical auctions literature (e.g., Laffont and Vuong (1995), and Guerre, Perrigne and Vuong (2002)) to estimate the distribution of valuations given the observed bids. In particular, the theory predicts a certain ordering between the distribution of bids under common valuation paradigm as the number of bidders varies, while the expected value of the object conditional on winning should not vary with the number of participants under PV. The problem they have to deal with though is the unobserved characteristics of the auctions, which in turn could influence the number of participating bidders. Our testing approach will not suffer
from this potential difficulty as it is based on dynamics of submitted bids within an auction.

As mentioned above our analysis involves a multi-unit environment. In particular, we look at auctions of divisible good, i.e., auctions of very large number of homogeneous units of a good, so that the quantity can be treated as continuous choice variable. The theory of such auctions has been laid out in Wilson (1979) and these auctions have generated a lot of interest recently, as they seem to be a fitting model for auctions of securities, electricity or emission permits. Empirical literature on divisible good auctions can be classified into two groups.

The first group of papers is interested in modelling behavior in electricity auctions (e.g., Wolak (2003, 2005), Hortaçsu and Puller (2005)). The private value framework seems like an appropriate setting for these auctions, and hence we will not be talking about these in more detail.

The second group consists of papers that aim to compare the revenue and efficiency of alternative auction mechanisms so that to provide a recommendation for the auctioneer. These papers usually use data from auctions of government treasury bills (e.g., Armantier and Sbai (2002), Fevrier, Preget and Visser (2002), Hortaçsu (2002), Kastl (2006a)). The only paper from this list that employs a common value framework is Fevrier, Preget and Visser (2002). They, however, look at the other extreme - pure common value environment, and they are able to make progress only by assuming a particular functional form for the distribution of private information because it allows for closed form solutions of equilibrium strategies. The other problem of their approach is that the implied equilibrium strategies are continuous downward sloping demand schedules, which is not what is observed in practice. Bidders are usually required to characterize their demands only by using a finite (and low) number of price-quantity pairs, which specify how much quantity they demand at a given price. Kastl (2006a) points out that ignoring this feature of bidding can have important consequences on the estimated valuations. All other papers in the list above look at a private value setting, and each provides some intuition as to why the private setting seems to be fitting. In our view a formal test for validity of this assumption conducted in a similar environment to provide supportive evidence for private values would be quite handy. On the other hand, should this test point towards an important common valuation component, then we should pay more attention to defending the private value paradigm in any given setting.
The remainder of the paper is organized as follows. In Section 2 we lay out the model of a discriminatory auction of a perfectly divisible unit good and characterize the necessary conditions for equilibrium bidding under private and common values. We use these necessary conditions to conduct structural estimation of bidders’ marginal valuations. We describe the actual test for common values in Section 3. To evaluate the performance of the proposed test, we conduct a Monte Carlo simulation in Section 4. In Sections 5 and 6 we describe our dataset and present the results. Finally, Section 7 concludes.

2 The Model and Test Description

The basic model underlying our analysis is based on share auction model of Wilson (1979) with private information, in which both quantity and price are assumed to be continuous. There are \( N \) bidders, who are bidding for a share of a perfectly divisible good. Each bidder receives a private (possibly multidimensional) signal, \( s_i \), which is the only private information about the underlying value of the auctioned goods. The joint distribution of the signals will be denoted by \( F(s) \).

Assumption 1 Bidder \( i \)'s signal \( s_i \) is drawn from a common support \( [0, 1]^M \) according to an atomless marginal d.f. \( F_i(s_i) \) with strictly positive density \( f_i(s_i) \).

Winning \( q \) units of the security is valued according to a marginal valuation function \( v_i(q, s_i, s_{-i}) \). In the special case of independent private values (IPV), the \( s_i \)'s are distributed independently across bidders, and bidders’ valuations do not depend on private information of other bidders, i.e., the valuation has the form \( v_i(q, s_i) \). At the estimation stage we will not impose full symmetry, since we will allow for different groups, within which the signal is distributed identically across bidders. We will impose the following assumptions on the marginal valuation function \( v(\cdot, \cdot, \cdot) \):

Assumption 2 \( v_i(q, s_i, s_{-i}) \) is measurable and bounded, strictly increasing in (each component of) \( s_i \) \( \forall (q, s_{-i}) \) and weakly decreasing in \( q \) \( \forall (s_i, s_{-i}) \).
Notice that we do not require any differentiability or continuity assumptions on \( v \). We will denote by \( V(q, s_i, s_{-i}) \) the gross utility: \( V(q, s_i, s_{-i}) = \int_0^q v_i(u, s_i, s_{-i}) \, du \).

Bidders’ pure strategies are mappings from private signals to bid functions: \( \sigma_i : S_i \rightarrow \mathcal{Y} \), where the set \( \mathcal{Y} \) includes all possible functions \( y : \mathbb{R}^+ \rightarrow [0, 1] \). A bid function for type \( s_i \) can thus be summarized by a function, \( y_i(\cdot|s_i) \), which specifies for each price \( p \), how big a share \( y_i(p|s_i) \) of the securities offered in the auction (type \( s_i \) of) bidder \( i \) demands. \( Q \) will denote the amount of T-bills for sale, i.e., the good to be divided between the bidders. \( Q \) might itself be a random variable if it is not announced by the auctioneer ex ante, or if the auctioneer has the right to augment or restrict the supply after he collects the bids. We assume that the distribution of \( Q \) is common knowledge among the bidders. Furthermore, the number of bidders participating in an auction, denoted by \( N \), is also commonly known. The natural solution concept to apply in this setting is Bayesian Nash Equilibrium. The expected utility of type \( s_i \) of bidder \( i \) who employs a strategy \( y_i(\cdot|s_i) \) in a discriminatory auction given that other bidders are using \( \{y_j(\cdot|\cdot)\}_{j \neq i} \) can be written as:

\[
EU_i(s_i) = E_{Q, s_{-i}|s_i} \left[ \int_0^{q_c^i(Q, s_i, y(\cdot|s_i))} v_i(u, s_i) \, du - \sum_{k=1}^K 1(q_k^i(Q, s_i, y(\cdot|s_i)) > q_k)(q_k - q_{k-1})b_k - \sum_{k=1}^K 1(q_k \geq q_c^i(Q, s_i, y(\cdot|s_i)) > q_{k-1})(q_t^i(Q, s_i, y(\cdot|s_i)) - q_{k-1})b_k \right]
\]

where \( q_c^i(Q, s_i, y(\cdot|s_i)) \) is the (market clearing) quantity bidder \( i \) obtains if the state (bidders’ private information and the supply quantity) is \( (s, Q) \) and bidders bid according to strategies specified in the vector \( y(\cdot|s) = [y_1(\cdot|s_1), ..., y_N(\cdot|s_N)] \), and similarly \( p_c^i(Q, s_i, y(\cdot|s)) \) is the market clearing price associated with state \( (s, Q) \). A Bayesian Nash Equilibrium in this setting is thus a collection of functions such that almost every type \( s_i \) of bidder \( i \) is choosing his bid function so as to maximize his expected utility: \( y_i(\cdot|s_i) \in \arg \max EU_i(s_i) \) for a.e. \( s_i \) and all bidders \( i \).

2.1 Equilibrium strategy of a bidder in a private value auction

In this subsection we describe equilibrium behavior of a bidder in a private value setting. The discriminatory auction version of Wilson’s model with private values has been previously studied in Hortaçsu (2001). Kastl (2006b) extends this model to empirically relevant setting, in which
bidders are restricted to use step functions with limited number of steps as their bidding strategies. He proves the following result summarizing necessary conditions for an equilibrium:

**Proposition 1** *(Kastl, 2006b)* Suppose values are private, rationing is pro-rata on-the-margin, and bidders can use at most $K$ steps. Then in any Bayesian Nash Equilibrium of a Discriminatory Auction, for almost all $s_i$, every step $k$ in the equilibrium bid function $y_i(\cdot|s_i)$ has to satisfy

$$v(q_k, s_i) = b_k + \frac{\Pr(b_{k+1} \geq p^c)}{\Pr(b_k > p^c > b_{k+1})} (b_k - b_{k+1})$$

Using these necessary conditions we can obtain point estimates of marginal valuations at submitted quantity-steps nonparametrically as described in Hortaçsu (2001) and Kastl (2006a). The resampling method that we employ in these papers is based on simulating different possible states of the world (realizations of vector of private information) using the data available to the econometrician and obtaining the corresponding market clearing prices. It works as follows:

Suppose there is $N_d$ potential dealers and $N_c$ potential customers and both types of players are (ex ante) symmetric within their respective group. Fix a dealer’s bid (or a customer). From the observed data, draw (with replacement) $N_d - 1$ actual bid functions submitted by dealers, and similarly draw $N_c$ bid functions submitted by customers. This simulates one possible state of the world - a possible vector of private information. Construct the aggregate bid function and intersect it with the supply function to obtain the market clearing price. Repeat this procedure large number of times in order to obtain an estimate of the distribution of the market clearing price conditional on the fixed bid. Using this simulated distribution of market clearing price, we can obtain our estimates of valuation at each step submitted by the bidder whose bid we fixed using (1).

One caveat that we need to be careful about is that a dealer can revise her bid after observing demand from her customer. Of course, such a dealer is no longer symmetric to her counterpart who does not posses this information. Therefore, when resampling, we do not want to pool all dealer bids together and draw from such a pool. Since customers actually participate in every auction, to simulate the states of the world correctly, we would have to perform conditional drawing. This works as follows:
Start drawing $N_c$ customer bids. Conditional on the bid drawn, draw a dealer’s bid. If a zero customer bid is drawn, draw from the pool of dealers’ bids, which have been submitted without observing any bid by the customers. If a non-zero customer bid is drawn, draw from the pool of dealers’ bids, which have been submitted having observed the same customer bid. After drawing $N_c$ customer bids, continue drawing from the pool of bids submitted by uninformed dealers until $N_d$ dealer bids are drawn. Obtain the market clearing price and repeat.

Performing such a conditional drawing procedure does, unfortunately, greatly reduce the number of states that can be simulated. As a robustness check, we can perform an uncoditional simulation, where among the dealer bids, we first flip a coin whether this dealer has hypothetically seen a customer’s bid or not, where the coin is biased such that it reflects the actual probability of a dealer observing a customer’s bid. If the coin determines a bid has been seen, then we draw an "updated" bid, otherwise we draw from an original dealer bid. This is performed for each of $N_d$ potential dealer draws, i.e., independently of the customer bids actually drawn in a given simulation round.

What would happen as additional information about a bid submitted by a rival becomes available to a bidder? In a private value setting, this bidder would simply update his belief about the distribution of the residual supply he will be facing in the auction. The following proposition states formally that using the conditional resampling procedure outlined above replicates a bidder’s updating process.

**Proposition 2** Under private values the conditional resampling procedure where the known rival’s bid is subtracted from the supply at each resampling draw and 1 less bids are drawn from the pool of potential bids leads to a consistent estimate of marginal valuation of a bidder with information about a rival’s bid.

Applying the conditional resampling procedure therefore results in two sets of marginal valuation estimates - before and after the information about rival’s bid arrives, and our test will be based on comparing the two sets of marginal valuation estimates. One caveat involved in constructing this test is that the bids before and after the information about rival’s bid arrives are not necessarily
submitted for the same quantities, and hence we will face an inference problem of how to compare the two sets of estimates. We will discuss these issues and the solutions in the section of the paper dealing with the test specification.

2.2 Equilibrium strategy of a bidder in an auction with affiliated values

If the valuation of a bidder has a common value component, then we will not be able to replicate the updating process of this bidder as new information becomes available to him. While the updating part due to better information about the location and shape of the residual demand is still the same as in the private value setting, there is a second updating component due to the additional information about the signal of a rival and hence about the common value. In particular, the necessary condition for optimality at $k^{th}$ step in an affiliated value environment is:

$$
\Pr (b_k > p^c > b_{k+1}) \left[ E \left[ v (q_k, s_i, s_{-i}) \mid \{b_k, q_k\}_{k=1}^K \right] - b_k \right] = \\
= \Pr (b_{k+1} \geq p^c) (b_k - b_{k+1}) + \frac{\partial E (p; b_k \geq p^c \geq b_{k+1})}{\partial q_k} \int_0^{q_k} \frac{\partial E \left[ v (u, s_i, s_{-i}) \mid \{b_k, q_k\}_{k=1}^K \right]}{\partial p} du
$$

In other words, we have the familiar trade-off in a discriminatory auction that occurs even with private values: marginally shading the quantity demanded at $k^{th}$ step results on the one hand in a loss of surplus of $E \left[ v (q_k, s_i, s_{-i}) \mid \{b_k, q_k\}_{k=1}^K \right] - b_k$ in the states that exactly that quantity would be won, but on the other hand it results in a saving of $b_k - b_{k+1}$ whenever the market clearing price is lower than the bid for the next higher quantity step. But now, because of the presence of the affiliation of values, there is an additional effect: marginally shading the quantity at $k^{th}$ step can lead to a different slope of expected market clearing price in the region where $k^{th}$ quantity demand affects the market clearing price or allocation and thus it can affect the way inference is drawn from the market clearing price realization on the unknown valuation (through updated information about rival’s signals).

Since we do not know enough about $E \left[ v (q_k, s_i, s_{-i}) \mid \{b_k, q_k\}_{k=1}^K \right]$, we cannot identify $v (q_k, s_i, s_{-i})$ non-parametrically. Therefore the main purpose of this paper is to construct a test that would enable us to empirically test whether or not the data are consistent with private values just using our
identification results from private value setting.

3 Test Specification

The fact that the dealers (large bidders) submit bids on behalf of customers (smaller clients) and that these bids are visible to the econometrician provides a unique environment for testing for the presence of a common component in bidders’ valuations which are not observed by the econometrician. In particular, dealers sometimes submit their own bids, but after fulfilling a request of one or more of their customers to submit a bid on their behalf, they decide to adjust their previously submitted bid. Since we observe the bid both before and after the additional information was made available to the dealer, we will now argue formally that we can potentially distinguish a setting in which common valuation component plays a key role from a private one. In a pure private valuation setting any such bid adjustment should be driven solely by more information about the residual supply that this bidder will be facing in the actual auction. In a setting with a common valuation component, the adjustment reflects both more information about the residual supply AND more information about the common valuation component, and hence these two adjustment results should be different. In Hortacşzu (2002) and Kastl (2006a) we proposed nonparametric methods based on simulating rivals’ strategies for estimation of marginal valuations in private value divisible good auctions. We will utilize these methods to estimate the marginal valuation schedules implied by the initial bid, and by the updated bid, taking into account the new information about the residual supply. In other words, as suggested in Proposition 2 we are able to mimic the bid updating process under private values hypothesis, but we are not be able to mimic it under common values. Therefore, under the null hypothesis of private valuation setting the estimates before and after the additional information should coincide. Should we find that the two marginal valuation schedules are significantly different, we would have to reject the null and conclude that the common valuation component plays an important role in these auctions.

The test for common values in a single unit setting proposed in Haile, Hong and Shum (2003) relies crucially on the ability of the auctioneer to observe repetitions of the same experiment over time, where the number of bidders varies exogenously. The problem of some auction characteristics
that are unobserved by the econometrician, but observed by (potential) bidders, would severely hamper their test. In our data, as we observe exact time of each bid submission, we can distinguish a change in bid due to more information coming from the bids by smaller bidders from a change in bid due to some new publicly available information. In the latter case, conditional on some small time window, all adjustments by large bidders should be positively correlated, whereas in the former case they should be independent. Therefore if we subject to our test only those changing bids that are not accompanied by similar changes in rival’s bids, we can be more confident that no commonly observed (but unobserved by us) piece of information is biasing our test as we do not need to compare estimates (such as valuation distributions) across auctions.

One important caveat of our approach is that since the bids in multiunit auctions are two-dimensional, and since bidders usually characterize their demand functions using only few points, bids submitted before and after the additional information becomes available can be quite different. But because there is an estimation error in the estimates of marginal valuations, we can write the estimated marginal valuation function of bidder $i$ as:

$$v_{ik} = f_k(q_{ik}) + \varepsilon_{ik} \quad \text{for} \quad k = B, W$$

where $B$ and $W$ stands for “before information” and “with information” respectively and $\varepsilon_{ik}$ is the estimation error in marginal valuation estimates, i.e., $v_{ik}(q_{ik}, \bar{s}) = \hat{v}_{ik}(q_{ik}, \bar{s}) + \varepsilon_{ik}$ with $\hat{v}$ being the estimates of marginal valuation from our resampling procedure. Since our estimate of marginal value at quantity $q_i$ is consistent, $E(\varepsilon|q) = 0$, and hence the level curve of the marginal valuation function at $\bar{s}$ $f_k(q_i)$ would be nonparametrically identified whenever the number of observed bids at different quantities for this particular signal level would go to infinity. It is reasonable to believe that this assumption which is necessary for consistency might be violated in practice, and hence in the subsequent section we discuss two alternative tests that can be performed. The first is based on testing for monotonicity of the estimated marginal valuation function and the second is based on comparing the two sets of estimates of marginal valuations, $k = B, W$. 
3.1 Test for Equality of Nonparametric Regressions

Under the null hypothesis of private values, $f_{BI} = f_{WI}$ and hence we can simply test for equality of two nonparametric regressions. Few of such tests have been proposed in the statistics literature on treatment evaluations (e.g., Koul and Schick, 1997).

Consider the statistic

$$T = \sqrt{\frac{n_B n_W}{n_B + n_W}} \frac{1}{n_B n_W} \sum_{i=1}^{n_B} \sum_{j=1}^{n_W} \frac{1}{2} \left( \eta(q_{B,i}) + \eta(q_{W,j}) \right) \rho(v_{B,i} - v_{W,j}) w_a(q_{B,i} - q_{W,j})$$

where $a$ is a small positive number depending on the sample sizes. $H_0$ is rejected for large values of $T$. Koul and Schick call this test a covariate-matched test.

The test statistic considered above assumes that for any given level curve of the marginal valuation function $v(q, \bar{s})$ at an unobserved signal $\bar{s}$, the set of quantities at which the value is estimated grows asymptotically, so that $v(q, \bar{s})$ can be identified nonparametrically. The test then rejects $H_0$ if the two estimated regression curves are sufficiently different. In practice, however, the number of steps in the observed bids is very low and there is no compelling reason to believe that it would vary much as the number of observed auctions increases (for a given unobserved signal realization $\bar{s}$). One possibility to obtain the asymptotic behavior consistent with the construction above is to assume private cost $c$ per bidpoint as in Kastl (2006a), which is drawn independently of $s$. As $c \downarrow 0$, bidders would submit bids with more and more steps (a continuous function in the limit of zero cost) for any $s$ and thus $v(q, \bar{s})$ would again be nonparametrically identified.

3.2 Non-parametric Test for Monotonicity

An alternative, and possibly more natural way to think about the asymptotics is to consider the number of steps and thus the number of quantities at which the marginal value can be estimated as fixed, and let just the number of auctions increase, which is necessary for these estimates to be consistent. Then we could test for monotonicity of the estimated marginal values at quantities submitted before and after the additional information. Order the quantities at which a bid was submitted by bidder $i$ either before the additional information was revealed or after it was revealed...
in an increasing order: $q_1 < q_2 < \ldots < q_K$ and let $\hat{v}_i, \ldots, \hat{v}_K$ denote the associated estimated marginal values. Consider the following test statistic:

$$S_i = \max_j \{\hat{v}_{ij+1} - \hat{v}_{ij}, 0\}$$

Clearly, when monotonicity is satisfied at all quantities, then $\hat{v}_{ij} \geq \hat{v}_{ij+1}$ and hence $S_i = 0$. On the other hand we could get violations of monotonicity due to the sampling error in a finite sample and hence $S_i > 0$ could be consistent with the null hypothesis. The major advantage of this approach is that it does not restrict the class of possible marginal valuation functions in any other way than that it be non-increasing in quantity.

**Critical Values**

We obtain the critical value for this test statistic using the subsampling approach developed in Politis, Romano and Wolf (1999). Let $b_N$ be a sequence of positive integers satisfying $b_N \to \infty$ such that $b_N / N \to 0$. Let $B_N$ be the set of all possible subsets of size $b_N$ of a dataset of size $N$. Let $	ilde{S}_{b_N,M}$ denote the statistic $S_i$ evaluated at a subset $M$ of the data of size $b_N$. The estimated $1 - \alpha$ quantile is then given by:

$$\tilde{c}_{1-\alpha} = \inf \left\{ x : \frac{1}{|B_N|} \sum_{M \in B_N} 1 \left\{ \tilde{S}_{b_N,M} \leq x \right\} \geq 1 - \alpha \right\}$$

**Discussion**

A short discussion of the testing approach is now necessary. Since our testing for monotonicity via the test statistic $S$ falls into the framework of partial identification, there could be situations in which the true model in fact has a common value component and thus the level curves of (expected) marginal valuation are different before and after the information is revealed, but our monotonicity test fails to reject the null hypothesis (possibly even asymptotically). While because of this reason rejecting the null hypothesis in itself is clearly not necessarily direct evidence for the private values model, it suggests that using the private value model to obtain point estimates of marginal valuation and then using the worst case bounds approach as proposed in Hortaçsu (2002)
and Kastl (2006a) would be a good enough approximation for the purposes of any counterfactual exercise. Therefore, while we acknowledge that the test may not be able to reject private values with probability one (not even asymptotically) if the truth is common values, we would like to stress that using the worst case bounds approach on point estimates from the private values model would be a sufficient approximation in this case to include the true (expected) marginal valuation function for each bidder in the identified set.

4 Monte Carlo

Our ability to test the performance of the above described testing procedure is limited by the fact that in most general cases we do not have closed form solutions for equilibrium strategies, either in the private or in the affiliated values settings. Hortaçsu (2002a) constructs an example of a discriminatory auction with private values, two bidders and exponential distribution of signals that has a closed form solution. We will use an extension of this example which involves also supply uncertainty to conduct a Monte Carlo experiment for our tests.

Scenario 1: Private Values Setting

Consider 2 bidders with true demands \(D(p, s_i) = \frac{1}{\beta} [\alpha - p + \gamma s_i]\) and the corresponding valuations \(v(q, s_i) = \alpha + \gamma s_i - \beta q\) where \(\alpha, \beta, \gamma > 0\). The signals are independently and exponentially distributed: \(F(s_i) = \exp[\theta s_i]\) with \(\theta > 0\) and \(s_i < 0\). In this setting there exists an equilibrium in linear strategies of the form: \(y(p, s_i) = \frac{1}{\beta} (\alpha + \gamma s_i - p - \frac{c}{\gamma})\). Guess symmetric strategies \(y(p, s_j) = a + bp + cs_j\)

\[
H(p, y) = \Pr(p^c < p|y) = \Pr(Q > y + a + bp + cs_j)
= \Pr\left(s_j < \frac{Q - y - a - bp}{c}\right)
= \exp\left[\theta \frac{Q - y - a - bp}{c}\right]
\]

Hence \(\frac{H}{H_p} = -\frac{c}{\beta} \frac{\theta}{\gamma}\). Using the optimality equation: \(v(y(p, s_i), s_i) = p + \frac{H}{H_p}\), a linear guess for the strategy \(y(p, s_i)\) and equating coefficients we obtain: \(y(p, s_i) = \frac{1}{\beta} (\alpha + \gamma s_i - p - \frac{c}{\gamma})\). Notice
that this equilibrium exhibits constant shading of \( \frac{2}{\beta} \) for every unit. Suppose furthermore that the auctioneer does not commit to a supply \( Q = 1 \) before the auction, but the supply is rather a random variable from perspective of the bidders which is distributed normally with mean 1 and variance \( \sigma^2 \).

The equilibrium strategy: Guessing the linear strategies \( y(p, s_i) = a + bp + cs_i \), the distribution of the market clearing price becomes:

\[
H(p, y) = \Pr(p^c < p|y) = \Pr(Q > y + a + bp + cs_j)
\]

\[
= \Pr\left(-\frac{Q}{c} + s_j < \frac{-y - a - bp}{c}\right)
\]

\[
= \Pr\left(u + s_j < \frac{-y - a - bp}{c} + \frac{1}{c}\right)
\]

where \( u = -\frac{Q}{c} + \frac{1}{c} \). The probability density of a sum of a normal random variable with \( \mu = 0 \) and variance \( \varphi^2 \) and an (negative) exponential r.v. with parameter \( \theta \) is (approximately) exponential (for \( x \ll -\varphi \)) with a cdf:

\[
F(x) = \frac{e^{\theta(x + \frac{\varphi^2}{4})}}{\varphi\sqrt{2\pi}}
\]

Since \( Q \sim N(1, \sigma^2) \), \( u \sim (0, \varphi) \), where \( \varphi = \frac{\sigma^2}{c} \) and thus we obtain:

\[
H(p, y) = \frac{e^{\theta(1 - y - a - bp + \frac{\varphi^2}{4})}}{\varphi\sqrt{2\pi}}
\]

Using (1), we get:

\[
v(y(p, s_i), s_i) = p - \frac{c}{\theta b}
\]

Hence equating coefficients we get exactly the same equilibrium bidding strategies as with no supply uncertainty. So the equilibrium demand function becomes: \( y(p, s_i) = \frac{1}{\beta} (\alpha + \gamma s_i - p - \frac{2}{\theta}) \). Since the true demand is: \( D(p, s_i) = \frac{1}{\beta} (\alpha - p + \gamma s_i) \) bidders are shading their demand by a constant \( \frac{2}{\theta} \).

To incorporate the feature of updating the bids, suppose that after submitting the bid described above, bidder 1 observes the realization of bidder 2’s signal \( s_2 \). In this case, the only remaining
uncertainty in his bid is the supply uncertainty. Since the supply is normally distributed, his
optimal bid function is defined implicitly:

\[ v(q, s_i) = p + \frac{1 - \Phi \left( \frac{q + y_2(p, s_2) - 1}{\sigma} \right)}{-\phi \left( \frac{q + y_2(p, s_2) - 1}{\sigma} \right) y'_2(p, s_2)} \]

\[ q = \frac{1}{\beta} \left( \alpha + \gamma s_1 - p - \frac{1 - \Phi \left( \frac{q + y_2(p, s_2) - 1}{\sigma} \right)}{-\phi \left( \frac{q + y_2(p, s_2) - 1}{\sigma} \right) y'_2(p, s_2)} \right) \]  

where \( \Phi(\cdot) \) is a standard normal CDF, \( \phi(\cdot) \) the corresponding PDF and \( \sigma \) is the standard deviation of the random supply. For the purposes of our Monte Carlo experiment, let us now focus on a partial
equilibrium analysis, in which bidder 2’s bid function remains linear (as derived above) even in the
case that bidder 1 can sometimes observe 2’s signal. In this case, \( y'_2(p, s_2) = -\frac{1}{\beta} \). We will generate
data from the model described above. The optimal bid function of player 1 has to satisfy:

\[ q = \frac{1}{\beta} (\alpha + \gamma s_1 - p) - \frac{1 - \Phi \left( \frac{q + \frac{1}{\beta} (\alpha + \gamma s_2 - p - \frac{\gamma}{\beta} - 1)}{\sigma} \right)}{-\phi \left( \frac{q + \frac{1}{\beta} (\alpha + \gamma s_2 - p - \frac{\gamma}{\beta} - 1)}{\sigma} \right) y'_2(p, s_2)} \]

In other words, his bid for \( q \) solves:

\[ p = \alpha + \gamma s_1 - \beta q - \beta \frac{1 - \Phi \left( \frac{q + \frac{1}{\beta} (\alpha + \gamma s_2 - p - \frac{\gamma}{\beta} - 1)}{\sigma} \right)}{-\phi \left( \frac{q + \frac{1}{\beta} (\alpha + \gamma s_2 - p - \frac{\gamma}{\beta} - 1)}{\sigma} \right) y'_2(p, s_2)} \]

Since our goal is to estimate the valuation of bidder 1, it does not really matter that the strategy
of bidder 2 we use in generating the data might not be truly a best response. We could think about
the game as bidder 1 being the only strategic player and bidder 2 being an automaton using the
linear strategies described earlier.

We will then try to infer the marginal valuation of bidder 1 from the data generated as described
above. Using the (observed) demands before and after bidder 1 observes bidder 2’s bid, we will
aim to compare the two sets of estimates of marginal valuations.

Figure 1 depicts the results for a particular bidder with signal draw \( s_i = -1.6 \) and for parameter
values $a = 10, b = 2, c = 2$, 20 signal draws for each bidder from exponential distribution with $\theta = 1$, random supply is distributed as $N(1, 0.04)$, each bid is discretized to 100 steps and there is 5000 resampling draws for the estimation.

![Figure 1: Monte Carlo with Private Values](image)

With finely specified bids (100 steps) the two estimates of marginal valuation curve for the given signal are very similar for all bidders that have drawn signals for which they find it worthwhile to submit a bid.

**Scenario 2: Affiliated Values Setting**

Consider 2 bidders with true demands $D(p, s_i, s_j) = \frac{1}{\beta} [\alpha + \gamma s_i + \delta s_j - p]$ and the corresponding valuations $v(q, s_i, s_j) = \alpha + \gamma s_i + \delta s_j - \beta q$ where $\alpha, \beta, \gamma, \delta > 0$. The signals are independently and exponentially distributed: $F(s_i) = 1 - \exp \left[ -\frac{s_i}{\theta} \right]$. Suppose again that the supply level is normal $(1, \sigma^2)$. If bidder 1 does not observe 2’s bid, he has to make inference about 2’s signal from the market clearing price. Hence the equilibrium strategy is implicitly characterized by:
$E[v \left( y(p|s_1), s_i, s_{-i} \right) | p = p^r] = y + \frac{H(p, y(p|s_1))}{H(p, y(p|s_1))} + H_y(p, y(p|s_1)) \int_0^{y(p|s_1)} \frac{\partial E[v(u, s_i, s_{-i}) | p = p^r]}{\partial p} \, du$

Whenever bidder 1 is able to observe the bid submitted by bidder 2, he will be able to infer 2’s signal and thus his own valuation schedule perfectly, and thus $\frac{\partial E[v(q, s_1, s_{-1}) | p = p^r]}{\partial p} \equiv 0$. Therefore he will submit a bid schedule that satisfies

$$v(q, s_1, s_2) = y + \frac{1 - \Phi(q + y_2(p, s_2))}{-\phi(q + y_2(p, s_2)) y_2'(p, s_2)}$$  \hspace{1cm} (3)

Again suppose that bidder 2 is an automaton which always submits a linear demand function $y(p, s_2) = \frac{1}{\beta} \left( \alpha - \frac{1}{\beta} - p + \gamma s_2 \right)$. Then a best response by bidder 1 which satisfies (3) is implicitly characterized by:

$$y(p, s_1, s_2) = \frac{1}{\beta} \left[ \alpha + \gamma s_1 + \delta s_2 - p \right] - \frac{1 - \Phi(q + y_2(p, s_2))}{\phi(q + y_2(p, s_2))}$$  \hspace{1cm} (4)

Notice that this construction implies different dynamics of bidder 1’s bid once he observes 2’s bid under PV and AV as under PV the updated bid satisfies (2) while under AV it satisfies (4).

5 Data and Background

Treasury bills and other Bank of Canada securities are issued in the primary market through sealed-bid discriminatory auctions. Bids are submitted electronically and can be revised at any point before the submission deadline. There are two major groups of potential bidders: primary dealers (PDs) and customers.

The major distinction between these two groups of potential bidders is that customers cannot bid on their own account in the auction, but have to route their bids through one of the dealers. The PDs are required to identify bids on behalf of the customers in the electronic bidding system. On average, there is about 2.5 primary dealers for one customer in an auction. In contrast, in
all auctions of Bank of Canada’s securities Hortaçsu and Sareen (2006) report that on average one dealer services 0.8 customers and that on average 8.6 customers participate. The auctions of treasury bills generate therefore less interest among the customers relative to the auctions of bonds and other securities.

In order to encourage liquidity provision and activity in the primary market, the rules of the auctions specify that a maximum amount a dealer can bid either for himself or his customers is based on his past primary market winning share and secondary market trading share, net of his current holdings of the auctioned security. However, there is also an institutionally set maximum of 25% of the issue amount for a bidder (dealer or customer individually) and 40% for a dealer (sum of all awarded bids submitted by the dealer including those on behalf of customers).

As usual in most government securities auctions, bids can be submitted both as competitive tenders and as noncompetitive tenders. Each participant is allowed to submit a single noncompetitive tender. A noncompetitive tender specifies a quantity that the bidder wishes to purchase at the price at which the auction clears. In our data, there are on average 3.6 noncompetitive tenders in an auction for on average 4.4% of the preannounced amount for sale.

Since there are no restrictions on how many times a primary dealer (or a customer) can revise her bid before the bid submission deadline, the information flow caused by customers’ routing their bids through dealers causes the dealers to update their bids exactly in the spirit of the test that we proposed in the previous section of the paper.


6 Results

7 Conclusion

[To be written]
Table 1: Data Summary

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Table 2: Summary of Noncompetitive Bids

|                  | 116 |
|-------------------|
| Auctions with NC bid | Mean | St.Dev. | Min | Max |
| Number of NC bids  | 3.6  | 1.1     | 2   | 7   |
| NC bid             | 0.044 | 0.08   | 0.00003 | 0.27 |

References


