Skill Premium, Schooling Decisions, Skill-Biased Technological and Demographic Change: A Macroeconomic Analysis*

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Abstract: This paper studies the driving forces behind the dynamics of the skill premium and the college enrollment rate in the postwar U.S. economy. I develop an overlapping generations general equilibrium model with endogenous discrete schooling choice. The production technology features capital-skill complementarity as in Krusell et al. (2000). Within this framework, I quantitatively examine the effects of two exogenous forces, investment-specific technological change (ISTC) and the demographic change known as “the baby boom and the baby bust” on the skill premium and the enrollment rate. I find that in terms of the skill premium, demographic change dwarfs ISTC before the late 1960s and contributes to the decline of the skill premium in the 1970s. However, after the late 1970s, ISTC takes over to drive the dramatic increase in the skill premium. ISTC also explains about 30% of the increases in the enrollment rate for the period 1951-2000, while demographic change does not have a significant effect on the enrollment rate.

JEL classification: E25 (Aggregate factor income distribution); I21 (Analysis of education); J24 (Human capital); J31 (Wage differentials by skill); O33 (Technological change).

Key Words: Skill Premium; Schooling Choice; Investment-Specific Technological Change; Capital-Skill Complementarity.

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1 Introduction

The skill premium, which is defined as the ratio of skilled labor wage to unskilled labor wage, has gone through dramatic changes in the postwar U.S. economy. As shown in Figure 1, starting from 1949, the evolution of the skill premium exhibited an “N” shape: it increased in the 1950s and 1960s, then decreased throughout the 1970s, and has increased dramatically since then. Meanwhile, as is also shown in the figure, the relative supply of skilled labor (the ratio of weeks worked by workers holding college degrees to those holding high school diplomas) has been increasing over time.

In the last two decades the literature on the skill premium has been growing. Researchers have been asking why does the pattern of the skill premium look like this. The popular explanations include investment-specific technological change through capital-skill complementarity (see Krusell, Ohanian, Rios-Rull and Violante (2000), hereafter KORV), international trade induced skill-biased technological change (Acemoglu (2003)), and skill-biased technological change associated with the computer revolution (Autor, Katz and Krueger (1998)). Probably the most popular story is the one proposed in Katz and Murphy (1992). They claim that a simple supply and demand framework works well to explain the dynamics of the skill premium. As they say: “A smooth secular increase in the relative demand for college graduates combined with the observed fluctuations in the rate of growth of relative supply could potentially explain the movements in the college wage premium from 1963 to 1987” (Katz and Murphy (1992), page 50). Table 1 (taken from Autor, Katz and Krueger (1998)) demonstrates the basic idea. There has been a steady growth of relative demand for skilled labor starting from 1950, but the pattern of the growth rate of relative supply of skilled labor has fluctuated. It was quite stable from 1940 to 1970 (around 2.5% per year), then dramatically increased to 4.99% per year during the 1970s. After 1980 it dropped to the original average. Therefore, if we put the supply and demand changes together, we will see that the relative price of skilled labor, which is the skill premium, dropped during the 1970s, since supply exceeded demand, while it increased in other decades.

Why did the relative supply of skilled labor increase dramatically during the 1970s? Katz and Murphy attribute it to the baby boom. High fertility rates in the U.S. from 1946 to around 1960 implied a huge increase of college graduates in the labor market since the late 1960s. In turn, the passage of the baby boom cohorts into mid-career, together with the accelerating skill-biased technological change in the 1980s, contributed to the rising college wage premium since 1980. In other words, the demographic change, together with the trend in skill-biased technological change, explains the dynamics of the skill premium.

On the other hand, KORV (2000) try to understand the source of the latent skill-biased technological change. They claim that with the capital-skill complementarity in a neoclassical aggregate production function, the growth in the stock of capital equipment will complementarily increase the marginal product of skilled labor and hence raise its relative demand. They also quantitatively evaluate how much this capital-skill complementarity has affected the skill premium from 1963 to 1992 and find that changes in observed factor inputs can account for most of the variations in the skill premium over these 30 years.

One should realize, however, that the success of KORV’s (2000) calibration depends on the pattern of

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1 Katz and Murphy (1992) find this secular growth in the relative demand for college graduates can be proxied by a linear time trend 3.33% per year, which can be viewed as a proxy for the skill-biased technological change (SBTC). But what drives this secular trend remains a “black box” in their paper and subsequent work along this line. For example, Bound and Johnson (1992) also attribute much of the variation in the skill premium to a residual trend component that is interpreted as SBTC.
fluctuations in the relative supply of skilled labor. They can generate the "V" shape of the skill premium from 1970 in their model since the so-called capital-skill complementarity effect is dominated by the relative quantity effect in the 1970s due precisely to high growth in the relative supply of skilled labor, while after 1980 the pattern is reversed. Thus the pattern of an inverse "V" shape of the growth rate of the relative supply of skilled labor becomes key to their paper. They take this pattern as given and focus on the relative demand side of skilled labor. The implicit assumption they impose is the orthogonality between the relative supply and demand of skilled labor.

However, from the rational expectations perspective, the supply of skilled labor is a response to the expected skill premium. If the expected skill premium increases, high school graduates will be more willing to go to college since they expect higher wages after graduation. The resulting college enrollment increase will in turn increase the future relative supply of skilled labor, which will also affect the skill premium in the future. In other words, the relative supply of skilled labor and the skill premium interact in a dynamic way. To understand the dynamics of the skill premium, we cannot ignore this interaction.

In this sense, the simple supply-demand framework as used in Katz and Murphy (1992) is also inadequate since it ignores this dynamic interaction between the relative supply of skilled labor and the skill premium. They both are equilibrium results emerging from a dynamic general equilibrium framework.

Therefore, looking at a single picture of the skill premium cannot tell U.S. the whole story. To understand better the dynamics of the skill premium, we should also look at the dynamics of the relative supply of skilled labor. To understand the dynamics of both the skill premium and the college enrollment rate within a dynamic general equilibrium framework is the task of this paper.

To achieve this goal, I develop a general equilibrium overlapping generations model with endogenous discrete schooling choice. The model includes three key features. First, with ex-ante heterogeneity in disutility cost of schooling, individuals in each birth cohort (high school graduates) choose to go to college or not based on their expected future wage differentials, the forgone wages during the college years, the tuition payments, and the idiosyncratic disutility cost. This microfoundation gives us the standard features as found in the human capital investment literature. (See for instance Ben-Porath (1967).) Second, the production technology has the feature of capital-skill complementarity as in KORV (2000): Capital is more complementary to skilled than unskilled labor. Third, following Greenwood, Hercowitz, and Krusell (1997), I assume that there exists a technological change on investment goods and call it investment-specific technological change.

Please refer to equation (13) in Section 3 for a detailed explanation of these two effects.
Under this theoretical framework, I calibrate the model and then quantitatively examine the effects of two widely discussed exogenous forces, investment-specific technological change (ISTC) as described above and the demographic change represented by the growth rate of the cohort size of high school graduates, on the skill premium and enrollment rate over the U.S. postwar period 1951-2000. I find that in terms of the skill premium, demographic change dwarfs ISTC before the late 1960s and accounts for about one third of the decline of the skill premium in the 1970s. However, after the late 1970s, ISTC takes over to drive the dramatic increase in the skill premium. ISTC can also explain about 30% of the increase in the enrollment rate for the period 1951-2000, while demographic change does not have a significant effect on the enrollment rate over time.

The reason why this model can generate co-rising skill premium and the enrollment rate, particularly after the early 1980s, is due to the following simple economic mechanism. When ISTC speeds up, investment becomes increasingly efficient over time, so the relative price of the capital stock falls. This encourages higher investment; hence the capital stock increases. Due to capital-skill complementarity, increase in the capital stock raises the relative demand of skilled labor, which raises the skill premium. Forward-looking people anticipate the rising skill premium which will increase the benefits of college education, thus they are more willing to go to college.

However, demographic change affects the skill premium and the enrollment rate through a different channel. For example, growing birth cohorts (“baby boom”) change the age structure in the economy and make it skewed towards younger (college age population) cohorts. On the one hand, more people stay in college and meanwhile more unskilled workers join the labor force, therefore the relative supply of skilled labor decreases, which tends to raise the skill premium. On the other hand, young people hold fewer assets over the life cycle, therefore, change in age structure also slows down asset accumulation. The decrease in capital stock, through capital-skill complementarity, tends to lower the skill premium. Thus the total effect of demographic change on the skill premium and the enrollment rate is ambiguous and has to be investigated quantitatively.

In quantitative terms, before the late 1960s, the U.S. had undergone dramatic population growth (See Figure 2), while the change of ISTC was only moderate (See Figure 15). Demographic change outweighed technological change, therefore it dominated the impact on the skill premium. After the late 1970s, the magnitude of the baby bust was much smaller compared to the baby boom, meanwhile ISTC speeded up dramatically to become the major driving force.

This paper extends the existing literature on the effects of the technological change on wage inequalities. In comparison to KORV(2000), I endogenize the supply of skilled labor and put their aggregate production function into a dynamic general equilibrium setup. Hence, I can test if the capital-skill complementarity is the driving force behind the skill premium and the relative supply of skilled labor within a more comprehensive framework. I also view this paper as a dynamic general equilibrium extension of the work by Katz and Murphy (1992).

Fernandez-Villaverde (2001) proposes a similar mechanism, but targeting on a totally different question. He tries to answer why growth in per capita income has coexisted with fall in fertility during the past 150 years. The idea is capital-specific technological change brings more productive capital, which through the capital-skill complementarity, raises the skill premium, i.e., return to human capital. The increment in the skill premium induces parents to choose more education instead of more children, hence causing the declining fertility.
In spirit, this paper is close to Heckman, Lochner and Taber (1998). They develop and estimate an overlapping generations general equilibrium model of labor earnings and skill formation with heterogenous human capital. They test their framework by building into the model a baby boom in entry cohorts and an estimated time trend of increase in the skill bias of aggregate technology. They find that the model can explain the pattern of wage inequality since the early 1960s. My model focuses on a different channel: capital-skill complementarity. In my model, ISTC is the source of skill-biased technological change. Thanks to the work by Greenwood, Hercowitz, and Krusell (1997) and Cummins and Violante (2002), I have data about this measured technological change. Therefore, instead of simulation, I am able to test my model by feeding in time series of ISTC and demographic change. This allows me to compare the model results with the data. A recent paper by Guvenen and Kuruscu (2006) presents a tractable general equilibrium overlapping-generations model of human capital accumulation which is consistent with several features of the evolution of the U.S. wage inequality from 1970 to 2000. Their work shares the similar microfoundation of schooling choice as in this paper. But they do not have capital stock in the production technology, and hence no capital-skill complementarity, either. The only driving force in their paper is the skill-biased technological change which is calibrated to match the total rise in wage inequality in the U.S. data between 1969 and 1995. They do not have ISTC in their model, therefore the mechanism for rising wage inequality is different.

My model also extends another strand of literature about the effect of cohort size on schooling choices. Ahlburg et al. (1981) find that there does appear to be a significant statistical relationship between cohort size and the educational attainment of the cohort. Flinn (1993) develops a perfect foresight overlapping generations model to investigate the effects of cohort size on schooling decisions and cohort-specific welfare measures in a partial equilibrium environment. Under a set of sufficient conditions, he finds the existence of a unique mapping from any cohort size sequence to both the human capital rental rate and schooling choice sequences. His calibration exercise shows that the equilibrium response of schooling to perturbations in the cohort size sequence is small. Based on the structural estimation framework developed in Keane and Wolpin (1997), Lee (2005) extends Flinn’s partial equilibrium schooling choice model to a dynamic general equilibrium model of career decisions (schooling, occupation and labor force participation choices). He then uses the model’s estimates to determine the impact of cohort size on human capital investment behavior and labor market outcomes. He has a Cobb-Douglas production and hence does not allow capital-skill complementarity in the technology. Capital stock, and capital and skilled labor share are all exogenously given. He also finds that the effect of demographic change (“the baby boom” and “the baby bust”) on the male college completion rate is insignificant (change as much as from -1% to 0.3%). This paper extends and enriches the production side of Lee (2005) by including ISTC and capital-skill complementarity. Capital is endogenous in my model and the accumulation of capital stock is key to driving the skill premium. Within this framework, I still find that demographic change does not have a significant effect on schooling choice.

The remainder of the paper is organized as follows. Section 2 documents some stylized facts about the dynamics of the cohort size of high school graduates, the college enrollment rate, and college tuition in the postwar U.S. economy. It also emphasizes the links among these facts. Section 3 presents my economic model of college going decisions, describes the market environment, and defines the general equilibrium in the model economy, thus laying out the theoretical foundation for the later data analysis and calibration exercise. Section 4 shows how to parameterize the model economy. Section 5 provides calibration results.
for the pre-1951 steady state. Section 6 computes the transition path of the model economy from 1951 to 2000 and compares the results with the data. It also conducts some counterfactual experiments to isolate the effects of investment-specific technological change and demographic change on the skill premium and the college enrollment rate, respectively. Finally, Section 7 concludes.

2 Stylized Facts

Figure 1 already shows the pattern of the skill premium and the relative supply of skilled labor. To understand the dynamics of the relative supply of skilled labor, we should ask the question of who provides skilled labor (workers with college degrees) and hence naturally turn our attention to the individual’s college education decisions. We should also be aware that the baby boom and baby bust only affect the cohort size of college age (in this paper I restrict the college age to be 18-21 years old). From 1970 to 1980 the college age cohort increased substantially due to high fertility in the 1950s. If the college enrollment rate did not change through the 1970s, the growth of relative supply should not have changed drastically. If the college enrollment rate increased dramatically as well, then by combining these two effects we would not be surprised to see that the relative supply of college educated workers increased greatly, as we observe in the data.

Figure 2 shows the cohort size of high school graduates. It was very stable before the early 1950s, then began growing, reached a peak around 1976, and has decreased since then. Since the common age of high school graduation is around 18, we can view this graph as a 18-year lag version of U.S. fertility growth, i.e., it reflects the baby boom and baby bust.4

Figure 3 measures the college-age population. I report the 17-21 and 18-21 age population in the U.S. since 1955. They follow a similar pattern as in Figure 2. The baby boom pushed the college-age population up until the fertility rate reached its peak around 1960, corresponding to the peak of the college-age population around 1980. The baby bust then dragged the population size down.

The above two figures show changes in the population base of potential college students, but does the proportion of people going to college change over time? Figure 4 shows the college enrollment rate of recent high school completers. It began growing from the early 1950s until 1968 and then went down; the entire 1970s was a depressed decade for college enrollment, and it was not until 1985 that the enrollment rate exceeded the level in 1968. Starting from 1980 (notice the timing), the enrollment rate kept increasing for nearly 20 years. This pattern is also confirmed by other studies. (See Macunovich (1996) Figure 1.a, 1.b, 2.a, 2.b, Card (2000) Figure 3.) By comparing the skill premium in Figure 1 and the college enrollment rate in Figure 4, I observe that interestingly they share a very similar pattern. This similarity implies a tight link between college-going decision and the expected skill premium. Future skill premium represents the expected gain from higher education. As the expected benefits increase, the enrollment rate increases.

To understand the behavior of the skill premium exhibited in Figure 1, we cannot ignore the behavior of schooling choice exhibited in Figure 4. The skill premium represents the benefit of a college education. To fully understand the determinants of schooling choice, we should also look at the cost side of college-going.

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4Cohort size has been increasing again since 1995 because the baby-boomer’s children reached college age since the mid 1990s.
In Figure 5 I report the real tuition, fee, room and board (TFRB) per student charged by an average 4-year institution (average means the enrollment weighted average of 4-year public and private higher education institutions, see Appendix A for details). Again, we see a pattern similar to that of the skill premium and the college enrollment rate. TFRB increased over time except in the 1970s. Starting from 1980 (notice this timing again), real TFRB has raised dramatically.

The similarity is not surprising since it reflects supply and demand in the higher education market. Higher demand for skilled labor in the 1980s and 1990s pushed up the skill premium as we see in Figure 1. More people wanted to go to college, hence the enrollment rate increased as shown in Figure 4. In turn, higher demand for college education raised the price of the college education, as shown in Figure 5.

The stylized facts relevant to this paper can be briefly summarized as follows:

1. The skill premium rose during the 1950s and 60s, then fell from 1971 to 1979, and has increased dramatically since 1980.

2. The relative supply of the skilled labor has increased since the 1940s.

3. The college enrollment rate exhibits the similar pattern as the skill premium, as do the tuition payments.

The stylized facts we observe from Figure 1 (skill premium) and 4 (enrollment rate) are the targets of this paper. To understand Figure 1, we also must understand Figure 4 because it tells us where the relative supply of skilled labor comes from. By modeling the dynamic interaction between them, I am able to examine the driving forces behind the dynamics of the skill premium and enrollment rate and answer an important quantitative question: By taking the demographic change as in Figure 3 and the measured investment-specific technological change as in Figure 15 exogenously given and feeding them into a dynamic general equilibrium model, what percentage of change in the skill premium and the enrollment rate can be explained by each of these two exogenous forces?

3 Model

In this section, I will present the economic model that will be used later for calibration. It is a discrete time overlapping generations (OLG) model. Individuals make the schooling choice in the first period. There is only one good in the economy that can be used either in consumption or investment.

3.1 Demographics

The economy is populated by overlapping generations. People enter the economy when they are 18 years old and finish high school, which I call the birth cohort and model as age $j = 1$. I assume people work up to age $J$, which is the maximum life span. To distinguish between the age of a cohort and the calendar time, I will use $j$ for the age, and $t$ for the calendar time. For example, $N_{j,t}$ is the population size of age-$j$ cohort at time $t$.

\[5\text{In another version of the paper, I extend current model to including retirement, social security system and lifetime uncertainty. The quantitative results are very similar to the ones presented here, while life cycle profiles of consumption and asset holdings are more realistic (hump-shaped). This version is available upon request from the author.}\]
In every period $t$ a new birth cohort enters the economy with cohort size $N_{1,t}$. It grows at rate $n_t$. Therefore, I have

$$N_{1,t} = (1 + n_t)N_{1,t-1}. \tag{1}$$

The fraction of age-$j$ cohort in the total population at time $t$ is

$$
\mu_{j,t} = \frac{N_{j,t}}{N_t} = \frac{N_{j,t}}{\sum_{i=1}^{J} N_{i,t}}. \tag{2}
$$

It will be used to calculate the aggregate quantities in the economy as cohort weights throughout the transition path.

The birth cohort in the model corresponds to the high school graduates (HSG) in Figure 2, and the growth rate of HSG cohort size is the data counterpart of $n_t$. Therefore, the “baby boom” corresponds to the 1951-1976 period when $n_t$ increased over time, while the “baby bust” period is from 1976 to 1990 when $n_t$ decreased over time.

### 3.2 Preferences

Individuals born at time $t$ want to maximize their discounted life time utility

$$J_{X_{j=1}^{J}} = \sum_{j=1}^{J} \beta^{j-1} u(c_{j,t+j-1}).$$

The period utility function is assumed to take the CRRA form

$$u(c_{j,t+j-1}) = \frac{c_{j,t+j-1}^{1-\sigma}}{1-\sigma}, \tag{3}$$

$\sigma$ is the coefficient of relative risk aversion, therefore $\frac{1}{\sigma}$ is the intertemporal elasticity of substitution. Since leisure does not enter into the utility function, each individual will supply all her labor endowment, which is normalized to be one.

### 3.3 Budget Constraints

An individual born at time $t$ chooses whether or not to go to college at the beginning of the first period. I use $s \in \{c, h\}$ to indicate this choice. If an individual chooses $s = h$, she ends up with a high school diploma and goes on the job market to work as an unskilled worker up to age $J$, and earns high school graduate wage sequence $\{w_{j,t+j-1}^h\}_{j=1}^{J}$. Or, she can choose $s = c$, spend the first four periods in college as a full time student, and pay the tuition. I assume she can always successfully graduate from college (there is no some college or college dropout in the model). After that, she goes on the job market to find a job as a skilled worker, and earns a college graduate wage sequence $\{w_{j,t+j-1}^c\}_{j=1}^{J}$. After this schooling choice, within each period, an individual makes consumption and asset accumulation decision according to her choice.

For $s = c$, the budget constraints of the cohort born at time $t$ are

$$
c_{j,t+j-1} + tuition_{t+j-1} + a_{j,t+j-1} \leq (1 + r_{t+j-1})a_{j-1,t+j-2} \quad \forall j = 1, 2, 3, 4 \tag{4}
$$

$$c_{j,t+j-1} + a_{j,t+j-1} \leq (1 + r_{t+j-1})a_{j-1,t+j-2} + w_{t+j-1}^c e_{j}^c \quad \forall j = 5, ..., J \tag{5}
$$

$$c_{j,t+j-1} \geq 0, a_{0,t-1} = 0, a_{f,t+J-1} \geq 0$$
where \( \{c_j^s\}_{j=0}^{s} \) is the age-efficiency profile of the college graduates. It represents the age profile of the average labor productivity for the college graduates. Notice that individuals have zero initial wealth and cannot die in debt\(^\text{6}\).

For \( s = h \), the budget constraints of the cohort born at time \( t \) are

\[
c_{j,t+j-1} + a_{j,t+j-1} \leq (1 + r_{t+j-1})a_{j-1,t+j-2} + \frac{w_{t+j-1}^h}{\prod_{i=2}^{4}(1 + r_{t+i-1})} \forall j, \quad \text{for } i = 1, \ldots, J \tag{6}
\]

\[
c_{j,t+j-1} \geq 0, a_{0,t-1} = 0, a_{J,t+J-1} \geq 0
\]

Similarly, \( \{c_j^h\}_{j=1}^{n-1} \) is the age-efficiency profile of high school graduates.

### 3.4 Schooling Choice

Next, I would like to explicitly model an individual’s schooling choice. In order to be able to generate a positive enrollment rate in the model, I need to introduce some ex-ante heterogeneity within each birth cohort. Without this within-cohort heterogeneity, the enrollment rate would be just only zero or one.

I assume that different individuals within each birth cohort are endowed with different levels of disutility cost of schooling. I index people by their disutility level \( i \in [0, 1] \), and the associated disutility cost that individual \( i \) bears is \( DIS(i) \). I assume \( DIS(i) < 0 \). The Cumulative Distribution Function (CDF) of disutility cost is denoted \( F, F(i_0) = \Pr(i \leq i_0) \). Now an individual \( i \) born at time \( t \) has her own expected discounted life time utility

\[
\sum_{j=1}^{J} \beta^{j-1}u(c_{j,t+j-1}) - I_i DIS(i) \tag{7}
\]

where

\[
I_i = \begin{cases} 
1 & \text{if } s_i = c \\
0 & \text{if } s_i = h 
\end{cases}
\]

subject to the conditional budget constraints (4)-(5) or (6), depending on individual \( i \)'s schooling choice \( s_i \). Notice that disutility \( i \) does not enter into the budget constraints, so everyone within the same cohort and with the same education status will have the same life-time utility derived from physical consumption.

I use \( UTIL_i^c \) to denote the discounted life-time utility derived from people who are born at time \( t \) and choose to go to college \( (s = c) \) and \( UTIL_i^h \) to denote the discounted life-time utility derived from people who choose not to go \( (s = h) \). Therefore, \( UTIL_i^c - UTIL_i^h \) represents the utility gain from attending college. Obviously, individual \( i \) will choose go to college if \( DIS(i) < [UTIL_i^c - UTIL_i^h] \), will not go if \( DIS(i) > [UTIL_i^c - UTIL_i^h] \), and is indifferent if \( DIS(i) = [UTIL_i^c - UTIL_i^h] \).

It is easy to show that this model implies

\[
UTIL_i^c - UTIL_i^h \overset{\text{\(\forall\)}}{\geq} 0 \text{ iff } NPV_i \overset{\text{\(\forall\)}}{\leq} 0
\]

where

\[
NPV_i = \sum_{j=5}^{J} \frac{w_{t+j-1}^c - w_{t+j-1}^h}{\prod_{i=2}^{4}(1 + r_{t+i-1})} - \sum_{j=1}^{4} \frac{w_{t+j-1}^h}{\prod_{i=2}^{4}(1 + r_{t+i-1})} - \sum_{j=1}^{4} \frac{tuition_{t+j-1}}{\prod_{i=2}^{4}(1 + r_{t+i-1})}. \tag{8}
\]

\(^6\)Notice that the model does not have exogenous borrowing constraints. However, the standard properties of the utility function and the restriction that the agent cannot die in debt impose a endogenous borrowing constraint at every period.
Here $NPV$ stands for the net present value of higher education. It consists of three terms. The first term represents the benefit of schooling, college graduates can earn more through the skill premium. The second term represents the opportunity cost of schooling. It is the four year foregone wage income for the college students. The third term is the present value of tuition paid during college time, which represents the direct cost of schooling. From this representation it is very clear how the skill premium is going to affect an individual’s schooling decision. Keeping other things equal, an increase in the skill premium will raise the benefit of schooling, thus raising $NPV$. A higher $NPV$ will induce higher utility gain from schooling $UTIL_t^c - UTIL_t^b$. If we assume that the distribution of disutility cost is stationary, higher utility gains from schooling means it is more likely that $DIS(i) < [UTIL_t^c - UTIL_t^b]$, which implies that more people would like to go to college. This mechanism is going to generate the co-movement between skill premium and enrollment rate as observed in the data.

### 3.5 Production

I close the model by describing the production side of the economy. The representative firm in the economy uses capital stock ($K$), skilled labor ($S$), and unskilled labor ($U$) to produce a single good. Here skilled labor consists of college graduates, and unskilled workers are high school graduates. Following KORV (2000), I adopt the form of aggregate production function with capital-skill complementarity as follows:

$$Y_t = A_t F(K_t, S_t, U_t)$$

$$= A_t [\mu U_t^\rho + (1-\mu)(\lambda((B_t K_t)^\rho + (1-\lambda)S_t^\rho\tau)^{\delta/\theta}]^{1/\theta}$$

where $A_t$ is the level of total factor productivity (TFP), $B_t$ is the level of capital productivity and represents capital-embodied technological change, and we have $0 < \lambda, \mu < 1$ and $\rho, \theta < 1$. This production technology is constant return to scale. The elasticity of substitution between capital-skilled labor combination and the unskilled labor is $\frac{1}{1-\rho}$ and the one between capital and skilled labor is $\frac{1}{1-\rho}$. For the capital-skill complementarity, we require $\frac{1}{1-\rho} < \frac{1}{1-\theta}$, which means $\rho < \theta$.

The difference between my production function and the one in KORV (2000) is that I do not distinguish between structure and equipment, so the capital $K$ in my model is just the total capital stock.

The representative firm rents capital, skilled labor and unskilled labor from households at the rates $r_t$, $w_c^t$ and $w_h^t$. Its profit maximization implies the first order conditions below

$$r_t = \lambda(1-\mu)A_t B_t^\rho H_t(\lambda(B_t K_t)^\rho + (1-\lambda)S_t^\rho)^{\frac{\theta}{\delta}}K_t^{\rho-1} - \delta$$

$$w_c^t = (1-\mu)(1-\lambda)A_t H_t(\lambda(B_t K_t)^\rho + (1-\lambda)S_t^\rho)^{\frac{\theta}{\delta}}S_t^{\rho-1}$$

$$w_h^t = \mu A_t H_t U_t^{\theta-1}$$

where $H_t = [\mu U_t^\rho + (1-\mu)(\lambda(B_t K_t)^\rho + (1-\lambda)S_t^\rho\tau)^{\delta/\theta}]^{\frac{1}{\delta-1}}$.

$\delta$ is the capital depreciation rate. Dividing (11) by (10), I derive the expression for the skill premium

$$\frac{w_c^t}{w_h^t} = \frac{(1-\mu)(1-\lambda)}{\mu} [\lambda(\frac{B_t K_t}{S_t})^\rho + (1-\lambda)] \frac{S_t^\rho}{U_t^\theta}.$$

Log-linearizing (13), differentiating it with respect to time, and using “hat” to denote the rate of change...
(\hat{X} = \hat{S})$, I obtain (ignoring the time subscript for the convenience)

$$\left(\hat{w}^{\circ}\right) \approx \lambda(\theta - \rho)(\frac{BK}{\hat{S}})^\rho[B + \hat{K} - \hat{S}] + (\theta - 1)[\hat{S} - \hat{U}]$$

(14)

This equation is exactly as same as in KORV (2000) except for the $B$ term. It says that the growth rate of the skill premium is determined by two components. One is the growth rate of the relative supply of skilled labor $[\hat{S} - \hat{U}]$. Since $\theta < 1$, relatively faster growth of skilled labor will reduce the skill premium. This term is called “relative quantity effect” in KORV (2000). Another term $\lambda(\theta - \rho)(\frac{S}{\hat{S}})^\rho[B + \hat{K} - \hat{S}]$ is called the “capital-skill complementarity effect”. If capital grows faster than skilled labor, this term will raise the skill premium due to $\rho < \theta$. The dynamics of the skill premium depend on the trade-off between these two effects.

The law of motion for the capital stock in this economy is expressed as

$$K_{t+1} = (1 - \delta)K_t + X_t q_t$$

where $X_t$ denotes capital investment. Following Greenwood, Hercowitz and Krusell (1997, GHK hereafter), I interpret $q_t$ as the current state of the technology for producing capital, hence changes in $q$ represent the notion of investment-specific technological change. When $q$ increases, investment becomes increasingly efficient over time.

To simplify the computation, following Fernandez-Villaverde (2001), I can map investment-specific technological change into the changes in the capital productivity level $B_t$. Therefore, increases in $q_t$ will transform into increases in $B_t$. As shown in equation (14), when $B_t$ increases, it will raise the skill premium. Investment-specific technological change thus is also skill-biased.

Finally, the resource constraint in the economy is given by

$$C_t + TUITION_t + X_t = Y_t$$

where $C_t$ is the total consumption and $TUITION_t$ is the total tuition payment.

### 3.6 The Recursive Competitive Equilibrium

The model above is a standard OLG setting with discrete schooling choices. I assume people have perfect foresight so that they can forecast all the future wages they are going to earn. Suppose an individual $i$ born at time $t$ has already made the schooling decision $s_{i,t}$. Conditional on this choice, I can present her utility maximization problem in terms of dynamic programming representation.

For $s_{i,t} = c$, let $V_{t+j-1}^c(a_{j-1,t+j-2}, j)$ denote the value function of an age-$j$ individual with asset holding $a_{j-1,t+j-2}$ at beginning of time $t + j - 1$, and it is given as the solution to the dynamic problem

$$V_{t+j-1}^c(a_{j-1,t+j-2}, j) = \max_{\{c_{j,t+j-1}, a_{j,t+j-1}\}} \left\{ u(c_{j,t+j-1}) + \beta V_{t+j}^c(a_{j,t+j-1}, j+1) \right\}$$

(15)

The transformation is $B_t = 1 + (1 - (1 - \delta)^{t-1})(\frac{q_1}{q_t} - 1)$. I normalize $q_1 = 1$ in initial steady state. Please refer to Fernandez-Villaverde (2001) for the details.

More accurately, it can be easily shown that an economy with investment-specific technological change $q$, but without capital-embodyed technological change $B$ (which is my benchmark economy), can be equivalent to another economy with capital-embodyed technological change $B$, but without ISTC in terms of allocation. (See Greenwood, Hercowitz and Krusell (1997) for details.) Since changes in $B_t$ will increase the skill premium, due to the equivalence, the ISTC in my benchmark economy has the same effect.
subject to (4)-(5).

For $s_{i,t} = h$, the corresponding value function is

$$V_{t+j-1}^h(a_{j-1,t+j-2}, j) = \max_{\{c_{j,t+j-1}, a_{j,t+j-1}\}}\{u(c_{j,t+j-1}) + \beta V_{t+j}^h(a_{j,t+j-1}, j + 1)\}$$

subject to (6).

Individuals solve their perfect foresight dynamic problem by using backward induction. Back to age 1, an individual with disutility index $i$ will choose $s_{i,t}$ based on the criteria below

$$s_{i,t} = c \text{ if } V_i^c(a_{0,t-1} = 0, 1) > DIS(i) \quad V_i^h(a_{0,t-1} = 0, 1)$$

$$s_{i,t} = h \text{ if } V_i^c(a_{0,t-1} = 0, 1) < DIS(i) \quad V_i^h(a_{0,t-1} = 0, 1)$$

$$s_{i,t} = \text{indifferent} \text{ if } V_i^c(a_{0,t-1} = 0, 1) = DIS(i) \quad V_i^h(a_{0,t-1} = 0, 1).$$

Based on the individuals’ dynamic program and schooling choice criteria above, the definition of the competitive equilibrium in this model economy is standard.

**Definition 1** Let $A = \{a: -b \leq a \leq a_{\text{max}}\}$, $S = \{h, c\}$, $J = \{1, 2, \ldots, J\}$, $D = \{0, 1\}$ and $T = \{1, 2, \ldots, T\}$. Given the age structure $\{\{\mu_{j,t}\}_{j=1}^J\}_{t=1}^T$, a Recursive Competitive Equilibrium is a sequence of individual value functions $V_t^s: A \times J \rightarrow \mathbb{R}$; individual consumption decision rules $C_t^s: A \times J \rightarrow R_+$; individual saving decision rules $A_t^s: A \times J \rightarrow A$ for $s \in S$ and $t \in T$; a period one individual $i$’s schooling choice $s_{i,t}$ for $s \in S$, $i \in D$ and $t \in T$; an allocation of capital and labor (skilled and unskilled) inputs $\{K_t, S_t, U_t\}_{t=1}^T$ for the firm; a price system $\{w_t^c, w_t^h, r_t\}_{t=1}^T$; and a sequence of measures of individual distribution over age and assets $\lambda_t^s: A \times J \rightarrow R_+$ for $s \in S$ and $t \in T$ such that:

1. Given prices $\{w_t^c, w_t^h, r_t\}$, the individual’s decision rules $C_t^s$ and $A_t^s$ solve the individual’s dynamic problems (15) and (16).

2. Optimal schooling choice $s_{i,t}^*$ is the solution to the schooling choice criteria (17) for each individual $i$.

3. Prices $\{w_t^c, w_t^h, r_t\}$ are the solutions to the firm’s profit-maximization problem (10)-(12).

4. The time-variant age-dependent distribution of individuals choosing $s$ follows the law of motion

$$\lambda_{t+1}(a, j + 1) = \sum_{a:a' \in A_t^s(a, j)} \lambda_t^s(a, j).$$

5. Individual and aggregate behaviors are consistent:

$$K_t = \sum_j \sum_a \mu_{j,t} \lambda_t^s(a, j) A_t^s(a, j - 1)$$

$$S_t = \sum_j \sum_a \mu_{j,t} \lambda_t^c(a, j) c_j^c$$

$$U_t = \sum_j \sum_a \mu_{j,t} \lambda_t^h(a, j) c_j^h.$$

\[
\sum_{j=1}^{J} \sum_{a} \sum_{s} \mu_{j,t}^{a} X_{s}^{a} (a,j) + \sum_{j=1}^{4} \sum_{a} \mu_{j,t}^{a} X_{s}^{a} (a,j) tuition_{t-1, t} + X_{t} = Y_{t}
\]

i.e.

\[
C_{t} + TUITION_{t} + X_{t} = Y_{t}.
\]

When ISTC and demographic change both stabilize at some constant levels, i.e., \( q_{t} = q \) and \( n_{t} = n \), the economy reaches a steady state. In such a steady state, the age structure, the distribution of individuals over assets and age and the individual decision rules are all age-dependant but time-invariant. Therefore, I can define the stationary competitive equilibrium accordingly.

4 Parameterization

In this section, I calibrate the model economy to replicate certain properties of the U.S. economy in the pre-1951 initial steady state. More specifically, my strategy is to choose parameter values to match on average features of the U.S. economy from 1947 to 1951.9

4.1 Data Work of Cohort-specific Skill Premium

The skill premium data I report in Figure 1 is the average skill premium across all age groups in a specific year. However, since the model presented here is a cohort-based OLG model, each cohort’s going-to-college decision is based on this cohort’s specific life time skill premium profile. For example, for the cohort born at time \( t \), the life time cohort-specific skill premium is \( \{ w_{c,t}^{a} + j - 1 w_{h,t}^{a} + j - 1 \}_{j=1}^{J} \). In order to understand the mechanism of schooling decision for each cohort, I need to find the data counterpart of this cohort-specific skill premium.

I use March CPS data from 1962 to 2003, plus 1950 and 1960 Census data to construct the cohort-specific skill premium profiles for the 1948-1991 cohorts. (I choose to end the sample in 1991 due to the data quality. 1991 cohort only has twelve year HSG wage and eight year CG wage data.) I follow Eckstein & Nagypál (2004) in restricting the data. I do this in order to make my results comparable. (please refer to their paper for the details). The sample includes all full-time full-year (FTFY) workers between age 18 and 65. To be consistent with the model, I only look at high school graduates (HSG) and college graduates (CG). The wage is the annualized real wage (in terms of year 2002 U.S. dollars). In Figure 6, I show the mean CG and HSG wages and the skill premium which is the ratio between these two means for the sample period 1949-2002. It is similar to the pattern of the skill premium shown in Figure 1 which includes post college graduates in the skilled labor group. However, the decline during the 1970s is flatter and the magnitude of the increase since 1980 is smaller. This is because the wages of post college graduates increase even faster than CG.10 Including them (as Autor and Katz (1999) do) further widens the wage gap between skilled and unskilled labor.

---

9I choose the U.S. economy from 1947 to 1951 as the initial steady state based on the observations that both the ISTC and demographic changes were quite stable for this time period.

10Eckstein & Nagypál (2004) find this fact. Please refer to their paper for more details.
Since CPS is not a panel data set, theoretically speaking I cannot keep track specific cohorts from it. However, since it is a repeated cross-sectional data set, I can use a so-called “synthetic cohort construction method” to construct a proxy of a cohort’s specific skill premium. For example, the 1962 cohort (18 year old HSG in 1962)’s lifetime (18-65 year old) HSG wage profile \( \{ w^h_{1961+j+18} \}_{j=1}^{48} \) is constructed as following: I take the 18 year old HSGs in 1962 and calculate their mean wage. Then I take the 19 year old HSGs in 1963 and calculate their mean wage. Next, the 20 year old HSGs in 1964, the 21 year old HSGs in 1965, and so on, until the 58 year old HSGs in 2002. Later I will show how to predict the mean HSG wage after age 58 to complete the life cycle wage profile for this cohort. See Figure 7 for the construction.

I perform a similar procedure to construct the 1962 cohort’s CG wage profile \( \{ w^c_{1961+j+22} \}_{j=1}^{48} \). But I start from 1966 because if someone from the 1962 cohort chose to go to college, she would spend four years in college, graduate in 1966, and start to earn CG wage thereafter. Therefore, I take the 22 year old CGs in 1966, calculate their mean wage, and then follow the above procedure again. See Figure 8 for the construction.

Using this method repeatedly for each birth cohort, I have the original data sequences of cohort-specific HSG and CG life-time wage profiles for the 1948-1991 cohorts. However, due to the time range of the CPS data, some data points are missing for a complete life-time profile for every cohort. For example, some cohorts are missing at the late age data points (cohorts after 1962) and some are missing at the early age data points (e.g., cohort 1948-1961). I use the econometric method to predict the mean wage at those specific age points to interpolate the missing data. I predict them by either second or third order polynomial, or conditional Mincer equation as follows

\[
\log[HSG\text{wage}(age)] = \beta_0^h + \beta_1^h\text{experience}_h + \beta_2^h\text{experience}_h^2 + \varepsilon^h, \quad \text{experience}_h=\text{age}-18
\]

\[
\log[CG\text{wage}(age)] = \beta_0^c + \beta_1^c\text{experience}_c + \beta_2^c\text{experience}_c^2 + \varepsilon^c, \quad \text{experience}_c=\text{age}-22.
\]

The criteria is basically the goodness of fit. I also check with the neighborhood cohorts to make sure the predicted value is reasonable. The “rule of thumb” of hump-shaped profile also applies here to help make choices. As an example, in Figure 9 and 10 I show the prediction for the 1955 cohort by using the trendlines of second and third order polynomial. Obviously the third order one fits the data better, therefore, I use it to predict the missing data points for this cohort.

Through this procedure, I obtain the complete life-time wage profiles for the 1948-1991 cohorts. As the first cut, I want to feed this data into the building block of the schooling choice problem (see Section 3.4) to see if it can shed some light on how people will react to these cohort-specific wage profiles. In Figure 11, I show the calculation of NPV (assume \( r_t+j = 6\% \forall t, j \)) for the 1948-1991 cohorts as in (8), together with the internal rate of return (IRR) to higher education for the same period. From this figure, we see both NPV and IRR have increased since 1970, but the biggest jump occurred in 1980. The tremendous fluctuation after 1986 is partially due to the bad data quality. (We have less data points for cohorts after 1986. For example, cohort 1987 only has 16 data points for HSG wage profile from age 18 to age 35 and 12 data points for CG wage profile.) If people react to this NPV pattern, we should expect that in the model the enrollment rate increases in 1950s, then beginning in 1960, declines slightly until 1970. After 1970, it should start to increase again and jump to a very high level around 1980.

\[11\] My estimation of IRR (range is between 9% and 14%) is closely in line with the mainstream research about the human capital investment, which claims IRR of higher education is around 9-16%.
Two findings need to be mentioned for the time path of the cohort-specific return to schooling. First, they show the possible Vietnam War Draft effect. If the schooling choice is purely based on a cost-benefit analysis as human capital investment theory suggests, we would see that the enrollment rate decreased in the 1960s, then increased since 1970. In this sense the spike centered around 1968 that we observe from data (Figure 4) could be related to the Vietnam War. To avoid the draft, males raised their enrollment into college.\footnote{Of course to clearly pin down this effect, I have to distinguish male enrollment rate from the female one. See He (2005) for the further work on this topic.} Second, the cohort-specific NPV captures the dramatic increase in the skill premium since 1980, so it also implies that the enrollment rate should follow, which is consistent with the data.

## 4.2 Distribution of Disutility Cost

Now the distribution of disutility cost becomes very crucial in my computation because it is this distribution that determines the enrollment rate in the model. The problem is how to obtain it.

The schooling choice criteria embodied in (17) actually sheds some light on how to compute the distribution of disutility cost. Note that the person $i^*$ who is indifferent between going to college or not will have

$$V_c^{t}(a_{0,t-1} = 0, 1) - DIS(i^*) = V_h^{t}(a_{0,t-1} = 0, 1),$$

i.e., her disutility cost is exactly the difference between two conditional value functions. Since disutility cost is a decreasing function of index $i$, people with disutility index $i > i^*$ will go to college. Therefore, for a specific cohort $t$, if we calculate the difference between two conditional value functions $V_c^{t}(a_{0,t-1} = 0, 1) - V_h^{t}(a_{0,t-1} = 0, 1)$, it gives us the cut-off disutility cost for this cohort. If we also know the enrollment rate of this cohort, it tells us the proportion of people in this cohort who have less disutility than $i^*$ at that specific cut-off point of disutility cost. By doing so, I can pin down one point on the CDF of disutility cost. Applying this procedure to different cohorts will give me a picture of how disutility cost is distributed.

Fortunately I have cohort-specific life time wage profile data from 1948 to 1991. I set interest rate equal $3\%$, discount parameter $\beta = 1.03$, and the preference parameter $\sigma = 1.5$.\footnote{Since I obtain the disutility cost from the partial equilibrium computation given the preference parameter $\sigma$, the discount rate $\beta$ and the interest rate $r$, later when I calibrate the model to match pre-1951 steady state, I need to make sure these three values are consistent with those used in the general equilibrium. The values I give here are consistent with those I obtain later in calibrating the initial steady state.} For each cohort born at time $t$, I normalize 18 year old HSG wage (which is $w_{h}^{t}$ in the model) to one and feed in the cohort-specific life time wage profiles. I go through the backward induction of Bellman equation as described in Section 3.6 to obtain the value function difference $V_c^{t}(a_{0,t-1} = 0, 1) - V_h^{t}(a_{0,t-1} = 0, 1)$ for every cohort $t$. In Figure 12, by plotting them against enrollment rate data in the same time range, I have 44 points on the possible CDF of the disutility cost. I then use OLS to estimate the CDF function locally. Later in the computation of the stationary equilibrium and transition path, during each iteration when I obtain factor prices $\{w_c^{t}, w_h^{t}, r_t\}$, I can go do backward induction of Bellman equations loop to get the conditional value functions. Feeding the difference between these two functions into the estimated CDF, I have the corresponding enrollment rate.\footnote{My transition path results also confirm this finding. See section 6.1 for more details.}
4.3 Demographic

The model period is one year. Agents enter the model at age 18 \((j = 1)\), work up to age 65 and die after that age \((J = 48)\).

The growth rate of cohort size \(n\) is calculated as the average growth rate of the HSG cohort size from 1948 to 1951, which is 0%.

4.4 Preferences and Endowments

I pick CRRA coefficient \(\sigma = 1.5\), which is in the reasonable range between 1 and 5 and is widely used in the literature (e.g., Gourinchas and Parker (2002), Attanasio et al. (1999), and Chen et al. (2004) among others).

The age-efficiency profile of high school graduates \(\{\varepsilon_h^j\}_{j=1}^J\) and college graduates \(\{\varepsilon_c^j\}_{j=1}^J\) are calculated as follows: from the 1962-2003 CPS and the 1950 and 1960 Census data I calculate the mean HSG and CG wages across all ages for the time period 1949-2002, then I obtain the mean HSG and CG wage in the same time period for each age group. Thus the age-efficiency profiles are expressed as

\[
\varepsilon_h^j = \frac{\text{HSG wage}_j}{\text{HSG wage}}, \varepsilon_c^j = \frac{\text{CG wage}_j}{\text{CG wage}}, \forall j = 1, \ldots, 48.
\]

The result is shown in Figure 13. Both profiles exhibit a clear hump shape and reach a peak around age 55. Also notice that \(\varepsilon_c^j = 0, \forall j = 1, \ldots, 4\) since CGs never work during study.

4.5 Technology

Two key elasticity parameters in the production function, the coefficient for elasticity of substitution between capital and skilled labor \(\rho = -0.495\) and the coefficient for elasticity of substitution between unskilled labor and capital-skilled labor combination \(\theta = 0.401\), are taken directly from KORV (2000). This implies the elasticity of substitution between capital and skilled labor is 0.67 and the one between unskilled and skilled labor is 1.67. Capital-skill complementarity is satisfied.

In the initial steady state, both TFP level \(A\) and capital productivity \(B\) are normalized to unity. I set the depreciation rate of capital \(\delta = 0.069\) by following İmrohoroğlu, İmrohoroğlu and Joines (1999). They calculated this parameter from the annual U.S. data since 1954.

Table 2 summarizes the choices of parameter values from the outside sources.

| \(J\) | maximum life span | 48, corresponding to age 65 in the real life |
| \(\{\varepsilon_s^j\}_{j=1}^J\), \(s = c, h\) | age-efficiency profiles | 1962-2003 CPS, 1950, 1960 Census Data |
| \(\sigma\) | CRRA coefficient | 1.5, Gourinchas and Parker (2002) |
| \(\theta\) | elasticity b/w \(U\) and \(K\) | 0.401, taken from KORV |
| \(\rho\) | elasticity b/w \(S\) and \(K\) | -0.495, taken from KORV |
| \(\delta\) | depreciation rate | 0.069, İmrohoroğlu, İmrohoroğlu and Joines (1999) |

Table 2: Parameter Values from Outside Sources
This leaves four parameter values to be calibrated. The subjective discount rate $\beta$ is set equal to 1.03 to replicate the target capital-output ratio 2.67 which is the average value from 1947 to 1951.\textsuperscript{14} The income share of capital in capital-skilled labor combination $\lambda = 0.645$ is chosen to match income share of capital in NIPA for the 1947-1951 period. The income share of unskilled labor $\mu = 0.418$ is chosen to match the average skill premium 1.4556 in the 1949 Census data. The scale factor of the disutility cost $sd$ (See the appendix B for the detail) is chosen to match the average enrollment rate between 1947 and 1951. Table 3 summarizes the discussion above.

The computation method of the steady state is described in detail in the Appendix B.

## 5 Steady State Results

In this section, I report the numerical simulations for the stationary equilibrium of the benchmark economy and compare the results with the pre-1951 U.S. data. The macro aggregates that the model generates are shown in Table 4.

The simulations show that the model does well in matching the data. It matches our targets—skill premium, enrollment rate, capital-output ratio ($K/Y$), and labor income-output ratio ($w^cU + w^hS)/Y$) by construction. Additionally, several key macro aggregate ratios such as consumption-output ratio ($C/Y$) and

\textsuperscript{14}It is plausible for a subjective discount factor greater than one in an overlapping generations setting. Please see İmrohoğlu et al. (1995) for detailed discussion.
investment-output ratio \((X/Y)\) are also in line with the U.S. average data. The model also matches the relative supply of skilled labor very well. The risk-free real interest rate is 3%. Average CG enjoys higher life-time utility than average HSG because the CG has a higher consumption over the life cycle. We can see this clearly in the life cycle profiles below.

### 5.1 Life Cycle Profiles

This model also generates the life-cycle profiles for CG and HSG, respectively. Figure 14 shows the life-cycle profile of wealth accumulation, consumption, and income for CG and HSG. Panel A shows that since CGs have no income in the first four periods, they have to borrow to pay for tuition and consumption. Therefore they accumulate negative wealth over the first four periods. After graduating from college, they start to earn the CG wage and are able to pay the loans. By age 34, CGs pay back all the loans borrowed from previous years and begin to accumulate positive wealth, reaching the peak around their mid-50s. At that time they begin to dissave. There is no bequest motive in this model, therefore people die with zero assets remaining. The same hump shape is also observed for HSG, except that they accumulate positive assets from the beginning.

The life-cycle profile of consumption in Panel B is worthy of explanation. It keeps increasing until the deaths of both CG and HSG. The reason is very clear when we look at the intertemporal Euler equation derived from the model

\[
\left(\frac{c_{t+1}}{c_t}\right)^\sigma = \beta(1 + r).
\]

(23)

Given \(\beta = 1.03\), \(r = 0.03\) as in the results, \(\beta(1 + r) = 1.061\). Therefore, the right hand side of the equation (23) is larger than 1, inducing a positive growth rate of consumption over the life cycle.

Although CGs do not have any income during first four periods, they have higher consumption than HSGs at any age. Because in this deterministic model, consumption path is determined by the permanent income, and CGs have higher discounted life time income.

Finally, panel C shows the hump-shaped life-time labor income profiles for CG and HSG, which are affected by the hump-shaped CG and HSG age-efficiency profiles \(\epsilon_{c,j}^j\) \(j=1\) and \(\epsilon_{h,j}^j\) \(j=1\).

### 5.2 Comparative Static Experiments

Based on the steady state results, I carries out some comparative static exercises to study the effects of the growth rate of the cohort size by changing \(n\) and the effects of investment-specific technological change by changing \(q\). I summarize the corresponding results in Table 5 and 6 respectively. In Table 5, 0% is the average growth rate of the HSG cohort size from 1947 to 1951, which is our benchmark case. 4.06% is the average growth rate of the HSG cohort size from 1952 to 1976, the “baby boom” period. -1.57% is the average growth rate from 1977 to 1991, the period when \(n\) continuously decreased. The results show that as the growth rate of the HSG cohort size increases, the skill premium and enrollment rate both increase, and vise versa. However, the effect is not quantitatively significant. In particular, the effect on the enrollment rate is very small.

Why does the increase in the HSG cohort size cause an increase in the skill premium and the college enrollment rate? The intuition is as follows: an increase in \(n\) will change the age structure \(\{\mu_j\}^j_{j=1}\) in the
Table 5: Effect of population growth on steady state economy, making it skewed towards younger cohorts. Keeping the enrollment rate unchanged, more people from the college age cohort stay in college. Meanwhile more people from the college age cohort also join the labor force as unskilled labor. It results in relatively less out-of-school skilled labor in the current labor market. This is shown in Table 5. When \( n \) increases up to around 4\%, the relative supply of skilled labor \( S/U \) decreases by 9.1\%. This change tends to raise the relative price of skilled labor which is the skill premium through the relative quantity effect. In turn, it will encourage people to go to college.\(^\text{15}\) However, change in age structure also has an impact on asset accumulation. Recall that the life cycle profile of asset holding for CG and HSG in Figure 14. People accumulate fewer assets during early working years. A shift towards younger cohorts in demographic structure thus decreases incentive of asset accumulation. As a result, the capital-output ratio \( (K/Y) \) decreases from 2.77 in the benchmark case to 2.57 in \( n = 4.06\% \) case. It also leads to a decrease in effective capital-skilled labor ratio \( (BK/S) \). It then, through capital-skill complementarity effect, tends to decrease the skill premium and thus the enrollment rate. Quantitatively, the change in age structure also has an impact on asset accumulation. As a result, the capital-output ratio \( (K/Y) \) decreases from 2.77 in the benchmark case to 2.57 in \( n = 4.06\% \) case. It also leads to a decrease in effective capital-skilled labor ratio \( (BK/S) \). It then, through capital-skill complementarity effect, tends to decrease the skill premium and thus the enrollment rate.

\(^{15}\) Alsalam (1985) also points out this mechanism and concludes the dynamic behavior of enrollment rate is derived from various time series of cohort size of HSGs. But his model is totally silent on the other features of my model, namely the capital-skill complementarity, and the investment-specific technological change. Therefore, his model does not have the interaction effect of demographic change on the demand side of the skilled labor as I mention in the text. He actually calls for a complete model of enrollment rates suitable for estimation and testing, and suggests that this model “would be flexible enough to allow for technological change or an increase in the demand for college educated labor relative to less educated workers.” My model does answer this call.

\[\begin{array}{cccccc}
 n \ (\%) & w^c / w^h & c \ (\%) & S/U \ (\%) & BK/S & UTIL^c \\
 0 \ (\text{benchmark}) & 1.4541 & 41.51 & 67.67 & 5.72 & -100.2 \\
 4.06 & 1.5085 & 41.88 & 61.54 & 5.43 & -100.9 \\
 -1.57 & 1.4400 & 41.44 & 69.55 & 5.83 & -100.0 \\
\end{array}\]

On the other hand, decrease in \( n \) will make age structure favor the older cohort, and hence will increase the relative supply of skilled labor and raise the incentive for asset accumulation. These two impacts again tend to offset each other. Quantitatively, a change in \( n \) from 0\% to -1.57\% only slightly decreases the skill premium and enrollment rate.

Next, I show the effect of a permanent change in \( q \) on the steady state. In this model, as shown in GHK(1997) and KORV(2000), due to the existence of the ISTC, the relative price of capital goods is equal to the inverse of the investment-specific technological change \( q \). Therefore I can use the relative price of capital to identify ISTC \( q \). I take the price index of the personal consumption expenditure from NIPA, and the quality-adjusted price index of the total investment (equipment and structure) from Cummins and Violante (2002) for the time period 1951-2000. I divide these two sequences to obtain the data counterpart of the \( q \). Figure 15 shows the log of time series of \( q_t \). It was fairly stable before 1957, then started to grow. The average growth rate of \( q \) in the 1960s and 1970s was 1.8\% and 1.7\%, respectively. It has speeded up since the early 1980s. The average growth rate in the 1980s was 3.2\% and it was even higher in the 1990s.
<table>
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<th>e (%)</th>
<th>S/U (%)</th>
<th>BK/S</th>
<th>UTIL(^c)</th>
<th>UTIL(^h)</th>
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<td>-125.3</td>
</tr>
<tr>
<td>2000 data</td>
<td>1.8357</td>
<td>63.33</td>
<td>163.06</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Effect of investment-specific technological change on steady state

(4.4%).

I then use the mapping mentioned before to transform the sequence of \( q_t \) to the changes in the capital productivity level \( B_t \). Normalizing the initial steady state value \( B_{1951} = 1 \), I have \( B_{2000} = 3.28 \). During these fifty years, the decline of the relative price of the capital goods is equivalent to increases in the capital productivity by approximately 3.3 times.

Suppose that the U.S. economy reaches the steady state again after 2000. Keeping other things equal, Table 6 shows the effects of this permanent change from \( B_{1951} \) to \( B_{2000} \).

Investment-specific technological change, through the capital-skill complementarity, increases both the skill premium and the enrollment rate significantly. The mechanism is as follows: investment-specific technological change raises the capital productivity \( B_t \), hence raising the effective capital stock \( B_tK_t \). Since capital is complementary with skilled labor, increases in effective capital also raise the demand for skilled labor. Therefore it tends to increase the skill premium. Forward-looking individuals predict that the skill premium will increase. The response by college age adults is to enroll in college. Thus we also observe the increases in the enrollment rate.

However, as more people go to college and earn degrees, the relative supply of skilled labor increases. The relative quantity effect thus dampens the increase of the skill premium. The quantitative results in Table 6 confirms that the first order impact of ISTC is on the demand side of skilled labor through capital-skill complementarity. This impact dominates the repercussion effect from the relative supply side. Hence the skill premium increases from 1.4541 to 1.8516 in the model which is quite close the data in 2000. The results also show that both CG and HSG significantly benefit from this technological change.

However, the model underpredicts the increase in the enrollment rate significantly. From 1951 to 2000, the enrollment rate increases from 41.54% (1947-1951 average) to 63.33%, while the model indicates only an increase from 41.51% to 47.46%. Increases in \( q_t \) can only explain around 27% of the increases in the enrollment rate from 1951 to 2000. The reason probably lies in the estimation of the CDF of the disutility cost. The OLS method I use allows me to obtain a fairly flat line for the CDF. The advantage of this method is that I obtain the convergence in steady state easily, but the disadvantage is that I cannot generate enough variations in the enrollment rate.

6 Transition Path

The comparative static exercises I have done above (especially the one with ISTC) show that we are on the right track to explain the increases in the skill premium and enrollment rate over time. However, to see how far the model can go to match the time series data of the skill premium and enrollment rate in the post war
U.S. economy, comparative static analysis is not enough. I have to solve the model along a time path.

Following the spirit of the computation method as in Chen, Imrohoroğlu and Imrohoroğlu (2004) and Conesa and Kreuger (1999), I compute the model along a transition path from initial pre-1951 steady state towards a final steady state in a far future. The computation algorithm is described in detail in Appendix C.

6.1 Benchmark Case

In the benchmark case, I feed into the model the exogenous path of capital-specific technological change \( \{B_t\}_{t=1951}^{2000} \) and demographic change embodied in the change in the growth rate of the HSG cohort size \( \{n_t\}_{t=1951}^{2000} \). I also feed in the normalized tuition payments \( \{tuition_t\}_{t=1951}^{2000} \). I then assume capital-specific technological change gradually decelerates until it becomes stable in 2030, continues at this constant level until 2050. For simplicity, I also assume that after 2000 there is no demographic change after 2000 and the tuition payment is constant at 2000 level. Since I want to focus on the effect of ISTC, the neutral TFP change has been normalized to unity all the time periods through the transition. I compute the transition path of the benchmark economy between 1951 and 2050 and truncate it to the 1951-2000 period. The results are shown in Figure 16.

In Figure 16, the simulated skill premium from the benchmark economy overshoots the actual data since 1965, but it captures the increase after 1980 very well. From 1951 to 2000, the data show that the skill premium increases from 1.4546 to 1.8357 and the average growth rate (per year) during these fifty years is 0.49%. In the model the skill premium increases from 1.4543 to 1.8793 and the average growth rate is 0.54%. If we compare the data to the model in detailed periods, from 1951 to 1959, the average growth rate of the skill premium in the data is 0.73%, while in the model is -0.32%. The model does not capture the increase through that decade. From 1963, I have annual data of the skill premium so the comparison between the data and the model performance is more accurate. From 1963 to 1969, the average growth rate is 1.30% in the data and 1.52% in the model. From 1969 to 1981, the skill premium decreases at the average rate of 0.45%, but the model misses this decline by predicting an almost flat skill premium over this period. The average growth rate is 0.02%. The skill premium starts to increase dramatically beginning in 1981. From 1981 to 1990, the average growth rate in the data is 1.45%, while the model predicts 0.82%. That is, the model captures 56% of the changes in the skill premium during this decade. From 1990 to 2000, the average growth rate in the data slows down to 0.96%, and the model predicts 1.01% which overshoots the growth. Overall, for the three episodes in the “N” shape of the skill premium from 1963 to 2000, the model captures 117% of the changes in the skill premium for the period 1963-1969 and 77% for the period 1981-2000. But the model fails to replicate the declining part of the skill premium from 1969 to 1981.

The model also raises the enrollment rate from 41.52% to 48.07% for the period 1951-2000; while the growth rate in the data is from 41.54% to 63.3%. In other words, the model can explain about 30% of the increases in the enrollment rate during this period. Breaking down into episodes, from 1951 to 1968, the enrollment rate grows at average rate 1.8% in the data. The average growth rate in the model is 0.48%. Therefore, the model accounts for about 27% of the growth in this period. From 1968 to 1980, the enrollment rate decreases. Its average growth rate is -0.88% in the data, while in the model the enrollment rate still increases slightly at a rate 0.34%. Since 1980, the enrollment rate starts to increase again at a average
growth rate 1.34% and the model predicts 0.12%, which means that the model can only account for around 9% of changes in the enrollment rate for the period 1980-2000.

Compared to the skill premium, the model does less successfully in accounting for the fluctuations in the enrollment rate. There are two possible reasons: the first is as I mentioned before, the OLS estimation of the disutility cost of schooling gives me an easy convergence, but at the expense of generating enough variations; the second is that the model here is a highly abstract one which excludes many of the important determinants of individual’s schooling choice. For example, from Figure 16, the effect of the Vietnam War Draft is obvious. It increased the college enrollment rate from 1963 to 1968, then the draft effect gradually phased out, dragging the enrollment rate down. This policy change is not included in our model. Therefore, it is not surprising that the model only predicts a moderate increase in the enrollment rate over this period. There were dramatic social norm changes since the 1970s that have affected especially the female college-going behavior and increased female enrollment rate. Increases in the female enrollment rate contribute to the large increase in college enrollment rate in the data. My model is also silent about this driving force behind rising enrollment rate.\textsuperscript{16}

As shown in equation (13), the skill premium is determined by the two competing effects: the \textit{relative quantitative effect} where the key determinant is the relative supply of skilled labor ($S/U$), and the \textit{capital-skill complementarity effect} where the effective capital-skilled labor ratio ($BK/S$) plays key role. In panel C and D of Figure 16, I show the model performance along these two dimensions. Panel C shows the effective capital-skilled labor ratio in the model and data. The data are taken from KORV (2000) and the time range is from 1963 to 1992. It is the ratio of the combination of real capital structure and quality-adjusted equipment stocks divided by total working hours of skilled labor. I normalize the data point in 1963 to be consistent with the one predicted in the model. The model does a good job in replicating the dramatic increase in this ratio. But since the model does less successfully in replicating changes in the enrollment rate, the relative supply of skilled labor is significantly underpredicted compared to the data. This could be the major reason why the model overshoots the skill premium data.

6.2 Counterfactual Decomposition

To answer the quantitative question raised in the introduction, I conduct the following counterfactual experiments to isolate each exogenous change and investigate its impact on the skill premium and enrollment rate.

I first shut down the capital-specific technological change, so the only exogenous force remaining is demographic change. The results are shown in Figure 17.

Panel A shows that the model fits the skill premium data fairly well until 1980, and it does generate the declining part of the skill premium during the 1970s. However, it cannot capture the dramatic increase since 1980 as shown in the data. More specifically, from 1963 to 1969, the model generates around 1% average growth rate in the skill premium, which can explain 77% of the changes in this period. The “demographic

\textsuperscript{16}Goldin (2006) calls the period since the late 1970s a “quiet revolution” for women’s increased involvement into the economy. She documents that during this period, women had expanded horizons, cared more about their individual identities, and faced shifts in relative earnings and occupations. It would be interesting to see the effects of these factors on the female enrollment rate in my dynamic general equilibrium framework.
change only” model also captures 31% of the decline of the skill premium from 1969 to 1981. But from 1981, it predicts a slight decrease in the skill premium (0.007% per year), in contrast to the dramatic increase shown in the data.

Additionally, Panel B depicts that the model generates little variation in the enrollment rate. From 1951 to 2000, model predicts that the enrollment rate slightly decreases from 41.52% to 40.88%, while the data are from 41.54% to 63.33%. The average growth rate of the enrollment rate is 0.95% in the data, while in the model is -0.03%. Demographic change does not have a significant effect on the enrollment rate over this period.

Panel C and D show the driving forces in shaping the pattern of the skill premium. In Panel C, we see that except for first few years (1963-1966) the “demographic change only” model is a total failure in replicating the effective capital-skilled labor ratio in the data. From 1966 to 1976, BK/S ratio in the model actually decreases from 5.83 to 5.29. Then from 1977 to 2000, it increases from 5.27 to 5.87. This pattern basically is consistent with the mechanism mentioned in the comparative static experiments before (see section 5.2), the “baby boom” decreases BK/S ratio while the “baby bust” increases it. But without ISTC, we cannot generate such a dramatic increase in BK/S ratio as shown in the data.

Panel D demonstrates the dynamics of the relative supply of skilled labor. Since the model fails to generate increases in the enrollment rate over time, hence it also fails to replicate the increasing trend of S/U over the period 1951-2000.

Panel C and D help us to understand the performance of the model in the skill premium. From 1966 to 1976, decreasing relative supply of skilled labor tends to raise the skill premium through the relative quantity effect, but the change in age structure also brings capital accumulation down. Decreases in BK/S ratio, through capital-skill complementarity, tends to decrease the skill premium. Things reverse after 1976. The shape of the skill premium thus is a result of the trade-off of these two forces.

Second, I shut down the demographic change. What remains is only the investment-specific technological change. Figure 18 shows that the results are similar to those in the benchmark case. The skill premium overshoots the data after the early 1970s and keeps increasing afterwards. Therefore it captures the dramatic increase since the late 1970s, but misses the declining part of the skill premium during the 1970s. From 1963 to 1969, the average growth rate of the skill premium in the model is 0.55%. The model thus can explain about 42% of the changes in the skill premium over this period. From 1969 to 1981, the data show that the skill premium decreases at an average rate 0.45%, while the model goes the wrong direction to predict a growth at a rate 0.26% on average. However, after 1981, ISTC only model generates a 0.89% average growth rate of the skill premium over the period 1981-2000, which can explain about 74% of the increases in the skill premium in the data.

In Panel B, the enrollment rate rises from 41.52% in 1951 to 48.07% in 2000, as same as in the benchmark case. The model also captures the dramatic rise in capital-skilled labor ratio. Since there is no demographic change in this model, changes in the relative supply of skilled labor are only driven by the increase in the enrollment rate. The model predicts continuously increasing S/U but much less drastic than in the data due to underprediction in the enrollment rate. This underprediction could contribute to the overshooting of the skill premium in the model both in this one and benchmark case. If the model could do better job to match the changes in the enrollment rate, S/U would increase and drag the skill premium down.
### Table 7: Average annual growth rate of the skill premium: model vs. data

<table>
<thead>
<tr>
<th>Period</th>
<th>Data (%)</th>
<th>Benchmark (%)</th>
<th>Demographic (%)</th>
<th>ISTC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1963-2000</td>
<td>0.68</td>
<td>0.73</td>
<td>0.11</td>
<td>0.63</td>
</tr>
<tr>
<td>1963-1969</td>
<td>1.30</td>
<td>1.52</td>
<td>1.00</td>
<td>0.55</td>
</tr>
<tr>
<td>1969-1981</td>
<td>-0.45</td>
<td>0.02</td>
<td>-0.14</td>
<td>0.26</td>
</tr>
<tr>
<td>1981-1990</td>
<td>1.45</td>
<td>0.82</td>
<td>0.06</td>
<td>0.78</td>
</tr>
<tr>
<td>1990-2000</td>
<td>0.96</td>
<td>1.01</td>
<td>-0.07</td>
<td>0.98</td>
</tr>
</tbody>
</table>

### Table 8: Decomposition of the contribution to the dynamics of the skill premium

The decomposition results will be more clear if we combine three cases in one graph. In Figure 19, I show the simulations of the skill premium in the benchmark, shutting down the technological change (“Demographic only”), and shutting down the demographic change (“ISTC only”) cases. We can see that the “Demographic only” outcome is very close to the benchmark outcome until 1965. Then they diverge. On the other hand, the “ISTC only” outcome closely follows the benchmark outcome since the late 1970s. From this observation, we can draw the conclusion that demographic change dwarfs the ISTC before 1966, and it does contribute to the decline of the skill premium in the 1970s. However, things reverse after the late 1970s. ISTC takes over to drive the increase in the skill premium.

Table 7 compares the average annual growth rate of the skill premium in the data and in the three model cases for different periods.

The contribution of each force to the dynamics of the skill premium is summarized in Table 8. Here the contribution is measured by the ratio of the average annual growth rate of the skill premium in the model and in the data. Overall, ISTC is much more important than demographic change in explaining the pattern of the skill premium from 1963 to 2000 (93% vs. 17%). But demographic change dwarfs ISTC in the 1960s (77% vs. 42%). It can also explain about one third of the declining part for the period 1969-1981 while ISTC goes into a wrong direction. The relative importance of demographic change decreases dramatically since 1981, while ISTC becomes the major driving force behind the skill premium.

We should also be aware of that these two exogenous forces are not mutually exclusive to each other. That’s the reason why the summation of each contribution in separate case is not equal to the one in the benchmark case. For example, in 1981-1990 period, the contribution by demographic change is 4% and the one by ISTC is 54%. We should expect the total effect by combining two forces together, as in the benchmark case, to be 58%, if the effects on the skill premium from these two forces are orthogonal, but the actual number is 56%. The reason lies in the interaction between these two forces (refer to section 5.2.
Table 9: Average annual growth rate of the college enrollment rate: model vs. data

<table>
<thead>
<tr>
<th>Period</th>
<th>Data (%)</th>
<th>Benchmark (%)</th>
<th>Demographic (%)</th>
<th>ISTC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1951-2000</td>
<td>0.95</td>
<td>0.30</td>
<td>-0.03</td>
<td>0.30</td>
</tr>
<tr>
<td>1951-1968</td>
<td>1.80</td>
<td>0.48</td>
<td>0.03</td>
<td>0.52</td>
</tr>
<tr>
<td>1968-1980</td>
<td>-0.88</td>
<td>0.34</td>
<td>0.00</td>
<td>0.29</td>
</tr>
<tr>
<td>1980-2000</td>
<td>1.34</td>
<td>0.12</td>
<td>-0.10</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 10: Decomposition of contribution to the dynamics of the college enrollment rate

<table>
<thead>
<tr>
<th>Period</th>
<th>Data (%)</th>
<th>Benchmark (%)</th>
<th>Demographic (%)</th>
<th>ISTC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1951-2000</td>
<td>100</td>
<td>31.47</td>
<td>-3.26</td>
<td>31.50</td>
</tr>
<tr>
<td>1951-1968</td>
<td>100</td>
<td>26.80</td>
<td>1.46</td>
<td>28.93</td>
</tr>
<tr>
<td>1968-1980</td>
<td>100</td>
<td>-38.85</td>
<td>-0.04</td>
<td>-32.67</td>
</tr>
<tr>
<td>1980-2000</td>
<td>100</td>
<td>8.98</td>
<td>-7.40</td>
<td>9.03</td>
</tr>
</tbody>
</table>

for the discussion). Both forces contribute to the dynamics of the capital-skilled labor ratio \(BK/S\) and relative supply of skilled labor \(S/U\). And the skill premium is nonlinearly determined by these two ratios.

Figure 20 shows the simulations of enrollment rate in these three cases. Clearly the “Demographic only” case generates little variation in enrollment rate. On the other hand, the “ISTC only” case is very similar to the benchmark one, which implies that investment-specific technological change, not the demographic change, is a driving force of the increases in enrollment rate. Table 9 summarizes the average annual growth rate of the enrollment rate in the data and in the three model cases for different periods. Table 10 shows the contribution of each force on the enrollment rate for each period.

Since the effective capital-skilled labor ratio \(BK/S\) and relative supply of skilled labor \(S/U\) are the two major determinants of the skill premium in the model, I also show simulations of both ratios in Figure 21 and 22, respectively. Figure 21 displays that \(BK/S\) in the benchmark case tracks closely with the one in “demographic change only” case from 1951 to 1965, which implies that demographic change dominates ISTC over this period. The ups and downs from 1959 to 1965 are due to the dramatic demographic change during that time. From 1959 to 1960, \(n_t\) drops from 14.2% to 5.7%. This decrease in the growth rate of HSG cohort size brings increase in \(BK/S\) (recall the mechanism for the “baby bust” in section 5.2). On the other hand, \(n_t\) increases from 1.3% in 1962 to 17.5% in 1963. This huge increase changes the age structure significantly and decreases the asset accumulation in the economy. Therefore, \(BK/S\) decreases drastically. Since 1965, \(BK/S\) in the benchmark case turns to follow closely with the one in “ISTC only” case. It shows that ISTC becomes the major driving force to affect this ratio. And it is the drastic rising capital-skilled labor ratio since the late 1970s, through capital-skill complementarity, that drives the rising skill premium.

Figure 22 shows the relative supply of skilled labor in benchmark and two decomposition cases. “Demographic change only” case predicts decreasing \(S/U\) from 1951 to 1976 and increasing \(S/U\) since then. This is consistent with the mechanism mentioned in section 5.2. “Baby boom” decreases \(S/U\), while “baby bust” does the opposite. In contrast to this case, “ISTC only” model shows a continuously rising \(S/U\) ratio by generating an increase in the enrollment rate over time. \(S/U\) in the benchmark case is in between two
decomposition cases. It decreases until 1966, then begins to increase and converges to the ratio in “ISTC only” model after 1990. This “J” pattern confirms that demographic change is a dominating force in driving the evolution of the relative supply of skilled labor before 1966, but the effect phases out after that. ISTC catches up to become the major driving force.

We should be aware that due to the weakness of the model in replicating enrollment rate data, the model underpredicts the growth of the relative supply of skilled labor in the postwar U.S. economy significantly. This weakness contributes to the overshooting of the skill premium compared to the data. Improvement in the enrollment rate will increase $S/U$ in the model and hence alleviate the overshooting problem.

6.3 Sensitivity Analysis

In this subsection, I show that my results are robust in the choice of the model setting, alternative values of key parameters, data sequences of the investment-specific technological change, and the timing of the final steady state.

My current model does not include retirement. Another model setting including retirement and social security system (people retire at age 66 and live up to age 100) gives very similar results.\footnote{This version of the model and the quantitative results are available on request from the author.}

Since the two elasticity parameters in the production function: $\theta$ and $\rho$ are the key parameters in the model, I also do the sensitivity analysis according to different values of these two parameters. More specifically, I use the values $\theta = 0.33$ and $\rho = -0.67$ which are taken from Fernandez-Villaverde (2001) since we share the same specification of the production function. This implies the elasticity of substitution between capital equipment and skilled labor is 0.60, the one between unskilled and skilled labor is 1.49. I then recalibrate the model according to these new values. All the comparative static and transition path results are similar to the benchmark one shown here.

The data sequence of the investment-specific technological change is taken from Cummins and Violante (2002). I also check the alternative sequence taken from GHK(1997). Their model divides the capital into two categories: structure and equipment. Investment-specific technological change is assumed to affect equipment only. Therefore, their $q$ is identified with a quality-adjusted relative price index of newly equipment, which is slightly different from our specification. Feeding in GHK’s $q$ data into the model we have similar results to the benchmark one. But now the magnitude of the technological change is bigger, therefore, the effects on the skill premium and the enrollment rate is higher than those in the benchmark case.

I set the timing of the final steady state in some arbitrary way. However, I notice that the choice of this timing does not affect my results significantly. For example, if the model reaches the final steady state right after 2000, I still obtain very similar results.

7 Conclusion

The skill premium (college wage premium) in the U.S. increased in the 1950s and 1960s, decreased in the 1970s, and has increased again dramatically since 1980. What are the driving forces behind this “N” shape? The previous literature has proposed several explanations including skill-biased technological change (SBTC)
and demographic change. In this paper, I find that the college enrollment rate shares a similar pattern as that of the skill premium, which motivates me to try to understand the dynamic interaction between the enrollment rate and the skill premium. To serve this goal, I establish and compute an overlapping generation general equilibrium model with endogenous schooling choice to answer an important quantitative question: what percentage of change in the skill premium and the enrollment rate can be explained by these two widely discussed exogenous driving forces: demographic change and investment-specific technological change (ISTC)?

In this model, ISTC and demographic change drive the equilibrium outcomes of the skill premium and the enrollment rate by dynamically affecting the relative demand and supply of skilled labor. ISTC, through the key feature of capital-skill complementarity in the production technology, increases the relative demand of skilled labor, thus raises the skill premium. The rising skill premium encourages skill formation and hence increases the relative supply of skilled labor. In contrast to ISTC, demographic change affects the age structure in the economy. Change in the age structure has a direct impact on the relative supply of skilled labor. Since people have different saving tendency along the life cycle, change in the age structure also has an influence on the relative demand of skilled labor through changing asset accumulation in the economy. The ultimate effects of these two forces on the skill premium and the enrollment rate depend on the quantitative magnitude of both demand and supply effects.

I calibrate the model to match U.S. data for the period 1947-1951 as the initial steady state. Then, by feeding in the investment-specific technological change data from Cummins and Violante (2002) and the growth rate of the HSG cohort size from 1951 to 2000, I conduct perfect foresight deterministic simulations to compare with the data of the period 1951-2000 and counterfactual decomposition experiments to identify the effects of each force.

My results show that in terms of the skill premium, demographic change dwarfs ISTC before the late 1960s and accounts for about one third of the decline of the skill premium in the 1970s. However, ISTC takes over to drive the dramatic increase in the skill premium since the early 1980s. It explains about three fourth of the increases in the skill premium since 1981. ISTC is also a driving force behind the increasing trend in the enrollment rate. It explains about 30% of the increases in the enrollment rate during the post war period, while demographic change does not have a significant effect on the enrollment rate.

As shown in the text, the benchmark model overshoots the skill premium data and can only capture limited variations of the enrollment rate. (See Figure 16.) The possible reason could lie in the model-generated disutility cost of schooling. Future work should include other important determinants of the schooling choice into the model to improve its performance. For example, we can add a time trend in the disutility cost associated with the change in social norm of schooling. By doing so, we would have a closer match with the enrollment rate data. This would increase the relative supply of skilled labor and hence alleviate the overshooting of the skill premium. Another extension is to allow for unemployment. The United States experienced very high unemployment rate in the 1970s. Adding in different unemployment shock associated with educational choice could help to explain the remaining part of the decline of the skill premium which cannot be captured by the demographic change.

The model I present here also provides a platform for further research topics in educational policy, health economics and population economics. First, one could study the general equilibrium effect of tuition policies.
on the skill premium and enrollment rate in this framework. Second, the empirical evidence suggests that survival probability is different across educational groups. In this model, one could endogenize the survival probability and link it to human capital investment and hence could open up another avenue through which schooling choice may affect the economy. Finally, many countries are undergoing dramatic demographic transition to an aging society (for example, Asian countries such as China and Japan). This model can also help to predict what would happen to wage inequality and educational attainment in these countries through the transition.
Reference


12. The College Board: Trends in College Pricing 2003


Appendix A. Data Sources and Construction

The skill premium and relative supply of skilled labor data in Figure 1 are taken from Katz and Autor (1999). Data are from the 1940, 1950, 1960 Censuses and the 1964-1997 March CPS. The skill premium in their paper is the coefficient on workers with a college degree or above relative to high school graduates in a log weekly wage regression. The sample includes full-time full-year workers aged between 18 and 65. The relative supply of skilled labor is the ratio between college equivalents and non-college equivalents, using weeks worked as weights. Here college equivalents=CG+0.5×workers with some college, non-college equivalents=High School Dropout+HSG+0.5×workers with some college. Figure 1 is as same as Figure 1 in Acemoglu (2003). Please refer to Katz and Autor (1999) or Data Appendix in Acemoglu (2003) for detailed data construction.

HSG cohort size data in Figure 2 are from National Center for Education Statistics (NCES): Digest of Education Statistics (DES) 2002, Table 103, data of 1941, 1943 and 1945 are from DES 1970 Table 66. In this figure year means school year, for example, 1939 means the school year 1939-1940.

The 18-21 year college age population data in Figure 3 are from NCES: DES 2002 Table 15 for 1970-2000, NCES: DES 1995 for 1960-1969, Standard Education Almanac 1968 Table 1 for 1955-1959. The 17 year old population data are from NCES: DES 2002 Table 103.

College enrollment rates of HSG in Figure 4 are from NCES: DES 2002 Table 183 for 1960-2001 data. 1948-1959 data are calculated by the author. To construct them, first I take the 1948-1965 first-time freshmen enrolled in institutions of higher education data from NCES: DES 1967 Table 86, divided by the HSG cohort size as in Figure 2. Since first-time freshmen are not necessarily from recent HSG, I use the overlapped year 1960-1965 to calculate the average difference between our own calculation and the true data, then adjust our own calculation for the 1948-1959 period according to this difference.

Average TFRB charges data in Figure 5 are constructed as following. First, I obtain data about estimated Average Charges to Full-Time Resident Degree-Credit Undergraduate Students between 1956-57 and 1966-67 school year from Standard Education Almanac 1969, Table 120; 1967-68 to 1973-74 data from Standard Education Almanac 1981-82, P. 231-232; 1974-75 to 1983-84 data from Standard Education Almanac 1984-85, P. 328-329; 1984-85 to 2003-04 four year institution data from “the trends in college pricing 2003”, the College Board, Table 5a, 5b. 1948-1955 data are from Table 102: “Estimated Costs of Attending College, Per Student: 1931-1981” on Standard Education Almanac 1968. To make it consistent with the data after 1955, I use the overlapped 1956 data to adjust. Second, I focus only on public or private four-year institutions. I obtain the TFRB charges for those institutions. Third, I calculate the enrollment share of public and private 4-year institutions respectively. For the 1948-1964 data, I obtain the total fall enrollment in degree-granting institutions by control of institution (private vs. public) from NCES: DES 2002 Table 172, noticing that it is for all higher education institutions. Then from Table 173, I have total fall enrollment in degree-granting institutions by control and type of institution from 1965 to 2000. Fourth, I weight the average TFRB charges of public and private 4-year institutions by the enrollment share, then use the Personal Consumption Expenditure deflator from NIPA to express them in constant 2002 dollars. Finally, the third order moving average method is used to smooth the data.

The construction of the cohort-specific skill premium data is in the text.

The skill premium data used in this paper is the ratio of the real mean annualized wage of CG and HSG.
as in Figure 7. The data counterpart of the relative supply of skilled labor \((S/U)\) is the ratio of weeks worked of CG plus some college and HSG. I obtain these data from Katz and Autor (1999)’s dataset.

**Appendix B. Computation Algorithm to Stationary Equilibrium**

Given the parameter values as shown in the text, I compute the stationary equilibrium as following:

1. Guess the initial values for (per capita, also aggregate since the measure of agents is one) capital stock \(K_0\) and initial enrollment rate \(e_0\).

2. Given initial guesses, I can calculate the skilled labor \(S_0\) and the unskilled labor \(U_0\). Notice that for every \(j\), \(\lambda^c(a, j)\)=enrollment rate, \(\lambda^h(a, j)=1\)-enrollment rate, so by (20) and (21) I have
   \[
   S_0 = (\text{enrollment rate}) \cdot \sum_j \mu_j e_j^c
   \]
   \[
   U_0 = (1 - \text{enrollment rate}) \cdot \sum_j \mu_j e_j^h.
   \]

   Given all the inputs, from the firm’s FOCs (10)-(12), I can compute interest rate \(r\), wage rates \(w^c\) and \(w^h\).

3. Discretize the asset level \(-b \leq a \leq a_{\text{max}}\) (make sure that the borrowing limit \(b\) and the maximum asset \(a_{\text{max}}\) will never be reached). Given prices \(\{w^c, w^h, r\}\), feed into the normalized tuition data. By using backward induction (remember \(a_J = 0\)), I can solve the conditional value function \(V_s(a, 1)\), for \(s = h, c\), therefore obtain the cut-off disutility cost \(DIS(i^*) = (V_c(a = 0, 1) - V^h(a = 0, 1))/sd\), where \(sd\) is the scale factor of disutility cost which is calibrated to replicate the enrollment rate data.

4. From the CDF function of disutility cost, corresponding to \(DIS(i^*)\), I will have the new enrollment rate \(e_1\). Check the convergence criteria \(\frac{|e_0 - e_1|}{e_0} \leq tol_e\). If it is not satisfied, update it by relaxation method
   \[
e_2 = \kappa_e e_0 + (1 - \kappa_e)e_1,
   \]
   where \(0 < \kappa_e < 1\) is the relaxation coefficient for the enrollment rate.

5. Using the decision rules I obtain from step 3, \(A^s(a, j) \forall j, \forall a\) and following (18), I compute the age-dependent distributions by forward recursion. Then I use these distributions \(\lambda^s(a, j)\) and the age shares \(\mu_j\) to compute the per capita (next period) capital stock \(K_1\), and new transfers \(\xi_1\) as follows
   \[
   K_1 = \left(\sum_j \sum_a \sum_s \mu_j \lambda^s(a, j) A^s(a, j)\right)/(1 + n)
   \]
   where \(n\) is the growth rate of cohort size. Check the convergence criteria \(\frac{|K - K_1|}{K} \leq tol_K\) to see if it needs to stop. If not, use the relaxation method to update \(K\)
   \[
   K_2 = \kappa_K K_0 + (1 - \kappa_K)K_1
   \]
   where \(0 < \kappa_K < 1\) are the relaxation coefficients for the capital. Then set \(K_0 = K_2\), \(e_0 = e_2\), and go to step 1. The iteration will stop once all errors fall into the tolerance ranges.
6. Compute the aggregate consumption, investment, tuition expense, and output by using the decision rules, age-dependent distributions and age shares

\[ C = \sum_{j=1}^{J} \sum_{a} \sum_{s} \mu_j \lambda^s(a,j) C^s(a,j) \]

\[ TUITION = \sum_{j=1}^{J} \sum_{a} \mu_j \lambda^s(a,j) tuition_j \]

\[ X = (n + \delta) K. \]

7. Finally, check if market clearing condition given by (22) holds. If it does, stop.

**Appendix C. Computation Algorithm to Transition Path**

In this paper I follow Chen, Imrohoroglu and Imrohoroglu (2004) (also see Conesa and Kreuger (1999)) in computing a transition path from initial pre-1951 steady state towards a final steady state. In this way, I view 1952-2000 as a part of the transitional path period. For notation of the time period, I have \( t = 1 \) for 1951, \( t = T \) for the final steady state, \( t = 2, \ldots, T - 1 \) for the transitional period. I take the following steps in computation.

1. Compute the pre-1951 initial steady state by following the method described in Appendix B. Save the distribution \( \lambda^s(a,j), \forall j, \forall s \) for the later calculation.

2. Feed in the exogenous changes in growth rate of HSG cohort size \( \{n_t\}_{t=1}^{T} \), transformed investment-specific technological change \( \{B_t\}_{t=1}^{T} \) and the tuition payments \( \{tuition\}_{t=1}^{T} \).

3. Compute the final steady state at \( t = T \). Save the value function \( V^*(a,j), \forall j, \forall s \) for the later calculation.

4. Take the initial and final steady state values of capital stock \( K \) and enrollment rate \( e \), use linear interpolation to guess the sequences of \( \{K_t\}_{t=1}^{T} \) and \( \{e_t\}_{t=1}^{T} \). This is the initial guess for the transition path computation.

5. Start from \( T - 1 \), take the value function of final steady state as the terminal values \( V^*(a,j,T) \), solve the individual optimization problem by backward induction, obtain the decision rules for all cohorts through the transition path.

6. Using the distribution of pre-1951 steady state as the initial asset distribution, together with the decision rules collected from step 5, calculate \( \{\lambda^s(a,j,t)\}_{t=2}^{T} \) by forward recursion. Then use them to calculate new \( \{K_t\}_{t=1}^{T} \) and \( \{e_t\}_{t=1}^{T} \).

7. Compare the new sequences of endogenous variables \( \{K_t\}_{t=1}^{T} \) and \( \{e_t\}_{t=1}^{T} \) with initial guess and iterate on them until convergence.
Figure 1. The Skill Premium and Relative Supply of the Skilled Labor: 1949-1996
Figure 2. High School Graduates Cohort Size

Figure 3. College Age Population: 1955-2000
Figure 4. College Enrollment Rate of High School Graduates: 1948-2001

Figure 5. Average Real TFRB charges: 1948-2001
Figure 6. Real Annualized Mean CG and HSG Wage: 1949-2002

| year | 1962 | 1963 | 1964 | ...........
|------|------|------|------|--------------
| Age  | 18   | 19   | 20   | ...........
| HSG  | mean | HSG  | mean | ...........
| wage | wage | wage | wage | ...........

Figure 7. Synthetic 1962 HSG Cohort
<table>
<thead>
<tr>
<th>year</th>
<th>1966</th>
<th>1967</th>
<th>1968</th>
<th>.........</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>.........</td>
</tr>
<tr>
<td>CG</td>
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<td>CG</td>
<td>.........</td>
</tr>
<tr>
<td>mean</td>
<td>mean</td>
<td>mean</td>
<td>mean</td>
<td>.........</td>
</tr>
<tr>
<td>wage</td>
<td>wage</td>
<td>wage</td>
<td>......</td>
<td>.........</td>
</tr>
</tbody>
</table>

Figure 8. Synthetic 1962 CG Cohort

Figure 9. Life-Cycle HSG Wage Profile: 1955 Cohort
\[ y = 0.624x^3 - 123.12x^2 + 7779.4x - 98023 \]
\[ R^2 = 0.8013 \]

\[ y = -40.294x^2 + 4281.6x - 51328 \]
\[ R^2 = 0.7937 \]

Figure 10. Life-Cycle CG Wage Profile: 1955 Cohort

Figure 11. Cohort-Specific Return to Schooling: 1948-1991
\[ y = 0.7679x + 34.923 \]
\[ R^2 = 0.4044 \]

Figure 12. CDF of the Disutility Cost

Figure 13. Age Efficiency Unit Profile: 1949-2002 Average
Figure 14. Life Cycle Profiles of HSG and CG

Figure 15: Log of Investment-Specific Technological Change

Source: Cummins and Violante (2002)
Figure 16. Model vs. Data: Benchmark Case
Figure 17. Model vs. Data: Demographic Change Only
Figure 18. Model vs. Data: ISTC Only
Figure 19. Skill Premium: Model vs. Data

Figure 20. College Enrollment Rate: Model vs. Data
Figure 21. Capital-Skilled Labor Ratio: Decomposition

Figure 22. Relative Supply of Skilled Labor: Decomposition