Anonymity and Individual Risk

Preliminary draft

January 2007

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Anonymity in the market place is a cornerstone of the standard competitive equilibrium framework in which agents are assumed to be price-takers. Anonymity is formally incorporated into a model of risk sharing by assuming that the central planner is unable to determine who is who, even though the planner knows the distribution of individual risks and characteristics in the economy. The constrained Pareto optimal allocation then has the property that it is envy-free in that there are fair net trades for all agents. This results in a conflict between equity and efficiency. The allocation is multilaterally incentive compatible and there are no arbitrage opportunities. The allocation, which also has the property that it is coalitionally fair, will result in a distribution of the intertemporal marginal rate of substitution across heterogeneous agents. This has strong implications for the stochastic discount factor used in dynamic, stochastic general equilibrium models.

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Implications for asset pricing, consumption and income distribution are explored in a simple model in which agents are ex ante heterogeneous because they face different income distribution risk. Agents are anonymous in the sense that the central planner is unable to determine who is who, even though the planner knows the distribution of individual risks and characteristics in the economy. Moreover, the central planner cannot prevent side payments and trading among agents. With side payments, the appropriate incentive compatibility constraint is the multilateral incentive compatibility constraint, as described by Hammond [1987] for example, and not the standard individual incentive compatibility constraint, which rules out side payments and trading coalitions. An allocation is multilateral incentive compatible (MIC) if no group or coalition can improve upon an allocation by misrepresentation and trading among itself. The result is that the allocations are envy free in that all agents face an identical generalized budget constraint for net trades.

To start, the first-best allocation is derived for two cases: pooled endowment across all agent types and nonpooled endowment in which each agent type trades only with agents of the same type. In the pooled endowment economy, the competitive equilibrium supporting the allocation requires that contingent claims prices differ by agent type for the same endowment realization, which is observable by all agents. The Second Welfare Theorem holds in this case as long as there is a redistribution of the endowment across agent types. In the nonpooled equilibrium, there are arbitrage profits opportunities across agent types, so that the allocation can be supported as a competitive equilibrium only if markets are exclusive, meaning that only agents of the same type are allowed to trade in a given market. Exclusivity of markets cannot be maintained if the agent’s type is private information. Under that assumption, the allocations of the pooled resource economy cannot be supported as a competitive equilibrium because the central planner cannot implement the required transfers. The allocations of the nonpooled resource economy cannot be supported because the arbitrage opportunities created by separating markets by type imply that agents have
an incentive to trade with agents of different types. Hence, under the assumption of anonymity, the central planner could still implement transfers that satisfy individual incentive compatibility constraints, but the central planner cannot force agents to consume the transfers nor can it prevent agents from forming coalitions to trade among themselves. Hence, the constrained Pareto optimal solution is one in which all net trades are envy-free. In this application, it means that all net trades are restricted by the same generalized budget constraint.

Agents only partially insure against endowment risk because a coalition comprised of agents of the same type will be blocked. Markets fail to separate because agents cannot be prevented from participating in a market in which they can fulfill all contracts they voluntarily enter because of their risk class or individual-specific attributes. Essentially ex ante heterogeneous agents can achieve full risk-sharing within a risk class if they participate in segregated markets, where markets are segregated by risk class. But an individual agent may be better off to buy or sell claims in another market. Since market participation cannot be limited only to agents from the same risk class, the prices for a contingent claim conditional on a particular endowment realization will be the same across risk classes. Agents are then able to insure against income shocks but not against differences in the distribution of income by risk class. As a result, the there is distribution of the intertemporal marginal rate of substitution (IMRS) across agents types. This has important implications for the pricing kernel in asset markets.

1 Basic Model

I start by examining a two period model and later extend it to an infinite horizon model. This is a pure endowment economy in which there is no storage between periods. Assume there is a countable infinity of agents indexed by \( i \in I = \{1, 2, 3, \ldots \} \) and let \( \varphi \) denote the finitely additive cumulative distribution function over \( I \). In the first period, the endowment for each agent is deterministic and
equal to \( y_0 > 0 \).

An agent’s endowment is random in the second period. Let \( \theta \in \Theta \) denote the random endowment, where \( \Theta \equiv \{\theta_1, \ldots, \theta_M\} \) is a discrete random variable such that \( \theta_1 \geq 0 \) and \( \theta_M \) is finite and the cardinality \( M \) is finite. Let \( \eta \in N \equiv \{\eta_1, \ldots, \eta_L\} \) be a discrete random variable and let \( f(\eta) \) denote the measure over \( N \), where \( L \) is finite. At the beginning of time, each agent draws an \( \eta \), which is fixed for rest of his lifetime; for reasons provided below the realization of this random variable is called the agent’s type.

The distribution of \( \theta \) is parameterized by \( \eta \) so that, for a particular \( \eta \in N \), the distribution of \( \theta \) is \( g(\theta \mid \eta) \). For example, if \( \eta_2 > \eta_1 \), \( g(\theta \mid \eta_2) \) might be a mean-preserving spread of \( g(\theta \mid \eta_1) \). More generally, \( \eta \) should be thought of as a vector of attributes. The key idea is that the distribution over the endowment sample space \( \Theta \) will differ across agents, with a fraction \( f(\eta) \) facing distribution \( g(\theta \mid \eta) \). Because the realization of \( \eta \) determines the endowment distribution, an agent with \( \eta = i \) and an agent with \( \eta = j \) such that \( i \neq j \) are said to be in different income distribution risk categories.

There is no aggregate uncertainty. Total endowment in the first period is \( y_0 \) where \( y_0 = \sum_{\eta \in N} f(\eta)y_0 \). Since there is a countable infinity of agents, the sample realization over the population in the second period is a pair \( \{\theta_i, \eta_i\} \) where \( i \in I \). Using the Law of Large Numbers, the frequency of observing the pair \( (\theta_i, \eta_j) \) \( \theta_i \in \Theta \) and \( \eta_j \in N \), is \( g(\theta_i \mid \eta_j)f(\eta_j) \). Total endowment in the second period is

\[
y = \sum_I \theta_i \varphi(di) = \sum_{\eta \in N} \sum_{\theta \in \Theta} f(\eta)g(\theta \mid \eta) .
\tag{1}
\]

The distribution of \( \theta \) for the economy is a mixture

\[
m(\theta) \equiv \sum_{\eta \in N} f(\eta)g(\theta \mid \eta) .
\tag{2}
\]
Define the mean of the endowment for type $\eta$ as

$$\theta_m(\eta) = \sum_{\theta \in \Theta} g(\theta | \eta) \theta.$$  

Before there is any trading, agents observe their realization of $\eta$. A type-$\eta$ agent has preferences

$$U(c_0) + \sum_{\theta \in \Theta} g(\theta | \eta) V(c_1).$$  

(3)

The functions $U, V$ are continuous and twice continuously differentiable, strictly increasing and strictly concave. Also, as $c \to 0$, $U'(c) \to \infty$ and $V'(c) \to \infty$. Notice the expectation in (3) uses the individual agent’s conditional probability of realizing $\theta$. Since there is no aggregate uncertainty, the only risk an agent faces after realizing $\eta$ is in second period endowment. A fraction $m(\theta)$ of agents have realization $\theta$. Of those, a fraction

$$\frac{f(\eta) g(\theta | \eta)}{m(\theta)}$$

are type $\eta$. Hence a fraction $m(\theta)$ of agents have endowment $\theta$ with certainty.

The commodity space is $\mathbb{R}^{M+1}_+$. A consumption allocation for a type $\eta$ agent is an $M + 1$-dimensional vector

$$c(\eta) \equiv (c_0(\eta), c(\theta_1 | \eta), \ldots, c(\theta_M | \eta)).$$

A feasible allocation is an $L$-tuple $(c(1), \ldots, c(N))$ such that $c(j) \in \mathbb{R}^{M+1}_+$ for $j \in N$ where

$$y_0 = f \cdot c^0 = \sum_{i=1}^{L} f(\eta_i)c^0(\eta_i)$$

(4)

and

$$\sum_{\eta \in N} f(\eta) \sum_{\theta \in \Theta} g(\theta | \eta) \theta \geq \sum_{N} f(\eta) \sum_{\Theta} g(\theta | \eta) c(\theta | \eta).$$  

(5)

Let $C \in \mathbb{R}^{M+1}_+$ be the set of all feasible allocations; this set is convex and closed.
2 Pareto Optimal Allocations

Two central planning problems will be described in this section. In the first, the central planner pools the resources of the economy and jointly maximizes the utility of the \( L \) types of agents. The result is the *pooled-resource allocation*. The pooled resource allocation requires lump-sum transfers across the different types of agents as well as transfers across agents of the same type. The competitive equilibrium supporting the Pareto optimal allocations requires that agents of different types (different \( \eta \) realizations) face different prices. In the second version, the central planner treats the \( L \) types of agents as if they are members of different economies; essentially the central planner doesn’t pool the resources of the \( L \) types of agents. The resulting allocation requires lump-sum transfers among agents of the same type but no resources are transferred across different types. This is referred to as the *nonpooling allocation*. As will be demonstrated, the competitive equilibria supported under these two cases requires *exclusivity* in markets: agents can trade only with agents of the same type.

**Pooled Resources**

Let \( \phi(\eta) \) denote the Pareto weight attached to a type \( \eta \) agent. Under these conditions, the problem solved can be stated as follows:

\[
\max \left\{ \sum_{\eta \in N} \phi(\eta) \left[ U(c_0(\eta)) + \sum_{\Theta} g(\theta \mid \eta)V(c(\theta \mid \eta)) \right] \right\},
\]

subject to

\[
y_0 \geq \sum_{\eta \in N} f(\eta)c_0(\eta), \tag{7}
\]

\[
y_1 \geq \sum_{\eta \in N} \sum_{\Theta} f(\eta)g(\theta \mid \eta)c(\theta \mid \eta). \tag{8}
\]
Let $\mu_0$ denote the Lagrange multiplier for the first resource constraint and $\mu_1$ for the second. The first-order conditions are

$$\frac{\phi(\eta)}{f(\eta)} U_1(c_0(\eta)) = \mu_0, \quad (9)$$

$$\frac{\phi(\eta)}{f(\eta)} V_1(c(\theta | \eta)) = \mu_1. \quad (10)$$

Hence marginal utility varies inversely with $\frac{\phi(\eta)}{f(\eta)}$ and consumption in the second period is invariant with respect to $\theta$. If $\phi(\eta) = f(\eta)$, as is often assumed, then $U_1(c_0(\eta)) = U_1(y_0)$ and $V_1(c(\theta | \eta)) = V_1(y_1)$. The central planner will optimally choose to ignore the idiosyncratic income distribution risk for each type, equalizing second-period consumption across types. The pooled-resources allocation requires lump-sum transfers equal to

$$\tau(\eta) = \theta_m(\eta) - y_1$$

across the $L$ different categories of agents. A type $\eta$ agent with $\theta$ receives

$$\tau_p(\theta) = \theta - y_1.$$

Notice that certain groups pay higher taxes than other groups since $\tau(\eta) > 0$ for some $\eta$ while $\tau(\hat{\eta}) < 0$ for others, although all agents with realization $\theta$ receive an identical transfer $\tau_p(\eta)$ regardless of type.

The associated competitive equilibrium supporting this allocation requires prices $q : \mathbb{R}_+^{M+1} \to \mathbb{R}_+$ to be indexed by $\eta \in N$. There must be exclusivity in markets in that a type $\eta$ agent can purchase contingent claims only in the $\eta$ market and cannot purchase claims in the $\hat{\eta}$ market, where $\hat{\eta} \neq \eta$. Let $q(\theta | \eta)$ denote the price of a contingent claim conditional on $\theta$ for a type $\eta$ agent. Define the net trades

$$x_0(\eta) = c_0(\eta) - y_0$$

$$x(\theta | \eta) = c(\theta | \eta) - [\theta - \tau_p(\theta)]$$
where the mandatory lump-sum state contingent taxes are incorporated. A type \( \eta \) agent faces lifetime budget constraint

\[
0 \leq y_0 + x_0(\eta) + \sum_{\theta} q(\theta \mid \eta)[\theta + x(\theta \mid \eta) - \tau_p(\theta)].
\]  

(11)

The agent maximizes his objective function (3) subject to the budget constraint (11). The simplified first-order conditions reduce to

\[
q(\theta \mid \eta) = \frac{g(\theta \mid \eta)V_1(x(\theta \mid \eta) + \theta)}{U_1(x_0(\eta) + y_0)}.
\]

If \( q(\theta \mid \eta) = g(\theta \mid \eta) \) then consumption is constant in the second period. In particular, \( c_0(\eta) = y_0 \) and \( c(\theta \mid \eta) = y_1 \), which is the central planner’s allocation when \( \phi(\eta) = f(\eta) \). Observe that the intertemporal marginal rate of substitution (IMRS)

\[
\frac{V_1(y_1)}{U_1(y_0)}
\]

is equalized across agents.

The maximization of the objective function (3) subject to the budget constraint (11), is dual to an expenditure minimization problem

\[
E(q(\eta), \bar{U}^*) \equiv \min\left[x_0 + \sum_{\theta} q(\theta \mid \eta)x(\theta \mid \eta)\right]
\]

subject to

\[
U(x_0 + y_0) + \sum_{\theta} g(\theta \mid \eta)V(x(\theta \mid \eta) + \theta) \geq \bar{U}^*
\]

(13)

where \( q_0 = 1, \bar{U}(\eta) \equiv U(y_0 + V(y_1) \) and \( q(\eta) = \{1, q(\theta_1 \mid \eta), \ldots, q(\theta_M \mid \eta)\} \). This defines the expenditure function \( E(\cdot, \cdot) : \mathbb{R}^{M+2} \rightarrow \mathbb{R}_+ \). All agents achieve the same level of lifetime utility because agents are offered a net trade choice set with prices indexed by type and there is a lump-sum redistribution of wealth. The type \( \eta \) agent is offered a net trade choice set \( B_x(\cdot \mid \eta) \) associated

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with the expenditure minimization problem where

\[ B_x(q(\eta)) = \inf q(\eta) \cdot x : x \in X \]

Hence, agents are offered different net trade choice sets based on type.

**Separate Resources**

If the central planner treats the \( L \) types of agents as separate groups, then for each \( \eta \in N \), the central planner maximizes the expected utility of the representative type \( \eta \) agent (3) subject to

\[
y_0 \geq c_0
\]

\[
\theta_m(\eta) \geq \sum_{\Theta} g(\theta \mid \eta)c
\]

Clearly \( c_0 = y_0 \) and

\[
g(\theta \mid \eta)V_1(c) = \mu(\eta)g(\theta \mid \eta),
\]

where \( \mu(\eta) \) is the Lagrange multiplier. It follows that \( c \) is invariant with respect to \( \theta \) and \( c = \theta_m(\eta) \).

A type \( \eta \) with endowment realization \( \theta \) will receive a transfer equal to

\[
\tau_n(\theta \mid \eta) = \theta_m(\eta) - \theta.
\]

No resources are transferred across groups. In particular

\[
\sum_{\Theta} \tau_n(\theta \mid \eta) = 0.
\]

The contingent claims market supporting this allocation is a familiar one. Let \( q(\theta \mid \eta) \) denote the price of a contingent claim for a type \( \eta \) conditional on \( \theta \). The type \( \eta \) household maximizes (3) subject to

\[
0 = [y_0 + x_0(\eta)] + \sum_{\Theta} q(\theta \mid \eta)[x(\theta, \eta) + \theta]
\]
If \( q(\theta \mid \eta) \) is proportional to \( g(\theta \mid \eta) \) for all \( \theta \) then all agents of the same type have identical consumption \( c_0(\eta) = y_0 \) and \( c(\theta \mid \eta) = \theta_m(\eta) \). The IMRS is now
\[
\frac{V_1(\theta_m(\eta))}{U_1(y_0)}
\]
and is not equal to the IMRS for the pooled resources economy with state-contingent lump-sum taxes as long as \( \theta_m(\eta) \) varies with \( \eta \). The intertemporal marginal rate of substitution (IMRS) for a type-\( \eta \) agent is
\[
q(\eta) \equiv \frac{V_1(\theta_m(\eta))}{U_1(y_0)}
\]
which will vary across groups, implying that the risk free rate, \([q(\eta)]^{-1}\) will be unequal across \( \eta \) types.

Define \( \bar{U}(\eta) = U(y_0) + V(\theta_m(\eta)) \). The expenditure function that is dual to the utility maximization problem is \( E(q(\eta), \bar{U}(\eta)) \), where the function \( E \) was defined earlier. Once again, the net trade choice set \( B_x(q \mid \eta) \) will differ across groups and exclusivity of markets is required. Notice that the prices faced by a type \( \eta \) agent in the two versions of the model are identical, however the level of utility is different because lump-sum redistribution of wealth is not allowed in the separate resource economy. Clearly some agents are better off in the pooled-resources allocation, while others are worse off \( (U^* > \bar{U}(\eta) \) for some \( \eta \).

The transfers can be implemented only if the central planner can identify an individual agent by his \( \eta, \theta \) pair. In this sense, the agents are not anonymous. In addition, both economies require

\(^2\)The first-order conditions are
\[
U_1(c^0(\eta)) = \lambda(\eta)
\]
\[
\lambda(\eta)q(\theta \mid \eta) = g(\theta \mid \eta)V_1(c(\theta \mid \eta))
\]
that the net trade set offered to agents be indexed by the agent’s type. In the next section, the concept of fair net trade and anonymity will be introduced.

3 Limited Redistribution

The notion of envy-free or an anonymous allocation is described in this section. To understand the constrained Pareto optimal solution, it is useful to summarize the five key features of the central planning problems just described:

1. The pooled resource allocation requires lump-sum transfers for the Second Welfare Theorem to apply. The central planner is able to identify the type of an agent as well as observe his second-period endowment realization.

2. The net trade choice set must be different for the \( L \) groups of agents to support the allocations chosen by the central planner for both the pooled resource and separate resource economies.

3. There must be exclusivity in the choice sets: a type \( \eta \) agent cannot choose from the net trade set \( B_x(q(\hat{\eta})) \) where \( \eta \neq \hat{\eta} \). Otherwise, there are arbitrage profit opportunities.

4. The intertemporal marginal rate of substitution varies across groups when resources are not pooled.

5. The utility maximization problem is dual to an expenditure minimization problem that requires a price for a unit of consumption in state \( \theta \) to differ across agent types: \( q(\theta \mid i) \neq q(\theta \mid j) \) for \( i \neq j \) in general. Hence the fraction \( m(\theta) \) of agents realization endowment \( \theta \) pay different prices for consumption insurance in state \( \theta \).

If we assume that the central planner is unable to redistribute or transfer resources across agents, then the Second Welfare Theorem will generally fail to hold. If the central planner is able to
maintain exclusivity in markets and thereby offer net trade sets that differ across agent type, then the allocation of the separated resource economy can be achieved, but notice that the risk free rate will differ across groups and there may be an arbitrage opportunity. If exclusivity of markets cannot be maintained, then the central planner is unable to offer different net trade sets and the allocations of either the pooled or separate resource economies cannot be supported by a competitive equilibrium. Exclusivity in markets may be difficult to maintain since the dual space to the utility maximization problem differs across agents. A type \( \eta \) agent may find that his net trades \( x_0(\eta), x(\theta_1 \mid \eta), \ldots, x(\theta_M \mid \eta) \) are minimized for some \( q(\hat{\eta}) \) such that \( \hat{\eta} \neq \eta \).

Let \( c(\eta) \in \mathbb{R}^{M+1} \) be a consumption vector for agent-type \( \eta \). The following definition is based on Mas-Colell \( \text{[1]} \) and earlier by \( \text{[2]} \).

**Definition 1** The feasible allocation \( c^* = (c(1), \ldots, c(L)) \) is self-selective (or anonymous or envy-free in net trades) if there is a set of net trades \( B \in \mathbb{R}^{M+1} \), called the generalized budget set, such that for every agent-type \( \eta \), \( x(\eta) = c(\eta) - \omega \) solves the problem

\[
\max \left[ U(x_0(\eta) + y_0) + \sum_\theta g(\theta \mid \eta)V(x(\theta \mid \eta) + \theta) \right] \quad (17)
\]

subject to

\[
x(\eta) \in B \quad (18)
\]

\[
x(\eta) + \theta \geq 0 \quad (19)
\]

This formulation requires that agents have the identical budget constraint regarding their net trades. The following assumption is imposed.

**Assumption 1** Agents are assumed to have anonymity.

Anonymity in the market-place, as defined above, implies that exclusivity in markets cannot be maintained. The utility maximization problem solved by a type \( \eta \) agent must be dual to the
expenditure function $E(q, \cdot)$, where $q$ is identical for all agents. Hence agents can achieve different levels of expected utility (although all agents of the same type must be identical) by choosing different bundles of net trades, subject to feasibility, but all agents face the identical price function $q$.

Anonymity can be motivated in several ways. One way is to assume that the agent’s type is private information. Then the central planner cannot distinguish among agents and is therefore unable to maintain exclusivity in markets or to implement lump-sum redistributions of wealth. Another way is to assume that the technology to maintain exclusivity in markets is unavailable, or that discrimination against different types of agents, in the form of offering different price functions $q(\eta)$ is illegal. Regardless of the reasons motivating the assumption, the result is a constrained Pareto optimal solution.

There is an extensive literature on optimal contracts when $\eta$ is private information. In general, the optimal contract has the property of truthful revelation. Incentive compatibility constraints are imposed along with the resource constraints. The resulting constrained Pareto optimal allocations cannot be achieved through a decentralized competitive equilibrium. The approach I am taking here is to assume that resources are allocated subject to the restriction of anonymity, as defined above. Agents then self-select in their choice of net trades and the resulting allocations are incentive compatible.

Agents are now assumed to be anonymous, in that the central planner knows the distribution $f$ over $N$ and the functions $g(\theta \mid \eta)$ but is unable to identify which agent is an $\eta, \theta$ type until after the net trades are made and, therefore, cannot implement the lump-sum transfers $\tau_p(\theta, \eta)$ defined above. The set of feasible net trades is defined as $X \in \mathbb{R}^{M+1}$ such that $x + \theta \geq 0$ and the resource constraints are satisfied. Define $B_x$ as the support of the distribution induced by the net trades $x$. 
Specifically, the support is defined for any \( q \in \mathbb{R}^{M+1} \) as

\[
B_x(q) = \inf\{q \cdot x : x \in X\}.
\]

For each \( x \in X \), the function \( B_x(\cdot) \) is a linear function of \( q \) while, for each element of \( q \),

\[
B_x(\bar{q}(\theta_1), \ldots, q, \ldots, \bar{q}(\theta_M))
\]

is the minimum value of the various linear functions \( B_x(\cdot) \) at \( \bar{q}(\theta_1), \ldots, q, \ldots, \bar{q}(\theta_M) \). When the set \( X \) is convex, the support function \( B \) provides a dual description of \( X \) since \( X \) can be reconstructed from \( B(q) \). In particular, \( \{ x \in \mathbb{R}^M : q \cdot x \geq B(q) \text{for every} q \} \). The function \( B \) is homogeneous of degree one and is concave. A feasible allocation \( c^0, c^1 \) is anonymous or envy-free in net trades if there is a set of net trades \( B \subset \mathbb{R}^{L \times L \times M} \), to be called the generalized budget set, such that for every \( \eta \in N \), \( x(\eta) = [c^0(\eta) - y_0, c(\theta_1 | \eta) - \theta_1, \ldots, c(\theta_M | \eta) - \theta_M] \) solves the problem

\[
\max \left[ U(x^0(\eta) + y_0) + \sum_{\theta \in \Theta} g(\theta | \eta)V(x(\theta | \eta) + \theta) \right]
\]

subject to

\[
0 \geq x_0 + \sum_{\theta \in \Theta} q(\theta)x(\theta) \tag{21}
\]

\[
0 \geq x(\theta) + \theta \tag{22}
\]

The second constraint is just the nonnegativity of consumption. Hence, all agents face the same budget constraint for their net trades - in particular this also means the endowment of an agent, say a type \( \eta \) agent realizing \( \theta \) in the second period will have his endowment valued at the same price as any other agent with endowment \( \theta \). Notice this is not true in the two versions of the Pareto optimal problem described earlier. A type \( \eta \) agent wishing to implement a particular set of net trades \( x(\eta) \) will face the same relative price as any other agent. An agent then self-selects
by choosing the set of net trades that maximizes his expected utility. Agents will reveal their type through the choices they make. The resulting allocation will be incentive compatible in that no agent would choose the net trade of any other agent of a different type.

Before the first-order conditions are derived, observe that the nonnegativity constraints will never bind because of the assumption that the Inada conditions hold. If marginal utility tends to infinity as consumption goes to zero, then if prices are strictly positive and finite, then agents will never choose negative or zero consumption in any state, as long as he faces a positive probability of being in that state. Let $\lambda(\eta)$ denote the Lagrange multiplier for the generalized budget constraint. Without a loss of generality, set $q_0 = 1$. The first order conditions are

\begin{equation}
\lambda(\eta) = U_1(x^0(\eta) + y_0),
\end{equation}

\begin{equation}
\lambda(\eta)q(\theta) = g(\theta \mid \eta)V_1(x(\theta \mid \eta) + \theta).
\end{equation}

Eliminate the multiplier to obtain

\begin{equation}
q(\theta) = \frac{g(\theta \mid \eta)V_1(x(\theta \mid \eta) + \theta)}{U_1(x^0(\eta) + y_0)}.
\end{equation}

The key property emerges: the intertemporal marginal rate of substitution (IMRS)

\[ \frac{V_1(x(\theta \mid \eta) + \theta)}{U_1(x^0(\eta) + y_0)} \]

is no longer equal across $\eta$. However, the weighted IMRS is equalized, where the weight is the conditional probability $g(\theta \mid \eta)$, which varies across $\eta$.

The equilibrium conditions are now discussed. The real value of the endowment, equal to $\sum_\theta q(\theta)\theta$ (aggregate demand), must equal the endowment, or

\begin{equation}
\sum_\eta \sum_\theta f(\eta)g(\theta \mid \eta)\theta = \sum_\theta q(\theta)\theta
\end{equation}
Multiply the budget constraint for a type \( \eta \) agent by \( f(\eta) \) and then sum over all \( \eta \), and rewrite to obtain

\[
\sum_\theta q(\theta) \theta = \sum_\eta f(\eta) \sum_\theta q(\theta) c(\theta | \eta)
\]

Substitute this into the previous equation

\[
\sum_\eta \sum_\theta f(\eta) g(\theta | \eta) \theta = \sum_\eta f(\eta) \sum_\theta q(\theta) c(\theta | \eta)
\]

or

\[
\sum_\eta \sum_\theta f(\eta) [g(\theta | \eta) \theta - q(\theta) c(\theta | \eta)]
\]

For each \( \theta \in \Theta \), market clearing in the \( \theta \) market requires

\[
\sum_\eta [f(\eta) g(\theta | \eta) - q(\theta)] [\theta - c(\theta | \eta)] = 0
\]

### 3.1 Equilibrium with Anonymity

To solve for the equilibrium, choose a numeraire consumer - say \( \eta = 1 \) - and define

\[
\bar{\lambda}(\eta) \equiv \lambda(\eta) / \lambda(1)
\]

Then (23) can be expressed as

\[
\bar{\lambda}(\eta) = \frac{U_1(x_0(\eta) + y_0)}{U_1(x_0(1) + y_0)}.
\]

Given the \( N - 1 \) values of \( \bar{\lambda} \) and \( x_0(1) \), define the function

\[
x_0(\eta) = H_0(\lambda, x_0) = (U_1)^{-1} (\lambda U_1(x_0 + y_0)) - y_0.
\]

Observe that \( H_0 \) is decreasing in its first argument and increasing in its second. Given the \( N - 1 \) values of \( \bar{\lambda} \), the equilibrium net trades in the first period can be determined from the resource constraint

\[
0 = f(1) x_0(1) + \sum_{\eta=2}^{N} f(\eta) H_0(\bar{\lambda}(\eta), x_0).
\]
The right side is strictly increasing in \( c_0 \), given the \( L - 1 \) values \( \bar{\lambda} \). Hence there exists a unique solution \( c_0(\bar{\lambda}) \). Next, solve (24) for the price \( q(\theta) \) and observe that
\[
q(\theta) = \frac{g(\theta \mid \eta)V_1(x(\theta \mid \eta) + \theta)}{\lambda(\eta)} = \frac{g(\theta \mid 1)V_1(x(\theta \mid 1) + \theta)}{\lambda(1)}.
\]

Define
\[
\bar{g}(\theta \mid \eta) = \frac{g(\theta \mid \eta)}{g(\theta \mid 1)}
\]
Then rewrite (33) as
\[
\bar{\lambda}(\eta) = \frac{\bar{g}(\theta \mid \eta)V_1(x(\theta \mid \eta) + \theta)}{V_1(x(\theta \mid 1) + \theta)}.
\]

Given the \( N - 1 \) values of \( \bar{\lambda} \) and \( M \) values of \( x(\theta \mid 1) \), define the function
\[
x(\theta \mid \eta) = H_\eta(\bar{\lambda}(\eta), \theta, x) - \theta
\]
as the solution to
\[
V_1(x(\theta \mid \eta) + \theta) = \frac{\bar{\lambda}(\eta)V_1(x(\theta \mid 1) + \theta)}{\bar{g}(\theta \mid \eta)}
\]
Substitute these functions into the lifetime budget constraint
\[
0 = H_0(\lambda, x_0) + \sum_\theta q(\theta)H_\eta(\lambda, \theta, x(\theta)))
\]
This equation can be solved for \( \lambda \), given \( x_0, x(\theta \mid 1) \) where
\[
q(\theta) = \frac{g(\theta \mid 1)V_1(x(\theta \mid 1) + \theta)}{U_1(x_0(1) + y_0)}
\]
The final step is to solve for the \( M + 1 \) values of \( x(1) = \{x_0, x(\theta_1 \mid 1), \ldots x(\theta_M \mid 1)\} \). Define the operator \( T_\eta : \mathbb{R}_{+}^{M+1} \to \mathbb{R}_{+} \) as
\[
\bar{\lambda}(\eta) = T_\eta(x)
\]
Hence given the vector \( x \in \mathbb{R}_{+}^{M+1} \), the operator \( T_\eta \) yields the value of the multiplier that solves the equation.
The next step is to incorporate equilibrium in each of the $M$ contingent claims markets. The payments made for a contract contingent on $\theta$ equals $\sum_\eta f(\eta)q(\theta | \eta)$. The total number of contingent claims held for $\theta$ equals $\sum_\eta f(\eta)g(\theta | \eta)x(\theta | \eta)$. In equilibrium,
\[
0 = \sum_\eta f(\eta)[q(\theta) - g(\theta | \eta)]x(\theta | \eta)
\] (38)
Substituting in for $q(\theta)$, and using the functions $H_\eta$,
\[
0 = \sum_\eta f(\eta)g(\theta | 1)[V(x(\theta) + \theta) - g(\theta | \eta)][\theta - H_\eta(T_\eta(x, \theta, x(\theta)))]
\] (39)
This established a mapping. Let $x^0 \in \mathbb{R}_+^{M+1}$ be given. Define $T : \mathbb{R}_+^{M+1} \rightarrow \mathbb{R}_+^{M+1}$ as the solution $x^1$ to the system of $M$ equations of the form (39).

**Incentive Compatibility**

Let $x^*(\eta)$ denote the solution to the maximization problem () with constraint (). Notice that there is no incentive compatibility constraint that is imposed on the opportunity set of the agent. The incentive compatibility constraint is
\[
U(y_0 + x(\eta)) + \sum_\Theta g(\theta | \eta)V(\theta + x(\theta | \eta)) \geq U(y_0 + x(\eta)) + \sum_\Theta g(\theta | \eta)V(\theta + x(\theta | \tilde{\eta}))
\] (40)
for all $\tilde{\eta} \neq \eta$ and $\tilde{\eta}, \eta \in N$. The question arises whether the net trade $x^*_\eta(\eta)$ satisfied the incentive compatibility constraint. The answer is that it does by construction. Since all net trades $x_0(\eta), x(\theta_1 | \eta), \ldots, x(\theta_M | \eta)$ for $\eta \in N$ lie along the same hyperplane, or equivalently all agents face the same generalized budget constraint, the net trades chosen by other agents $x^*_\eta(\tilde{\eta})$ where $\tilde{\eta} \neq \eta$ are within the generalized budget constraint faced by a type $\eta$ agent. Hence, the solution to the maximization problem will satisfy (40). Hence the agent’s actions, summarized by his net trades $x^*(\eta)$, will truthfully reveal his type. As long as the agent faces the generalized budget constraint, he has no incentive to deviate from his actions. The approach taken here differs from
the standard one in which a principal designs an incentive compatible contract. The allocations are **multilaterally incentive compatible**, using the terminology of Hammond and Haubrich.

### 3.2 Coalitions and Market Structure

Boyd, Prescott and Smith model economic organizations as coalitions of agents instead of exogenously imposing an organizational structure and, by doing so, show that the pattern of trade and resulting allocations will be affected. In the simple economy studied above, if we assume that there is an insurance industry selling insurance under a zero-profit assumption, then markets will be separated by type. If we drop the assumption that there is an insurance industry and allow agents of different types to trade directly, the equilibrium allocations are changed. The goal then is to determine what sort of organization will emerge endogenously, where an organization is a coalition of agents, and to determine what type of trading structure will result.

A coalition is a subset of agents $S \in I$. Agents form coalitions with the goal of pooling risk and exploiting gains from trade. The economy with separated markets, in which insurance firms sell insurance and separate markets, can be viewed as $N$ separate coalitions (the number of agent types) where all members within a coalition are identical. It is convenient for discussion’s sake to assume that $N = 2$. Let $\tilde{\psi} = (\psi_1, \psi_2)$ denote the measure of each type in a coalition. In the separated markets example, there are two coalitions $\tilde{\psi}_1 = (f(\eta_1), 0)$ and $\tilde{\psi}_2(0, f(\eta_2))$. An allocation $c(\theta | \eta_1), c(\theta | \eta_2)$ is feasible for a particular coalition $\tilde{\psi}$ if

$$\sum_{\theta} \{\psi_1[g(\theta | \eta_1)[\theta - c(\theta | \eta_1)] + \psi_2[g(\theta | \eta_2)[\theta - c(\theta | \eta_2)] \geq 0.$$  

An arrangement or a sharing rule $A$ is mapping that specifies an allocation for a coalition as a function of its membership. A feasible arrangement $A$ is *blocked* by another feasible arrangement $B$ if some agent is made better off under arrangement $B$ than $A$ and no other agent is made worse off. In the discussion above, in which there was retrading among agents of different types after the
first round of trading with full insurance, is an example where another coalition \( \bar{\psi}(f(\eta_1), f(\eta_2)) \) (the grand coalition of all agents) blocked the \( N \) separate coalitions.

A core arrangement is an unblocked feasible coalition. Let \( \bar{\psi} \) be a coalition such that \( 0 < \psi_1 \leq f_1 \) and \( 0 < \psi_2 \leq f_2 \). The coalition maximizes the expected utility of the representative agent. The probability of a representative coalition member realizing endowment \( \theta_1 \) is

\[
\rho_1 = \psi_1 g_{11} + \psi_2 g_{21},
\]

where \( \rho_1 = h(\theta_1) \) if \( \psi_1 = f_1 \), and \( \rho_2 = 1 - \rho_1 \) is the probability of a representative coalition member realizing endowment \( \theta_2 \). It is also the frequency of observing \( \theta_1 \) or \( \theta_2 \) within the coalition. Let \( \phi_{i,j} \) denote the coalition weight for a type \( j \) agent with endowment realization \( \theta_i \), for \( i = 1, 2 \) and \( j = 1, 2 \). The coalition \( \bar{\psi} \) solves the maximization problem

\[
\max_{\{c_{i,j}\}} \left\{ \sum_{j=1}^{2} \sum_{i=1}^{2} \phi_{i,j} U(c_{i,j}) \right\}
\]

subject to the resource constraint

\[
\rho_1 \theta_1 + \rho_2 \theta_2 \geq \rho_1 [\psi_1 c_{11} + \psi_2 c_{12}] + \rho_2 [\psi_2 c_{21} + \psi_2 c_{22}],
\]

and the participation constraints

\[
\sum_{i=1}^{2} g_{i,j} U(c_{i,j}) \geq U(\theta_m(\eta_j)),
\]

for \( j = 1, 2 \). Let \( \lambda \) denote the multiplier for the resource constraint. The first-order conditions take the form

\[
\phi_{i,j} U_1(c_{i,j}) = \lambda \rho_i \psi_j.
\]

Observe that if \( \rho_1 = f_1 \) and

\[
\phi_{i,j} = \frac{f_j g_{i,j}}{\mu_j},
\]

that the allocation coincides with the allocation achieved when retrading is allowed.
4 Infinite-Horizon Markets

Assume now that an agent has an infinite time horizon. As before, at time 0, an agent observes the realization of a random variable \( \eta \) that affects the distribution of his endowment for all future periods. Let \( \theta^t(i) = \{\theta_1(i), \ldots, \theta_t(i)\} \) denote the history of endowment realizations for a particular agent \( i \in I \). Let \( g_t(\theta^t(i) | \eta) \) denote the probability of the history \( \theta^t(i) \) under the assumption that agent \( i \) is a type \( \eta \) agent. For the first period, the allocations across different \( \eta \) agents with different endowments \( \theta \) lie in \( N \times M \). The history of allocations for agents from periods 1 to \( t \) is an element of \( (N \times M)^t \). There is no aggregate uncertainty and the history of the system is the entire space \( (N \times M)^t \) at time \( t \) and hence is deterministic. Initially prices are allowed to be functions of history \( \theta^t \), since the history of endowments is observable for any agent and, moreover, it is possible for \( \theta^t \) to be identical for two agents of different types \( \eta_i \neq \eta_j \).

A stationary solution is an allocation in \( N \times M \). Under the assumption that only stationary solutions are considered, the history of the system does not affect the price of a contingent claim in market \( \theta \). The distribution of the endowment remains unchanged so that the probability that a random agent has realization \( \theta \) is \( \sum_N f(\eta)g(\theta | \eta) \). I assume that there is a clearing house announcing prices that clear the market for each \( \theta \), with no price discrimination based on \( \eta \) type.

4.1 Integrated Markets in the Infinite Horizon

Under the assumption that prices are stationary, agents will have an incentive to borrow and lend in other \( \eta \)-agent markets if agents are allowed to retrade. In that case, where markets cannot be segregated, agents solve

\[
\max \sum_{t=0}^{\infty} \sum_{\theta^t} g_t(\theta_t | \eta) \beta^t U(c_t(\theta_t | \eta))
\] (44)
subject to
\[ 0 \leq \sum_{t=0}^{\infty} \sum_{\theta} p_t(\theta_t) [\theta_t - c_t(\theta^t | \eta)] \]  \hspace{1cm} (45)

Let \( \hat{\mu}_\eta \) denote the Lagrange multiplier. The first-order condition is
\[ \sum_{\theta_{t-1}} g_t(\theta^t | \eta) \beta U'(c_t(\theta^t | \eta)) = \hat{\mu}_\eta \sum_{\theta_{t-1}} p_t(\theta_t) = \hat{\mu}_\eta p_t(\theta_t) \]  \hspace{1cm} (46)

Consider only stationary solutions. Let
\[ q(\theta_t) = \frac{p_t(\theta_t)}{\beta^t} \]

Observe that, if we assume only stationary solutions are to be considered so \( c(\theta | \eta) = c_t(\theta^t | \eta) \) and since
\[ \sum_{\theta_{t-1}} g_t(\theta^t | \eta) = g(\theta_t | \eta), \]
then the first-order condition becomes
\[ \frac{U_1(c(\theta | \eta))g(\theta | \eta)}{\hat{\mu}(\eta)} = q(\theta). \]  \hspace{1cm} (47)

The construction of the equilibrium allocation given the endowments proceeds in the same way as described in the one period model because this is an endowment economy with no capital accumulation decision and we focus on the stationary solution. The intertemporal model has the property that the stationary consumption allocation is identical with the allocation in the one-period model. Since the endowment is nonstorable and there are no capital market frictions such as borrowing constraints, this is not surprising.

The infinite horizon problem can also be expressed as a sequential dynamic programming problem and doing so provides insight into how the integration of markets affects agents facing uninsurable income distribution risk. The key is to note that the conditional probability of moving to a state \( \theta' \in \Theta \) for agent \( \eta \) with endowment \( \theta \) differs across different \( \eta \)-types. As mentioned earlier,
the property that consumption insurance isn’t actuarially fair creates a wedge between the trade-off over states offered by the market and the marginal rate of transformation of marginal utility across states for the agent. Let \( \hat{q}(\theta_{t+1}, \theta_t, \eta) \) denote the effective price of a unit of consumption in state \( \theta_{t+1} \) for a type \( \eta \) agent with current endowment \( \theta_t \). The Bellman equation is

\[
V(z, \theta_t \mid \eta) = \max \left[ U(c) + \beta \sum_{\Theta} g(\theta_{t+1} \mid \eta)V(z(\theta_{t+1}, \theta_{t+1} \mid \eta), \theta_{t+1} \mid \eta) \right]
\]

subject to

\[
\theta + z \geq c + \sum_{\Theta} \hat{q}(\theta_{t+1}, \theta_t, \eta)z(\theta_{t+1}, \theta_t \mid \eta)
\]

Let \( \lambda(\theta, \eta) \) denote the Lagrange multiplier. The first-order conditions and envelope condition are

\[
U'(c) = \lambda(\theta, \eta)
\]

\[
\hat{q}(\theta_{t+1}, \theta_t, \eta)\lambda(\theta, \eta) = \beta g(\theta_{t+1} \mid \eta)V_z
\]

\[
V_z = \lambda(\theta, \eta)
\]

These conditions can be simplified as

\[
U'(c_t)\hat{q}(\theta_{t+1}, \theta_t, \eta) = \beta g(\theta_{t+1} \mid \eta)U'(c_{t+1})
\]

For any \( \theta \in \Theta \),

\[
\mu(\eta) = \frac{g(\theta \mid \eta)U_1(c(\theta \mid \eta))}{q(\theta)}
\]

or

\[
U'(c(\theta_t \mid \eta)) = \frac{\mu(\eta)q(\theta_t)}{g(\theta_t \mid \eta)}.
\]

Update the time subscript in (54) by one and substitute into (53) to obtain

\[
0 = \left[ \frac{\mu(\eta)q(\theta_t)}{g(\theta_t \mid \eta)} \right] \hat{q}(\theta_{t+1}, \theta_t, \eta) - \beta g(\theta_{t+1} \mid \eta) \left[ \frac{\mu(\eta)q(\theta_{t+1})}{g(\theta_{t+1} \mid \eta)} \right]
\]
so that

\[ q(\theta_{t+1}, \theta_t, \eta) = \beta \left[ \frac{g(\theta_{t+1})g(\theta_t | \eta)}{q(\theta_t)} \right] \]  

The price \( \hat{q} \) measures the intertemporal terms of trade for a type \( \eta \) agent with current endowment \( \theta_t \). Notice that (55) also implies

\[
U'(c(\theta_t | \eta)g(\theta_t | \eta)) = U'(c(\theta_{t+1} | \eta))g(\theta_{t+1} | \eta).
\]

Notice the similarity between this equation and (??); the price \( \hat{q} \) takes into account the wedge created by the property that consumption insurance is actuarially unfair.

Observe that by averaging over the different agent types

\[
\sum_{\eta} \hat{q}(\theta_{t+1}, \theta_t, \eta) = \beta \sum_{\eta} \left[ \frac{g(\theta_{t+1})g(\theta_t | \eta)}{q(\theta_t)} \right] \]

\[ = \beta \left[ \frac{q(\theta_{t+1})}{q(\theta_t)} \right] \] (58)

As a result, the intertemporal marginal rate of substitution for an individual type \( \eta \) agent is

\[
m(\theta_{t+1}, \theta_t, \eta) \equiv \beta \frac{U'(c(\theta_{t+1} | \eta))}{U'(c(\theta_t | \eta))} (59)
\]

For any \( \theta', \theta \) pair in \( \Theta \), observe that there is a distribution of IMRS \( m(\theta', \theta, \eta) \).

To understand the implications for the agent’s budget constraint, solve (49) for \( z(\theta_t | \eta) \),

\[
z(\theta_t | \eta) = c(\theta_t | \eta) - \theta_t + \sum_{\theta_{t+1}} \hat{q}(\theta_{t+1}, \theta_t, \eta)z(\theta_{t+1} | \eta)
\]

Define

\[
q_c(\theta_{t+1}, \theta_t) \equiv \frac{q(\theta_{t+1})}{q(\theta_t)}.
\]

Make the substitution for \( \hat{q} \) and solve recursively forward to obtain

\[
z(\theta_t | \eta) = c(\theta_t | \eta) - \theta_t + \beta \sum_{\theta_{t+1}} q_c(\theta_{t+1}, \theta_t)g(\theta_t | \eta)c(\theta_{t+1} | \eta) - \theta_{t+1} + \sum_{\theta_{t+2}} \hat{q}(\theta_{t+2}, \theta_{t+1}, \eta)z(\theta_{t+2} | \eta)
\]
\[ c(\theta_t | \eta) - \theta_t + \sum_{j=1}^{\infty} \beta^j \left[ \sum_{\theta_{t+j} \in \Theta} \left( \prod_{i=0}^{j-1} q_c(\theta_{t+i+1} | \theta_{t+i}) g(\theta_{t+i+1} | \eta) \right) \left[ c(\theta_{t+j} | \eta) - \theta_{t+j} \right] \right] \]

5 Aggregate Risk

The model so far has deterministic output; there is idiosyncratic risk for an individual agent, and because the idiosyncratic endowment distribution risk is not insurable at a actuarially fair price, agents experience consumption fluctuations that average out in the aggregate. The next step is to introduce aggregate uncertainty in a tractable way.

Let \( S = \{s_1, \ldots, s_S\} \) be a discrete random variable forming a Markov chain with

\[ \pi(s_{t+1}, s_t) \]

denoting the one-step ahead probability of the state next period equaling \( s_{t+1} \) conditional on starting in state \( s_t \). Now assume that the individual agent’s probability of an endowment realization \( \theta \) conditional on the aggregate state \( s \) and type \( \eta \) is

\[ g(\theta | \eta, s) \]

For example, there may be some type \( \eta \) agents that have an endowment distribution that is unaffected by an aggregate shock while other type \( \eta \) agents have endowment distributions that are affected by the aggregate shock. If \( s_2 > s_1 \) then an example would be \( g(\theta | \eta, s_2) \) is a mean-preserving spread of \( g(\theta | \eta, s_1) \). Let \( s_0, s_1, \ldots, s_t \) be a history of realizations of the aggregate shock. Denote the probability of the history as \( \pi_t(s^t) \).

The type \( \eta \) agent solves

\[ \max_{\pi_t(s^t)} \left[ \sum_{t=0}^{\infty} \sum_{s^t} \sum_{\theta^t} \beta^t \pi_t(s^t) g_t(\theta^t | s^t, \eta) U(c_{t}(\theta^t, s^t | \eta)) \right] \]  

(60)
subject to
\[ 0 = \sum_t \sum_{s^t} \sum_{\theta_t} p_t(\theta, s^t)(\theta_t - c_t(\theta_t, s^t | \eta))] \tag{61} \]

The first-order condition is
\[ \beta_t \pi_t(s^t) \sum_{\theta^{t-1}} g_t(\theta^t | s^t, \eta)U'(c_t(\theta^t, s^t | \eta)) = \mu(\eta) \sum_{\theta^{t-1}} p_t(\theta^t, s^t) \tag{62} \]

Only stationary equilibria will be examined. Let \( c(\theta_t, s_t | \eta) \) denote a solution. Define
\[ p(s_t, \theta_t) \equiv \frac{\sum_{\theta^{t-1}} p_t(\theta^t, s^t)}{\beta_t \pi_t(s^t)} \]
so that the first-order condition can be expressed as
\[ \pi(s_t)g(\theta_t | s_t, \eta)U'(c(\theta_t, s_t | \eta)) = \mu(\eta)p(s_t, \theta_t). \tag{63} \]

For any two aggregate states \( \hat{s}, \bar{s}, \) observe that
\[ \mu(\eta) = \frac{\pi(\hat{s})g(\theta | s, \eta)U'(c(\theta, \hat{s} | \eta))}{p(\hat{s}, \theta)} = \frac{\pi(\bar{s})g(\theta | s, \eta)U'(c(\theta, \bar{s} | \eta))}{p(\bar{s}, \theta)}. \]

The marginal rate of substitution across aggregate states, given \( \theta \),
\[ \frac{U'(c(\theta, \hat{s} | \eta))}{U'(c(\theta, \bar{s} | \eta))} \]
is not equalized across states since the marginal utility is weighted by the individual’s conditional probability of \( \theta, s \). The agent’s intertemporal marginal rate of substitution is
\[ m(\theta_{t+1}, \theta_t, s_{t+1}, s_t, \eta) \equiv \beta g(\theta_{t+1} | s_{t+1}, \eta)U'(c(\theta_{t+1}, s_{t+1} | \eta)) \frac{U'(c(\theta_t, s_t | \eta))}{U'(c(\theta_t, s_t | \eta))} \tag{64} \]

Observe that given \( \theta_{t+1}, \theta_t, s_{t+1}, s_t \), the IMRS is a distribution over \( N \).
6 Conclusion

Under full consumption insurance, heterogeneous agents are able to equalize their marginal rate of substitution (MRS) state by state. Hence the associated pricing kernel and stochastic discount factor for the heterogeneous agent model are identical with the representative agent model. Since the representative agent model has performed poorly in empirical testing, there have been many efforts to enrich the model by introducing uninsurable idiosyncratic risk. In many instances, agents are modeled as being identical ex ante and become differentiated only over sample paths. If there is a countable infinity of agents, the model has the property that the cross sectional distribution of idiosyncratic risk is identical to the endowment risk an agent faces over his lifetime. In the absence of capital market frictions, this means that the consumption insurance offered by a standard Arrow-Debreu contingent claims market allows complete smoothing of idiosyncratic risk. Restrictions on the asset market structure, such as simple borrowing constraints, have not substantially improved the empirical performance of the model.

There are two main results. The first is related to Boyd, Prescott and Smith: The exogenously imposed market structure results in allocations that are very different from those that emerge when organizations are formed as endogenous coalitions of agents. In the example developed here, agents will choose not to separate into coalitions comprised of identical agents. The market structure achieving the identical allocation as the grand coalition is a clearing house of the form modeled by Wright. A second result is that agents trade not just to achieve risk pooling but also to exploit gains from trade with different types of agents. As a result, agents optimally choose to only partially insure. By introducing ex ante heterogeneous income distribution risk that is only partially insured, the asset market implications of the model are significantly altered. The stochastic discount factor is now a distribution over different categories of income distribution risk and the pricing kernel is a weighted average of the individuals’ pricing kernels.
References


