Money and nominal bonds∗

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Abstract

This paper studies an economy with trading frictions, ex post heterogeneity and nominal bonds in a model à la Lagos and Wright [16]. It is shown that a strictly positive interest rate is a sufficient condition for the allocation with nominal bonds to be welfare improving. This result comes from the protection against the inflation tax.

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1 Introduction

Berentsen, Camera and Waller [2] (hereafter, BCW) show that in new generation models of monetary economics with preference shocks the existence of a banking sector can help to reduce the inefficiency generated from the fact that some agents are cash constrained while others hold idle money. This source of inefficiency has been investigated by Bewley [8], Green and Zhou [11] and Levine [17]. Other attempts to address this inefficiency include models with illiquid assets (Kocherlakota [14]), collateralized credit (Shi [21]), or inside money (Cavalcanti and Wallace [9], Cavalcanti, Erosa and Temzelides [10] and He, Huang and Wright [12]).

BCW demonstrate that financial intermediation improves allocation and welfare. This is due to the fact that sellers can deposit idle cash (and earn an interest) and not from relaxing borrowers’ liquidity constraints.

An alternative approach to reduce the above mentioned inefficiency consists of replacing banks with nominal risk-free bonds. Using the basic framework of BCW and Lagos and Wright [16] (hereafter, LW) this paper does this by assuming that agents can acquire nominal government-issued bonds once they realize that they have idle money. A crucial assumption here is that individuals cannot sell bonds, i.e. they cannot borrow, which will make clear that the welfare improving role of bonds comes from the protection of the inflation tax and not that it may relax agents’ cash constraints. As in Kocherlakota [14], it is assumed that bonds are illiquid in the sense that they are not accepted in exchange for goods.

The LW framework is useful because it allows one to introduce heterogeneous preferences for consumption and production while keeping the distribution of money holdings analytically tractable. Shi [22] also gets money holdings degenerate but by different means. He assumes that the fundamental decision-making unit is not an individual, but a household with a continuum of agents. For a detailed discussion of the two approaches see Lagos and Wright [15].

The main result of the paper is that a strictly positive interest rate is a sufficient condition for the allocation with nominal bonds to be welfare improving.

The paper is organized as follows. Section 2 describes the basic framework and the agents’ decision problem. Stationary equilibria are characterized in Section 3. Section 4 states the results. Section 5 examines a modification of the tax system. The conclusions end the paper.
2 The model

The basic set up is LW. Time is indexed by \( t = 1, 2, \ldots, \infty \) and in each period \( t \) there are two perfectly competitive markets that open sequentially.\(^1\) There is a \([0, 1]\) continuum of infinitely-lived agents and one perishable good that can be produced and consumed by all agents. At the opening of the first market agents get a preference shock such that they can either consume or produce. With probability \( n \in \mathbb{R} (0, 1) \) an agent can produce but cannot consume while with probability \( 1 - n \) the agent can consume but cannot produce. We refer to consumers as buyers and producers as sellers. Some recent attempts to endogenize the fraction of agents entering in the market include Berentsen, Rocheteau and Shi [4], Li [18, 19] and Shi [22].

Agents get utility \( u(q) \) from \( q \) consumption in the first market, where \( u'(q) > 0, \ u''(q) < 0, \ u'(0) = \infty, \) and \( u'(\infty) = 0 \). Furthermore, we assume that the elasticity of utility \( e(q) = qu'(q)/u(q) \) is bounded. Producers incur utility cost \( c(q) \) from producing \( q \) units of output with \( c'(q) > 0 \) and \( c''(q) \geq 0 \). Let \( q^* \) denote the solution to \( u'(q^*) = c'(q^*) \). Buyers in the first market are anonymous. Consequently, trade credit is ruled out and transactions are subject to a *quid pro quo* restriction so there is a role for money (Kocherlakota [13] and Wallace [23]).

In the second market all agents consume and produce, getting utility \( U(x) \) from \( x \) consumption, with \( U'(x) > 0, \ U'(0) = \infty, \ U'(\infty) = 0 \) and \( U''(x) \leq 0 \). Let \( x^* \) be the solution to \( U'(x^*) = 1 \). The difference in preferences over the good sold in market 2 allows us to impose technical conditions such that the distribution of money holdings is degenerate at the beginning of each period. All agents can produce consumption goods from labor using a linear technology. This implies that all agents will choose to carry the same amount of money out of market 2, independent of their trading history. Agents discount between market 2 and the next-period market 1, but not between market 1 and market 2. This is not restrictive since as in Rocheteau and Wright [20] all that matters is the total discounting between one period and the next.

At the beginning of market 1, after the idiosyncratic shocks are realized, sellers hold idle cash while buyers may want more money than what they are carrying. Before trade of goods takes place in the first market, sellers can invest (they will) their money in a risk-free asset \( b \) bearing the gross nominal

\(^1\)Competitive pricing in LW is a feature of Rocheteau and Wright [20] and BCW.
rate of return $1 + i$ with $i \geq 0$.\(^2\)

As in Zhu and Wallace [24], this asset is a one-period, risk-free bond that matures (automatically turns into money) in the second market; suppose that there are vending machines maintained by the government which offer such bonds in exchange for money. It is assumed that these vending machines have a record-keeping technology of their activity and they can observe the owner’s name and address which is printed on the certificate. That claims can be costlessly counterfeit, and counterfeits automatically perish after they change hand. It is also assumed that the technology for detecting counterfeits is not available in the good market so agents do not accept bonds in transactions. In this sense bonds are illiquid and money is the only medium of exchange.\(^3\)

It is assumed that $b \in \mathbb{R}_+$, so that individuals can invest but not borrow. Interest payments are financed by lump-sum taxes levied by the government in market 2. The change in the nature of taxes does not affect the main results of the analysis and will be discussed later in the paper.

It is assumed a central bank exists that controls the money supply at time $t$, $M_t > 0$. We also assume that $M_t = \gamma M_{t-1}$, where $\gamma > 0$ is constant and new money is injected, or withdrawn if $\gamma < 1$, as lump-sum transfers $\pi M_{t-1} = (\gamma - 1) M_{t-1}$ to all buyers; things are basically the same if transfers also go to sellers, as long as they are lump-sum (i.e. they do not depend on agents’ behavior). We restrict attention to policies where $\gamma \geq \beta$, with $\beta \in \mathbb{R} (0,1)$ denoting the discount factor. Let $\pi_b M_{t-1} = \pi M_{t-1} / (1 - n)$ be the per buyer money transfer. The time subscript $t$ is omitted and shorten $t + 1$ to +1, etc. in what follows.

The timing of the events is shown in Figure 1. At the beginning of market 1 agents observe their preference shock and buyers receive the lump-sum money transfers $\pi_b$. Then, sellers have the opportunity to invest their cash in nominal bonds before trade of goods begins. In the second market agents produce, pay taxes, receive the principal plus interest on bonds, and consume. The structure of this economy is shown in Figure 2.

In period $t$, let $\phi = 1 / P$ be the real price of money and $P$ the price of goods in market 2. We study steady state equilibria, where aggregate real

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\(^2\)A similar framework in which agents can either lend or borrow is in Berentsen, Camera and Waller [3] and Berentsen and Waller [5].

\(^3\)An exhaustive discussion of illiquid bonds is in Kocherlakota [14]. Restrictions on bond circulation have been introduced also in Andolfatto [1], Berentsen and Waller [6] and Boel and Camera [7].
Figure 1: Timing of events

Figure 2: Money, nominal bonds and taxation
money balances are constant. We refer to this as stationary equilibrium
\[ \phi M = \phi_{-1} M_{-1} \] (1)
which implies that \( \phi_{-1}/\phi = M/M_{-1} = \gamma \); the Fisher equation holds, hence it is equivalent to set the nominal interest or inflation here.

In nominal terms, the government budget constraint is
\[ PG + Bi = T \] (2)
where \( B \) is the government debt outstanding at the beginning of market 2, \( T \) is a lump-sum nominal tax, and \( PG \) is spending for government consumption. Equation (2) states that the government expenditure \( (PG + Bi) \) is financed by tax revenues \( (T) \). To simplify the analysis, we assume \( G = 0 \).

### 3 Stationary equilibria

Consider a stationary equilibrium. Let \( V(m_1) \) denote the expected value from trading in market 1 with \( m_1 \) money balances conditional on the idiosyncratic shock. Let \( W(m_2, b) \) denote the expected value from entering the second market with \( m_2 \) units of money and \( b \) units of nominal bonds. In what follows, we look at a representative period \( t \) and work backwards from the second to the first market.

In the second market agents produce \( h \) units of good using \( h \) hours of labor, pay taxes, receive repayment of the investment plus interest, consume \( x \), and adjust their money balances. The real wage per hour is normalized to one. Hence, the representative agent’s problem is

\[ W(m_2, b) = \max_{x,h,m_1,1} [U(x) - h + \beta V_{+1}(m_{1,1})] \] (3)
such that
\[ x = h + \phi (m_2 - m_{1,1}) + \phi (1 + i) b - \phi T \] (4)
where \( m_{1,1} \) is the money taken into period \( t + 1 \). Eliminate \( h \) from (3) using (4) and get
\[
W(m_2, b) = \phi [m_2 + (1 + i) b - T] \\
+ \max_{x,m_{1,1}} [U(x) - x - \phi m_{1,1} + \beta V_{+1}(m_{1,1})].
\] (5)
The first order conditions (FOCs) with respect to \( x \) and \( m_{1,1} \) are
\[ U'(x) = 1, \quad \beta V_{+1}'(m_{1,1}) = \phi \] (6)
where the term $\beta V_{m_1+1}^+$ is the marginal benefit of taking money out of market 2 and $\phi$ is its marginal cost. In competitive markets (i.e., under price taking), uniqueness of $m_{1,+1}$ is a direct consequence of $u''(q) < 0$, so all agents in the second market choose the same $m_{1,+1}$.

There are two main results from (6). First, the quantity of goods $x$ consumed by every agent is equal to the efficient level $x^*$ where $x^*$ is such that $U'(x^*) = 1$. Second, $m_{1,+1}$ is independent of $b$ and $m_2$. As a result, the distribution of money holdings is degenerate at the beginning of the following period. This is due to the quasi-linearity assumption in (3), which eliminates the wealth effects on money demand in market 2. Agents who bring too much cash into the second market spend some buying goods, while those with too little cash sell goods.

The envelope conditions are

$$W_m(m_2, b) = \phi, \quad W_b(m_2, b) = \phi(1 + i). \quad (7)$$

Let $q_b$ and $q_s$ denote the quantities consumed by a buyer and produced by a seller trading in market 1, respectively. Let $p$ be the nominal price of goods in market 1. It is straightforward to show that agents who are buyers will never acquire nominal bonds. We drop the argument $b$ in $W(m_2, b)$ where relevant for notational simplicity.

An agent who has $m_1$ money at the opening of market 1 has expected lifetime utility

$$V(m_1) = (1 - n)[u(q_b) + W(m_1 + \pi_b M_{-1} - pq_b, 0)] + n[-c(q_s) + W(m_1 - b + pq_s, b)]$$

where $pq_b$ is the amount of money spent as a buyer, and $pq_s$ the money received as a seller. From linearity of $W(m, b)$, expression (5) can be rewritten as

$$W(m_2, b) \equiv W(0, 0) + \phi[m_2 + (1 + i) b]$$

which can be used to rewrite the indirect utility function as follows

$$V(m_1) = W(m_1, 0) + (1 - n)[u(q_b) + \phi(\pi_b M_{-1} - pq_b)] + n[-c(q_s) + \phi(pq_s + ib)]. \quad (8)$$

Once the production and consumption shocks occur, agents become either a buyer or a seller.

4See LW under bargaining and Rocheteau and Wright [20] under price posting.
If an agent is a seller in the first market, his problem is
\[
\max_{q_s,b} \left[ -c(q_s) + W(m_1 - b + pq_s, b) \right]
\] (9)
such that
\[
b \leq m_1.
\] (10)
The FOCs are
\[
\begin{align*}
-c'(q_s) + pW_m &= 0, \\
-W_m + W_b - \lambda_b &= 0 \\
\end{align*}
\] (11)
where \(\lambda_b\) is the Lagrangian multiplier on the bonds constraint. By virtue of (7), if \(i > 0\) then \(\lambda_b > 0\) hence (10) binds. So sellers invest all their money in government bonds. Again, using (7) the FOC for \(q_s\) reduces to
\[
c'(q_s) = p\phi.
\] (12)
Sellers produce a quantity such that the ratio of marginal costs across markets \((c'(q_s)/1)\) is equal to the relative price of goods \((p\phi)\). Due to the linearity of the envelope conditions, \(q_s\) is independent of \(m_1\) and \(b\). Consequently, each seller in market 1 produces the same amount of goods no matter how much money he holds or what financial decisions he makes.

If an agent is a buyer in the first market, his problem is:
\[
\max_{q_b} \left[ u(q_b) + W(m_1 + \pi_b M_{-1} - pq_b) \right]
\] (13)
such that
\[
pq_b \leq m_1 + \pi_b M_{-1}
\] (14)
where (14) means that buyers cannot spend more money than what they bring into the first market, \(m_1\), plus the transfer \(\pi_b M_{-1}\). Using (7) the buyer’s FOC is
\[
\begin{align*}
u'(q_b) - \phi p - \lambda_c p &= 0 \\
\end{align*}
\] (15)
then eliminate \(p\) using (12) and get
\[
\begin{align*}
u'(q_b) &= \left[ 1 + \frac{\lambda_c}{\phi} \right] c'(q_s) \\
\end{align*}
\] (16)
where \(\lambda_c\) is the multiplier on the cash constraint.

If the constraint (14) is not binding (i.e. \(\lambda_c = 0\)), condition (16) reduces to \(u'(q_b) = c'(q_s)\), so trade is efficient. Conversely, if \(\lambda_c > 0\) then the constraint binds and \(u'(q_b) > c'(q_s)\). Hence, no trade is efficient and the buyer consumes \(q_b = (m_1 + \pi_b M_{-1})/p\).
Differentiating (8) with respect to $m_1$ yields

$$V'(m_1) = W_m(m_1) + (1 - n) \left[ u'(q_b) \frac{\partial q_b}{\partial m_1} - \phi p \frac{\partial q_b}{\partial m_1} \right] + n \left[ -c'(q_s) \frac{\partial q_s}{\partial m_1} + \phi \left( p \frac{\partial q_s}{\partial m_1} + i \frac{\partial b}{\partial m_1} \right) \right]$$

(17)

where $V'(m_1)$ is the marginal value of money. Because the quantity of goods produced by sellers is independent of their money holdings, it holds that $\partial q_s/\partial m_1 = 0$. Note that sellers can derive no benefits from holding cash in the first market, so they always spend all their balances in nominal bonds if $i > 0$, this means $\partial b/\partial m_1 = 1$. (If $i > 0$ then $W_b > W_m$, hence (10) binds.)

### 4 Welfare analysis

Using (7), (12) and rearranging, equation (17) can be rewritten as

$$V'(m_1) = \phi \left[ (1 - n) \frac{u'(q_b)}{c'(q_s)} + n (1 + i) \right].$$

(18)

The first term within brackets, $(1 - n) u'(q_b)/c'(q_s)$, refers to buyers and is the same as in the basic LW model. Now, the second term, $n (1 + i)$, refers to sellers and indicates that they can invest a unit of money and receive $1 + i$. Hence, the effect of nominal bonds on the marginal value of money is positive since sellers can earn an interest on idle balances.

Before pursuing monetary equilibria, we have to derive hours of work in the second market. Since all buyers have the same amount of money at the opening of market 1 and face the same problem $q_b$ coincides for all of them. In a symmetric equilibrium the same applies to sellers. Hence, clearing condition in market 1 implies

$$q_s = \frac{1-n}{n} q_b$$

(19)

then, efficiency is achieved at

$$u'(q^*) = c' \left( \frac{1-n}{n} q^* \right)$$

(20)

where $q^*$ is the quantity such that (20) is satisfied. The buyer’s hours of work in the second market are

$$h_b = x^* + \phi m_{1,+1} + \phi T$$

(21)

where $x^*$ is the quantity of goods such that the first equation in (6) is satisfied. A buyer enters the second market with no cash, hence he has to work $x^* +$
\( \phi m_{1,+1} + \phi T \) hours in order to consume \( x^* \) quantity of goods, pay taxes \( T \), and take \( m_{1,+1} \) units of money out of the second market. Similarly, hours of work for a seller are

\[
h_s = x^* + \phi m_{1,+1} + \phi T - \phi [pq_s + (1 + i)b]. \tag{22}
\]

A seller enters the second market with \( pq_s \) units of money and he receives interest plus notional \((1 + i)b\), while he consumes \( x^* \), pays taxes \( T \), and takes \( m_{1,+1} \) units of money into the next period. Directly from (21) and (22), it holds that sellers work less than buyers in market 2, i.e. \( h_s < h_b \).

Aggregate hours of work in the second market are

\[
h = nh_s + (1 - n)h_b \tag{23}
\]

which, using (19), (21), (22) and rearranging, can be rewritten as

\[
h = x^* - \phi iB + \phi T \tag{24}
\]

by virtue of \( M = [1 + (1 - n)\pi_b]M_{-1} \), symmetric conditions \( m_{1,+1} = M \), \( b = m_1 = M_{-1} \), \( nb = nM_{-1} = B \), and using the fact that buyers in market 1 spend all their money, i.e. \( pq_b = (1 + \pi_b)M_{-1} \).

Now, use the budget constraint (2) to eliminate \( B \) from (24), and impose symmetric conditions \( h = H \) and \( x = X \) to get aggregate hours of work in market 2

\[
H = X^*
\]

where \( X^* \) is such that \( U'(X^*) = 1 \).

In steady state monetary equilibria, inflation equals the money growth rate (i.e., \( \gamma = 1 + \pi \)), and the real interest rate is \( i_R = 1/\beta - 1 \). Substitute these terms directly into the Fisher equation, \( 1 + i = (1 + i_R)(1 + \pi) \), and get

\[
i = \frac{\gamma - \beta}{\beta}. \tag{25}
\]

Now, use the second expression in (6) lagged one period, and (19) to rewrite (18) as follows

\[
\frac{\phi_{-1}}{\beta} = \phi \left\{ (1 - n) \frac{u'(q_{b})}{v'(\frac{1}{1-n}q_{b})} + n (1 + i) \right\}
\]

then take the steady state, eliminate \( i \) using (25) and rearrange to get the equilibrium condition

\[
\frac{\gamma - \beta}{\beta} = \frac{u'(q_b)}{v'(\frac{1}{1-n}q_b)} - 1. \tag{26}
\]
Definition 1 A symmetric steady state monetary equilibrium is an interest rate \( i \) satisfying (25) and a quantity \( q_b \) satisfying (26).

At this point of the analysis, the main result of the paper can be introduced:

Proposition 1 A strictly positive interest rate is a sufficient condition for the allocation with nominal bonds to be welfare improving.

Proof. Assume a strictly positive interest rate, i.e. \( i > 0 \). Now, let \( \tilde{q}_b \) denote the quantity of goods consumed in an economy without nominal bonds (see LW). This implies

\[
\frac{\gamma - \beta}{\beta} = (1 - n) \left[ \frac{u'(\tilde{q}_b)}{c'(\frac{1}{n} \tilde{q}_b)} - 1 \right]. \tag{27}
\]

Since \( n \in \mathbb{R} (0, 1) \), the expression within brackets must be lower, for given \( \gamma > \beta \), in an economy with nominal bonds than without. Comparison of equations (27) and (26) implies \( \tilde{q}_b < q_b \) for any \( i > 0 \).

BCW get exactly the same allocation with financial intermediation. Now, buyers can (they will) borrow in BCW, but they are not allowed to do so in our framework. Thus it is clear that the welfare improving role of bonds comes from the protection of the inflation tax and not that it may relax agents’ liquidity constraints.

5 Tax system

In this section we explore a modification of the tax system. Instead of lump-sum taxes, it is assumed that interest payments are financed by distortionary labor income taxes. This affects many of the results, such as the inefficient level of consumption in market 2, but is not crucial for the main story.

As before, we assume \( G = 0 \). Thus, the government budget constraint (2) becomes

\[
Bi = Pt_h H \tag{28}
\]

where \( t_h \in \mathbb{R} (0, 1) \) is the proportional income tax on aggregate hours of work in market 2. By working backwards from the second to the first market, it is straightforward to show that the marginal value of money is

\[
V'(m_1) = \phi \left[ (1 - n) \frac{u'(q_s)}{c'(q_s)} + n (1 + i) \right] \tag{29}
\]

which differs from (18) as we have distortionary taxes here.
The agent’s hours of work in market 2 are

\[ h_b = \frac{x + \phi m_{1,t+1}}{1 - t_h} \]

if he is a buyer, and

\[ h_s = \frac{x + \phi m_{1,t+1} - \phi[pq_b + (1+i)b]}{1 - t_h} \]

if he is a seller. Consequently, using (23) and rearranging, one gets

\[ h = \frac{x - \phi i B}{1 - t_h} \tag{30} \]

then eliminate \( B \) using the budget constraint (28), impose symmetric conditions \( h = X \) and \( x = X \), and obtain aggregate hours of work in the second market

\[ H = X \tag{31} \]

where \( X \) in (31) is such that \( U'(X) = 1/(1 - t_h) \), with \( X < X^* \).

The modification of the tax system does not affect the other equilibrium conditions, which we rewrite here for convenience

\[ i = \frac{\gamma - \beta}{\beta} \tag{32} \]

and

\[ \frac{\gamma - \beta}{\beta} = \frac{u'(q_b)}{c'(\frac{1 + \alpha}{\alpha} q_b)} - 1. \tag{33} \]

As in the case of lump-sum taxes, a strictly positive interest rate is a sufficient condition for the allocation with nominal bonds to be welfare improving; to see this note that equations (33) and (26) are identical. It then follows that the main result of the paper (Proposition 1) is robust to alternative specifications of the tax system.

6 Conclusions

This paper studied an economy with trading frictions, ex post heterogeneity and nominal bonds in a model à la Lagos and Wright [16]. It is shown that a strictly positive interest rate is a sufficient condition for the allocation with nominal bonds to be welfare improving. This result comes from the protection of the inflation tax.
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