Abstract

Between 1952 and 1982, the cyclical correlation between household mortgage debt and aggregate economic activity was around 0.80; in the last 25 years, this correlation has been almost zero. During the same period, the volatility of residential investment has sharply fallen, accounting for a large fraction of the Great Moderation.

In this paper, we provide an explanation for these facts. We argue that a common factor can explain at the same time the declining cyclicality of household debt and the reduced volatility of household investment. This factor is the larger household income volatility of the last decades: larger income volatility at the individual level makes individual more cautious about changing their stock of durables, thus making purchases of durable goods less sensitive to given size aggregate shocks.

We formalize these points using a calibrated version of the stochastic growth model with housing investment, collateralized household debt and uninsurable idiosyncratic and aggregate shocks. We distinguish between an early period (1950s through 1970s), when individual risk was relatively small, aggregate risk was relatively high, and maximum loan-to-value ratios were low; and a late period (1980s through today), with high individual risk, low aggregate risk, and high loan-to-value ratios.

In the early period, precautionary saving is small, wealth-poor people are close to the maximum borrowing limit, and housing investment and household debt closely track aggregate productivity. In the late period, as a consequence of higher idiosyncratic risk, precautionary saving becomes larger, wealth-poor people borrow less than the maximum, and become more cautious and able to self-insure in response to aggregate shocks. As a consequence, the correlation between debt, household investment and aggregate productivity is lower, as in the data. Quantitatively, larger idiosyncratic shocks can explain: (1) 5 percent of the reduction in total GDP volatility since the mid 1980s; (2) more than one half of the reduction in the volatility of household investment; (3) the sharp decline in the correlation between household debt and economic activity.

Keywords: Household investment, Mortgage Debt, Great Moderation; Volatility
JEL: E32, E65
1. Introduction

The Question. The goal of this paper is to understand how uninsurable idiosyncratic risk affects the business cycle properties of U.S. macroeconomic time series. More in detail, we want to explore the extent to which the rise in earnings volatility of the last three decades has affected the cyclical properties and the volatility of household debt, household and business investment, consumption and labor supply. While our model nests the traditional real business cycle model as a special case, our emphasis is mostly on household debt and household investment: in a model in which households face shocks to their endowment of efficiency labor units, debt (or positive assets) are the crucial ingredient that households use to smooth consumption over time; in addition, since a large fraction of household debt in the U.S. is backed by housing collateral, we find it natural to focus jointly on debt and housing.

The Facts. Lucas (1977) stated that “Business cycles are all alike”, but a closer look at residential investment and household debt illustrates how their cyclical properties have changed since the mid-1980’s: in particular, the volatility of residential investment has sharply fallen (from 8.24% to 3.51%), and the correlation of household debt with GDP has sharply decreased (from 0.78 to 0.04). In addition, a model that explores the link between idiosyncratic risk and aggregate outcomes has to acknowledge that business cycles have become milder, while at the same time individual households seem to face more volatile economic circumstances. We use the changing volatility of the shocks as one of the inputs of our calibration exercise.

Our model. Our benchmark economy is a version of the stochastic growth model with heterogeneous consumers, extended to allow for housing investment, collateralized borrowing and housing adjustment costs. There is a continuum of infinitely lived agents who receive shocks to their endowment of labor. To generate a realistic distribution of wealth, our baseline model features preference heterogeneity, as in Krusell and Smith (1998). In the stationary equilibrium, this simple modeling device results in a very unequal yet realistic wealth distribution: very few households - the “patients” - save a lot and hold a disproportionate share of total wealth; a large majority of households - the “impatient” - hold little wealth, with a portfolio made by a home and a mortgage.

There are no state contingent markets for hedging against idiosyncratic risk, and only self-insurance through a risk-free bond is possible. Agents can borrow up to a fraction of their housing wealth, and incur a cost in adjusting the housing stock. Finally, aggregate uncertainty is introduced in the form of a shock to total factor productivity. The model uses as inputs the exogenous aggregate and idiosyncratic uncertainty, and delivers as output the endogenously derived dynamics of housing and aggregate debt over the business cycle.
**Our Findings.** We calibrate our model economy in a way to replicate key business cycle facts for the period 1952-1982. Our model does a good job in accounting for U.S. business cycle facts over the period in question: in particular, it replicates the observation that residential investment is procyclical and more volatile than business investment; in addition, our model does a good job in matching the distribution of wealth and the procyclical behavior of household debt. We then use the model to characterize the business cycle implications of increasing microeconomic volatility, lowering down-payment constraints, and decreasing volatility of macroeconomic (TFP) shocks. While smaller macroeconomic shocks play the lion’s share in explaining the reduced volatility of GDP, a surprising implication of our model is that larger idiosyncratic risk can account for a non-negligible fraction of the reduced volatility of housing investment and for the reduced cyclicality of household debt. This happens because larger idiosyncratic risk impacts on how agents respond to aggregate disturbances: when income profiles become more volatile, individuals become more cautious following aggregate productivity shocks; as a consequence, durable good purchases become less volatile. Quantitatively, this mechanism can explain 50 percent of the reduction in volatility of housing investment, and 5 percent of the reduction in total GDP volatility.

**Previous Answers.** Our model is part of a large and growing literature that analyzes the aggregate behavior of economies with heterogeneous agents and incomplete markets. Most of these model economies feature an “approximate aggregation” result: that is, their business cycle properties can be described with reference to a small set of aggregate variables, such as the mean of the wealth distribution. This is not to say, however, that the business cycle properties of these economies are invariant to any changes in the underlying structure of the economy. We are not aware of studies that have looked at this question in detail in the context of heterogeneous-agents, incomplete market models. An exception is represented by Silos (2007), who analyzes the relationship between macroeconomic shocks and household portfolio choice adopting a similar framework to ours. His focus is on the impact of aggregate shocks on the wealth distribution and portfolio composition. On the opposite, we concentrate on how individual risk affects macroeconomic fluctuations through household portfolio choices.

Closest to our work are the papers by Fisher and Gervais (2007) and Campbell and Hercowitz (2006), who propose alternative explanations to the increased macroeconomic stability. Fisher and Gervais (2007) find that the decline in residential investment volatility is driven by a change in the demographics of the population together with an increase in the cross-sectional variance of earnings. However, theirs is in a certain sense a partial equilibrium framework in which the interest rate does not react to aggregate fluctuations. Campbell and Hercowitz (2006) study the impact of financial innovation on macroeconomic volatility, and their mechanism is through the labor supply: less tight collateral constraints weaken the connection between constrained households’ housing investment and their hours worked. Our quantitative results suggest that although financial innovation plays an important role, at least half of the reduction in housing investment volatility can be explained by the higher idiosyncratic risk only.
2. Facts

In this section, we present some basic facts on the aggregate time series for the U.S. economy that are relevant to our analysis.

All our series are quarterly and have been HP-filtered with a smoothing parameter of 1,600. We compare two periods. The first period begins in 1952 and ends in 1982. The second period goes from 1984 to 2006.

The sample split that we - and many others - choose corresponds to three important developments - for the purposes of our study - in the U.S. Economy:

1. **The Great Moderation**: several authors\(^1\) have noted that, starting from the mid-1980s, macroeconomic aggregates have become much more stable. As shown by the Table, the standard deviation of HP-filtered GDP was 2.09 percent in the early period, and 1.21 in the late period. Different measures of volatility portray a similar picture.

2. **Financial Innovation**: Several financial reforms have made it easier for households to access the credit market. See for instance Campbell and Hercowitz (2005), Dynan, Elmendorf and Sichel (2005) and Guerron-Quintana (2007).

3. **Increase in individual earnings volatility**: although aggregate economic activity has become less volatile, individual households appear to have faced more volatile economic circumstances over time. See for instance Krueger and Perri (2006).

For our purposes, we single out two additional features of the data:

1. Over time, the **correlation between household mortgage debt** (which makes up for 90% of total household liabilities in the US) and GDP has fallen substantially. Household debt was very procyclical in the early period, and it has become acyclical in the late period.

2. The reduction in aggregate volatility has been non-uniform across the different categories of expenditure. The standard deviation of consumption has slightly fallen in proportion to that of GDP. Looking at investment, the **relative standard deviation of residential investment has fallen** from a multiple of 3.9 to 2.9 relative to the standard deviation of GDP. Instead, the **relative standard deviation of business investment has risen** from 2.4 to 3.5 times that of GDP.

\(^1\)See for instance the survey paper by Stock and Watson (2003) and references therein.
3. The Model Economy

In the following, we extend an otherwise standard heterogeneous agents, real business cycle model to allow for housing investment and collateralized borrowing. We describe the economy’s main features, introduce the individual problem, and define a recursive competitive equilibrium. Appendix A at the end provides the details on the computational approach used for the solution.

3.1. The Environment

The economy is populated by a continuum of infinitely lived agents of measure one. Time is discrete. One single good is produced, which can be used either as non-durable or as durable (housing), or can be saved.

**Preferences** Let $\tilde{t}$ denote each agent’s total time endowment. Households derive utility from leisure $(\tilde{t} - t)$, non-durable consumption $c$, and service flows from durables, which are assumed to be proportional to the housing stock $h$. Their expected discounted utility can be written as

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t, l_t)$$

where $E_0$ denotes expectations at $t = 0$, and the per-period utility function $u$ takes the following functional form:

$$u(c_t, h_t, l_t) = \log c_t + j \log h_t + \tau \log (\tilde{t} - l_t)$$

with $j, \tau > 0$. 

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Notes: (i) The ratio is the standard deviation of the variable scaled by that of GDP; (ii) $C$, $IH$ and $IK$ are chain-weighted consumption, residential investment and business investment respectively; $GDP$ is the sum of the nominal series ($C + IH + IK$) divided by the GDP deflator; (iii) Durables good are considered part of $IH$, not of $C$; (iv) Debt is gross household mortgage debt outstanding deflated by the GDP deflator. Prior to detrending, all series are scaled by civilian non-institutional population.

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<td>$Debt_{iv}$</td>
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Household $i$’s discount factor $\beta_i$ can be either low, $\beta_L$, or high, $\beta_H$, with $0 < \beta_L < \beta_H < 1$. One’s degree of patience is innate, and does not change over time. We refer to households characterized by a lower discount factor as impatient. As mentioned above, preference heterogeneity is introduced in the model in order to obtain a realistically unequal distribution of wealth (see, among others, Krusell and Smith, 1998, and Venti and Wise, 2001).

**Individual Labor Productivity Endowments** Each unit of time endowment supplied in the labor market is paid at the wage rate $w_t$. At each $t$, households receive a shock to their efficiency units of labor, $z_t \in \tilde{Z} \equiv \{z^1, \ldots, z^n\}$. The shock follows a Markov process with transition matrix $\pi_{z,z'} = \Pr (z_{t+1} = z' / z_t = z)$, $\pi_{z,z'} > 0$ for every $z, z' \in \tilde{Z}$, with $\sum_{z'} \pi_{z,z'} = 1$ for every $z \in \tilde{Z}$. By a law of large numbers, $\pi$ also represents the fraction of agents experiencing a transition from $z$ to $z'$ between any two periods, with $z$ and $z' \in \tilde{Z}$.

The volatility of the idiosyncratic shock implied by the Markov process will be key to explaining different household portfolio composition choices, which in turn will have important quantitative implications on business cycle dynamics.

Let $\Pi$ be the unique stationary distribution associated with the transition probability $\pi$. Again, by a law of large numbers, at each period there are $\Pi(z)$ agents characterized by labor productivity $z$. The total amount of labor efficiency units $\sum_{z \in \tilde{Z}} z \Pi(z)$ is constant and normalized to one for convenience.

**Financial Market** Households can self-insure through a risk-free bond, $b$, which pays a gross interest rate $R_t$ in period $t$. For convenience, let positive amounts of this bond denote a net debt position. Only collateralized borrowing is possible. At each period $t$, the maximum net debt that households can incur is a fraction $m$ of the housing stock:

$$b_t \leq mh_t$$

The intermediary financial sector collects deposits from some households, and lends both to other households and to firms. In the calibration exercises below, we analyze the impact of a financial innovation through a change (an increase) in the loan-to-value ratio $m$.

**Production** The goods market is perfectly competitive and characterized by constant returns to scale. The representative firm produces the good according to the Cobb-Douglas technology:

$$Y = AK^\alpha L^{1-\alpha}$$

where $L$ and $K$ denote aggregate labor and aggregate capital respectively, $\alpha \in (0, 1)$ is the capital share of aggregate income. Business cycle fluctuations correspond to stochastic shocks to the total factor productivity $A \in \tilde{A} \equiv \{A^1, \ldots, A^n\}$. This aggregate shock is assumed to follow a finite-state Markov
process with transition matrix \( \pi_{A,A'} = \Pr (A_{t+1} = A' / A_t = A) \), with \( \pi_{A,A'} > 0 \) for every \( A, A' \in \tilde{A} \), and \( \sum_{A'} \pi_{A,A'} = 1 \) for every \( A \in \tilde{A} \).

The representative firm maximizes its profits setting the marginal product of labor and capital equal to the wage rate and interest rate (net of capital depreciation), respectively.

### 3.2. The Household Problem

Households maximize their expected discounted utility by choosing the levels of housing and financial asset holdings, hours worked and non-durable consumption.

Denote with \( \Phi_t (z_t, b_{t-1}, h_{t-1}; \beta) \) the distribution over individual productivity shocks, asset holdings, durable wealth, and discount factors in period \( t \). Given aggregate volatility, this distribution, and thus prices, will change over time, depending on the evolution of aggregate shocks and individual states.

In order to predict future prices, each household would need to know the distribution \( \Phi_t (z_t, b_{t-1}, h_{t-1}; \beta) \) and its law of motion, very complex objects. However, as in Krusell and Smith (1998), we assume that only the first moments of this distribution are known, and that they are sufficient to predict future prices.² In particular, agents observe aggregate durable wealth \( H_{t-1} \) and aggregate capital \( K_{t-1} \) at the beginning of period \( t \), and approximate the evolution of each of these variables and of aggregate labor with a linear function that depends on the aggregate shock \( A_t \).

We can thus write the household problem in an "approximate" recursive formulation, where the aggregate state variables are represented by the economy's capital and housing stock at the beginning of the decision period (keeping in mind that the "true" aggregate state would be the entire distribution \( \Phi_t \)). The relevant individual state variables are the productivity shock \( z_t \), the net assets position \( b_{t-1} \), and the stock of durable wealth \( h_{t-1} \) owned at the beginning of period \( t \). Denote with \( x_t \equiv (z_t, b_{t-1}, h_{t-1}, A_t, K_{t-1}, H_{t-1}) \) the vector of individual and aggregate state variables. In recursive form, the dynamic problem of a household with discount factor \( \beta_i \) can be stated as follows:

\[
V (x_t; \beta_i) = \max_{c_t, b_t, h_t, l_t} \left\{ u (c_t, h_t, l_t) + \beta_i \sum_{z_{t+1}, A_{t+1}} \pi_{A_t, A_{t+1}} \pi_{z_t, z_{t+1}} V (x_{t+1}; \beta_i) \right\} 
\]

s.t. 
\[
c_t + h_t + \Psi (h_t, h_{t-1}) = w_t z_t l_t + b_t - R_t b_{t-1} + (1 - \delta_H) h_{t-1} \\
b_t \leq mh_t, \ c_t \geq 0, \ l_t \in (0, \bar{l}) \\
(K_t, H_t, L_t) = F (K_{t-1}, H_{t-1}, A_t)
\]

where \( F \) is a linear function in \( K_{t-1} \) and \( H_{t-1} \), whose parameters depend on the aggregate shock \( A_t \), and denotes the law of motion of the aggregate states, which agents take as given. The house value depreciates

²Krusell and Smith's approach can be seen as a mere computational device to solve for an "approximate" equilibrium in this kind of models. In a different interpretation, agents could be thought of as having "partial information", or being characterized by "bounded rationality". In any case, Krusell and Smith (1998) show that their methodology is accurate enough so to have very small forecasting errors and an "approximate" equilibrium that is very close to the exact one.
at rate $\delta_H$. The agent sustains an additional cost, proportional to his initial housing stock, only if the adjustment in durables is big enough: $\Psi(h_t, h_{t-1}) = \psi h_{t-1}$ if $\frac{h_t - h_{t-1}}{h_{t-1}} > \phi_h$, with $\psi, \phi_h \in (0, 1)$. No expense is incurred if the proportional change in durables is lower than $\phi_h$.

### 3.3. The Equilibrium

A ("approximate") recursive competitive equilibrium is a value function $\{V(x_t; \beta)\}_{t=0}^{\infty}$, a set of policy functions $\{h(x_t; \beta) , b(x_t; \beta), c(x_t; \beta)\}_{t=0}^{\infty}$ for each $\beta$, prices $\{R_t\}_{t=0}^{\infty}$ and $\{w_t\}_{t=0}^{\infty}$, aggregate variables $K_t$, $L_t$, and $H_t$ for each period $t$, and a law of motion $F(K, H; A)$ such that:

**Definition 3.1.**

1. **Agents optimize:** Given $R_t$, $w_t$, and the law of motion $F$, the value functions are solution to the individual's problem in (1), with the corresponding policy functions;

2. **Prices are determined competitively at any $t$:**

   $$R_t - 1 + \delta_K = \alpha A_t (L_t/K_{t-1})^{(1-\alpha)}$$
   $$w_t = (1 - \alpha) A_t (K_{t-1}/L_t)^0$$

3. **Assets and Labor Markets clear at any $t$:**

   $$\int b(x_t; \beta) \partial \Phi_t (z_t, b_{t-1}, h_{t-1}; \beta) + K_t = 0$$
   $$L_t = \int l(x_t; \beta) z_t \partial \Phi_t (z_t, b_{t-1}, h_{t-1}; \beta)$$

As a consequence, the goods market satisfies the resource feasibility constraint at any $t$:

$$C_t + H_t - (1 - \delta_H) H_{t-1} + \Omega(H_t, H_{t-1}) + K_t - (1 - \delta_K) K_{t-1} = Y_t$$

where $\Omega(H_t, H_{t-1})$ denotes the total transaction costs incurred for adjustments to the housing stock, while $\delta_K$ is the depreciation rate of capital.

4. **The law of motion for aggregate capital, housing wealth and labor** is given by

   $$(K_t, H_t, L_t) = F(K_{t-1}, H_{t-1}, A_t)$$

Notice that individual labor supply is an intra-temporal decision, and the first order conditions that characterize the optimal $l(x_t; \beta)$ are the following:

$$\frac{w_t z_t}{c(x_t; \beta)} = \frac{\tau}{l - l(x_t; \beta)}$$
if $l(x_t; \beta) > 0$

$$\frac{w_t z_t}{c(x_t; \beta)} < \tau$$
if $l(x_t; \beta) = 0$
4. Parameterization

Our model period is assumed to be a year. We assume that two thirds of households have a discount factor of 0.95, and one third of households have a discount factor of 0.97. The high discount factor pins the average real interest rate down to an average value around 3 percent, as in the data. The low discount factor is in the range of estimates in the literature, see for instance Hendricks (2007) and references therein. The shares of patient and impatient agents imply that 1/3 of people hold most of wealth, and imply a Gini coefficient for wealth of around 0.8, roughly in line with the data.

We set the capital share $\alpha = 0.33$ and its depreciation rate $\delta_K = 0.10$. In all the economies we consider, these values yield average capital to output ratios around 2.6 and average investment to output ratios around 25% on an annual basis.³

We set the weight on housing in the utility function $j = 0.15$, and the depreciation rate for housing $\delta_H = 0.04$. These values yield average housing capital to output ratios of around 1.4 and average housing investment to output ratios around 5% on an annual basis. All these values are roughly in accordance with the National Income and Product Accounts and the Fixed Assets Tables.

The household incurs a cost equal to $\psi = 3\%$ of its current house value if its net housing investment exceeds 3% of the beginning of period housing stock ($\phi_h = 0.03$). This assumption reflects the fact that home improvements are a way to adjust housing consumption by small amounts without incurring in transaction costs.

We set $\tau = 1$ and the total endowment of time $t = 2.06$: these two parameters imply that average time worked (around unity) is slightly less than half the available time. As explained in King and Rebelo (2000), this is equivalent to assuming a labor supply elasticity of around unity.

Our calibration of the aggregate shock is meant to reproduce a standard deviation of output roughly in line with the data for the period 1952-1982 and 1984-2006. For the aggregate productivity shock $A_t$ we use a Markov-chain specification with five states to match the following first-order autoregressive representation for the logarithm of total factor productivity

$$\log A_t = \rho_A \log A_{t-1} + \sigma (1 - \rho_A^2)^{1/2} \varepsilon_t, \quad \varepsilon_t \sim \text{Normal} (0, 1).$$

We set $\rho_A = 0.90$. In the early period, we set $\sigma_A = 0.015$. In the late period, we set $\sigma_A = 0.009$.

In a similar vein, we specify the idiosyncratic labor efficiency shock using a three-state Markov chain to match:

$$\log z_t = \rho_Z \log z_{t-1} + \sigma_Z (1 - \rho_Z^2)^{1/2} \varepsilon_t, \quad \varepsilon_t \sim \text{Normal} (0, 1).$$

We set $\rho_Z = 0.925$. Our calibration of the idiosyncratic shock is meant to proxy for a reasonable degree of income uncertainty at the household level. The numbers we choose are in line with several microecono-

³Our definition of output excludes the value of imputed rents on housing services, which account for about 10% of GDP in the United States.
metric studies (for instance, see Aiyagari (1994) and references therein). We choose \( \sigma_Z = 0.14 \) in the early period, and we double this number (\( \sigma_Z = 0.28 \)) in the late period.

Finally, we set the maximum loan-to-value ratio \( m \) at 0.75 in the early period, and to 0.95 in the late period.

5. Model Results

In this section, we report our main findings for the baseline parameterization of the model. In light of our introduction, we focus on three main moments generated by the model: the overall volatility of output, the volatility of residential investment, and the correlation between aggregate debt and aggregate output.

In the calibration for the early period, the standard deviation of GDP generated by the model is 2.22 percent (the standard deviation of HP-filtered GDP is 2.09, as in the data counterpart). The implied correlation between debt and GDP is 0.61, only slightly lower than in the data (simulated aggregates are plotted in figure 1). As reflected in figure 2, this positive correlation is mainly the result of the behavior of the impatient agents, who borrow close to the maximum allowed, hold a disproportionate share of total debt in the economy (about 90 percent), and borrow more in good times, while increasing their holdings of housing in order to relax their borrowing constraint. As in the data, residential investment is very volatile, with a volatility which is about four times as large as that of total output. As reported in the table, the model does also a good job in accounting for the relative volatility of the other components of aggregate demand, namely consumption (although it slightly over predicts the volatility of consumption) and business investment (although it slightly under predicts its volatility).

Next, we change only one parameter of the calibration exercise, by doubling the volatility of the idiosyncratic shock from 0.14 to 0.28. Two interesting findings emerge:

1. larger idiosyncratic risk implies that aggregate debt becomes countercyclical (with the correlation between debt and GDP decreasing from 0.61 to −0.34);  
2. the volatility of residential investment is sharply reduced, by about 50 percent.

What is the mechanism at work? Figure 3 illustrates the simulated behavior of a patient agent (left column) and an impatient agent (right column), in the benchmark economy (low aggregate volatility, low idiosyncratic risk) as well as in the model with a higher idiosyncratic risk (but still low aggregate volatility). The mechanism at work mainly comes from the impatient. Because these agents face more volatile circumstances, they now reduce their holdings of debt in good times, effectively becoming prudent and starting to save for a rainy day. Of course, at the individual level they still borrow more when receiving a good idiosyncratic shock. However, this logic is reversed in good aggregate times, when they save more (or borrow less) for precautionary reasons. In fact, while in the benchmark economy an impatient agent
Data and Model.

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<tr>
<th>corr</th>
<th>sd(GDP)</th>
<th>sd(IH)</th>
<th>sd(IK)</th>
<th>sd(C)</th>
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<tr>
<td>Debt, GDP</td>
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**DATA**

(I) 1952.I-1982.IV  
0.78   2.09   8.24   5.04   1.19

(II) 1984.I-2006.IV  
0.04   1.21   3.51   4.21   0.59

**MODEL**

(I) Large agg. risk, small idio risk  
0.61   2.22   9.72   3.94   1.74

Large agg. risk, large idio risk  
-0.34  2.19   6.21   3.69   1.81

(II) Small agg. risk, large idio risk, high m  
0.05   1.32   4.49   2.33   1.08

Data and Model

Notes:

is almost always constrained (the ratio \( m_i = \frac{b_i}{h_i} \) is equal to \( m \), or 0.75), when facing higher risk the same agent decides not to reach the borrowing limit. Moreover, because impatient agents are less leveraged in good times, they also prefer not to change their holdings of housing collateral too much over the cycle.

In the last row of the table, we also allow for smaller aggregate shocks and for larger loan-to-value ratios, which capture in a crude way financial innovation episodes. As the table shows, smaller aggregate shocks are necessary to account for the overall decline in GDP volatility, but it is surprising how larger idiosyncratic risk can account for the reduced cyclicality of debt, for smaller volatility of housing investment, and for about 5% of the total reduction in GDP volatility observed in the data.

We illustrate our findings in the next three Figures.

1. Figure 1: Simulated aggregates

   By changing the variance of the individual shocks, the volatility of housing capital falls substantially

2. Figure 2: Simulated aggregates by group

   Debt of impatient agents becomes acyclical

3. Figure 3: Simulated individual variables
Impatients (left panel) engage in precautionary behavior by borrowing less than the maximum.

![Graphs of simulated aggregate variables](image)

Figure 1: Simulated aggregate variables. Thick line: Simulation with small idiosyncratic risk; Thin line: Large idiosyncratic risk.
Figure 2: Simulated variables aggregating across all patient (left column) and impatient agents (right column).
Figure 3: Simulated individual variables. Left column: Patient agent; Right column: Impatient agent.

6. Conclusions

[ TBA ]
References


Appendix A: Computational Details

We numerically solve for the model equilibrium using a computational method similar to the one used in Krusell and Smith (1998). The value and policy functions are computed on grids of points for the state variables, and then approximated with linear interpolation at points not on the grids. The algorithm consists of the following steps:

1. Specify grids for the state space of individual and aggregate state variables.

As a result of robustness checks, the number of grid points was chosen as follows: 5 points for the aggregate shock, 3 values for the idiosyncratic shock, 100 and 125 points for the housing stock of impatient and patients respectively, 800 points for the financial asset chosen by the patient agent, and 200 points for the b grid (mainly debt) of the impatient. We regress the aggregate variables $K, H$ and $L$ on aggregate capital only, for which we choose a grid of 12 equally spaced points in the range $[0.9K^*, 1.1K^*]$, where $K^*$ denotes the steady state aggregate capital. The aggregate housing stock seems to have negligible power for predicting future aggregate states.

2. Guess initial coefficients \( \{ \omega_{is}^A \}_{A \in A, i=0, \ldots, 2, s \in \{K, H, L\}} \) for the linear functions that approximate the laws of motion of aggregate capital, durables and labor:

\[
K_t = \omega_{0K}^A + \omega_{1K}^A K_{t-1} \\
H_t = \omega_{0H}^A + \omega_{1H}^A K_{t-1} \\
L_t = \omega_{0L}^A + \omega_{1L}^A K_{t-1}
\]

3. Use value function iteration in combination with Howard’s algorithm to compute optimal policies as a function of the individual and aggregate states.\(^4\) Notice that the intra-temporal optimal value for labor hours as a function of consumption and productivity shock is the following:

\[
l_t = \bar{l} - \frac{\tau c_t}{w_t z_t}
\]

which allows one to derive consumption directly from the budget constraint as follows:

\[
c_t = \frac{w_t z_t \bar{l} - R_t b_{t-1} + b_t + (1 - \delta_H) h_{t-1} - h_t - \Psi(h_t, h_{t-1})}{1 + \tau}
\]

and to write the per-period utility function as

\[
u = (1 + \tau) \log c_t + j \log h_t + \tau \log \tau - \tau \log(w_t z_t)
\]

\(^4\)In computation, we exploit the strict concavity of the value function in the choice for assets as well as the monotonicity of the policy function in assets (given any choice for the durable good).
As a consequence, the individual dynamic optimization problem entails solving for policy functions for $b$ and $h$ only.

For $l_t = 0$, we have

$$\max_{b_t, h_t} \{ \log c_t + j \log h_t + \beta_t E_t V (x_{t+1}; \beta_t) \}$$

s.t. $c_t = h_{t-1} (1 - \delta_H) - h_t - \Psi (h_t, h_{t-1}) - R_t b_{t-1} + b_t$

In practice, we prevent individuals from choosing zero hours.

4. Draw a series of aggregate and idiosyncratic shocks, and use the (approximated) policy functions and the predicted aggregate variables to simulate the optimal decisions of a large number of agents for many periods. We simulate 12,000 individuals for 10,000 periods, discarding the first 200.\(^5\)

Compute the aggregate variables $K_t$, $H_t$ and $L_t$ at each $t$.

5. Run a regression of the simulated aggregate capital, durables and labor on past aggregate variables $K$, retrieving the new coefficients $\{ \omega^A_{it} \}$ for the law of motion for $H_t$, $L_t$ and $K_t$. We repeat steps 3 and 4 until convergence over the coefficients of the regressions. We measure goodness of fit using the $R^{\text{ squared}}$ of the regressions (which are always equal to 0.999 or higher at convergence).

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\(^5\)We “enforced” the law of large numbers by making sure that the simulated fractions of labor productivity shocks corresponded to the theoretical ones, by randomly adjusting the values of the shocks.
Appendix B: Data Construction

[ TBA ]