Abstract

In this paper we document a new fact about the cyclical behavior of productivity and wages. Using industry-level time series on wages and labor productivity, we show that in high-wage industries, wages respond relatively little to industry productivity shocks, whereas in low-wage industries, productivity movements result in a relatively large movements in wages. In other words, wages are substantially "smoother" than productivity over time in high-wage industries, while wages are comparatively less smooth in low-wage industries. To explain this fact we develop a variant of the Thomas & Worrall (1988) wage contracting model. The two key features of our model are match-specific skills, which serve to increase wage smoothing in the contract, and exogenous match separations, which serve to reduce smoothing. We show that, empirically, a higher fraction of the skills of the high-wage workers are match-specific than the skills of low-wage workers, and that job separation rates are lower for high-wage workers than low-wage workers. A calibrated version of the model accounts quite well for the facts at hand.

JEL Codes: E24, E32, J24

Keywords: Wage Smoothing, Wage Rigidity, Risk Sharing, Match-Specific Capital

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1 Introduction

In this paper we document a new fact about the cyclical behavior of productivity and wages. Using industry-level time series on wages and labor productivity, we show that in high-wage industries, wages respond relatively little to innovations in industry productivity, whereas in low-wage industries, productivity movements result in relatively large movements in wages. In other words, wages are substantially "smoother" than productivity over time in high-wage industries, while wages are comparatively less smooth in low-wage industries. We show that this finding is robust within manufacturing and service industries, and in both the US and the majority of OECD countries for which we have data. To the best of our knowledge we are the first to document this fact.

We find this fact puzzling in light of standard implicit-contract theory (Azariadis, 1975 and Baily, 1974). According to implicit-contract theory, workers and firms agree to a wage contract in which the worker receives wages that are smoother over time than the worker’s marginal product, and in return the firm pays the worker wages that are on average lower than the worker would earn in spot markets. The key assumption in this theory is that workers are more risk averse than firm owners. This assumption is typically justified by arguing that workers have worse access to asset markets than firm owners, and hence have less means of otherwise smoothing their consumption. However, since low-wage workers generally have less access to asset markets than high-wage workers, the theory suggests that low-wage workers would have the smoother wages rather than high-wage workers. Our findings show exactly the opposite.

To help resolve this puzzle, we develop a variant of the Thomas & Worrall (1988) wage contracting model, in which a risk-neutral firm and risk-averse worker agree upon an optimal wage contract under limited commitment. The limited commitment is two-sided: both the worker and the firm can renege on the contract after any history. The state of the world in each period is characterized by the worker’s productivity, which evolves exogenously. Following Thomas and Worrall, we restrict our study to wage contracts that are self-enforcing, meaning that neither party has an incentive to renege on the contract in any state of the world.

The optimal contract in this environment specifies wage smoothing: wages move as little as possible after any productivity realization to keep both parties at least indifferent to remaining in the match. The amount of wage smoothing sustainable in equilibrium de-
pends on the outside options of the worker and firm. The better the outside options, the less smoothing can be sustained. In our model, the firm and worker have the option of leaving the match and going to spot markets in any period. If the worker goes to spot markets she earns her marginal product in every subsequent period. If the firm fires its current worker, it may match up with another worker, but competition among firms leads to zero expected profits from any given match.

We depart from Thomas & Worrall by adding two new features into the environment, each of which qualitatively affects the amount of wage smoothing sustainable in the optimal wage contract. The first feature is match-specific skills, which are lost if either party leaves the match. In the model, wage smoothing is increasing in the fraction of the worker’s skills that are match specific, since the worker can take fewer of her skills to a new firm, and hence is less productive in any new firm. The second feature we add is the possibility of an idiosyncratic, exogenous job separation. In the model, smoothing is decreasing in the probability of a separation to the match. Intuitively, the more likely the worker and firm are to separate, the less either party will value promises of higher future payoffs in exchange for lower payoffs in the present, as is required in order to smooth wages.

Our hypothesis about why the high-wage industries get relatively more wage smoothing is as follows. First, a higher fraction of the skills of high-wage workers are match-specific compared to low-wage workers. In other words, high-wage workers stand to lose relatively more from leaving their current job, and hence have relatively worse outside options than low-wage workers. Second, the probability of a job separation is higher for low-wage workers. In our model, both of these features lead qualitatively to smoother wages for high-wage workers.

Since our theory rests on these two differences between high and low-wage jobs, we document that both components are in line with empirical evidence. The more well-known of the two is the differences in separation rates across the two sectors. In the literature on job turnover, numerous studies document higher separation rates for low-wage industries; two recent examples include Davis, Faberman, and Haltiwanger (2006) and Fallick & Fleishman (2004). Regarding industry average wages and the fraction of skills that are match specific, we provide our own supporting evidence using estimates of wage losses for workers separated in mass layoffs as a proxy for match-specific skills. Carrington and Zaman (1994) estimate the average wage losses for displaced workers by detailed indus-
try, which we match with our own measures of industry average wages. We show that workers in high-wage industries tend to lose a higher fraction of their wages than low-wage workers after a large layoff, which we interpret as evidence that a higher fraction of the skills of high-wage workers are match-specific.

As a test of our theory, we ask whether a calibrated version of our model can match the degree of wage smoothing we observe in the cross section of industries. Specifically, we calibrate two versions of our model: one to represent a typical high-wage industry and one to represent a typical low-wage industry. We treat the empirical match-specificity of skills and separation rates as exogenous characteristics of the two sectors. We find that the two calibrated versions of our model predict degrees of wage smoothing that are quite similar to their empirical counterparts.

Our paper contributes to two distinct literatures. The first is the recent labor search and matching literature in macroeconomics, which seeks to explain equilibrium unemployment through matching frictions. A major challenge in this literature has been to explain how relatively small exogenous productivity shocks translate into relatively large movements in hiring, and hence equally large movements in the unemployment rate (Shimer, 2005). One potential resolution of the puzzle, explored by Hall (2005), Menzio (2005) and Rudanko (2006), among others, centers around wage contracts in which wages move little in response to a productivity shock, giving firms large incentives to hire new workers after small exogenous increases in productivity.

Our paper contributes to this literature in two ways. First, we document the cross-industry variation in the response of wages to productivity, which can be used to test among existing theories of why wages respond little to productivity shocks, such as those described above. Second, we propose an alternative mechanism for the response of wages to productivity that is grounded in the empirical response observed in the cross section of industries. Our paper contrasts with those of Hall and others in that we focus specifically on explaining how and why wages respond to productivity shocks rather than the implications for unemployment volatility. In a related paper (Lagakos & Ordonez, 2007) we relate our empirical findings directly to the unemployment volatility puzzle using an

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1 Using a stochastic version of the Mortensen-Pissarides (1994) model calibrated to match important moments of the US labor productivity series, Shimer finds that the model predicts just 10% of the volatility of unemployment and vacancy postings seen in the data.

2 This literature describes the unresponsiveness of wages to productivity as “wage rigidity” or “wage stickiness.”
industry version of Shimer’s (2005) model.

The second literature to which our paper is related is the one on risk sharing among private agents in the economy. This literature has focused in large part on impediments to risk sharing and the welfare implications of imperfect risk sharing. Recent papers by Krueger and Perri (2004), and Heathcote, Storesletten and Violante (2004, 2005) have demonstrated that the wage volatility has first order effects on consumption volatility and large effects on welfare more generally. Our paper contributes to this literature by analyzing how wage volatility arises in the first place for employed workers. Since the firm is an important vehicle for sharing risk, it is important to understand the impediments to risk sharing between firms and workers. Our paper highlights two specific impediments: a lack of match-specific skills for the worker, which forces wages to respond to changes in either party’s outside option, and the likelihood of an exogenous separation, which serves to discount the future of the match.

The remainder of the paper is as follows. In Section 2 we discuss our industry-level wage and productivity data, our measure of wage smoothing, and our empirical findings about the pattern of wage smoothing across industries. In Section 3 we present our wage contracting model, and in Section 4 we calibrate the model and describe our quantitative findings. In Section 5 we conclude.

2 Wage Smoothing: The Industry-Level Facts

2.1 Description of data

We use two main sources of data on industry productivity and wages in the study. The first source, available for the US, is the value-added by industry data constructed by the US Bureau of Economic Analysis (BEA). We use two different BEA data sets, each with annual industry-level measures of value added, employment, and compensation of labor. Our longest data set uses the 1972 Standard Industry Classification (SIC) codes, covers the entire private economy, and is available annually from 1947-1987. The second uses the 1987 SIC codes, and covers the shorter period from 1987-1997.3 Unfortunately we cannot conduct a similar analysis for the North American Industrial Classification System (NAICS), which is the BEA's preferred industry definition, because the BEA has not yet released historical employment data by NAICS industries.

3Unfortunately we cannot conduct a similar analysis for the North American Industrial Classification System (NAICS), which is the BEA's preferred industry definition, because the BEA has not yet released historical employment data by NAICS industries.
these time series into one long panel of industries, since several important industries changed definitions in 1987.

Our second data source is the OECD Structural Analysis (STAN) database, which is constructed using the national accounts of major OECD nations, including the US, and supplemented with data from national surveys of firms. For each country, our data set contains annual industry-level measures of value added, employment, and compensation of employees. The data is available from 1970 or later to 2000, depending on the country. The industries comprise all sectors of the economy, and are standardized across countries according to 2-digit International Standard Industrial Classification (ISIC) codes.4

The two main variables of interest are labor productivity and average compensation of labor. In each of our data sets, total compensation of labor consists of all salaries, bonuses, contributions to medical and pension plans, and any other compensation that is not in-kind. Our measure of average compensation per employee is total industry labor compensation divided by full-time-equivalent (FTE) employees. Throughout, we use the term ‘wages’ to mean ‘average compensation of employees’ for expositional purposes. Our measure of labor productivity is real industry value added divided by FTE employees.5 Employees consist of both production and non-production workers. For all countries we create real wage and value added data by deflating nominal values by a national consumer price index or its closest equivalent. To capture the relatively high-frequency component in our variables we de-trend productivity and wages in each industry, in each country, using an HP filter with smoothing parameter $\lambda = 100$.

Value added is difficult to measure in industries that do not have market-determined output prices. For this reason we drop any industry that consists of non-market activities, or in which market prices are not readily available.6 In particular, we drop public administration, defense, and compulsory social security; education; health and social work; real-estate activities; other community, social, personal services; and private household employees. We also drop agriculture since problems in measuring employment (particu-

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4The STAN dataset is available for purchase online directly from the OECD at http://www.oecd.org/sti/stan/. Documentation can be freely downloaded from the same site. The list of industries comprising the STAN database can be found at http://www.oecd.org/dataoecd/33/19/1830838.html.

5For countries in which FTE employment is not available, we use the total number of employees instead. The choice of labor measure does not seem to drive any of our results: for countries with both labor input measures the results were very similar under the two measures.

6Here we follow the STAN documentation. See pages 6-9.
larly for non-paid family workers) exist in several countries.

2.2 Our Measure of Wage Smoothing

We measure the amount of smoothing in industry $j$ as $\varepsilon_j$ in the linear model:

$$w_{j,t} = \varepsilon_j p_{j,t} + u_{j,t}$$

where $w_{j,t}$ and $p_{j,t}$ are average wages and labor productivity in industry $j$ at time $t$, each expressed in log deviation from trend, and $u_{j,t}$ is an error term. Our wage-smoothing measure $\varepsilon_j$ has the interpretation of the elasticity of wages in deviation from trend with respect to productivity in deviation from trend.\footnote{We omit the constant term in this regression under the assumption that when productivity is at trend, then wages will be at trend as well.} We refer to this elasticity as the wage-productivity elasticity throughout the paper. Industries with wage-productivity elasticities close to one are industries with little wage smoothing, and industries with elasticities close to zero have a high degree of smoothing.

To illustrate this measure of wage smoothing, we plot de-trended wages and productivity in two select US industries in Figure 1. Each plot in the figure shows value added per worker (green lines with +'s) and compensation per worker (blue lines with x’s) expressed in log deviations from trend. For expositional purposes we selected two industries that display clear differences in their degrees of wage smoothing. The wage-productivity elasticity in Rubber & Plastics manufacturing is 0.18, indicating a high degree of smoothing, while in Textile manufacturing, the wage-productivity elasticity is 0.51.\footnote{We omit the standard errors of industry elasticities here and elsewhere. We find that they are relatively small: on the order of 0.05 or smaller in almost all cases, without substantial variation across industries.} As is apparent in the figure, wages in textiles are considerably more responsive to a change in productivity than in Rubber & Plastics.

Figure 1 presents summary statistics for these elasticities for all industries, as well as for just service industries and just manufacturing industries. We define services here to be all industries in our data set that do not constitute manufacturing, mining, or mining-related industries. The statistics are all weighted by industry employment, which is crucial since industry sizes vary substantially.\footnote{To save space we omit summary statistics for US industries in the OECD STAN database and for the SIC 1987 industry definition. These statistics were extremely similar.} The first thing to take away from this table is that mean
(and median) elasticities are roughly comparable across manufacturing and services, with services above the overall average at around 0.5, and manufacturing below at around 0.3. The second finding, and perhaps the more interesting one, is that both services and manufacturing exhibit large variation in elasticities, with standard deviations of around 0.15 and 90-percentile ranges from around 0 to 0.5 in manufacturing and from around 0 to 0.8 in services. These results show that there have been vast differences in wage smoothing across industries of all types over the post-war period. We now turn to the question of which types of industries tend to have the highest degrees of smoothing.

\[ \varepsilon^{w,p} = 0.18 \]

\[ \varepsilon^{w,p} = 0.51 \]

Figure 1: Wage-Productivity Elasticities in Two Select US Industries.

\[ \text{Rubber & Plastics Manufacturing} \]

\[ \text{Textile Manufacturing} \]

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\[ ^{10}\text{The first paper we are aware of to measure the wage-productivity elasticity in the US is by Hagedorn and Manovskii (2006), who arrive at an estimate 0.45 for aggregate BLS data. This estimate is entirely in line with our findings for the average US industry in each of our data sets.} \]
### Table 1: Summary Statistics for US Wage-Productivity Elasticities, 1947-1987.

<table>
<thead>
<tr>
<th>Industry Type</th>
<th>Mean</th>
<th>Median</th>
<th>Standard Deviation</th>
<th>5th Percentile</th>
<th>95th Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>0.41</td>
<td>0.39</td>
<td>0.18</td>
<td>0.06</td>
<td>0.73</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.28</td>
<td>0.27</td>
<td>0.12</td>
<td>0.05</td>
<td>0.51</td>
</tr>
<tr>
<td>Services</td>
<td>0.47</td>
<td>0.46</td>
<td>0.17</td>
<td>0.07</td>
<td>0.79</td>
</tr>
</tbody>
</table>

2.3 **High-Wage Industries Have the Most Smoothing**

In this section we detail our main empirical finding with regard to wage-productivity elasticities, namely that elasticities tend to be lower in industries with high average wages. We document this pattern first for US industries, using our three different data sets, and then for industries in OECD countries, using the STAN data set. Within the US, we show that this pattern shows up within manufacturing industries as well as within service industries.

Figure 2 displays our main result for the US, using the BEA time series from 1947-1987. Each “bubble” on the figure represents one industry, where the size of each bubble is the industry employment weight. The $y$-axis represents the wage-productivity elasticity and the $x$-axis represents the industry average wages in 1987 in thousands of 2005 dollars.\(^{11}\) The main feature of the graph is the strong negative relationship between industry average wages and elasticities, demonstrating that higher wage industries tend to get the most smoothing. The employment-weighted correlation across industries is -0.53, indicating a robust negative relationship between the industry average wage and the wage-productivity elasticity. We obtained similar results in the STAN dataset for the US, with a correlation of -0.66, and in the shorter BEA series, with a correlation of -0.38.

We explore the robustness of this result in two ways. First, we show that the negative correlation between the average wage and the wage-productivity elasticity appears within both manufacturing and service industries. Table 2 shows the correlations in each data set for just the manufacturing industries, and just the service industries (as well as for the whole economy). In all cases the correlation between the wage productivity elasticity and

\(^{11}\)We choose 1987 because it is the latest year of our series.
average wage is negative. The second way we check robustness is to regress the wage-productivity elasticity on the industry average wage and other salient industry characteristics, using our entire sample of industries as observations. Table 3 shows the results of this regression, where the independent variables are a manufacturing dummy plus (the logs of) industry average wages, the volatility of industry productivity, the autocorrelation of industry productivity, and the industry’s labor share in value added. As is evident from the regression results, the only industry characteristic that turns out to be statistically significant from zero is the industry wage. Furthermore, it has an economically significant coefficient. A hypothetical doubling of an industry’s average wage holding all else constant yields an elasticity that is lower by 0.19, which constitutes roughly 25% of the entire range in elasticities we see in the data. We conclude from the regression results that whatever drives wage smoothing is closely related to the average industry wage.

Next we examine whether this result holds in other OECD countries. Table 4 shows the
Table 2: Correlations of Wage-Productivity Elasticities and Average Wages in US Industries.

<table>
<thead>
<tr>
<th>Data Source</th>
<th>Industry Definition</th>
<th>Industries</th>
<th>Correlation of $\epsilon^{w,p}$ and $w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OECD STAN, 1970-2000</td>
<td>ISIC, Rev 3.</td>
<td>All</td>
<td>-0.66</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Manufacturing</td>
<td>-0.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Services</td>
<td>-0.58</td>
</tr>
<tr>
<td>US BEA, 1947-1987</td>
<td>SIC 1972 definition</td>
<td>All</td>
<td>-0.53</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Manufacturing</td>
<td>-0.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Services</td>
<td>-0.42</td>
</tr>
<tr>
<td>US BEA, 1987-1997</td>
<td>SIC 1987 definition</td>
<td>All</td>
<td>-0.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Manufacturing</td>
<td>-0.27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Services</td>
<td>-0.33</td>
</tr>
</tbody>
</table>

same correlations over other OECD countries in our sample, using the STAN data. Just as for the US, we compute the correlations weighing each industry by total employment in the last year in which data is available. Due to short samples in some countries for some industries, we drop any industry whose elasticity standard error was greater than 0.1. Next, we drop any country with less than 10 industries, so as not to generalize about too few particular industries.\textsuperscript{12}

\textsuperscript{12}While these choices are fairly arbitrary, we find that the results do not change in any important way for other similar choices. For instance when taking 0.05 as our cutoff for the standard error, and 20 for our cutoff on the number of industries, we end up with more industries and fewer countries but the same overall result.
<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Coefficient (standard error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log(Industry Average Wage, 1987)</td>
<td>-0.19** (0.07)</td>
</tr>
<tr>
<td>Log(Volatility of Industry Productivity)</td>
<td>-0.07 (0.09)</td>
</tr>
<tr>
<td>Log(Autocorrelation of Industry Productivity)</td>
<td>0.08 (0.06)</td>
</tr>
<tr>
<td>Manufacturing Dummy</td>
<td>-0.08 (0.05)</td>
</tr>
<tr>
<td>Log(Industry Labor Compensation / Value Added)</td>
<td>-0.07 (0.10)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.95* (0.56)</td>
</tr>
</tbody>
</table>

R-squared 0.46
Number of Observations (Industries) 52
Dependent Variable: Wage-Productivity Elasticity

**significant at 5% level, *significant at 10% level

Table 3: Regression of Wage-Productivity Elasticity on Industry Characteristics, US Industries.

The results for OECD countries largely mimic the US in that there is a negative correlation between the wage level and the wage-productivity elasticity. For 8 countries out of 17, the correlation is below -0.2, indicating a reasonably strong negative relationship, for an additional 6 countries the relationship is weaker, but still negative, and for just 3 of the 17 countries the relationship is positive. The main limitation of this analysis is that the time series are relatively short for most countries, with just 20 years of data or less available in most cases. Even so, at the very least our findings suggest that the negative relationship

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13 A more comprehensive comparison of results across OECD and developing countries, using longer time series, would be both useful and interesting, although it is outside of the scope of this paper. In particular, it would be interesting to relate this correlation and the elasticities more generally to differences in labor regulations and unionization rates across these countries, which are likely to play an important role.
<table>
<thead>
<tr>
<th>Country</th>
<th>Time Period</th>
<th>Correlation of $\epsilon^{w:p}$ and $w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>1970 - 2001</td>
<td>-0.66</td>
</tr>
<tr>
<td>Spain</td>
<td>1986 - 2000</td>
<td>-0.61</td>
</tr>
<tr>
<td>Belgium</td>
<td>1980 - 1999</td>
<td>-0.45</td>
</tr>
<tr>
<td>Netherlands</td>
<td>1980 - 2000</td>
<td>-0.44</td>
</tr>
<tr>
<td>Sweden</td>
<td>1980 - 1999</td>
<td>-0.43</td>
</tr>
<tr>
<td>Germany</td>
<td>1991 - 2000</td>
<td>-0.39</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>1985 - 2001</td>
<td>-0.36</td>
</tr>
<tr>
<td>Austria</td>
<td>1980 - 1999</td>
<td>-0.24</td>
</tr>
<tr>
<td>Portugal</td>
<td>1988 - 1999</td>
<td>-0.18</td>
</tr>
<tr>
<td>Finland</td>
<td>1975 - 2001</td>
<td>-0.18</td>
</tr>
<tr>
<td>Norway</td>
<td>1980 - 1997</td>
<td>-0.13</td>
</tr>
<tr>
<td>France</td>
<td>1980 - 2000</td>
<td>-0.08</td>
</tr>
<tr>
<td>Japan</td>
<td>1980 - 1998</td>
<td>-0.08</td>
</tr>
<tr>
<td>Korea</td>
<td>1980 - 1997</td>
<td>-0.01</td>
</tr>
<tr>
<td>Australia</td>
<td>1980 - 1999</td>
<td>0.10</td>
</tr>
<tr>
<td>Italy</td>
<td>1980 - 2001</td>
<td>0.11</td>
</tr>
<tr>
<td>Denmark</td>
<td>1980 - 2001</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Table 4: Cross-Industry Correlations of Wage-Productivity Elasticities and Average Wages, OECD Countries.

between elasticities and wage levels is not idiosyncratic to the US, but in fact exists in a number of modern economies.

3 Model

In this section we develop a model of wage contracting between a worker and firm, and we use the model to demonstrate how match-specific capital and separation rates influence the degree of wage smoothing present in the optimal wage contract. We show that a higher degree of match-specific skills leads to smoother wages, as does a lower probability of an exogenous separation. In the calibration section to follow, we document empirically that a higher fraction of the skills of the high-wage workers are match-specific than the skills of low-wage workers, and that job separation rates are lower for high-wage workers than low-wage workers. The model therefore predicts that high-wage workers will have smoother wages than low-wage workers.


3.1 Environment

A risk-averse worker and risk-neutral firm are matched. We assume that the worker prefers higher values of

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

(2)

where $E_0$ is the expected value operator at time 0, $c_t$ is consumption, and $\beta \in (0, 1)$ is the worker’s discount factor. The utility function $u(\cdot)$ is assumed to be strictly increasing and strictly concave. Workers are endowed with one unit of labor each period which they supply inelastically to the firms for wage $w_t$. We abstract from asset markets or storage possibilities, and so the worker’s consumption each period equals her wage: $c_t = w_t$.

The firm operates a constant-returns technology that uses labor from one worker as the sole input to produce output $y_t$. The firm keeps the output, which it sells for a (normalized) price of 1, and pays the worker a wage $w_t$. The firm prefers higher values of

$$E_0 \sum_{t=0}^{\infty} \beta^t (y_t - w_t)$$

(3)

A worker matched to a firm can either be skilled in the match, or unskilled in the match. We let $\theta \in \{0, 1\}$ represent the possession of skill in the match, where $\theta = 1$ represents a worker possessing match-specific skills and $\theta = 0$ represents a worker without match-specific skills. We let $m \in [0, 1]$ represent the fraction of a worker’s skills that are match-specific. Finally, let $p \in \mathbb{R}^+$ represent the current realization of productivity in the match; we will explain how productivity evolves in detail below. The production function is given by:

$$y = F(p, m, \theta) = p(1 - m(1 - \theta))$$

(4)

In other words when the worker possesses match-specific skills, output from the match is simply $p$, whereas without the match specific skills output is $p(1 - m)$. Productivity $p$ is stochastic and takes on one value each period in the set $\mathbb{P} \equiv \{p^1, p^2, ..., p^S\}$ where $p^i < p^j$ for $i < j$. We assume that productivity evolves as a first-order markov chain, where $\alpha_{p', p}$ is the probability of transitioning to state $p'$ from state $p$.

At the beginning of each period $t$ the productivity state $p_t$ is realized. If they both decide to stay in the match then output $y_t$ is produced, and the worker gets wage $w_t$. Either party may leave the match after $p_t$ realizes, however, in which case they both get their respective
outside options. Let $\Pi(p)$ denote the value of the firm’s outside option in productivity state $p$. We assume that if the firm breaks the match it can match up with a new worker, but the new worker does not trust the firm and will only accept a wage equal to her output in each period. In this case the firm will earn zero profits in each period. So $\Pi(p) = 0 \; \forall p \in P$.

The worker’s outside option is to join another firm. Let $V(p)$ denote the worker’s outside option in state $p$. We assume that if the worker leaves the match she becomes unskilled in the new match for $n$ periods, and only afterwards does she gain match-specific skills in the new match. In addition, the new firm will not trust the worker and will pay her a wage each period exactly equal to her output. We can express her outside option in state $p$ as:

$$V(p) = u(p(1-m)) + E \left[ \sum_{t=1}^{n-1} \beta^t u(p_t(1-m)) + \sum_{t=n}^{\infty} \beta^t u(p_t) \right]. \quad (5)$$

The final component of the environment is the possibility of an exogenous separation, which we assume occurs with probability $s \in [0,1)$ each period. If an exogenous separation occurs in productivity state $p$ then the worker gets $V(p)$ and the firm gets $\Pi(p)$.

### 3.2 Wage Contracting Problem

We now consider the optimal wage contracting problem in this environment. For now assume that at the initial period the worker is entitled to a particular utility promise $v$, and the worker begins as skilled in the match. Following Thomas & Worrall (1988) we consider only contracts that are self-enforcing, in the sense that in no state of the world does either party have incentive to break the contract.

Let $\Pi(v,p)$ be the firm’s value function given a promised utility $v$ for a worker in productivity state $p$, which represents the maximized expected discounted profits from the match. The firm’s problem can be written as

$$\Pi(v,p) = \max_{w \in \{v'(p') \}_p} \left\{ p - w + \beta \sum_{p'} \alpha_{p'|p} \Pi(v'(p'),p') \right\} \quad (6)$$
subject to a promise-keeping constraint

\[ v = u(w) + \beta \sum_{p'} \alpha_{p'\mid p} v'(p') \]  

(7)

to worker self-enforcement constraints for every future state:

\[ v'(p') \geq V(p') \forall p', \]  

(8)

and to firm self-enforcement constraints

\[ \Pi(v'(p'), p') \geq \Pi(p') \forall p' \]  

(9)

The self enforcing constraints guarantee that neither party ever wants to leave the contract. Separations only occur exogenously. As in Thomas & Worrall, the optimal wages in the contract will be functions of current and one-period-prior productivities \((p, p_{\cdot 1})\), and the optimal wages will move as little as possible to satisfy the self-enforcing constraints.

**Proposition 1** Thomas & Worrall (1988). Let \((p_{\cdot 1}, p, p')\) be any productivity history in \(\mathbb{P} \times \mathbb{P} \times \mathbb{P}\), and let \(w \equiv w(p, p_{\cdot 1})\) and \(w' \equiv w(p', p)\) be the optimal wage after history \((p, p_{\cdot 1})\) and \((p', p)\). Then

1. if \(w' > w\) then \(v(p') = V(p')\)

2. if \(w' = w\) then \(v(p') > V(p')\) and \(\Pi(v, p') > \Pi(p)\)

3. if \(w' < w\) then \(\Pi(v, p') = \Pi(p)\)

**Proof** See Appendix A.1.

The proposition says that if wages rise from one period to the next, they do so just to the point where the worker is indifferent between staying in the match. Similarly, if wages fall they do so until the firm is indifferent. Finally, if wages stay the same then it must be the case that both parties strictly prefer the match to their respective outside options. In short, wages are smoothed as much as possible such that both parties are willing to stay in the match. This result highlights the fact that the amount of wage smoothing will depend in large part on the outside options for each party.
Also just as in Thomas & Worrall we have the following corollary about the domain on which optimal wages must lie.

**Proposition 2** For all \( p \in \mathbb{P} \) there exists an interval \([w_p, \bar{w}_p}\) such that

1. \( w(p, p_{-1}) \in [w_p, \bar{w}_p] \) \( \forall p_{-1} \)
2. when \( w(p, p_{-1}) = w_p \) then \( v = \nabla(p) \), and
3. when \( w(p, p_{-1}) = \bar{w}_p \) then \( \Pi(v, p) = \Pi(p) \)

This result says that the range of optimal wage always lives in an interval where the worker is indifferent between staying in the contract or not at the lowest wage in the interval, and the firm is indifferent at the highest wage in the interval. This result will be used to greatly simplify the quantitative analysis to come later.

### 3.3 Two-state Version

For the remainder of the paper we consider a two-state version of the model. As we will show below, two features of the productivity process in the model have direct implications for the amount of wage smoothing present in the optimal contract. These are the volatility and autocorrelation of the productivity series. We note that the two-state version of the model will not restrict our quantitative analysis with regards to autocorrelation and volatility: both can be captured with a 2-state Markov chain representing the productivity process.

In this version we consider the set of productivity states to be \( \mathbb{P} = \{p_L, p_H\} \) where \( p_L < p_H \). Let \( \alpha \) be the persistence parameter in the transition matrix. Denote the two intervals described in Proposition 2 by \([w_L, \bar{w}_L]\) and \([w_H, \bar{w}_H]\) in states \( p_L \) and \( p_H \). We immediately get the following two corollaries, which depend on whether or not the two intervals \([w_L, \bar{w}_L]\) and \([w_H, \bar{w}_H]\) overlap or not.

**Corollary 1** If \( \bar{w}_L \geq \bar{w}_H \) then the optimal wages are constant after the first time productivity switches states.
To see this result, let $\bar{w}_L > \bar{w}_H$, and take an arbitrary initial state (for exposition say $p_L$) and an arbitrary initial wage $w_0$ that satisfies Proposition 2. Once the state switches to $p_H$, we know by Proposition 1 that if $w_0 < \bar{w}_H$ then the wage rises until the worker’s self-enforcement constraint binds, i.e. until $w = \bar{w}_H$. But this wage is now incentive compatible in both states, and hence by Proposition 1 it remains constant for all future periods. If on the other hand $w_0 \geq \bar{w}_H$ then it is incentive compatible to both parties in both states to begin with, and hence it remains constant.

**Corollary 2** If $\bar{w}_L < \bar{w}_H$ then the optimal wages $(w_L, w_H)$ after the first time productivity switches states are given by $w_L = \bar{w}_L$ and $w_H = \bar{w}_H$ for all remaining periods.

The intuition for this corollary is seen as follows. Take an arbitrary initial state (for exposition again say $p_L$) and an arbitrary wage that satisfies Proposition 2. Once the state switches to $p_H$, we know by Proposition 1 that the wage rises until the worker’s constraint binds, i.e. until $w = \bar{w}_H$. By Proposition 1 again we know that while at $p_H$ the wage remains constant. When $p_L$ realizes the wage must fall until the firm’s constraint binds, i.e. until $w = \bar{w}_L$. Similarly, wages remain constant while in $p_L$. When $p_H$ realizes again we have $w = \bar{w}_H$.

These corollaries describe the two possible types of wage dynamics in the model: perfect smoothing and imperfect smoothing. So far we have said nothing about the initial wage, or alternatively the initial utility promise to the worker. However, the initial split of the surplus is not of particular importance in this model, since the wage dynamics are set as soon as the productivity switches states. Therefore, we focus on the long-run implications of the wage contract, which we define to be after the state has switched at least once. We also focus on the case of imperfect wage smoothing (i.e. $\bar{w}_L < \bar{w}_H$), since this is the empirically relevant case. Now, by Corollary 2, we conclude the wages in the optimal contract are $w_L = \bar{w}_L$ and $w_H = \bar{w}_H$ in all periods.

Let $V_L(w_L, w_H) \in \mathbb{R}^+$ and $V_H(w_L, w_H) \in \mathbb{R}^+$ be the worker’s expected discounted utilities in states $p_L$ and $p_H$ under wages $(w_L, w_H)$. Similarly, let $\Pi_L(w_L, w_H) \in \mathbb{R}^+$ and $\Pi_H(w_L, w_H) \in \mathbb{R}^+$ be the firm’s expected discounted profits in the optimal contract in states $p_L$ and $p_H$. The optimal contract can then be pinned down by the following system of two equations and two unknowns, $w_L$ and $w_H$:

$$V_H(w_L, w_H) = \bar{V}(p_H) \quad (10)$$

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We now turn to how wages in the optimal contract depend on fundamentals of the environment, in particular the match-specific capital \( m \), separation rate \( s \), and volatility and autocorrelation of the productivity process.

**Proposition 3** Wage smoothing is increasing in \( m \), i.e.
\[
\frac{\partial w_H}{\partial m} < 0 < \frac{\partial w_L}{\partial m}.
\]

The intuition for this result is that losing match specific skills has a first-order negative effect on the value of the worker’s outside option in all states. Reducing this outside option allows the firm to obtain lower average wages that are smoother.

**Proposition 4** Wage smoothing is decreasing in \( s \), i.e.
\[
\frac{\partial w_L}{\partial s} < 0 < \frac{\partial w_H}{\partial s}.
\]

The separation rate acts to discount future utility promises for both parties, since there is less expected future in the match. But discounting the future of the match more heavily reduces the willingness of either party to sacrifice in the present in exchange for future payoffs. Hence, the worker and firm can only agree upon wages that are relatively close to the worker’s marginal product.

The final proposition gives the model’s implications for the autocorrelation and volatility of productivity on wage smoothing.

**Proposition 5** Wage smoothing is decreasing in the autocorrelation of productivity (\( \alpha \)), i.e.
\[
\frac{\partial w_L}{\partial \alpha} < 0 < \frac{\partial w_H}{\partial \alpha},
\]
and increasing in the volatility of productivity (\( p_H/p_L \)).

The intuition for the autocorrelation result is that the higher the autocorrelation, the higher is the worker’s value from deviating in the high state, since she can expect to be in the high state for longer. Thus her wage in the high state must be higher to keep her in the contract, and her low-state wage lower to keep the firm in the contract. The volatility result is straightforward: the higher is the volatility of shocks, the worse the value of the worker’s outside option relative to the contract, which allows for more smoothing.

Recall that our theory about why high-wage industries had smoother wages than low-wage industries is that high-wage industries have a higher degrees of match-specific skills and lower separation rates than low-wage industries. Our theory predicts that these two
differences between high and low-wage industries will result in more wage smoothing in high-wage industries. In the following section we provide empirical support for these two differences between high and low wage industries, and we compare the quantitative implications of the model to the facts documented in Section 2.

4 Quantitative Analysis

In this section we parameterize the model developed above and assess its quantitative predictions. In parameterizing the model, we also document directly that high-wage industries have a higher degree of match specific capital than low-wage industries and lower separation rates than low-wage industries. While the previous section demonstrated that match-specific capital and separation rates lead qualitatively to more smoothing in high-wage industries, this section assess whether the model can match the quantitative degrees of wage smoothing found in the data. In particular, we quantify two versions of our model, one to represent the average high-wage industry and one to represent the average low-wage industry. We allow three key differences between these two sectors: to the fraction of skills that are match-specific $m$, to the separation rate $s$, and to the productivity series themselves. Quantitative success will be if the calibrated versions can match the average wage-productivity elasticity for low-wage industries, which is 0.34, and the average elasticity for high-wage industries, which is 0.59.

4.1 Differences Between High and Low-Wage Industries

The first part of our hypothesis is that a higher fraction of the skills of high-wage workers are match-specific than the skills of low-wage workers. However, measuring the extent to which skills are match specific is not a straightforward exercise. How does one distinguish between match-specific and general skills? A seminal paper by Jacobson, Lalonde and Sullivan (1993) uses worker wage loss after a mass layoff as a proxy for the value of skills that are match specific. They treat mass layoffs as an event unrelated to worker ability, and compare the post-layoff wages of workers that were laid off (leavers) to those that stayed with their respective companies (stayers). If all skills were general, rather than specific to the match, there is little reason to expect that the leavers would have substan-
tial wage reductions after a layoff— they could simply take their (general) skills to a new match and earn a comparable wage as in their old match.

In fact Jacobson et al find that wages of stayers and leavers, while almost indistinguishable before the layoff period, depart drastically after the layoff. The wages of stayers stay roughly constant while the wages of leavers fall by around 40% or more. Furthermore, the wages of leavers stay depressed for a long period after their layoff, returning to their pre-layoff wage only after around 4 years. The authors conclude that these drastic layoffs provide direct evidence that match-specific skills form a very large fraction of a worker’s total human capital. One caveat here is that the authors consider only the losses of workers with 6 years of tenure or more, who are much more likely to have acquired match-specific skills than workers with lower tenure. In a similar study using different data and a different period, Schoeni and Dardia (1996) corroborate almost all of the findings of Jacobson et al.¹⁴

Following the work of Jacobson et al, Carrington and Zaman (1994) measure the costs of job displacement by industry, using the percentage wage loss after a mass layoff as a measure of match-specific portion of a worker’s overall human capital. Fortunately for our study, Carrington and Zaman conduct their analysis using the same 2-digit industry classification as ours, which allows us to compare their displacement cost estimates to our industry characteristics, especially average industry wages. We present these findings in Figure 3. As can be seen on the graph, there is a significant positive correlation between Carrington & Zaman’s estimates of displacement costs (in percentage terms) by industry and the industry average wage. For low-wage industries, displacement costs run from around 6% to 18% of average wages. On the other hand workers in high-wage industries tend to lose between 10% to 26% of their average wage. This finding suggests that the skills of high-wage workers are generally more match specific than low-wage workers.

Regarding separation rates, recent estimates by Davis, Faberman, and Haltiwanger (2006) provide evidence that separation rates are indeed higher in low-wage industries. Davis et al construct industry measures of job separation rates using micro data from the Job Opening and Labor Turnover Survey (JOLTS) for 2001-2005, which contains detailed measures of job separations by broad industry groups. They find that the highest separation rates occur in industries with the lowest average wages, such as retail trade and hospitality & leisure. Retailing for example has a monthly separation rate of 3.9%, which is equivalent

¹⁴For an engaging overview of the literature on worker displacement see Kletzer (1998).
to a 41% annual rate of separation. On the other hand, high wage industries have relatively lower rates of separation. In manufacturing, for example, the monthly separation rate is 2.7%, which is a yearly rate of 28%.

### 4.2 Calibration & Simulation

We begin our calibration with our measures of match specific capital and separation rates, which correspond to $m$ and $s$ in the model. For $m$, rather than picking one particular value for high-wage industries and a second value for low-wage industries, we solve each version of the model over the ranges of $m$ observed in the data. The empirically-observed ranges of $m$ are 6% to 18% for low-wage industries and 10% to 26% for high-wage industries. For separation rates, for the low-wage-industry version we take the observed 41% annual rate of separation in retail trade, an industry which had an average
wage in 1987 of around 50% of the average industry. For our high-wage-industry version we take the 28% rate of separation observed in manufacturing, which has average wages roughly 50% higher than average in our sample of industries.

Other parameters are calibrated as follows. For the household’s preferences we choose CRRA utility with risk-aversion equal to 1, and for the discount factor $\beta$ we choose 0.95 as is typical in annual data. We choose the length of time the worker is unskilled in a new match following separation to be 4 years as a benchmark, which is consistent with the findings of Jacobson et al and Schoeni and Dardia mentioned above. For the productivity series we estimate, for each industry, an AR(1) process for productivity of the form

$$p_{i,t} = \phi_i p_{i,t-1} + u_{i,t}$$  \hspace{1cm} (12)$$

where we assume that $u_{i,t} \sim N(0, \sigma_i^2)$, and where $p_{i,t}$ is the logarithm of detrended productivity in industry $i$. We take the average $\hat{\phi}_i$ and $\hat{\sigma}_i$ for low-wage industries and high-wage industries and approximate each sector by a 2-state Markov chain. Normalizing long-run productivity to be 1, we end up with states of $p_{H,LW} = 1.042$ and $p_{L,LW} = 0.958$ for the low-wage sector and $p_{H,HW} = 1.063$ and $p_{L,HW} = 0.937$ for the high-wage sector. The persistence parameters for the transition matrices are $\alpha_{LW} = 0.71$ for the low-wage sector and $\alpha_{HW} = 0.69$ for the high-wage industries.

For each sector, we simulate the model over the range of $m$ values described above. For each $m$, we simulate 10,000 paths for productivity. Each path consists of 1,040 periods where the first 1,000 are discarded to avoid any influence of the initial state, and the next 40 (representing 1947-1987) are kept. For these 40 periods we calculate the wage-productivity elasticity in the same way as in our empirical analysis, and we take the mean elasticity over all 10,000 simulations.

### 4.3 Simulation Results

Our simulations results are shown in figures 4 and 5. Figure 4 shows the results for the low-wage sector. The $x$-axis represents $m$ and the $y$-axis represents the elasticities generated by the model and in the data. The blue horizontal line (at $\epsilon_{w,p} = 0.59$) is the average low-wage elasticity. The thicker downward-sloping blue line is the elasticity generated by the model for each given $m$. At $m = 0$ the elasticity is 1, meaning that the worker...
earns his marginal product in every period. At $m = 0.22$ we get an elasticity of 0, or perfect wage smoothing. For intermediate $m$ values we get imperfect smoothing. The red shaded box on the $x$-axis represent the empirically relevant range of match specific skills taken from Carrington & Zaman. For the empirically plausible range of $m$ we get elasticities of between 0.2 and 0.75, which are distinguished by the red box along the $y$-axis. These elasticities are largely in line with the range of low-wage elasticities seen in the data.\textsuperscript{15} The median low-wage industry (retail trade) has an estimated $m$ of 0.12 – for this value the model yields an elasticity of 0.5 (shown as a red dotted line), which is close to the true value of 0.59 but too much smooth relative to the data. We will return to this sector shortly.

Figure 4: Simulated Wage-Productivity Elasticities, Low Wage Sector, n=4.

For the high-wage sector (Figure 5) we have an empirically plausible range of $m$ of 10% to 26%, which the model maps into elasticities of 0 to 0.6. These are also very much in line with the elasticities found in the data for high-wage industries. The median high-wage industry has $m$ of around 16%, which gives an elasticity of 0.37 (shown as a red dotted

\textsuperscript{15}This can be seen in Figure 2.
Figure 5: Simulated Wage-Productivity Elasticities, High Wage Sector, n=4.

(Description of the diagram: The graph shows the simulated wage-productivity elasticities for the high-wage sector, with a comparison to the empirical data. The median high-wage industry is marked with a horizontal line, almost exactly the 0.35 seen in the data. We conclude that this baseline version of the model does very well in matching the high-wage sector but predicts a bit too much smoothing in the low-wage sector.

How can we decrease the smoothing in the model low-wage sector? A natural choice is to reduce \( n \), the number of years for which the low-wage work is not skilled in a match. This seems sensible, since low-wage jobs generally require lower (specific and general) human capital levels than high-wage jobs. When we decrease \( n \) to 3 years, we do much better in matching the data. In this case the median low-wage \( m \) of 12\% implies an elasticity of 0.62, which is closer to the empirical value of 0.59. A lower \( n \) works just as expected.

We conclude from the quantitative portion of the paper that this simple wage contracting model does surprisingly well in matching the facts documented in the empirical section of the paper. This suggests that wage smoothing in the cross-section of industries is well explained by differences in the degree of match specific skills and separation rates.
5 Conclusion

In this paper we document a new fact about the cyclical behavior of wages and productivity: in high-wage industries, average wages move relatively little in response to a change in labor productivity, whereas the opposite is true in low-wage industries. In other words, wages are substantially smoother than productivity in high wage industry and considerably less so in lower-wage industries. This finding appears for the US within both manufacturing and service industries and in a majority of OECD countries for which we had data. The finding also appears robust to controlling for other important industry characteristics such as the volatility of industry productivity and the industry labor share in value-added.

Our explanation of this fact is that the response of wages to productivity is determined by an optimal wage contract between a worker and firm under two-sided limited commitment. In high wage industries, high levels of match-specific capital and low separation rates lead to a greater degree of wage smoothing in the optimal contract. We provide direct empirical support for our hypothesis using industry-level data on displacement costs and separation rates, and we formalize our hypothesis in a model based on the Thomas & Worrall (1988) study of wage contracting under limited commitment. We find that the calibrated model performs quite well in explaining the facts at hand, using empirically justifiable measures of match-specific skills and separation rates by industry. Future work will explore the quantitative performance of the model in greater detail.
A Proofs of Propositions

A.1 Proof of Proposition 1

Fix a state \((v, p)\) and let \(\eta\) be the Langrange multiplier on the promise keeping constraint (7). For the worker and firm self-enforcing constraints (8) and (9) let the multipliers be \(\beta \alpha_{p'} \lambda_e(p')\) and \(\beta \alpha_{p'} \lambda_f(p')\). The first order conditions are for choice of \(w\) and each \(v'(p')\) imply

\[
\frac{1}{u_w(w)} = \frac{1}{u_w(w')}(1 + \lambda_f(p')) - \lambda_e(p') \quad \forall p'
\]

(13)

If \(w' = w\) then it must be true that \(\lambda_f(p') = \lambda_e(p') = 0\), which implies that \(v(p') > \bar{V}(p')\) and \(\Pi(v, p') > \Pi(p)\). If \(w' > w\) then \(u_w(w') < u_w(w)\) by concavity, which by (13) implies that \(\lambda_e(p') > 0\) and hence \(v(p') = \bar{V}(p')\). By a similar argument \(w' < w\) implies that \(\lambda_f(p') > 0\), and hence \(\Pi(v, p') = \bar{\Pi}(p)\). ■

A.2 Proof of Proposition 3

First we establish that \(\frac{\partial \Pi}{\partial m} > 0\). Note that increasing \(m\) has a first-order negative effect on the outside options: \(\frac{\partial \bar{V}(p_H)}{\partial m} < 0\) and \(\frac{\partial \bar{V}(p_L)}{\partial m} < 0\). With this in mind, imagine that \(m\) increases but that the contract wages \(w_H\) and \(w_L\) stayed the same. Then \(V_H(w_H, w_L)\) would remain the same while \(\bar{V}(p_H)\) would fall, implying that (10) would no longer hold. On the other hand it would still be true that \(\Pi_L(w_H, w_L) = 0\), in other words (11) would still hold. It follows that leaving that leaving \(w_L\) and \(w_H\) the same is clearly not optimal when \(m\) increases, and more importantly that the firm could reduce average wages in order to reduce \(V_H(w_H, w_L)\) and make (10) hold once again. With lower average wage, it follows then that \(\frac{\partial \Pi}{\partial m} > 0\).

Now there are two logical possibilities to reduce average wages: (1) the firm could reduce \(w_H\) while increasing \(w_L\) by a smaller magnitude, or (2) it could increase \(w_H\) and reduce \(w_L\) by a larger magnitude. We show that (1) is in fact the case. From the definition of \(\Pi_L\) in the imperfect smoothing case we have that

\[
\frac{\partial \Pi_L}{\partial m} = -\frac{\partial w_L}{\partial m} + \beta(1 - \alpha)(1 - s) \frac{\partial \Pi_H}{\partial m}
\]

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and hence
\[
\frac{\partial w_L}{\partial m} = \beta (1 - \alpha)(1 - s) \frac{\partial \Pi_H}{\partial m}
\] (14)
using the fact that \(\frac{\partial \Pi_L}{\partial m} = 0\) from (11). From this we conclude that \(\frac{\partial w_L}{\partial m} > 0\). Using the definition of \(\Pi_H\) we have that
\[
\frac{\partial \Pi_H}{\partial m} = -\frac{1}{1 - \beta \alpha (1 - s)} \frac{\partial w_H}{\partial m}
\]
Combining this with (14) we get that
\[
\frac{\partial w_L}{\partial m} = -\frac{\beta (1 - \alpha)(1 - s) \frac{\partial w_H}{\partial m}}{1 - \beta \alpha (1 - s)}
\]
from which we conclude that \(\frac{\partial w_H}{\partial m} < 0\) as we claimed would be the case. ■

A.3 Proof of Proposition 4

First we show that \(\frac{\partial w_L}{\partial s} < 0\). We start with the fact that \(\Pi_L = 0\) in the optimal contract no matter what \(s\) is, so
\[
\frac{\partial \Pi_L}{\partial s} = -\frac{\partial w_L}{\partial s} + \beta (1 - \alpha) \left[ (1 - s) \frac{\partial \Pi_H}{\partial s} - \Pi_H \right] = 0
\] (15)
which implies that
\[
\frac{\partial w_L}{\partial s} = \beta (1 - \alpha) \left[ (1 - s) \frac{\partial \Pi_H}{\partial s} - \Pi_H \right] = 0.
\]
Again by the fact that \(0 = \Pi_L = p_L - w_L + \beta (1 - s) \Pi_H\) we have that
\[
\Pi_H = (\beta (1 - \alpha)(1 - s))^{-1} (w_L - p_L)
\]
and hence that
\[
\frac{\partial \Pi_H}{\partial s} = (\beta (1 - \alpha)(1 - s))^{-1} \left[ -(w_L - p_L)/(1 - s) + \frac{\partial w_L}{\partial s} \right]
\] (16)
Combing (15) and (16) we get that
\[
\frac{\partial w_L}{\partial s} = (\beta (1 - \alpha))^{-1} \left[ -\Pi_H - (w_L - p_L)/(1 - s) \right].
\]
Since \( \Pi_H \geq 0 \) and \( w_L - p_L > 0 \) the left-hand side must be negative, which implies that \( \frac{\partial w_L}{\partial s} < 0 \) as well.

Second, we show that \( \frac{\partial w_H}{\partial s} > 0 \). Using the fact that \( V_H = \tilde{V}_H \) in the optimal contract for any \( s \), it follows that \( \frac{\partial V_H}{\partial s} = 0 \), which gives

\[
0 = \frac{\partial V_H}{\partial s} = \beta(1 - \alpha)(\tilde{V}_L - V_L) + \beta(1 - s)(1 - \alpha)\frac{\partial V_L}{\partial s} + u_w(w_H)\frac{\partial w_H}{\partial s}
\]

and hence

\[
\frac{\partial V_L}{\partial s} = \frac{\beta(1 - \alpha)(V_L - \tilde{V}_L) - u_w(w_H)\frac{\partial w_H}{\partial s}}{\beta(1 - \alpha)(1 - s)}. \tag{17}
\]

From the definition of \( V_L \) we get

\[
\frac{\partial V_L}{\partial s} = \frac{\beta \alpha(V_L - \tilde{V}_L) - u_w(w_H)\frac{\partial w_H}{\partial s}}{1 - \beta \alpha(1 - s)}
\]

which can be combined with (17) to give

\[
(\gamma - \alpha \beta)(V_L - \tilde{V}_L) = \gamma u_w(w_H)\frac{\partial w_H}{\partial s} + u_w(w_L)\frac{\partial w_L}{\partial s} \tag{18}
\]

for \( \gamma \equiv \frac{(1 - (1 - s)\alpha \beta)}{(1 - \alpha)(1 - s)\beta} \). It can be shown that \( \gamma > 1 \), which implies that the left-hand side of (18) is greater than or equal to zero. Using our result that \( \frac{\partial w_L}{\partial s} < 0 \) it follows that in order to satisfy (18) that \( \frac{\partial w_L}{\partial s} \) must be > 0. □

A.4 Proof of Proposition 5

We start with the proof that \( \frac{\partial w_H}{\partial s} > 0 > \frac{\partial w_L}{\partial s} \) under imperfect smoothing. Note that increasing \( \alpha \) increases the worker’s outside option in \( p_H \) and reduces it in \( p_L \); \( \frac{\partial V(p_H)}{\partial \alpha} > 0 \) and \( \frac{\partial V(p_L)}{\partial \alpha} < 0 \). As in the proof of Proposition 3, consider an increase in \( \alpha \) while the firm hypothetically keep contract wages the same. Then \( V_H(w_H, w_L) \) would remain the same while \( \tilde{V}(p_H) \) would increase, and on the other hand \( \Pi_L(w_H, w_L) = 0 \). So (10) would fail to hold while (11) would still hold. It follows that leaving that leaving \( w_L \) and \( w_H \) the same is not the optimal, and that the firm would need to raise average wages in order to increase \( V_H(w_H, w_L) \) to make (10) satisfied. With a higher average wage, it follows then that \( \frac{\partial \Pi}{\partial \alpha} < 0 \).
We can use the definition of $\Pi_H$ to get

$$\frac{\partial \Pi_H}{\partial \alpha} = -\frac{1}{1 - \beta \alpha (1 - s)} \frac{\partial w_H}{\partial \alpha}$$

which tells us that $\frac{\partial w_H}{\partial \alpha} > 0$. From the definition of $\Pi_L$ and using the fact that $\frac{\partial \Pi_L}{\partial \alpha} = 0$, we get that

$$\frac{\partial w_L}{\partial \alpha} = \beta (1 - s) \left[ \frac{-1(1 - \alpha)}{1 - \beta \alpha (1 - s)} \frac{\partial w_H}{\partial \alpha} - \Pi_H \right]$$

Since $\Pi_H \geq 0$ we conclude that $\frac{\partial w_L}{\partial \alpha} < 0$, which completes the proof.

Finally, we argue that wage smoothing is increasing in the volatility of shocks, which we capture by $p_H/p_L$. For brevity we keep this argument short and informal as it follows almost identically the logic of the proof of Proposition 3. Increasing the volatility reduces the worker’s outside options in both states, which allows the firm to reap higher profits from the match. Profits are higher the smoother the wages, which means that $w_H$ falls and $w_L$ rises, rather than the other way around. ■
References


