Ambiguity, Information Quality and Credit Risk*

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Abstract

This paper studies the implications of ambiguity for the credit spreads. We consider two ways of incorporating ambiguity into the Duffie and Lando [2001] model of credit spreads under incomplete information. We begin by examining the credit spreads in a setting where news about the asset value are of an uncertain quality. In this setting, ambiguity-averse investors act as if they take the worst-case assessment of quality. Thus, they react more strongly to bad news than to good news. As a result, changes in the information environment can lead to a widening of credit spreads and implied default intensities, even if firm fundamentals do not change. If the underlying asset value dynamics are also ambiguous, then the ambiguity-averse investors in the secondary debt market act as if the drift rate of the underlying asset value is lower than it actually is. We calibrate the two competing models to match the observed increase in credit spreads after August 9, 2007. Comparing the implications of the two models for default spreads, we find that ambiguity about the underlying asset value dynamics is better able to match the patterns observed in the data.

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1 Introduction

In the summer of 2007, the world financial markets entered into a severe liquidity crisis. On August 9, 2007, France’s largest bank BNP Paribas announced that it was having difficulties because two of its off-balance-sheet funds had loaded up on securities based on American subprime mortgages. But Paribas was not alone in its troubles: a month before, the German bank IKB announced similar difficulties, and the Paribas announcement was followed the next day by Northern Rock’s revelation that it had only had enough reserve cash to last until the end of the month. These and other similar announcements lead to a freeze of the credit markets as banks lost faith in each other’s balance sheets. The situation was particular surprising considering the market conditions shortly before the crisis began. At the beginning of 2007, financial markets were liquidity-unconstrained and credit spreads were at historical lows (see Fig. 1). Even as late as May 2007, it would have been hard to predict the magnitude of the response that the losses on subprime mortgages had generated. Compared to the total value of financial instruments traded worldwide, the subprime losses were relatively small: even the worst-case estimates put them at around USD 250 billion \(^1\). Further, for investors familiar with the instruments, the losses were not unexpected. By definition, the subprime mortgages were part of the riskiest segment of the mortgage market, so it was hardly surprising some borrowers would default on the loans. Yet, despite their predictability, the defaults had precipitated the current liquidity crisis that spread between the credit markets.

As Caballero and Krishnamurthy [2008a] argue, the crisis was primarily caused by the rise in ambiguity that followed the subprime defaults. That is, although the value of the underlying assets did not necessarily change, the credit market participants were taken by surprise at how the complex credit derivatives were reacting and became uncertain about the quality of their investments. The rise of the complex credit derivatives in the previous five years proliferated the problem. Because investors had no prior experience with how these instruments would behave in a time of crisis, they did not know how to interpret the securities’ response to mortgage failure. The complexity of the instruments involved made it almost impossible to calculate the proper response to changes in the underlying. Thus, when even AAA-rated subprime tranches suffered

\(^1\)Source: Caballero and Krishnamurthy [2008b]
losses, the resulting increase in ambiguity about the quality of investments in place lead market participants to make decisions based on their perceived worst-case scenario. The revelations by major banks about their exposure to subprime mortgages through off-balance sheet funds only served to reinforce the increase in ambiguity about signals from the major market participants.

The current paper takes the Caballero and Krishnamurthy [2008a] argument – that the credit freeze was precipitated by a sudden increase in ambiguity – as given and considers the implications of an increase in ambiguity for credit spreads. We extend the model of Duffie and Lando [2001] by introducing ambiguity into the secondary debt market in two different ways. First, we assume that the participants in the secondary debt market do not know the information quality of the signal about the underlying asset value. This assumption generates an asymmetric response to positive and negative news, with negative news being interpreted by the agents as having the highest possible precision and positive as having the lowest possible. As an alternative specification, we also consider the case when investors are ambiguous about the asset value dynamics itself. To capture the Caballero and Krishnamurthy [2008b] observation that the increase in ambiguity has lead investors to evaluate investment under the worst-case scenario, we specify the debt holder’s aversion to ambiguity by a max-min expected utility optimization problem. The axiomatic basis for preferences of this type was first introduced by Gilboa and Schmeidler [1989] and extended to the case of intertemporal utility maximization by Epstein and Schneider [2003].

A rapidly growing literature studies the behavior of asset prices in the presence of ambiguity in dynamic economies. A substantial part of this literature considers investor ambiguity about the data-generating model. Anderson et al. [2003] derive the pricing semigroups associated with robust perturbations of the true state probability law. Trojani and Vanini [2002] use their framework to address the equity premium and the interest rate puzzles, while Leippold et al. [2008] consider also the excess volatility puzzle. Gagliardini et al. [Forthcoming] study the term structure implications of adding ambiguity to a production economy. This setting has also been used to study the portfolio behavior of ambiguity-averse investors and the implications for the options markets (see e.g. Trojani and Vanini [2004] and Liu et al. [2005]).

The second strand in the literature, however, assumes that, although the agents in the economy know the “true” data-generating model, they face uncertainty about the quality of the
observed signal about an unobservable underlying. Chen and Epstein [2002] study the equity premium and the interest rate puzzles in this set-up, and Epstein and Schneider [2008] consider the implications for the excess volatility puzzle. The portfolio allocation implications of this setting have also been studied extensively in e.g. Uppal and Wang [2003] and Epstein and Miao [2003].

However, none of these papers study the relationship between ambiguity aversion and the term structure of credit spreads. We begin by introducing ambiguity aversion using the simple setting of Epstein and Schneider [2008], who measure ambiguity by the size of the set of possible signal precisions. In this way, we can easily study the impact of an increase in the signal quality ambiguity on the term structure of credit spreads, in dependence on the boundaries of the set of possible signal precisions. The preferences underlying the max-min expected utility problem solved by our representative agent are of the recursive multiple priors utility type, which, in the terminology of Epstein and Schneider [2003], implies a “rectangular” set of relevant likelihoods and admits an axiomatic foundation.

Using this setting to extend the model of Duffie and Lando [2001], we find that, under the max-min preferences, the ambiguity-averse participants in the secondary debt market will always choose the lowest signal precision in evaluating positive news about the corporate bonds and the highest signal precision when evaluating negative news. This is exactly in line with the observation of Caballero and Krishnamurthy [2008b] that the current credit crisis prompted investor to consider the worst-possible case. Notice also that this coincides with the result of Epstein and Schneider [2008] who find that, when evaluating signals about a company’s cash-flows, equity investors interpret negative signals as very precise and positive signals as having low precision. Hence, in our case, since the accounting signals received were negative, the lower bound of signal variance needs to decrease to generate changes in the credit spreads. Using the calibration of Duffie and Lando [2001] as the parameters of the economy before August 9, 2007, we find that, in order to generate the observed changes in the credit spreads, the lower bound of signal variance had to decrease to at least 1%. Further, the decrease in signal quality was asymmetric between the Aaa and Baa bonds, with the quality of information decreasing more for the Aaa bonds.

Next, we consider the setting where investors know the signal precision but are unsure about
the model for the underlying asset value dynamics. In this setting, participants in the secondary debt market will always choose the likelihood corresponding to the lowest possible expected drift rate. As before, we use the calibration of Duffie and Lando [2001] as the base case, and find that the ambiguity tolerance (that is, the size of the set of likelihoods considered plausible by the debt holders) of the debt holders has to increase substantially to match the observed changes in the credit spreads.

To distinguish between the two sources of ambiguity, we consider their implications for the default spreads of financial institutions. In particular, we calibrate the model to the balance sheet data of several prominent financial institution and repeat the above exercise to find the implied changes in ambiguity following the BNP Paribas announcement. We find that, although ambiguity about signal quality is better able to match the changes in the 1 year default spread, changes in the ambiguity about the asset dynamics are better able to match the changes in the overall term structure of default spreads. Thus, we conclude that ambiguity about the underlying is more suitable in this setting.
2 Model

2.1 The firm

The model considered in this paper extends the model of Duffie and Lando [2001] to allow for ambiguity aversion on the part of the participants in the secondary debt market. Denote by $A_t$ the stock of assets of a given firm and assume that it evolves according to a geometric Brownian motion. In particular, let $A_t = e^{Z_t}$ and assume:

\[ dZ_t = m dt + \sigma dW_t, \quad t \geq 0, \]

where $W_t$ is a standard Weiner process. The above specification implies that the stock of assets of the firm grows at an expected rate of $\mu = m + \frac{1}{2} \sigma^2$.

While the firm is in operation, it generates cash flows equal to a constant fraction, $\delta$, of the asset value, so that at time $t$ the cash flows of the firm are given by $\delta A_t$. At time 0, the firm issues debt for some amount $D$ with a constant coupon rate $C > 0$. As in Duffie and Lando [2001], we assume that the coupon rate $C$ is chosen to maximize the total value of the firm (the market value of equity plus the market value of debt) at time 0. The amount of debt issued, $D$, is then calculated as the expected present value of the cash-flows to the (infinitely-lived) bond. To simplify the analysis, we assume that the debt is non-callable and that the tax rate is a constant $\theta \in (0, 1)$.

2.2 Equity owners’ problem

The equity owners’ problem considered in the paper is the same as in Duffie and Lando [2001]. Here, we recall the main points of the set-up and provide the optimal decisions rule.

The firm is operated by risk-neutral equity owners, who discount the future at a constant rate $r$. The equity owners are assumed to have complete information about the value of the firm’s assets, $A_t$, at all times $t$. That is, the equity owners’ information set is $\mathcal{F}_t = \sigma\{A_s : s \leq t\}$. Given the filtration $\mathcal{F}_t$, the equity owners decide when to liquidate the firm at an $\mathcal{F}_t$-stopping time $\tau : \Omega \rightarrow [0, \infty]$. The firm is assumed to be liquidated at the expected present value of the discontinued cash-flows:

\[ \frac{\delta A_\tau}{r - \mu} \]
but a fraction $\alpha$ of the assets is lost in liquidation. The cash generated by the liquidation, $(1 - \alpha)\delta A_t / (r - \mu)$, is distributed among the debt holders. Finally, it is assumed that the proceeds from the initial sale of debt are paid as a cash distribution to the initial shareholders. Thus, the equity holders solve:

$$\sup_{\tau} F(A_0, C, \tau) = \mathbb{E} \left[ \int_0^\tau e^{-rt} (\delta A_t + (1 - \theta)C) \, dt \right].$$

Duffie and Lando [2001] show the following result.

**Proposition 2.1. (Proposition 2.1 of Duffie and Lando [2001])**

Suppose $r > \mu$. Let $a_B(C)$ and $d(A_0, C)$ be defined by:

$$a_B(C) = \frac{(1 - \theta)C\gamma(r - \mu)}{r(1 + \gamma)\delta},$$

$$d(a, C) = \frac{(1 - \alpha)a_B(C)\delta}{r - \mu} \left( \frac{a}{a_B(C)} \right)^{-\gamma} + \frac{C}{r} \left[ 1 - \left( \frac{a}{a_B(C)} \right)^{-\gamma} \right],$$

where

$$\gamma = \frac{m + \sqrt{m^2 + 2r\sigma^2}}{\sigma^2}.$$

Then the optimal liquidation problem (2.2) is solved by the first time $\tau(a_B(C))$ that $A$ is at or below $A_B = a_B(C)$. The associated initial values of equity and debt are $v(A_0)$ and $d(A_0, C)$, respectively, with $v$ given by:

$$v(a) = \left\{ \frac{\delta a}{r - \mu} - \frac{a_B(C)\delta}{r - \mu} \left( \frac{a}{a_B(C)} \right)^{-\gamma} + (\theta - 1)C \left[ 1 - \left( \frac{a}{a_B(C)} \right)^{-\gamma} \right] \right\} 1_{\{a \geq a_B(C)\}},$$

where $1_{\{\}}$ is the indicator function.

### 2.3 Debt holder’s problem

Unlike the equity owners, the participants in the secondary market do not observe the asset valuation process, $A_t$, directly. Instead, they receive imperfect accounting signals, $\hat{A}_t$, at selected times $t_1 < t_2 < \ldots$. More specifically, we assume that the signal observed by the debt holders is given by $Y(t) \equiv \log \hat{A}_t = Z(t) + U(t)$, where $U(t)$ is normally distributed and independent of $Z(t)$. In the following section, we examine the debt owners’ problem in the presence of ambiguity about the signal quality and the underlying asset dynamics. Below, we present the problem of participants in the secondary debt market for a single likelihood.
The participants in the secondary debt market understand that the optimizing equity owners will force liquidation if the equity value falls below \( A_B \). Further, in addition to the accounting signals, they observe at each \( t \in [0, \infty) \) whether the firm has been liquidated. Thus, the information set of the debt holders is given by:

\[
\mathcal{H}_t = \sigma \{ Y(t_1), \ldots, Y(t_n), 1_{\{\tau \leq s\}} : 0 \leq s \leq t \},
\]

for the largest \( n \) such that \( t_n \leq t \) and \( \tau = \tau(V_B) \). As in Duffie and Lando [2001], we assume that equity is not traded in the public market and that equity owners are precluded from trading in the public debt markets. This assumption allows us to keep the simple structure (2.6) for the information set of the secondary market and avoid solving the rational-expectations equilibrium problem with asymmetric information.

We assume that the signal errors are serially uncorrelated, so that \( U_t \sim \text{i.i.d} \ N(\bar{\mu}, \sigma_u^2) \). We further assume that the agents believe the accounting signal to be unbiased, so that \( \mathbb{E}[e^{U_t}] = 1 \). Thus, for a given level of the signal variance, \( \sigma_u^2 \), the quality of the signal is given by \( 1/\sigma_u^2 \) and the mean error is \( \bar{\mu} = -\sigma_u^2/2 \).

Denote by \( Z^{(n)} \) a history of the true realizations of the (log) asset value up to time \( t_n \) and \( Y^{(n)} \) the history of signals:

\[
Z^{(n)} = (Z_{t_1}, \ldots, Z_{t_n}) \quad \text{and} \quad Y^{(n)} = (Y_{t_1}, \ldots, Y_{t_n})
\]

and by \( b_n(\cdot|Y^{(n)}) \) the conditional probability of observing the history \( Z^{(n)} \):

\[
b_n \left( \cdot |Y^{(n)} \right) = \mathbb{P} \left( z^{(n)} \in dz^{(n)}, \tau > t_n |Y^{(n)} \right).
\]

Then, applying Bayes’ rule, we have:

\[
b_n \left( z^{(n)} | y^{(n)} \right) = \frac{\mathbb{P} \left( Z^{(n)} \in dz^{(n)}, \tau > t_n, Y^{(n)} \in dy^{(n)} \right)}{\mathbb{P} \left( Y^{(n)} \in dy^{(n)} \right)}.
\]

Introduce two events:

\[
A = \{ \tau > t_n, Z_{t_n} \in dz_{t_n}, Y_{t_n} \in dy_{t_n} \}
\]

\[
B = \{ \tau > t_{n-1}, Z_{t_{n-1}} \in dz_{t_{n-1}}, Y_{t_{n-1}} \in dy_{t_{n-1}} \}.
\]
Then we can represent:

\[ \psi_{\beta} (z_{n-1} - a, z_n - a, \sigma \sqrt{\Delta t}) \psi (z_{t_n} | z_{t_{n-1}}) \frac{p_Y (y_{t_n} - z_{t_{n-1}} | y_{n-1})}{p_Y (y_{n-1})} \times b_{n-1} (z^{(n-1)} | y^{(n-1)}) . \]

Here, \( \psi(z_0, x, \sigma \sqrt{t}) \) is the probability, conditional on \( Z \) starting at some given level \( z_0 \) at time 0 and ending at some level \( x \) at a given time \( t \), that \( \min \{Z_s : 0 \leq s \leq t\} > 0 \); \( p_Y (u_t | u_{t_n-1}) \) is the transitional density of \( U_{t_1}, U_{t_2}, \ldots \); \( p_Z (z_{t_n} | z_{t_{n-1}}) \) is the transitional density of \( Z_{t_1}, Z_{t_2}, \ldots \); \( p_Y (y_{t_n} | y^{(n-1)}) \) is the conditional density of \( Y_{t_n} \) given \( Y^{(n-1)} \), \( a = \log A_B \).

After conditioning on \( Y^{(n)} \) and also on survival to time \( t_n \), we have that the conditional density \( g_n (z^{(n)} | y^{(n)}) \) of \( Z^{(n)} \) is given by:

\[ g_n (z^{(n)} | y^{(n)}) = \frac{b_n (z^{(n)} | y^{(n)})}{\int_{R_n} b_n (z | y^{(n)}) \, dz}, \]

where the region of integration is given by:

\[ R_n = \{ z \in \mathbb{R}^n : z_1 \geq a, \ldots, z_n \geq a \} . \]

Notice that, although we do not have an explicit formula for \( g_n \), we can use the recursive solution (2.7) to do the numerical integration recursively, one dimension at a time.

In the rest of the paper, we focus on the special case of observing only one period of accounting signals. Since we are assuming that the signal errors are serially uncorrelated, this is equivalent to assuming that, after each accounting report, participants in the secondary debt market update the last “known” observation of \( Z \) to the new posterior mean. Denote: \( \bar{y} = y - a - \bar{u} \), \( \bar{z} = z - a \), and \( \bar{z}_0 = z_0 - a \). Then, from Duffie and Lando [2001], we know that, conditionally on having observed a single noisy observation \( y \) at time \( t = t_1 \) and on \( Z \) starting at some given level \( z_0 \) at time 0, the time \( t \) distribution of \( z \) is given by:

\[ g(z | y, z_0, t) = \exp \left( \frac{\beta_3}{4 \beta_0} - \frac{\beta_3}{4 \beta_0} - \beta_3 \right) \left[ 1 - \exp \left( -\frac{2 \bar{z}_0 \bar{z}}{\sigma^2 t} \right) \right] \Phi \left( \frac{\beta_1}{\sqrt{2 \beta_0}} - \beta_3 \Phi \left( \frac{\beta_2}{\sqrt{2 \beta_0}} \right) - \beta_3 \right), \]
where

\[
J(\tilde{y}, \tilde{z}, \tilde{z}_0) = \frac{(\tilde{y} - \tilde{z})^2}{2\sigma^2_u} + \frac{(\tilde{z}_0 + mt - \tilde{z})^2}{2\sigma^2_t}
\]  

(2.10)

\[
\beta_0 = \frac{\sigma^2_u + \sigma^2_t}{2\sigma^2_u\sigma^2_t}
\]  

(2.11)

\[
\beta_1 = \frac{\tilde{y}}{\sigma^2_u} + \frac{\tilde{z}_0 + mt}{\sigma^2_t}
\]  

(2.12)

\[
\beta_2 = -\beta_1 + 2\frac{\tilde{z}_0}{\sigma^2_t}
\]  

(2.13)

\[
\beta_3 = \frac{1}{2} \left( \frac{\tilde{y}^2}{\sigma^2_u} + \frac{(\tilde{z}_0 + mt)^2}{\sigma^2_t} \right)
\]  

(2.14)

Notice that we can interpret \(\tilde{y}\) as the new information contained in the signal \(y\). In particular, since we do not update the mean of the accounting error, \(\bar{u}\), and since the liquidation threshold \(\underline{a}\) is given exogenously, \(\tilde{y}\) measures the expected distance to the liquidation barrier, conditional on the signal \(y\). Thus, the sign of \(\tilde{y}\) indicates the “direction” of the news. If \(\tilde{y} < 0\), then the expected distance to the liquidation threshold is negative so the news contained in the signal is bad. Similarly, if \(\tilde{y} > 0\), then the signal carries good news as the expected distance to liquidation is positive.

2.4 Default and credit spreads

We turn now to the implications of the above model for the term structure of credit spreads of the modeled firm. We begin by deriving the term structure of default swap spreads. As noted in Duffie and Lando [2001], this formulation is more convenient as it allows us to not make assumptions about the recovery rates on zero coupon bonds while allowing us to still recover the term structure of credit spreads.

Let \(T\) be the maturity of the swap and denote by \(X\) the payment if default occurs before the maturity date \(T\). Then, assuming that the default-swap annuity payments are made semi-annually, we have:

\[
X = 1 - \frac{(1 - \alpha)\delta A_B}{(r - \mu)D}.
\]

Since the swap’s value at inception is 0, we have that the time \(t\) default-swap spread on a bond
maturing at $T$ is:

$$c(t, T) = \frac{2X \mathbb{E} \left[ e^{-r(\tau - t)} \mathbf{1}_{\tau \leq T} \mid \mathcal{H}_t \right]}{\sum_{i=1}^{k} e^{-\frac{r}{2}} \mathbb{E} \left[ \mathbf{1}_{\tau > t + \frac{1}{2}i} \mid \mathcal{H}_t \right]}.$$  

The implicit discount curve is then given by:

$$Z(t, T_i) = \frac{1 + c(t, T_i)/2}{1 + c(t, T_i)/2} Z(t, T_i) = \frac{1 - \frac{c(t, T_i)}{2} \sum_{j=1}^{i-1} Z(t, T_j)}{1 + c(t, T_i)/2},$$

with the credit spread on a bond maturing at date $T_i$, $\eta_i$, defined by:

$$Z(t, T_i) = \exp \left( -(r + \eta_i)(T_i - t) \right).$$

Consider now calculating the expectations in (2.15). Notice that the expectation in the denominator can be rewritten as:

$$\mathbb{E} \left[ \mathbf{1}_{\tau > t + \frac{1}{2}i} \mid \mathcal{H}_t \right] = \mathbb{E} \left[ 1 \mid \tau > \left( t + \frac{1}{2}i \right), \mathcal{H}_t \right]$$

$$= \mathbb{P} \left( \tau > t + \frac{1}{2}i \mid \mathcal{H}_t \right)$$

$$= \int_{\mathbb{R}^n} \left( \prod_{j=1}^{n} \left[ 1 - \pi(s - t, z_j - a) \right] \right) g_n \left( z \mid y^{(n)} \right) dz.$$  

That is, the expectation in the denominator is the probability of surviving up to time $t + \frac{1}{2}i$. Here, $\pi(t, x)$ denotes the probability of the first passage of a Brownian motion with drift $m$ and volatility parameter $\sigma$ from initial condition $x > 0$ to a level below 0 at time $t$. The expression for $\pi$ is known in closed form, which we provide in the appendix; for a derivation, see e.g. Harrison [1985] Chapter 1.

The expectation in the numerator, however, is the expected discounted loss due to default. We can rewrite this as:

$$\mathbb{E} \left[ e^{-r(\tau - t)} \mathbf{1}_{\tau \leq T} \mid \mathcal{H}_t \right] = \mathbb{E} \left[ e^{-r\tau} \mid \mathcal{H}_t \right] - \mathbb{E} \left[ e^{-r\tau} \mathbf{1}_{\tau \geq T} \mid \mathcal{H}_t \right]$$

$$= \mathbb{E} \left[ e^{-r\tau} \mid \mathcal{H}_t \right] - \mathbb{E} \left[ e^{-r\tau} \tau \geq T \mid \mathcal{H}_t \right]$$

$$= \left( 1 - e^{-r(T-t)} \right) \mathbb{E} \left[ e^{-r\tau} \mid \mathcal{H}_t \right]$$

$$= \left( 1 - e^{-r(T-t)} \right) \int_{\mathbb{R}^n} \left( \prod_{j=1}^{n} \exp \left[ -\alpha_s(r)(z_j - a) \right] \right) g_n \left( z \mid y^{(n)} \right) dz.$$  

(2.18)
where (see e.g. Harrison [1985])

\[ \alpha_*(r) = \frac{1}{\sigma^2} \left[ \sqrt{m^2 + 2\sigma^2 r + m} \right]. \]

### 3 Ambiguity and default-swap spreads

In this section, we consider the implications of investor ambiguity for default spreads and perceived default intensities. In particular, assume that in addition to the uncertainty about the true realization of the asset value, \( A_t \), the participants in the secondary debt market are uncertain about the overall information quality in the economy. In the following subsections, we focus on two particular sources of ambiguity: accounting signal quality and the dynamics of the true asset value. Here, we focus on the general case, without specifying the source of ambiguity. That is, the secondary debt market believes that the signal \( \hat{A}_t \) is related to the true realization \( A_t \) by a family of likelihoods \( \Xi \), parametrized by some parameter \( a \).

We assume that, unlike the equity owners, the secondary debt holders have the intertemporal version of the Gilboa and Schmeidler [1989] preferences axiomatized by Epstein and Schnei- der [2003]. In particular, keeping the assumption that the debt holders are risk-neutral (but ambiguity-averse), the representative agent’s utility from a consumption stream \( C_t \) is given by:

\[
U(C_t, t) = \min_{\Xi} \mathbb{E} \left[ \int_0^{+\infty} e^{-ru} C_{t+u} du | \mathcal{H}_t \right].
\]

Notice that, although in general we need to formulate the problem (3.1) recursively to maintain time consistency, Chen and Epstein [2002] show that, for the case of time-separable utility, the above representation holds. Since, in equilibrium, the representative participant in the secondary debt market will hold all the secondary market debt, it follows that the worst-case conditional probability minimizes the conditional expected payoff. Thus, in the presence of ambiguity, the time \( t \) credit default spread for a swap with maturity \( T \) is given by:

\[
c(t, T) = \max_{\Xi} \sum_{i=1}^{2k} e^{-\frac{\sigma^2}{2} T} \mathbb{E}^a \left[ 1_{T > t+rac{1}{2}i} | \mathcal{H}_t \right].
\]

To see this, recall that we can express the credit default spread as:

\[
c(t, T) = \frac{1 - Z(t, T)}{\sum_{i=1}^{k} Z(t, t + \frac{1}{2}i)},
\]
where $Z(t, T)$ is the time $t$ price of a bond maturing at date $T$. Since the worst-case conditional probability minimizes the expected payoffs, it follows that we must solve:

$$\min_{\Xi} Z(t, T).$$

This in turn implies (3.2). Intuitively, under the worst-case scenario implied by ambiguity preferences, investors must be compensated more for the perceived threat of default.

### 3.1 Ambiguity and Information Quality

Assume now that the ambiguity in the economy is due to the precision of the accounting signals. That is, the set of possible likelihoods, $\Xi$, is parametrized by the signal quality $\text{var}(U_t) = \sigma_u^2 \in [\sigma_u^2, \sigma_u^2]$. The informational quality of the accounting signal is captured by a range of precisions $[1/\sigma_u^2, 1/\sigma_u^2]$. Thus, in this case, the participants in the secondary debt market solve:

$$c(t, T) = \max_{\sigma_u^2 \in [\sigma_u^2, \sigma_u^2]} 2X\mathbb{E}_{\sigma_u^2} \left[ e^{-r(T-t)} \mathbf{1}_{\tau \leq T} | \mathcal{H}_t \right].$$

We have the following result:

**Proposition 3.1.** The worst-case likelihood solving the problem (3.3) is given by:

$$\tilde{\sigma}_u^2 = \begin{cases} \sigma_u^2, & y - \bar{u} > a \\ \sigma_u^2, & y - \bar{u} < a \end{cases}$$

*Proof.* See the Appendix.

Thus, participants in the secondary debt market treat positive signals (i.e. signals which are above the default threshold even after accounting for the mean signal error) as very imprecise and negative signals as very precise. Intuitively, since $y - \bar{u} - a$ measures the expected distance to default, a negative signal should be perceived as being very precise under the worst-case scenario interpretation implied by ambiguity preferences. On the other hand, a positive expected distance to default should be taken by the ambiguity-averse agents with a grain of salt: the worst possible scenario in this case is that the signal is very imprecise and the firm in fact is much closer to default than appears otherwise. This result is in line with the conclusion of Epstein and Schneider [2008], who find that, when evaluating signals about a company’s cash flows in
an uncertain informational environment, equity investors believe negative signals to contain the
highest possible informational content and positive signals to have the lowest. Notice that this
results differs fundamentally from the standard Bayesian learning. In the presence of multiple
priors, the agent who uses Bayesian updating would update the probability of each prior using
Bayes’ rule. Thus, no one prior would receive full probability weight with finite amount of data,
unless the initial probability distribution over possible priors had the same property. Further,
with Bayesian updating, negative and positive signals would not necessarily be perceived to have
radically different informational quality.

3.2 Ambiguity and Fundamentals

Unlike the discussion above, assume now that the signal quality is known (so that $\sigma_u^2$ is fixed)
but, instead, the participants in the secondary debt market fear misspecification of the firm
asset dynamics itself. That is, the representative agent fears misspecifications of the asset value
transition law that are sufficiently small so that they are difficult to detect in the data because
they are obscured by the random noise, $U_t$. In particular, we consider the specification in (2.1)
as the reference model for the asset value dynamics and assume that the equity owners make the
liquidation decisions based on the belief that this is the correct specification. The representative
participant in the secondary debt market, however, assumes that the true asset value dynamics
are of the form:

\begin{equation}
(3.5) \quad dZ_t = (m + \sigma h(m, t)) dt + \sigma dW_t,
\end{equation}

for all $t \geq 0$ and some $h(m) \in \Xi(m)$. Similarly to Leippold et al. [2008], we assume that the set
of admissible disturbances, $\Xi(m)$, is given by:

\begin{equation}
(3.6) \quad \Xi(m) \equiv \left\{ h(m) : \frac{1}{2} h^2(m, t) \leq \eta \quad \forall \quad t \geq 0 \right\},
\end{equation}

where $\eta \geq 0$ is the ambiguity tolerance of the debt holder. The above assumption implies that,
for finite values of the ambiguity tolerance, the discrepancy between the reference model-implied
distributions and those implied by the $h$-likelihood is constrained to be statistically small. Recall
that Anderson et al. [2003] define:

\begin{equation}
(3.7) \quad \epsilon(h)(m) = \frac{1}{2} h^2(m, t)
\end{equation}
to be the relative entropy between the two probability laws and show this to bound model
detection probabilities.

Since we assume that the drift rate of the log asset value, $m$, is known to the debt holders, some interpretation of the above setting is in order. In this section, we assume that $m$ is a parameter reported to the secondary market by the equity owners. Thus, although the participants in the secondary debt market directly observe $m$, they do not know for sure that the underlying asset value grows at the rate $\mu = m + \frac{1}{2}\sigma^2$, as reported by the equity owners. Instead, they assume that the asset value evolves according to one of the models parametrized by the set of admissible likelihoods $\Xi(m)$. The underlying asset value then grows at the rate $\mu^h = m + \sigma h(m) + \frac{1}{2}\sigma^2$ for a given likelihood $h(m) \in \Xi(m)$. Notice also that, in some situations, it might be plausible to allow the ambiguity tolerance of the debt holders to depend on the reported value $m$. For example, it is plausible to assume that $\eta$ is (weakly) increasing in $m$. That is, if the equity owners report that the asset value is growing at a low rate, the secondary debt market will consider this to be more likely to be the true rate of asset value growth than if the equity holders report a high rate of growth. In the following, however, we derive the worst-case likelihood without imposing any assumptions on the form of $\eta$.

Notice that, since the expected asset growth rate under the perturbed model is still independent of the asset value realization, the filtering solution from Section 2 carries through. That is, for a given likelihood $h(m) \in \Xi(m)$, the conditional distribution of $z$ at time $t$ is given by the expression in (2.9) but with $m$ replaced by $m + \sigma h(m)$. It is worth noting that, unlike the case of ambiguity about signal quality, ambiguity about the asset dynamics impacts the debt holders’ perceptions about the liquidation decision rule of the equity owners. In particular, in the presence of ambiguity about the underlying, the (log) liquidation boundary, $a$, depends on the likelihood chosen:

$$(3.8) a^h_B(C) = \frac{(1 - \theta)C\gamma^h(r - \mu^h)}{r(1 + \gamma^h)\delta},$$

where we now have:

$$\gamma^h = \frac{m + \sigma h(m) + \sqrt{(m + \sigma h(m))^2 + 2r\sigma^2}}{\sigma^2}$$

$$\mu^h = m + \sigma h(m) + \frac{1}{2}\sigma^2.$$
With the above assumptions in place, we have that, in the presence of ambiguity about the fundamental asset value dynamics, the participants in the secondary debt market solve:

\[
c(t, T) = \max_{h \in \Xi(m)} \frac{2 \sum_{i=1}^{k} e^{-r \tau} \mathbb{E}^{h} \left[ e^{-r(t-t)}1_{\tau \leq t} \mathcal{H}_{t} \right]}{\sum_{i=1}^{k} e^{-r \tau} \mathbb{E}^{h} \left[ 1_{\tau > t + \frac{1}{2}} \mathcal{H}_{t} \right]}.
\]

**Proposition 3.2.** The worst-case likelihood solving problem (3.9) is given by:

\[
h^{\ast}(m, t) = -\sqrt{2\eta}.
\]

*Proof.* See the Appendix.

Thus, in the presence of ambiguity about the asset value dynamics, the participants in the secondary debt markets choose the lowest possible drift rate for the underlying. Intuitively, because we consider the case \(m > 0\), a lower drift rate implies that the firm is less likely to recover from a negative shock to the asset value. Thus, in following the worst-case scenario, ambiguity-averse investors compute the filtered asset value assuming the lowest possible drift rate. Notice that this result is in line with the results of Leippold et al. [2008] who find that, in the presence of ambiguity about the dividend growth dynamics, ambiguity-averse investors always choose the lowest admissible rate.

### 4 Credit Crunch

In this section, we examine the behavior of credit and default spreads in response to a change in the information quality in a market. As Caballero and Krishnamurthy [2008b] argue, when France’s largest bank BNP Paribas revealed that two of its off-balance-sheet funds had loaded up on sub-prime mortgage securities, it not only sent a negative signal about its own performance, but also increased the ambiguity present in the market as a whole. Similar revelations by IKB, a German bank, and Northern Rock, a British bank, reinforced the shock to the information quality that the credit market received. Because these revelations dealt with off-balance-sheet debt, banks reacted by losing trust in each other’s balance sheets, thereby freezing the credit markets. The effect of the Paribas announcement is well illustrated in the behavior of both the credit and the default spreads. For example, in Fig. 1 we can see that both the Aaa and
The Baa spread above the 1 month T-bill rate increased dramatically immediately following the announcement. Likewise, Table 2 show dramatic increases in the default spreads for financial institutions. A similar effect is observed following the bail-out of Bear Stearns on March 16, 2008. Both of these occurrences suggest a strong response of the credit spreads to deterioration in the market information quality.

Notice that the revelation that banks were exposed to defaults in the sub-prime mortgage market through their off-balance-sheet debt increased ambiguity in the credit markets in two ways. First, these revelations made financial reports seem less plausible, thereby increasing ambiguity about the information quality. The complexity of the securities involved, however, also lead to an increase in the ambiguity about the true “asset” value growth rates for financial institutions. In the following subsections, we perform two calibration exercises. First, we use the parameters from the calibration of Duffie and Lando [2001] as the reference model parameters and look at how much ambiguity about either signal quality or fundamentals had to increase to match the observed changes in the Aaa and Baa spreads. The second exercise consists of using balance sheet data of several major financial institutions to calibrate the reference model parameters and then determining how much ambiguity about either signal quality or fundamentals had to increase to match the observed changes in the default spreads.

### 4.1 Ambiguity and Credit Spreads

As the base calibration, we use the parameters from Duffie and Lando [2001]. In particular, we assume that, before August 9, 2007, for Aaa-rated firms we have:

\[
\theta = 0.35; \quad \sigma = 0.05; \quad r = 0.06; \quad m = 0.01; \quad \delta = 0.05; \quad \alpha = 0.3.
\]

For the Aaa-rated firms, we have that \( C_{\text{Aaa}} = 8.00 \), the liquidation boundary is \( V_{\text{Aaa}} = 78.0 \), and the initial par debt level \( D_{\text{Aaa}} = d(V_0, C) = 129.4 \). For the Baa-rated firms these values are, respectively, \( C_{\text{Baa}} = 7.91 \), \( V_{\text{Baa}} = 63.37 \) and \( D_{\text{Baa}} = 121.75 \).

With these values, we have that the yield to debt for Aaa-rated firms is \( C/D = 6.18\% \) and
for Baa-rated firms 6.50%. Recovery of the debt at default, as a fraction of face value, is \( \delta(r - \mu)^{-1}(1 - \alpha)V_B/D = 43.3\% \) for Aaa and 40.5\% for Baa bonds. For comparison, the corresponding average recovery of all defaulted bonds monitored by the rating agency Moody’s, for 1920 to 1997, is 41\%. Finally, we assume that, initially, the participants in the secondary debt market assumed that the standard deviation of the error in the accounting signal was \( \sigma_u = 10\% \), for both the Aaa- and the Baa-rated firms and that, initially, the debt owners had no ambiguity about the asset dynamics (so that \( \eta = 0 \)). While there is no direct empirical evidence for these parameters, Duffie and Lando [2001] demonstrate that this parameter combination generates plausible credit spreads for various levels of the initial asset value and observed signals. The full list of calibrated parameters is presented in Table 1.

Before we proceed, consider the observed changes in the credit spreads, documented in Fig. 2. From the upper panel, we see that after the BNP Paribas announcement on August 9, 2007, the Aaa spread increased by 100\% in the first three days and the Baa spread increased by 50\%. The Baa-Aaa spread increased by 10\%. Following the Bear Stearns bailout, the spreads increased by a further 20\% percent for Aaa bonds and 15\% for Baa bonds. The Baa-Aaa spread increased by a further 3\%.

Consider first the case of ambiguity about the signal quality. In Fig. 3, we plot the model-implied changes in the credit spreads for different levels of \( \sigma \) as we vary \( \sigma_u^2 \). Recall that we identify Aaa-rated firms with \( \sigma = 5\% \) and Baa-rated firms with \( \sigma = 10\% \). From the figure, we can see that, to match the observed 100\% increase in the Aaa spread following August 9, 2007, the lower bound of the asset volatility, \( \sigma_u^2 \), would have to decrease to less than 1\% (from 10\%). Thus, following the BNP Paribas announcement, investors interpreted negative signals about companies as being extremely precise. Likewise, \( \sigma_u^2 \) would have to decrease to less than 1\% to match the observed increased in the Baa spread. Notice that, however, we need to decrease \( \sigma_u^2 \) less to match the observed increase in the Baa spreads. Thus, not only did the ambiguity increase after August 9, 2007, but the change in ambiguity was not the same for Aaa- and Baa-rated bonds. The greater increase in the Aaa spread corresponds to the Caballero and Krishnamurthy [2008b] observation that losses to the Aaa-rated tranches of subprime securities brought the stability of all Aaa-rated securities into question. Finally, notice from Fig. ?? that, to match the observed change in the Baa-Aaa spread, \( \sigma_u^2 \) only had to decrease to 7\%. Thus, in
terms of our model, the Baa-Aaa spread did not increase enough following the August 09, 2007 announcement.

Consider now the case of ambiguity about the fundamentals. We assume that the debt holders’ ambiguity tolerance is given by:

\[ \eta = \eta m^2. \]  

In particular, we have that, before the current crisis began, \( \eta = 0 \). Consider once again the changes in the credit spreads over the risk-free rate first. From Fig. 5, we can see that to match the 100% change in the Aaa spread after the Paribas announcement, \( \eta \) needs to increase to more than 10; to match the 50% increase in the Baa spread, \( \eta \) needs to increase to around 3. Thus, just as with ambiguity about signal quality, the increase in the ambiguity about asset value dynamics for Aaa-rated bonds is much higher than the increase in the ambiguity about Baa-rated bonds. Intuitively, because the current credit crisis has brought into question the Aaa rating of complex securities, debt holders’ would have higher ambiguity about the true growth rate of the asset value of such securities. Looking at the Baa-Aaa spread as a function of \( \eta \), from Fig. 6, we can see that the observed 10% increase can be generated by an increase in \( \eta \) to just 0.5. Thus, similarly to the case of ambiguity about the signal quality, the Baa-Aaa spread did not increase enough following the August 09, 2007 announcement.

### 4.2 Ambiguity and Default Spreads

As an alternative calibration exercise, consider the increases in the default spreads for financial institutions following the BNP Paribas announcement (Table 2) and following the bailout of Bear Stearns (Table 3). From Table 2, we can see that the default spreads increased by as much as 244% for 1 year swaps between July and August 2007 and as much as 165% for 10 year swaps. Similarly, the default spreads for 1 year swaps increased by as much as 170% following the bailout of Bear Stearns in March 2008 and by as much as 140% for 10 year swaps. Below, we calibrate the model to the balance sheet data of several financial institutions and consider how much ambiguity had to increase to match the observed changes in default spreads.

We begin the calibration by identifying the firm’s asset value, \( A_t \), with observations of book equity. Running a simple OLS regression on 1 quarter differences in the log book equity allows
us to identify $m$ and $\sigma$ and, hence, $\mu$. Following the model assumption that equity holders receive a constant fraction $\delta$ of assets as a cash-flow stream, we identify $\delta$ as the time series average of total earnings as a fraction of total assets. We use the last available (pre-August 2007) value of long-term debt as an estimate of $D$ since, in terms of our model, $D$ is the face value of infinitely-lived debt. Then, using the Duffie and Lando [2001] calibrated parameters for $\theta$, $r$ and $\alpha$, we can determine $C$ as the value that minimizes the distance between the model-implied and calibrated $D$. Finally, as before, we assume that the initial value of signal volatility is $\sigma_u^0 = 10\%$ and the initial value of $\eta$ is 0. The results of this calibration are presented in Table 4 and the corresponding quantities of interest in Table 5.

Notice that there are several major differences between the results of this calibration and the calibration of Duffie and Lando [2001]. First, the mean growth rate is much higher (around 3-5%) than their calibrated value of 1%. Further, the long-term bond yield is much smaller (ranging from 0.14% to 2% as opposed to 6%) despite the fact that the expected recovery as a fraction of face value in case of default is much lower as well: the Duffie and Lando [2001] parameters imply recovery rates of around 41-43% whereas our calibrated parameters yield recovery rates of at most 8%. The lower bond yield is due to the fact that the financial institutions are initially further away from the default boundary than the firm in the Duffie and Lando [2001] calibration; the lower expected recovery rate at default is due to the relatively high $D$ value for the financial institutions. In the interest of brevity, we will use the parameters calibrated for a particular financial institution (Goldman Sachs) for the remainder of the discussion.

Consider first the impact of ambiguity about signal quality on the default spreads. From Fig. 7 we see that, to match the observed increase of 126% in the 1 year default spread, the lower bound of the signal variance, $\sigma_u^2$, would need to decrease to less than 1%. Notice, however, that even for this (very low) level of signal variance, the model-implied change in the 10 year default spread is only a few percent, not the 92% observed in the data. Thus, although we can decrease the lower bound of the signal variance enough to generate the observed changes in the 1 year spreads, ambiguity about signal quality is not enough to generate the observed changes in the spreads for longer maturities. Intuitively, we can expect signal variance to have more of an impact in the short term than in the longer term, when investors can extract additional information about the firm’s asset from observing if the firm is liquidated.
Consider now the case of ambiguity about the fundamentals. As before, we parametrize the debt holders' ambiguity tolerance as:

\[ \eta(m) = \eta m^2 \]

and assume that, before the crisis began, we had \( \eta = 0 \). From Fig. 8, we see that, to match the observed increase of 126% in the 1 year default spreads, \( \eta \) needs to increase to more than 10. Notice that, although increases in ambiguity about the signal quality match the observed increases in 1 year default spreads better, increases in ambiguity about the fundamentals match better the overall profile of the changes in credit spreads. Intuitively, while the signal quality has a sizable impact in the short run default spreads, it is the underlying asset dynamics that dominate in determining the long-term default spreads. This also corresponds to the fact that the BNP Paribas announcement made investors reevaluate their beliefs about the expected distance to default of financial institutions.

5 Conclusion

In this paper, we have considered the implications of ambiguity for credit and default swap spreads. Using the Duffie and Lando [2001] model of credit spreads under incomplete information as a starting point, we introduced ambiguity in two different ways. First, we examined the behavior of default spreads in the presence of ambiguity about the information quality of the accounting signals that the participants in the secondary debt market receive. We find that, since the ambiguity-averse investors take the worst possible assessment of signal quality, they react more strongly to bad news than to good news. Thus, a deterioration of the information environment leads to a widening of credit spreads, even if the firm fundamentals do not change. Further, even if the informational environment remains constant, ambiguity about signal quality causes an asymmetry in the response of credit spreads to negative and positive signals, with the increase in credit spreads following negative news larger than the corresponding decrease following positive news of similar magnitude.

We continued by introducing ambiguity about the dynamics of the underlying asset value. In this setting, participants in the secondary market as if they believe the asset value growth
rate to be the lowest feasible one. Thus, in this setting, it is possible for the credit spreads on Aaa-rated bonds to behave similarly to the credit spreads on Baa-rated bonds, if there is relatively greater ambiguity about the Aaa-rated firm.

We then proceeded to examine the implications of these two sources of ambiguity in the data. We began with the calibrated parameters of Duffie and Lando [2001] and considered how much ambiguity about either the signal quality or the asset value dynamics had to increase to match the observed increase in credit spreads after August 9, 2007. We find that, for both models, ambiguity about Aaa-rated firms had to increase more than the ambiguity about Baa-rated firms. Further, both models are able to match the increases in credit spreads for reasonable values of model parameters. Next, we calibrated the model parameters to the balance sheet data of several prominent financial institutions and considered once again the increases in ambiguity necessary to generate the observed increases in the term structure of default swap spreads for these institutions. We find that, although ambiguity about signal quality is better able to match the changes in the 1 year default spread, changes in the ambiguity about the asset dynamics are better able to match the changes in the overall term structure of default spreads. This leads us to conclude that ambiguity about the underlying asset value dynamics is a better model of ambiguity in the context of credit spreads.
A Technical Appendix

A.1 One-sided first passage time distribution

Recall from Harrison [1985], that if we define $T(x)$ the first time at which a random variable $Y_t$ reaches $x > 0$ from below, with $Y_t = 0$, then we have:

$$
\mathbb{P}\{T(x) > t\} = \Phi\left( \frac{x - \mu t}{\sigma \sqrt{t}} \right) - e^{2 \mu x / \sigma^2} \Phi\left( \frac{-x - \mu t}{\sigma \sqrt{t}} \right),
$$

where $\mu$ and $\sigma^2$ are the drift and variance of the Brownian motion, $Y_t$, respectively. In our case, we are interested in the mirror-opposite of the above problem: we would like to know the distribution of the first passage hitting time of when a process $Y_t$, with initial condition $x > 0$, reaches 0 from above. Notice, however, that the one-dimensional Brownian motion is time-reversible. Hence, in the notation of the paper, we have:

$$
(A.1) \quad \pi(t, x) = \Phi\left( \frac{x - mt}{\sigma \sqrt{t}} \right) - e^{2 \mu x / \sigma^2} \Phi\left( \frac{-x - mt}{\sigma \sqrt{t}} \right).
$$

A.2 Proof of Proposition 3.1

Recall that, in the presence of ambiguity about signal quality, the default spread on a bond with maturity $T$ is given by:

$$
c(t, T) = \max_{\sigma^2 \in [\sigma_u^2, \sigma_u^2]} \frac{2X\mathbb{E}\sigma_u^2 \left[ e^{-r(\tau - t)} \mathbf{1}_{\tau \leq T} \right] }{\mathcal{H}_t}. \sum_{i=1}^{k} \frac{\partial g}{\partial \sigma_u^2} dz
$$

Taking the derivative with respect to $\sigma_u^2$, we have:

$$
(A.2) \quad \frac{\partial c}{\partial \sigma_u^2} = \int_{\frac{x}{\sigma}}^{+\infty} \left\{ 2X \left[ 1 - e^{-r(T - t)} \right] \exp \left[ -\alpha^*(r)(z - \rho) \right] - c(t, T) \sum_{i=1}^{k} e^{-\frac{\mu_i}{x^2}} \left[ 1 - \pi \left( \frac{1}{2}, z - \rho \right) \right] \right\} \frac{\partial g}{\partial \sigma_u^2} dz.
$$

Notice that:

$$
2X \left[ 1 - e^{-r(T - t)} \right] \exp \left[ -\alpha^*(r)(z - \rho) \right] c(t, T) \sum_{i=1}^{k} e^{-\frac{\mu_i}{x^2}} \left[ 1 - \pi \left( \frac{1}{2}, z - \rho \right) \right] = 2X \left[ 1 - e^{-r(T - t)} \right] \sum_{i=1}^{k} \frac{\exp \left[ -\alpha^*(r)(z - \rho) \right]}{\mathcal{H}_t} \left[ 1 - \pi \left( \frac{1}{2}, z - \rho \right) \right] \times
$$

$$
\left\{ \sum_{i=1}^{k} e^{-\frac{\mu_i}{x^2}} \left[ 1 - \pi \left( \frac{1}{2}, z - \rho \right) \right] \right\}
$$

Since $c(t, T) < 1$, this implies that

$$
2X \left[ 1 - e^{-r(T - t)} \right] \exp \left[ -\alpha^*(r)(z - \rho) \right] c(t, T) \sum_{i=1}^{k} e^{-\frac{\mu_i}{x^2}} \left[ 1 - \pi \left( \frac{1}{2}, z - \rho \right) \right] < 0.
$$

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Thus, to determine the sign of the derivative, we need to consider the sign of $\partial g / \partial \sigma_u^2$. Recall that:

$$g(z|y, z_0, t) = \frac{b(z|y, z_0, t)}{\int_{-\infty}^{+\infty} b(z|y, z_0, t) dz},$$

where:

$$b(z|y, z_0, t) = \exp \left[ -\frac{(y - \bar{\pi} - z)^2}{2\sigma_u^2} - \frac{(z_0 + mt - z)^2}{2\sigma_t^2} \right] \left[ 1 - \exp \left( -\frac{2(z_0 - a)(z - a)}{\sigma^2 t} \right) \right].$$

Hence:

$$\frac{\partial g}{\partial \sigma_u^2} = \frac{\partial b / \partial \sigma_u^2}{\int_{-\infty}^{+\infty} b dz} - g(z|y, z_0, t) \frac{\int_{-\infty}^{+\infty} \frac{\partial b}{\partial \sigma_u^2} dz}{\int_{-\infty}^{+\infty} b dz}$$

$$= \frac{1}{2(\sigma_u^2)^2} g(z|y, z_0, t) \left[ (y - \bar{\pi} - z)^2 - \int_{0}^{+\infty} (y - \bar{\pi} - z)^2 g(x|y, z_0, t) dx \right]$$

$$= \frac{1}{2(\sigma_u^2)^2} g(z|y, z_0, t) \left[ (y - \bar{\pi} - z)^2 - \mathbb{E}_t[(y - \bar{\pi} - z)^2] \right].$$

Notice that:

$$(y - \bar{\pi} - z)^2 = (y - \bar{\pi} - a)^2 - 2(y - \bar{\pi} - a)(z - a) + (z - a)^2,$$

so, substituting into (A.2), we obtain:

$$\frac{\partial c}{\partial \sigma_u^2} = -2^2 \sigma_u^2 \left[ \left\{ 2X \left[ 1 - e^{-r(T-t)} \right] \exp \left[ -\sigma_u^2 \left( z - a \right)^2 \right] - c(t, T) \sum_{k=1}^{k} e^{-r(T-t)} \mathbb{E}_t \left[ \left( z - a \right)^2 \right] \right\} \right]$$

$$+ \left\{ 2X \left[ 1 - e^{-r(T-t)} \right] \exp \left[ -\sigma_u^2 \left( z - a \right)^2 \right] - c(t, T) \sum_{k=1}^{k} e^{-r(T-t)} \mathbb{E}_t \left[ \left( z - a \right)^2 \right] \right\} \mathbb{E}_t \left[ \left( z - a \right)^2 \right].$$

From the above, we can see that, if $y - a - \bar{\pi} > 0$, the derivative is positive for all values of $\sigma_u^2$ and the worst-case likelihood is given by $\sigma_u^2 = \sigma_u^2$. Similarly, if $y - a - \bar{\pi} < 0$, the derivative is negative for all values of $\sigma_u^2$ and the worst-case likelihood is given by $\sigma_u^2 = \sigma_u^2$.

### A.3 Proof of Proposition 3.2

Recall that, in the presence of ambiguity about the firm’s asset dynamics, the default spread on a bond with maturity $T$ is given by:

$$c(t, T) = \max_{h \in \Xi(m)} \frac{2X \mathbb{E}_h \left[ e^{-r(T-t)} \mathbb{1}_{\tau \leq T} \right] \mathcal{H}_t}{\sum_{i=1}^{k} e^{-\frac{\tau_i}{2}} \mathbb{E}_h \left[ \mathbb{1}_{\tau > t + \frac{1}{2}} \right] \mathcal{H}_t}.$$
Notice that the choice of $h$ is equivalent to the choice of $m$, with the feasible set appropriately defined. Thus, we can solve the above problem by maximizing over the set of feasible $m$’s.

Taking the derivative with respect to $m$, we obtain:

$$
\frac{\partial c}{\partial m} = -\frac{\int_{a}^{+\infty} \left\{ 2X \left[ 1 - e^{-r(T-t)} \right] \left( z - a \right) \frac{\partial \alpha^*}{\partial m} \right\} \exp \left[ -\alpha^*(r)(z - a) \right] - c(t, T) \sum_{i=1}^{k} e^{-\frac{a}{\sigma}} \frac{\partial \pi}{\partial m} \right\} g(z|y, z_0, t) dz}{\sum_{i=1}^{k} e^{-\frac{a}{\sigma}} \mathbb{P}_h \left[ 1_{r > t + \frac{1}{2} i} \right] \mathbb{H}_t} + \frac{\int_{a}^{+\infty} \left\{ 2X \left[ 1 - e^{-r(T-t)} \right] \exp \left[ -\alpha^*(r)(z - a) \right] - c(t, T) \sum_{i=1}^{k} e^{-\frac{a}{\sigma}} \left[ 1 - \pi \left( \frac{1}{2} i, z - a \right) \right] \right\} \frac{\partial \pi}{\partial m} dz}{\sum_{i=1}^{k} e^{-\frac{a}{\sigma}} \mathbb{P}_h \left[ 1_{r > t + \frac{1}{2} i} \right] \mathbb{H}_t} = \frac{-s_1(m) + s_2(m)}{\sum_{i=1}^{k} e^{-\frac{a}{\sigma}} \mathbb{P}_h \left[ 1_{r > t + \frac{1}{2} i} \right] \mathbb{H}_t}.
$$

Consider the first summand in the derivative, $s_1(m)$. We have:

$$
\frac{\partial \alpha^*}{\partial m} = \frac{1}{\sigma^2} \left( 1 + \frac{m}{\sqrt{m^2 + 2\sigma^2 r}} \right) = \frac{\alpha^*}{\sqrt{m^2 + 2\sigma^2 r}},
$$

$$
\frac{\partial \pi}{\partial m} = -\sqrt{t} \left[ \phi \left( \frac{x - mt}{\sigma \sqrt{t}} \right) - e^{2mx/\sigma^2} \phi \left( \frac{-x - mt}{\sigma \sqrt{t}} \right) \right] - 2 \frac{x}{\sigma} e^{2mx/\sigma^2} \Phi \left( \frac{-x - mt}{\sigma \sqrt{t}} \right).
$$

Thus, the first summand can be expressed as:

$$
\begin{align*}
\frac{\partial c}{\partial m} & = 2X \left[ 1 - e^{-r(T-t)} \right] \mathbb{E}_m \left[ \left( z - a \right) \frac{\alpha^*}{\sqrt{m^2 + 2\sigma^2 r}} \exp \left[ -\alpha^*(r)(z - a) \right] \right] \mathbb{H}_t \\
& \quad + 2c(t, T) \sum_{i=1}^{k} e^{-\frac{a}{\sigma}} \mathbb{E}_m \left[ \frac{z - a}{\sigma} e^{2m(z - a)/\sigma^2} \Phi \left( \frac{-z - a - \frac{mt}{2}}{\sigma \sqrt{t}/2} \right) \right] \mathbb{H}_t \\
& \geq 0.
\end{align*}
$$

Consider now the second summand, $s_2(m)$. We have:

$$
\frac{\partial g}{\partial m} = -\frac{1}{\sigma^2} g(z|y, z_0, t) \left[ \left( z_0 + mt - z \right) - \mathbb{E}_t \left[ z_0 + mt - z \right] \right]
$$

$$
= \frac{1}{\sigma^2} g(z|y, z_0, t) \left[ z - a - \mathbb{E}_t \left[ z - a \right] \right].
$$

Thus, we can express the second summand as:

$$
\begin{align*}
\frac{\partial c}{\partial m} & = \mathbb{E}_m \left\{ 2X \left[ 1 - e^{-r(T-t)} \right] \exp \left[ -\alpha^*(r)(z - a) \right] - c(t, T) \sum_{i=1}^{k} e^{-\frac{a}{\sigma}} \left[ 1 - \pi \left( \frac{1}{2} i, z - a \right) \right] \right\} \left[ z - a - \mathbb{E}_t \left[ z - a \right] \right] \mathbb{H}_t \\
& \leq 0.
\end{align*}
$$
Thus, \( \partial c / \partial m \) is negative for all values of \( m \) and the maximum is obtained by taking the lowest admissible value of \( m \). Hence, the worst-case likelihood is

\[
h^* = -\sqrt{2\eta}.
\]
References


M. Leippold, F. Trojani, and P. Vanini. Learning and asset prices under ambiguous information. 


### Common Parameters

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$r$</th>
<th>$m$</th>
<th>$\delta$</th>
<th>$\alpha$</th>
<th>$\sigma_u^0$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>35%</td>
<td>7%</td>
<td>1%</td>
<td>5%</td>
<td>30%</td>
<td>10%</td>
<td>0</td>
</tr>
</tbody>
</table>

### Aaa parameters

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$C$</th>
<th>$V_B$</th>
<th>$D$</th>
<th>$X$</th>
<th>$C/D$</th>
<th>Recovery</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>8</td>
<td>78</td>
<td>129.4</td>
<td>97.38%</td>
<td>6.18%</td>
<td>43.3%</td>
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</tbody>
</table>

### Baa parameters

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$C$</th>
<th>$V_B$</th>
<th>$D$</th>
<th>$X$</th>
<th>$C/D$</th>
<th>Recovery</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>7.91</td>
<td>63.37</td>
<td>121.75</td>
<td>97.35%</td>
<td>6.50%</td>
<td>40.5%</td>
</tr>
</tbody>
</table>

Table 1: Reference parameters for the credit spread calibration and associated base quantities

<table>
<thead>
<tr>
<th></th>
<th>1 year</th>
<th>2 year</th>
<th>3 year</th>
<th>4 year</th>
<th>5 year</th>
<th>7 year</th>
<th>10 year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank of America</td>
<td>156.36</td>
<td>145.88</td>
<td>140.87</td>
<td>150.00</td>
<td>154.55</td>
<td>68.67</td>
<td>82.35</td>
</tr>
<tr>
<td>Citi Corp</td>
<td>184.48</td>
<td>183.70</td>
<td>185.60</td>
<td>184.47</td>
<td>184.26</td>
<td>140.50</td>
<td>116.19</td>
</tr>
<tr>
<td>Goldman Sachs</td>
<td>126.52</td>
<td>122.22</td>
<td>117.72</td>
<td>122.37</td>
<td>125.95</td>
<td>103.55</td>
<td>92.53</td>
</tr>
<tr>
<td>JP Morgan</td>
<td>174.24</td>
<td>189.36</td>
<td>201.67</td>
<td>182.35</td>
<td>173.85</td>
<td>75.85</td>
<td>80.25</td>
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<tr>
<td>Lehman Bros.</td>
<td>206.76</td>
<td>203.92</td>
<td>200.00</td>
<td>203.00</td>
<td>206.31</td>
<td>142.58</td>
<td>164.34</td>
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<tr>
<td>Merrill Lynch</td>
<td>118.31</td>
<td>118.04</td>
<td>112.79</td>
<td>119.20</td>
<td>123.71</td>
<td>90.00</td>
<td>95.03</td>
</tr>
<tr>
<td>Wells Fargo</td>
<td>243.75</td>
<td>263.89</td>
<td>274.07</td>
<td>256.19</td>
<td>244.96</td>
<td>94.77</td>
<td>109.60</td>
</tr>
</tbody>
</table>

Table 2: Change in default spreads after August 9, 2007, as compared to one month previous. Changes reported in percentage terms. Source: Bloomberg
<table>
<thead>
<tr>
<th>Financial Institution</th>
<th>1 year</th>
<th>2 year</th>
<th>3 year</th>
<th>4 year</th>
<th>5 year</th>
<th>7 year</th>
<th>10 year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank of America</td>
<td>37.08</td>
<td>59.67</td>
<td>73.39</td>
<td>69.24</td>
<td>59.47</td>
<td>68.81</td>
<td>76.80</td>
</tr>
<tr>
<td>Citi Corp</td>
<td>104.82</td>
<td>113.21</td>
<td>101.38</td>
<td>89.21</td>
<td>85.19</td>
<td>81.49</td>
<td>77.57</td>
</tr>
<tr>
<td>Goldman Sachs</td>
<td>124.19</td>
<td>137.29</td>
<td>133.16</td>
<td>128.87</td>
<td>96.81</td>
<td>135.75</td>
<td>141.42</td>
</tr>
<tr>
<td>JP Morgan</td>
<td>170.88</td>
<td>107.47</td>
<td>104.11</td>
<td>96.06</td>
<td>88.25</td>
<td>84.78</td>
<td>93.08</td>
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<tr>
<td>Lehman Bros</td>
<td>148.02</td>
<td>140.93</td>
<td>133.53</td>
<td>129.71</td>
<td>127.10</td>
<td>132.89</td>
<td>130.23</td>
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<tr>
<td>Merrill Lynch</td>
<td>-83.14</td>
<td>81.24</td>
<td>80.68</td>
<td>76.37</td>
<td>70.48</td>
<td>80.81</td>
<td>121.83</td>
</tr>
<tr>
<td>Wells Fargo</td>
<td>71.91</td>
<td>72.88</td>
<td>69.95</td>
<td>68.68</td>
<td>87.56</td>
<td>69.02</td>
<td>74.68</td>
</tr>
</tbody>
</table>

Table 3: Change in default spreads after Bear Stearns bailout, as compared to one month previous. Changes reported in percentage terms. Source: Bloomberg

<table>
<thead>
<tr>
<th>Financial Institution</th>
<th>m</th>
<th>μ</th>
<th>σ</th>
<th>δ</th>
<th>D</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank of America</td>
<td>4.85</td>
<td>5.58</td>
<td>12.09</td>
<td>3.81</td>
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<td>2063.91</td>
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<tr>
<td>Merrill Lynch</td>
<td>3.56</td>
<td>3.63</td>
<td>3.54</td>
<td>3.76</td>
<td>185292.00</td>
<td>240.97</td>
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<tr>
<td>JP Morgan</td>
<td>3.42</td>
<td>4.21</td>
<td>12.57</td>
<td>2.22</td>
<td>172163.00</td>
<td>2420.22</td>
</tr>
<tr>
<td>Citi Corp</td>
<td>3.99</td>
<td>5.16</td>
<td>15.33</td>
<td>3.88</td>
<td>507216.00</td>
<td>10268.65</td>
</tr>
<tr>
<td>Wells Fargo</td>
<td>3.26</td>
<td>3.80</td>
<td>10.34</td>
<td>3.51</td>
<td>93830.00</td>
<td>939.54</td>
</tr>
<tr>
<td>Goldman Sachs</td>
<td>5.93</td>
<td>6.08</td>
<td>5.57</td>
<td>5.03</td>
<td>167253.00</td>
<td>428.35</td>
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<tr>
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<td>3.54</td>
<td>3.61</td>
<td>3.73</td>
<td>3.61</td>
<td>100819.00</td>
<td>145.40</td>
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</table>

Table 4: Calibrated parameters for financial institutions. Calibration done using balance sheet data from COMPUSTAT. m, μ, σ, and δ reported in percentage terms.
Table 5: Quantities of interest for financial institutions. Calibration (results in Table 4) done using balance sheet data from COMPUSTAT. $X$, $C/D$ and Recovery reported in percentage terms.
Figure 1: The daily credit spread for Aaa (dash-dot) and Baa (dash) bonds over the one month T-bill rate for the time period January 2007 - August 2008. Source: Moody's.
Figure 2: The percent change in the credit spreads after August 9, 2007 (upper panel) and after March 16, 2008 (lower panel). In each case, the change is calculated relative to the level on the previous available day. Source: Moody’s.
Figure 3: Percent change in the credit spreads relative to the level corresponding to $\sigma_u = 10\%$ for different levels of the asset value volatility, $\sigma$. Following Duffie and Lando [2001], the time since the debt valuation is known for sure, $t$, is set to 1 and the signal value to $A_0$. The maturity of the bond is set $T = 5$ to match the duration of the Moody reported bonds. The other values are as reported in Table 1.
Figure 4: Percent change in the spread between $\sigma = 5\%$ and $\sigma = 10\%$ relative to the level corresponding to $\sigma_u = 10\%$. Following Duffie and Lando [2001], the time since the debt valuation is known for sure, $t$, is set to 1 and the signal value to $A_0$. The maturity of the bond is set $T = 5$ to match the duration of the Moody reported bonds. The other values are as reported in Table 1.
Figure 5: Percent change in the credit spreads relative to the level corresponding to $\eta = 0$ for different levels of the asset value volatility, $\sigma$. The standard deviation of the observation error is assumed to be $\sigma_u = 10\%$. Following Duffie and Lando [2001], the time since the debt valuation is known for sure, $t$, is set to 1 and the signal value to $A_0$. The maturity of the bond is set $T = 5$ to match the duration of the Moody reported bonds. The other values are as reported in Table 1.
Figure 6: Percent change in the spread between $\sigma = 5\%$ and $\sigma = 10\%$ relative to the level corresponding to $\eta = 0$. The standard deviation of the observation error is assumed to be $\sigma_u = 10\%$. Following Duffie and Lando [2001], the time since the debt valuation is known for sure, $t$, is set to 1 and the signal value to $A_0$. The maturity of the bond is set $T = 5$ to match the duration of the Moody reported bonds. The other values are as reported in Table 1.
Figure 7: Percent change in the default spread relative to the level corresponding to $\sigma_u = 10\%$ for different maturities. The time since the debt valuation is known for sure, $t$, is set to 1 month and the signal value to $A_0$. 
Figure 8: Percent change in the default spread relative to the level corresponding to $\eta = 0$ for different maturities. The time since the debt valuation is known for sure, $t$, is set to 1 month and the signal value to $A_0$. 