Matching with phantoms*

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Preliminary draft

Abstract: Searching partners involves informational persistence that lowers future traders’ matching probability. In this paper, traders that are no longer available but left tracks on the labor market are called phantoms. I examine a discrete-time matching market in which phantom traders are a by-product of search activity, no coordination frictions are assumed, and non-phantom traders may lose time trying to match with phantom traders. The resulting aggregate matching technology features increasing returns to scale in the short run, but has constant returns to scale in the long run. I discuss the labor market evidence and argue that there is observational equivalence between phantom unemployed and on-the-job seekers.

Keywords: Endogenous matching technology; Intertemporal and intratemporal congestion externalities; Information persistence

JEL classification: J60

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*This paper has benefited from the comments of seminar participants in GREQAM and in the University of New South Wales. I wish to thank Mohamed Belhaj, Gautam Bose, Frédéric Deroian, Cecilia García-Penalosa, Alain Trannoy, and Alain Venditti.

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1 Introduction

Searching partners involves informational externalities. In this paper, I build on the idea that matching frictions can result from information persistence on the market about traders that have already found a match. I refer to those traders as phantom traders, or phantoms for short. Phantoms are a by-product of search activity: while exiting the market, each trader may leave a track that tends to disappear with time. Phantoms result in a loss of time and resources for the future traders to find an adequate partner. The research question I address in this paper is: Can this only source of information imperfection result in a matching technology that is well-behaved?

There are various reasons why there may be phantom traders on the market. First, search strategies have some involuntary persistence. To recruit workers, firms post ads that convey some information on job offers. This information is useful to attract potential employees, who can direct their search towards such jobs as a result. What happens to this information once a worker get recruited? The ad is likely to last for some additional time. This may be misleading for the workers who lose time and effort to prospect a job that does no longer exist. Similarly, workers send applications and register on websites. Firms may deal with such applications or consult websites after workers’ recruitment. Second, match makers may voluntarily delay the moment they delete the information about traders that have left the market. Dating websites may keep online profiles for months or even years since the person has logged in for the last time. Real estates may showcase sold houses or rented flats. In both cases, the goal is to attract customers by making the number of potential traders bigger than it really is. Third, matched traders may be incited to go on searching even though they do not want to find another trading partner. Firms that have filled in their jobs may post ads to accumulate a stock of potential applicants in case they have new vacancies. Married persons may enter a romantic online relationship without willing to go further. Finally, on-the-match seeking is a particular way of haunting the matching market.

I consider a generic situation. Time is discrete and buyers and sellers try to contact each other on a unique search place. To disentangle the impacts of phantoms on the search market from more standard congestion externalities, I assume that each buyer meets one seller at most, and every trader on the short side of the market is ensured to meet someone. Unfortunately, that someone may be a phantom buyer, or a phantom seller. No trade takes place in such a case. I suppose that the populations of phantoms obey simple flow-stock equations, the inflow of new phantoms being proportional to the past outflow of successful traders. I examine the resulting matching pattern between the two populations of traders.

I refer to the aggregate matching technology as the phantom matching technology, or PMT for short. The PMT features intratemporal and intertemporal externalities. Intratemporal externalities result from the fact that an increase in the number of agents on the long side of the market reduces the proportion of phantom traders. A larger proportion of contacts gives birth to matches as a result. This implies that the PMT displays increasing returns to scale in the short run. Intertemporal externalities result from the fact that current matches fuel future phantom traders. Although period-$t$ number of traders may have an ambiguous impact on period-$t + k$ number of matches, intertemporal externalities combine so as to negatively affect the current number of matches. Intratemporal and intertemporal externalities balance each other, so that the PMT features constant returns to scale vis-à-vis the whole set of current and past traders.

The interplay between intratemporal and intertemporal matching externalities has two implications.
First, I discuss the stationary phantom matching technology (SMPT) that emerges as the steady-state PMT of an environment where the populations of traders are themselves stationary. The SMPT obeys a simple parametric form that depends on the entry rate of new phantoms and phantom death probability. This exhibits constant returns to scale. The elasticity of the matching technology vis-à-vis the number of traders on the short side of the market depends on the ratio of sellers to buyers (negatively if sellers are on the short side, and positively otherwise). This elasticity belongs to the intervalle \((1/2, 1)\). Second, I examine the effects of a temporary increase in the number of traders on the short side of the market. Owing to short-run increasing returns to scale, this generates a matching boom in the short run. Then, the matching boom originates phantom traders that alter the matching pattern. As the boom stops, the market is left with many more phantoms and fewer matches take place than prior to the shock. Matching probabilities gradually converge towards their steady-state values.

I further discuss the PMT through three extensions to the basic model. The first extension is devoted to the honeymoon effect that benefits new markets. New markets have no history, and feature no phantoms. Matching probabilities start very high as a result. Then, phantoms accumulate and matching probabilities deteriorate. The second extension considers another popular source of market frictions, namely coordination frictions. This allows me to distinguish the respective contributions of phantom traders and coordination frictions to overall matching frictions. The final extension jointly discusses the labor market evidence and on-the-job search. I argue that on-the-job seekers can be considered as phantoms in the PMT framework. This implies that phantoms and on-the-job seekers are observationally equivalent. Consequently, papers that show the extent of on-the-job search provide indirect evidence about phantom traders.

By design, this paper completes the literature on endogenous matching frictions. I now compare my framework to the other papers.

Mismatch explanations rely on the idea that the matching market is composed of many micro markets with imperfect mobility of traders between the micro markets. The distribution of traders across micro markets governs the shape of the aggregate matching technology (see Drèze and Bean, 1990, Lagos, 2000, 2003, and Shimer, 2007). The PMT framework is orthogonal and therefore complements this line of research. Note, however, that geographic mismatch and phantom matching involve imperfect mobility between micro markets or alternative partners. Trying to match requires time, time that is not available to search in another marketplace.

Stock-flow matching models postulate that existing stocks of traders can only match with newcomers (see e.g. Taylor, 1995, Coles and Muthoo, 1998, Coles and Smith, 1998, Coles and Petrongolo, 2008, and Ebrahimy and Shimer, 2008). There are no restrictions on trader mobility. Buyers can visit as many sellers as they want. However, there is only a probability that trade can take place with each seller. Agents who do not match are agents that cannot match with each other. This is why they have to wait new agents. Although the two matching technologies are based on very different ideas, they both feature increasing returns vis-à-vis current stock of traders.

The idea that some applications get lost as they reach non-available traders can also be found in the telephone-line matching technology of Stevens (2007). Buyers try to contact sellers by phone calls, but sellers may either be processing or waiting. Only waiting sellers can answer the phone call. Processing sellers can be viewed as temporary phantom traders. Similarities stop there as Stevens focuses on a steady-state environment.
The fact that agents lose time as they try to get matched with traders that are already matched is very related to urn-ball matching models (see e.g. Butters, 1977, Hall, 1977, and Albrecht et al, 2006, 2007, with multiple applications). Each buyer observes all possible sellers and sends a buy order to one of them. As buyers do not coordinate, some sellers receive several buy orders, while others do not receive any order. This coordination friction implies that some buyers end up unmatched because the targeted seller get matched to another buyer. To some extent, intra-period matches (of the other agents) lower one’s own matching probability through phantom creation. From that perspective, the main difference relies on the timing of matching externalities. Those externalities are static in the urn-ball matching model, while they are dynamic in the phantom matching model. There is no memory in the former model, while market history affects current and future matches in the latter.

The rest of the paper is organized as follows. Section 2 introduces the model and computes the resulting matching technology. Section 3 analyzes the interplay between intratemporal and intertemporal externalities. Section 4 discusses the honeymoon effect, studies the interplay between phantom traders and coordination frictions, and analyzes on-the-job search.

2 The model

Time is discrete and denoted by $t$. A population of buyers and sellers want to trade with each other. But they have to meet prior trade takes place. Matching takes place every period. Each time a buyer and a seller meet and agree on match formation, they exit the market.

Let $B$ denote the (mass) number of buyers, $S$ the number of sellers, $P_B$ the number of phantom buyers, and $P_S$ the number of phantom sellers.

The matching mechanism takes place in two steps. In a first step, each trader on the short side of the market is assigned to a trader on the long side. This results in the following number of contacts:

$$
\min \{ B_t + P^B_t, S_t + P^S_t \}
$$

In a second step, matches are derived from contacts. The rule is that only contacts between non-phantom traders lead to effective trade. The number of matches is

$$
M_t = \frac{B_t}{B_t + P^B_t} \frac{S_t}{S_t + P^S_t} \min \{ B_t + P^B_t, S_t + P^S_t \}
$$

The number of contacts is multiplied by the product of the two proportions of non-phantom traders. This assumes that phantoms cannot be distinguished from non-phantoms by the matching mechanism.

Matching probabilities are

$$
\mu_t = \frac{M_t}{B_t} = \frac{S_t}{S_t + P^S_t} \min \left\{ 1, \frac{S_t + P^S_t}{B_t + P^B_t} \right\}
$$

$$
\eta_t = \frac{M_t}{S_t} = \frac{B_t}{B_t + P^B_t} \min \left\{ B_t + P^B_t, S_t + P^S_t, 1 \right\}
$$

The numbers of phantoms obey the following laws of motion:

$$
P^B_t = \beta^B M_{t-1} + \left( 1 - \delta^B \right) P^B_{t-1}
$$

$$
P^S_t = \beta^S M_{t-1} + \left( 1 - \delta^S \right) P^S_{t-1}
$$
with $\beta^j > 0$, and $0 < \delta^j \leq 1$, $j = B, S$. The inflow of new phantoms is proportional to former matches. The parameter $\beta^j$ can be interpreted as the probability that a match gives birth to a phantom trader, or as the relative search efficiency of phantoms vis-à-vis non-phantoms. In the former case, $\beta^j \leq 1$. In the latter case, there is no additional restriction on $\beta^j$. The outflow results from a constant depreciation rate $\delta^j$. Phantoms face a constant probability of dying $\delta^j$ each period. Life expectancy follows a Poisson law.

**Proposition 1** In each period $t$, the number of matches is given by

$$\ln M_t = \ln S_t + \ln B_t - \ln \left[ X_t + \beta_t \sum_{k=0}^{\infty} (1 - \delta_t)^k M_{t-k-1} \right]$$

(PMT)

with

1. $\beta_t = \beta^B$, $\delta_t = \delta^B$ and $X_t = B_t$ if $\min \{ B_t + P_t^B, S_t + P_t^S \} = S_t + P_t^S$
2. $\beta_t = \beta^S$, $\delta_t = \delta^S$ and $X_t = S_t$ if $\min \{ B_t + P_t^B, S_t + P_t^S \} = B_t + P_t^B$.

Proof. Suppose that $\min \{ B_t + P_t^B, S_t + P_t^S \} = S_t + P_t^S$. This yields the following matching technology

$$M_t = \frac{S_t B_t}{B_t + P_t^B}$$

(7)

We have

$$P_t^B = \beta^B \sum_{k=0}^{\infty} (1 - \delta^B)^k M_{t-k-1}$$

(8)

This gives

$$\ln M_t = \ln S_t + \ln B_t - \ln \left[ B_t + \beta^B \sum_{k=0}^{\infty} (1 - \delta^B)^k M_{t-k-1} \right]$$

(9)

Now suppose that $\min \{ B_t + P_t^B, S_t + P_t^S \} = B_t + P_t^B$. This yields the following matching technology

$$M_t = \frac{B_t S_t}{S_t + P_t^S}$$

(10)

We have

$$P_t^S = \beta^S \sum_{k=0}^{\infty} (1 - \delta^S)^k M_{t-k-1}$$

(11)

This gives

$$\ln M_t = \ln S_t + \ln B_t - \ln \left[ S_t + \beta^S \sum_{k=0}^{\infty} (1 - \delta^S)^k M_{t-k-1} \right]$$

(12)

This closes the proof.

The phantom matching technology (PMT) collapses to the usual non-frictional technology whenever $\beta_t = 0$. The novelty comes from the inclusion of the weighted sum of former matches in the last term. The weights depend on survival probabilities $(1 - \delta_t)^k$ and entry rate of new phantom $\beta_t$.

Market history may start at a finite date, say $t = 0$ without loss of generality. Equation (13) must be modified accordingly:

$$\ln M_t = \ln S_t + \ln B_t - \ln \left[ X_t + \beta_t \sum_{k=0}^{t-2} (1 - \delta_t)^k M_{t-k-1} + \beta_t (1 - \delta_t)^{t-1} P_0^t \right]$$

(13)

where the relevant initial number of phantoms is $P_0^t = \begin{cases} P_0^S & \text{if } \min \{ B_t + P_t^B, S_t + P_t^S \} = S_t + P_t^S \\ P_0^B & \text{if } \min \{ B_t + P_t^B, S_t + P_t^S \} = B_t + P_t^B \end{cases}$.
The role played by market history is parameterized by $\delta_t$. As $\delta_t$ tends to 0, phantoms are almost infinite-lived and old phantoms have a large impact on current matches. Conversely, with full depreciation $\delta^B = \delta^S = 1$, phantoms live only for one period and the PMT reduces to

$$\ln M_t = \ln S_t + \ln B_t - \ln [x_t + \beta_t M_{t-1}]$$  \hspace{1cm} (14)

### 3 Intertemporal vs intratemporal externalities

In this section, I examine the matching externalities featured by the phantom matching technology. The combination of intratemporal and intertemporal externalities implies that the technology has increasing returns to scale in the short run and constant returns in the long run. I study those properties in three steps.

#### 3.1 Intratemporal externalities

**Proposition 2** Without loss of generality, assume that $S_t + P_t S < B_t + P_t B$. In each period $t$,

1. $\frac{d \ln M_t}{d \ln S_t} = 1$
2. $\frac{d \ln M_t}{d \ln B_t} = \frac{\beta_t \sum_{k=0}^{\infty} (1-\delta_t)^k M_{t-k-1}}{B_t + \beta_t \sum_{k=0}^{\infty} (1-\delta_t)^k M_{t-k-1}}$

Proof. This results from direct computation.

The phantom matching technology has constant returns vis-à-vis the number of traders on the short side of the market. This property is typical of non-frictional models of matching assignment. Meanwhile, the PMT has positive returns vis-à-vis the number of traders on the long side. The reason is that additional traders reduce the proportion of phantom traders. This effect is all the higher than the number of phantoms is large.

This intratemporal externality implies that the matching technology exhibits increasing returns to scale in the short run. Indeed, $\frac{d \ln M_t}{d \ln S_t} + \frac{d \ln M_t}{d \ln B_t} > 1$. The magnitude of IRS is parameterized by $\beta_t$ (that drives the creation of new phantoms), $\delta_t$ (that governs phantom death), and by the history of matching flows $\{M_{t-k-1}\}_{k=0}^{\infty}$ (that fuels potential phantoms).

#### 3.2 Intertemporal externalities

**Proposition 3** Without loss of generality, assume that $S_t + P_t S < B_t + P_t B$. In each period $t$,

1. $\frac{\partial \ln M_t}{\partial \ln M_{t-k-1}} = -\frac{\beta_t (1-\delta_t)^k M_{t-k-1}}{B_t + \beta_t \sum_{k=0}^{\infty} (1-\delta_t)^k M_{t-k-1}}$
2. $\frac{d \ln M_{t+k}}{d \ln B_t} = \sum_{j=0}^{k-1} (\frac{\partial \ln M_{t+k}}{\partial \ln M_{t+j}}) (d \ln M_{t+j} / dB_t)$
3. $\frac{d \ln M_{t+k}}{d \ln S_t} = \sum_{j=0}^{k-1} (\frac{\partial \ln M_{t+k}}{\partial \ln M_{t+j}}) (d \ln M_{t+j} / dS_t)$
\[ (iv) \sum_{k=1}^{\infty} \left( d\ln M_t / d\ln B_{t-k} + d\ln M_t / d\ln S_{t-k} \right) = - \frac{\beta^B \sum_{k=0}^{\infty} (1 - \delta^B)^k M_{t-k-1}}{B_t + \beta^B \sum_{k=0}^{\infty} (1 - \delta^B)^k M_{t-k-1}} \]

Proof. Points (i) to (iii) result from direct computation. Point (iv) results from the fact that
\[ d\ln M_t / d\ln M_{t-1} = \sum_{k=1}^{\infty} \left( d\ln M_t / d\ln B_{t-k} + d\ln M_t / d\ln S_{t-k} \right). \]

Former matches generate phantom traders. In turn, phantoms deteriorate the current matching process. This intertemporal externality implies that the whole market history affects current matches. Intertemporal externalities are characterized by points (ii) and (iii). The main lesson is that current matches may positively or negatively alter future matches. To understand this property, I consider the case where phantoms only last one period, i.e. \( \delta^B = \delta^S = 1. \) Then,
\[ d\ln M_{t+k} / d\ln B_t = (-1)^k \prod_{j=0}^{k-1} \frac{\beta^B M_{t+j}}{B_{t+j} + \beta^B M_{t+j}} \] (15)

The magnitude of this elasticity decreases with horizon period \( k. \) Its sign depends on \( (-1)^k, \) which is negative for even \( k \) and positive for odd \( k. \) An increase in the number of period-\( t \) traders increases the number of period-\( t+1 \) phantoms, thereby reducing the flow of matches in period \( t+1. \) For a similar reason, this increases the flow of matches in period \( t+2. \)

Point (iv) shows that the sum of intertemporal externalities is negative. This compensates for the positive intratemporal externality discussed in Proposition 3. Intratemporal and intertemporal externalities combine so that the matching technology has constant returns to scale vis-à-vis the whole set of current and former traders.

### 3.3 Stationary phantom matching technology

I assume that each time a buyer and a seller get matched they are replaced by a similar pair of agents. I show that the phantom matching technology (PMT) converges towards a stationary technology, the stationary phantom matching technology (SPMT).

The number of traders follows \( B_t = B \) and \( S_t = S \) for all \( t. \) The number of matches follows the PMT. Without loss of restriction, sellers are on the short side of the market and
\[ \ln M_t = \ln S + \ln B - \ln \left[ B + \beta^B \sum_{k=0}^{\infty} \left( 1 - \delta^B \right)^k M_{t-k-1} \right] \] (16)

Proposition 4 The sequence \( M_t \) converges towards the stationary number of matches
\[ M = m(B, S) = B - \frac{1 + 4\beta^B / \delta^B (S/B)}{2\beta^B / \delta^B} \] (SPMT)

Proof. In steady state, \( M_t = M \) and solves
\[ \beta^B M^2 / \delta^B + BM - BS = 0 \]

The solving gives (SPMT).
The SPMT features standard properties. First, it is strictly increasing in the numbers of traders on each market side. Second, it has constant returns to scale. This property results from the constant intertemporal returns to scale discussed previously. Third, the elasticity of the matching technology with respect to the ratio of sellers to buyers is \( \varepsilon (S/B) = \frac{2(\beta^B/\delta^B)S/B}{-(1+4(\beta^B/\delta^B)S/B)^2+(1+4(\beta^B/\delta^B)S/B)} \in (1/2, 1) \).

This elasticity decreases with \( (\beta^B/\delta^B) S/B \). If buyers were on the short side of the market, the elasticity would be decreasing in \( (\beta^B/\delta^B) S/B \).

### 3.4 Dynamic implications

I consider a temporary increase in the number of sellers. This allows me to illustrate the results shown by Proposition 2 to Proposition 4. From time \( t_0 \) to time \( t_1 > t_0 \), the number of sellers goes from \( S \) to \( S(1+\varepsilon) \). Then, it goes down to \( S \). Initial numbers of phantoms are set at their stationary numbers. I distinguish three different matching technologies. In all cases, I consider deviations vis-à-vis the log of the stationary number of matches \( \ln M_t = \ln S_t/S + \ln |B + M| - \ln \left[ B + \frac{5}{2} \sum_{k=0}^{\infty} (5)^k M_{t-k-1} \right] \) (PMT1)

\[
\ln M_t/M = \ln S_t/S + \ln |B + M| - \ln [B + M_{t-1}] \quad \text{(PMT2)}
\]

\[
\ln M_t/M = \ln S_t/S + \ln |B + M| - \ln [B + M_{t-1}] - \ln S - \ln |B + M| \quad \text{(SPMT)}
\]

In technology PMT1, half of the matches gives birth to phantom traders and the depreciation rate is 50\%, i.e. \( \beta^B = .5 \) and \( \delta^B = .5 \). This technology has unlimited memory. In technology PMT2, all matches originate phantom traders, but phantoms only last one period, i.e. \( \beta^B = 1.0 \) and \( \delta^B = 0.0 \). This technology has limited memory. The technology SPMT is the stationary phantom matching technology corresponding to PMT1 and PMT2. This technology does not depend on the past number of matches.

The stationary numbers of traders are \( B = 2.0 \) and \( S = 1.0 \). The shock consists of a 10\% increase in the number of sellers, i.e. \( \varepsilon = .1 \). Initial numbers of phantoms are set at their stationary values.

I first consider a one-period shock. The shock takes place at period \( t_0 = 3 \). Figure 1 depicts the resulting trajectories of buyers’ matching probabilities. With the SPMT, the matching probability increases at the time of the shock, and goes down to its stationary value afterwards. The elasticity of the matching probability with respect to the ratio \( S/B \) is about .7. With the other technologies, Proposition 2 shows that the short-run elasticity of the matching probability with respect to \( S/B \) is one. This explains why the spike at the time of the shock is higher than with the SPMT. Changes in the phantom proportion then alter the matching probabilities, which converge towards the SPMT. This implies that the matching probability undershoots its long-run value at period \( t_0 + 1 = 4 \). With PMT1, phantoms die at constant rate, and there is monotonic convergence towards the steady-state value. With PMT2, phantoms only last one period. This implies oscillations of decreasing magnitude around the steady-state value, as discussed below Proposition 3.

I then consider a five-period shock. The shock occurs from \( t_0 = 3 \) to \( t_1 = 7 \). Figure 2 shows that the phantom matching technologies rapidly converge towards the SPMT. This implies oscillations with PMT2, and monotonic convergence with PMT1. Noteworthy, both technologies originate the same negative effect in period \( t = 8 \), that is once the negative shock has elapsed. Technology PMT1 compensates a lower
Figure 1: Changes in buyers’ matching probability following a one-period shock – The shock takes place at time $t_0 = 3$. Initial conditions: $S = 1.0$, $B = 1.0$, $M_t = M$ for all $t < t_0$. Shock $\varepsilon = .1$. Parameters are $\beta^B = .5$ and $\delta^B = .5$ in the case MPT1, and $\beta^B = 1.0$ and $\delta^B = 0$ in the case MPT2.

Figure 2: Changes in buyers’ matching probability following a five-period shock – The shock takes place at time $t_0 = 3$ and lasts until $t_1 = 7$. Initial conditions: $S = 1.0$, $B = 1.0$, $M_t = M$ for all $t < t_0$. Shock $\varepsilon = .1$. Parameters are $\beta^B = .5$ and $\delta^B = .5$ in the case MPT1, and $\beta^B = 1.0$ and $\delta^B = 0$ in the case MPT2.
phantom birth rate than technology PMT2 by a larger survival probability. Overall, the stock of phantoms is the same in \( t = 8 \).

Those examples illustrate two general phenomena. First, the matching technology has IRS in the short run. This tends to magnify temporary shocks with respect to matching technologies that have constant returns to scale in the short run. Second, the accumulation of phantoms and the resulting negative intertemporal externality imply that matching probabilities fall below their stationary level after the shock has elapsed.

4 Discussions

I discuss four aspects of the phantom matching technology (PMT). First, I argue that a new market place benefits from a honeymoon effect because there are no phantoms haunting this market. Second, I augment the model with another source of matching frictions, namely coordination frictions. Finally, I compare the PMT framework to matching technologies that account for on-the-match search.

4.1 Market birth and the honeymoon effect

Given increasing returns to scale in the short run, a new market benefits from a honeymoon effect. Without phantoms in the very beginning of market history, traders get matched easily. However, phantoms start accumulating and the matching technology deteriorates.

I develop this idea in the case where the total population of matched and unmatched agents is fixed. This corresponds to the marriage market, with an equal number of men and women. Suppose that a new marketplace opens. \( N \) men and \( N \) women enter the market, with \( S_0 \) individuals unmatched (singles) and \( N - S_0 \) matched (in couple) on each side of the market. Matched men and matched women originate phantoms with equal probability \( \beta \), and phantoms of both gender die with equal probability \( \delta \). The initial number of phantoms is 0. Once matched, men and women enjoy the benefits from male-female relationships until they separate. The separation rate is \( q \).

Populations of traders obey the following motions:

\[
S_t = S_{t-1} - M_{t-1} + q (N - S_{t-1}) \tag{17}
\]

\[
\ln M_t = 2 \ln S_t - \ln \left[ S_t + \beta \sum_{k=0}^{t-2} (1 - \delta)^k M_{t-k-1} \right] \tag{18}
\]

In steady-state, \( S_t = S \) and \( M_t = M \). This gives \( M = m(S, S) \) and \( S = (qN - M) / q \). It follows that

\[
\frac{M}{S} = \frac{-1 + (1 + 4\beta/\delta)^{1/2}}{2\beta/\delta} \tag{19}
\]

\[
S = qN / \left[ -1 + (1 + 4\beta/\delta)^{1/2} \frac{2\beta/\delta}{2\beta/\delta} + q \right] \tag{20}
\]

I assume that the initial population of singles is the steady-state population, i.e. \( S_0 = S \). The total population \( N \) of each gender is normalized to 1. I consider the two matching technologies PMT1 and PMT2 used in subsection 3.4. PMT1 corresponds to \( \beta = 1 \) and \( \delta = 1 \). PMT2 corresponds to \( \beta = .5 \) and \( \delta = .5 \). Figure 3 depicts the resulting patterns of the matching probabilities. Without phantoms, the matching probability is one the first period. This is the honeymoon effect. The matching probability
falls the second period and converges towards its stationary value. The honeymoon effect may apply to various match-making industries, like online dating, real estates, or even temporary work agencies. This predicts that newcomers in those markets may build on their initial advantage and easily conquer market shares in a first step. However, they should suffer from negative intertemporal externalities in a second step, leading to high mortality rates. The honeymoon effect may also contribute to explaining why public employment agencies are less efficient than private agencies. Public agencies are typically older and have many phantoms, while younger private agencies have not.

4.2 Phantoms and coordination frictions

I examine how phantom traders interact with an alternative source of market frictions. In the urn-ball matching (UBM) model, agents on one side of the market try to contact agents on the other side. However, they do not coordinate, resulting in coordination frictions. The PMT framework and the UBM model complete each other so as to offer a rich description of market frictions.

Assume that each buyer, including phantoms, sends a buy order to one of the sellers, including phantoms too. The probability that a particular seller receives a buy order from a particular buyer is $1/(S + P_S)$. In case a seller receives multiple offers, two cases must be analyzed. Either the seller can distinguish a phantom buyer from a non-phantom buyer or he cannot.

In the former case, the number of matches is

$$M = S \left[ 1 - \left( 1 - \frac{1}{S + P_S} \right)^B \right]$$

As $B, S, P_S \to \infty$, this gives

$$M = S \left[ 1 - \exp \left( -\frac{B}{S + P_S} \right) \right]$$

(22)
This technology still features increasing returns to scale vis-à-vis $B$ and $S$, as an increase in $S$ allows to reduce the phantom proportion on sellers’ side. In the long run, the SPMT is

$$M = S \left[ 1 - \exp \left( - \frac{B}{S + \beta^S M/\delta^S} \right) \right]$$  \hspace{0.5cm} (23)

This equation implicitly defines $M = m(B, S)$. The SPMT has constant returns to scale.

In the latter case, the number of matches is

$$M = \frac{B}{B + P^B S} \left[ 1 - \exp \left( \frac{B + P^B}{S + P^S} \right) \right]$$  \hspace{0.5cm} (24)

The corresponding SPMT is

$$M = \frac{B}{B + \beta^B M/\delta^B} S \left[ 1 - \exp \left( - \frac{B + \beta^B M/\delta^B}{S + \beta^S M/\delta^S} \right) \right]$$  \hspace{0.5cm} (25)

The implicit function $M = m(B, S)$ also features constant returns to scale. Noteworthy, the latter technology highlights the contributions of phantom traders and coordination frictions to overall market frictions. The term $BS/(B + \beta^B M/\delta^B)$ captures the direct role played by phantom traders, while the term $1 - \exp \left( - \frac{B + \beta^B M/\delta^B}{S + \beta^S M/\delta^S} \right)$ relies on coordination frictions. Coordination frictions themselves are parameterized by the stocks of phantoms on each market side.

4.3 Labor market evidence and on-the-job search

On-the-match seekers consist of a particular type of phantom traders. In this subsection, I elaborate on this idea. I make two points. On the one hand, the PMT is a natural framework to analyze on-the-match search. On the other hand, usual empirical strategies to account for on-the-job search fail to identify on-the-job seekers from other types of phantom traders.

On-the-match search occurs when matched traders go on searching for alternative partners. They may do so for various reasons largely discussed in the literature, as expanding their information set, changing partner, or bargaining a larger share of match surplus. On-the-match seekers may alter the search of unmatched agents through congestion or crowding-out effects. On-the-match search is usually captured as follows. Let $E$ denote the number of matched traders. The number of matches between unmatched traders is $m(B, S; E)$. The dependence vis-à-vis $E$ is typically nonpositive.

Adopting the terminology in use in this paper, on-the-job seekers can be seen as phantoms. Hereafter, the total population of matched and unmatched agents are $N^B_t = B_t + E_t$ and $N^S_t = S_t + E_t$, where $E_t$ denotes the total number of matched agents. Matched agents separate with probability $q$. There are no other phantoms than matched agents.

To begin with, I assume that matched agents always go on searching alternative partners. This may be so to improve their information on the distribution of potential partners, or to increase their share of match surplus through alternative offers and counteroffers. I also assume that matched agents provide $\beta$ efficient units of search. In the PMT framework, this corresponds to $\beta^B = \beta^S = \beta$ and $\delta^B = q$. The PMT is

$$\ln M_t = \ln S_t + \ln B_t - \ln \left[ B_t + \beta \sum_{k=0}^{\infty} (1 - q)^k M_{t-k-1} \right]$$  \hspace{0.5cm} (26)
By definition, the total number of matches is the sum of all former matches weighted by the probability that they have not separated. Therefore, \( E_t = \sum_{k=0}^{\infty} (1 - q)^k M_{t-k-1} \) and

\[
\ln M_t = \ln S_t + \ln B_t - \ln [B_t + \beta E_t]
\]  

This matching technology directly derives from the PMT. As such, this features intratemporal increasing returns to scale vis-à-vis \( B \) and \( S \). And this has intertemporal constant returns to scale once the negative dependence vis-à-vis \( E_t \) is taken into account.

This technology could be confronted to labor market data. Assuming that (i) the number of matches that takes place in \( t \) can only be observed in \( t + 1 \), (ii) \( S_t \) is proportional to the actual stock of vacancies, and (iii) there is unbiased measurement error on the number of sellers/vacancies, the statistical model would write

\[
\ln M_{t+1} = \alpha_0 + \alpha_1 \ln S_t + \alpha_2 \ln B_t - \alpha_3 \ln [B_t + \beta E_t] + \omega_t
\]

where \( \omega_t \) is iid. The empirical restriction would be \( \alpha_1 = \alpha_2 = \alpha_3 = 1 \).

However, this restriction should not hold, partly because this technology abstracts from any other source of market friction, and partly for reasons already discussed in the empirical literature. xx argue that nonregistered vacancies are imperfectly correlated with registered vacancies, resulting in a mis specification bias. Similarly, Burgess (1993) and Anderson and Burgess (2000) argue that a large proportion of employees do not seek jobs. The proportion that seeks jobs is likely endogenous, which requires an elaborated empirical strategy. Meanwhile, some employees do search alternative matches, but on markets that unmatched traders cannot access. For instance, suppose that employees need to find a single alternative employer to receive their marginal contribution to output (net of matching costs). Therefore, they go on searching after they have found a first job, and stop searching once they have found a second job. Phantom birth rate is still \( \beta^B = 1.0 \). Phantom dying rate is computed as follows. With probability \( 1 - q \) the worker stays employed. With probability \( M_t/B_t \), he receives a second offer. Therefore, \( \delta^B_t = q + (1 - q) M_t/B_t \). The PMT is

\[
\ln M_{t+1} = \ln S_t + \ln B_t - \ln \left[ B_t + M_{t-1} + \sum_{k=2}^{\infty} M_{t-k} \prod_{j=1}^{k-1} [q + (1 - q) M_{t-j}/B_{t-j}] \right]
\]

The stock of phantoms does no longer coincide with employment. This implies that employment may be a poor proxy for the negative externality caused by employed job-seekers.\(^1\)

The fact that there is observational equivalence between phantoms and on-the-match seekers casts some doubt on the interpretation of estimated matching technologies that explicitly account for on-the-job search. The general problem is that the number of phantoms is correlated with recent hires. The fact that traders of the past affect current hires does not prove that employees create congestion effects for the unemployed. This may also result from the type of informational persistence that is advocated in this paper.

Suppose for instance that there are two types of phantoms: on-the-match seekers and regular phantoms. The birth rate of regular phantoms (on-the-match seekers) is \( \beta^R (\beta^O) \), while the dying rate is \( \delta^R \) (\( q \)). The PMT is

\[
\ln M_{t+1} = \ln S_t + \ln B_t - \ln \left[ B_t + \beta^O \sum_{k=0}^{\infty} (1 - q)^k M_{t-k-1} + \beta^R \sum_{k=0}^{\infty} (1 - \delta^R)^k M_{t-k-1} \right]
\]

\(^1\)See Sunde (2007) for a discussion on the implications of endogenous search behavior.
It is very difficult to disentangle the impact of regular phantoms from the impact of on-the-job seekers. One may try to use data on job-to-job movements to control for the effects of on-the-job seekers. However, this strategy is misleading. On the one hand, many employed job-seekers do not necessarily want to change jobs. They may simply try to get an alternative offer so as to obtain a counter-offer from their current employer. On the other hand, employed job-seekers do not necessarily compete with the unemployed, while regular phantoms do.

A key difference between phantom traders and on-the-match seekers is the fact that phantoms consist of a backward variable, while part of on-the-match seekers consist of a forward variable. As noted by Burgess, on-the-match seekers tend to seek more intensively during booms. Of course, phantoms cannot adapt to changing market conditions. That may offer an empirical strategy to distinguish phantoms from non-phantoms. Parameter $\beta^O$ should depend on workers expectations on the future number of matches, that is $\beta^O = \beta^O (E_{t-1}M_t)$.


