Endogenous Market Segmentation and the Volatility of House Prices

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ABSTRACT

We study an economy where households face transactions costs of participating in the housing market. In response to these costs, households choose to buy and sell houses infrequently. This, in turn, implies that the value of the typical transaction is large relative to income. In this way our model captures two essential features of household-level adjustments to residential capital. Moreover, it implies that, in any period, only a fraction of households are active in the housing market, and that this fraction evolves with the aggregate state of the economy. We find that this endogenous market segmentation amplifies and propagates the response in the relative price of housing following aggregate and sectoral shocks. Because those households currently active in the real estate market must absorb changes to the aggregate housing stock in equilibrium, market segmentation exacerbates changes in house prices. Moreover, it implies large and persistent changes in the distribution of households following an aggregate shock, which themselves cause additional movements in prices. Thus households’ optimal (S,s) policies, driven by nonconvex transactions costs, not only induce lumpy adjustment at the individual level, but also explain a nontrivial fraction of observed excess price volatility in real estate markets.

KEYWORDS: Housing in general equilibrium, segmented markets, house price volatility, generalized (S,s) policies

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1 Introduction

Buying or selling a house is a lumpy investment activity. While over sixty percent of households in the U.S. economy own their residences, the typical household buys or sells infrequently. Moreover the value of housing transactions are large relative to other consumption and investment expenditures. These salient characteristics of the housing market have been largely ignored by macroeconomists. Given the problems of solving discrete choice models in dynamic stochastic general equilibrium, there are few attempts to model lumpy housing transactions.\footnote{See, for example, the survey of general equilibrium models of housing by Jeske (2005).} Turning to aggregate behavior, both residential investment and house price indices are far more volatile than GDP. Indeed, the high volatility of house prices has proven a challenge for general equilibrium models of the housing market, thus far eluding them.

We develop a generalized (S,s) model wherein households choose to make infrequent and lumpy adjustments to their housing stock in response to fixed costs associated with the sale or purchase of a house. Thus the model captures an essential aspect of housing that, at the micro-level, distinguishes it from most nondurable goods. Further, the aggregate implication of households’ decisions is that the market for real estate is endogenously segmented. In any period, only a fraction of households are either buying or selling their house. This limited participation amplifies responses in house prices following shocks to the supply of houses, when compared to a setting where all households participate in the real estate market in every period. Moreover, resulting changes to the distribution of households lead to persistent movements in house prices. As such, the model not only amplifies, but also propagates, changes in house prices.

In recent months, a sharp rise in foreclosures following mortgage defaults (widely attributed to a fall in loan standards resulting from increases in securitization) have caused a sharp drop in house price indices. As a large fraction of household wealth is held in owner-occupied residential property, the effects of such price changes on consumer expenditures are believed to be significant (see, for example Campbell and Cocco (2008) and Li and Yao (2007)). While large fluctuations in house prices may have important aggregate effects, we have few quantitative equilibrium models that can help us understand the determinants of these prices. Most DSGE models with an explicit role for housing assume that, at the individual level, residential investment is a frictionless choice that evolves smoothly with changes in household state variables. At the same time, such models have found it difficult to reproduce the empirical volatility of house prices. For example, consider the work of Davis and Heathcote (20005); while explaining the volatility of residential investment, their model is not able to reproduce the variability observed in house price indices.

We find that the frictions we introduce to explain lumpy household-level adjustments to residential capital can imply large and persistent movements in equilibrium house prices following shocks. In our model households hold two assets, financial wealth and housing.
Each period, each household receives endowment income and, in the absence of a change in residence, it allocates this income between current nonhousing consumption goods and bonds. In periods when a household decides to move, it must pay a transactions cost. After the payment of the cost, it is able to sell its current house. The funds from this sale, alongside the value of wealth held in bonds, in whole or in part, are used to pay for the purchase of a new residence. If current wealth is insufficient to fully finance this investment, the household will obtain a mortgage.

Transactions costs lead households to adopt generalized (S,s) policies in determining the timing of their housing adjustments. In the steady state, the typical household moves roughly every five years. While a household lives at one address, its house depreciates. Thus, over time, the difference between the household’s target house size and its actual one increases. When the difference is sufficiently large, the household decides to move. On average, when a household moves, it acquires a larger house. During times when it is not active in the housing market, the household saves using financial assets that help fund the next purchase.

As mentioned above, households’ infrequent buying and selling of residences implies that the housing market is endogenously segmented; in any period, only a fraction of households participate in the market. Consequently, any shock to the supply of new houses causes a large response in the relative price of housing. This is because the aggregate shock initially must be absorbed by the subset of households that are active in the market. To induce these households to move into unusually large houses, prices must fall much further than they would in a benchmark model without segmentation. Exploring a simple example, we find there is a long-lived response in prices following a purely transitory shock to the supply of housing. At the date of the shock, house prices change 50 percent more than does the quantity of houses. Moreover, while a temporary rise in the supply of housing reduces its relative price at the time of the shock, it thereafter drives an increase in house prices as large as the initial shock to quantities. Thus, in response to shocks to the housing stock, our model predicts that house prices are volatile and their correlation with sales is low.

In a second example, we examine the response in house prices following a persistent rise in the aggregate endowment. Here, we find a rich response in prices that is almost four times more volatile than the change in quantities. This volatility is characterized by episodes where house prices fall, followed by episodes where they are higher than average. These movements in house prices are driven by changes in the distribution of households over housing and financial assets. As this distribution is part of the aggregate state of the economy, its disruption following a housing shock generates further movements in the relative price of housing in subsequent periods. Moreover, because households make infrequent adjustments to their housing stocks, changes in the distribution are long-lived. This, in turn, increases the persistence of movements in house prices. Overall, the non-monotone response of house prices implies a correlation with sales that is quite small.

There are very few dynamic stochastic general equilibrium models that directly address the infrequent buying and selling of houses. Two important recent exceptions are Iacoviello
and Pavan (2008) and Rios-Rull and Sanchez-Marcos (2008). In contrast to our work, both of these papers address the joint distribution of housing and financial wealth as measured in the U.S. economy. Iacoviello and Pavan focus on the effects of a rise in individual income risk, and a fall in mortgage down payments, on the implied volatility and cyclicality of residential investment and the cyclicality of mortgage debt. Studying a production economy, they assume that the marginal rate of transformation between consumption and residential investment is one; housing and non-housing consumption are perfect substitutes in production. This implies that the relative price of housing is fixed and constant over time; their model predicts no movement in real house prices. Nonetheless, in their paper, as in ours, households face a fixed cost of moving.

The primary focus of Rios-Rull and Sanchez-Marcos (2008) is the volatility of house prices. They document that the prices of existing and new homes are more volatile than GDP. The standard deviation of the HP-filtered median price of existing home sold, relative to GDP, is 1.287; the corresponding value for new home sales is 2.274 (Table 1, p. 4, Rios-Rull and Sanchez-Marcos (2008)). The standard deviation of the quantity of houses sold, relative to GDP, is 6.767. In an endowment economy, Rios-Rull and Sanchez-Marcos capture the lumpiness of housing transactions by assuming that there are two house sizes in the economy. They also allow individual households to hold bonds, with these representing mortgages when they take on negative values. Importantly, a fixed percentage of the house price must be paid as a downpayment. Examining shocks to earnings, interest rates, and the mortgage premium, Rios-Rull and Sanchez-Marcos find that house prices generally move far less in their model than they do in the data.

Households in our model economy are confronted with a maximum borrowing limit. While the presence of idiosyncratic risk makes them reluctant to approach the limit on debt (as this precludes further borrowing), they are able to finance house purchases using debt. As a result, young, relatively poor households buy small houses. Thus, we generate house price volatility in the absence of the mechanism stressed by Ortalo-Magne and Rady (2006) and adopted by Rios-Rull and Sanchez-Marcos (2008), the inability of young households to afford down payments.

While our model is designed to reproduce the empirical distribution of housing and financial wealth, we have not yet done so. Our first goal is to explore the effects on house prices arising from a defining feature of real estate markets, households’ infrequent participation in these markets. In contrast to Rios-Rull and Sanchez-Marcos, we do not assume that houses are indivisible. In fact, the divisibility of housing is important in our primary result.

As discussed above, changes in the distribution of housing in our model economy drive long-lived movements in prices. These movements are the result of periodic re-entry into the housing market of those households that were active at the date of an aggregate shock. The initial fall in house prices at the date of a temporary rise in the supply of available housing implies a large wealth effect for households active in the housing market at that time. Several years later, when most of these households return to the market, their preference for smooth
consumption profiles implies a desire to maintain above-average sized houses. Because the quantity of available housing is monotonically reverting to its long-run level, this means that there must be a second round of price changes as these initially active households re-enter the market. Had we assumed that housing was indivisible, target house sizes could not respond to the shock, and much of this rich dynamic response in prices would have been eliminated. Thus we see that maintaining an intensive margin decision, the size of the new house chosen by active households, is itself essential in the model’s ability to propagate shocks to the housing sector. Of course, the presence of this second margin of adjustment complicates the model solution somewhat; conditional on the decision to pay a transactions cost and move, each of our households also faces a continuous choice in selecting its next residence.

The present paper is also related to our previous work on endogenous market segmentation in Khan and Thomas (2007). There, examining a model where risk-averse households faced transactions costs of adjusting their portfolios between high-yield assets (bonds), and low-yield assets (money), we found that endogenous market segmentation implied persistent movements in real interest rates that were absent in an economy without transactions costs. This earlier work extended the study of dynamic stochastic general equilibrium (S,s) economies, previously confined to the study of firms subject to fixed costs of changing prices, investing or ordering goods (Dotsey, King and Wolman (1999), Thomas (2002), Khan and Thomas (2003, 2008), Khan and Thomas (2007)). Here, as we further extend the environment to examine the housing market, we present two models. The first is closely related to our existing work and heavily exploits risk-sharing. The second goes a step beyond the existing literature, relaxing the assumption of risk-sharing. We believe this to be a necessary step in the eventual development of a model capable of explaining the observed joint distribution of housing wealth and financial wealth.

2 The model

The economy is populated by a large number of long-lived households, each of whom has a risky income process. Income is received in units of a non-storable consumption good which, alongside housing services, is one of two goods valued by households. There are two assets: bonds denominated in units of consumption and houses. Bonds represent households’ financial wealth and may be held in positive or negative quantity. Each household’s bond holdings are bounded below by a borrowing limit, and bonds may be traded every period without cost. By contrast, houses are bought and sold in a market that is subject to transactions costs borne by buyers and sellers. Households that are not presently active in the housing market avoid these costs.
2.1 Housing choices with contingent-claims

We begin with a model wherein households have access to a full set of contingent claims. The results that follow are useful in providing a benchmark that is closely related to previous work examining households confronted with transactions costs. Let \( s \) describe the state of the economy in period \( t = 0, 1, \ldots \), and let \( s^t \) denote the date-event history \( (s_0, s_1, \ldots, s_t) \). The unconditional probability of \( s^t \) is written as \( \pi (s^t) \), while the conditional probability of \( s^{t+n} \) given \( s^t \), where \( s^t < s^{t+n} \) is \( \pi (s^{t+n} | s^t) \).

Let \( y_i (s^t) \) be the income of household \( i = 0, 1, \ldots \), in period \( t \). Households face transactions costs, \( \xi \geq 0 \), which they must pay if they are to participate in the housing market. These costs are idiosyncratic across households and over time, each period being drawn from a time-invariant distribution, \( G \). As we exploit the assumption that \( \xi \) is i.i.d., it is useful to distinguish it from \( s \). Following this approach, we describe each household using the pair \((s, \xi)\). Let \( b_i (s^t, \xi^t) \) describe a household’s financial wealth at the beginning of the period and \( c_i (s^t, \xi^t) \) denote its current consumption of the non-storable consumption good. For simplicity, we abstract from the home production of housing services and simply equate the current stock of housing, \( h_i (s^t, \xi^t) \), to its consumption.

Let \( q (s^{t+1}) = q(s^t, s_{t+1}) \) be the price of a contingent-claim that will pay 1 unit of the consumption good next period if the current state of the economy is \( s^t \) and the state next period is \( s^{t+1} = (s^t, s_{t+1}) \). Competitive financial intermediaries issue households claims that are contingent on both \( s \) and \( \xi \) at the composite price \( q (s_{t+1}) G (\xi) \) (see Lemma 1 in Khan and Thomas (2007)). The unit price of housing, which we treat as a homogenous, divisible commodity, is \( q^h (s^t) \).

Here, we abstract from any time costs associated with moving. As such, a household may adjust its housing capital immediately upon payment of its current transactions cost. Any given household enters period \( t \) with housing \( h_i (s^{t-1}, \xi^{t-1}) \) and financial wealth \( b_i (s^t, \xi^t) \). It must then choose its current nondurables consumption and the level of housing services it wishes to consume. If the latter implies an active change in its housing capital, that is if the household decides to move, then it pays its transactions cost \( \xi \), sells its existing house, which has size \( 1 - \delta_s^h \), \( h_i (s^{t-1}, \xi^{t-1}) \), and then purchases the house it chooses to inhabit in the present period, \( h_i (s^t, \xi^t) \).

The budget constraint is shown below.

\[
c_i (s^t, \xi^t) + q^h (s^t) h_i (s^t, \xi^t) + \int_S q (s^{t+1}) b_i (s^{t+1}, \xi^{t+1}) G (d\xi_{t+1}) ds_{t+1} \\
+ \chi (h_i (s^t, \xi^t) - (1 - \delta_s^h) h_i (s^{t-1}, \xi^{t-1})) \xi_t \leq y_i (s^t) + q^h (s^t) (1 - \delta_s^h) h_i (s^{t-1}, \xi^{t-1})
\]

In equation (1), the indicator function \( \chi \) takes on a value of 1 if its argument is not 0; otherwise it is equal to 0. Its presence reflects the fact that transactions costs \( \xi_t \) are incurred only if the household chooses a stock of housing for this period that is not equal to its

\textsuperscript{2}The depreciation of houses is an assumption that implies every household will eventually move. An alternative assumption that would achieve the same result would be growth in individual incomes.
previous stock net of depreciation. The household is also subject to the limit condition on debt,
\[ \lim_{t \to \infty} \int_{s^t} q(s^t) b_i(s^t) ds^t = 0. \] (2)

Each household chooses (nondurables) consumption, bonds and housing at each date and state to solve:

\[
\max_{\{c_i(s^t, \xi^t), b_i(s^t, \xi^t), h_i(s^t, \xi^t)\}} \sum_{t=0}^{\infty} \pi(s^t) \int u(c_i(s^t, \xi^t), h_i(s^t, \xi^t)) G(\xi^t)
\]

subject to (1) and (2),
given \( b_i(s_{-1}) = 0 \) and \( h_i(s_0) = h_0 \).

As indicated in the household’s problem above, we assume that all households are initially identical. Each begins its life with no debt and a common-sized house. Beyond this, households’ access to a full set of state-contingent bonds provides them insurance against their income and transaction cost risk. In particular, by assuming households can purchase these bonds in an initial period, \( t = -1 \), wherein there are no transactions costs draws, endowment incomes, consumption or housing market trades, we ensure that they each adopt a common state-contingent lifetime plan for their consumption and housing purchases over the subsequent dates when their transactions costs and incomes begin to distinguish them.

Given the insurance noted above, the marginal utility of consumption will be equated across all households in all states of nature. However, the presence of transactions costs will imply that, in general, households do not fully insure themselves with respect to their housing. Thus, at any given time, households will have residences of differing sizes. As a result, the efficient allocation of consumption, conditional on the distribution of housing, will imply a nontrivial distribution of consumption across households. Nonetheless, in periods when they are active in the housing market, households’ state-contingent claims allow them to eliminate differences in financial wealth and housing capital. A set of households active at the same date may enter the period with different size houses, and thus differences in their bond holdings. However, upon entering the real estate market, they will all move to the same priced house, choose the same consumption level, and leave the period with the same portfolio of state-contingent claims.

Define \( p(s^t) \) as date \(-1\) relative price of a unit of the nondurable consumption good delivered at date-event \((s^t)\). To establish a finite-memory property of this discrete choice economy, we derive the lifetime budget constraint of the household listed below.

\[ 0 \leq \sum_{t=0}^{\infty} p(s^t) \left( y_i(s^t) - c_i(s^t, \xi^t) - \chi(e(s^t, \xi^t)) \left[ \xi_t - q^{h}(s^t) e_i(s^t, \xi^t) \right] \right) + q^{h}(s_0) h_0. \] (3)
In the above, \( e_i(s^t, \xi^t) \) is the gross investment in housing capital implied by selling the existing house of size \( (1 - \delta h) h_i(s^{t-1}, \xi^{t-1}) \) and moving to a new house of size \( h_i(s^t, \xi^t) \). It follows that each household faces period-by-period constraints,

\[
h_i(s^t, \xi^t) \leq (1 - \delta h) h_i(s^{t-1}, \xi^{t-1}) + e_i(s^t, \xi^t),
\]

that determine its stock of housing at any time.

Let \( \Lambda \) denote the multiplier on (3) and \( G(\xi^t) \gamma(s^t, \xi^t) \) be the multiplier for (4). The optimal allocation for the household satisfies the following conditions.

\[
\beta^t \pi(s^t) G(\xi^t) D_1 u(c_i(s^t, \xi^t), h_i(s^t, \xi^t)) = \Lambda p(s^t)
\]

If \( e_i(s^t, \xi^t) > 0 \), then \( \chi(e(s^t, \xi^t)) = 1 \), and we have the first-order condition for housing expenditure,

\[-\Lambda p(s^t) q^h(s^t) + G(\xi^t) \gamma(s^t, \xi^t) = 0.\]

Finally, whether or not the household participates in the housing market in a given period, the following efficiency condition holds for \( h_i(s^t, \xi^t) \).

\[
\beta^t \pi(s^t) D_2 u(c_i(s^t, \xi^t), h_i(s^t, \xi^t)) = \gamma(s^t, \xi^t)
\]

While \( \gamma(s^t, \xi^t) \) varies by household, \( \Lambda \) is the same for all. Given the same initial stocks of financial and housing wealth, all households face the same time-0 lifetime budget constraint. Consider then a state \( (s^t) \) when the household \( (s^t, \xi^t) \) is active in the housing market. As \( \chi = 1 \), we have

\[
\beta^t \pi(s^t) D_2 u(c_i(s^t, \xi^t), h_i(s^t, \xi^t)) = \Lambda p(s^t) q^h(s^t).
\]

This implies that all active households choose the same new house, \( h_a(s^t) \) and the same consumption level, \( c_a(s^t) \). Returning to the period-by-period budget constraint, it follows almost immediately that \( b_i(s^{t+1}, \xi^{t+1}) = b_a(s^{t+1}, \xi_{t+1}) \), so these households also choose the same portfolio of state-contingent bonds for the next period.

In concluding this section, we emphasize that transactions costs associated with housing trades eliminate the appearance of complete risk sharing even when households are allowed access to a complete set of state-contingent claims. Nonetheless, that access to insurance limits heterogeneity in an important sense. When households choose to pay transactions costs to move, they erase all effects of their past idiosyncratic shocks to income and transactions costs. Thus, those active in the housing markets together in any one date become effectively identical. Combining this with the finite delay between moves implied by the depreciation of each owner-occupied house, the relevant history of the economy is truncated to the maximum number of periods since any given group of households last moved.
2.2 Housing decisions with idiosyncratic risk

The formulation above yields a tractable setting in which to explore fluctuations in house prices, and it will serve as a useful reference as we go forward. However, it is deficient in one essential respect. If we are to reproduce the joint distribution of financial and housing wealth, we must adopt an environment that can accommodate persistent differences in households’ wealth. Thus, in this section, we move to consider an alternative setting where households do not have access to risk-sharing. While eliminating the full set of Arrow-Debreu securities introduces several complications for the analysis of our model, we show that its solution can still be obtained without excessive computational burden. This may be of independent interest beyond our current application, given that previous studies of dynamic (S,s) economies in general equilibrium have always relied on complete risk-sharing. It is also worth noting that our method does not require the complete absence of risk-sharing; nothing prevents a slight modification introducing limited insurance to the following model.

Aside from the lack of insurance, there is one important new feature distinguishing our current model from the environment described in the section above. We now assume that every household leaves the economy with probability one in finite time. Specifically, each household has probability $\pi_s \in (0,1)$ of surviving from any one period to the next. We introduce this assumption to conveniently capture certain demographic characteristics of the data (for instance, young households entering the economy with little wealth) without having to follow an explicit age structure.

Let $S \in S$ denote the aggregate state vector for the economy, evolving according to a law of motion $S' = F(S, S')$. Each household has access to claims that are contingent on the aggregate state, $S$, but has no way to insure against individual risks to income or transactions costs. Individual household income follows a Markov Chain, $\Pr \{y' = y_j \mid y = y_i\} = \pi_{ij}$, where $\sum_{j=1}^{N(y)} \pi_{ij} = 1$ for each $i = 1, \ldots, N(y)$. Meanwhile, transactions costs are drawn from the same distribution described in the previous section.

A household begins the period with financial and housing wealth $(b_0, h_0)$. Bond holdings may be positive or negative; the household has debt whenever $b_0 < 0$. While debt may be associated with smoothing the effects of uninsurable risk, we associate debt with mortgages and assume a maximum borrowing limit, $b_l(S) \leq 0$, that may vary with the aggregate state of the economy. Alongside beginning of period bond holdings, a household’s current resources are determined by its income and transaction cost.$^3$

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$^3$Given risky income, it is possible that a household may not be able to maintain positive consumption and housing in the presence of the common borrowing limit above. If there was a positive probability of default, the competitive financial intermediaries would adjust the borrowing rate for each household according to its individual characteristics. Households with higher debt levels, smaller houses and lower income would carry a higher probability of default, and thus face greater costs of borrowing. This is a problem that has been carefully characterized by Chatterjee et al (2007) in the absence of housing decisions. We abstract from this complication in our current model. The problem does not arise in the examples we have solved thus far, because debt is secured by house value. While the revenue from the sale of a house may be less
We model the housing decision as taken at the beginning of a period prior to the consumption-savings decision, which is conditional on the stock of housing capital. This is entirely for expositional simplicity; nothing prevents the household from simultaneously choosing its consumption of nondurables and housing, alongside its bond holdings for next period. Regardless, let \( W(x, h, y, S) \) denote the middle of period value, prior to the savings decision, of a household with mid-period financial resources \( x \) and living in a house of size \( h \), given current income \( y \) and aggregate prices determined by \( S \).

Stepping back to the beginning of the period, define \( V_0 (b_0, h_0, y_i, \xi, S) \) as the expected-discounted lifetime utility of a household entering the period with \( (b_0, h_0, y_i) \), current transaction cost \( \xi \), and facing a moving decision. If the household chooses to move, it will sell its current house, which has value \( q^h h_0 \), and simultaneously purchase a new house. Because houses are perfectly divisible in our model, the size of the next house, \( h_1 \), is itself a choice variable. However, the maximum value of the next house is limited by the household’s current wealth and its ability to borrow. In particular, given the borrowing limit, \( b_l(S) \leq 0 \), the household may purchase a house costing no more than \( b_0 + y_i - \xi + q^h h_0 - b_l(S) \), or it may retain its current house \( h_0 \). Taking all of this into consideration, the household’s moving decision at the start of the period solves the following problem.

\[
V_0 (b_0, h_0, y_i, \xi, S) = \max \left\{ W (b_0 + y_i, h_0, y_i, S), \right. \\
\left. \max_{h_1 \in H(x_0, \xi; S)} W \left( b_0 + y_i - \xi + q^h (S) (h_0 - h_1), h_1, y_i, S \right) \right\}
\]

where \( H(x_0, \xi; S) = \left[ 0, \frac{x_0 - b_l(S) - \xi}{q^h(S)} \right] \), and where \( x_0 \equiv b_0 + y_i - \xi + q^h(S) h_0 \) is a summary variable describing the household’s total wealth at the beginning of the current period.

Once the moving decision has been made, the household next allocates its mid-period financial wealth between current consumption and bonds purchased for the next period.

\[
W(x, h, y_i, S) = \max_{c \in \{b(S')\}} \left( u(c, h) + \beta \pi^a \int_S V(b'(S'), h', y_i, S') F(S, dS') \right)
\]

subject to

\[
c + \int_S q(S, S') b(S') dS' \leq x \\
h' = (1 - \delta_h) h.
\]

than a household’s mortgage, it is still sufficient to cover any transaction cost and some positive level of consumption when it is augmented by even the lowest income level. Thus, proceeds from the forced sale of a house help borrowers avoid bankruptcy. We allow the borrowing limit to vary with the aggregate state to ensure that the effects of equilibrium changes in house prices and interest rates do not force households into bankruptcy when, for example, \( q^h \) falls.
Notice that the survival probability, $\pi^s \in (0, 1)$ appends the subjective discount factor, $\beta$. It is useful at this stage to define $c(x, h, y_i, S)$ as the decision rule for consumption, and $g(x, h, y_i, S)$ as the decision rule determining end-of-period bond holdings, for a household that has net wealth $x$ after the housing decision.

We complete our step-by-step description of each household’s optimization problem by defining the final value function, $V$, that appeared in the consumption-savings problem (6). This function determines the household’s expected value as it exits any period, taking expectations over all possible realizations of the income and transactions cost it may encounter in the next period.

$$V(b, h, y_i, S) = \sum_{j=1}^{N(y)} \pi_{ij} \int_\xi V_0(b, h, y_j, \xi, S) G(d\xi).$$  \hspace{1cm} (7)

Together, (5) - (7) define a functional equation in $V_0$ that fully describes the lifetime utility maximization problem of any household in our economy. Given the aggregate law of motion $F$ and the equilibrium price functions $q(\cdot, S')$ and $q^h$, this functional equation allows us to derive the household’s decision rules as a function of its individual state, $(b, h, y_i)$, and its current cost of participating in the housing market, $\xi$.

2.2.1 Threshold decision rules

We now characterize households’ housing adjustment decisions - that is, their decisions of whether and where to move. Suppressing the dependence of the relative price of housing, and the dependence of the borrowing limit, on the aggregate state, let $W_0(x_0, y, \xi)$ represent the solution to the housing problem conditional on a move,

$$W_0(x_0, y, \xi; S) = \max_{h_1 \in H(x_0, \xi; S)} W \left( x_0 - \xi - q^h h_1, h_1, y, S \right),$$  \hspace{1cm} (8)

and define $h^*_1 \equiv h_1(x_0, y, \xi; S)$ as the corresponding choice of $h_1$. Next, let $\tilde{\xi}^T$ describe that level of current transactions cost that leaves a household of type $(b, h, y)$ indifferent between actively participating in the housing market and deferring its participation for at least one more period. This cost solves $W_0 \left( x_0, y, \tilde{\xi}^T; S \right) = W \left( b + y, (1 - \delta h) h, y, S \right)$, or:

$$W \left( b + y - \tilde{\xi}^T + q^h (h - h^*_1), h^*_1, y; S \right) = W \left( b + y, (1 - \delta h) h, y, S \right).$$  \hspace{1cm} (9)

Using the cost isolated in (9), we define $\xi^T(b, h, y; S) = \min \left\{ \tilde{\xi}^T, \xi^T \right\}$. This threshold cost separates those households of a given type $(b, h, y)$ that adjust their housing (those drawing costs at or below $\xi^T$) from those that do not. Thus, we can use it to determine the fraction of all type $(b, h, y)$ households that participate in the housing market in the current period: $G \left( \xi^T(b, h, y; S) \right)$. 

10
Turning next to the intensive margin decision of which house to purchase conditional on an adjustment, consider any set of participating households that began the current period sharing a common type, \((b, h, y)\). Notice that the absence of insurance against the idiosyncratic transactions cost implies these households exit the period with differences in their wealth, \(b + y - \xi + q^h (h - h_1)\). As a result, the optimal house size, \(h_1 (b, h, y, \xi)\), varies across this group of households. Those with lower costs will have larger non-housing wealth remaining after the sale of their house, and thus tend to buy larger houses. This aspect of our current model distinguishes it from previous limited participation studies wherein all households becoming active at a given date adopted a common target value for the individual state variable in question, regardless of their type. Here, by contrast, the target house size is not the same even for the group of households that are ex-ante identical and vary only in their iid transaction costs. As such, our current model does not exhibit the uniformity of action levels present in Dotsey, King and Wolman (1999), Thomas (2002), King and Thomas (2006) and many related studies.

Beyond the elimination of uniformity of action levels, the absence of insurance against transaction cost draws and idiosyncratic income differences in our model implies that individual differences will persist. Thus, we also forfeit the finite memory property (a finite number of distinct types in the distribution entering the economy’s aggregate state vector) inherent in the studies mentioned above, as well as in Khan and Thomas (2007) and the perfect risk-sharing model described in the previous section. This means that our model is also distinguished by the fact that its aggregate state cannot be described simply in terms of vectors with lengths corresponding to the maximum number of periods between adjustments. Hence, standard linear methods cannot be adapted to the study of our environment.

In closing this section, we must specify what happens when households not surviving a given period exit the economy. Each period, as fraction \(1 - \pi^s\) of all households leave the economy, they are replaced by an equal number of young households. We assume that there are no bequests, so young households are born with zero bond account balance and no house. Thus, there is no inheritance of property, and the residences of exiting households are not redistributed to surviving households. With these assumptions set out, we are now in a position to describe competitive equilibrium.

### 2.2.2 Competitive equilibrium

Let \(\mu (b, h, y)\) define the beginning of period distribution of households over bond holdings, houses and income. The aggregate state is \(\mu\) alongside an exogenous shock, \(z\), that evolves over time according to a Markov Process, \(Q (z, z')\). We assume that the aggregate supply of non-durables is given by \(Y (z)\) and that gross residential investment goods are exogenously supplied as \(Y^h (z)\). Given \(b_t (S)\), a recursive competitive equilibrium is a set of functions, \((c, g, \xi^T, h_1, q, q^h, V_0)\), \(\Gamma\) satisfying the following conditions.

1. \(V_0\) solves (5) - (7).
2. Given $V$, and $W$ defined by (6), $(c, g)$ are the associated policy functions that attain the optimum, while $h_1$ attains the optimum in (8) and $\xi^T$ solves (9).

3. The markets for nondurable consumption goods, bonds and houses clear.

4. Individual decision rules are consistent with the aggregate law of motion for $\mu$, $\Gamma$.

3 Results

At this preliminary stage, we eliminate variability in endowment income and assume $y_i = y$ for all households to explore two illustrative examples of our model. Despite the absence of income risk, the model generates a rich distribution of housing and non-housing wealth. Several forces continue to drive differences across households in this special-case setting, thus influencing the economy’s endogenous wealth distribution. First, the level of housing capital affects the marginal utility of nondurable consumption and thus influences the savings behavior of households. Second, transactions costs drive differences in mid-period wealth among households currently active in the housing market (hereafter, active households). Third, as households age, they accumulate assets.

3.1 Parameterization

We may view income risk in our current setting as effectively captured by random differences in transactions costs. While our model is designed to allow for additional sources of income variation, we can isolate the effect of segmentation in relatively simple examples with a two-dimensional distribution of wealth, eliminating one household-level state variable, by abstracting from such uncertainty here. In these examples, we assume that the length of a period is one year, and we set the average annual real interest rate at 4 percent. The survival probability for any household is set at 98 percent. This implies an expected remaining lifespan of 50 years for adult households and, assuming that households form when their adult members are of age 20, an overall lifespan of 70 years.

As mentioned above, an important difference between our current work and previous general equilibrium studies of housing is that, subject to the payment of the transactions costs, our active households face a continuous choice in selecting their next residences. Thus, in our model economy, the lumpiness evident in individual behavior is endogenously driven by transactions costs and the resulting infrequent adjustments to housing. In specifying our households’ preferences over nondurables and housing, we adopt the following CES utility function.

$$U(c, h) = \left( \left( (1 - \omega) \frac{c^{\frac{1}{\sigma}}}{\tau} + \omega h^{\frac{1}{\sigma}} \right)^{\frac{1-\sigma}{\tau-1}} \right)^{1-\sigma}$$
Our momentary utility function above is the same as that considered by Piazessi, Schneider and Tuzal (2007). Thus, we may use some of their empirical observations to guide our parameter selection. Examining data between 1929 and 2001, Piazessi, Schneider and Tuzal find that the share of non-housing consumption has changed over time, but its variance is small. At the aggregate level, they recover a correlation between $c$ and $h$ at 0.75; this suggests that, at least for a representative household, the elasticity of substitution should exceed 1. Finally, they also find that, in the aggregate, the CEX data suggest that expenditure shares are not volatile over time. They estimate $\varepsilon$ to be close to, but above, 1.\footnote{In their home production model, McGrattan, Rogerson and Wright (1997) estimate $\varepsilon = 1.75$, while Ogaki and Reinhart (1998) place the elasticity of substitution between 1.04 and 1.43, based on aggregate consumer durables data.} In our baseline parameter set, we adopt their benchmark value, $\varepsilon = 1.05$. (We have also explored a parameterization using their alternative value, 1.24; for the examples we report here, the results are similar.) Given that we do not explicitly model the production of housing services here, the share of housing must be set fairly low; otherwise, because houses are long-lived assets, the model would predict very large house-value to consumption ratios among households. We normalize the relative price of housing in the steady state to 1, and thereafter set $\omega = \frac{1}{4}$ so as to imply a reasonable aggregate ratio of housing wealth to consumption.

We assume that transactions costs are uniformly distributed between 0 and an upper bound representing 40 percent of steady-state annual income. While this upper bound may seem large, note that the distribution $G$ implies that the typical active household pays a far lower transactions cost when buying a house. It is also important to remember that randomness in transactions costs is the only source of heterogeneity in this example; thus we are implicitly subsuming all idiosyncratic risk into this single shock. Given the transactions cost distribution, we assume a depreciation rate of 10 percent in order to generate a plausible degree of segmentation in the housing market. A more realistic value would be somewhat lower, but a lower $\delta$ would require additional sources of idiosyncratic risk to ensure sufficient turnover in houses. At present, our depreciation rate is a temporary stand-in for such additional shocks; we could alternatively assume that individual incomes grow over time.

We solve for the equilibrium $q$ that implies that bonds are in zero net supply in the steady state. As noted above, the equilibrium real interest rate is 4 percent, unadjusted for survival probability. Taking into account mortality risk, the effective real interest rate available to borrowers and lenders is 6.75 percent. Given our choice of $q_h = 1$ in the steady state, the parameterization adopted here implies an average aggregate ratio of housing capital to consumption near that in the data, at roughly 1.35. On average, a household moves every 5 years in our model economy, which is also empirically reasonable. Thus, while our environment is somewhat stylized, it does capture the appropriate magnitude of housing in the economy as well as the lumpy nature of stock adjustments at the individual level.
3.2 Solution

The task of computing dynamic stochastic general equilibrium in our model is somewhat involved given our inclusion of discrete choices among risk-averse households alongside uninsurable risk. As has been mentioned above, the model is related to Khan and Thomas (2007) in that risk-averse households make discrete choices. However, as has also been noted above, there are some important differences here that necessitate an entirely different solution method. In particular, because we allow for uninsurable idiosyncratic risk that generates persistent differences in wealth, the distribution of wealth affects not only the extensive margin (the fractions of households becoming active), but also the intensive margin, the choices of new housing among those active. To cope with the increased heterogeneity in our model, we adopt methods similar to those we previously used to search for nonlinearities in aggregate investment arising from discrete choices among firms differing in their capitals and relative productivities (Khan and Thomas (2008)). However, we must further extend these methods to accommodate our shift from a setting where discrete choices are made by risk-neutral firms to one where risk-averse households make such choices in the face of uninsurable risk. In our current application, we must confront the fact that the threshold adjustment cost, $\xi^T(b,h)$, is determined by an implicit equation, as well as the fact that the target house size, $h_1(b,h,\xi)$, varies not only with exogenous shocks but also net wealth.

Given the (S,s) nature of household decisions, we use multivariate spline interpolation to approximate household value functions between gridpoints in solving our model, since these are smoother objects than the decision rules. Figure 1 illustrates the value functions $v(b,h)$ and $w(b,h)$ in the steady state over the knot points used for the bivariate splines. As elsewhere, we adopt spline interpolation here to improve the accuracy of our solution; given the splines, we need not restrict households’ choices to a finite set of points. Nonetheless, as we aggregate these choices in the process of computing recursive equilibrium, we store the distribution of households using a two-dimensional grid with 9000 points. Many points on this grid have zero mass; decisions are set to 0 at these irrelevant points.

3.3 Steady state

Figure 2 illustrates the end of period bond holdings among active households drawing $\xi^T(b,h)$, the highest adjustment cost tolerated by members of their type $(b,h)$, and for inactive households, in the model’s steady state. The horizontal $y$–axis measures start-of-period house size, the horizontal $x$–axis is beginning of period financial wealth, and the vertical axis is the corresponding end of period saving (or borrowing) in bonds. Note that, among households of any given $(b,h)$ type, those that are active have lower savings, or more borrowing, than those that are inactive. This is a reflection of the lumpy expenditure that an active household chooses to undertake when adjusting its housing capital. Although the current purchase is partly funded by the sale of an existing residence, most active households replace their houses with larger ones. This is in part a response to anticipated depreciation
in their stock over coming periods and in part because, on average, households accumulate wealth over time.

Older households tend to begin the period with positive bond holdings. Conditional on age, households with larger residences are usually those that have more recently purchased a house and are holding lower bond wealth. Households generally remain in the same house for several periods (5 on average), during which time the house depreciates. Throughout such episodes of inactivity, households accumulate bonds to help finance their next move. As such, individual house size and financial wealth are negatively correlated during episodes when a household’s address remains the same. Nonetheless, as we shall see below, this correlation becomes positive when we look across households.

Figure 3 illustrates the steady state adjustment hazard determining the probability that any given household will participate in the real estate market in the current period, conditional on its type, \((b, h)\). Overall, 20 percent of households are active in the market each year. From the figure, we see that wealthier households are more likely to participate, and that, given wealth, those in smaller residences are more likely to buy a new one. As depreciation pulls \(h\) downward relative to \(b\), the ratio of nondurables consumption relative to housing grows ever larger. Corresponding to this, as their durations of inactivity extend, households become more and more willing to absorb any given level of transactions cost to adjust their housing stocks.

Across all levels of financial wealth, individuals with the largest houses are unwilling to pay substantial transactions cost to move. Thus their probabilities of adjustment are very low. Notice that bond holdings are measured on the right axis of figure 3, and consider the leftmost curve in the figure, beginning around \(h = 0.33\) and \(b = -0.475\). This curve reflects the population of households most recently active in the real estate market, and reveals that the chosen house size rises with financial wealth. Near the top of the curve, we see older households who had accumulated more wealth by last period, and thus could afford a larger house, relative to the younger households near the bottom. Looking rightward from this curve, we have a series of successive curves describe the housing market participation probabilities for households that last moved 2, 3, and more years ago. Over time, as a household shifts from one curve to the next, its bond holdings grow while its unadjusted stock of housing decays; thus, the strong positive relation between financial wealth and housing disappears. Finally, notice that the maximum time between housing adjustments is more than twice the 5-year mean duration.

The stationary distribution of households over house size and bond holdings is plotted in figure 4. There, we see that older households with large houses tend to carry high levels of \(b\), while young households tend to carry very negative values of \(b\). Although they begin their lives with no bonds and no housing, almost all young households pay their transactions costs and purchase a house. To do so, however, they must take on large mortgages. Thus, young households tend to be at the bottom edge of each time-since-active curve, clustered around the most negative values of \(b\). Thereafter, as they age, these households accumulate
bonds. Hence, as any one of them shifts rightward from one time-since-active curve to the next while its initial house depreciates, its location on each subsequent curve grows ever higher.

Notice that, along each curve, households at the boundary associated with the lowest levels of $b$ also have the smallest $h$. As explained above, these are mostly young households. However, they are joined in this region by older households who have suffered very large transactions costs. As we look upward along any one curve, we see both $b$ and $h$ rising. We have noted before that most older households are wealthier, and they live in larger houses. Moreover, given their high levels of wealth, they will once again purchase large houses when they next choose to move. That noted, the differences in house size among a group of households that last moved at the same date are not very large in this example. They would be greater if the differences in wealth arising here from differing ages and transactions costs were accompanied by those arising from differences in incomes. Nonetheless, if our environment allowed households infinite lives and full insurance against their transactions costs, the rich distribution shown in our current figure would be replaced by a single-line from the bottom left corner to the top-right. The dispersion we observe in house size and bond holdings is driven by transactions cost risk, alongside the borrowing constraints that prevent young households from borrowing as much as they would like against their future income. Interestingly, the young do not borrow all the way to their limit, because to do so would eliminate the possibility of increasing debt in the event of a very low effective income level in the future.

### 3.4 Dynamic examples

Moving from the steady state analysis above, we next examine how limited participation can alter the dynamics of house prices following aggregate shocks. For the moment, we further simplify our model environment to undertake a preliminary exploration of the implications segmented housing markets may have for real estate price volatility. In particular, we study a time-dependent adjustment version of the economy where the adjustment hazard governing household market participation rates is assumed to remain fixed at the steady state depicted in figure 3 throughout every period. Our findings here serve as a temporary stand-in for general results to come, where the endogeneity of adjustment choices in response to shocks will be reinstated. Nonetheless, as will be explained below, they provide strong evidence that segmented markets will have important implications for the dynamics of house prices in our full model.

To develop a time-dependent adjustment version of the model, we use figure 3 to compute an average adjustment hazard derived from the steady state of our full model wherein households follow optimal generalized $(S,s)$ policies in determining the timing of their moves. Next, we impose this hazard in an otherwise identical model to our own, but one where households face exogenous probabilities of being able to participate in the housing market, with these
probabilities rising in a household’s time-since-last adjustment exactly as in figure 3. In this
time-dependent adjustment setting, we consider two sets of impulse responses, one following
a purely transitory rise in the supply of new houses and one following a persistent rise in
housing supply. Each exercise is, for now, partial equilibrium in nature in that we maintain
the real interest rate for bonds, \( \frac{1}{q} - 1 \), at its steady state value and solve for the value of \( q_h^t \)
that clears the housing market inside each period.

We first study the dynamic response of our time-dependent model to a purely transitory
1 percent rise in the supply of new houses. Because there is limited participation in the real
estate market, we know that the entire rise in available housing must be absorbed by the
subset of households active in the market at the date of the shock. This means that each
active household must buy a larger house than usual. To induce households to purchase these
larger houses, given their wealth, the relative price of housing clearly must fall. However,
the necessary adjustments in house prices do not end when the stock returns to normal in
the next date, as may be seen in figure 5.

Figure 5 shows the deviation of the relative price of housing from its steady-state value
following the transitory supply shock, plotting the ratio \( \frac{q_h^t}{q_h^0} \) where \( q_h^0 \) is the steady state
relative price. At the impact date, note that the drop in the house price is 1.5 times the size
of the shock. This amplification comes from the fact that a small fraction of all households
must initially absorb the entire rise in housing supply. As suggested above, these active
households require a large downward movement in the relative price of housing to induce
them to purchase the disproportionate quantities of housing implied. In the next period,
although the supply of houses has returned to its steady state level, our nontrivial distribution
of households over \((b, h)\) continues to propagate the effects of the shock through the economy.
At date 2, notice that house prices actually rise as far above trend as did the original supply
shock. Looking across the two dates, we see that the one period growth rate is 2.5 times the
initial shock to quantity.

The echo effect seen in date 2 is a familiar result in \((S, s)\) adjustment settings, stemming
entirely from the fact that adjustments at the impact date of the shock alter the subsequent
period’s distribution of households relative to its long-run shape. Entering date 2, those
households that were active at the time of the shock each hold unusually large housing
capital relative to that held by households of time-since-last-active 1 in ordinary periods. As a result, the house size of the typical household active in this year after the shock must be smaller than usual. In equilibrium, this requires a rise in the relative price of housing to reduce the target size among households buying houses in this date. In subsequent periods,
as the distribution of households slowly returns to its average shape, we see additional,
dampened responses in house prices. As this suggests, the non-monotone response in house
prices is not an artifact of the transitory nature of the shock.

Figure 6 shows the response in prices following a persistent rise in the housing supply
of 0.5 percent. Here, we set the persistence of the shock equal to \( 1 - \delta_h \) so as to capture a
one-time shock to residential investment whose effects on the housing stock decay over time.
through depreciation. For reasons noted above, the response in the relative house price at the impact of this shock is twice as large as the shock itself. In contrast to our previous example, however, there is no perceptible rise in house prices for roughly three periods. Over these periods, the supply of houses remains high while a relatively small fraction of households enter the market to purchase a new residence.

The cumulative effect of moves undertaken throughout the first few periods following the shock is a significant fraction of households holding unusually large housing stocks. As a result of this reshaping of the household distribution, we see a sharp rise in house prices in period 5, despite the fact that supply remains above its steady state level. This episode of above-average house prices persists for several periods. Thereafter, beginning with the drop in house prices in period 8, we see the cycle repeating itself in a dampened fashion. Note that, over the entire 15 periods shown in figure 6, the quantity of housing is above its steady state and monotonically reverting to that level. Despite this smooth response in quantities, house prices shift between periods when they co-move with quantities, and periods when they move oppositely. As a result, the correlation between the quantity of housing and its relative price falls to $-0.3$. Moreover, the large swings in house prices seen here imply a standard deviation that is 4 times the changes in quantity.

In our discussion above, we have noted a repeated cycle in house prices arising from the limited participation in our model, wherein initial periods of below-trend prices, followed by several dates of above-trend prices, are echoed by a subsequent series of below-trend prices, and so forth. These episodes are largely driven by the consumption smoothing behavior of households active at the time of the shock, and their duration is roughly determined by the average number of periods a typical household spends in a given house, delaying its next move. As they return to the real estate market, households active at the date of the shock (and those active in the subsequent two periods) seek to maintain unusually high housing stocks in effort to spread the benefits of the shock out over their lifetimes. Given the monotonic return of the housing supply toward steady state, this can be achieved only through further adjustments in the relative price of housing. On the whole, we see that the correlation between house price movements and the supply of houses is low following the persistent shock to supply, and this happens as the result of rich dynamic responses in the distribution of houses. Furthermore, just as we saw in the case of the transitory shock above, here too the response in the house price series exhibits considerable and long-lived volatility, substantially exceeding the supply shock that initiated it.

We close this section by commenting on how the suggestive exercises above are likely to compare to the dynamic responses we will ultimately uncover in our full dynamic stochastic general equilibrium model with state-dependent household participation rates. In one respect, by examining a partial equilibrium setting, the exercises seen here likely overstate the overall volatility in house prices generated by our model. When interest rates are allowed to adjust in response to either supply shock considered above, the wealth effect on nondurables consumption should imply an interest rate drop at the date of the shock. This, in turn, will
make it less difficult to induce households participating in the real estate market at the date of the shock to absorb the extra housing stock. Thus, we expect that the initial drop in the relative price of housing, and hence the subsequent echo effects, will be dampened relative to what we have seen here.

At the same time, however, our exploration of the special case time-dependent adjustment setting where household participation rates are not allowed to change over time likely understates our model’s ability to propagate shocks. This observation is derived from our experience with models of limited participation in asset markets, where we have seen substantial dynamic changes in moving from an setting with a fixed set of time-dependent adjustment rates versus a setting where households optimally choose the timing of their adjustments in response to both individual and aggregate conditions. In Khan and Thomas (2007), we found that endogenizing market segmentation introduced echoes through the distribution of households over time that themselves induced persistent changes in real interest rates following even a purely transitory shock to the money growth rate. This persistence was entirely absent in an otherwise identical time-dependent adjustment economy where participation rates were fixed at the steady state of the endogenous segmentation model. By contrast, we have seen above that a time-dependent variant of our current model is capable of protracting the adjustments in house prices following a shock to the supply of housing. While we anticipate the amplitude of responses will be reduced in our full DSGE model, allowing time-variation in household participation rates should only serve to strengthen the propagation we have seen here.
References


Figure 1: the value function $v(b,h)$ and $w(b,h)$.
Figure 2: decision rules for current adjustors with $i(x) = 20$

decision rules for nonadjustors
Figure 4: distribution of households over houses and financial wealth
Figure 5: Response in house price to a transitory shock

House price relative to steady state price
Figure 6: Response in house prices to a persistent shock