What is the Relationship between Competition and Productivity?

Yan Bai† and Berthold Herrendorf‡
(Arizona State University)

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Abstract
What is the relationship between the degree of competition in an industry and its average productivity? We study this question in a version of the Spence–Dixit–Stiglitz model with many industries, which each produce finitely many varieties of a different good. We assume that firms pay a fixed entry cost to become a monopolist for one variety and that they make a costly productivity choice. We show that the relationship between competition and productivity depends critically on why competition differs across industries. If competition in an industry is larger because it has a larger market, then the firms in the industry choose higher productivity. If competition in an industry is larger because entry costs into it are lower, then the firms in the industry choose lower productivity. We use these results to shed light on the findings of the empirical literature.

Keywords: Competition; Mark Ups; Productivity

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†Email: Yan.Bai@asu.edu
‡Email: Berthold.Herrendorf@asu.edu
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Appendix
1 Introduction

What is the relationship between the degree of competition and average labor productivity? Perhaps surprisingly, the profession has not reached a consensus about this classic question. Starting with Schumpeter (1943) and Arrow (1962), the theoretical literature has disagreed whether more competition in an industry decreases or increases average productivity in the industry. Recently, Vives (2008) and Schmutzler (2007) have even concluded that theoretically there is no hope to establish a robust relationship. The empirical literature has also disagreed. While studies like Syverson (2004), Schmitz (2005), and Lagakos (2007) found a positive relationship between competition and productivity, Aghion et al. (2005) found an inverted U shape between competition and the number of patents, which they take as a proxy for productivity.

In this paper, we revisit the relationship between the degree of competition and average productivity in a cross section of industries. We study the two most obvious reasons why competition differs across industries: differences in the market sizes of the industry or differences in the entry costs into an industry. We show that the answer to the question of this paper depends critically on which of the two reasons drives the differences in competition.

We use the Spence–Dixit–Stiglitz (SDS) model, because it has become the workhorse for modeling deviations from perfect competition in macroeconomics.¹ We develop a version in which the price elasticity of demand responds to changes in market size and entry costs. Specifically, we assume that there is a measure one of goods and that individuals have log preferences over them. This implies that the market size of each good is a parameter that we can vary exogenously. We also assume that each good comes in finitely many varieties and that individuals have CES preferences over these varieties. Since there are finitely many varieties of each good, CES preference imply that the price elasticity of demand for each good increases in the number of varieties. Lastly, we assume that firms pay an entry cost to become the monopolist for one variety. After they have entered,

¹See Spence (1976) and Dixit and Stiglitz (1977).
firms make a costly choice of their productivity.

We show that there is a unique symmetric subgame–perfect Nash equilibrium. The equilibrium the number of varieties of each good is finite. We call an industry the set of firms that produce the varieties of a good and we focus on parameter values for which each industry has at least two firms. We employ the two measures of competition that are most commonly used in the empirical literature: the inverse of average mark ups (“Lerner index) and the inverse of the two–firm concentration ratio in an industry.\(^2\) We show that in our model both measures of competition report that an industry with larger market size or lower entry costs has a higher degree of competition. The reason is that a larger market size or lower entry costs both increase the number of firms in the industry. This decreases the two–firm concentration ratio. Since there are finitely many firms in the industry, the increases in the number of firms also increases the price elasticity of demand for each firm’s output. This decreases the mark up in the industry.

We show that the relationship between competition and productivity across industries depends critically on why competition differs. On the one hand, if a larger market size leads to more competition in an industry, then the firms of this industry choose larger productivity. In this case, there is a positive relationship across industries between competition and productivity across industries. On the other hand, if lower entry costs lead to more competition among the firms in an industry, then the firms choose lower productivity. In this case, there is a negative relationship between competition and productivity.

The intuition for our results is as follows. Consider first an increase in the market size of an industry. As a result, additional firms will enter into the industry. This increases the price elasticity of demand, which decreases the mark up and the price that each firm chooses. To satisfy the zero–profit condition, each firm must have higher output. Having higher output increases the marginal benefit of choosing a higher productivity while leaving the marginal costs unaffected. Thus, the firms in the industry choose higher

\(^2\)The literature considers anything between two– and n–firm concentration ratios. Since in symmetric equilibrium, all firms in an industry make the same choices, here using the two–firm concentration ratio is without loss of generality.
productivity. Consider instead the implications of lower entry costs into an industry. As a result, additional firms will enter into the industry, which leads to two opposing effects. First, given the same market size, a larger number of firms decreases the output per firm. Second, a larger number of firms increases the price elasticity of demand, which increases the output per firm. It turns out that if there are at least two firms in the industry, then the first effect dominates and the marginal benefit of choosing a higher productivity decreases while the marginal costs remain unaffected. Thus, the firms in the industry choose lower productivity.

Our work is closely related to that of Desmet and Parente (2006). They departed from the observation that the SDS model with a continuum of varieties has a constant elasticity of demand, which right from the start shuts down effects that work through changes in the elasticity. In reaction to this, Desmet and Parente turned away from the SDS model and instead studied the Lancaster (1979) model with costly productivity choice. They showed that a larger population implies a larger number of firms, a higher elasticity of demand, and higher productivity. While these results are similar to our results on the effects of market size, there are two important differences. First, we obtain our results in the SDS model with finitely many variety instead of a continuum of varieties. Although this version is harder to analyze than the version with a continuum of varieties, it is much easier to analyze than the Lancaster model. Second, we study the effects of changes in market size separately from those of changes in the entry costs. In contrast, Desmet and Parente studied only the effects of changes in the population. Interestingly, we find that the two changes have the opposite implications for the relationship between competition and productivity.

Our work is also closely related to that of Jaimovich and Floetotto (2008), who used the SDS model with finitely many varieties to generate counter-cyclical mark ups. The key channel in their paper is the same as here: with finitely many varieties, an increase in the number of firms increases the price elasticity of demand, and so it decreases mark ups. The main difference is the focus: While we are interested in the implications of
changes in the price elasticity of demand for the choice of productivity in a cross-section of industries, they were interested in the implications for the time series of mark ups over the business cycle.

We demonstrate the usefulness of our results by using them to shed light on the existing empirical evidence. In particular, we argue that the studies of Syverson (2004), Schmitz (2005), and Lagakos (2007) are examples of how differences in the market size affect productivity. Our results are consistent with the positive relationship between competition and productivity that these studies find. In contrast, the study of Aghion et al. (2005) is an example of how differences in entry costs affect productivity. Our results are consistent with the inverted U shape relationship that this study find.

The rest of the paper is organized as follows. In Section 2, we present the environment and define the equilibrium. In Section 3, we show the existence and uniqueness of equilibrium. Section 4 characterizes the relationship between competition and productivity when changes in productivity result from changes in industry markets size or from changes in the entry costs into an industry. We discuss the empirical evidence in light of these results in Section 6. ... We conclude in Section 8.

2 Model

2.1 Environment

There is a measure one of goods indexed by $i \in [0,1]$. Good $i$ comes in finitely many varieties indexed by $n_i$ where $n \in \{1, ..., N_i\}$.

There is a measure one of individuals. Each individual is endowed with one unit of labor. Preferences are described by the utility function

$$\int_0^1 \alpha_i \log(C_i)di,$$ (1)
where

\[ \int_0^1 \alpha_i di = 1, \]

\[ C_i \equiv \left( \sum_{n=1}^{N_i} \frac{c_{ni}^{\sigma-1}}{\sigma} \right)^{\frac{\sigma}{\sigma-1}} \]

and \( \sigma > 1. \)

For each good, there are many potential firms. If a firm pays an entry cost of \( \kappa_i \) labor units, then it becomes the monopolist for a variety of good \( i \). We refer to the set of firms/varieties of good \( i \) as industry \( i \).

The production function for a variety is given by:

\[ y = Al \]

where \( y \) is output, \( A \) is (labor) productivity, and \( l \) is labor. Producing with productivity \( A \) requires paying an adoption cost of \( \phi A^\psi \) labor units where \( \phi, \psi > 0. \)

We can think of entering into an industry as a product innovation that creates a new variety at the costs \( \kappa_i \). We can think of adopting of a higher \( A \) as a process innovation that reduces the marginal costs \( 1/A \) at the costs \( \phi A^\psi \).

### 2.2 Equilibrium Definition

There are three stages:

1. firms choose whether and which industry to enter
2. for each industry, the entrants choose their productivity, price, and labor
3. individuals choose their consumption.

---

3Note that for \( A = 0 \) the adoption cost is zero, it increases in \( A \), and it is convex. For most of our results, we could use other adoption cost functions with these properties. However, for the derivations they would not be as convenient as our adoption cost function.
The equilibrium concept is symmetric subgame–perfect Nash. In particular, the firms of a given industry choose the same productivity, price, and labor taking as given the wage and the choices of the other firms in the industry. One might think that monopolistic competition would be the natural equilibrium concept for our environment. However, since there are finitely many varieties in each industry each firm’s choice affects the demand for the output of the other firms in the industry. So, one key characteristics of monopolistic competition does not hold in our environment.\(^4\)

We start with stage 3. Let the wage be the numeraire, \(w = 1\). Taking prices as given, individuals solve

\[
\max_{\{c_n\}} \int_0^1 \alpha_i \log \left( \frac{\sum_{n=1}^{N_i} c_n^{\sigma-1}}{\sum_{n=1}^{N_i} c_n^{\sigma-1}} \right) di \quad \text{s.t.} \quad \int_0^1 \sum_{n=1}^{N_i} p_n c_n di = 1 + \int_0^1 \sum_{n=1}^{N_i} \pi_{n} di. \tag{2}
\]

where \(\pi_{ni}\) denotes the profit of firm \(ni\). The solution implies a demand function for each variety. The assumption of log–preferences implies that the individuals spend a share \(\alpha_i\) of their expenditures on varieties of good \(i\).

We continue with stage 2. Consider a particular firm that entered industry \(i\). We denote its price by \(p_i\) and the prices of the other \(N_i - 1\) firms by \(P_i\). Moreover, we denote by \(c_i(p_i, P_i)\) the demand function for the variety produced by the particular firm. Taking as given \(w = 1\), \(P_i\), and \(c_i(\cdot)\), the particular firm solves:

\[
\max_{A_i, p_i, l_i} \pi_i \equiv p_i c_i(p_i, P_i) - \phi A_i^\phi - l_i \quad \text{s.t.} \quad c_i(p_i, P_i) = A_j l_i. \tag{3}
\]

We finish with stage 1. Free entry implies that the equilibrium number of firms \(N_i\) satisfies two conditions: if \(N_i\) firms produce in industry \(i\) each firm’s profit from stage (2) is larger or equal to the entry costs, \(\pi_i \geq \kappa_i\); if \(N_i + 1\) firms produced in industry \(i\), then each firm’s profit from state 2 would be smaller than the entry costs.

\(^4\)For a definition of monopolistic competition, see for example page 287 of Tirole (1988).
Lastly, we turn to market clearing. Given that the entry costs \( \kappa_i \) and the adoption costs for \( A_i \) are in labor units, labor–market clearing requires:

\[
\int_0^1 N_i [\kappa_i + \phi A_i^e + l_i] di = 1. \tag{4}
\]

Since all firms are monopolists for their variety, we do not have impose that the goods markets clear.

**Definition 1**

A symmetric, subgame perfect Nash equilibrium is

\[
\{ N_i, P_i, A_i, l_i, c_i \}_{i \in [0,1]}
\]

such that

- \( \{ c_i \}_{i \in [0,1]} \) solves the individual problem (2)
- \( (A_i, P_i, l_i) \) solve problem (3) of a particular firm in industry \( i \)
- \( N_i \) satisfies the free–entry condition
- the labor market clears, i.e. (4) is satisfied

### 3 Existence and Uniqueness of Equilibrium

To simplify matters, we allow \( N_i \) to be a real number instead of restricting it to be an integer. The zero–profit condition then holds with equality for all industries and the equilibrium number of firms is the largest integer that does not exceed the solution to the zero–profit condition.

We first solve the problem of an individual. To this end, we define the price index for industry \( i \):

\[
P_i \equiv \left( \sum_{n=1}^{N_i} \frac{x_n}{c_{ni}^e} \right)^{\frac{x}{x-1}} . \tag{5}
\]
The first-order conditions imply that:

\[
\frac{c_i}{C_i} = \left( \frac{P_i}{p_i} \right)^\sigma
\]

Substituting this back into the budget constraint with \(\pi_i = 0\), we get that \(P_iC_i = 1\). The individual demand function follows as:

\[
c_i = \alpha_i \frac{P_i^{\sigma-1}}{p_i^\sigma}.
\] (6)

Next, we solve the problem of a particular firm in industry \(i\). The first order-conditions with respect to \((p_i, A_i)\) imply:

\[
p_i = \frac{\epsilon_i}{\epsilon_i - 1} \frac{1}{A_i},
\] (7)

\[
\phi \psi A_i^{\psi-1} = \left( -\frac{\partial}{\partial A_i} \frac{1}{A_i} \right) c_i(p_i, P_i).
\] (8)

Condition (7) says that the price \(p_i\) is the monopoly mark up over marginal costs. Condition (8) says that the productivity \(A_i\) is such that the marginal cost of increasing \(A_i\) (left-hand side) equals the reduction in the unit cost times the number of units produced (right-hand side).

To ensure the existence of an interior equilibrium with at least two firms, we restrict the parameters.

**Assumption 1** The parameters of the model are such that

\[
\alpha_i > \kappa_i(\sigma + 1),
\] (9)

\[
\psi > \frac{\alpha_i(\sigma - 1)}{2[\alpha_i - \kappa_i(\sigma + 1)]},
\] (10)

We start characterizing the equilibrium by deriving the price elasticity of demand for varieties of good \(i\). The proof is delegated to the appendix.
Lemma 1 In symmetric equilibrium the price elasticity of demand for varieties of good $i$ is given by:

$$
\epsilon_i = \sigma - \frac{\sigma - 1}{N_i} \in [1, \sigma].
$$

(11) shows that the price elasticity increases with the number of firms in industry $i$. The reason is that with finitely many firms in each industry, each firm takes into account the effect that changing its price has on the price index of the industry. The more firms an industry has the smaller is this effect. In the limit, $\epsilon_i = \sigma$ and we are back to the standard SDS case with a continuum of firms in each industry.

Imposing zero profits and combining the first–order conditions (7)–(8), we find two equations that characterize the equilibrium $(A_i, N_i)$:

$$
\psi \phi A_i^\psi = \left[1 - \frac{1}{\epsilon(N_i)}\right] \frac{\alpha_i}{N_i},
$$

$$
\frac{\alpha_i}{N_i \epsilon(N_i)} - \phi A_i^\psi = \kappa_i.
$$

Simplifying, we obtain:

Lemma 2

The equilibrium $(A_i, N_i)$ is characterized by:

$$
A_i = \left[\frac{\kappa_i[\epsilon(N_i) - 1]}{\phi[\psi - [\epsilon(N_i) - 1]]}\right]^{\frac{1}{\psi}},
$$

$$
\alpha_i[\psi - [\epsilon(N_i) - 1]] = \psi \kappa_i \epsilon(N_i) N_i.
$$

Proposition 1

There is a unique equilibrium.

Proof. See the appendix.
4 What is the Relationship between Competition and Productivity?

4.1 Measuring competition

We use the two measures of competition that are most commonly used in the empirical literature: the inverses of average mark ups and of the two–firm concentration ratio.

The average mark up in industry \( i \) is defined as the percentage difference between average price and average marginal costs. In symmetric equilibrium, the inverse average mark up is given as:

\[
\mu_i^{-1} = \frac{A_i^{-1}}{p_i - A_i^{-1}}.
\]

Using (7), this becomes:

\[
\mu_i^{-1} \equiv \epsilon_i - 1.
\]

According to this first measure of competition, an industry is more competitive if it has a higher price elasticity of demand.

The \( n \)-firm concentration ratio is defined as the percentage of total sales in industry \( i \) that is accounted for by the \( n \) largest firms. Since in symmetric equilibrium, there are at least two firms in each industry and since all firms have have the same sales, we work with the two–firm concentration ratio. In symmetric equilibrium, the inverse of the two–firm concentration ratio is given as

\[
\text{CR}_i^{-1} \equiv \frac{N_i P_i c_i}{2P_i c_i} = \frac{N_i}{2}.
\]

According to this second measure of competition, an industry is more competitive if it has more firms.

As we showed in Lemma 1, in equilibrium the price elasticity is monotonically increasing in the number of firms. In our environment both measures of competition will therefore agree on whether a given industry is more or less competitive than another in-
dustry, so it does not matter which one we use. We recognize that in other environments these measures of competition may not be appropriate to use. For example, Holmes et al. (2008) study a discrete adoption choice under Bertrand competition and show that an increase in the degree of competition can lead to a discrete increase in mark ups. While this is interesting, it cannot happen in our environment with a continuous adoption choice.

In what follows, we will study the cross sections of industries in two economies. In the first economy, we exogenously vary the market size \( \alpha_i \) while keeping the entry costs \( \kappa_i \) the same. In the second economy, we exogenously vary the entry costs while keeping the market size the same.

4.2 Varying market size

Proposition 2 (Positive Relationship Between Competition and Productivity)
If \( \alpha_i > \alpha_{i'} \), then \( N_i > N_{i'} \), \( \epsilon_i > \epsilon_{i'} \), and \( A_i > A_{i'} \).

Proof. See the appendix.

In words, industries with larger market sizes are more competitive and have higher productivity. Therefore, the relationship between the degree of competition and average productivity is positive if differences in market size cause the differences in the degree of competition. Figure 1 illustrates this.

Intuitively, this can be understood as follows. An increase in the market size of an industry implies that additional firms enter. This increases the price elasticity of demand, which decreases the mark up and the price that each firm chooses. To satisfy the zero–profit condition, each firm must have higher output. Thus, the marginal benefit of choosing a higher productivity increases while the marginal costs remain unaffected and the firms in the industry choose higher productivity.

A different way of saying this is that the number of firms increases less than proportionally to the market size, so \( \alpha_i/N_i > \alpha_{i'}/N_{i'} \). To see this, suppose that the price elasticity of demand did not change. The number of firms would then increase proportionally to the the market size and \( \alpha_i/N_i = \alpha_{i'}/N_{i'} \). Now take into account that the price
elasticity increases in the number of firms. If it was still the case that $\alpha_i/N_i = \alpha_i'/N_i'$, then the firms in the industry would make negative profits. To satisfy the zero-profit condition, the number of firms must increase less than proportionally to the market size so that $\alpha_i/N_i > \alpha_i'/N_i'$.

The result of Proposition 2 is similar to the result of Desmet and Parente (2006). These authors studied how changing the population size on a Lancaster circle changes firms’ costly choice of productivity. They showed that larger population implies that firms choose higher productivity. There are two important differences between their and our work.

First, Desmet and Parente observed that in the standard SDS model with a *continuum* of varieties, differences in market size have no effect on productivity. The reason is that the price elasticity equals $\sigma$, which is the limiting case of our model when the number of firms in an industry goes to infinity. They concluded from this observation that SDS model is not helpful for studying their question, and so they turned to the Lancaster
model. We show that this conclusion does not follow if each industry has finitely many firms. In this case, the market size does have a positive effect on productivity. This is important because the SDS model has become the standard model of noncompetitive macroeconomics, and so it is well understood. Moreover, compared to the Lancaster model, our version of the SDS is considerably easier to solve and to generalize to dynamic applications.\footnote{In fact, in a first version of this paper, we did work with a Lancaster model. Although we derived similar results as in this paper, the first was considerably longer than the current one.}

Second, Desmet and Parente assume that the costs of entry and adoption equal $\phi \exp(A)$. This implies that if a firm chooses $A = 0$, then it pays the entry costs $\phi$. If it chooses productivity $A > 0$, then it pays the additional adoption costs $\phi[\exp(A) - 1]$. The limitation of this functional form is that the entry costs are closely tied to the marginal adoption costs. Our specification is more general in that the entry costs $\kappa_i$ are different from the marginal adoption costs $\psi \phi A^{\psi - 1}$. In the next subsection, we will vary $\kappa_i$ without changing the adoption costs. We will see that this leads to drastically different conclusions from those obtained by Desmet and Parente.

4.3 Varying entry costs

Proposition 3 (Negative Relationship Between Competition and Productivity)

If $\kappa_i > \kappa_i'$, then $N_i < N_i'$, $\epsilon_i < \epsilon_i'$, and $A_i < A_i'$.

Proof. See the appendix.

In words, industries with larger entry costs are more competitive and have lower productivity. Therefore, the relationship between the degree of competition and productivity is negative if differences in entry costs cause the differences in the degree of competition. Figure 2 illustrates this.

Intuitively, this can be understood as follows. Lower costs of entry into an industry imply that firms will enter into the industry, which leads to two opposing effects. First, given the same market size, a larger number of firms decreases the output per firm.
Second, a larger number of firms increases the price elasticity of demand, which increases the output per firm. It turns out that if there are at least two firms in the industry, then the first effect dominates and the marginal benefit of choosing a higher productivity decreases while the marginal costs remain unaffected. Thus, the firms in the industry choose lower productivity.

5 Extensions

5.1 Different measures of productivity

So far, we have defined labor productivity as output divided by production labor:

\[
LP_{1i} \equiv \frac{A_i l_i}{l} = A_i.
\] (16)
Alternatively, we could define labor productivity as output divided by the sum of the labor that goes into production and technology adoption:

\[ \text{LP}_{2i} \equiv \frac{A_i l_i}{l_i + \phi A_i^\psi} = \frac{1}{1 + \frac{\phi A_i^\psi}{l_i}} A_i. \tag{17} \]

Fortunately, our results would not change if we did this. To see this, we rewrite \( \text{LP}_{2i} \) by using (8) and that \( c_i = A_i l_i \):

\[ \text{LP}_{2i} = \frac{\psi}{\psi + 1} A_i. \]

Thus, \( \text{LP}_2 \) goes up if and only if \( \text{LP}_{2i} \) goes up.

We could also take into account the labor input into setting up the firms:

\[ \text{LP}_{3i} \equiv \frac{A_i l_i}{l_i + \phi A_i^\psi + \kappa_i}. \tag{18} \]

Zero profits imply that

\[ l_i + \phi A_i^\psi + \kappa_i = p_i A_i l_i = \frac{\epsilon_i}{1 - \epsilon_i} l_i. \]

Thus,

\[ \text{LP}_{3i} = \left(1 - \frac{1}{\epsilon_i}\right) A_i. \]

Since both \( \epsilon_i \) and \( A_i \) increase in \( \alpha_i \), a larger market size implies higher productivity also according to this third measure of productivity. Since \( \epsilon_i \) decreases in \( \kappa_i \) but \( A_i \) increases in \( \kappa_i \), however, the relationship between \( \text{LP}_{1i} \) and \( \text{LP}_{3i} \) is no longer monotonic when we vary \( \kappa_i \). We are not very concerned by this, because most empirical studies calculate labor productivity according to the other two measures. Moreover, in dynamic environments, \( \kappa_i \) would only be paid by the firms that enter in the current period, so \( \text{LP}_{1i} \) or \( \text{LP}_{2i} \) would still be the correct measure of labor productivity for the firms that entered in previous periods.
5.2 SDS model with a continuum of varieties

It is instructive to compare our results with those from the SDS model with a continuum of varieties/firms in each industry. Each firm then is small and takes the price index in its industry as given. As a result, the price elasticity of demand equals \( \sigma \) and is independent of the number of firms. We can think of this case as the limit of our model when the entry costs converge to zero and the number of firms in each industry converges to infinity; compare (11). It is straightforward to show the following results for the SDS model with measures of firms in each industry.

Increasing the market size of an industry increases the number of firms proportionally while leaving each firm’s output the same. Thus, it has no effect on the productivity in the industry. With regards to the degree of competition, our two measures give different answers: the inverse mark up reports that the degree of competition remains unchanged, as the elasticity remains unchanged; the inverse two–firm concentration ratio reports that competition increases, as the number of firms increases. So, depending on the measure of competition, we would conclude that changes in market size neither affect the degree of competition nor productivity or that changes in market size increase competition but do not affect productivity. This is different from the case of finitely many firms we studied above, in which increasing the market size of an industry increases the degree of competition and productivity in that industry irrespective of the measure of competition.

Increasing the entry costs into an industry in the SDS model with a continuum of firms decreases productivity. The reason is the same as in our model. The market size remains the same while the number of firms increases. Since the price elasticity of demand remains unchanged, each firm produces fewer units. Therefore, the marginal benefit from increasing productivity falls while the marginal cost remains the same. With regards to the degree of competition, however, the two measures again give different answers: the inverse mark up reports that the degree of competition remains unchanged, as the elasticity remains unchanged; the inverse two–firm concentration ratio reports that competition increases, as the number of firms increases. So, depending on the measure
of competition, we would either conclude that changes in the entry costs do not affect
the degree of competition but reduce productivity or that they increase the degree of
competition and reduce productivity. Again, this is different from the case of finitely
many firms in which increasing the entry costs of an industry increases the degree of
competition and decreases productivity in that industry irrespective of the measure of
competition.

5.3 CES preferences over goods

In this subsection, we show that our results do not depend on the assumption of log–
preferences over the goods. To this end, we replace (1) by the more general utility
function

\[ \left( \int_0^1 \alpha_i C_i^\eta \frac{\eta - 1}{\eta} \, di \right)^{\frac{\eta}{\eta - 1}}. \]

where \( \eta \in (0, \sigma] \). We do not consider \( \eta > \sigma \) because that would mean that the substi-
tutability is larger between different goods than between the varieties of a given good. This does not make sense.

We can solve the consumer problem as a two–step problem. The first step is to choose
the consumption of the goods

\[ \max_{C_i} \quad C = \left( \int_0^1 \alpha_i C_i^\eta \frac{\eta - 1}{\eta} \, di \right)^{\frac{\eta}{\eta - 1}} \quad \text{s.t.} \quad \int_0^1 P_i C_i \, di = 1. \]

This gives the demand function for goods \( i \):

\[ C_i = \alpha_i^{\eta} \frac{C_i^{1-\eta}}{P_i}. \] (19)

The second step is to choose the varieties of a given good:

\[ \max \left\{ c_{ni} \right\} \quad \left( \sum_{n=1}^{N_i} c_{ni}^{\frac{\eta}{\sigma - 1}} \right)^{\frac{\sigma}{\sigma - 1}} \quad \text{s.t.} \quad \sum_{n=1}^{N_i} p_{ni} c_{ni} = P_i C_i. \]
This gives the demand for variety \( n \):

\[
c_{ni} = \frac{C_i P_i^\sigma}{p_{ni}^\sigma}.
\]

Plugging equation (19) into the above equation, we have

\[
c_{ni} = \alpha_i \eta C_{1}^{1-\eta} P_{i}^{\sigma-\eta} p_{ni}^{\sigma}.
\] (20)

This implies that in symmetric equilibrium

\[
\epsilon_i = \sigma - \frac{\sigma - \eta}{N_i},
\] (21)

\[
c_{ni} = \alpha_i \eta C_{1}^{1-\eta} N_i^{\sigma-\eta} p_{ni}^{\sigma},
\]

\[
P_i = N_i^{\frac{1}{1-\sigma}} p_{ni}.
\]

In order to ensure that \( \epsilon_i > 1 \), we assume that

**Assumption 2** The elasticities satisfy

\[
\frac{\eta + \sigma}{2} > 1.
\] (22)

In the Appendix, we prove that Proposition (2) and (3) still hold.

## 6 Empirical evidence

Our results are useful to shed light on the diverse empirical evidence about the relationship between competition and productivity. Some studies find that this relationship is positive while others find that it is an inverted U shape.

The result of Proposition 3 that a larger market size increase both competition and
productivity is consistent with several studies. For example, Syverson (2004) studied the ready–mixed concrete industry in U.S. and found that markets with higher demand have higher average productivity. Lagakos (2007) studied the retail sector and again found that markets with high demand have higher productivity than markets with low demand.

Our result that lower entry costs can increase or decrease productivity can help us understand the several empirical cases in which the relationship between competition and productivity does not monotonically increase. The deregulation of the U.S. banking sector provides perhaps the most interesting example. In the 1980s, the U.S. banking sector was very regulated: Most states had restrictions on the geographic areas in which banks could operate and all banks were prohibited from operating across state lines. This resulted in artificially low competition across markets. Within the framework of our model, this would be reflected by high entry costs.

Hannan and McDowell (1984) measured the degree of competition in U.S. banking with the three–firm market concentration ratio (CR3), which is the percentage of market output generated by the three largest firms in the industry. The higher this ratio the lower is competition. Hannan and McDowell found that ATM machines diffused faster in markets with less competition. This would be an example for Proposition 4.

Aghion et al. (2005) found that the relationship between competition and productivity has an inverted U shape for a large panel data set of U.K. firms. Our results can explain this. When an industry has very large entry costs, reducing them initially increases productivity because the Arrow effect dominates. When the entry costs have come down sufficiently, however, the Schumpeter effect starts dominating; productivity declines when entry costs fall further.
7 Discussion

8 Conclusion

To answer what is the relationship between competition and productivity, we have developed a model with Lancaster preferences and a costly productivity choice. We have shown that the relationship between competition and productivity depends critically on why competition differs. In particular, if a larger market size implies more competition in an industry, then industry productivity is higher. In contrast, if lower entry costs imply more competition, then industry productivity may be lower or higher and an inverted U shape relationship between competition and productivity may emerge. We have used these results to shed light on the diverse findings on the empirical literature on competition and productivity.

References


Appendix

In this appendix, we drop the industry index $i$ to lighten notation.

Proof of Lemma 1

Using (6), we find that in symmetric equilibrium the price elasticity of demand is given by:

$$\epsilon = \frac{\partial c}{\partial p} = \sigma - (\sigma - 1) \frac{\partial P}{\partial p} \frac{p}{P}.$$

Using (5), we find that

$$\epsilon = \sigma - (\sigma - 1) \left( \frac{P}{p} \right)^{\sigma - 1}.$$

In symmetric equilibrium with $p = P$, this becomes

$$\epsilon = \sigma - (\sigma - 1) \frac{1}{N}.$$

Proof of Proposition 1

We first prove equation (15) has a unique solution of $n \geq 1$. Equation (14) then has a unique solution for $A$. Define the function $M(n)$ as:

$$M(N) \equiv \alpha [\psi - [\epsilon(N) - 1]] - \psi \kappa \epsilon(N) N.$$

$M(N) = 0$ if and only if $N$ solves (15).
$M(\cdot)$ has the following properties:

\[
M'(N) = -\frac{\alpha(\sigma - 1)}{N^2} - \psi\kappa\sigma < 0, \\
M(2) = \psi[\alpha - \kappa(\sigma + 1)] - \frac{\alpha(\sigma - 1)}{2} > 0, \\
\lim_{N \to \infty} M(N) < 0.
\]

According to the Intermediate Value Theorem, there is a unique $N \geq 2$ such that $M(N) = 0$.

**Proof of Proposition 2**

The derivative of $N$ with respect to $\alpha$ is given by:

\[
\frac{\partial N}{\partial \alpha} = \psi - \frac{[\epsilon(N) - 1]}{M'(N)}.
\]

Since $\psi > \epsilon(N) - 1$ and $M'(N) < 0$, this implies that

\[
\frac{\partial N}{\partial \alpha} > 0.
\]

So, a larger market implies more firms.

The derivative of $\epsilon$ with respect to $\alpha$ is given by:

\[
\frac{\partial \epsilon}{\partial \alpha} = \frac{\partial \epsilon}{\partial N} \frac{\partial N}{\partial \alpha}.
\]

Equation (11) implies that

\[
\frac{\partial \epsilon}{\partial N} = \frac{\sigma - 1}{N^2} > 0.
\]

Thus,

\[
\frac{\partial \epsilon}{\partial \alpha} > 0.
\]

So, a larger market implies a higher price elasticity.
The derivative of $A$ with respect to $\alpha$ is given by:

$$\frac{\partial A}{\partial \alpha} = \frac{\partial A}{\partial \epsilon} \frac{\partial \epsilon}{\partial N} \frac{\partial N}{\partial \alpha}.$$  

Equation (14) implies that

$$\frac{\partial A}{\partial \epsilon} = \frac{\kappa}{\phi[\psi - [\epsilon(N) - 1]^2]A^{1-\psi}} > 0.$$  

Thus,

$$\frac{\partial A}{\partial \alpha} > 0.$$  

So, a larger market implies higher productivity.

**Proof of Proposition 3**

The derivative of $N$ with respect to $\kappa$ is given by:

$$\frac{\partial N}{\partial \kappa} = \frac{\psi \epsilon(N) N}{M'(N)} < 0,$$

Since $M'(N) < 0$, we have:

$$\frac{\partial N}{\partial \kappa} < 0,$$

So, lower entry cost imply more firms.

The derivative of $\epsilon$ with respect to $\kappa$ is given by:

$$\frac{\partial \epsilon}{\partial \kappa} = \frac{\partial \epsilon}{\partial N} \frac{\partial N}{\partial \kappa}.$$  

Thus,

$$\frac{\partial \epsilon}{\partial \kappa} < 0.$$  

So, larger entry costs imply a lower price elasticity.
The derivative of $A$ with respect to $\kappa$ is given by:

$$\frac{\partial A}{\partial \kappa} = \frac{\partial A}{\partial N} \frac{\partial N}{\partial \kappa}.$$  

Substituting equation (15) into equation (14), we obtain:

$$A = \left[ \frac{\alpha}{\phi \psi} \frac{\epsilon(N) - 1}{N \epsilon(N)} \right]^{\frac{1}{2}}.$$

The derivative of $A$ with respect to $N$ is given by:

$$\frac{\partial A}{\partial N} = \frac{\alpha A^{1-\psi}[-\epsilon(N)^2 + N\epsilon'(N) + \epsilon(N)]}{\phi \psi^2 [N\epsilon(N)]^2}.$$

$\partial A/\partial N < 0$ if and only if

$$-\epsilon(N)^2 + N\epsilon'(N) + \epsilon(N) < 0.$$

A sufficient condition for this to hold is $N \geq 2$, which is ensured by Assumption 1. Thus,

$$\frac{\partial A}{\partial \kappa} > 0.$$

**Proof of Proposition 4**

Equations (12)–(13) become:

$$\phi \psi A^\psi = C^{1-\eta} \alpha^{\eta} N^{\frac{\sigma - \eta}{\tau - \eta}} \left( \frac{\epsilon(N) - 1}{\epsilon(N)} \right)^{\eta} A^{\eta - 1},$$

$$\frac{1}{\epsilon(N) - 1} C^{1-\eta} \alpha^{\eta} N^{\frac{\sigma - \eta}{\tau - \eta}} \left( \frac{\epsilon(N) - 1}{\epsilon(N)} \right)^{\eta} A^{\eta - 1} = \kappa + \phi A^\psi.$$
This can be simplified to

$$A = \left[ \frac{\kappa[\epsilon(N) - 1]}{\phi[\psi - (\epsilon(N)) - 1]} \right]^{\frac{1}{\eta}},$$

(23)

$$(\phi_\psi)^\frac{1}{\eta} A^{\psi-\eta-1/\eta} = \alpha \frac{C^{\frac{1-\eta}{\eta} \frac{x-\eta}{\eta}} \epsilon(N) - 1}{\epsilon}.$$  

(24)

For convenience, let’s define the following functions:

$$g(N, \kappa) \equiv \left[ \frac{\kappa[\epsilon(N) - 1]}{\phi[\psi - (\epsilon(N)) - 1]} \right]^{\frac{1}{\eta}},$$

$$f(N, \kappa, \alpha) \equiv (\phi_\psi)^\frac{1}{\eta} A^{\psi-\eta-1/\eta} - \alpha C^{\frac{1-\eta}{\eta} \frac{x-\eta}{\eta}} \frac{\epsilon(N) - 1}{\epsilon(N)}.$$  

It is tedious but straightforward to show that

$$\frac{\partial g}{\partial N} = \frac{A^{1-\eta} \kappa \epsilon'}{\phi[\psi - (\epsilon(N)) - 1]^2} \geq 0,$$

$$\frac{\partial g}{\partial \kappa} = \frac{A^{1-\eta} \psi}{\phi[\psi - (\epsilon(N)) - 1]} \geq 0,$$

$$\frac{\partial f}{\partial g} = \frac{(\psi - \eta + 1)(\phi_\psi)^\frac{1}{\eta} A^{\psi-\eta-1/\eta}}{\eta} \geq 0,$$

$$\frac{\partial f}{\partial \alpha} = -C^{\frac{1-\eta}{\eta} \frac{x-\eta}{\eta}} \frac{\epsilon(N) - 1}{\epsilon(N)} \leq 0,$$

$$\frac{\partial f}{\partial N} = \frac{(\phi_\psi)^\frac{1}{\eta} A^{\psi-\eta-1/\eta}}{\epsilon - 1} \frac{(\sigma - \eta)\epsilon(\epsilon - 1) - (\sigma - 1)\eta N \epsilon'}{(\sigma - 1)\eta N \epsilon}.$$  

We start by showing that larger market share implies higher productivity. Note that:

$$\frac{\partial A}{\partial \alpha} = \frac{\partial g}{\partial N} \frac{\partial N}{\partial \alpha},$$

$$\frac{\partial N}{\partial \alpha} = -\frac{\partial f}{\partial \alpha} + \frac{\partial f}{\partial A} \frac{\partial A}{\partial N}.$$  

This implies

$$\frac{\partial f}{\partial N} + \frac{\partial f}{\partial A} \frac{\partial A}{\partial N} = \frac{(\phi_\psi)^\frac{1}{\eta} A^{\psi-\eta-1/\eta}}{\epsilon - 1} \frac{(\psi - \epsilon + 1)(\sigma - \eta)\epsilon(\epsilon - 1) + (\psi + 1)(\sigma - 1)N \epsilon'}{(\psi - \epsilon + 1)(\sigma - 1)\eta N \epsilon}.$$  

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This is larger than zero if $\epsilon > 1$, which is ensured by Assumption 2.

We continue by showing that lower entry cost implies lower productivity. Note that:

$$\frac{\partial A}{\partial \kappa} = \frac{\partial g}{\partial N} \frac{\partial N}{\partial \kappa} + \frac{\partial g}{\partial \kappa},$$

$$\frac{\partial N}{\partial \kappa} = -\frac{\frac{\partial f}{\partial A} \frac{\partial g}{\partial \kappa}}{\frac{\partial f}{\partial A} \frac{\partial g}{\partial N} + \frac{\partial f}{\partial N}}.$$

This implies that

$$\frac{\partial A}{\partial \kappa} = \frac{\frac{\partial g}{\partial \kappa}}{\frac{\partial f}{\partial A} \frac{\partial g}{\partial N} + \frac{\partial f}{\partial N}}.$$

The denominator in the above equation is positive, as we showed above. The relation between $A$ and $\kappa$ therefore depends on the sign of $\partial f/\partial N$. This in turn depends on the sign of

$$(\sigma - \eta)\epsilon(\epsilon - 1) - (\sigma - 1)\eta N \epsilon'$$

which equals

$$\frac{\sigma - \eta}{(\sigma - 1)\eta} (\epsilon - 1) - \frac{N \epsilon'}{\epsilon}.$$

Thus, we have to show that

$$\frac{\sigma - \eta}{(\sigma - 1)\eta} (\epsilon - 1) - \frac{N \epsilon'}{\epsilon} > 0. \quad (25)$$

If $\eta \leq 1$, then $(\sigma - \eta)/(\eta(\sigma - 1)) \geq 1$. A sufficient condition for (25) is that

$$\epsilon - 1 \geq \frac{N \epsilon'}{\epsilon}.$$

This is equivalent to

$$N \geq \frac{\sigma - \eta}{\sigma - 1} (1 + \sigma^{-1/2}).$$

Since $\eta \leq 1$, this condition holds for sure.
If $\eta \in (1, \sigma]$, we plug the definition for $\epsilon$ into (25):

$$\left(\frac{\sigma - \eta}{\sigma - 1}\right) \left(\frac{\sigma - 1}{N} - \frac{\sigma - \eta}{N}\right) > \frac{\sigma - \eta}{N} \left(1 - \frac{\sigma - \eta}{N}\right).$$

This is equivalent to

$$[N(\sigma - 1) - (\sigma - \eta)][N(\sigma - (\sigma - \eta))] > N\eta(\sigma - 1),$$

or

$$N\sigma(\sigma - 1) + \frac{(\sigma - \eta)^2}{N} > \sigma[(\sigma - 1) + (\sigma - \eta)].$$

Since $\eta > 1$ and $N \geq 2$, we have:

$$N\sigma(\sigma - 1) + \frac{(\sigma - \eta)^2}{N} > 2\sigma(\sigma - 1) > \sigma[(\sigma - 1) + (\sigma - \eta)].$$