Human Capital, Product Quality, and Bilateral Trade

Michael E. Waugh*
Federal Reserve Bank of Minneapolis
New York University

Email: waugh@minneapolisfed.org

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Abstract

In this paper, I develop a quantitative, general equilibrium theory of product quality and international trade. In the model, producers make choices regarding the quality/technology of their intermediate inputs given the set of endowments they have access to. This choice affects the producers ability to produce goods domestically and internationally, thus shaping the pattern of bilateral trade. In otherwise identical countries, optimizing behavior results in: (i) the high human capital country importing a relatively small volume of goods from the low human capital country and (ii) the low human capital country importing a relatively large volume of goods from the high human capital country — qualitatively consistent with the observed volume of bilateral trade between rich and poor countries. I quantify the theory for a sample of 77 countries and show that it explains up to 90 percent of the variation in bilateral trade; twice the amount of alternative models with no role for human capital and product quality.

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1 Introduction

In this paper, I develop a quantitative general equilibrium theory of human capital, product quality, and international trade. The idea is simple: firms within a country make decisions regarding the choice of the quality of inputs used in production given the human capital of workers in that country. These decisions affect a firm’s productive ability to export across all destinations—influencing bilateral trade volumes. I quantify the theory and show that it explains up to 90 percent of the variation in bilateral trade; twice the amount of alternative models with no role for human capital and product quality.

In its essence, my theory is about the ability of human capital to use technology and the choice or adoption of technology and how these forces affect bilateral trade volumes. In the model, there is a continuum of intermediate goods produced and consumed at different quality levels with an input-output production structure, i.e., intermediate goods are used to produce other intermediate goods. By quality I mean that an intermediate good of higher quality is more productive at producing other intermediate goods, holding all else constant. In other words, quality equals technology. Furthermore, the quality of an intermediate good is complementary to the human capital of the workers employed in the spirit of Griliches (1969). The idea being that skilled workers are better able to use higher quality intermediate goods.

The general equilibrium nature of the model is made explicit in the demand and supply of quality type. On the demand side, firms in each country face a choice regarding the quality of intermediate goods as inputs. Since quality is complementary to human capital, this choice depends upon the human capital of workers in each country. On the supply side, firms are perfect competitors and can produce any quality level at increasing cost to supply the requisite demand for quality type. In a competitive equilibrium, the result is that firms in countries with higher human capital chose to employ higher quality intermediate goods relative to low human capital countries. This equilibrium result is an outcome analogous to the ideas of Nelson and Phelps (1966) emphasizing the role of human capital in choosing or adopting the appropriate technology. The novelty of my theory is that because quality is a form of technology and because quality is complementary to human capital—the equilibrium choice provides high human capital countries an advantage at producing all goods and quality levels influencing bilateral trade volumes between countries with different levels of human capital.

Armed with this theory, I ask two quantitative questions: First, the model makes specific predictions regarding how disaggregate relative prices should covary across countries depending on the human capital of an importing country, so I ask do we see these predictions in the data? Second, if yes, then is this mechanism quantitatively important to understanding bilateral trade volumes? With regards to the first question, I find a positive

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1 Yi (2003), Eaton and Kortum (2002) are different examples of this structure. Hummels, Rapoport, and Yi (1998) and Hummels, Ishii, and Yi (2001) are studies that document the importance of trade in intermediate goods.
correlation between unit values and the importing countries level of human capital as the theory predicts.\(^2\) Regarding the second questions, I calibrate the model using parameter values picked in a manner such that bilateral trade volumes are not explicitly targeted. Hence the ability of the model to replicate the variation in bilateral trade volumes is a test of the theory. I find that the model can explain up to 90 percent of the variation in bilateral trade volumes. This is almost twice the amount that alternative models with no role for human capital and product quality can achieve.

My theory relates to two classic theories of international trade articulated by Linder (1961) and Vernon (1966). The basic idea in Linder (1961) is that because rich countries have similar tastes they will trade more and because rich and poor countries have dissimilar tastes they will trade less. What differences in tastes means, the sources of these differences, and hence the exact predictions for bilateral trade were never formalized in Linder (1961). In my theory, differences in the taste for quality arise because of differences in the skill level of workers across countries. These differences affect bilateral trade because the quality choice affects the ability of firms to be the least cost supplier. And, I quantify my theory and demonstrate that it is consistent with observed bilateral trade volumes between rich and poor countries.

Vernon’s (1966) theory of the “product cycle” argued that recent vintages of goods are typically first produced and consumed in advanced countries because of a greater demand for labor saving technologies and familiarity with the tastes of buyers. Prominent formalizations of his theory are articulated in Flam and Helpman (1987) and Stokey (1991). Like these models, the equilibrium demand for quality generates a product cycle with rich countries consuming and producing most of the high quality goods. However, my theory is distinguished by the sources of demand for quality and the mechanism with which it affects trade. Rich countries consume higher quality goods because they complement the human capital of workers and most high quality goods are produced by rich countries because this choice endows them with an absolute advantage to produce all goods. More important, because I build upon theories of capital-skill complementarity and skill in adoption, the mechanism I emphasize is backed by the large amount of micro-evidence supporting these theories.

Empirical focus on product quality and international trade grown since the results of Schott (2004) and Hummels and Klenow (2005) showing how unit values (prices) systematically covary across exporters. Models using this evidence are those of Baldwin and Harrigan (2007) and Johnson (2008) which build upon the framework of Melitz (2003) with the quality of a good produced exogenously driven by the assumed distribution of productivity to help explain this evidence. Hallak (2006) uses this evidence in a model along the lines of Krugman (1980) with exogenous preferences for quality to study bilateral

\(^2\)This result is similar in spirit to the results of Schott (2004) and Hummels and Klenow (2005), though I focus on how prices covary across destinations while these papers focused on a given destination (the U.S.) and how prices covary across sources.
trade patterns at the sectoral level. Kugler and Verhoogen (2007) and Verhoogen (2008) use a modified Melitz (2003) model with input choice, similar in spirit to this paper, and provide evidence from firm level data supporting the theory. Relative to these studies the distinguishing characteristics of this paper are my contributions: I provide a novel theory of human capital and product quality and I demonstrate that my theory quantitatively accounts for the bilateral trade data.

2 The Model

Consider a world with \( N \) countries. Each country has two sectors, an intermediate goods sector and a final goods sector with intermediate goods used in the production of both sectors. Thus the model employs an input-output structure similar to that used the trade models of Yi (2003), Eaton and Kortum (2002), Alvarez and Lucas (2007), and Waugh (2007).3 Intermediate goods can be produced and consumed at different quality levels and all are potentially traded. Within each country \( i \), there is a measure of homogenous consumers \( L_i \) with the same human capital level \( h_i \). Each consumer has one unit of time supplied inelastically in the domestic labor market. Furthermore, each consumer has preferences only over the non-traded final good. In the following, all variables are normalized relative to the work force in country \( i \).

2.1 Intermediate Goods Sector

Intermediate goods are indexed by two variables: \( x \in [0, 1] \) denoting the good as in Dornbusch, Fischer, and Samuelson (1977) and quality level \( q' \in [\bar{q}, \infty] \) of the good. The factors of production are labor \( n_i \) with human capital level \( h_i \) and the aggregate intermediate good \( y_i(q) \) of quality level \( q \). I take human capital \( h_i \) to mean the maximum potential effectiveness or skill level of the raw labor. The factors of production are combined in the following manner to produce quantity \( m_i(q', x) \) of quality level \( q' \) of good \( x \):

\[
m_i(q', x) = z_i(x)^{-\theta} \left[ \left( \frac{q}{q'} \right)^{\rho} + \left( \frac{h_i}{q'} \right)^{\rho} \right]^{\frac{1}{\beta}} n_i^\beta y_i(q)^{1-\beta}.
\]

Let’s walk through each of these terms in (1). The right most term is a Cobb-Douglas production function relating the physical units of labor and the aggregate intermediate good. Power term \( \beta \) controls factor shares and is common to all countries. The left most term reflects technological differences specific to the production of good \( x \). Across production of quality levels \( q' \), all firms enjoy the same productivity level \( z(x)^{-\theta} \). Across goods \( x \), production technologies differ only in their efficiency level \( z(x)^{-\theta} \).

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3Hummels, Rapoport, and Yi (1998) and Hummels, Ishii, and Yi (2001) are studies that document the importance of trade in intermediate goods and specifically vertical specialization.
The CES term in the middle defines quality by relating the efficiency of labor $h_i$, the quality of intermediate inputs $q$, and the quality of the output $q'$. In general I assume that $\rho < 0$ implying that human capital $h_i$ and the quality of the aggregate intermediate $q$ are complements. Notice a producer of any good $x$ of quality type $q'$ is more productive, ceteris paribus, as a result of: (i) higher skill level of its workers or (ii) higher quality type $q$ of intermediates. With $\rho = -\infty$, the efficiency of the intermediate input is only as productive as the skill level of its workers.

Now let the quality level produced $q'$ vary and hold a firms input type constant. The functional form assumption implies that the productivity of the firm depends on the quality level produced relative to the inputs. Hence, for fixed input types, a firm is less productive at producing a high quality good relative to producing a low quality good. Furthermore, this term is taken to the power $\gamma$ encompassing the possibility that: (i) quality does not matter with $\gamma = 0$, (ii) there is curvature in the production of quality levels different than those employed by the firm with $\gamma \neq 0$.

### 2.1.1 Discussion

The production function in (1) has antecedents in two theories regarding the interaction of skill and technology: capital-skill complementarity and skill in adoption. Capital-skill complementarity was first discussed in Griliches (1969). The underlying idea is that skilled workers are better able to use capital goods; see Greenwood and Jovanovic (2000). Krusell, Ohanian, Rios-Rull, and Violante (2000) and Caselli (1999) are modern studies focusing on the theory’s implications for changes in the skill premium within a country. To connect this production structure with these studies, compare the marginal product of human capital relative to the marginal product of raw labor:

$$
\frac{\partial m/\partial h_i}{\partial m/\partial n} = \frac{\gamma}{\rho \beta} \left[ \left( \frac{q}{h} \right)^{\rho} + 1 \right]^{-1} \frac{n}{h}
$$

Equation (2) is basically statement regarding the skill premium, i.e. the return to skilled labor relative to unskilled labor. Similar to the mechanism in Krusell, Ohanian, Rios-Rull, and Violante (2000) an increase in the quality $q$ of intermediate inputs increases the return to human capital relative to raw labor. I must emphasize, however, that this is an artificial concept since all labor within a country has the same skill level $h_i$. Hence this model is, by assumption, not intended for the study of skill premium within a country.

The second theory relates to role of skill in choosing or adopting the appropriate technology as first discussed in Nelson and Phelps (1966); Bils and Klenow (2000) and Greenwood and Yorukoglu (1997) are modern incarnations. Nelson and Phelps (1966) provide the following quote summarizes their theory:

“It is clear that the farmer with a relatively high level of education has tended to adopt productive
innovations earlier than the farmer with relatively little education... The less educated farmer, for whom the information in technical journals means less, is prudent to delay the introduction of a new technique...”

This theory is more fully articulated in the firms’ problem in (5). Since the firm has access to all quality types of intermediate inputs, the firm faces a choice regarding the appropriate technology depending on the price-quality schedule and the human capital of its workers. As an equilibrium outcome, the model predicts that countries with workers which have high human capital will choose to employ more productive types of intermediate inputs relative to countries with lower human capital.

There is ample empirical evidence regarding both these theories. Studies which show human capital is complementary to technology use are Doms, Dunne, and Troske (1997), Autor, Katz, and Krueger (1998), and Goldin and Katz (1998). Studies which show correlations between adoption of technology and human capital are Bartel and Lichtenberg (1987), Doms, Dunne, and Troske (1997) as well, and Caselli and Coleman (2001) which focuses on cross-country evidence regarding the adoption of computers and human capital. In independent work, Kugler and Verhoogen (2007) provide supporting evidence for a similar mechanism as well. They hypothesize that input quality at the plant level and plant level productivity are complements in producing output quality within a modified Melitz (2003) model. Examining plant level data in Columbia they find support for this hypothesis.

2.2 Aggregation of Intermediate Goods

Individual intermediate goods of quality level $q'$ are aggregated according to a standard symmetric Dixit-Stiglitz technology producing the aggregate intermediate good of quality level $q'$:

$$ y(q') = \left[ \int m(q', x)^{\frac{1}{\eta}} dx \right]^{\frac{\eta}{\eta - 1}} $$

(3)

with elasticity of substitution $\eta > 0$. Note no substitution is allowed in between different quality levels of intermediate inputs to produce the aggregate quality intermediate good.

2.3 The Distribution of Productivity Across Goods

Following Eaton and Kortum (2002), I parameterize the model by treating $z_i(x)$ as an idiosyncratic random variable. I follow Alvarez and Lucas (2007) and assume that $z_i(x)$ is

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4Manuelli and Seshadri (2003), studying the diffusion of tractor technology in a frictionless environment, summarize this view succinctly: “the adoption decision is equivalent to picking a point on the appropriate isoquant.”
distributed independently and exponentially with parameter $\lambda_i$ differing across countries. Because good level productivity is $z_i(x)^{-\theta}$ this formulation is equivalent to a Type II extreme value distribution or Fréchet distribution used by Eaton and Kortum (2002).\(^5\) Castillo, Hadi, Balakrishnan, and Sarabia (2005) is a useful resource regarding extreme value distributions. The $\lambda_i$s and $\theta$ play the following roles regarding how productivity varies across goods:

- $\lambda_i$ governs each country's efficiency average level. One can show that each country's mean productivity is proportional to $\lambda_i^\theta$, with the constant of proportionality not depending upon the country. So a country with a relatively larger $\lambda_i$ is, on average, more efficient at producing all tradable goods.

- $\theta$ controls the dispersion of efficiency levels. Mechanically, a larger (smaller) $\theta$ yields more (less) variation in efficiency levels relative to the mean. As $\theta$ increases, the likelihood that two countries productivity at producing the same good is different increases yielding more incentives to trade. In contrast, as $\theta$ approaches zero, the likelihood that the two countries productivity at producing the same good is different decreases yielding less incentives to trade because they are increasingly similar. In this sense $\theta$ controls the degree of comparative advantage.

2.4 Final Goods Sector

I assume there is a final goods sector producing a homogenous good not distinguished by quality level. I introduce this sector into the model only to facilitate calibration. Each firm has access to the following production function mapping labor $n_i$ of human capital level $h_i$ and the aggregate intermediate good $y_i(q)$ of quality level $q$ produce quantity $y_i^f$ in the following manner:

$$y_i^f = h_i^{1-\alpha-\gamma} \left[ \omega q^\rho + h_i^\rho \right] \eta_i n_i^{1-\alpha} y_i(q)^{1-\alpha}. \tag{4}$$

Similar to (1), the right most term is a Cobb-Douglas production function with power term $\alpha$ controlling factor shares. Furthermore, the quality of intermediate inputs and human capital affect production in a complementary way with $\rho < 0$ and firms will face a choice over the appropriate technology to use.

There are two important distinctions between equation (1) and (4) which are simplifying assumptions. The first distinction is the term $h_i^{1-\alpha-\gamma}$. The rationale for the formulation of production in the following manner, the direct influence of human capital on income per-worker is only $h_i^{1-\alpha}$ as in a standard neoclassical growth model; see Hall and Jones (1999). Section (6) discuss this aggregation result. The second distinction is the term $\omega$

\(^5\)Kortum (1997) shows how a model of innovation and diffusion consistent with balanced growth can give rise to this distribution.
which I will assume is equal to \( \frac{1-\alpha}{\beta} \). The rationale for this assumption is that it simplifies the equilibrium outcome for demands of quality.

2.5 Trade Costs

To model trade costs, the standard iceberg assumption is made, i.e. \( \tau_{ij} > 1 \) of good \( z \) must be shipped from country \( j \) for one unit to arrive in country \( i \) in which \( (\tau_{ij} - 1) \) “melts away” in transit. Trade costs \( \tau_{ij} \) are thought to be some function of distance and other policy and non-policy related variables; the quantitative section is more specific. In addition, \( \tau_{ii} \) is normalized to equal one for each country.

3 Equilibrium

In the economy, firms are perfect competitors and take prices as given and make choices regarding the appropriate quality choice. Below, I discuss the representative firms problem, I define a competitive equilibrium, and I discuss properties of the competitive equilibrium.

3.1 Firms Problems

The representative firm producing quality level \( q' \) of good \( x \) in country \( i \) faces the following problem:

\[
\min_{n,y,q} \left[ w_i n_i + p_i(q) y_i(q) \right] \quad \text{s.t.} \quad \tag{5}
\]

\[
z_i(x)^{-\theta} \left[ \left( \frac{q}{q'} \right)^{\rho} + \left( \frac{h_i}{q'} \right)^{\rho} \right] \frac{\rho}{\lambda} n_i^\beta y_i(q)^{1-\beta} \geq m_i(q', x).
\]

Since labor is homogenous with human capital level \( h_i \), the firm has no choice regarding the skill level of its employees, it only has a choice over the number of workers. The firm does have a choice over the quality type \( q \) of its intermediate inputs and in a sense has a choice over the productivity of its employees. The optimal choices of \( n \) and \( y \) are standard. The optimal choice of the quality type \( q \) is:

\[
q = \arg \min \left\{ z_i(x)^{\theta} \left[ \left( \frac{q}{q'} \right)^{\rho} + \left( \frac{h_i}{q'} \right)^{\rho} \right] \frac{\rho}{\lambda} w_i^\beta p_i(q)^{1-\beta} \right\} \quad \tag{6}
\]

where the term in the bracket is a unit cost function for a fixed \( q \). Notice the tradeoff the firm faces in (6). Firms must weigh the efficiency gain from employing higher quality intermediate inputs: \( \left[ \left( \frac{q}{q'} \right)^{\rho} + \left( \frac{h_i}{q'} \right)^{\rho} \right] \frac{\rho}{\lambda} \), versus potentially higher costs for higher quality intermediate inputs \( p_i(q)^{1-\beta} \). Hence, the optimal choice of \( q \) depends on how the price schedule changes for different quality levels relative to the efficiency gain.
A representative firm producing the final good $y^f_i$ in country $i$ faces a similar problem:

$$\min_{n,q} \left[ w_i n_i + p_i(q) y_i(q) \right] \quad \text{s.t.}$$

$$h^{1-\alpha-\gamma} \left[ \omega q^\rho + h_i^o \right] \tilde{r} n_i^{1-\alpha} y_i(q)^{1-\alpha} \geq y^f_i.$$  \hspace{1cm} (7)

Similar to the intermediate goods problem, the firm has no choice regarding the skill level of its employees, it only has a choice over the number of workers. But it does have a choice over the quality type $q$ of its intermediate inputs. The optimal choice of the quality type $q$ is:

$$q = \arg\min \left\{ h^{1-\alpha-\gamma} \left[ \omega q^\rho + h_i^o \right] \tilde{r} w_i^{1-\alpha} p_i(q)^{1-\alpha} \right\} \quad \text{\hspace{1cm} (8)}$$

Again, when selecting $q$ the firm faces a tradeoff between an efficiency gain from employing higher quality intermediate inputs and potentially higher costs for higher quality inputs.

To solve both these problems one needs more information regarding the price-quality schedule. With a definition of a competitive equilibrium, I can derive some properties regarding this price-quality schedule without determining the optimal quality choice for a firm. Then given the properties of the price-quality schedule, I can then analytically determine the optimal quality choice.

### 3.2 The Competitive Equilibrium

As mentioned, the definition of a competitive equilibrium allows me to characterize the price-quality schedule and then analytically determine the optimal quality choice of a firm. Below, I provide a sufficient definition. After the optimal quality choice is determined, I will characterize the competitive equilibrium as a solution to specific functions later in this section.

**Definition 1** A competitive equilibrium is a vector of wages $(w_1, \ldots, w_N)$, price indexes $[p_1(q), \ldots, p_N(q)]$ for all $q$, final goods prices $(p^f_i, \ldots, p^f_N)$ and a production plan for each firm such that:

1. The firms production plan solves the problems in (5) and (7).

2. Markets clear with $m_i(q, x)$, $y_i(q)$, $y^f_i$, and $L_i$ in zero excess supply and demand.

This definition is standard—firms are optimizing and markets clear. Given this definition, the next section determines the optimal quality choice.

### 3.3 The Optimal Quality Choice

This section proves that the optimal choice of quality level is such that $q_i \propto h_i$. As discussed, this outcome depends upon the equilibrium price-quality schedule that firms face. The
following proposition describes a property of an equilibrium price-quality schedule:

**Proposition 1** In any competitive equilibrium, the price of the aggregate intermediate good of quality level \( q \) is increasing in quality level at rate \( \gamma > 0 \).

**Proof**: First, in any competitive equilibrium, the price a firm in country \( i \) can sell good \( x \) of quality level \( q' \) at is:

\[
p_i(q', x) = (q')^\gamma z(x)^\theta \min_q \left\{ \left[ q^\rho + h_i^\rho \right]^\frac{\theta}{\rho} c(q) \right\}.
\]  

(9)

with \( c_i(q) = w_i^\beta p(q)^{1-\beta} \). Note that in any competitive equilibrium, it must be the case that in country \( i \) the last term in equation (9),

\[
\min_q \left\{ \left[ q^\rho + h_i^\rho \right]^\frac{\theta}{\rho} c(q) \right\},
\]

is constant across any producer of any good in country \( i \). If this does not hold, then this implies that a firm could produce a good with a strictly lower unit cost contradicting the definition of an equilibrium. This does not imply that all firms make the same choice regarding \( q \), just that their unit cost—adjusted for the good they are producing—must be the same. As a result of this observation define \( c_i = \min_q \left\{ \left[ q^\rho + h_i^\rho \right]^\frac{\theta}{\rho} c(q) \right\} \).

Given this observation and the distributional assumptions on \( z_i(x) \), the price of the aggregate intermediate good in country \( i \) for an arbitrary quality level \( q' \) is:

\[
p_i(q') = (q')^\gamma \Omega \left\{ \sum_{j=1}^N c_j r_{ij} \frac{\theta}{\rho} \lambda_j \right\}^{-\theta}.
\]  

(10)

where \( \Omega \) is a collection of constants. Given equation (10), now compare the aggregate price of intermediate goods for an alternative quality level \( q' > q'' \). Since the bracketed term in (10) is independent of the quality level produced this implies that:

\[
\frac{p_i(q')}{(q')^\gamma} = \frac{p_i(q'')}{(q'')^\gamma} \Rightarrow \frac{p_i(q')}{p_i(q'')} = \left( \frac{q'}{q''} \right)^\gamma
\]  

(11)

which shows that if \( \gamma > 0 \), then the price of the aggregate intermediate goods is increasing in quality level at rate \( \gamma \).

This proposition is informative because says that in any competitive equilibrium the price-quality schedule is increasing at rate \( \gamma \). With this information, I can determine the optimal quality choice of firms in country \( i \) which I summarize in the following propositions for intermediate goods producers and final goods producers:

**Proposition 2** In any competitive equilibrium, it is optimal for producers of good \( x \) and quality level \( q' \) in country \( i \) to set \( q_i = \left( \frac{\beta}{\rho} \right)^{\frac{1}{\rho}} h_i \).
**Proof:** From equation (6), the optimal \( q \) is the argument which minimizes:

\[
q = \arg \min \left\{ z_i(x)^\rho \left[ \left( \frac{q}{q'} \right)^\rho + \left( \frac{h_i}{q} \right)^\rho \right] w_i^\beta p_i(q)^{1-\beta} \right\}.
\]

From Proposition 1, the problem to finding the optimal \( q \) can be restated as:

\[
\min_q (q^\rho + h_i^\rho) \frac{\gamma}{\rho} q^{\gamma(1-\beta)},
\]

with terms irrelevant to the optimization problem are abstracted from. This optimization problem yields the following first order condition:

\[
-\gamma (q^\rho + h_i^\rho) \frac{\gamma}{\rho} q^{\gamma(1-\beta)} + (q^\rho + h_i^\rho) \frac{\gamma}{\rho} q^{\gamma(1-\beta) - 1} = 0
\]

\[
\Rightarrow \quad q = \left( \frac{\beta}{1-\gamma} \right) \frac{\gamma}{\rho} h_i
\]

where the solution implies that \( q \propto h \). \( \square \)

**Proposition 3** In any competitive equilibrium, it is optimal for final goods producers in country \( i \) to set \( q_i = \left( \frac{\beta}{1-\gamma} \right) \frac{\gamma}{\rho} h_i \).

**Proof:** Similar arguments as in Proposition 2, see appendix.

The following theorem and corollary summarize the optimal quality choice in any competitive equilibrium:

**Theorem 1** In any competitive equilibrium and for all firms in country \( i \), the optimal quality choice of intermediate inputs is \( q_i = \left( \frac{\beta}{1-\gamma} \right) \frac{\gamma}{\rho} h_i \).

**Proof:** Propositions 1-4 together prove the result.

**Corollary 1** In any competitive equilibrium, country \( i \) only imports one quality level \( q_i = \left( \frac{\beta}{1-\gamma} \right) \frac{\gamma}{\rho} h_i \).

Corollary 1 tells us that each country will only import one quality level and that quality level is proportional to its human capital. This does not imply that a country produces only one quality level. Each country exports a finite set of quality levels for a certain range of goods. That is the production basket will be diverse while the consumption basket is homogenous in quality. This equilibrium outcome makes the model particularly tractable despite the complex product space. Countries demand only a point in quality space leaving only the range of goods imported to be determined in equilibrium.

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6A sufficient condition for this solution to be an minimum is that \( \rho < 0 \) and \( \gamma(1-\beta) > 1 \), which is valid for calibrated/estimated parameter values.
3.4 The Competitive Equilibrium Revisited

Characterizing a competitive equilibrium are the functions which determine the aggregate price of tradable goods, trade shares, and wages. From these functions all other equilibrium objects are determined.

**Price Index:** Each country faces the following price of aggregate intermediate inputs of quality level $q'$:

$$p_i(q') = \Omega \left\{ \sum_{j=1}^{N} \left[ c_j(h_{ij}) \tau_{ij} \left[ \left( \frac{\kappa h_{ij}}{\kappa h_i} \right)^{\rho} + \left( \frac{h_{ij}}{h_i} \right)^{\rho} \right]^{\frac{1}{\rho}} \right] \lambda_j \right\}^{-\theta}. \quad (12)$$

where $\Omega$ is a collection of constants and $\kappa = \left( \frac{\varphi}{1 - \beta} \right)^{\frac{1}{\rho}}$. Other than the effects from quality, this expression is standard in multi-country Ricardian models with Fréchet distributed technologies.

**Trade Shares:** $M_{ij}(h_i)$ is the fraction of all goods country $i$ imports from country $j$ of quality level $q_i = \kappa h_i$. For notational clarity, I will index demands of quality level by each country’s human capital level. Since there is a continuum of goods, computing this fraction boils down to finding the probability that country $j$ is the low-cost supplier to country $i$ given the joint distribution of efficiency levels, prices, trade costs, and quality choices for any good $z$. The expression for a trade share is:

$$M_{ij}(h_i) = \frac{\left\{ c_j(h_{ij}) \tau_{ij} \left[ \left( \frac{\kappa h_{ij}}{\kappa h_i} \right)^{\rho} + \left( \frac{h_{ij}}{h_i} \right)^{\rho} \right]^{\frac{1}{\rho}} \right\}^{\frac{1}{\theta}} \lambda_j}{\sum_{\ell=1}^{N} \left\{ c_{\ell}(h_{\ell}) \tau_{\ell} \left[ \left( \frac{\kappa h_{\ell}}{\kappa h_i} \right)^{\rho} + \left( \frac{h_{\ell}}{h_i} \right)^{\rho} \right]^{\frac{1}{\rho}} \right\}^{\frac{1}{\theta}} \lambda_{\ell}}. \quad (13)$$

Again this expression is standard except for the effects from quality. Note that formally the set of demands are with $M_{ij}(q) = 0, \forall q \neq \kappa h_i$, and $M_{ij}(h_i) > 0$.

**Wage Function:** An equilibrium wage vector is computed given trade shares and imposing balanced trade. Imports are defined as

$$\text{Imports} = L_i p_i(h_i) y_i(h_i) \sum_{j \neq i}^{N} M_{ij}(h_i),$$

which is the total value of all goods country $i$ consumes from abroad. Similarly, exports are defined as

$$\text{Exports} = \sum_{j \neq i}^{N} M_{ji}(h_j) L_j p_j(h_j) y_j(h_j),$$
which is the total value of all goods countries abroad purchase from country \(i\).

Imposing balanced trade and including each country \(i\)'s consumption of goods produced at home implies the following relationship must hold:

\[
L_i p_i(h_i) y_i(h_i) \sum_{j=1}^{N} M_{ij}(h_i) = \sum_{j=1}^{N} L_j p_j(h_j) y_j(h_j) M_{ji}(h_j).
\]

which says the aggregate value of intermediate goods purchased by country \(i\) is to equal the value of intermediate goods all \(N\) countries purchase from country \(i\).

The appendix derives the allocations of factors across the intermediate goods sector and final goods sector. As in Alvarez and Lucas (2007), this provides the following relationship between wages and total value of intermediate goods purchased: \(w_j = \frac{\beta}{(1-\alpha)} p_j(h_j) y_j(h_j)\). Since factor shares are assumed constant across countries, this provides a simplified expression for balanced trade:

\[
w_i = \frac{N}{L_i} \sum_{j=1}^{N} L_j w_j M_{ji}(h_j).
\]

At this point, the three key pieces of the model have been derived. Equation (12) describes the equilibrium price of intermediate goods, equation (13) describes the fraction of goods countries purchase from each other, and equation (14) describes the equilibrium wage rate for each country. From these functions, all other prices and quantities are determined and an equilibrium constructed.\(^7\)

The following definition summarizes this by characterizing a competitive equilibrium:

**Definition 2** A competitive equilibrium is characterized by a vector of wage rates \(w = (w_1, \ldots, w_N)\) that satisfy (14), demands for quality type \(q\) that satisfy Theorem 1, functions \(p(q_i)\) that satisfy (12), the functions \(M_{ij}(q_i)\) that satisfy (13), and optimal allocations of factors.

4 Qualitative Features of the Model: A Two Country Example

In this section I discuss some qualitative features of the model. First, I illustrate how quality affects bilateral trade and then discuss some implications for the data that I will use to calibrate the model. Second, I illustrate how my model generates a “product cycle” first articulated by Vernon (1966).

\(^7\)Alvarez and Lucas (2007) provide theorems proving the existence and uniqueness under certain conditions. Simply relabel the trade costs as \(\tau_{ij} = \tau_{ij} \left[ \left( \frac{\nu h_j}{h_i} \right)^{\rho} + \left( \frac{h_j}{h_i} \right)^{\rho} \right]^{\frac{\gamma}{\rho}}\) and the results from Alvarez and Lucas (2007) apply.
4.1 How Does Quality Affect Bilateral Trade?

To illustrate how quality choice and human capital affects bilateral trade consider an
economy with only two countries denoted country 1 and 2. Assume that there are no costs
to trade, that $\lambda_1 = \lambda_2$, and both countries have the same labor endowments.\(^8\)

First, consider the case with $h_1 = h_2$. Each country has the same human capital level
and hence selects the same quality level. This case is equivalent to a model with no
quality choice ($\gamma = 0$) and hence it is a useful benchmark to contrast other cases with. To
illustrate the implications for bilateral trade, divide equation (13) for country 1’s imports
from country 2 by the equation for country 2’s imports from country 1 yielding the following
expression for relative trade shares:

$$\frac{M_{12}(h_1)}{M_{21}(h_2)} = 1,$$

implying each country purchases half their goods from each other. The top panel of Figure
1 illustrates this case. In Figure 1, the left y-axis is quality level demanded and the right
y-axis is a country’s human capital level. The x-axis is good $x$ indexed on the zero-one
interval. In this case both countries have the same human capital level, hence firms demand
the same quality level, and both locate on the same point in the quality dimension. This
has no affect on the pattern of trade and each country half their goods from each other.
This is simply Dornbusch, Fischer, and Samuelson (1977) in a completely symmetric world.

Consider the case with $h_1 > h_2$. Because of the differences in human capital, the
high human capital country selects a higher quality level relative to country 2. Using
balanced trade and dividing equation (13) for country 1’s imports from country 2 by the
same equation for country 2’s imports from country 1 yields the following implication for
bilateral trade shares between the countries:

$$\frac{M_{12}(h_1)}{M_{21}(h_2)} = \left( \frac{h_2}{h_1} \right)^{\frac{\gamma \beta}{\delta + \beta}} < 1.$$

Country 1 purchases a smaller share from country 2 and country 2 purchases a larger
share from country 1 relative to the case when human capital and quality choice play
no role. The bottom panel of figure 1 illustrates this outcome. Here the two countries
locate at different points in the quality dimension because of differences in human capital.
Basically, this choice results in a productivity difference between the two countries. The
difference in productivity affects trade shares by decreasing the number of goods—the
extensive margin—that the human capital poor country can competitively export to the
human capital rich country. Similarly, the difference in productivity increases the number
of goods that the human capital rich country can export to the human capital poor country.

This qualitative result resembles the arguments in Linder (1961). He argues that be-

\(^8\) The appendix provides the algebraic details.
cause rich countries have similar tastes they will trade more and because rich and poor countries have dissimilar tastes they will trade less. My theory has a similar flavor to it. But the distinctive feature of my model is that the demand for quality arise because of differences in the skill level of workers across countries. These differences affect bilateral trade because the quality choice affects the ability of firms to be the least cost supplier.

This simple example may provoke several questions. For example, how is this different from simple absolute advantage? What about alternative human capital stories? These are important questions, below I answer them.

4.1.1 A Discussion

What distinguishes this from absolute advantage? From an observer studying bilateral trade shares, nothing. In terms of bilateral trade shares, it appears as if country 1 is more productive relative to country 2. One could imagine relabeling $\lambda$ appropriately generating the same pattern of trade.

The two models are distinguishable by the observed prices transacted at between the two countries. Table 1 presents the prices that both country 1 and country 2 pay in the case where quality matters and where it does not (i.e. $\gamma = 0$ or $h_1 = h_2$) but with $\lambda$’s adjusted to generate the same trade flows. Table 1 presents prices for a good $x$ which country 1 has the comparative advantage in and hence produces it for consumption at home and abroad. The quality model predicts that country one will pay a higher price relative to country 2—even though they are consuming the same good $x$ but at different quality levels.$^9$ In the model with $\gamma = 0$, prices in both countries are the same. This is an important observation because it suggest a way to validate the structure of the model. Given highly disaggregated price data, I can estimate the parameter $\gamma$; Section 5 follows this path.

---

$^9$Notice how quality’s interaction with human capital creates “wedges” in prices. For example, if an observer was unable to measure differences in quality, the relative price differences across destinations may be attributed to trade costs, taxes, or pricing to market. Yet, there are no distortions, it is just that they are different goods produced with different technologies. In this sense the equilibrium outcome of the model here resembles the model of Matsuyama (2007) who assumes countries have different technologies depending if the goods are exported or not. In this model, a priori, all firms have the same technologies for export. In equilibrium, they chose to employ different quality types of intermediate inputs, which affects their ability to export depending on their human capital stock.
What distinguishes this from an alternative human capital story? In the model described above, human capital takes on a rather unorthodox role. The traditional approach is to treat a country’s labor endowment as \( \tilde{L} = h \times L \). Here, human capital only changes the number of labor units available in a country. With balanced trade and holding all else constant, this makes a human capital rich country appear larger lowering the wage in the human capital rich country allowing it to produce goods cheaper relative to a human capital poor country. Hence, this simple story does generate a similar bias in bilateral trade between human capital rich and poor countries. However, similar to the previous discussion above and table 1, the product quality/human capital story has different predictions for relative prices compared to a traditional human capital story. In the traditional human capital story, relative export prices for a given good \( x \) should be the same across destinations. In the product quality/human capital model prices differ across destinations and are positivity correlated with the stock of human capital in that destination.

Second, the distinction between the two in its ability to fit bilateral trade shares in the data is a quantitative question. That is how well does this alternative story account for the pattern of trade relative to the model described here. Section 6.3 revisits this question and shows the two stories have different quantitative implications.

4.2 A Product Cycle

My model generates a product cycle as well. Vernon (1966) formulated the idea by arguing that recent vintages of goods are typically first produced and consumed in advanced countries.\(^ {10} \) On both the demand side and on the production side, my model generates this same outcome.

On the demand side my model clearly generates a product cycle. Corollary 1 says each country imports and consumes only one quality level. This implies that high human capital countries will demand higher quality/vintage goods relative to countries with lower human capital. If one thinks of human capital as growing according to a deterministic and common trend to all countries, then as the economy moves through time high human capital countries will always first consumer high quality products. Low human capital countries consume these quality types until the growth trend moves them to the appropriate level.

The left panel of Figure 2 illustrates this scenario. At time \( t \), the high human capital country consumes \( q_1 \) and the low human capital country consumes \( q_2 \). At \( t+1 \) each countries human capital grows by a common amount. Figure 2 illustrates a scenario with the low human capital country consuming the same quality level as the high human capital country in \( t \). And the high human capital country is consuming an even higher quality level in \( t+1 \).

\(^{10}\)Prominent articulations of product cycles are Flam and Helpman (1987) and Stokey (1991). My theory is distinguished by the sources of demand for quality and the mechanism which builds upon theories of capital-skill complementarity and skill in adoption and hence backed by the large amount of micro-evidence supporting these theories.
On the production side, my model generates a product cycle with high human capital countries producing mostly high quality goods. First, I will define a simple index of quality produced as:

\[ \text{Index of Quality Produced} = q_i^p = M_{ii}(q_i)q_i + M_{ji}(q_j)q_j. \]

There are a variety of represent the production basket, but this makes sense: if a country is in autarky then its production basket is equivalent to its consumption basket, if a country produces very little goods for domestic consumption than its production basket will be closer to the consumption basket of its trading partner. Given the assumptions on this two country example, I show in appendix that,

\[ q_1^p > q_2^p, \]

meaning high human capital countries’ production basket is of higher quality/more advanced than low human capital countries. The right panel of Figure 2 illustrates this scenario. At time \( t \), the high human capital country index of quality produced is \( q_1^p \) and the low human capital country index of quality produced is \( q_2^p \). At \( t + 1 \) each countries human capital grows by a common amount, and the production baskets each advance in unison. Similar to the arguments of Vernon (1966) higher quality/newer vintage goods are mostly produced by advanced/high human capital countries.

Differences in production baskets between the two countries are less than consumption baskets. The reason is that the producers in the low human capital country produce some high quality goods for export to the high human capital country and vice versa. Though the quality choice reduces producers productivity in the low human capital country relative to producers in the high human capital country, their are still some goods the low human capital country is able to export for idiosyncratic reasons.\(^{11}\) The key is that number of goods exported—the extensive margin—decreases in equilibrium. This then reduces the weight placed on high quality goods within a low human capital country’s production basket making it less “advanced” then the rich country.

5 Estimating \( \gamma \)

Given the theory I have outlined, in the next sections I will study the quantitative nature of the theory. I will ask two questions. First, per the discussion in Section 4.1.1, do we see these predictions on how prices at a disaggregate level should covary by destination, i.e. is \( \gamma = 0 \) or is \( \gamma > 0 \)? This is the defining quantitative property of my theory. In this

\(^{11}\)There is anecdotal evidence that poor countries producing high quality goods for export only; see Verhoogen’s (2008) discussion of Volkswagen producing new Beatles for export and old Beatles for domestic consumption in Mexico.
section I estimate $\gamma$ from disaggregate price data which will provide a test of my theory. Given these results I ask a second question: is this mechanism quantitatively important to understanding bilateral trade flows between rich and poor countries? Section 6 answers this questions.

The foundation for estimating $\gamma$ comes from the discussion in Section 4.1.1 and how relative export prices at a disaggregate level should be related to the relative levels of human capital in a destination countries with elasticity $\gamma$:

$$\frac{p_{j}^{fob}(x)}{p_{\ell}^{fob}(x)} = \left( \frac{h_{j}}{h_{\ell}} \right)^{\gamma}$$

in which superscript $fob$ denotes the free on board price. I will use the equilibrium relationship specified in (15) to estimate $\gamma$. Before proceeding, several decisions are necessary: First, I must take a stand on the data that corresponds with the prices and human capital data in (15). Second, I must turn (15) into a probability model and determine an appropriate estimator of $\gamma$. The next two subsections discuss each of the topics.

5.1 Export Unit Value Data

To compute f.o.b. prices for the estimation of $\gamma$ I will use data available for U.S. exports at Robert Feenstra’s website: http://cid.econ.ucdavis.edu/. Descriptions of this data are available in Feenstra (1997) and Feenstra, Romalis, and Schott (2002); see Schott (2004) for a useful discussion as well. In this data set, data on U.S. exports are categorized by 10 digit Harmonized System (HS) number including both raw quantities and dollar values for most categories. Furthermore, dollar values are reported as free alongside ship which includes charges associated with getting the shipment to the dock, but excluding shipping costs between countries.

To make use of this data, I will assume a 10 digit category is a good. This is a simplification but necessary. Armed with this assumption, I can compute unit values for shipments from the U.S. to each destination by dividing the total dollar value for each good by the quantity reported for categories that report quantity data. I will take these unit values to be a proxy for the prices on the left hand side of (15). Because I will test my model on its ability to replicate bilateral trade flows in manufactures for the year 1996, I restrict the sample to only manufactures which I define as a good belonging to one of 34 BEA manufacturing industry categories. The total amount of good categories used are 16,803. Furthermore, my sample of bilateral trade flows includes 77 countries and the sample of prices is restricted only to this set of countries as well.

12Since countries are points in space, f.a.s. and f.o.b. are the same, hence this distinction is not relevant.
5.2 Human Capital Data

I will use human capital data from Caselli (2005). This data is constructed using schooling data from Barro and Lee (2001) and then turned into a measure of human capital by using the formulae: 

\[ h_i = \exp(\phi(s_i)) \]

where \( h_i \) is human capital and \( s_i \) is average years of schooling for the population 25 years or older. Because of the assumptions made on the production of the final goods sector, in my model a workers log real wage will be proportional to his log human capital; see equation (17). The relationship between the wages and schooling is thought to be log-linear hence the rational for the assumed functional form. Finally, Caselli (2005) assumes \( \phi(s_i) \) is piecewise linear in \( s \) as in Hall and Jones (1999).

To be consistent with the assumptions behind this formulation of human capital, the implicit assumption I make is that agents are myopic with respect to their schooling choice and how it affects aggregate total factor productivity. As I show in equation (17), aggregate total factor productivity is endogenous depending upon the pattern of trade. Since the schooling choice influences the pattern of trade, the schooling choice has an indirect effect on total factor productivity. My assumption is that agents do not recognize this indirect effect.

5.3 An Estimate of \( \gamma \)

My benchmark approach uses a Poisson pseudo maximum likelihood (PPML) estimator to estimate \( \gamma \) from (15).\(^{13}\) The alternative is to take the logarithm of both sides of (15) and use ordinary least squares. By adopting a PPML estimator, I am making an assumption on how the conditional variance relates to the conditional expectation. The appropriate specification ultimately is an empirical question which I will test. Silva and Tenreyro (2006) advocates employing this estimator rather than simply log-linearizing models and applying least squares. They demonstrate the quantitative importance of this estimator by studying the results from estimating reduced form gravity models under alternative probability models.

Table 2 presents the estimation results. The first row presents the benchmark estimate. The point estimate is 7.11 with a standard error of 1.37. Through the standard errors are wide, the estimate is significantly different from zero in which product quality does not matter. The results from log-linear probability model in the bottom row of Table 2 provides a conflicting view point with a point estimate that is basically zero. Which probability model is appropriate? I follow Silva and Tenreyro (2006) and perform two tests to confirm the adequacy of my benchmark specification.

\(^{13}\)A pseudo maximum likelihood estimator is one where the true probability distribution function may not be of the family chosen to maximize the likelihood function. Gouriou, Monfort, and Trognon (1984) identify several special cases when consistent estimators are available, yet the true probability distribution function is not the family chosen. Of particular interest here is the PPML estimator. They show that the data need not be poisson and that \( y_i \) do not have to be integers for the estimator based on the poisson likelihood function to be consistent.
Table 2: Estimates of $\gamma$

<table>
<thead>
<tr>
<th>Approach</th>
<th>Estimate</th>
<th>Robust Std. Err.</th>
<th># of Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Levels, PPML</td>
<td>7.11</td>
<td>1.37</td>
<td>105,660</td>
</tr>
<tr>
<td>Log-linear, OLS</td>
<td>-0.04</td>
<td>0.05</td>
<td>105,660</td>
</tr>
</tbody>
</table>

**Specification Tests**

<table>
<thead>
<tr>
<th>Test</th>
<th>P-Value</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Park” Test (Log Linear, OLS is Valid)</td>
<td>0.000</td>
<td>Reject Log-Linear Model</td>
</tr>
<tr>
<td>GNR Test (Var(yi</td>
<td>x) $\propto b_0 E(yi</td>
<td>x)^{b_1}$)</td>
</tr>
</tbody>
</table>

The first test checks whether the log linear model using OLS is appropriate or not. Following Park (1966), Manning and Mullahy (2001) suggest running the following regression

$$\ln(y_i - \hat{y}_i)^2 = \ln b_0 + b_1 \ln \hat{y}_i + v_i$$

where $y_i$ are the data and $\hat{y}_i$ are the predicted values. Testing the null hypothesis that $b_1 = 2$ is a test of the log-linear model because this probability model implies that $Var(y_i|x) = b_0 E(y_i|x)^{b_1}$ with $b_1 = 2$. The first row in the bottom panel in Table 2 presents the results from this test by reporting the p-value. This test decisively rejects the log linear model.

The second test checks the adequacy of the PPML estimator by checking if $Var(y_i|x) = b_0 E(y_i|x)^{b_1}$ with $b_1 = 1$ as this probability model implies. Silva and Tenreyro (2006) provide the details, but the approach is to run the regression

$$(y_i - \hat{y}_i)^2 / \sqrt{y_i} = b_0 \sqrt{y_i} + b_0(b_0 - 1)(\ln \hat{y}_i) \sqrt{\hat{y}_i} + \epsilon_i.$$  

If the estimate of $b_0(b_0 - 1)$ is not statistically different from zero, than this implies the PPML assumption with $Var(y_i|x) \propto E(y_i|x)$ can not be rejected by the data. The second row in the bottom panel in Table 2 presents the results from this test. This test indicates that I can not reject the hypothesis behind the PPML probability model.

To summarize, these results support the my benchmark probability model generating an estimate of $\gamma = 7.11$. This means that the price variation across destinations is consistent with and supports my theory. I will use this value for $\gamma$ throughout the rest of the paper.
6 Product Quality and Bilateral Trade Flows

As another test of the theory, I ask if product quality is quantitatively important to understanding bilateral trade flows between rich and poor countries? Let me be more explicit regarding my goal here. I want to calibrate the model using data from outside sources without explicitly targeting bilateral trade volumes. Given the calibrated model, I will compute an equilibrium and compare the results from the calibrated model to the observed pattern of bilateral trade and contrast these results with alternative models. Below I describe my calibration approach.

6.1 Calibration

The objects to calibrate are trade costs, technology parameters, common factor shares, labor endowments, and $\theta$. I discuss each in turn below.

6.1.1 Calibrating Trade Costs

To calibrate trade costs I estimate a function relating trade costs to distance from trade flows for only OECD countries and then use these estimates and distance data to impute trade costs for the entire sample of countries I study. The reason I restrict my attention to only OECD countries is so I can test the model’s ability to replicate the entire pattern of trade.

I implement this approach by deriving the following expression for trade costs and trade data from the model:

$$\left( \frac{M_{ij} M_{ji}}{M_{jj} M_{ii}} \right)^{-\theta} = \tau_{ij} \tau_{ji}$$

(16)

I will assume that trade costs take on the functional form with $\tau_{ij} = \pi d_{ij}^{\rho}$ which is standard in the gravity literature. Given this functional form assumption and a stand on $\theta$, I estimate the parameters $\pi$ and $\rho$ for only OECD countries using equation (16), data on bilateral trade shares (discussed below), and data on the distance between OECD countries. The distance measures are in miles from capital city in country $i$ to capital city in country $j$ calculated by the great circle method and are from from Centre D’Etudes Prospectives Et D’Informations Internationales (http://www.cpeii.fr). Given the estimates for $\pi$ and $\rho$ I then impute the set of trade costs for all 77 countries I study.

6.1.2 Calibrating Technology Parameters

I calibrate each countries technology parameter, $\lambda_i$, by recovering it as a residual from a mapping in the model between real GDP per worker data, other observable data, and $\lambda_i$. 

21
I show in the appendix that real income per worker in the model can be expressed as:

$$\text{Real GDP Per Worker} = \lambda_i^\theta \alpha \beta_i M_{ii}^{\theta \alpha} h_i^{1-\alpha}$$ \hspace{1cm} (17)$$

Given data on income per worker, human capital, home trade shares, and the calibrated common parameter values, I can recover each country’s $\lambda_i$ as a residual to satisfy (17).

### 6.1.3 Factor Shares and Other Data

Given the model’s structure resulting in equation (17), I set $\alpha$ to be consistent with the exercises in the income accounting literature. To do so, I set $\alpha$ equal to 1/3. Gollin (2002) provides an argument for setting $\alpha$ equal to 1/3 by calculating labor’s share for a wide cross-section of countries and finding it equal to around 2/3.

$\beta$ controls value added in tradable goods production. Since tradable goods are assumed to correspond with manufactures, one measure of $\beta$ is manufacturing value added relative to gross manufacturing production. Using manufacturing value added and gross production data from UNIDO (1996–), 0.33 is the average across 61 of the countries with data available. Across all countries, there is no correlation between the calculated $\beta$ and a country’s level of development. Based on this evidence, 0.33 is a reasonable value.

I followed Alvarez and Lucas (2007) in selecting the value for $\eta$. Other than satisfying the necessary assumptions detailed in the appendix, this value plays no quantitative role. To calibrate $\theta$, I use the value estimated from Waugh (2007) which is 0.1818 for the same set of countries.

To compute an equilibrium I need measures of labor endowments. I use labor endowments from Caselli (2005) which are from information in Heston, Summers, and Aten (2002) as well. Note, that given my interpretation of labor any differences in capital stocks across countries will be manifested in my measure of $\lambda_i$.

### 6.2 Moments of Interest

To test the theory, I will compare data bilateral trade shares $M_{ij}$ between all countries in the sample to the predicted bilateral trade shares of the model. I constructed the data on bilateral trade shares following Waugh (2007):

$$M_{ij}(h_i) = \frac{\text{Imports}(h_i)_{ij}}{\text{Gross Mfg. Production}(q_{j \in \{h_j\}})_i - \text{Total Exports}(q_{j \in \{h_j\}})_i + \text{Imports}(h_i)_i}.$$

The terms in parenthesis denotes the theory implied quality of these flows. In the numerator is the the aggregate value of manufactured goods country $i$ imports from country $j$ which is implied by the theory to be quality $h_i$. This data is from Feenstra, Lipsey, and Bowen (1997) and manufactures is defined to be the aggregate across all 34 BEA manufacturing industry.
Table 3: Implications for Bilateral Trade

<table>
<thead>
<tr>
<th></th>
<th>$\text{Var}(\log(M_{ij}))$</th>
<th>Intercept: $\hat{\beta}_0$</th>
<th>Slope: $\hat{\beta}_1$</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>10.92</td>
<td>—</td>
<td>—</td>
<td>4,242</td>
</tr>
<tr>
<td>Baseline, $\gamma = 7.11$</td>
<td>9.97</td>
<td>-1.31</td>
<td>0.73</td>
<td>4,242</td>
</tr>
<tr>
<td>No Product Quality, $\gamma = 0$</td>
<td>4.78</td>
<td>-2.46</td>
<td>0.50</td>
<td>4,242</td>
</tr>
<tr>
<td>$\gamma = 0$ and $\bar{L} = L \times H$</td>
<td>5.33</td>
<td>-2.27</td>
<td>0.54</td>
<td>4,242</td>
</tr>
</tbody>
</table>

Note: Intercept and Slope values are from a regression of the logarithm of $M_{ij}$ generated from the model on the logarithm of the data.

There are several terms in the denominator. First there is gross manufacturing production minus total manufactured exports (for the whole world). Note that a country may be producing many different quality types of goods, but this procedure leaves only the value of quality $h_i$ goods produced for home. Finally, this value is added to manufactured imports (for only the sample) of quality $h_i$. The result is an expenditure share by dividing the value of inputs country $i$ imports from country $j$ divided by total value of inputs in country $i$—all of quality type $h_i$ goods. Gross manufacturing data is either from UNIDO (1996-), OECD, or the World Bank.

With this data, the question is: Can the calibrated model quantitatively account for the variation in bilateral trade shares? The specific moment of interest I have in mind is the variance the logarithm of $M_{ij}$. My reasoning follows from the observation that if all countries were the same, this value would be zero. Countries have differences labor endowments, calibrated trade costs, and technology parameters generating some variation in bilateral trade shares. My exercise asks: How much more of the variation in bilateral trade shares can be understood by differences in human capital affecting the choice of quality?

6.3 Results

Table 3 presents the results. The first row depicts the variance the logarithm of $M_{ij}$ data. The second row presents the results for the calibrated model with product quality. Here the model generates nearly 90 percent of the observed variation in bilateral trade. Despite my focus on the variance, one may wonder about the performance of the model with respect to the means. The second and third column presents the intercept and slope coefficient from a regression of the logarithm of $M_{ij}$ generated from the model on the logarithm of the data. If the model fit the data perfectly, then these values should be 0 and 1. For the baseline model these values are -1.31 and 0.73.

Table 3 illustrates the quantitative importance of product quality relative to alternative models. The third row presents the results for the calibrated model as described above,
but here product quality plays no role with $\gamma$ set equal to zero. In this case, the model generates only 43 percent of the observed variation in bilateral trade. More telling, the model with product quality generates 2 times more of variation in the bilateral pattern of trade relative to the no product quality model. Furthermore, the intercept and slope coefficients tell a similar story—the product quality model performs significantly better in replicating bilateral trade flows relative to the restricted model.

The bottom row of Table 3 presents the result for a model with no product quality, but human capital differences incorporated in a standard way, see the discussion in Section 4.1.1. This model generates a bit more variation in bilateral trade than the model with no human capital differences and no product quality. The reason is because more variation in fundamentals across countries maps into more variation in bilateral trade shares. However, the product quality model still performs significantly better in replicating the variation in bilateral trade shares.

7 Conclusion

In this paper, I developed a simple model: firms make choices of the quality/technology of their intermediate inputs given the set of endowments they have access to. This choice affects the firms ability to produce goods domestically and internationally, thus shaping the pattern of trade. Like traditional endowment driven Heckscher-Ohlin trade theory my theory puts endowment differences at the center of the model, albeit in a different manner. Unlike the difficulties Heckscher-Ohlin trade theory have faced explaining the data, I demonstrated the quantitative importance of my theory in understanding the variation in bilateral trade volumes across countries. Moving forward, research studying the time-series implications of my model may be yield interesting results. For example, the dramatic growth in the Korea’s trade volumes that accompanied Korea’s growth “miracle” is an episode my model have much to say about. I am currently exploring this topic.
References


8 Appendix: Omitted Proofs and Derivations

Below is the proposition showing that final goods firms in country $i$, demand quality type $q_i = h_i$ of the intermediate good.

**Proposition 4** In any competitive equilibrium, it is optimal for final goods producers in country $i$ to set $q_i = \left(\frac{\beta}{1-\beta}\right)^{\frac{1}{\alpha}} h_i$.

**Proof:** From equation (8), the optimal $q$ is the argument which minimizes:

$$q = \arg \min \left\{ h^{1-\alpha-\gamma} \left[ \omega q^\rho + h_i^\rho \right]^\frac{1}{\rho} w_i^{1-\alpha} p_i(q)^{1-\alpha} \right\}.$$

From proposition ??, the problem to finding the optimal $q$ can be restated as:

$$\min_q (\omega q^\rho + h_i^\rho)^\frac{1}{\rho} q^{\gamma(1-\alpha)},$$

in which terms irrelevant to the optimization problem are abstracted from. This optimization problem yields the following first order condition:

$$\frac{-\gamma}{\rho} [\omega q^\rho + h_i^\rho]^{-\frac{1}{\rho}} - 1 \omega q^\rho - 1 q^{\gamma(1-\alpha)} + [\omega q^\rho + h_i^\rho]^{-\frac{2}{\rho}} \gamma(1-\alpha) q^{\gamma(1-\alpha)} = 0$$

$$\Rightarrow q = \left(\frac{\beta}{1-\beta}\right)^{\frac{1}{\alpha}} h_i$$

which shows that $q_i \propto h_i$ in the final goods sector and is the same choice as in the intermediate goods sector. ■

8.1 Factor Allocations

This section builds on the observations in Álvarez and Lucas (2007) and extends them to the case of the model with product quality. With the Cobb-Douglas technologies, it is straightforward to show that:

$$Total \ Factor \ Payments = FactorShare \times TotalRevenue$$

From this relationship we can show that fraction $\alpha$ of labor will be allocated to the final goods sector and fraction $\beta$ of the aggregate intermediate good to the final goods sector.

To show this assume the given country has a labor endowment of 1. For the production of good $x$ of quality level $q'$ the following relationship must hold:

$$w_i n_i(q', x) = \beta p(q', x) y(q', x)$$
where \( n(q', x) \) is the fraction of workers employed in the production of good \( x \) of quality level \( q' \). Aggregating over all goods \( x \) and all quality levels produced yields:

\[
\frac{w_i(1 - n_i^f)}{\beta} = \sum_{j=1}^{N} \int p(q_j, x) y(q_j, x) dx
\]

(18)

in which \( n_f \) is the fraction of workers employed in the production of final goods. By similar arguments, one can show that:

\[
\frac{p(q_i)[y(q_i) - y^f(q_i)]}{1 - \beta} = \sum_{j=1}^{N} \int p(q_j, x) y(q_j, x) dx
\]

(19)

in which \( y^f(q_i) \) are the units of the aggregate intermediate good employed in the production of final goods. Similarly for the final goods sector, we have the following expressions:

\[
\frac{w^f}{\alpha} = p^f y^f \quad \text{and} \quad \frac{p(q_i) y^f(q_i)}{1 - \alpha} = p^f y^f
\]

(20)

Notice that from equations (18) - (20) one can arrive at two equations to solve for the allocations of factors across sectors. From here it is straightforward to show that labor is allocated such that \( \alpha \) is allocated towards the final goods sector and that \( \beta \) of the aggregate intermediate good is allocated towards the final goods sector.

### 8.2 Two Country Example

This section derives the equations for a two country example. First note that from equation 14 and normalizing the wage in country one equal to one we have:

\[
w_2 = \frac{M_{12}(h_1)}{M_{21}(h_1)}.
\]

Now let’s study each country’s relative import share using equation (13) yielding:

\[
\frac{M_{12}(h_1)}{M_{21}(h_2)} = w_2^{-\frac{\beta}{\rho}} \left( \frac{p_2(h_2)}{p_1(h_1)} \right)^{\frac{\beta - 2}{\rho}} \left( \left[ \frac{\kappa h_2}{\kappa h_1} \right]^\rho + \left( \frac{h_2}{h_1} \right)^\rho \right)^{-\frac{\gamma}{\rho}}
\]

(21)

where the observation that the denominator of equation (13) is equivalent to each country’s price index to the power \( -\frac{1}{\rho} \). Next make the observation that \( p_1(h_1) = h_1^\gamma p \) and likewise \( p_2(h_2) = h_2^\gamma p \). This allows us to simplify equation (21):

\[
\frac{M_{12}(h_1)}{M_{21}(h_2)} = w_2^{-\frac{\beta}{\rho}} \left( \frac{h_2}{h_1} \right)^{\frac{\beta}{\rho}}
\]
then substituting in for $w_2$ from balanced trade one finds that

$$\frac{M_{12}(h_1)}{M_{21}(h_2)} = \left(\frac{h_2}{h_1}\right)^{\frac{\gamma^3}{\eta + \eta^3}} < 1$$  \hspace{1cm} (22)$$

To verify the claim in Section 4.2, note the two production indexes for each country may be expressed as:

$$q_1^p = q_1^d - M_{12} \left(q_1^d - \frac{M_{21}}{M_{12}} q_2^d\right)$$

$$q_2^p = q_2^d + M_{12} \left(q_1^d - \frac{M_{21}}{M_{12}} q_2^d\right).$$

Subtract $q_1^p$ from $q_2^p$ and by contradiction assume my claim is not true. This implies the following expression must hold:

$$q_1^d - q_2^d < 2M_{12} \left(q_1^d - \frac{M_{21}}{M_{12}} q_2^d\right) \implies$$

$$q_1^d (1 - 2M_{12}) < q_2^d \left(1 - 2\frac{M_{21}}{M_{12}}\right).$$

Now it is straightforward to show that $\frac{M_{12}(h_1)}{M_{22}(h_2)} = 1$. This fact and that $\frac{M_{12}(h_1)}{M_{21}(h_2)} < 1$ implies $M_{12} < 0.50$. Revisit the inequality above. The left-hand side is positive but $\frac{M_{12}(h_1)}{M_{21}(h_2)} < 1$ implies the right-hand side is negative resulting in a contradiction. Thus the claim that $q_1^p > q_2^p$ is proved.
Figure 1: Quality Model versus No Quality Model

\[
q_1 = q_2 \\
h_1 = h_2
\]
Figure 2: A Product Cycle