Foreclosures and House Price Dynamics in Local Housing Markets

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Abstract

We develop a model of collaterized lending (mortgages) with the risk of default. The focus of our analysis is on the interaction between home price fluctuations in local housing markets (metropolitan areas) and the frequency of mortgage defaults. There are three distinguishing features of our model. First, default on mortgages occur only when there is a fall in the value of local housing stock and a borrower’s current earnings is low. Second, mortgages are priced taking into account household risk characteristics and the amount put forward as downpayment (which can be freely chosen). Third, the price of housing as well as mortgages is affected by a city-specific (aggregate) shock. Preliminary results indicate that if households have high discount factors (low discount rates) and rational expectations, the possibility of capital loss or gain on housing leads to offsetting movements in downpayments, with little or no change in equilibrium default frequency.

Key Words: Foreclosures, Default, Mortgages, House Prices, Lending Standards

JEL:
1 Motivation

We develop a model of collaterized lending (mortgages) with the risk of default. The focus of our analysis is on the interaction between home price fluctuations in local housing markets (metropolitan areas) and the frequency of mortgage defaults. Our analysis is motivated by the recent experience of declining house prices and rising defaults on mortgages in the United States. As is well known, metropolitan areas that have witnessed the largest decline in home prices are also the ones experiencing the most increase in mortgage defaults. Our goal is to study an economic environment in which house price declines can trigger default and use it to study how lenders and borrowers adjust their behavior in mortgage markets in response to the possibility of capital gain and loss.

There are three distinguishing features of our model. First, in our model default on mortgages occur only when the mortgagee’s current earnings are low and there is a decline in the value of the local housing stock due to some adverse city-specific income or population shock. This in contrast to most studies on mortgage default among where default is triggered by idiosyncratic shocks to the value of pledged property and household earnings (see, for instance, Chambers, Garriga and Schlagenhauf (2008)).

Second, mortgages are priced taking into account household risk characteristics and the amount of funds put forward as downpayment – which can be freely chosen. Thus, unlike many studies in this literature, we do not impose exogenous downpayment requirements on borrowers. We allow endogenous downpayments because in reality lenders can choose to lend the full value of the property through various means (a primary mortgage supplemented by a home equity line of credit, for instance). Making downpayments endogenous allows us to analyze variations in lending standards associated with variations in housing demand.

Third, the price of housing as well as the price of mortgages is affected by a city-specific (aggregate) shock. Thus, a methodological contribution of our paper is to develop a tractable framework to analyze the equilibrium interactions between aggregate shocks and housing market outcomes in a model of uninsured income risk and collaterized borrowing and default.

Preliminary results indicate that if households have high discount factors (low discount rates) and
rational expectations, the possibility of capital loss or gain on housing leads to offsetting movements in downpayments, with little or no change in equilibrium default frequency. We suspect, however, that introducing households with lower discount factors and offering them mortgages with low initial payments would result in more risky mortgages being written with consequent increase in default in the event of a decline in house prices.

2 Environment

We will study the housing equilibrium in a representative city. Time is discrete and indexed by \( t = 0, 1, 2, \ldots \) The city has a fixed stock of housing space \( H > 0 \). The city is subject to shocks \( z \) that affects its population level and the productivity of its residents. The shocks \( z \) are drawn independently from a Markov process with finite support \( Z \) and \( P(z_{t+1} = z' | z_t = z) \) is \( p_{zz'} \geq 0 \). We will assume that this process implies a unique invariant distribution \( p_z \) where \( p_z > 0 \) for all \( z \in Z \).

2.1 People

There are two types of people who live in the city. One type are permanent residents whose measure is normalized to 1. The other type are transient residents whose measure \( N(z) \) fluctuates with the city shock \( z \). In what follows, we will analyze the decision problem of permanent residents. Transient residents are assumed to rent a certain given amount of housing. Thus, the size of the transient population just affects the supply of housing available to permanent residents.

There are two goods in this economy, a homogenous consumption good and the service flow from housing space. We permit the service flow from owner-occupied housing space to be potentially higher than the the service flow from rented housing space. Let \( c_t \) denote consumption of the homogeneous good in period \( t \), let \( h_t \) denote the consumption of housing space in period \( t \) and let \( e_t \) be an indicator variable that is 1 if the person owns the housing space \( h_t \) and 0 otherwise. Then a permanent resident values the consumption stream \( c = \{c_0, c_1, c_2, \ldots \} \), \( h = \{h_0, h_1, h_2, \ldots \} \).
and $e = \{e_0, e_1, e_2, \ldots, \}$ according to

$$U(c, h, e) = \sum_{t=0}^{\infty} \beta^t u(c_t, h_t, e_t), 0 < \beta < 1. \tag{1}$$

Given $e_t$, $u(\cdot, \cdot, e_t) : \mathbb{R}^2_+ \rightarrow \mathbb{R}$ is continuous, bounded, strictly concave. We assume that $u(c_t, h_t, 0) = u(c_t, h_t) = \left( c_t^{1-\eta} h_t^n \right)^{1-\sigma} / (1 - \sigma)$ and $u(c_t, h_t, 1) = u(c_t, h_t) + \theta$, where $\theta \geq 0$. Note that housing space used by a resident must be either fully owned or fully rented.

Each (permanent) resident independently draws a earnings level $w$ from a finite-state Markov process with non-negative positive support $W \subset \mathbb{R}_+$. The shock $z$ affects this earnings draw. Given $z$ and the $w_{t-1} = \bar{w}$, the probability that $w_t = w$ is $F_z(w, \bar{w})$.

### 2.2 Market Arrangement

The homogeneous consumption/endowment good is the *numeraire* good. There are four markets in this economy.

- There is a market for owner-occupied housing in which the price per unit of housing space is $P(S)$ where $S$ denotes the aggregate state of the economy (to be defined more precisely below). An individual who buys $k'$ units of housing pays a purchase price of $P(S) \cdot k'$. We assume that owner-occupied housing comes in discrete sizes given by the finite set $K$.
- Second, there is a market for rental housing in which the rent per unit of housing space is $R(S)$. An individual who rents $h$ units of housing space pays $R(S) \cdot h$ as rent.
- Third, there is a market for risk-free deposits. The interest rate on deposits is $r > 0$. An individual who saves $a$ units receives $a(1 + r)$ next period.
- Finally there is a market for mortgages where individuals can borrow to fully or partially fund the purchase of a house. For tractability we assume that all mortgages are perpetuities – in the sense that each mortgagor pays some agreed to amount $x$ each period unless the mortgagor defaults or the mortgage is paid off. In case of default, the financial intermediary gets ownership of $h$. In case the mortgage is paid off, the lender receives present value of the
promised payment $x$ discounted at the risk-free rate. Because of the possibility of default, the (unit) price $q$ of a mortgage depends on the amount of housing pledged as collateral $h$, the payment amount $x$ promised in perpetuity, the individual’s post-purchase savings $y$, his current earnings $w$, and the current aggregate state $S$. An individual who takes out a mortgage with perpetuity payment $x$ receives $q(x, h, y, w, S) \cdot x$ in the current period.

### 2.3 Government Sector

There is also a government sector that levies taxes on income. For simplicity, we assume that government consumption of goods does not have provide any benefits to households. The amount of taxes $g$ to be paid by an individual is modeled after the US tax code. An individual’s tax liability depends on the individual’s taxable income which is calculated as the sum of earnings and interest income minus the greater of (i) mortgage payment $x$ or (ii) the standard deduction $s$. That is:

$$ g(w, a, x) = \tau(w + ra - \max[x, s]) \cdot (w + ra - \max[x, s]). $$

Here the tax rate, or bracket, $\tau(\cdot)$ is weakly increasing in taxable income.

We can now make precise the aggregate state for a city. Let $\mu(k, x, a, w)$ be the distribution of people over owned housing (zero for renters), mortgage payments (zero for renters), savings (excluding interest earnings) and current earnings. Then, the aggregate state $S$ is summarized by the pair $S = \{\mu(k, x, a, w), z\}$.

### 2.4 Financial Intermediaries

Financial intermediaries take in deposits, sell mortgages, and own the housing space rented by people. All intermediaries can borrow or lend funds in a world credit market at a given risk-free interest rate $\bar{r} > 0$. We will assume that there is one representative risk-neutral intermediary that takes all prices as given.
3 Decision Problems

Before we state the decision problem of people and financial intermediaries, it is helpful to state the timing of events in a period.

- At the start of a period the shock $z$ is realized.
- Then, permanent residents independently draw their earnings $w$. As noted earlier, the earnings distribution depends on current $z$ and the resident’s previous period earnings $\tilde{w}$.
- All permanent residents choose $h$, $a'$, $k'$ and $x'$. For tractability, we assume that those permanent residents who currently do not own a house are the only ones who can choose to purchase a house. Current owners can keep their house and make their mortgage payment, or they can sell their house and pay off their mortgage, or they can default on the mortgage and forfeit their house. If they choose to either sell or default, they must rent in that period – meaning, that the sell or default decision occurs at the start of the period and selling or defaulting household cannot live in the property the sold or defaulted on.

3.1 People

The state variables for individuals are $k, x, a, w$ and the state variable for the city is $S = (\mu(k, x, a, w), z)$. Consider first the decision problem of a household that does not own housing. For this household $k = 0$ and $x = 0$. The household may choose to buy a house or may choose to rent. If he chooses to buy, he solves:

$$
M_1(0,0,a,w;S) = \max_{c \geq 0, k' \in K, x \geq 0, a' \geq 0} \{U(c,k') + \theta + \beta E_{w', S'}|w, S V(k', x, a', w', S')\}
$$

$$
c = w - g(w, a, x) + a(1 + r) - a' - P(S) \cdot k' + q(x, k', a', w, S) \cdot x
$$

$$
q(x, k', a', w, S) \cdot x \leq P(S) \cdot k'
$$
If he chooses to rent he solves

\[ M_0(0, 0, a, w; S) = \max_{c \geq 0, a' \geq 0, h \geq 0} \{ U(c, h) + \beta E_{w', S'} w, s V(0, 0, a', w', S') \} \]
\[ c = w - g(w, a, 0) + a(1 + r) - a' - R(S) \cdot h \]

Define \( M(0, 0, a, w; S) \) as:

\[ M(0, 0, a, w; S) = \max \{ M_1(0, 0, a, w; S), M_0(0, 0, a, w; S) \} \]

Consider next the decision problem of a household that does own a house and has an outstanding mortgage. The household may choose to keep the current house, sell it or default on the mortgage.

If he chooses to keep his house, he solves:

\[ K_1(k, x, a, w; S) = \max_{c \geq 0, a' \geq 0} \{ U(c, k) + \theta + \beta E_{w', S'} w, s V(k, x, a', w', S') \} \]
\[ c = w - g(w, a, x) + a(1 + r) - a' - x \]

If he chooses to sell he solves:

\[ K_0(k, x, a, w; S) = \max_{c \geq 0, h \geq 0, a' \geq 0} \{ U(c, h) + \beta E_{w', S'} w, s V(0, 0, a', w', S') \} \]
\[ c = w - g(w, a, 0) + a(1 + r) - a' + P(S) \cdot [k - \chi_0] - (1 + 1/\bar{r}) \cdot x - R(S) \cdot h \]

Here \( \chi_0 \) is the cost of selling a house and is some percentage of the sale price of the house \( P(S) \). Selling the house requires the individual to buy back the perpetuity \( x \) at the risk-free interest rate.\(^1\)

If the household chooses to default on the mortgage, he solves:

\[ K_D(k, x, a, w; S) = \max_{c \geq 0, h \geq 0, a' \geq 0} \{ U(c, h) - \Delta + \beta E_{w', S'} w, s V(0, 0, a', w', S') \} \]
\[ c = w - g(w, a, 0) + a(1 + r) - a' - \chi_D - R(S) \cdot h \]

Here, \( \Delta > 0 \) is the of stands in for the non-pecuniary costs of foreclosure and \( \chi_D \) for the pecuniary costs.

\(^1\)Ideally we should require the individual to buy back the perpetuity at \( \tilde{q} \cdot x \) where \( \tilde{q} \) is the price the individual paid when he issued the perpetuity. But to implement this would require expanding the state vector.
costs. Foreclosure results in the individual losing the house as well as the mortgage. Since there is no unobserved heterogeneity in the model, there are no adverse “reputation effects” of default. Consequently default does not affect an individual’s ability to acquire a mortgage in the future.

The standard contraction mapping argument establishes that $V(k, x, a, w, S)$ exists and is unique. Let $c(k, x, a, w, S)$, $k'(k, x, a, w, S)$, $x'(k, x, a, w, S)$, $h(k, x, a, w, S)$, and $d(k, x, a, w, S)$ denote the optimal decision rules for consumption, size of owned housing, size of mortgage payments, housing services, and default ($d = 1$ indicates default and $d = 0$ indicates no default). Also, let $s(k, x, a, w, S) = 1$ indicate the case where the individual sells the house, i.e., $s(k, x, a, w, S) = 1$ if and only if $k > 0$, $d(k, x, a, w, S) = 0$, and $k'(k, x, a, w, S) = 0$.

As noted earlier, we do not model the decision problem of transient residents. We simply assume that each transient resident rents $h$ of housing space.

### 3.2 Financial Intermediaries

The representative financial intermediary accepts deposits, buys mortgages, and buys and rents housing. We will assume that the intermediary chooses how much to engage in these activities on the basis of the expected return in each of these activities. Denote the intermediary’s expected net rate return (profits) on a deposit of size $a$ by $\pi(a)$, the net expected return on a mortgage with characteristics $k'$, $x'$, $a'$, $w$ and $S$ by $\pi(k', x', a', w, S)$ and the net expected return on the purchase of $h$ units of housing in state $S$ by $\pi(h, S)$. Correspondingly, let $m(a)$, $m(k', x', a', w, S)$ and $m(h, S)$ denote the measure (more precisely, the density or pdf) of such contracts acquired by the financial intermediary. Then, the financial intermediary’s decision problem is:

$$\Pi(S) = \max_{\{m(a), m(k', x', a', w, S), m(h, S)\}} \left\{ \int \pi(a)m(da) + \int \pi(k', x', a', w, S)m(dk', dx', da', dw, S) + \int \pi(h, s)m(dh, S) \right\}$$

For there to be a (bounded) solution to this problem the net expected returns on each type of asset must be non-positive. For deposits this requires $\pi(a) = a - [a(1 + r)]/(1 + r) \leq 0$ for all $a$. This
requirement reduces to

\[(1 + r) \geq (1 + \bar{r}).\] \hspace{1cm} (3)

For housing, this requires that \( \pi(h, S) = R(S) \cdot h + E_{S'|S}[P(S') \cdot (1 - \delta)h]/(1 + \bar{r}) - P(S)h \leq 0 \) for all \( h \) (financial intermediaries do not pay any cost for selling houses), where \( \delta \) is the depreciation on rental properties. This requirement reduces to

\[P(S) \geq R(S) + E_{S'|S}[P(S')(1 - \delta)/(1 + \bar{r})] \] \hspace{1cm} (4)

For mortgages, the expression for net return is more involved. When the intermediary acquires a mortgage it gives up \( q(k', x, a', w, S) \cdot x \) in goods. Next period, if the individual defaults the intermediary receives \( [P(S') - \chi_{LD}] \) where \( \chi_{LD} \) is the cost of foreclosure to the intermediary; if the individual sells the property the intermediary receives \( x \cdot (1 + 1/\bar{r}) \); and if he neither defaults nor sells then the intermediary receives \( x \) plus the value of the continuing mortgage which is then given by \( q(k', x, a'', w', S') \cdot x \). Recalling the definition of \( d(k, x, a, w, S) \) and \( s(k, x, a, w, S) \), the requirement that the expected net return from a mortgage \( \pi(k', x', a', w, S) \) be non-positive becomes

\[q(k', x', a', w, S) x \geq E_{w', S'|w,S} \left\{ d(k', x', a', w', S')[P(S') - \chi_{LD}] + s(k', x', a', w', S')[x(1 + 1/\bar{r})] + (1 - d(k', x', a', w', S'))(1 - s(k', x', a', w', S'))[P(S') - \chi_{L,D}] \right\} \hspace{1cm} (5)

4 Equilibrium

An equilibrium consists of rental pricing function \( R^*(S) \), a housing price function \( P^*(S) \), a deposit interest rate \( r^* \), a mortgage price function \( q^*(k, x, a, w, S) \), a law of motion for the distribution \( \mu^* = \Gamma^*(S) \), and a set of decision rules \( c^*(k, x, a, w, S), h^*(k, x, a, w, S), k^*(k, x, a, w, S), x^*(k, x, a, w, S), d^*(k, x, a, w, S) \) and \( s^*(k, x, a, w, S) \) such that:

1. The decision rules are optimal given \( R^* \), \( P^* \), \( q^* \) and \( \Gamma^* \).
2. The net returns (3)-(5) are zero.

3. The law of motion $\Gamma^*$ is implied by the decision rules.

4. Demand for housing equals supply, that is, $\int h^*(k, x, a, w, S) d\mu = H - \bar{h} \cdot N(S)$.

5 Parameter Selection and Calibration

The preference parameter $\sigma$ was set to 2, which is the standard value in models of consumer debt and default. The exponent to $h$ in the momentary utility function was set to 0.25 to be consistent with observed average share of rent payments in household budgets of around 25 percent (Jacob and Shipp (1990)). The discount factor $\beta$ is set to 0.91. The benefit from homeownership parameter $\theta$ is set to 0 for the time being. In the future, this parameter will be selected to match the average homeownership rate.

The earnings process for households $F_z(w, \bar{w})$ has idiosyncratic and city-level components. The interaction between the city-level and idiosyncratic shocks, and the consequences of such interaction for the housing market equilibrium, is an important focus of this paper. Thus, rather than selecting this process to match one set of facts, we study with several different processes. In all these experiments, we assume that the set $W$ (from which $w$ is drawn) contains 5 elements. These elements, and the associated transition matrix, is chosen to deliver an individual earnings process with first-order autocorrelation of 0.9, an innovation of variance of 0.0076 and a median earnings level of 1 (which is a normalization). The city-level shock either affects the size of these elements (multiplicatively), or it affects the probability with which these elements can be drawn.

- A process where the city-level shock has a level effect on individual earnings. In this example, $Z = \{0, 1\}$ and the probability of switching between the two states is given by $p(z', z) > 0$. When the state is $z$ and the selection from the idiosyncratic set $W$ is $w$, the household’s (before tax) earnings is given by $[1 + \alpha z - \alpha (1 - z)] w$. Thus, in the high city shock state, all incomes rise by $\alpha$ percent above the mean (roughly) and in the low city shock state, all incomes decline by $\alpha$ below the mean.
A process where the city level shock temporarily affects the growth of incomes. In the example, \( Z = \{0, 1, \ldots, T\} \), where \( T \geq 2 \). The state \( z = 0 \) is an absorbing state, meaning that if \( p(z', 0) = 0 \) for all \( z > 0 \). Also, \( p(0, T) = 1 \), meaning that the city shock goes to 0 from \( T \) for sure (and then stays at zero forever). For \( z \) other than 0 or \( T \), \( p(z+1, z) = 1 - p(0, z) = p > 0 \). When the state is \( z \) and the selection from the idiosyncratic set \( W \) is \( w \), the household’s (before tax) earnings is given by \( [1 + \alpha]^z w \). This process allows us to study the situation where the city has been in state 0 forever but unexpectedly switches to state 1. Once it is in state 1 it may continue to grow with probability \( p \) at the rate \( \alpha \) for at most \( T \) periods.

A process where the city level shock affects the probability with which \( w \) are drawn. In this example, \( Z = \{0, 1\} \) and \( p(z', z) > 0 \). The shock \( z \) does not affect the value of household earnings, but affects the probability with which the worst realization of earnings is drawn. If we interpret the worst realization of earnings as unemployment, it allows us to study the interaction between fluctuations in the unemployment rate and housing market outcomes.

The tax code is chosen to resemble the US tax code (Gouviea and Strauss (1994)). Normalizing median earnings to 1, the tax brackets are \( [0, 0.192], (0.192, 1.177], (1.177, 2.850], (2.850, 5.946], (5.946, 12.926] \) and \( (12.926, \infty] \). The corresponding tax rates are \( \{0, 0.15, 0.25, 0.28, 0.33, 0.35\} \). The standard deduction, normalized by median earnings, is set at 0.21.

The cost of selling a house varies from one locality to another but, an average selling cost of 5 percent seems reasonable. However, since we have set the value of homeownership parameter to 0, we assume for the moment that the selling cost parameter \( \chi_0 = 0 \) also. The psychological costs of default are hard to measure and the pecuniary costs seem to be mostly related to the adverse effects of a lowered credit score. For the moment, we assume that there are no pecuniary or psychological costs to default and thus \( \Delta \) and \( \chi_D \) are both set to 0. Default, however, is costly to the lender. Estimates of the cost of foreclosure as a percentage of the market value of a house range from 20 percent to 50 percent. We set \( \chi_{LD} = 0.20 \).

The set of house sizes, \( K \), is taken to be the set of 21 equally spaced points between 3 and 8. The exact sizes are not important, since the choice of this set amounts to a normalization of the total housing stock \( H \).
Finally, the risk free interest rate available to the financial intermediary is set at 4 percent (annualized).

6 Computational Strategy

The exact computation of the model is challenging because of the presence of the distribution of people over individual states as part of the aggregate state of the economy and the fact that we need to compute many, many equilibrium prices. Recall that the equilibrium requires us to compute the price of housing $P(S)$, the rental rate $R(S)$, and the entire menu of mortgage interest rates (or prices) $q(k, x, y, w; S)$.

The typical approach is to assume that what matters for the determination of equilibrium prices are some specified moments of this distribution and to postulate a forecasting equation for the evolution of these moments whose accuracy can then be checked via simulation (Krusell and Smith (1997)). An alternative approach is due to Colussi (2006). In this approach, one attempts to forecast equilibrium prices directly via a forecasting equation that contains current prices and the current aggregate shock. In effect, current prices are taken to be the relevant “summary statistic” of the current distribution of people over individual states. This approach seems more suited to our problem and is the approach followed in this paper.

Exactly what forecasting equation works best can only be determined via experimentation. The form of the forecasting equation will certainly depend on the way in which the aggregate shock interacts with individual earnings process. Here, we describe our initial approach for the case where the aggregate shock affects the level of earnings (the first example discussed in the previous section). At this point, our computational strategy is to assume that the forecasting equation for the price of housing is given by

$$\ln(P') = \rho \ln(P - \bar{P}) + \nu z$$  \hspace{1cm} (6)

Then, the net return equation for housing (4) implies an (easily computable) equilibrium relationship between $P$ and $z$ and the current rental price $R$. We will denote this relationship as
Finally, note that the assumption that $P$ is the "summary statistic" of the distribution means that the price of mortgages depend on $P$ and $z$. Thus, the equilibrium requires solving for $q(k, x, y, w; P, z)$ satisfying the zero profit condition (5). Needless to say, if equation (6) performs poorly, the default risk premium implicit in the solution $q(k, x, y, w; P, z)$ will not accurately reflect the actual default frequency on mortgages.

7 Some Partial Equilibrium Results for the Case of Population Shocks

Consider the situation where $Z = \{0, 1\}$ and the shock $z$ affects the size of the transient population only (and does not affect earnings in any way). In particular, $N(1) > N(0)$ and $p(0, 0) = p(1, 1) = 0.95$.

The full solution of the model is not yet available. We report the results from assuming a very specific forecasting equation, namely, $\ln(P) = \bar{P} + \nu z$. The results, although not an equilibrium, provide some insights into the forces at work in the model.

First we discuss the "steady-state" results. This is the situation when the economy is on one state for a long time. Tables 1-3 gives the values of the key variables of interest for the high and low population states.

Table 1 (Panel A) The high population state ($z = 1$)

<table>
<thead>
<tr>
<th>Earnings</th>
<th>Prop of homeowners</th>
<th>Size of owned houses</th>
<th>Size of rentals</th>
<th>Downpayment ÷ Price of house</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3312</td>
<td>0</td>
<td>-</td>
<td>0.0297</td>
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<td>0.5562</td>
<td>0</td>
<td>-</td>
<td>0.1288</td>
<td>-</td>
</tr>
<tr>
<td>0.9917</td>
<td>0.0033</td>
<td>3</td>
<td>0.4760</td>
<td>0.0857</td>
</tr>
<tr>
<td>1.7684</td>
<td>0.9722</td>
<td>3.2798</td>
<td>1.0001</td>
<td>0.1144</td>
</tr>
<tr>
<td>3.1533</td>
<td>0.9659</td>
<td>7.6839</td>
<td>2.4139</td>
<td>0.1445</td>
</tr>
</tbody>
</table>
Table 1 (Panel B) The low population state \((z = 0)\)

<table>
<thead>
<tr>
<th>Earnings</th>
<th>Prop of homeowners</th>
<th>Size of owned houses</th>
<th>Size of rentals</th>
<th>Downpayment÷Price of house</th>
</tr>
</thead>
<tbody>
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<td>-</td>
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<td>-</td>
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<td>8</td>
<td>5.0267</td>
<td>0.0434</td>
</tr>
</tbody>
</table>

Table 2 (Panel A) Proportion of down payment relative to house price: High state

home size

<table>
<thead>
<tr>
<th>Earnings</th>
<th>3</th>
<th>4.25</th>
<th>5.5</th>
<th>6.75</th>
<th>8</th>
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Table 2 (Panel B) Proportion of down payment relative to house price: Low state

home size

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<th>5.5</th>
<th>6.75</th>
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Table 3: Default spreads in high population state (default spreads in low state are 0).
Home sizes

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In steady state people only people with relatively high income own homes. Since there is no utility gain from ownership, a household will choose renting unless ownership gives it a strong enough tax advantage. The benefits of renting is that the household can choose exactly the size of housing it desires given its income. Thus if its income changes it can costlessly change the amount of housing it consumes. In contrast, owner-occupied housing is costly to change because selling a house involves significant transaction costs as does defaulting on a house. Also in the experiments reported here the size of owner-occupied housing is discrete and so cannot be freely chosen. However, owner-occupancy gives the household a tax benefit. If it chooses to do so, the household can deduct the mortgage payment before calculating its taxable income (whereas rents cannot be so deducted). This benefit tends to be large for high income people for two reasons: first, their income is taxed at a higher marginal rate and, second, they tend to take out bigger mortgages to purchase bigger homes (hence the deduction for mortgages exceeds the standard exemption). Because ownership is tied to income in this way, when incomes change the ownership decision changes as well. For this reason there are transactions in the housing market each period – people whose income drop sell their homes and those whose income rise buy them.

Observe that the average units of rented housing space is considerably smaller than average owner-occupied house. This is again a consequence of the fact that ownership is a good deal for high income people. The result seems consistent with the fact that rental housing in the US is generally smaller and of poorer quality than owner-occupied housing. In other words, we do not see small owner-occupied houses because there is no demand for such housing.

Now we turn to how houses are financed in the two states. Let’s begin with the low state when price of a unit of housing space is low but, with some probability, is expected to rise in the future
(when the economy switches to the high population state). Observe that equilibrium interest rate on mortgages is the same as the risk-free rate – i.e. the default spread is zero. This is a consequence of people choosing to put down enough of a down payment so that they do not have an incentive to default if their income falls in the future. To understand this, note that even when prices are constant, people have an incentive to change their housing demand when there is a change in income. If they wish to change their demand for housing consumption they have a choice of selling their home – which involves a selling cost – or of defaulting on their mortgage. The latter saves them the selling cost but imposes on them the costs of exclusion. If the down-payment is too low (i.e., the mortgage is too high) then defaulting becomes a better option and the intermediaries respond by increasing the spread because the foreclosure cost borne by the intermediaries in case of default is very large. To avoid this increase in the cost of borrowing households put enough down-payment so that walking away from the mortgage (and their home equity) is not optimal. Notice that the down payment is lower for highest income households because exclusion costs are highest.

In contrast, in the high population state, the equilibrium down payment are much higher. This is a consequence of the fact that now there is some probability that prices will decline in the future. If a person’s income changes and he wishes to change their housing consumption, the option to default becomes much more attractive since he has a large mortgage (taken out when prices were high) which will not be covered by the sale of his house when prices are low. In addition, there is another factor raising the possibility of default. Even if the person’s income does not change and he does not wish to change his housing consumption he can reduce his expenditures by defaulting on the old mortgage and taking out a new, smaller mortgage. For both reasons, the likelihood of default on a mortgage with low downpayment becomes quite high. Of course, this makes borrowing very costly and households respond by increasing their downpayment substantially. But they do not raise it by enough to completely eliminate the spread, so there is a default risk on mortgages in equilibrium.  

Given the above discussion, we can understand why the ownership rates among high income people differ between the high and low population states. Because of relatively high downpayment

\[ \text{equation} \]
requirements, high income households have to spend some time accumulating assets in order to afford the downpayment. Thus, on average, the ownership rate is lower in the high population state. Overall, the average homeownership rate is 28%. Thus, it seems that just the tax advantages of homeownership is not enough to induce the level of homeownership we see in the US economy. This indicates that we should consider making $\theta$ positive.

8 References

(To be added)