Choice of Product under Government Regulation:
The Case of Chile’s Privatized Pension System

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Abstract

Chile was one of the first countries to implement pension reform by transiting from a pay-as-you-go system to a fully funded individual accounts system in 1980. When designing the pension system, the Chilean government sought to restrict the downside market risk of pension investment by regulating the investment activities of pension fund management companies, called AFPs. In particular, the AFP firms were required to guarantee a return within 2% of the industry average. We study the effect of this government regulation on the performance of pension funds, on the fees, and on pension accumulations. We develop and estimate a model of balance accumulation where the consumer chooses an AFP firm to manage his pension investments and invests 10% of monthly earnings, as mandated under Chilean law. The supply side is modeled as an oligopolistic environment in which AFPs sequentially choose product location (mean and variance of the return), and the fees they charge for service, taking into account consumers' preferences for risk. The model is used to assess the consequences of the current regulations and analyze alternative regulations that the government could use to achieve the same goal of protecting pension investors from market risk.
1 Introduction

Many pay-as-you-go social security systems in the United States and Europe face impending insolvency as the number of pensioners per worker rises. The kinds of reforms being considered include increasing the required contribution per worker, raising the standard retirement age, or completely overhauling the system by transiting to a fully funded pension system. Chile has been at the forefront of pension reforms, having switched to a fully funded private retirement accounts system twenty eight years ago.¹ Under the Chilean system, workers are required to pay 10% of their monthly earnings into one of the privately managed and licensed pension funds. They are permitted to invest with only one firm at a time and must move all their pension accruals to a new firm if they wish to switch money managers.² The Chilean pension fund system, called the Administradoras de Fondos de Pensiones (AFPs), has been a model for pension reform in a number of Latin American countries, most recently Mexico. Policy makers in the U.S. are also looking with keen interest at the Chilean experience, which has been suggested as a possible prototype for pension reform in the U.S.

Previous research on Chile has mainly examined the impact of pension reforms on the macro-economy, capital markets and aggregate savings. That research finds substantial benefits from moving to a private retirement accounts system in terms of developing well-functioning capital markets and stimulating economic growth. However, there continues to be a heated debate about other relative merits of a decentralized, private accounts system vis-à-vis a more centralized system. Critics of the privatized system point to some problematic aspects, such as low coverage rates and commissions and fees that some consider to be excessive. Initially, it was thought

¹University of Chicago economists played a role in the early adoption of the privatized account system under Pinochet’s regime.
²This rule is intended to help participants keep track of their accounts and to prevent "orphaned" accounts, observed in other countries including the U.S. The rules governing switching have changed some over the years, but starting in 1984 switching does not incur any monetary costs on the part of the participant.
that competition among the pension management firms would keep fees low. There is also concern that workers face greater market risks under a privatized account system than under a pay-as-you-go system, although the Chilean pension system includes several features designed to mitigate the risks of investment. The focus of this paper is on studying whether and to what extent governmental regulations imposed on a privatized account system can protect investors from risk without too greatly compromising investment returns, a question that is pertinent not only for Chile but also for any other country considering a privatized account system.

The Chilean pension system consists of a privatized and nearly universal pension account system, paired with a minimum pension benefit guarantee for those who contribute for 20 years. The guarantee protects against low pension accumulations either because of a low wage or because of poor pension fund performance. In addition to the guarantee, the Chilean government put in place regulations on the investment activities of the AFPs aimed at limiting the downside market risk of pension investment. One of the most significant of these regulations requires the AFPs to deliver returns which are within 2% of the industry average, which essentially shifts most of the market risk of investment to the firms. Firms that do not meet the regulatory requirements are forced to exit the market and the accounts are typically absorbed by another AFP firm.

This paper examines the consequences of different kinds of governmental regulations on the choice of products offered by AFP firms and on their pricing strategies as well as on the pension plan savings of the pension system participants. To this end, we specify and estimate a demand and supply model for the AFP market. Our analysis combines data from multiple sources: longitudinal household survey data gathered in 2002 and 2004, administrative data on fund choices from 1981-2004 that was obtained from the pension fund regulatory agency, market data on the performance of the various funds, and a data series on the fees/commissions charged by
funds.

The household survey data come from the 2002 Historia Laboral y Seguridad Social (HLLS) survey and the 2004 Enuesta Proteccion Sociale (EPS) follow up survey.\textsuperscript{3} The data contain demographic and labor market information on 17,246 individuals age 15 or older, including information on household demographics, work history, pension plan participation, and savings, as well as more limited information on health, assets, disability status and utilization of medical services. There are also questions that are aimed at assessing financial literacy and eliciting individual’s attitudes toward risk. We complement the survey data with linked administrative data on pension contributions and fees paid that we obtained from the pension fund regulatory agency, market data on the performance of various pension funds (their returns, costs and profits), and data on the fees/commissions of pension funds that were in operation at different points in time. Through the linking of the household demographic data to the administrative history of individual pension fund decisions, we are able to carry out a detailed micro-economic analysis of consumer’s pension investment decisions.

The consumer’s choice of investment fund at a given point of time is assumed to depend on the product’s characteristics (mean return and risk) and on the fees charged by that fund. Chilean pension funds charge fixed and variable fees that depend on account balances and contribution levels. Our modeling framework also allows for observable and unobservable sources of heterogeneity in risk preferences across consumers. It also incorporates unobservable attributes of pension fund firms that may, in addition to fund performance, affect perceptions of fund quality. Repeated pension fund choices over time determine the consumer’s balance accumulation. Aggregation over consumers generates the market demand for an AFP product.

The supply side of the market is modeled as an oligopolistic environment

\textsuperscript{3}These data were gathered in 2002 and 2004 by the Microdata Center of the Department of Economics of the Universidad de Chile under the direction of David Bravo.
in which AFPs sequentially choose location (mean return and risk) and fees while taking into account the distribution of consumers’ preferences and consumer types as defined by consumer characteristics. The game of location choice with pricing has been the subject of a number of theoretical investigations. The modeling origins of this game lie with Hotteling’s model of a linear city. It was initially shown that for some locations’ configurations in the game, the equilibrium of the pricing game may not exist. However, more encouraging results were later obtained by Caplin and Nalebuff (1991) who showed that the equilibrium of pricing game does exist for any location configuration in the large class of models that satisfy some broad restrictions on the interplay of product locations and consumer tastes and on the distribution of consumer tastes. We believe that our environment can be summarized by a model for which these restrictions are satisfied. Caplin and Nalebuff also show that the equilibrium is not unique, in general, which influences our choice of estimation strategy.

After estimating the parameters of the demand and supply model, i.e. the distribution of consumers’ tastes and companies’ cost functions, we use the equilibrium model to conduct counterfactual experiments that introduce alternative regulatory schemes. For example, instead of requiring AFP firms to deliver returns close to the industry average, an alternative regulation would explicitly regulate the choice of investment instruments. We evaluate the effectiveness of alternative designs of the pension system and compare them to the current regulatory environment using average life-time pension accumulation of individuals as a criterion for choosing among regulatory schemes. We find that the current Chilean regulatory rule creates incentives for AFPs to invest in the riskier portfolios than they would otherwise choose were the riskiness of the portfolio explicitly regulated. It also leads to relatively lower levels of participation in the pension plan. Also, the choice of the portfolios under the current regulation is riskier than selection of portfolios the social planner would choose. Not surprisingly, it leads to a higher than desirable (by a social planner)
volatility in accumulated balances.

The paper is organized as follows. Section two provides some background information on the Chilean private accounts system. Section three describes the consumer’s choice problem and the oligopolistic model of firm price and location decisions. Section four describes the estimation strategy. Section five presents the empirical results and section six concludes.

2 Industry Description

Chile’s personal accounts pension system was launched in 1981 as a replacement of a number of bankrupt pay-as-you-go defined benefit systems. As described in the introduction, pension investors are permitted to hold their money in only one AFP at a time. The exact rules governing switching between money managers changed several time over the years, but beginning in 1984 investors can switch funds without incurring any monetary costs.

Pension funds charge fees for their services. Initially, the fee was a three part non-linear tariff which consisted of a fixed fee, a variable fee proportional to the participant’s contribution, and a fee proportional to participant’s balance. Some companies also charged fees for funds withdrawal, but the government passed regulation in 1984 to disallow fees on the balance or on withdrawal. Currently, most AFPs charge a two-part tariff consisting of a fixed fee and a fee that is proportional to participant’s contribution.

From the beginning of the private accounts system, the government exerted strong control over the investment choices. Initially, the pension investments could only be held in government bonds, but over time the investment options were expanded to include riskier assets (stocks and a higher degree of foreign investments). Nonetheless, the government maintained the requirement that AFPs deliver a real
return that is within 2% of the industry average, which essentially makes the AFP firms responsible for covering unexpectedly low realizations of returns with their own capital. During the period after 1987, a number of AFPs had financial difficulties because of these restrictions and had to exit the market. In each case, the exit was organized as a merger with one of the existing AFPs. The clients of an exiting AFP were transferred to its merging partner, though they could easily switch funds afterwards.

From the inception until 2000, each AFP essentially offered a single investment product. Starting in 2000, however, they were allowed to offer four instruments which differ according to the riskiness of the investment.4 We use data from the time period prior to offering multiple investment instruments, and therefore only model the choice of AFP firm with all firms offering a single investment product.

3 Model

This section describes the demand side model of consumer’s choice of AFP fund and the supply side model of AFP funds pricing and location decisions in an oligopolistic environment. Subsequently, we derive the equilibrium conditions for the location-pricing game in this environment.

3.1 Demand side model of consumer’s choice of AFP

Under the Chilean private accounts system, consumers can freely switch AFP funds without incurring monetary switching costs. We, therefore, model the consumer’s problem as a choice of fund in a given time period.

A consumer \( i \) in period \( t \) is characterized by the tuple \((b_{it}, y_{it}, \gamma_{it}, \epsilon_{it}) \in B_t\) where \( J \) is the total number of funds available and \( B_t \) is a convex subset of \( \mathbb{R}^{3+J} \).

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4 Each of these instruments has a targeted age group. An investor’s contributions are allocated by default into an age-appropriate fund unless he/she gives instructions to the contrary.
$b_{it} \geq 0$ is the pension balance of consumer $i$ at time $t$, $y_{it} \geq 0$ is the size of his pension contribution in period $t$ which is equal to 10% of his current labor income, $\gamma_{it} \geq 0$ reflects consumer $i$’s attitude towards risk, and $\epsilon_{it}$ represents the unobservable part of consumer preferences for pension funds. The risk attitude parameter is assumed to depend on gender, age and marital status, which is why we allow it to vary with $t$. In the following we refer to a consumer characterized by $(b_{it}, y_{it}, \gamma_{it}) \in B_t$ as consumer $i$.

If consumer $i$ is a pension plan participant at time $t - 1$, then at the beginning of time period $t$ he chooses a fund to which to allocate his pension savings. If he does not yet belong to pension system at time $t - 1$, which usually occurs because he has not worked in the formal sector since 1981, then at the beginning of period $t$ he has to decide whether to enter pension system and if so where to allocate his contributions.

A consumer considers fund $j$ as being characterized by its return, $R_{jt}$, fees, and other non-pecuniary features related to the convenience of obtaining service from this fund. The consumer regards $R_{jt}$ as a random variable and forms beliefs about the distribution of $R_{jt}$ on the basis of past realizations of returns. We assume that all consumers have access to the same information and, therefore, form the same beliefs about the return distribution. To formalize belief formation, we assume that consumers use a GARCH forecasting technique to predict the distribution of returns. Further, if government requires AFP to guarantee annual return within 2% of industry average, then consumers incorporate this restriction into their decision making. More specifically, a consumer recognizes that if he deposits his money with fund $j$, his actual return would be $\tilde{R}_{jt} = \max(R_{jt}, \sum(R_{it})/J - 0.02)$. Finally, we assume that consumer utility in a given time period is quadratic in retirement wealth and additive in other non-pecuniary costs. The fees associated with participating in various funds differ by consumers, because they depend on contribution levels.
The preferences of consumer $i$ from choosing fund $j$ in period $t$ are given by:

$$U_{ijt} = w_{ijt}R_{jt} - \gamma_{it}s(w_{ijt})w_{ijt}^2R_{jt}^{-2} + \Xi_j + \epsilon_{ijt},$$  \hspace{1cm} (1)$$

where $s(w_{ijt}) = w_{ijt}^{-\alpha}$, with $\alpha > 1$, and $w_{ijt} = (b_{it} + y_{it} - p_{ijt})$ denotes the net retirement wealth (balance plus new contributions minus fees) of consumer $i$ given his choice of company $j$ at time $t$. The variable $\Xi_j$ represents the unobserved product-specific fixed effect. Thus, the expected utility of consumer $i$ is given by:

$$Eu_{ijt} = w_{ijt}\mu_{jt} - \gamma_{it}w_{ijt}^{2-\alpha}((\mu_{jt})^2 + (\sigma_{jt})^2) + \Xi_j + \epsilon_{ijt}.$$  \hspace{1cm} (2)$$

Consumers choose to invest their retirement savings at the company which offers the highest contemporaneous utility. Let $M_{jt} \subset B \subset \mathbb{R}^3$, where

$$M_{jt}(p_{jt}, p_{-jt}, R_{jt}, R_{-jt}) \equiv \{(b_{it}, y_{it}, \gamma_{it}, \epsilon_{ijt}) : Eu_{ijt} \geq Eu_{ikt} \text{ for } k \neq j | (p_{jt}, p_{-jt}, x_{jt}, x_{-jt})\}$$  \hspace{1cm} (3)$$

denote the set of consumers that prefer AFP $j$ over its competitors $-j$. Accordingly the demand for AFP $j$ is given by

$$D_{jt}(p_{jt}, p_{-jt}, R_{jt}, R_{-jt}) = \int_{B} M_{jt}(p_{jt}, p_{-jt}, x_{jt}, x_{-jt}) dG(\gamma; b, y),$$  \hspace{1cm} (4)$$

where $G(\cdot)$ denotes the joint cdf of consumer’s risk attitude, balance, contribution, and unobservable tastes. We assume that the joint distribution of consumer characteristics satisfies the following condition:

**Condition 1: [Caplin and Nalebuff (1991)]:** In a given period the joint probability density of consumers’ risk attitude, balance, and contribution, $g(\gamma; b, y)$ and unobservable tastes, $\epsilon_{ijt}$, satisfies $g(\gamma; b, y)^{-\frac{1}{\alpha+1}}$, is a concave function over its support $B$, which is a convex subset of $\mathbb{R}^3 + J$ with positive volume. In addition $RW' = \sup_{(\gamma, b, y) \in B} RW$ is finite.  

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5Pedersen and Satchell (2003) show that this modification of the quadratic utility function removes its implausible IARA and bliss point property
3.2 Supply side

Supply side of the market is represented by \( J \) pension funds operating in the market. Consumers have quadratic utility function with respect to the retirement wealth. In this context the mutual fund separation theorem indicates that money managers – the AFPs – will choose a convex combination of the risk-free asset and the market portfolio. The fund’s portfolio, therefore, can be summarized by the share that funds invests into market portfolio (or correlation between the market excess return and funds excess return). We refer to this share as fund’s location and denote it by \( x_{jt} \). Next we compute the effective mean and variance of the fund’s return with different from the actual ones because of the regulatory restrictions imposed by the government. Consumers that choose fund \( j \) receive an expected return equal to \( \mu_{jt} \) only if

\[
x_{jt}(\tilde{R}_{mt} - R_{0t}) \geq \frac{\sum_{k=1}^{J_t} x_{kt}}{J_t}(\tilde{R}_{mt} - R_{0t}) - 0.02,
\]

which is equivalent to

\[
\tilde{R}_{mt} - R_{0t} \geq u_t \equiv -\frac{0.02J_t}{\kappa_t}, \quad \text{with} \quad \kappa_t \equiv J_t x_{jt} - \sum_{k=1}^{J_t} x_{kt}
\]

if \( \kappa_t > 0 \). If \( \kappa_t < 0 \) then an investor will earn an expected return equal to \( E[\tilde{R}_{jt}] \) if \( \tilde{R}_{mt} - R_{0t} < u_t \).

This implies that due to the restriction the expected return and the variance of fund \( j \) at time \( t \) are given by

\[
\mu_{jt}^a \equiv E[\tilde{R}_{jt}|\text{restr}, \kappa_t > 0] = A_t^a + \frac{\sum_{k=1}^{J_t} x_{kt}}{J_t}\mu_{mt} + \frac{\kappa_t}{J_t} \int_{u_t}^{\infty} R_{mt} f(R_{mt}) dR_{mt},
\]

and

\[
(\sigma_{jt}^a)^2 \equiv Var(\tilde{R}_{jt}|\text{restr}, \kappa_t > 0) = E \left[ (\tilde{R}_{jt} - \mu_{jt}^a)^2 | \text{restr}, \kappa_t > 0 \right]
\]

where

\[
A_t^a = 1 - 0.2F(u_t) + R_{0t} \left( 1 - x_{jt} + F(u_t) \frac{\kappa_t}{J_t} \right),
\]
if $\kappa_t > 0$. Whereas the expected return and the variance of fund $j$ at time $t$ are given by

$$\mu^b_{jt} \equiv E[R^b_{jt}| \text{restr}, \kappa_t < 0] = A^b_t + x_{jt} \mu_{mt} - \frac{\kappa_t}{J_t} \int_{u_t}^{\infty} R_{mt} f(R_{mt}) dR_{mt},$$

(9)

and

$$\left(\sigma^b_{jt}\right)^2 \equiv \text{Var}(R^b_{jt}| \text{restr}, \kappa_t < 0) = E \left[ \left( R^b_{jt} - \mu^b_{jt} \right)^2 \mid \text{restr}, \kappa_t < 0 \right]$$

(10)

where

$$A^b_t = 0.8 + 0.2F(u_t) + R_{ot} \left( 1 - \frac{\sum_{k=1}^{n_t} x_{kt}}{J_t} - F(u_t) \frac{\kappa_t}{J_t} \right),$$

(11)

if $\kappa_t < 0$. If $\kappa_t = 0$ the restriction is not binding and a consumer will earn an expected return equal to $\mu_{jt}$ at a variance of $(\sigma_{jt})^2$.

In our market regulatory agency does not allow AFPs to charge fee on balance or withdrawal of funds. Therefore, we restrict our attention to the two part-tariff fee structure of the type:

$$p_{ijt} = p_{j0t} + p_{j1t} y_{it},$$

(12)

where $p_{j0t}$ denotes company $j$’s fixed fee and $p_{j1t}$ is company $j$’s percentage fee on consumer $i$’s pension contribution in period $t$, $y_{it}$.

To describe the decision problem of the AFP firm, we need to make some assumptions about the production process AFP uses to provide its services. We think of the final output of AFP as being two-dimensional; it consists of the number of customers that receive service from a given AFP as well as the total balance which the AFP manages for them. We assume that all AFP’s have access to the same technology, which may have economies of scale related to both inputs as well as their scales relative to each other. Additionally, we assume that these economies of scale may depend on the location choice of the fund.

The cost function of the fund $j$ is given by a non-linear function $C(D_{jt}, TB_{jt}, x_{jt})$ where

$$TB_{jt} = \int_{B} \left( b_{it} + y_{it} - p_{ijt} \right) M_{jt}(p_{jt}, p_{-jt}, x_{jt}, x_{-jt}) dG(\epsilon_t, \gamma_t; b_t, y_t)$$

(13)
Additionally, the regulatory return restriction imposes expected cost on the AFP which are given by

\[
E[C_{jt}^a] = \int_B TB_{st} M_{jt}(p_{jt}, p_{-jt}, x_{jt}, x_{-jt})dG(\epsilon_t, \gamma_t; b_t, y_t)F(u_t)\left( u_t - (E[\tilde{R}_{mt}] - R_{0t}) \right),
\]

if \( \kappa_t > 0 \) and

\[
E[C_{jt}^b] = \int_B TB_{st} M_{jt}(p_{jt}, p_{-jt}, x_{jt}, x_{-jt})dG(\epsilon_t, \gamma_t; b_t, y_t)(1 - F(u_t))
\]

\[
\left( (E[\tilde{R}_{mt}] < u_t) - R_{0t} \right),
\]

if \( \kappa_t < 0 \). If \( \kappa_t = 0 \) the expected cost with regulation are equal to \( E[C^UR_{jt}] \) which is equal to zero.

Therefore, the expected profit of AFP \( j \) that chooses the location \( x_{jt} \) and charges \( p_{jt} = (p_{0jt}, p_{1jt}) \) is given by

\[
E[\Pi_{jt}(p_{jt}, p_{-jt}, x_{jt}, x_{-jt})] = \int_B M_{jt}(p_{jt}, p_{-jt}, x_{jt}, x_{-jt})p_{jt}dG(\epsilon_t, \gamma_t; b_t, y_t)
\]

\[
-C(D_{jt}, TB_{jt}, x_{jt}),
\]

in the unregulated system if its competitors locate at \( x_{-jt} \) and charge \( p_{-jt} \). The corresponding expected profit under regulation is

\[
E[\Pi^R_{jt}(p_{jt}, p_{-jt}, x_{jt}, x_{-jt})] = \int_B M_{jt}(p_{jt}, p_{-jt}, x_{jt}, x_{-jt})p_{jt}dG(\epsilon_t, \gamma_t; b_t, y_t)
\]

\[
(C(D_{jt}, TB_{jt}, x_{jt}) + C^l_{jt}), \text{ where } l \in \{UR, a, b\}.
\]

### 3.2.1 Equilibrium conditions of the location-then-price game

At a given point in time, competition between AFPs takes the form of a two-stage game. In the first stage, the AFP firms simultaneously and irrevocably choose their locations. Each AFP observes the choices of the other firms and selects a price in second stage that is contingent on the chosen locations. Thereafter, the rate of return
on the market portfolio is realized, interest is paid on consumers’ net retirement wealth, and AFPs’ profits accrue. We solve for the subgame perfect equilibrium by backward induction.

**The Pricing Stage**  Taking the other AFPs’ prices as given, firm $j \in \{1, \ldots, n_t\}$ maximizes its expected profit for every location vector if it selects $p_{0jt}$ and $p_{1jt}$ such that

$$\frac{\partial E[\Pi^z_{jt}(p_{jt}, p_{-jt}, x_{jt}, x_{-jt})]}{\partial p_{ojt}} = 0,$$

$$\frac{\partial E[\Pi^z_{jt}(p_{jt}, p_{-jt}, x_{jt}, x_{-jt})]}{\partial p_{1jt}} = 0,$$

where $z \in \{R, UR\}$.

Because our setup meets the regularity conditions specified in Caplin and Nalebuff (1991), we know that a price equilibrium exists for every location choice. However, the pricing equilibrium is not necessarily unique, because the uniqueness conditions derived by Caplin and Nalebuff (1991) do not extend to a model with nonexclusive consumption, multiple firms, preference and income heterogeneity. We take the potential non-uniqueness of second stage equilibria into account in our choice of estimation strategy and when we perform the counterfactual experiments by adopting the convention that firms select the payoff dominant equilibrium if multiple equilibria were to exist. The solution of the system of first order conditions (22) yields optimal prices as a function of AFPs location, i.e. $p^*_jt(x_{jt}, x_{-jt})$.

**The Location Stage**  Given the other AFP’s locations and taking into account the impact on prices, firm $j \in \{1, \ldots, n_t\}$ maximizes its expected profit by choosing the location $x_{jt} = x^*_jt$, where $x^*_jt$ solves

$$\frac{\partial E[\Pi^z_{jt}(p_{jt}(x_{jt}, x_{-jt}), p_{-jt}(x_{jt}, x_{-jt}))]}{\partial x_{jt}} = 0,$$

(23)
where $z \in \{R, UR\}$. Unlike the price setting game, there is hardly any literature on the existence of equilibria in a location-then-pricing game for general distributions of consumer tastes with simultaneous location choice. To the best of our knowledge, there are no general regularity conditions that guarantee the existence of a location equilibrium in the setup that we employ. Thus we resort to numerical methods to show that the subgame perfect equilibrium of the two-stage location-then-price game exists.

4 Estimation Methodology

This section describes the methodology that we use to estimate the parameters of consumer preferences and of the industry cost structure from the data. An important component of the analysis is determining firm’s location choices, as represented by riskiness of the portfolios that they offer to their customers. We assume that fund’s location can be summarized by a CAPM beta, that is, by the correlation between fund’s excess return with the excess return of the market. The details are provided below.

4.1 Consumer Preferences

In our model, a consumer chooses one fund out of multiple discrete alternatives. His preference for alternative funds are described by a random utility model described in the previous section, where utility depends on balances, current contribution level, fees, and the location of the firm, as characterized by expected return and variance. Consumers differ in their risk aversion in a way that depends on observable demographics (age, gender, marital status, education) as well as on unobservables (random coefficients). The demand model also allows for sources of alternative-specific fixed

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6 Anderson, Goaree, and Ramer (1997) are a notable exception. They, however, establish an existence result that only applies in a duopoly setting with one-dimensional heterogeneity.
effects to accommodate unobserved differences in the perception of fund quality.

The model is estimated using simulated method of moments, as proposed in McFadden (1989). The parameter vector $\theta$ is recovered as

$$\theta = \arg\min_\theta (d - P(\theta)/W'W(d - P(\theta)))$$  \hspace{1cm} (24)

Here $d$ denotes $Jn \times 1$ vector of consumer choices with $d_{ij} = 1$ if individual $i$ chose alternative $j$ and $P(\theta)$ represents the predicted choice given a vector of coefficients $\theta$. Therefore, $d - P(\theta)$ is a vector of residuals stacked by individual and by alternatives for a given individual. The matrix $W$ is $K \times Jn$ array of instruments of rank $K \geq k$ where $k$ is the length of parameter vector.

The choice probability is estimated using a frequency simulator. McFadden (1989) shows that with a suitable choice of a simulator and matrix of instruments proportional to $\partial \ln(P(\theta^*))/\partial \theta$, this method can obtain good asymptotic efficiency. He also shows that similar level of efficiency can be obtained with frequency estimator for sufficient number of draws ($= 90$). We implement the method using an iterative process. First, we find an ‘initial consistent’ estimator of $\theta^*$ using a matrix of non-optimal instruments $(X_{kij}, X_{kij}^2)$. Then we use the first-stage estimator to construct nearly optimal instruments which are used to obtain final estimate of $\theta^*$. As required by the method, we use two independent sets of random draws in estimation: the first set to construct the instruments and the second set to simulate choice probabilities.

Finally, we compute variance - covariance matrix of estimated coefficients according to

$$\Sigma = (R'R)^{-1}R'GR(R'R)^{-1}$$  \hspace{1cm} (25)

where

$$R = \lim_{n \to \infty} n^{-1} WP_\theta(\theta^*)$$  \hspace{1cm} (26)

and

$$G = \lim_{n \to \infty} n^{-1} (1 + r^{-1}) \sum_{t=n}^{t=1} \sum_{j=J}^{j=1} (P(j|\theta^*, X)W_{jt}W_{jt}' - W_{jt}W_{jt}').$$  \hspace{1cm} (27)
Here $r$ is the number of draws used in the frequency simulator.

4.2 Cost Function

We use annual data on firms’ operational costs to recover parameters of the costs function. One of the goals in this part of analysis is to measure potential scale effects both with respect to the number of customers served and with respect to the total balance managed by a particular fund, while allowing for the interdependence of these two effects. Further, we aim to model the cost side in a flexible way, which would allow these effects to change with the riskiness of the product the fund offers. To this end, we estimate the cost function using ‘translog’ functional form, where costs depend on the number of customers ($N_{jt}$), the total balance managed ($B_{jt}$) and the fund’s location ($x_{jt}$). More specifically, we assume that

$$C(N_{jt}, B_{jt}, x_{jt}) = \gamma_1 N_{jt} + \gamma_2 B_{jt} + \gamma_3 N_{jt}^2 + \gamma_4 B_{jt}^2 + + \gamma_5 N_{jt}B_{jt} + \gamma_6 N_{jt}x_{jt} \quad (28)$$

$$+ \gamma_7 B_{jt}x_{jt} + \nu_j + \gamma_{19} D_{1981} + \cdots + \gamma_{28} D_{1999} + \epsilon_{jt}$$

where $\nu_j$ are firm fixed effects and $D_y$ are year effects. We estimate parameters of this function using standard panel data methods.

4.3 Recovering Fund’s Locations

The funds’ location or riskiness of fund’s portfolio constitute an important component of both the demand and supply side models. However, this variable is not directly observable in the data and needs to be inferred from information on the fund’s history of returns. We use fund’s CAPM beta to proxy for it in estimation. More specifically, we work with a model of time varying beta and GARCH errors to recover funds’ betas to use in cost function estimation. We also use this model to approximate consumer’s forecast of the funds’ expected returns and their volatilities. More specifically, our model is based on Bollerslev at al(1989) CAPM model with time varying covariances.
The details of this model are given below. Denote by $Y_{j,t}$ an excess return of fund $j$ at time $t$ and by $Y_{m,t}$ an excess market return at time $t$. We assume that vector $(Y_{j,t}, Y_{m,t})$ changes over time according to

$$
Y_{j,t} = b_j + \delta h_{jm,t} + \epsilon_{jt}
$$

$$
Y_{m,t} = b_m + \epsilon_{mt}
$$

Additionally, $\epsilon_t = (\epsilon_{jt}, \epsilon_{mt})$ is distributed according to $N(0, H_t)$ with

$$
h_{jj,t} = \gamma_{jj} + \alpha_{jj}\epsilon_{jt-1}^2 + \beta_{jj}h_{jj,t-1}
$$

$$
h_{mm,t} = \gamma_{mm} + \alpha_{mm}\epsilon_{mt-1}^2 + \beta_{mm}h_{mm,t-1}
$$

$$
h_{jm,t} = \gamma_{jm} + \alpha_{jm}\epsilon_{jt-1}\epsilon_{mt-1} + \beta_{jm}h_{jm,t-1}
$$

We estimate this model using maximum likelihood methodology. The values of beta and forecasts are obtained using rolling 18 months window.

5 Empirical Results

5.1 Descriptive statistics

Table 1 presents some descriptive statistics derived from the administrative pension data, which includes men and women. The pension plan participant sample is fairly young, with a median age of 34 and an interquartile range of 27-42. The median years of contribution is 3.83 years with an interquartile range of 1.41-7.58. The median balance is close to the median of one year’s annual income.

Figure 1 plots the 25, 50, and 75th percentiles of the balance distribution conditional on age. There is rising dispersion with age, particularly over the age 35-45 range. At subsequent ages, the disparity remains roughly constant. Figure 2 plots the sample percentiles of the income distribution conditional on age. There is increasing dispersion in income up through age 40, as exhibited by the interquartile range.
range, after which it declines. The conditional on age income distribution is skewed with a long right tail, as commonly observed in income data. The dispersion in income is not as large as the dispersion in balances, which might be expected given that balances represent a stock measure and income a flow.

We next turn to descriptive characteristics of the AFP firms. Table 2 shows the fixed and variable fees charged by the AFP firms in year 1999 and reveals substantial variation in the fees charged across firms. A number of funds do not charge any fixed fee. The AFP firm Habitat has the lowest variable fee at 2.84% of monthly contributions and no fixed fee. The firm Concordia has the highest fixed fee at 3.48% and also a relatively high fixed fee at 230 pesos per month.

In Table 3, we compare the market shares of the different funds in terms of their share of clients and in terms of the share of the total market balance under each firm’s management. The table also shows the estimated CAPM-beta, with lower betas indicating lower risk. The fund with the largest market share both in terms of share of customers and share of balances managed is Provida, which manages pensions for about one-third of all pension plan participants. Provida also is one of the least risky funds. The AFP firm Santa Maria has the second largest market share in terms of clients but ranks lower in terms of balance shares. Its portfolio allocation is in the median risk range. The firm with the lowest fees, Habitat, is relatively low ranking in terms of numbers of share of customers but is in the top three in terms of share of total balances. There are also a number of funds in the market with very low shares of customers and of balances. For example, Fomenta has the riskiest portfolio, measured in terms of the beta, and also attracts few clients.

5.2 Model Estimates

Tables 4, 5 and 6 present estimated model parameters and evidence on goodness of fit. As described above, we allow risk aversion (the $\gamma$ parameter) to depend on
demographics and also allow for a random coefficient component to risk aversion to reflect unobservable sources of heterogeneity. Theoretical models of dynamic savings accumulation decisions would suggest that age is an important demographic characteristic, although its net effect on risk aversion is ambiguous. Older individuals are typically less willing to take on investment risk, because of a shorter time horizon until retirement, but may also be more willing to take on risk, because they have higher balances. Our estimates related to risk aversion indicate that older individuals are more risk averse and that younger individuals under the age of 35 are the least risk averse.

Table 5 presents estimates of absolute and of relative risk aversion at different ages. The Arrow (1965)-Pratt (1964) measure of absolute risk aversion (as a function of consumption) is given by

\[
- \frac{u''(c)}{u'(c)}
\]

and the measure of relative risk aversion by

\[
- \frac{cu''(c)}{u'(c)}.
\]

These are standard measures of risk aversion that stay constant up to affine transformations of the utility function.\(^7\)

In Table 6, we examine the importance of unobservables to the model fit. Specifically, we evaluate the fit of the moments under the original model, under a restricted version that sets the alternative-specific fixed effects to zero (i.e. shuts down permanent unobservable firm heterogeneity) and under on a restricted version that, in addition, suppresses the utility shock. We find that the fit of the moments is not greatly compromised by shutting down unobservable sources of heterogeneity, although the fit is certainly improved by including these components.

\(^7\)The advantage of the relative measure vis-a-vis the absolute measure is that it accommodates the situation varying degrees of risk aversion at different levels of \(c\) (for example, switching from being risk averse to risk loving and then back to risk averse).
Table 7 compares the model’s aggregate predicted shares of annual contributions to pension funds to the empirical shares. Recall that in estimation we only used moments pertaining to shares of customers and share of balances. The moments related to shares of contributions were not used in estimating the model parameters, so this comparison could be viewed as a form of model validation. Generally, the model is able to identify the top four AFP firms in terms of shares of contributions and is fairly accurate in terms of predicting the actual contribution share for three of the four funds. For the fourth (Concordia), the model substantially overpredicts the contribution share. The AFP firm Provida had the largest contribution share in the data, but the model predicts Provida to be the firm with the second-largest share.

Table 8 provides AFP cost function estimates that are derived from panel data on firms, costs and cost components. We assume that the cost depends on the number of clients served, the total balance under management, and the firm’s location choice. We specify the cost function flexibly as a function of linear and interaction effects in these variables. To better visualize the returns to scale, Figure 3 plots the cost as a function of the balance and total customers, for a firm with the median location choice. The figure suggests that after the pension fund reaches a certain size, there are decreasing returns to scale.

6 Policy Experiments

When the Chilean pension system was designed, the government sought to limit downside market risk by placing restrictions on AFP investment behavior. Under the current regulations, AFPs are required to guarantee a market return that is within 2% of the industry average. We next use the estimated demand and supply-side models to evaluate the effects of the current regulation on AFP location choices, on individual pension performance and balance accumulations, and on participation rates in the pension fund program. At present, our simulation is based on three firms
only, but we plan to extend it to include more firms. Our simulation compares the AFP location decisions and individual pension fund investment decisions under the current regulatory environment to those that would be realized (i) under a social planner and (ii) under an alternative form of regulation that would place an upper limit on the riskiness of the firm’s portfolio (the CAPM beta). The social planner problem maximizes the sum of individuals’ balances over a ten year period.

Table 9 shows the average location decisions of firms under the three different scenarios (the social planner, the restriction on riskiness of the portfolio and the restriction on returns (the current regulation)). Interestingly, the current regulation that requires that returns not fall too far below the industry average leads to substantially riskier firm location decisions.

Table 10 compares the expected coverage rates of the pension system under the three different cases. A major concern of the Chilean government today is low coverage rates, with a substantial fraction of workers opting to work outside the formal sector of the economy and therefore to not participate in the pension program. The estimates in table 10 indicate that coverage under the current regulation is about 60%, which is significantly lower than it would be under a social planner or under the alternative regulation that restricts portfolio risk directly. The lower coverage arises because some individuals opt not to participate in the higher risk environment.

Lastly, Table 11 examines longer run implications of the regulation for the number of firms participating in the pension market. We assume a fixed cost of entry, so that a firm enters the market if its expected profits exceed the costs of entry. Both of the types of regulations we consider result in a number of firms that is greater than under the social planner solution.
7 Conclusions

Chile has one of the oldest individual-account pension systems and therefore provides a unique opportunity to study firm and consumer behavior under a well established private accounts system. When designing the pension system, the Chilean government introduced regulations that were intended to restrict the downside of pension investment. We study a model of balance accumulation where consumer chooses an AFP to manage his investments. The amount of monthly contribution is regulated and restricted to 10% of labor market earnings. The supply side is modeled as an oligopolistic environment in which AFPs sequentially choose product location (mean and variance of the return), and fee they charge for service while taking into account consumers’ preferences for risk. We assess the impact of the current regulation on the funds’ choices of locations. We also study implications of this regulations for the consumer accumulated balance and its volatility. We find that Chilean regulatory rule that mandates firms to guarantee returns within 2% of the industry average creates incentives for the AFP firms to invest in the riskier portfolios than they would choose were the riskiness of their portfolio limited directly. Surely, this is an unanticipated effect of the regulation. Because the portfolio location choices that firms make are riskier, we find that fewer people participate in the pension program, a particularly worrisome finding considering that increasing coverage rates are a high priority of the government. Also, the choice of the portfolios under the current regulation is riskier than would be the selection of portfolios that a social planner would choose. Not surprisingly, it leads to a higher than desirable (by social planner) volatility in accumulated balances.
References


Table 1: Summary of Consumer Characteristics

<table>
<thead>
<tr>
<th></th>
<th>25% centile</th>
<th>median</th>
<th>75% centile</th>
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<tbody>
<tr>
<td>age</td>
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<td>34</td>
<td>42</td>
</tr>
<tr>
<td>annual income, $</td>
<td>20</td>
<td>1342</td>
<td>3196</td>
</tr>
<tr>
<td>balance, $</td>
<td>283</td>
<td>1090</td>
<td>3381</td>
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<tr>
<td>years contr</td>
<td>1.41</td>
<td>3.83</td>
<td>7.58</td>
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</table>

Figure 1: Centiles of Balance Distribution Conditional on Age
Figure 2: Centiles of Participants’ Income Distribution Conditional on Age
<table>
<thead>
<tr>
<th>Institution</th>
<th>% fee</th>
<th>Fixed Fee</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concordia</td>
<td>3.48</td>
<td>230</td>
</tr>
<tr>
<td>Cuprum</td>
<td>2.99</td>
<td>0</td>
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<tr>
<td>Habitat</td>
<td>2.84</td>
<td>0</td>
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<tr>
<td>Planvital</td>
<td>3.45</td>
<td>280</td>
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<tr>
<td>Provida</td>
<td>2.85</td>
<td>195</td>
</tr>
<tr>
<td>Santa Maria</td>
<td>3.15</td>
<td>100</td>
</tr>
<tr>
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<td>230</td>
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<tr>
<td>Union</td>
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</tr>
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<td>Proteccion</td>
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<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>Fomenta</td>
<td>3.25</td>
<td>0</td>
</tr>
<tr>
<td>Company</td>
<td>N_Shares</td>
<td>B_Shares</td>
</tr>
<tr>
<td>---------------</td>
<td>----------</td>
<td>----------</td>
</tr>
<tr>
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<td>0.191</td>
<td>0.223</td>
</tr>
<tr>
<td>Cuprum</td>
<td>0.018</td>
<td>0.005</td>
</tr>
<tr>
<td>Habitat</td>
<td>0.058</td>
<td>0.133</td>
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<tr>
<td>Planvital</td>
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<td>0.011</td>
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<td>0.450</td>
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<td>0.053</td>
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<td>Proteccion</td>
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<td>0.013</td>
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<td>0.001</td>
</tr>
<tr>
<td>Fomenta</td>
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<td>0.005</td>
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Note: N_Shares denotes shares in the number of customers served. B_Shares denote the share of the overall balance under management
Table 4: Estimated Coefficients (Demand System)

<table>
<thead>
<tr>
<th></th>
<th>Coefficients</th>
<th>Std. Errors</th>
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<td>age &lt;35</td>
<td>5.36</td>
<td>1.23</td>
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<td>35&lt;age &lt;50</td>
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<td>1.34</td>
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<tr>
<td>age&gt;50</td>
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<td>-1.51</td>
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<tr>
<td>sigma</td>
<td>2.05</td>
<td>0.95</td>
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Table 5: Implied Risk Aversion

<table>
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<th>absolute risk aversion</th>
<th>relative risk aversion</th>
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<tr>
<td>age=20</td>
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<td>age=30</td>
<td>-3.00E-03</td>
<td>-1.24</td>
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<td>-1.98</td>
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<tr>
<td>age=50</td>
<td>-3.10E-03</td>
<td>-1.93</td>
</tr>
<tr>
<td>age=60</td>
<td>-4.10E-03</td>
<td>-2.1</td>
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</table>

Table 6: Role of Unobservables

<table>
<thead>
<tr>
<th>Model</th>
<th>% of Emp Moment Cond</th>
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<tr>
<td>deterministic part</td>
<td>89%</td>
</tr>
<tr>
<td>deterministic part + alt. fixed effects</td>
<td>93%</td>
</tr>
<tr>
<td>deterministic part + alt. fixed effects + weibull errors</td>
<td>97%</td>
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Table 7: Aggregate Fit

<table>
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<th>AFP</th>
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<tbody>
<tr>
<td>Concordia</td>
<td>0.185</td>
<td>0.428</td>
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<td>0.008</td>
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<td>0.070</td>
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<td>Planvital</td>
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<td>Provida</td>
<td>0.290</td>
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<tr>
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<td>Balance</td>
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<td>Balance2</td>
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<td>0.0007</td>
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<td>Balance*N_customers</td>
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<td>0.0001</td>
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<tr>
<td>Location*N_customers</td>
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<tr>
<td>Location*Balance</td>
<td>0.003</td>
<td>0.0006</td>
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### Table 9: Policy Experiment: Locations (3 firms)

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std.Dev</th>
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<tr>
<td>Social Planner</td>
<td>0.36</td>
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</tr>
<tr>
<td>Restriction on Portfolio</td>
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<td>Restriction on Returns</td>
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<td>0.05</td>
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### Table 10: Policy Experiment: Consumers

<table>
<thead>
<tr>
<th></th>
<th>Coverage</th>
<th>Mean (balance)</th>
<th>Std.Dev (balance)</th>
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<tr>
<td>Social Planner</td>
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<td>35mln</td>
<td>13mln</td>
</tr>
<tr>
<td>Restriction on Portfolio</td>
<td>75%</td>
<td>30mln</td>
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<tr>
<td>Restriction on Returns</td>
<td>60%</td>
<td>39mln</td>
<td>21mln</td>
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</table>

### Table 11: Policy Experiment: Number of Firms

(hypothetical fixed costs)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
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<tr>
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<td>Restriction on Returns</td>
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