Liquidity and Growth*

Aleksander Berentsen
Department of Economics, University of Basel, Switzerland
(aleksander.berentsen@unibas.ch)

Mariana Rojas Breu
Department of Economics, University of Basel, Switzerland
(mariana.rojas-breu@unibas.ch)

Shouyong Shi
Department of Economics, University of Toronto, Canada
(shouyong@chass.utoronto.ca)

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Abstract

Many countries simultaneously suffer from high rates of inflation, low growth rates of per capita income and poorly developed financial sectors. In this paper, we integrate a microfounded model of money and finance into a model of endogenous growth to examine the effects of inflation and financial development. We address two quantitative issues. One is the effects of an exogenous improvement in the productivity of the financial sector on welfare and per capita growth. The other is the effects of inflation on welfare and growth, with an emphasis on how these effects depend on a country’s financial development. Consistent with the data, the growth gains of reducing the inflation rate by 10% are highly nonlinear: for a low inflation rate (10%) the gain is 0.4 percentage points while for a high rate (40%) the gain is only 0.13 percentage points. In contrast, the growth gain of an exogenous increase in financial market development is independent of the level of inflation.

Keywords: Money; Credit, Innovation; Growth.
JEL Classification:

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1 Introduction

Many countries simultaneously suffer from high rates of inflation, low growth rates of per capita income and poorly developed financial sectors. For example, during the period from 1960-1995, Bolivia had an average annual inflation rate of 50%, a low growth rate of per capita income of 0.36%, and a share of the financial sector in GDP that was about 5 times smaller than the share in the US. In this paper, we integrate a microfounded model of money and finance into a model of endogenous growth to examine the effects of inflation and financial development. After calibrating the model, we address two quantitative issues. One is the quantitative effects of an exogenous improvement in the productivity of the financial sector on welfare and the growth rate of per capita income. The other is the quantitative effects of inflation on welfare and growth, with an emphasis on how these effects depend on a country’s financial development.

These issues are motivated by the empirical literature that has documented that financial development has a robust and positive effect on economic growth and that inflation has robust and negative effects on financial development and growth (e.g., Levine, Loayza and Beck, 2000, Boyd, Levine and Smith, 2001, and King and Levine, 1993a,b). These effects are sizable, even after controlling for country-specific factors such as the level of a country’s development, political factors, trade and price distortions, and fiscal policy. For example, the regression coefficients in Levine, Loayza and Beck (2000) suggest that an exogenous improvement in financial intermediation from the level in India to the sample average in the period 1960-1995 (i.e., an increase of 28%) can increase annual growth rate of per capita income by 0.6 percentage point. The regression coefficients in Boyd, Levine and Smith (2001) suggest that an increase in inflation by the median value in the sample (9%) can reduce financial intermediation by 26% in low-inflation countries.

Table 1 (Data): Inflation, growth and financial development

<table>
<thead>
<tr>
<th>Inflation</th>
<th>Per cap. growth</th>
<th>Bank credit</th>
<th>Liquid Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low inflation</td>
<td>5.71</td>
<td>2.41</td>
<td>0.44</td>
</tr>
<tr>
<td>Med inflation</td>
<td>9.21</td>
<td>1.91</td>
<td>0.27</td>
</tr>
<tr>
<td>High inflation</td>
<td>31.6</td>
<td>1.28</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Table 1 displays the relationship between inflation, real per capita growth, and two measures of financial sector size: bank credit (claims on private sector by deposit money banks as share of GDP) and liquid liabilities (liquid liabilities as share of GDP). To construct Table 1, we have used the same cross-country data as Levine, Loayza and Beck (2000) and sorted the 73 countries into inflation tertiles. We then calculated for each country type the average inflation rate, the average real per capita growth rate, the average of bank credit and the average of liquid liabilities. Bank credit and liquid liabilities are measures of financial sector size and are used in many studies as indicator for financial development. Table 1 clearly indicates that countries with high real GDP per capita growth
tend to have both larger financial sectors and lower rates of inflation. This suggests that one needs to examine the effects of both - financial sector size and inflation - on economic growth.

Although these empirical findings are suggestive, it is not clear how to interpret them. One possible interpretation is that the statistical relationships are causal. That is, low inflation rates foster financial market development, financial market development promotes economic growth, and hence low rates of inflation promote economic growth (Altig, 2003). If so, then the empirical findings suggest that monetary policy can help financial market development, which in turn can increase economic growth. The competing interpretation is that the statistical relationships may not indicate any causality, because financial development is endogenous; in particular, a poorly developed financial sector may be a result of a weak real economy.¹

In contrast to such ambiguity in the interpretation, a general equilibrium model makes the causality explicit. Thus, it is important to use a general equilibrium model to quantify the effects of inflation and financial market development on economic growth. Moreover, the empirical literature cannot evaluate the welfare consequences of inflation and financial development. How large are the welfare cost of inflation? Does the cost depend on the degree of financial development? These questions are important for designing policies, and they can explicitly be addressed by a general equilibrium model, as we do in this paper.

We construct a model of endogenous growth with microfoundations for the demand for money and for financial intermediation. A search model with large households, as developed by Shi (1997), is used to give fiat money a useful role in the equilibrium, and the model is extended to allow for financial intermediation and a balanced growth path. A representative household in the model consists of a continuum of members who are allocated to different tasks. They can work in an innovation sector, a goods sector, a financial sector, or enjoy leisure. The outcome of the innovation sector determines productivity of the goods sector. Long-run growth is sustained by non-diminishing marginal productivity in the innovation sector and depends positively on the fraction of labor allocated to innovation. Following the tradition of the money and search literature, we assume random matching and anonymity in the innovation sector which, due to a double coincidence of real wants problem, makes money essential for trade in the innovation sector. Moreover, innovators have heterogeneous liquidity needs and financial intermediaries emerge endogenously to offer liquidity services. As in Berentsen, Camera and Waller (2007), these intermediaries behave like banks since they take deposits and make loans.

We use the model to quantify the effects of financial market reform and inflation on growth and welfare. The model is consistent with the above mentioned stylized facts. First, an exogenous increase in financial sector size - generated by higher efficiency of financial intermediaries - increases per capita growth and

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¹ For this classic debate on whether financial development has a causal effect on growth or whether finance simply responds to changing demand from the real sector, see Levine 2004 for a discussion of this debate.
welfare. Second, the model generates a negative relationship between inflation and financial sector size. Third, the model displays a negative relation between inflation and real growth.

To quantify the welfare and growth effects of financial market reform and inflation, we calibrate our model to the average low-inflation country and perform several counterfactual experiments. First, for each country type we ask how much would the representative household pay in terms of consumption for a zero inflation policy. Second, we calculate the welfare and growth effects of a financial market reform. Table 2 reports our main simulation results. For example, the average low inflation country could increase its per capita growth rate by 0.2628 percentage points by following a zero percent inflation rate. In contrast, financial market reform would increase its per capita growth rate by 0.1033 percentage points. For the average high inflation country, the growth gains are much larger. Such a country could increase its per capita growth rate by 0.857 percentage points by following a zero percent inflation rate. In contrast, financial market reform would increase its per capita growth rate by only 0.0999 percentage points.

Table 2 (Results): Price stability vs financial market reform

<table>
<thead>
<tr>
<th></th>
<th>Low (5.71%)</th>
<th>Mid (9.21%)</th>
<th>High (31.6%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Welfare</td>
<td>Growth</td>
<td>Welfare</td>
</tr>
<tr>
<td>Zero inflation</td>
<td>1.0894</td>
<td>0.2628</td>
<td>1.686</td>
</tr>
<tr>
<td>Market reform</td>
<td>1.0208</td>
<td>0.1033</td>
<td>1.0283</td>
</tr>
</tbody>
</table>

Table 2 reveals that the growth benefits of a zero inflation policy are much higher than those of financial market reform for all country types (note that Table 2 reports an upper bound on the possible gains from financial market reform since it reports the gain of a reform that yields a perfectly efficient financial market). Our results demonstrate that the poor growth performance by high-inflation countries is mainly caused by inflation. They support the theory that low inflation rates foster financial market development, financial market development promotes economic growth, and hence low rates of inflation promote economic growth (Altig, 2003). Nevertheless, they also support financial market reform because more efficient financial intermediation generates substantial welfare gains. As Table 2 reveals, the welfare gains of financial market reform are approximately 1% for all country types.

**Literature on finance and growth** There are numerous theoretical and empirical contributions that investigate the relation between finance and growth.\(^2\)

\(^2\)Early empirical studies on the relation of finance and growth are Goldsmith (1969), Shaw (1973) and MCKinnon (1973). More recent theoretical and empirical contributions are Greenwood and Jovanovic (1990), Levine (1991), King and Levine (1993a,b), Bencivenga and Smith (1993), Jones and Manuelli (1995), Acemoglu and Zilibotti (1997), Acemoglu, Aghion, and
A comprehensive and very useful survey is Levine (2004). Levine (2004) characterizes the literature according to the functions that the financial intermediaries perform. The functions are: (1) production of information for investors which improves the allocation of capital and hence growth; (2) monitor investments and improve corporate governance which improves the use of capital and hence growth; (3) risk amelioration which affects portfolio decisions which can affect real growth; (4) pooling of savings which improves real activities because it allows, for example, to finance large investment project; and (5) easing exchange which allows for a greater degree of specialization which fosters growth. The upshot of all these financial activities is that they allow for a better resource allocation which can improve economic growth.

Our paper combines elements of (3) and (5). Our financial intermediaries take deposits and make loans and thereby they reallocate liquidity to where it is needed. Hence, financial intermediation facilitates exchange (5). Our paper is also related to (3) because the financial intermediaries effectively act as a risk sharing arrangement. Households after choosing their money holdings face trading shocks which results in an inefficient allocation of money holding across buyers in the innovation sector. The financial intermediaries allow household to readjust their money holdings after the shocks occur which improves the allocation in the innovation sector leading to higher growth.

Literature on inflation and growth There is a large literature that studies the effect of inflation on growth and welfare in endogenous growth models. These models typically combine a variant of an endogenous growth model with a cash in advance constraint or with a shopping time model to generate a demand for money. The models are then used to obtain estimates of the cost of inflation and/or the effect on growth. The first attempts along these lines are Gomme (1993), Ireland (1994), Dotsey and Ireland (1996), or Chari, Jones and Manuelli (1996). The common result in this literature is that inflation has a negligible effect.
on per capita growth (e.g. Gomme (1993), Dotsey and Ireland (1996) or Chari, Jones and Manuelli (1996)). These models have in common that the financial sector provides consumption loans. In contrast, in our model money is needed to finance innovation goods which more directly affects the productivity in the economy. Our modelling choice results in large and nonlinear growth effects of inflation that are consistent with the empirical evidence. For example, we find that the growth gain of reducing the inflation rate from 10% to 0% is 0.4 percentage points while a reduction from 40% to 30% only yields a gain of 0.13 percentage points. The small growth effects in the previously mentioned papers, therefore, provide evidence that financial intermediation that provides credit to consumers is not important for growth (it is however important for welfare as Dotsey and Ireland (1996) show). Hence, the key to understand the effects on inflation on growth is the need for liquidity and intermediation in the innovation sector.

Many of the previous studies calculate the growth effects and welfare benefits of reducing the rate of inflation from 10% to 0%. In order to relate our findings to some of these previous studies we report in Table 3 the results of a selected number of these studies and of our model. All entries report the welfare gains (in percentage of consumption) and per capita growth gains (in percentage points) of reducing the inflation rate from 10% to 0% if not indicated differently. We report that gains for two models. Model 1 reports our estimate of the benchmark economy; Model 2 estimates the gains of the benchmark economy in the absence of financial intermediation. This latter model therefore highlights the role of finance for growth and the welfare cost of inflation. The growth gains for the economy that has no financial market (Model 2) are more than 30% higher whereas the welfare gains are 50% larger than those of an economy with financial intermediation. This clearly demonstrates that countries with a well-developed financial system suffer less severely from inflation than countries with more primitive finance.

Table 3: Welfare and growth gains of reducing inflation from 10%-0%

<table>
<thead>
<tr>
<th></th>
<th>Traditional(^5)</th>
<th>Gomme(^6)</th>
<th>Dot./Ire.(^7)</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth</td>
<td>-</td>
<td>0.056</td>
<td>0.05</td>
<td>0.409</td>
<td>0.548</td>
</tr>
<tr>
<td>Welfare</td>
<td>0.3 - 0.45</td>
<td>0.024</td>
<td>0.915</td>
<td>1.811</td>
<td>2.77</td>
</tr>
</tbody>
</table>

Table 3 shows that Gomme (1993) and Dotsey and Ireland (1996) find equally small growth gains but their estimates of the costs of inflation differ widely. For our benchmark economy, our estimates for both the growth and

\(^5\) The traditional approach, pioneered by Bailey (1956) and Friedman (1969), estimates the welfare cost by computing the appropriate area under the money demand curve.

\(^6\) Gomme (1993) considers a 10% money growth rate (8.5% inflation rate).

\(^7\) The welfare cost is 0.92% of output per year if the model is calibrated to M0 and 1.7% if it is calibrated to M1.
welfare gains are much higher. We get higher estimates because in our model money and finance facilitates trade of innovation goods that directly affect productivity in the goods sector. The welfare gains are also larger than those presented by Fisher (1981) and Lucas (1981). They are higher since the traditional approach does not capture the cost of inflation that origins in a lower real growth rate.

The proofs of all lemmas and propositions in this paper appear in the Appendix, unless specified otherwise.

2 The model

Consider a discrete-time economy with many households. The number of households in each type is large and normalized to one. All households have the same discount factor $\beta \in (0, 1)$. Denote $R \equiv (1 - \beta) / \beta$. The households of each type are specialized in producing a good which they do not consume; instead, they consume a set of goods produced by other types of households. Goods are perishable between periods. We pick an arbitrary household as the representative household and use lower-case letters to denote its decisions. The decisions of other households and the aggregate variables are denoted with capital-case letters. The representative household takes all capital-case variables as given.

The utility function in each period is $u(q_b, s, n)$, where $s$ is the measure of shoppers, $q_b$ indicates consumption goods per shopper, so that $sq_b$ is consumption, and $n$ is leisure. For the model to generate balanced growth, we let $u(q_b, s, n) = \ln(sq_b) + \ln(n^\theta s^\varphi)$. The representative household consists of a large number of members, who share consumption and regard the household’s utility as the common objective. The measure of members in a household is normalized to one. The household divides the members into five groups: a fraction $l$ of members are innovators; a fraction $k$ work in the intermediation/financial sector; a fraction $h$ produce consumption-goods; a fraction $n$ enjoy leisure and a fraction $s$ are buyers in the consumption-goods market. This allocation of members to the five activities (or time) is one of the links between inflation and economic growth which we want to focus on in this paper.

We define the production function of consumption goods as $q(h)$ where the input $h$ is the fraction of agents who are producers in the consumption-goods market. We assume that the function $q(h)$ is linear so that $q(h) = ah_0h$, where

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Fisher (1981) and Lucas (1981) estimate the cost of inflation by calculating the appropriate welfare cost under the money demand curve. Fisher (1981) estimates the cost of increasing the rate of inflation from 0% to 10% to be 0.3% and Lucas (1981) to be 0.45%. For a discussion of this estimation procedures and more recent estimates (e.g. Lucas 2000) see Craig and Rocheteau (2005).

Shouyong: The shopping time does not do anything to this version of the model and so we could take it out. But if we include it we need that $s$ provides some utility to avoid that the household chooses $s = 0$ (one agent does all the shopping).

The device of a household is used here to maintain tractability, as it enables us to smooth the matching risk within a household and hence to obtain a degenerate distribution of money holdings across households. See Shi (1997).
\( a \) is the level of productivity of the household’s technology to be described later and \( h_0 \) is a constant, with \( h_0 > 0 \).\(^{11}\)

Innovators produce innovation goods which are not useful for their own household but may be useful for other households. Hence, innovation goods are traded in a frictional market where innovators meet at random in bilateral meetings. When two innovators meet, the first agent desires the innovation good produced by the second agent with probability \( \sigma \); with the same probability, the second agent desires the innovation good produced by the first agent. No double-coincidence meeting occurs. In a single coincidence meeting, the buyer makes a take-it-or-leave-it offer to the seller which consists of an amount of money to be paid by the buyer and a quantity \( y \) of the innovation good to be produced by the seller. Producing \( y \) represents a disutility equal to \( c(y) \) to the seller, with \( c(0) = 0, c’ > 0, c” > 0 \) and \( c'(0) = 0 \). We will assume the following functional form: \( c(y) = c_0 y^\alpha \) with \( \alpha > 1 \) and \( c_0 > 0 \).

Finally, two additional features of the innovation market are important. First, agents are anonymous and no form of record-keeping is feasible in the innovations market. Therefore, a medium of exchange is needed for transactions to take place. This medium is fiat money, a perfectly storable object which is intrinsically worthless. The need for liquidity to improve the households’ technology is the second link between inflation and economic growth that we intend to address. Second, in order to generate a role for financial intermediation we assume - as in Rocheteau and Wright (2005) or in Berentsen, Mentio and Wright (2008) - that innovators get into the innovation market only probabilistically: only one half of the innovators can enter the innovation market. This assumption generates different liquidity needs among innovators. Those who cannot enter have "idle" money which they would like to lend earn interest and those who can enter demand more liquidity and are willing to pay for it as we show below in more detail.

If \( a \) is the current productivity of the household’s technology in producing goods, then the productivity of the technology in the next period is:

\[
a_{t+1} = a + a \sigma (l/2) y.
\]

The subscripts "+1" indicate the next period throughout the paper. One interpretation of \( a \) is that it is the stock of human capital, as in Lucas (1988). Like many endogenous growth models, our model generates long-run growth from the non-diminishing marginal productivity of \( a \) in the innovation process. We specify the innovation sector to deliver long-run growth, rather than using the so-called AK model, because we want to emphasize the allocation of time between different sectors. The modeling is also convenient for finding identification restrictions in the calibration, as illustrated later.

We assume that in the market for consumption goods a form of record-keeping is available, so that buyers get credit. Hence, they do not need a medium of exchange to carry out transactions, but they may buy goods provided that the

\(^{11}\)Shouyong: We could also model a concave production function \( q(h) = ah^\eta h^\eta \). However, this would require an additional target.
household repays them at the end of the period. The purpose of this assumption is to focus on the role of liquidity in the innovation market, which is supposed to affect growth, instead of studying how liquidity affects the consumption-goods market. Moreover, we assume that the participants of the goods market take prices as given. This particular pricing mechanism streamlines the characterization of the equilibrium and simplifies the algebra; however, we could use alternative pricing mechanisms, such as a form of bilateral bargaining as we do for trades among innovators.

2.1 The social planner’s allocation

To provide a benchmark against which to measure the efficiency of the equilibrium, let us first consider the allocation of a social planner. Assume that the social planner can dictate all quantities and the allocation of time subject to feasibility constraints. In this case, there is no need for a credit market, and so \( k = 0 \). Denote the planner’s allocation as \( P = \{ q_b, q, y, l, n, h, s \} \), where \( q_b \) is the quantity of goods given to a buyer, \( q \) is the quantity of goods produced by a household, \( y \) is the amount of innovations produced in each bilateral single coincidence meeting and \( l, n, h \) and \( s \) correspond to the allocation of time in each period. Denote the maximized social welfare function as \( W(a) \). Then, the planner’s allocation solves:

\[
W(a) = \max_P u(q_b, s, n) - \sigma \left( \frac{l}{2} \right) c(y) + \beta W(a + 1)
\]

subject to the following constraints:

\[
sq_b \leq q \tag{1}
\]
\[
q \leq ah_0h \tag{2}
\]
\[
l + n + s + h \leq 1 \tag{3}
\]
\[
a_{+1} = a + a \sigma \left( \frac{l}{2} \right) y \tag{4}
\]

The first term in the welfare function is a household’s total utility from consumption and leisure and the second term is the disutility from the production of innovation goods by \( \left( \frac{l}{2} \right) \) members who produce with probability \( \sigma \). The constraints (1) and (2) state that the total amount of goods consumed must be equal to the total amount produced, which is determined by the fraction \( h \) of agents and the level of productivity \( a \). The constraint (3) states that the sum of members allocated to the different activities must be equal to 1. Equation (4) is the law of motion of productivity.

Denote the solution to the above problem by adding the superscript \( P \) to the variables. The following proposition describes the socially optimal allocation:

**Proposition 1** A unique social optimum exists, which is the solution to the
The socially efficient rate of growth is:

\[ g^p = 1 + \sigma \left( \frac{l^p}{2} \right) y^p \]  

The quantity \( y^p \) is increasing in \( \beta, \theta \) and \( \vartheta \) and decreasing in \( \sigma \). Moreover, we can verify \( dl^p/d\beta > 0, \, dn^p/d\beta < 0, \, dh^p/d\beta < 0, \, ds^p/d\beta < 0 \) and \( dq^p/d\beta < 0 \). Therefore, the optimal growth rate is increasing in \( \beta \).

The equation (5) comes from the first-order condition on \( l \) whereas (6) comes from the first-order conditions for \( n, h \) and \( s \), respectively. (7) results from combining the envelope condition on \( a \) and the first-order condition on \( y \). Finally, (8) is the constraint (2) which binds in the planner’s solution.

### 2.2 Financial intermediation

Now we return to the market economy and describe financial intermediation. Because the household allocates money to the innovators before they learn whether they have can enter the innovation market, innovators have different liquidity needs after the entry selection is revealed. This generates a role for financial intermediaries that reallocate liquidity from those innovators that have no access to the innovation market to those that have access. The intermediation/financial sector is perfectly competitive in the sense that there is free entry of banks.

Financial intermediaries have no ability to keep records on transactions in the innovation market. This assumption prevents banks from issuing credit that supersedes money or directly intermediating the trade in the innovation market. However, banks are able to keep financial records on monetary loans and repayments, at a cost. So, borrowing and lending are in terms of money. If a buyer fails to repay a loan, the bank can confiscate money holdings of the buyer’s household, which ensures that loans are always repaid. Banks take the deposit rate as given and compete in the loan market. Depositors have perfect information about the banks’ financial state and trading histories, which induces the banks to always repay the depositors.\(^{12}\)

We assume that the production function in the financial sector is such that labor input required to produce loans is proportional to the number of loans.\(^{13}\)

That is,

\[ l/2 = \phi K. \]  

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\(^{12}\)Our assumptions of perfect monitoring by the banks on the borrowers and by the depositors on the banks simplify the analysis and enable us to focus on growth. For a relaxation of this assumption, see Berentsen, Camera and Waller (2007).

\(^{13}\)There are several possible alternative specifications. For example, we could also assume that the labor needed is proportional to the real stock of loans; i.e., \( Br/p = a\phi K \).
where \( l/2 \) is the number of loans, \( \phi \) measures the productivity of the financial system and \( K \) is the aggregate measure of agents per household working in the financial sector. We refer to the case \( \phi \to \infty \) as a perfect loan market, i.e. one in which financial intermediation requires no resources.

We indicate the economy-wide average of the amount of borrowing and lending per household as \( B_t \) and \( B_d \), respectively. The wage bill paid to the financial workers is the cost of operating the loans in nominal terms. Let \( w \) denote the nominal wage rate in the economy. Then, the total nominal cost of loans is \( wK \).

A financial intermediary covers the cost of loans with a spread between the loan rate, \( r_c \), and the deposit rate, \( r_d \). Therefore, banks’ profits are

\[
r_t B_t - r_d B_d - wK,
\]

where \( r_t B_t \) is the interest paid by borrowers to banks, \( r_d B_d \) is the interest paid by banks to depositors and \( wK \) is the wage bill paid to workers in the financial sector. In a competitive equilibrium with free entry of banks, banks’ profits are zero, so that

\[
r_t B_t - r_d B_d = wK. \tag{11}
\]

Financial intermediaries take the loan rate and the deposit rate as given. There is no strategic interaction among financial intermediaries or between financial intermediaries and agents. In particular, there is no bargaining over terms of the loan contract.

For clarity, let us describe the sequence of events in a period as follows. At the beginning of the period, each household chooses \((l,k,n,s,h)\), and divides its holdings of money among the innovators. Then, shoppers and innovators leave the household. Each innovator learns if he can enter the market for innovation goods and decides whether to borrow from or lend to intermediaries. Then, bilateral meetings occur among innovators. Simultaneously, shoppers buy consumption goods in the final goods market. Once trades are completed, all members return home and give their holdings of money, innovation goods and consumption goods to the household. Before the period ends, all debts are settled and households receive a lump-sum monetary transfer from the government.

### 2.3 The representative household’s decisions

At the beginning of the period, the household chooses the division of the members into workers in the financial sector, \( k \), innovators, \( l \), producers \( h \), members who enjoy leisure, \( n \), and buyers who enjoy leisure as well \( s \). It allocates money among innovators by giving \( m/l \) units of money to each of them. The household decides the quantity of consumption goods per buyer \( q_b \) and determines the offer made by innovators who buy in the innovation market, which consists of a quantity \( y \) of the innovation good and an amount \( d \) of money to be given in exchange. After leaving the household, innovators face equal probability of entering and not entering the market for innovation goods. This friction creates a role for the financial system, since innovators who enter the market may have
incentives to borrow money whereas innovators who cannot enter want to de-
posit their money holdings. We indicate by $b_t$ and $b_d$ the amounts borrowed and
deposited by an innovator, respectively. At the end of the period, the household
chooses future holdings of money, $m_{t+1}$, and the future productivity, $a_{t+1}$. The
decision variables of the household are then:

$$z \equiv [q_t, l, n, k, h, s, d, b_t, b_d, m_{t+1}, a_{t+1}] .$$

Let $m$ denote a representative household’s holdings of money at the begin-
ing of a period. The household’s value function is $V(a, m)$. Define:

$$\omega \equiv \beta \frac{\partial V(a_{t+1}, m_{t+1})}{\partial m_{t+1}}$$

and

$$\lambda \equiv \beta \frac{\partial V(a_{t+1}, m_{t+1})}{\partial a_{t+1}} .$$

The variable $\omega$ is the shadow value of money next period and $\lambda$ the shadow
value of future productivity, both of which are discounted to the current period.

The representative household’s problem is to choose $z$ to solve the following
problem:

$$V(a, m) = \max_z u(q_t, s, n) - \sigma (l/2) c(Y) + \beta V(a_{t+1}, m_{t+1})$$

subject to the following constraints:

$$1 \geq l + k + s + h + n \quad (\pi)$$

$$d \leq \frac{m}{l} + b_t \quad [\sigma (l/2) \mu]$$

$$b_d \leq \frac{m}{l} \quad [(l/2) \psi]$$

$$\delta \Omega \geq c(y) \quad [\sigma (l/2) \varsigma]$$

$$a_{t+1} = a + \sigma (l/2) ya$$

$$m_{t+1} - m = p(h_0ah - sq_b) + \sigma (l/2) (-d + D)$$

$$+ (l/2) (b_d r_d - b_c r_c) + w k + T$$

where $T$ is a lump-sum transfer or tax and $p$ is the nominal price of con-
sumption goods. The first term in the objective function represents the utility of
consuming the goods brought by buyers to the household and enjoying leisure.
The second term is the disutility from producing innovation goods and the third
term is the discounted future value. Notice that the amount of production by
an innovator is not subject to the decision of the household since it is proposed
by buyers in each meeting (we denote the amount produced with a capital letter
$Y$).

The constraint (12) is the time constraint of the household. The constraint
(13) specifies that an innovator who buys from another innovator cannot spend
more money than the sum of his money holdings, $m/l$, and the money borrowed,
$b_t$. According to (14), the amount of deposits of an agent is bounded by his
money holdings. (15) is the seller’s participation constraint: for a seller to be
willing to produce $y$, the value of the money received, $d \Omega$, must be at least as
high as the disutility $c(y)$. The constraint (16) is the law of motion of produc-
tivity described earlier. The law of motion of the household’s holding of money
is (17). The term \( p (sq_b - ph_0 ah) \) is the net income from trading in the final goods market. The term \( \sigma (l/2) (-d + D) \) is the net income from trading in the innovation goods market since \( (\sigma/2) \) innovators spend an amount \( d \) to buy innovation goods \( (\sigma/2) \) of them receive an amount \( D \) when selling innovation goods. The term \( (l/2) (h_{rd} - h_{rt}) \) is the net income from borrowing and lending since an equal number of innovators borrow money and deposit money. Finally, the household receives wage payments for workers in the financial sector, \( wk \), and lump-sum monetary transfers from the government, \( T \).

In the bilateral bargaining in the market for innovation goods we assume that the buyer has all the bargaining power. Therefore, the buyer solves:

\[
\max_{d,y} (a\lambda y - d\omega)
\]

s.t. \( d\Omega \geq c(y), d \leq m/l + b_t \)

The pay-off to the buyer is the gain from acquiring a quantity \( y \) of goods to improve the household’s productivity, measured by \( a\lambda y \), minus the value of the money paid to the seller, \( d\omega \). When solving this problem, the buyer must satisfy the seller’s participation constraint and the cash constraint.

### 2.4 Equilibrium definition and optimal conditions

We focus on the monetary equilibrium which is symmetric in the sense that the decisions are the same for all households. Throughout this paper, monetary policy is such that monetary transfer maintains the gross rate of money growth at a constant level \( \gamma \geq \beta \).

With the above focus, a monetary equilibrium consists of the representative household’s decisions, \( z \), other household’s decisions, \( Z \), and interest rates, \( r_d \geq 0 \) and \( r_c \geq 0 \), which meet the following requirements: (i) \( z \) solves the representative household’s maximization problem above; (ii) the decisions are symmetric across households: \( z = Z \); (iii) the credit market and the consumption-good markets clear; and iv) the seller’s participation constraint in the innovation market is non-negative.

We state the market clearing condition for consumption goods as:

\[
h_0 ah = sq_b \tag{18}
\]

The first-order condition on \( q_b \) is:

\[
\frac{1}{q_b} = \omega ps \tag{19}
\]

That is, the higher utility from increasing \( q_b \) and, hence, consumption, should be offset by the value of money used to settle the purchase.

Denote \( \sigma (l/2) \mu \) and \( (l/2) \psi \) the multipliers associated to (13) and (14), respectively. The first-order conditions on \( b_d \) and \( b_t \) are as follows:

\[
\psi = r_d\omega \tag{20}
\]

\[
\sigma \mu = r_c\omega \tag{21}
\]
The shadow prices associated to the deposit constraint and the cash constraint are given by the deposit and borrowing interest rates, respectively. We indicate by \( \sigma (l/2) \varsigma \) the multiplier associated to (15). The first-order condition on \( d \) and \( y \) are the following:

\[
\begin{align*}
\omega (\varsigma - 1) - \mu &= 0 \\
a\lambda - c'(y)\varsigma &= 0
\end{align*}
\]

Using (21), we rewrite them as follows:

\[
\begin{align*}
\varsigma &= \frac{r}{\sigma} + 1 \\
a\lambda &= c'(y) \left( \frac{r}{\sigma} + 1 \right)
\end{align*}
\] (22) (23)

Next, we calculate the optimal conditions regarding the allocation of time. We call \( \pi \) the multiplier associated to (12). The first-order condition on \( l \) is:

\[-\sigma (1/2) c(Y) - \pi - (1/2) (\sigma \mu + \psi) (m/l) + \lambda a (\sigma/2) y + \sigma (1/2) (-d + D) + (1/2) \omega (bdrd - brrt) = 0 \]

This condition says that the marginal cost of allocating an additional member of the household to the innovation market is given by a higher cost of producing innovation goods, the opportunity cost of not allocating an extra member to the rest of the activities and a decrease in the amount of money allocated to each innovator. It also says that the marginal benefit of an additional innovator comes from a higher productivity in the future. In addition, allocating an extra member to the innovation sector affects the money holdings of the household at the end of the period, as it is described by the other terms: it increases the money spent and received in the market for innovation goods and entails higher interest for the household’s deposits as well as higher interest payments.

In a symmetric equilibrium, \( y = Y \) and \( d = D \). These equalities allow us to write the first-order condition on \( l \) as follows:

\[-\sigma (1/2) c(y) - \pi - (1/2) (\sigma \mu + \psi) (m/l) + \lambda a (\sigma/2) y + (1/2) \omega (bdrd - brrt) = 0 \] (24)

The first-order condition on \( n \) is:

\[
\pi = \theta/n
\] (25)

Similarly, the first-order condition on \( s \) is:

\[
\frac{1 + \vartheta}{s} - \pi - \omega pq_{b} = 0
\]

According to (25), the opportunity cost of a member enjoying leisure should be equal to the marginal utility that he provides to the household. In the condition for \( s \) the utility from a greater quantity of goods for consumption and the value
of money to buy them are considered as well. This condition may be simplified as follows, using (19):

\[ \pi = \psi/s \]

The optimal condition for \( k \) is:

\[ \pi = \omega w \] (26)

The first-order condition on \( h \) is:

\[ \pi = \omega ph\alpha \]

Using (19) and (18), it reduces to:

\[ \pi = 1/h \]

Finally, we calculate the envelope conditions for \( a \) and \( m \). The envelope condition for \( a \) is

\[ \lambda_{-1}/\beta = \lambda [1 + \sigma (l/2)y] + \omega ph\alpha h \] (27)

The envelope condition for \( m \) is:

\[ \omega_{-1}/\beta = \omega + (1/2) (\sigma \mu + \psi) \] (28)

Envelope conditions state that the current value of an asset (\( m \) or \( a \)) is equal to the future value of the asset plus the additional value of the asset in the current exchange. According to (27), a marginal unit of productivity results in \([1 + \sigma (l/2)y]\) units of future productivity, the value of which is \( \lambda [1 + \sigma (l/2)y] \). In addition, a marginal unit of productivity saves production cost in the exchange, as it is computed in the second term of the condition for \( a \). Similarly, in (28), \( \mu \) is the shadow value of money that reflects how the household’s constraint on money is alleviated when an additional unit of money is acquired.

3 Balanced Growth

In this section, we characterize the balanced growth path of the economy. A balanced growth path is defined as an equilibrium in which productivity, \( a \), grows at a constant gross rate \( g \), and interest rates are non-negative and finite constants (i.e., \( 0 \leq r_d \leq r_c < \infty \)). It is clear from (16) that

\[ g = 1 + \sigma (l/2)y. \] (29)

Lemma 1 The balanced growth path has the following properties: (i) \( l \), \( k \), \( h \), \( s \) and \( n \) are constants in \((0,1)\) with \( l + k + h + n + s = 1 \); (ii) \( q_b \) grows at rate \( g \); (iii) the marginal value of money, \( \omega \), decreases at rate \( \gamma \); and (iv) the marginal value of productivity, \( \lambda \), falls at rate \( g \). Interest rates, \( r_d \) and \( r_c \), satisfy:

\[ \frac{(\gamma - \beta)}{\beta} = \frac{(r_c + r_d)}{2} \] (30)

\[ (r_c - r_d)\phi = (\alpha - 1)(\sigma + r_c) \] (31)
The equilibrium solution for \( \{\pi, y, l, n, s, k, h\} \) is determined by:

\[
\begin{align*}
\pi &= (\alpha - 1) c(y) \left( \frac{\sigma}{2} + r_t/2 \right) \quad (32) \\
\pi &= \theta/n = 1/h = \vartheta/s \quad (33) \\
1/R &= c'(y) [1 + \sigma (l/2) y] \left( 1 + r_t/\sigma \right) \quad (34) \\
1 &= l + n + s + h + k \quad (35) \\
1/2 &= \delta k \quad (36)
\end{align*}
\]

The value of money and consumption satisfy \( \omega_m = \left( \frac{l}{2} \right) c(y) \) and \( sq_b = h_0 a h \). Moreover, \( dr_d/d\gamma > 0, dr_t/d\gamma > 0 \) and \( dy/d\gamma < 0 \).

Equation (30) comes from (28), which is the envelope condition of money holdings. The condition (31) comes from the zero-profit condition of intermediation. Equation (32) comes from the first-order condition for \( l \) whereas equations in (33) come from the first-order conditions for \( n, h \) and \( s \), respectively. The equation in (34) is the envelope condition for \( a \) which comes from (27). Finally, (35) is the time constraint of the household and (36) replicates the production function of the financial sector.

The reminder of this section needs more work....

Denote \( g^C \) and \( g^N \) the real growth rates that prevail in an economy with credit market and in an economy without credit market, respectively. Similarly, let \( y^C \) and \( y^N \) be the respective quantities of \( y \) traded in these economies.

**Lemma 2** When \( \gamma > \beta \) and there is no credit market, the equilibrium allocation \( \{\pi, y, l, n, s, h\} \) satisfies the following equations:

\[
\begin{align*}
\pi &= (\alpha - 1) c(y) \left[ \frac{\sigma}{2} + \frac{\gamma - \beta}{2\beta} \right] \quad (37) \\
\pi &= \theta/n = 1/h = \vartheta/s \quad (38) \\
1/R &= c'(y) [1 + \sigma (l/2) y] \left[ 1 + 2 \left( \frac{\gamma - \beta}{\beta\sigma} \right) \right] \quad (39) \\
1 &= l + n + s + h + k \quad (40)
\end{align*}
\]

Moreover, \( \sigma (y^C g^C) > \sigma (y^N g^N) \) for \( \gamma > \beta \).

To examine how monetary policy affects long-run growth, let us first analyze an economy with a perfect loan market (i.e., with \( \phi \to \infty \)) and then an economy with an imperfect loan market (i.e., with \( \phi < \infty \)). Taking the limit \( \phi \to \infty \) in Lemma 1, it is straightforward to prove the following proposition on an economy with a perfect loan market:

**Proposition 2** For \( \gamma > \beta \) and \( \phi \to \infty \), the equilibrium allocation \( \{r_d, r_t, \pi, y, l, n, s, k, h\} \) satisfies \( r_d = r_t = r = \left( \gamma - \beta \right) / \beta \) and \( k = 0 \). Moreover, \( \{\pi, y, l, n, s, h\} \) satisfy

\[
\begin{align*}
\pi &= (\alpha - 1) c(y) \left[ \frac{\sigma}{2} + \frac{(\gamma - \beta)}{2\beta} \right] \quad (41) \\
\pi &= \theta/n = 1/h = \vartheta/s \quad (42) \\
1/R &= c'(y) \left[ 1 + \frac{\sigma}{2} l y \right] \left[ 1 + \frac{1}{2} \left( \frac{\gamma - \beta}{\beta\sigma} \right) \right] \quad (43) \\
1 &= l + n + s + h + k \quad (44)
\end{align*}
\]
The balanced growth path depends on money growth as follows: \( dr/d\gamma > 0 \)
\( dy/d\gamma < 0 \).

**Corollary 1** For any \( \gamma > \beta \), the quantity of innovation goods traded in an economy with a perfect credit market is strictly larger than in an economy without credit market.

### 4 Quantitative Analysis

In this section we calibrate the model to quantify the welfare and growth effects of inflation and an improvement in the productivity of financial intermediation.

#### 4.1 Calibration

We choose the following functional forms:

\[
\begin{align*}
    u(q_b, n, s) &= \ln (sq_b) + \ln (s^\beta n^{\theta}) , \\
    c(y) &= c_0 y^\alpha,
\end{align*}
\]

With the above functional forms, the parameters to be identified are as follows: (i) preference parameters: \( (\beta, \theta, \vartheta, h_0, c_0, \alpha) \); (ii) technology parameters: \( (\phi, \sigma) \); (iii) policy parameters: the money growth rate \( \gamma \). To identify these parameters, we calibrate the model to US data. Table 4 lists the identification restrictions and the identified values of the parameters.

<table>
<thead>
<tr>
<th>parameters</th>
<th>values</th>
<th>identification restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.9824</td>
<td>deposit rate = 0.075</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.0826</td>
<td>inflation rate = 0.057</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.0225</td>
<td>bank credit = 0.44</td>
</tr>
<tr>
<td>( h_0 )</td>
<td>1</td>
<td>normalization to 1</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>27.628</td>
<td>working time/total time = 0.2</td>
</tr>
<tr>
<td>( c_0 )</td>
<td>( 3.45 \times 10^{-18} )</td>
<td>shopping time/working time= 0.1116</td>
</tr>
<tr>
<td>( \phi )</td>
<td>112.225</td>
<td>loan officers/employment = 0.0025</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.256</td>
<td>per capita growth rate = 0.024</td>
</tr>
<tr>
<td>( \vartheta )</td>
<td>8.911</td>
<td>R&amp;D expenditure to GDP ratio= 0.02</td>
</tr>
</tbody>
</table>

The identification restrictions come from two sources. First, the inflation rate, bank credit and per capita growth rate matches the ones of the average low inflation country (see Table 2).\(^{14}\) Second, all other restrictions are from US data: the deposit rate matches the average interest rate on the US 3-month deposit certificates from 1965 -1995,\(^{15}\) as in King and Rebelo (1993), we choose the balanced-growth fraction of time working to be 20%; according to a report

---

\(^{14}\)We use the measure of the size of the financial sector, \( \text{priv} \), from Levine, Loayza and Beck (2000). The measure \( \text{priv} \) contains all claims on the private sector of banks that take deposits divided by GDP. From Table 2, the average \( \text{priv} \) among low-inflation countries is \( \text{priv} = 0.44 \).

\(^{15}\)A certificate of deposit is a time deposit. It is insured and thus virtually risk-free.
on occupational employment and wages by the Bureau of Labor Statistics (BLS, 2006), in May 2005 the fraction of loan officers to total employment was 0.0025;\textsuperscript{16} from the time-use diary reported in Juster and Stafford (1991), the ratio of shopping time to working time is 11.16%; from the OECD data reported in Grossman and Helpman (1995, p10), the average ratio of R&D expenditure to GDP is about 2% in the US.

In the Appendix we detail the calibration procedure which uses these restrictions to compute the parameter values listed in Table 4.

Table 5: Calibration

<table>
<thead>
<tr>
<th></th>
<th>Inflation</th>
<th>Growth</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>Data</td>
<td>0.057</td>
<td>0.024</td>
</tr>
<tr>
<td>Mid</td>
<td>Data</td>
<td>0.092</td>
<td>0.019</td>
</tr>
<tr>
<td>High</td>
<td>Data</td>
<td>0.316</td>
<td>0.013</td>
</tr>
</tbody>
</table>

Table 5 compares the model’s predictions on inflation, the growth rate and the size of the financial sector with the data. We have calibrated the model to the average low-inflation country which explains why the model and the data are the same in the row labelled "Low".

Our model predicts an non-linear relationship between the financial sector measure bank credit and inflation as Figure 1 shows. At low levels of inflation an increase in inflation reduces the size of the financial sector more than at high level of inflation.

\textsuperscript{16}According to this report, in May 2005, 332690 people were working as loan officers and the total employment was 130307850. The ratio 0.0025 is similar to the 0.0028 reported in Dotsey and Ireland (1996)
A highly interesting aspect of the model, consistent with the data, is the non-linear relationship between inflation and real growth. At high rates of inflation, 10% reduction in inflation has almost no growth effects while at low rates of inflation a 10% reduction of inflation has large growth effects. We will discuss this property of the model in subsection 4.5.

**4.2 Welfare analysis**

With the identified model, we now quantify the cost of inflation, $\Pi$, and the benefit of an exogenous improvement in financial productivity, $\phi$. We focus on the balanced growth path.

Following the literature (e.g. Lucas, 2000 or Lagos and Wright, 2005), we measure the welfare cost of inflation at $\Pi$ relative to $\Pi'$ by asking how much consumption in percentage agents would be willing to give up in order to change inflation from $\Pi$ to $\Pi'$. Similarly, we measure the welfare benefit of an improvement in financial productivity from $\phi$ to $\phi'$ by asking how much consumption in percentage agents would be willing to give up for the improvement. To express these measures formally, let $\Pi$ be any given inflation rate, $\phi$ any level of the exogenous component of financial productivity, and $(1 - \Delta)$ any rate of a consumption tax. Slightly abusing an earlier notation, we write the household’s expected discounted utility under $(\Pi, \Delta, \phi)$ as:

$$W(\Pi, \Delta, \phi) = \sum_{t=1}^{\infty} \beta^{t-1} \left[ \ln (\Delta s q_{b,t}) + \ln \left( n^\theta s^\delta \right) - \sigma \left( l/2 \right) c(y) \right]$$

where all the quantities $(y, q_{b}, l, n, s)$ take their equilibrium values when $\Delta = 1$.  

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Note that $q_b$ is indexed by $t$ because it evolves over time according to the growth rate $g$. This expression can be rewritten as follows:

$$W(\Pi, \Delta, \phi) = \frac{1}{1 - \beta} \left[ \ln \left( n^0 s^0 \right) + \ln (\Delta s q_b) - \frac{\sigma}{2} c(y) + \frac{\beta}{1 - \beta} \ln (g) \right]$$

For any fixed $\phi$, the welfare cost of inflation at $\Pi$ relative to $\Pi'$ is the value of $\Delta$ that solves $W(\Pi', \Delta, \phi) = W(\Pi, 1, \phi)$. Similarly, for any fixed $\Pi$, the welfare benefit of improving the exogenous component of financial productivity from $\phi$ to $\phi'$ is the value of $\Delta$ that solves $W(\Pi, \Delta, \phi') = W(\Pi, 1, \phi)$.

### 4.3 The growth and welfare effects of inflation

Table 6 characterizes the balanced-growth equilibrium calibrated to the average low-inflation country. It compares this equilibrium to those obtained with zero inflation, the average inflation rate of the medium-inflation countries (9.21%), and the average inflation rate of the high-inflation countries (31.6%).

It also reports for each country type, the welfare and per capita growth gains from moving to a zero inflation rate policy.

<table>
<thead>
<tr>
<th>Table 6: Effects of inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time innovating</td>
</tr>
<tr>
<td>Time working in finance</td>
</tr>
<tr>
<td>Shopping time</td>
</tr>
<tr>
<td>Leisure</td>
</tr>
<tr>
<td>Time producing</td>
</tr>
<tr>
<td>Interest rate wedge</td>
</tr>
<tr>
<td>Bank credit (priv)</td>
</tr>
<tr>
<td>Participation rate</td>
</tr>
<tr>
<td>Innovation goods (y)</td>
</tr>
<tr>
<td>GDP</td>
</tr>
<tr>
<td>Growth rate</td>
</tr>
<tr>
<td>Growth loss (percentage points)</td>
</tr>
<tr>
<td>Welfare loss (% of consumption)</td>
</tr>
</tbody>
</table>

An increase in inflation changes total labor supply and its composition. Table 6 shows that as the inflation rate rises, the household reduces the time spent in the innovation sector. It also slightly reduces the time working in finance. Inflation changes the distribution of time spent in leisure and between work and leisure. The increase in inflation also reduces the time spent producing and increases the time spent shopping. The interest rate wedge and bank credit also increase with inflation. The participation rate decreases with inflation. The growth rate decreases with inflation, and the growth loss increases with inflation. The welfare loss also increases with inflation.
contrast, the household increases leisure, shopping time and the time producing consumption goods. Thus, any additional labor allocated to either leisure, production or shopping as a result of higher inflation comes mainly from reducing labor in the innovation sector. The substitution of labor out of innovation and into leisure, shopping and production reduces the per capita growth rate.

The effects of inflation on growth are large. For the average medium-inflation country, reducing its inflation from the average (9.2%) to zero increases the long-run growth rate by 0.385 percentage points. For the average high-inflation country, reducing its average inflation to zero increases the long-run growth rate by 0.86 percentage points. These large effects of inflation on per capita growth fall into the range of empirical estimates obtained from cross-country studies (see the references in the Introduction). They distinguish the quantitative performance of our model from those of previous models such as Gomme (1993) or Dotsey and Ireland (1996), who find that reducing inflation to zero in a similar experiment increases growth by only 0.05 percentage points.

Higher rates of inflation also affect the financial market. Table 6 shows that the interest rate wedge increases with inflation. At 9.2 percent inflation the wedge is 6.5%. The wedge becomes 14% for the average high inflation country. Participation in the financial market, measured by the fraction of innovators who take out loans, and the ratio of loans to GDP strongly decrease with inflation showing a reduction of the financial market size. Finally, aggregate output and the amount of innovation goods traded decrease as inflation increases.

4.4 Decomposing the welfare and growth effects: allocation of time

Table 6 shows that the welfare cost of inflation are 1.053% of consumption for the average low-inflation country, 1.622% for the average medium inflation country and 4.024% for the average high-inflation country. Moreover, inflation has large negative effects on growth. In this subsection, we investigate how the distortions of inflation on various margins of labor supply contribute to the welfare and growth effects of inflation.
Table 7: Constant labor in the innovation sector

<table>
<thead>
<tr>
<th></th>
<th>0%</th>
<th>5.71%</th>
<th>9.21%</th>
<th>31.6%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time innovating</td>
<td>0.1122</td>
<td>0.1122</td>
<td>0.1122</td>
<td>0.1122</td>
</tr>
<tr>
<td>Time working in finance</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
</tr>
<tr>
<td>Shopping time</td>
<td>0.0223</td>
<td>0.0223</td>
<td>0.0223</td>
<td>0.0223</td>
</tr>
<tr>
<td>Leisure</td>
<td>0.7777</td>
<td>0.7777</td>
<td>0.7777</td>
<td>0.7777</td>
</tr>
<tr>
<td>Time producing</td>
<td>0.0873</td>
<td>0.0873</td>
<td>0.0873</td>
<td>0.0873</td>
</tr>
<tr>
<td>Interest rate wedge</td>
<td>0.0361</td>
<td>0.0545</td>
<td>0.0662</td>
<td>0.1477</td>
</tr>
<tr>
<td>Bank credit (priv)</td>
<td>0.6322</td>
<td>0.4408</td>
<td>0.3729</td>
<td>0.1939</td>
</tr>
<tr>
<td>Participation rate</td>
<td>0.0561</td>
<td>0.0561</td>
<td>0.0561</td>
<td>0.0561</td>
</tr>
<tr>
<td>Innovation goods (y)</td>
<td>4.8034</td>
<td>4.7396</td>
<td>4.7105</td>
<td>4.5989</td>
</tr>
<tr>
<td>GDP</td>
<td>1.0351</td>
<td>1.0261</td>
<td>1.0229</td>
<td>1.0146</td>
</tr>
<tr>
<td>Growth rate</td>
<td>0.0245</td>
<td>0.0241</td>
<td>0.024</td>
<td>0.0234</td>
</tr>
<tr>
<td>Growth loss (percentage points)</td>
<td>0.</td>
<td>0.0328</td>
<td>0.0477</td>
<td>0.105</td>
</tr>
<tr>
<td>Welfare loss (% of consumption)</td>
<td>0.</td>
<td>0.8834</td>
<td>1.3791</td>
<td>3.6323</td>
</tr>
</tbody>
</table>

First, let us examine whether the choice of labor in innovation is important for the effects of inflation. To do so, Table 7 considers a version of the model with exogenous time devoted to innovation. This version of the model keeps the fraction of agents working in innovation constant at its calibrated value $l^{\ast} = 0.1122$.\(^ {18}\) In contrast to Table 6, the allocation of time in Table 7 is inelastic. The quantity of innovation goods traded in the innovation sector $y$ is only slightly smaller in Table 7 than in Table 6. As a result, the growth losses of inflation are substantially smaller when $l$ is constant. Moreover, the real output and the measure priv turn out to be less sensitive to inflation. Another contrast of Table 7 with Table 6 is that the welfare cost of inflation is smaller. For example, 9.21% inflation costs 1.379% of consumption under constant $l$, rather than 1.622% as under endogenous $l$. This clearly indicates that the effect of inflation on the choice for $l$ matters for generating the welfare cost estimates reported in Table 6.

\(^{18}\)Tables 6-10 are constructed as follows. First, we replace one equilibrium condition by setting $x = x^{\ast}$ where $x$ is the variable under consideration and $x^{\ast}$ its calibrated value. Second, we simulate the model for different values of inflation. For example, for table 6, $x = l$ and $x^{\ast} = l^{\ast}$.

Note that we do not recalibrate the model since this not necessary. A recalibration of the model for each experiment - using the same targets - would yield the same values for the calibrated parameters as in the baseline case. The reason is that $x$ is held constant at its equilibrium value obtained from the baseline calibration.
Table 8: Constant leisure

<table>
<thead>
<tr>
<th></th>
<th>0%</th>
<th>5.71%</th>
<th>9.21%</th>
<th>31.6%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time innovating</td>
<td>0.1136</td>
<td>0.1122</td>
<td>0.1116</td>
<td>0.109</td>
</tr>
<tr>
<td>Time working in finance</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
</tr>
<tr>
<td>Shopping time</td>
<td>0.022</td>
<td>0.0223</td>
<td>0.0225</td>
<td>0.023</td>
</tr>
<tr>
<td>Leisure</td>
<td>0.7777</td>
<td>0.7777</td>
<td>0.7777</td>
<td>0.7777</td>
</tr>
<tr>
<td>Time producing</td>
<td>0.0861</td>
<td>0.0873</td>
<td>0.0878</td>
<td>0.0899</td>
</tr>
<tr>
<td>Interest rate wedge</td>
<td>0.0368</td>
<td>0.0545</td>
<td>0.0657</td>
<td>0.1424</td>
</tr>
<tr>
<td>Bank credit (priv)</td>
<td>0.6371</td>
<td>0.4408</td>
<td>0.3714</td>
<td>0.1895</td>
</tr>
<tr>
<td>Participation rate</td>
<td>0.0568</td>
<td>0.0561</td>
<td>0.0558</td>
<td>0.0545</td>
</tr>
<tr>
<td>Innovation goods (y)</td>
<td>4.8026</td>
<td>4.7396</td>
<td>4.7107</td>
<td>4.5999</td>
</tr>
<tr>
<td>GDP</td>
<td>1.0355</td>
<td>1.0261</td>
<td>1.0227</td>
<td>1.014</td>
</tr>
<tr>
<td>Growth rate</td>
<td>0.0248</td>
<td>0.0241</td>
<td>0.0238</td>
<td>0.0227</td>
</tr>
<tr>
<td>Growth loss (percentage points)</td>
<td>0.0635</td>
<td>0.0926</td>
<td>0.2041</td>
<td></td>
</tr>
<tr>
<td>Welfare loss (% of consumption)</td>
<td>0.9031</td>
<td>1.4062</td>
<td>3.6671</td>
<td></td>
</tr>
</tbody>
</table>

Second, let us analyze whether the leisure choice is important for the effects of inflation. To do so, Table 8 considers a version of the model where the amount of leisure is fixed at the calibrated level $n^* = 0.777$. It is clear from Table 8 that the allocation of time is again rather inelastic, because a large part of it (i.e., leisure) is fixed. A comparison with Table 6 makes it more evident that the increase in leisure in the general model is at the expense of innovation rather than reducing the time producing, shopping or trading, since the resources assigned to these three activities are similar in Tables 6 and 8. This explains the large differences in the growth effects between the complete model and the version with $n$ constant: in the former case, growth losses are more than 4 times higher for the three groups of countries. In addition, from Table 8 it is noticeable that endogenizing the choice of leisure is important for determining the welfare cost of inflation, since this is smaller for $n$ fixed. Actually, its estimate is very close to the one for $l$ fixed, which is consistent with the fact that, when inflation changes, households adjust their choices basically through leisure and innovation.
Table 9: Constant shopping time

<table>
<thead>
<tr>
<th></th>
<th>0%</th>
<th>5.71%</th>
<th>9.21%</th>
<th>31.6%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time innovating</td>
<td>0.1225</td>
<td>0.1122</td>
<td>0.1073</td>
<td>0.0878</td>
</tr>
<tr>
<td>Time working in finance</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0004</td>
</tr>
<tr>
<td>Shopping time</td>
<td>0.0223</td>
<td>0.0223</td>
<td>0.0223</td>
<td>0.0223</td>
</tr>
<tr>
<td>Leisure</td>
<td>0.7684</td>
<td>0.7777</td>
<td>0.7821</td>
<td>0.7998</td>
</tr>
<tr>
<td>Time producing</td>
<td>0.0862</td>
<td>0.0873</td>
<td>0.0878</td>
<td>0.0898</td>
</tr>
<tr>
<td>Interest rate wedge</td>
<td>0.0374</td>
<td>0.0545</td>
<td>0.0654</td>
<td>0.1403</td>
</tr>
<tr>
<td>Bank credit (priv)</td>
<td>0.675</td>
<td>0.4408</td>
<td>0.3592</td>
<td>0.155</td>
</tr>
<tr>
<td>Participation rate</td>
<td>0.0613</td>
<td>0.0561</td>
<td>0.0537</td>
<td>0.0439</td>
</tr>
<tr>
<td>Innovation goods (y)</td>
<td>4.7999</td>
<td>4.7396</td>
<td>4.7115</td>
<td>4.602</td>
</tr>
<tr>
<td>GDP</td>
<td>1.0378</td>
<td>1.0261</td>
<td>1.022</td>
<td>1.0114</td>
</tr>
<tr>
<td>Growth rate</td>
<td>0.0267</td>
<td>0.0241</td>
<td>0.0229</td>
<td>0.0183</td>
</tr>
<tr>
<td>Growth loss (percentage points)</td>
<td>0.</td>
<td>0.2575</td>
<td>0.377</td>
<td>0.8421</td>
</tr>
<tr>
<td>Welfare loss (% of consumption)</td>
<td>0.</td>
<td>1.0489</td>
<td>1.6163</td>
<td>4.0143</td>
</tr>
</tbody>
</table>

Table 9 shows the simulation results with constant shopping time $s^* = 0.0223$. If we compare the results with the benchmark model (Table 6) one can see that the differences are small. The reason is that the $s$ is not very sensitive to changes in inflation. Moreover, its calibrated value is low.\textsuperscript{19} Finally, we have also studied a version of the model where the time for production of consumption goods $h$ is held constant. We obtain similar results as shown in Table 9 and we therefore omit to present them.

4.5 Why is there a non-linear relation between inflation and growth?

As we mentioned before the effects of inflation on per capita growth are highly nonlinear. At high rates of inflation, 10% reduction in inflation has almost no growth effects while at low rates of inflation a 10% reduction of inflation has large growth effects (see Figure 1 below). In this section we show that the non-linearity is driven by the choice of $y$ which is affected by inflation and not by the allocation of time. Inflation decreases the value of money and hence the willingness of a seller to produce goods in a bilateral meeting. Therefore, the quantity $y$ traded per buyer in the innovation sector decreases with inflation. In order to estimate the importance of this channel, Table 10 presents a version of the model in which $y$ is taken as exogenous. It is kept at its calibrated value $y^* = 4.7396$ for all inflation rates considered.

\textsuperscript{19}This result is different from the findings by Love and Wen (1999), who attribute important welfare effects of inflation to the time-costs of transacting featured by their model.
Interestingly, the growth loss when $y$ is constant is similar to the estimate of the general model for the high-inflation countries, but it is considerably smaller for the low-inflation and medium-inflation countries. For the low-inflation countries, the loss in the growth rate is equivalent to 0.174 percentage points for $y$ fixed against 0.263 percentage points for $y$ endogenous. These figures are 0.278 and 0.385 percentage points, respectively, for the medium inflation countries. This result clearly suggests that the non-linear relationship between inflation and growth predicted by the model is due to changes in $y$ across inflation rates. To illustrate this, we have plotted in Figure 2 the growth gains predicted by the model that result from a reduction in inflation of 10 percentage points at each inflation rate. In the same figure, we have plotted the growth gains predicted by the version of the model with $y$ constant. Comparing both curves, it is clear that the non-linearity of the relation between inflation and per capita growth is entirely driven by the choice of $y$ that changes in inflation.

In previous models (Gomme, 1993; Dotsey and Ireland, 1996; and others) the only mechanism which affected the per capita growth rate was the allocation of time. These authors found very small growth effects of inflation and, moreover, these effects were linear. If we hold constant $y$ at its calibrated value in our model, so that only the allocation of time affects the growth rate, we still have large effects of inflation on growth but they become linear. Thus, our model not only predicts large effects of inflation on growth but, in addition, they are highly non-linear. The reason why we get these non-linear effects is that the cost of producing $y$ is convex and, further, the bargaining solution for $y$ depends on the value of money, which is not linear in the inflation rate.
In the case of low-inflation and medium-inflation countries, the effect of inflation through the choice of $y$ is crucial to account for the relation between inflation and growth, as it can be seen from Figure 2 and Tables 6 and 10. In the case of the high-inflation countries, this effect becomes less important relative to the effect of inflation through the allocation of time. If households cannot adjust $y$, they set $l$ to a value lower than the one in the general model in this case, as we can check comparing Tables 6 and 10. As a result, the growth loss for the high-inflation group is slightly higher when $y$ is constant.

Finally, a major contrast between Tables 6 and 10 is that in the former the welfare losses owing to inflation are substantially higher when $y$ is allowed to adjust and this is verified for the three average inflation rates considered.

4.6 Exogenous development in financial intermediary

In this subsection, we change the parameter $\phi$ which captures what Levine, Loayza and Beck (2000) call the exogenous component of financial intermediary development to analyze its impact on welfare and growth. Table 11 displays the effects of reducing $\phi$ by 50% or increasing it by 100% for each country type. First, for each country type higher efficiency in the financial sector increases the time spent in the innovation sector. Moreover, the amount traded per innovator $y$ is also increasing in $\phi$. Therefore, a larger financial productivity entails gains in the growth rate. For instance, for the average low-inflation country, the growth loss of cutting $\phi$ by one half is equivalent to 0.1165 percentage points and the growth gain of doubling $\phi$ is equal to 0.0535 percentage points. The effect of financial productivity in growth does not greatly differ across inflation rates, although the gain from a more productive financial sector is slightly higher for lower inflation rates. In addition, welfare losses from having a financial
productivity that is half the calibrated productivity are similar among country types and represent about 1% of consumption.

Table 11: Exogenous increase in productivity

<table>
<thead>
<tr>
<th>Low inflation</th>
<th>Medium inflation</th>
<th>High inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta^2 )</td>
<td>( 2\delta )</td>
<td>( \delta^2 )</td>
</tr>
<tr>
<td>Time innovating</td>
<td>0.1074</td>
<td>0.1144</td>
</tr>
<tr>
<td>Time working in finance</td>
<td>0.001</td>
<td>0.0003</td>
</tr>
<tr>
<td>Shopping time</td>
<td>0.0224</td>
<td>0.0223</td>
</tr>
<tr>
<td>Leisure</td>
<td>0.7815</td>
<td>0.776</td>
</tr>
<tr>
<td>Time producing</td>
<td>0.0877</td>
<td>0.0871</td>
</tr>
<tr>
<td>Interest rate wedge</td>
<td>0.125</td>
<td>0.0256</td>
</tr>
<tr>
<td>Bank credit (priv)</td>
<td>0.365</td>
<td>0.4798</td>
</tr>
<tr>
<td>Participation rate</td>
<td>0.0537</td>
<td>0.0572</td>
</tr>
<tr>
<td>Innovation goods (( \gamma ))</td>
<td>4.7151</td>
<td>4.7906</td>
</tr>
<tr>
<td>GDP</td>
<td>1.0278</td>
<td>1.025</td>
</tr>
<tr>
<td>Growth rate</td>
<td>1.0057</td>
<td>1.0061</td>
</tr>
<tr>
<td>Growth loss (pp)</td>
<td>0.1165</td>
<td>–0.0535</td>
</tr>
<tr>
<td>Welfare loss (% of c)</td>
<td>1.0632</td>
<td>–0.5115</td>
</tr>
</tbody>
</table>
Appendix

Proof of Proposition 1

The planner chooses \( P = \{q_b, q, y, l, n, h, s\} \) to maximize the following welfare function:

\[
W(a) = \max_P u(q_b, s, n) - \sigma (l/2) c(y) + \beta W(a_{+1}) \tag{45}
\]

subject to the following constraints:

\[
\begin{align*}
sq_b & \leq q \tag{46} \\
q & \leq ah_0h \tag{47} \\
1 & \geq l + n + s + h \tag{48} \\
a_{+1} & = a + a\sigma (l/2)y \tag{49}
\end{align*}
\]

We rewrite (45) using (46), (47) and the functional form of \( u(q_b, s, n) \):

\[
W(a) = \max_P \ln (ah_0h) + \ln (s^n n^s) - \sigma (l/2) c(y) + \beta W(a_{+1})
\]

subject to the constraints:

\[
\begin{align*}
1 & \geq l + n + s + h \\
a_{+1} & = a + a\sigma (l/2)y
\end{align*}
\]

The first-order condition on \( y \) is:

\[
\lambda a = c'(y) \tag{50}
\]

The first-order conditions for \( n, h \) and \( s \) are as follows:

\[
\begin{align*}
\pi & = \theta/n \tag{51} \\
\pi & = 1/h \tag{52} \\
\pi & = \vartheta/s \tag{53}
\end{align*}
\]

The first-order condition on \( l \) is:

\[
\lambda ay = c(y) = (2/\sigma) \pi
\]

Using (50), it becomes

\[
\pi = (\alpha - 1) c(y) (\sigma/2) \tag{54}
\]

The envelope condition for \( a \) is:

\[
\frac{\lambda_{-1}}{\beta} = \lambda [1 + \sigma (l/2)y] + 1/a
\]

We rewrite it using (50):

\[
\frac{1}{R} = c'(y) [1 + \sigma (l/2)y] \tag{55}
\]
Finally, constraint (48) binds in the planner’s choice, so that:

$$1 = l + n + s + h$$

Equations (51)-(56) form the planner’s solution stated in Proposition 1. To see that a unique solution to the above system exists, we rewrite (55). First, we replace $l$ with (56) and then $n, h$ and $s$ with (51), (52) and (53) as follows:

$$1/R = c'(y) \left[ 1 + (\sigma/2) y \left( 1 - \frac{\theta + \vartheta + 1}{\pi} \right) \right]$$

Then, we replace $\pi$ using (54) and rearrange to get:

$$c'(y) [1 + (\sigma/2) y] = 1/R + \frac{\alpha (\theta + \vartheta + 1)}{\alpha - 1}$$

(57)

From (57) we obtain the solution for the optimal value of $y$. The left-hand side in (57) is monotonously increasing in $y$, is equal to 0 when $y = 0$ and approaches $+\infty$ as $y$ approaches $+\infty$. Since the right-hand side is constant, given $\sigma, \beta, \alpha, \vartheta$ and $\theta$, a unique solution for $y$ exists. The solution for $n$ comes from (51) and it is unique since the solution for $y$ is unique. Analogously, the solutions for $h, s$ and $l$, which are recursively obtained from (52), (53) and (56), are unique. Finally, it is straightforward to verify that $y$ is increasing in $\beta, \vartheta$ and decreasing in $\sigma$ and that $dn/d\beta < 0, dh/d\beta < 0, ds/d\beta < 0$ and $dl/d\beta > 0$.

**Proof of Lemma 1**

To get a two-equation system in $r_d$ and $r_l$, we first rewrite the envelope condition on $m$ (28) using the first-order conditions on $b_d$ and $b_l$ (20) and (21) to get (30). Second, we rewrite the banks’ zero-profit condition (11). The total amount of borrowing is equal to the total amount of deposits, so that $B_l = B_d$. In addition, $B_l$ is equal to $(l/2) b_l = (l/2) b_d$, which we can rewrite as $(l/2) (m/l)$. Therefore, (11) becomes:

$$\omega (m/l) \phi (r_l - r_d) = \pi$$

(58)

where we have used the production function of loans (10) and the first-order condition on $k$ (26). We set $\omega (m/l) = \omega (d/2) = c(y)/2$ to get:

$$c(y) \phi (r_l - r_d)/2 = \pi$$

(59)

Besides, we rewrite the first-order condition on $l$ (24) using (20), (21) and (23)

$$(\alpha - 1) (r_l + \sigma) (1/2) c(y) = \pi$$

(60)

Combining (59) and (60) yields the second equation in $r_l$ and $r_d$ (31). (32) and (33) have already been derived whereas (27) is the envelope condition for $a$ that results from using (23). To see that $\omega m = (l/2) c(y)$ simply combine (58)
with (59). To see that \( dr_d/d\gamma > 0 \) and \( dr_t/d\gamma > 0 \) we pin down \( r_t \) and \( r_d \) from (30) and (31) as follows:

\[
\begin{align*}
    r_d &= 2 \frac{(\gamma - \beta)}{\beta} - r_t \\
    r_t &= \frac{(\alpha - 1)\sigma + 2\phi (\gamma - \beta)}{2\phi - (\alpha - 1)}
\end{align*}
\]  

(61) (62)

From (61) and (62), we know that the solution for \( r_t \) and \( r_d \) is unique. In addition, we verify:

\[
\begin{align*}
    dr_t/d\gamma &= \frac{2\phi}{\beta} > 0 \\
    dr_d/d\gamma &= \frac{2/\beta}{2\phi - (\alpha - 1)} = \frac{2\phi (\sigma + r_d)}{\beta [2\phi - (\alpha - 1)] (\sigma + r_t)} > 0
\end{align*}
\]

where we have used (31). To verify that \( dy/d\gamma < 0 \) we use the system in Lemma 1 to get an equation in \( y \). First, we get an expression for \( l \) from the time constraint (35), replacing \( h, n \) and \( s \) with the first-order conditions on them stated in (33) and \( k \) with the production function of loans (36):

\[
l = \frac{1 - (\theta + \vartheta + 1)}{1 + 1/(2\phi)}
\]

Then, we replace \( l \) in the envelope condition for \( a \) (34). We also use (60) to replace \( \pi \) and (62) to replace \( r_t \):

\[
\frac{1}{R} + \frac{\alpha (\theta + \vartheta + 1)}{(\alpha - 1) [1 + 1/(2\phi)]} = \frac{c'(y)}{c(y)} \left( 1 + \frac{(\sigma/2)y}{1 + 1/(2\phi)} \right) \left( 1 - (\gamma - \beta)/(\sigma \beta) \right)
\]

From this equation it is straightforward to compute \( dy/d\gamma < 0 \). It is also possible to verify that a unique solution for \( y \) exists. As a result, the solution is unique for \( \pi, l, n, s \) and \( h \).

**Proof of Lemma 2**

In the absence of credit market, the representative household solves the following problem:

\[
V(a, m) = \max_z u (q_b, s, n) - \sigma (l/2) c(Y) + \beta V(a_{+1}, m_{+1})
\]

subject to the following constraints:

\[
\begin{align*}
    1 & \geq l + s + h + n \quad (\pi) \\
    d & \leq m/l \quad [\sigma (l/2) \mu] \\
    d\Omega & \geq c(y) \quad [\sigma (l/2) \varsigma] \\
    a_{+1} & = a + \sigma (l/2) ya \\
    m_{+1} - m & = p (h_0 a h - s q_b) + \sigma (l/2) (-d + D) + T
\end{align*}
\]
The first-order conditions on \( q_b, s, n \) and \( h \) are as in the case with financial market. The first-order condition on \( y \) is
\[
\lambda a = c'(y) \varsigma
\]  
(63)

The first-order condition on \( l \) is:
\[
-\sigma \left( \frac{1}{2} \right) c(Y) - \pi - \left( \frac{\sigma}{2} \right) \mu \left( \frac{m}{l} \right) + \lambda a \left( \frac{\sigma}{2} \right) y = 0
\]

The optimal condition for the choice of \( d \) is:
\[
\mu = \omega (\varsigma - 1)
\]  
(64)

The envelope condition for \( m \) is:
\[
\omega^{-1/\beta} = \omega + \sigma \left( \frac{1}{2} \right) \mu
\]

Using (64), it becomes
\[
\varsigma = \frac{2(\gamma - \beta)}{\beta \sigma} + 1
\]  
(65)

The envelope condition for \( a \) is:
\[
R \left[ 1 + \sigma \left( \frac{l}{2} \right) y \right] = \frac{1}{\lambda a}
\]

Using (63) and (65), it is:
\[
\frac{1}{R} = c'(y) \left[ 1 + \sigma \left( \frac{l}{2} \right) y \right] \left[ 2(\gamma - \beta) / (\beta \sigma) + 1 \right]
\]  
(66)

which is equation (39) in Lemma 2. We rewrite the first-order condition on \( l \) using (64), (65) and \( \omega (m/l) = \omega d = c(y) \):
\[
\pi = (\alpha - 1) c(y) \left[ \sigma / 2 + (\gamma - \beta) / \beta \right]
\]  
(67)

which is equation (37). Equations (38) and (40) are as in the case with financial market and equation except that \( k = 0 \).

To verify that \( \sigma \left( y^C \right) g^C > \sigma \left( y^N \right) g^N \), pin down \( r_t \) from (30) and insert it in (34) as follows:
\[
1/R = c'(y) \left[ 1 + \sigma \left( l/2 \right) y \right] \left[ 1 + 2(\gamma - \beta) / (\beta \sigma) - (r_d/\sigma) \right]
\]  
(68)

Comparing (68) with the equation (66) that corresponds to the case without financial market, it is immediate that \( \sigma \left( y^C \right) g^C > \sigma \left( y^N \right) g^N \).

**Proof of Proposition 2**

First, since the functioning of a perfect credit market requires no resources, \( k = K = 0 \). From (11), \( r_t = r_d = r \). Then, from the envelope condition for \( m \) as stated in (30) in Lemma 1 we obtain \( r = (\gamma - \beta) / \beta \) from which we can check that \( dr/d\gamma > 0 \).
(42) and (44) are straightforward. To get (41), we replace \( r_t = r = (\gamma - \beta) / \beta \) in the analogous equation in Lemma 1 (32). To get (43), we also replace \( r_t \) in (34). That \( dy/d\gamma < 0 \) may be shown as in the system in Lemma 1. When the financial market is perfect, the equation that solves for \( y \) reduces to:

\[
c' (y) [1 + (\sigma/2) y] [1 + (\gamma - \beta) / (\beta \sigma)] = 1/R + \frac{\alpha (\theta + \vartheta + 1)}{\alpha - 1} \tag{69}
\]

**Proof of Corollary 1**

To get to an equation in \( y \) in the case with no credit market, we rewrite (66) replacing \( l \) with the time constraint (40) and then \( n, h \) and \( s \) as we did above, so that we have:

\[
1/R = c' (y) \left[ 1 + \left( \frac{\theta + \vartheta + 1}{\pi} \right) y \right] \left[ 2 \left( \frac{\gamma - \beta}{\beta \sigma} + 1 \right) \right]
\]

Then, we replace \( \pi \) with (67):

\[
c' (y) [1 + (\sigma/2) y] [1 + 2 (\gamma - \beta) / (\beta \sigma)] = 1/R + \frac{\alpha (\theta + \vartheta + 1)}{\alpha - 1}
\]

Comparing the solution for \( y \) from this equation without credit market with the one in (69) it is immediate to check that \( y \) is higher in an economy with a perfect credit market than in an economy with no credit market.

**The calibration procedure**

We use the following calibration strategy:

1) We first determine the allocation of time between leisure, goods production, shopping, innovation and working in the financial sector by using three targets: the ratio of shopping time to working time, 11.16%, the balanced-growth fraction of working time, 20% and the fraction of people working as loan officers to total employment, 0.0025. This yields \( n^*, s^* \) and \( k^* \).

2) We set the model’s net real growth rate, \((g - 1)\), equal to the average US per capita real growth rate. This yields \( g^* \).

3) We calibrate the growth rate of the money supply as \( \gamma^* = (1 + \Pi) g^* \), where \( \Pi \) is the average US net inflation rate. We set the deposit rate equal to the average interest rate of 3-month deposit certificates and the loan rate equal to the sum of the deposit rate and the wedge in interest rates (set to 7%). Then, from (30) we get \( \beta^* \).

5) We normalize \( h_0^* = 1 \). A change in \( h_0^* \) does not affect the calibration results.

6) We are left with five parameters to identify: \( \{c_0, \alpha, \sigma, \theta, \vartheta\} \). We start by getting \( y \) as a function of \( \sigma \) and \( l \) with the growth equation (29):

\[
y = 2 (g - 1) / (\sigma l) \tag{70}
\]

20 The relation between inflation \( \Pi \) and money growth \( \gamma - 1 \) is given by: \( \gamma = (1 + \Pi) g \). Our calibration implies a money growth rate of 7.6%. For comparison, the growth rate of \( M0 \) was 7.2% between 1960-1995, and the growth rates of \( M1 \) and \( M2 \) were 5% and 7%, respectively.
From the production function of loans (36) we pin down \( \phi \) as a function of \( l \):

\[
\phi = \frac{l}{2k}
\]

We insert this expression for \( \phi \) in the zero-profit condition for financial intermediation (31) to get \( \sigma \) as a function of \( l \), \( \alpha \) and values already known:

\[
\sigma = l (r_e - r_d) [2k (\alpha - 1)]^{-1} - r_e
\]

Next, we use the R&D expenditure to output ratio, \( \rho \). In the model, \( \rho \) is

\[
\rho = \frac{\sigma (l/2) d}{\sigma (l/2) + \rho \sigma y} = \frac{\sigma (l/2) c(y)}{\sigma (l/2) c(y) + 1}
\]

From the R&D ratio (72), we express \( c_0 \) as a function of \( a \), \( y \) and \( \sigma \):

\[
c_0 = \frac{2\rho}{\sigma ly^{\alpha} (1 - \rho)}
\]

Then, we replace \( h \) in the time constraint (35) using the first-order condition on \( h \) replicated in (33):

\[
1 = l + n + s + \frac{1}{\pi} + k
\]

With (32), it becomes:

\[
1 = l + n + s + k + \frac{1}{(\alpha - 1) c(y) (\sigma/2 + r_e/2)}
\]

In addition, we use the envelope condition on \( a \):

\[
1/R = c'(y) [1 + \sigma (l/2) y] (1 + r_e/\sigma)
\]

In (75) and (74), we replace \( y \), \( \sigma \) and \( c_0 \) using (70), (71) and (73) to get two equations in \( l \) and \( \alpha \) that give \( l^* \) and \( \alpha^* \). Then, with (70), (71) and (73 we obtain \( c_0^* \), \( \sigma^* \) and \( y^* \). Finally, we get \( \pi^* \) from (32). This allows us to get \( \theta^* \) and \( \theta^* \) using the first-order conditions on \( s \) and \( n \) stated in (33).
References


