Age-Design Employment Protection

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Abstract

This paper aims at examining the age-design of firing taxes by extending the theory of job creation and job destruction to account for a finite working life-time. We first argue that the potential employment gains related to employment protection is large for older workers, but higher firing taxes for these workers increase job destruction rates of younger generations. On the contrary, age-decreasing firing taxes can account for lower job destruction rates at all ages. Furthermore, from a normative standpoint, because firings of older (younger) workers exert a negative (positive) externality on the matching process, we find that the first best age-dynamics of firing taxes and hiring subsidies is typically hump-shaped. Taking into account distortions related to unemployment benefits and bargaining power shows the robustness of this result, in contradiction with what is done in most OECD countries.

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1 Introduction

Faced to low employment rates for older workers, most OECD countries have experimented with a older worker-specific employment protection in the form of specific firing taxes and hiring subsidies targeted to workers over the age of 50 (see OECD [2006]). Additional penalties for firms that lay off older workers have been introduced either in the form of having a tax or higher social security contributions (e.g., Austria, Finland, France and Spain) or in the form of paying part or all of the costs of outplacement services to help workers find new jobs (e.g., Belgium and Korea). Older workers are also protected to a greater extent than younger workers by job protection legislation as a result of tenure-related provisions: workers with longer tenure (more likely to be older workers) are often required to be given longer notice periods in the case of dismissals and higher severance payments. In Sweden, the Last In-First Out rule implies that older workers are more protected in the event of lay-offs than younger workers since they usually have longer tenure. In parallel, various hiring and wage subsidy schemes to hire and to retain older workers have been introduced. The hirings of older workers lead to permanent reduction or exonerations in social security contributions in Austria, Belgium, Netherlands, Norway and Spain. Direct subsidies to employers who hire older workers also exist in Denmark, Germany, Japan and Sweden. Employment subsidy schemes for older workers (in-work benefits) also exist in Austria, Germany, Japan, United Kingdom and the United-States. The main objective of these policies is to prevent older workers from unemployment because these workers suffer from longer durations of unemployment spells. The aim of our paper is to determine the optimal age-design employment protection, taking into account that the hiring and the separation decisions lead to discriminate against older workers. In other words, is it optimal to increase the firing taxes with the age of the workers as it is observed in large set of OECD countries?

The generalization of higher firing taxes and hiring subsidies for older workers in most developed countries calls for a theory of age-design employment protection. Since the seminal works of Mortensen and Pissarides [1999], it is well-established that the employment protection reduces job destructions but also job creations. Combining firing taxes with hiring subsidies is then thought of as corresponding to a consistent policy set to boost employment rate. This paper calls into question the higher employment protection for older workers by extending the theory of job creation and job destruction to account for a finite working life-time. What are the predicted effects of this policy on the older workers? If older worker employment should benefit from the higher protection put in place in most developed countries, what
are the consequences of this policy on younger workers? Can this policy be legitimated by welfare arguments? Surprisingly enough, the theoretical foundations and implications of the age-design employment protection have not been yet addressed. From this point of view, our paper fills a gap.

Our analysis of employment protection includes both positive and normative issues which are successively addressed. Unlike the large literature following MP, we consider a life cycle setting characterized by a deterministic age at which workers exit the labor market. The only heterogeneity across workers is the distance to retirement. We also assume that firms cannot ex-ante age-direct their search, that is vacancies cannot be targeted at a specific age group. However, once the contact with a worker is made, according to the observed productivity of the job-worker pair, the firm can make the choice of not recruiting the worker, and this productivity threshold decision obviously depends on the age of the worker. These assumptions allow us to be consistent with existing legislation prohibiting age-discrimination such as in the US and European countries, together with the observed discrimination against older workers (Neumark [2001]).

The shorter distance to retirement is then the key point to understand the economics of older employment workers. We think that it is the only intrinsic characteristics of the older workers common to any countries. We then consider that this is a natural starting point, to which other potential sources of heterogeneity across workers of different ages could be added\(^1\).

Because the horizon of older workers is shorter, the life-cycle labor market equilibrium shows that firms invest less in labor-hoarding activities at the end of the life cycle. This explains that the separation rate increases with the age of the worker and that there exists a demand for protecting the older worker employment.

We then propose a positive analysis of the older worker employment protection. We first argue that the impact of a firing tax is larger for older workers than for younger ones. Indeed, at the end of the working cycle, introducing a firing tax increases the present firing cost without any future consequences on the job value as the worker will be retired in the next periods. For younger workers, the present firing cost increases, but the job value also decreases, as the firm rationally expects the future cost of the firing tax. In some sense, retirement allows firms to avoid the firing tax, leading them to more labor hoarding of older workers.\(^2\) Ultimately, this tax can be

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\(^1\)For instance, one could think that older workers have much more job-specific skills and suffer more from losing their job. The amount of idiosyncratic uncertainty could be weaker for older workers. The bargaining power of younger and older workers is not necessarily the same.

\(^2\)In an infinite horizon economy such as MP, a firing tax has no impact on job destruct-
high enough so that it implies a decreasing age-dynamics of job destruction rates, at the opposite of the laissez-faire equilibrium. We secondly put emphasis on the age-differentiated effects of higher employment protection for older workers. Even though higher employment protection for older workers decreases job destruction rate for this age group, it can increase firings of younger ones.\textsuperscript{3} On the contrary, we show that an age-decreasing path would allow to unambiguously decrease job destruction rates for all workers’ ages, because it gives to firms incentives to keep a worker by expecting lower firing taxes in the future, whatever the age of the worker. This shape of the firing taxes allows the policy maker to reach his main objective: to decrease the unemployment spell duration.

The second step of our analysis is related to normative considerations. In the context of matching frictions and wage bargaining, it is now well-established that the decentralized equilibrium is in general not optimal, except when the Hosios [1990] condition holds.\textsuperscript{4} In our finite working cycle setting, we consider that each firm is engaged in a non age-directed search. In that context, the age distribution of the unemployed workers determines the return on vacancies. We show that older worker job destructions then exert a negative externality on the employment of the younger unemployed workers which is not internalized by firms in the decentralized equilibrium. This is why the Hosios condition is not enough to restore the social optimality of the labor market equilibrium: there are too many (not enough) older (younger) worker job destructions even though the optimal profile of job destructions is typically increasing with age as in the equilibrium outcome.

This result provides welfare foundations for age-design employment protection. In a first best perspective, it is optimal to implement a hump-shaped age-dynamics for the firing taxes and hiring subsidies which is at odds with the existing policies. We then further explore the optimal age-design employment protection in the context of distortions related to bargaining power and unemployment benefits. We then show that high unemployment benefits (or high worker bargaining power) can even require strictly age-decreasing firing taxes and hiring subsidies. This result reflects the fact that distortions related to unemployment benefits are higher for younger workers, since it

\textsuperscript{3}These results are consistent with the recent empirical evidence of Behagel, Crépon and Sédillot [2008] which stresses perverse effects related to the French experimentation of higher employment protection for workers over the age of 50.

\textsuperscript{4}This condition states that the elasticity of the matching friction with respect to vacancies should be equal to the worker’s bargaining power (Hosios [1990]). This efficiency result could also be obtained in a competitive search equilibrium (Moen [1997]).
actually corresponds to the discounted sum of distortions until retirement.

Overall, the existing higher employment protection for older workers seems to be at odds with the policy recommendations which can be deduced from the MP’s model. Age-decreasing employment protection from 50 until retirement could be more efficient both in terms of employment and welfare.

The next section presents the benchmark model and the age-dynamic properties of the equilibrium. The third section addresses the impact of age-design employment protection on employment. The fourth section deals with the social efficiency of the equilibrium and presents the optimal age-design employment protection both in a first and second best environments. The final section concludes.

2 A Finite-Horizon Economy with Endogenous Job Creations and Job Destruc-
tions

The primary objective of this section is to show that extending the job cre-
ation - job destruction approach to account for a finite life-time horizon of workers give rise to increasing (decreasing) age-dynamics of job destruc-
tions (creations). This provides some foundations to the observed low employment rate of older workers which lead some OECD countries to implement firing taxes and hiring subsidies targeted toward these workers.

We consider an economy with labor market frictions à la Mortensen 
- Pissarides [1994] with endogenous job creation and job destruction decisions, extended to account for a finite life time horizon of workers. That is, instead of assuming infinite-lived agents, our setting is characterized by a determin-
istic age $T$ at which workers exit the labor market. Workers only differ respectively in their age $i$, and so in their distance to deterministic retirement. The model is in discrete time and at each period the older worker generation retiring from the labor market is replaced by a younger worker generation of the same size (normalized to unity) so that there is no labor force growth in the economy. The economy is at steady-state, and we do not allow for any aggregate uncertainty. We assume that each worker of the new generation enters the labor market as unemployed.

We consider (un)employment policies $(i)$ a firing cost $F_i$ which refers both to the implicit costs in mandated employment protection legislation and to the experience-rated unemployment insurance taxes, $(ii)$ a hiring subsidy $H_i$, that is a lump sum paid to the employer when a worker of age $i$ is hired, $(iii)$ unemployment benefits $z$. 
2.1 Shocks and workers flows

Firms are small and each has one job. The destruction flows derive from idiosyncratic productivity shocks that hit the jobs at random. Once a shock arrives, the firm has no choice but either to continue production or to destroy the job. Then, for age \( i \in (2, T - 1) \), employed workers are faced with layoffs when their job becomes unprofitable. At the beginning of each age\(^5\), a job productivity \( \epsilon \) is drawn in the general distribution \( G(\epsilon) \) with \( \epsilon \in [0, 1] \). The firms decide to close down any jobs whose productivity is below an (endogenous) productivity threshold (productivity reservation) denoted \( R_i \).

Job creation takes place when a firm and a worker meet. The flow of newly created jobs result from a matching function, \( M(v, u) \), the inputs of which are vacancies \( v \) and unemployed workers \( u \). \( M \) is increasing and concave in both its arguments, and with constant returns-to-scale. We assume that firms cannot ex-ante age-direct their search and that the matching function embodies all unemployed workers. The flow of newly created jobs also depends on productivity thresholds \( R_i^0 \) because it is assumed that productivity values \( \epsilon \) are known after firm and worker have met. \( R_i^0 \) may differ from \( R_i \) since firms are not liable for the firing cost at this stage.

Let \( \theta = v/u \) denote the tightness of the labor market. It is then straightforward to define the probability for unemployed workers of age \( i \) to be employed at age \( i + 1 \), as \( j_{c_i} = p(\theta)[1 - G(R_i^0)] \) with \( p(\theta) = \frac{M(u,v)}{u} \). Similarly, we define the job destruction rate for an employed worker of age \( i \) as \( j_{d_i} = G(R_i) \).

At the beginning of their age \( i \), the realization of the productivity level on each job is revealed. Workers hired when they were \( i - 1 \) years old (at the end of the period) are now productive. Workers whose productivity is below the reservation productivity \( R_i \) (\( R_i^0 \)) are laid off (not hired, for those previously unemployed). For any age \( i \), the flow from employment to unemployment is then equal to \( G(R_i)(1 - u_{i-1}) \). The other workers who remain employed \((1 - G(R_i))(1 - u_{i-1})\) can renegotiate their wage. The age-dynamics of unemployment is then given by:

\[
  u_{i+1} = u_i \left[ 1 - p(\theta)(1 - G(R_i^0)) \right] + G(R_i)(1 - u_i) \quad \forall i \in (1, T - 1) \quad (1)
\]

for a given initial condition \( u_1 = 1 \). The overall level of unemployment is \( u = \sum_{i=1}^{T-1} u_i \), so that the average unemployment rate is \( u/[T - 1] \).

\(^5\)This assumption is made to allow for analytical results. The persistency of shocks is left for a quantitative empirical investigation of the model performance.
2.2 Hiring and firing decisions

Any firm is free to open a job vacancy and engage in hiring. $c$ denotes the flow cost of recruiting a worker and $\beta \in [0, 1]$ the discount factor. Let $V$ be the expected value of a vacant position and $J^0_i(\epsilon)$ the value of a filled job with productivity $\epsilon$:

$$V = -c + \beta q(\theta) \sum_{i=1}^{T-2} \left[ \frac{u_i}{u} \left( \int_{R_{i+1}} J^0_{i+1}(x) + H_{i+1} \right) dG(x) + G(R_{i+1}^0) V \right] + \beta(1-q(\theta))V$$

where the hiring subsidy $H_i$ is perceived by the firm when the job becomes productive. At this time, the age of the hired worker is perfectly observed.

Beyond the traditional matching externality, the heterogeneity across ages in filled job values and in productivity thresholds imply the existence of intergenerational externalities in the search process: the more older unemployed workers there are, the less is the expected return on a vacancy.

The zero-profit condition $V = 0$ allows us to determine the labor market tightness from the following condition:

$$\frac{c}{q(\theta)} = \sum_{i=1}^{T-2} \left[ \frac{u_i}{u} \left( \int_{R_{i+1}} J^0_{i+1}(x) + H_{i+1} \right) dG(x) \right]$$

(2)

We follow MP by considering that the wage structure that arises as a Nash bargaining solution has two tiers.\(^6\) The first tier wage reflects the fact that the hiring subsidy is directly relevant to the decision to accept a match and that the possibility of incurring firing costs in the future affects the value the employer places on the match. In turn, the second tier wage applies when firing costs are directly relevant to a continuation decision. For a bargained outsider wage $w^0_i(\epsilon)$, the expected value $J^0_i(\epsilon)$ of a filled job by a worker of age $i$ is defined, for $\forall i \in [1, T-1]$, by:

$$J^0_i(\epsilon) = \epsilon - w^0_i(\epsilon) + \beta \int_{R_{i+1}} J_{i+1}(x) dG(x) + \beta G(R_{i+1}) (V - F_{i+1})$$

(3)

\(^6\)Recently, this wage setting rule has been somewhat disputed. Shimer [2005] argue that, in the conventional matching model, the wage rate is close to being as cyclical as productivity, so that the model does not have enough power to generate the observed cyclical volatility of unemployment. Using microeconometric evidence, Pissarides [2008] however shows that the cyclical volatility of wages in the canonical matching model is about the same as the one estimated for new matches. Furthermore, alternative “insider wage” rule is discussed in Appendix B and it shows that our main results are robust to the wage setting process.
whereas for a bargained insider wage \( w_i(\epsilon) \), the expected value \( J_i(\epsilon) \) of a filled job by a worker of age \( i \) is defined, for \( \forall i \in [1, T - 1] \), by:

\[
J_i(\epsilon) = \epsilon - w_i(\epsilon) + \beta \int_{R_{i+1}}^{1} J_{i+1}(x) dG(x) + \beta G(R_{i+1}) (V - F_{i+1}) \tag{4}
\]

Optimal decisions of the firm are then characterized by productivity thresholds \( \{ R_i, R^0_i \} \), who are solving:

\[
J^0_i(R^0_i) = -H_i ; \quad J_i(R_i) = -F_i
\]

Adding the free entry condition, \( V = 0 \), it is straightforward to find:

\[
\begin{align*}
R_i &= w(R_i) - F_i - \beta \left[ \int_{R_{i+1}}^{1} J_{i+1}(x) dG(x) - G(R_{i+1}) F_{i+1} \right] \tag{5} \\
R^0_i &= R_i + F_i - H_i + w(R^0_i) - w(R_i) \tag{6}
\end{align*}
\]

The equation (5) gives the lowest level of productivity \( R_i \) allowing to prevent for a separation (firing decision) and the equation (6) determines the lowest productivity level \( R^0_i \) for a newly created job (hiring decision). The productivity threshold governing the hiring decision must be at least equal to the outsider wage \( (w(R^0_i)) \) net of the hiring subsidies \( (H_i) \). But firm also expects profits (continuation value). This continuation value is the same than for a job occupied by an insider: \( \beta \left[ \int_{R_{i+1}}^{1} J_{i+1}(x) dG(x) - G(R_{i+1}) F_{i+1} \right] \). Because the labor hoarding has the same value for a newly created job than for a pre-existing one, the link between \( R^0_i \) and \( R_i \) is static (see equation (6)). The higher the wage, the higher the reservation productivity \( R_i (R^0_i) \), and hence the higher (lower) the job destruction (creation) flows. On the other hand, the higher the option value of filled jobs (expected gains in the future), the weaker the job destructions and the greater the job creations. Because the job value vanishes at the end of the working life, labor hoarding of older workers is less profitable. It is worth determining the terminal age conditions: \( R_{T-1} = w_{T-1}(R_{T-1}) - F_{T-1} \) and \( R^0_{T-1} = R_{T-1} + F_{T-1} - H_{T-1} + w(R^0_{T-1}) - w(R_{T-1}) \).
2.3 Nash bargaining

Values of insiders, outsiders (on a job of productivity $\epsilon$) and unemployed workers of any age $i$, $\forall i < T$, are respectively given by:\footnote{We assume that $W_T = U_T$ so that the social security provisions do not affect the wage bargaining and the labor market equilibrium.}:

\[
W_i^0(\epsilon) = w_i^0(\epsilon) + \beta \left[ \int_{R_{i+1}}^{1} W_{i+1}(x) dG(x) + G(R_{i+1}) U_{i+1} \right] \tag{7}
\]

\[
W_i(\epsilon) = w_i(\epsilon) + \beta \left[ \int_{R_{i+1}}^{1} W_{i+1}(x) dG(x) + G(R_{i+1}) U_{i+1} \right] \tag{8}
\]

\[
U_i = b + z + \beta \left[ p(\theta) \int_{R_{i+1}}^{1} W_{i+1}^0(x) dG(x) + p(\theta) G(R_{i+1}) U_{i+1} + (1 - p(\theta)) U_{i+1} \right] \tag{9}
\]

with $b \geq 0$ denoting the opportunity cost of employment.

For a given bargaining power of the workers, considered as constant across ages, the global surplus generated by a job, is divided according to the following two sharing rules which are the solution of the conventional Nash bargaining problems in the context of two-tier contracts:\footnote{Again, see Appendix B for an examination of alternative “insider wage” rule.}:

\[
W_i^0(\epsilon) - U_i = \gamma [J_i^0(\epsilon) + H_i + W_i^0(\epsilon) - U_i] \tag{10}
\]

\[
W_i(\epsilon) - U_i = \gamma [J_i(\epsilon) + F_i + W_i(\epsilon) - U_i] \tag{11}
\]

so that the equations for the initial and subsequent wage bargaining are (see Appendix for details on derivation):

\[
w_i^0(\epsilon) = \gamma (\epsilon + c \theta \tau_i + H_i - \beta F_{i+1}) + (1 - \gamma) (b + z) \tag{12}
\]

\[
w_i(\epsilon) = \gamma (\epsilon + c \theta \tau_i + F_i - \beta F_{i+1}) + (1 - \gamma) (b + z) \tag{13}
\]

where $\tau_i$ is defined by:\footnote{To derive this expression, let notice that $J_i^{0}(\epsilon) = 1 - \gamma$ and $J_i^{0}(R_i^{0}) = -H_i$ implies that $J_i^{0}(\epsilon) = (1 - \gamma) (\epsilon - R_i^{0}) - H_i$. Furthermore, integrating by parts, it comes that $\int_{R_{i+1}}^{1} (x - R_{i+1}^{0}) dG(x) = \int_{R_{i+1}}^{1} [1 - G(x)] dx$.}

\[
\tau_i \equiv \frac{\int_{R_{i+1}^{0}}^{1} [J_{i+1}^{0}(x) + H_{i+1}] dG(x)}{\sum_{i=1}^{T-1} \left( \frac{u_i}{u} \int_{R_{i+1}^{0}}^{1} [J_{i+1}^{0}(x) + H_{i+1}] dG(x) \right)} = \frac{\int_{R_{i+1}^{0}}^{1} [1 - G(x)] dx}{\sum_{i=1}^{T-1} \left( \frac{u_i}{u} \int_{R_{i+1}^{0}}^{1} [1 - G(x)] dx \right)}
\]

and $\tau_{T-1} = 0$ by definition. $\tau_i$ gives the value of a worker hired at age $i$ relative to the expected value of a job according to the age distribution of
unemployed workers. $\tau_i$ decreases with age $^{10}$ Despite the assumption of undirected search, implying a homogenous search cost for each match, wages are age-specific.

Contrary to Mortensen and Pissarides’ framework, the equilibrium is no longer symmetrical. Due to the finite-lived agents assumption, the way turnover costs interact with the wage bargaining process depends on the age of the workers through the variable $\tau_i$. This means that ending up with a young worker is more worth while than hiring an older one for the firm; the job value is greater than the average job value. Hence, younger workers capture a larger fraction of the search costs than the older worker. A younger worker has then to be rewarded for more than the saving of the average search costs ($c\theta$).$^{11}$

As in Mortensen and Pissarides [1999], the difference between the initial wage and subsequent renegotiation arises because hiring subsidies are sunk in the latter case but on-the-table in the former, and termination costs are not incurred if no match is formed initially but must be paid if an existing match is destroyed. From that point of view, effects of firing taxes and hiring subsidies on these wage rules are conventional:

(i) The hiring subsidy $H_i$ increases the initial wage $w_i^0(\epsilon)$ because this gain is conditional on agreement to form the match, whereas the discounted firing tax $\beta F_{i+1}$ decreases this wage because it reduces expected match surplus at the creation date. Workers then get a share $\gamma$ of the expected net subsidy of the job, $H_i - \beta F_{i+1}$.

(ii) Once the job is formed, the hiring subsidy no longer influence wages in continuing jobs $w_i(\epsilon)$, but in turn the employer is now liable for the firing tax $F_i$ and this strengthens the workers’ hand in the wage bargain. Accordingly, wages for continuing workers is increased (decreased) by firing costs if firm expects that keeping the worker account for an additional cost (gain), that is if $F_i < \beta F_{i+1}$ ($F_i > \beta F_{i+1}$).

\begin{footnotesize}
\begin{itemize}
\item[$^{10}$] As shown hereafter, and at least in an economy without labor market policy, $\tau_1 > 1$ for the youngest workers, $\tau_{T-1} = 0$ for the oldest ones, and $\tau_{i+1} \leq \tau_i \forall i$.
\item[$^{11}$] For a given productivity level $\epsilon$, the wage is lower for a worker of age $i + 1$ than for a worker of age $i$, $w_{i+1}(\epsilon) \leq w_i(\epsilon)$. Age-decreasing dynamics of wages is obviously at odds with empirical findings. This shortcoming could easily be overcome by allowing the model to account for exogenous human capital accumulation. For instance, consider $h_{i+1} = (1 + \mu)h_i$, where the productivity of the job is now given by $h_i\epsilon$, it can be the case that the growth rate $\mu \geq 0$ is high enough to imply $w_{i+1}(\epsilon) \geq w_i(\epsilon)$. Furthermore, even though higher growth rate would account for higher labor market tightness, the shape of the age-dynamics of job creations and job destructions would be left unaltered.
\end{itemize}
\end{footnotesize}
2.4 The labor market equilibrium

**Proposition 1.** A labor market equilibrium with wage bargaining exists and it is characterized by:

\[
\frac{c}{q(\theta)} = \beta(1 - \gamma) \sum_{i=1}^{T-2} \left( \frac{u_i}{u} \int_{R^0_{i+1}}^1 [1 - G(x)] dx \right) \tag{14}
\]

\[
b + z + \frac{\gamma}{1 - \gamma} \epsilon \theta \tau_i = R_i + \beta \int_{R_{i+1}}^1 [1 - G(x)] dx + F_i - \beta F_{i+1} \tag{15}
\]

\[
R^0_i = R_i + F_i - H_i \tag{16}
\]

\[
u_{i+1} = u_i \left[ 1 - p(\theta)(1 - G(R^0_{i+1})) \right] + G(R_{i+1})(1 - u_i) \tag{17}
\]

where \( \tau_i = \frac{\int_{R^0_{i+1}}^1 [1 - G(x)] dx}{\sum_{i=1}^{T-1} \left( \frac{u_i}{u} \int_{R^0_{i+1}}^1 [1 - G(x)] dx \right)} \), with terminal conditions \( R_{T-1} = b + \)
\( z_{T-1} - F_{T-1} \), \( R^0_{T-1} = b + z_{T-1} - H_{T-1} \) and a given initial condition \( u_1 \).

**Proof.** Combining (2), (5), (6), (7) (13) and noticing that \( J^{R^0}_{i+1}(\epsilon) = 1 - \gamma \) and \( J^0_i(R^0_i) = -H_i \) implies that \( J^0_i(\epsilon) = (1 - \gamma)(\epsilon - R^0_i) - H_i \), as well as \( J^{R^0}_{i+1}(\epsilon) = 1 - \gamma \) and \( J_i(R_i) = -F_i \) implies \( J_i(\epsilon) = (1 - \gamma)(\epsilon - R_i) - F_i \). Furthermore, integrating by parts, it comes that \( \int_{R^0_{i+1}}^1 (x - R^0_{i+1}) dG(x) = \int_{R^0_{i+1}}^1 [1 - G(x)] dx \) and \( \int_{R_{i+1}}^1 (x - R_{i+1}) dG(x) = \int_{R_{i+1}}^1 [1 - G(x)] dx \).

In particular, the equation (15) shows that a job is destroyed when the expected profit from the marginal job -current product plus option value from expected productivity shocks- (right side of (15)) falls to cover the worker’s reservation wage (left side). Without any fiscal distortions, the reservation productivity governing the hiring decision \( (R^0_i) \) is equal to the reservation productivity determining the firing decision because insider and outsider wages are equal and there is no termination and opening costs/gains. At the opposite, it is obvious that the age-dynamics of job creations and job destructions depends on employment protection.

2.5 The age-dynamics of job creations and job destructions in a laissez-faire economy

As a benchmark case, we first consider the equilibrium age-dynamics of job creations and job destructions without any labor market policies. This allows us to show that older workers face a higher (lower) probability of exit from employment (unemployment). Let us denote \( \hat{R}_i \) and \( \hat{\theta} \) the productivity
thresholds and the labor market tightness without respectively in the case of no labor market policy \((z = 0\) and \(H_i = F_i = 0\ \forall i)\).

**Proposition 2.** The equilibrium without any labor policies is characterized by \(\hat{R}_{i+1} \geq \hat{R}_i\ \forall i \in [2, T - 1]\).

**Proof.** The age-dynamics of job creations and job destructions is governed by the sequence \(\{\hat{R}_i\}_{i=2}^{T-1}\) which solves:

\[
\hat{R}_i = b - \beta [1 - \gamma p(\hat{\theta})] \int_{\hat{R}_{i+1}}^{1} [1 - G(x)]dx
\]

with terminal conditions \(\hat{R}_{T-1} = b\), and where \(\hat{\theta}\) is defined by \(\frac{d_i}{q(\hat{\theta})} = \beta (1 - \gamma) \sum_{i=1}^{T-1} \left( \frac{m_i}{n_i} \int_{\hat{R}_{i+1}}^{1} [1 - G(x)]dx \right) + G(\hat{R}_{i+1})(1 - u_i)\). Solving backward this equation (with \(\beta [1 - \gamma p(\hat{\theta})] < 1\)) and starting with terminal condition \(\hat{R}_{T-1} = b\), we obtain \(\hat{R}_{i+1} \geq \hat{R}_i\). \(\square\)

Because the horizon of older workers is shorter, firms invest less in labor-hoarding activities at the end of the life cycle, and older workers are more vulnerable to idiosyncratic shocks. Otherwise stated, this reflects the fact that labor-hoarding decreases with worker’s age. It is then straightforward to see that this increasing (decreasing) age-dynamics of job destructions (creations) is, at least qualitatively, able to account for observed low employment rate of older workers.\(^{12}\)

### 3 The Impact of Firing Taxes Revisited

Face to the low employment rate of older workers, most developed countries have experimented with higher employment protection combined with subsidies targeted toward these workers. The main objective of this section is to question, from a positive point of view, the effectiveness of such policies.

It is well-known following Mortensen and Pissarides [1999] that more severe employment legislations protect workers who already have a job, but at the expense of those without a job. The overall impact on employment is then theoretically ambiguous unless hiring subsidies offset perverse effects of employment protection on job creation. It is straightforward to see that such a result also holds in our framework because the firing tax increases productivity thresholds at the time of job creation, whatever worker’s age (condition (16)). But in addition to this conventional wisdom, the incidence of firing taxes on job creation and job destruction is age-dependant.

\(^{12}\)Quantitative assessment of the model is beyond the scope of that paper (see Chéron, Hairault and Langot [2008]).
3.1 On the age-differentiated effect of firing taxes

As a preliminary step, our objective is first to examine the age-differentiated effect of a constant firing tax \( (F_i = F \; \forall i) \) on the age-dynamics of job flows. Without loss of generality, we consider also that \( H_i = F \; \forall i \) so that firing tax unambiguously increases employment rates at all ages. However, the impact on the job destruction and job creation rates remains age-dependent. We argue indeed that, in a finite horizon setting, there is a specific intertemporal trade-off related to the introduction of a firing tax.

**Proposition 3.** Consider \( H_i = F_i = F > 0 \; \forall i \), the labor market equilibrium is characterized by:

\[
0 \geq \frac{dR_2}{dF} > \frac{dR_i}{dF} \cdots > \frac{dR_{T-1}}{dF} \quad \forall i \in [2, T-1] \text{ and } \frac{d\theta}{dF} > 0
\]

*Proof.* Consider \( z = 0 \), \( F_{i+1} = F_i \equiv F \) and \( H_i = F \) so that \( R_i^0 = R_i \) in Proposition 1, it comes that \( \frac{dR_0}{dF} = -(1-\beta) + \frac{dR_{i+1}}{dF} \beta (1-\gamma p(\theta)) [1 - G(R_{i+1})] + \frac{d\theta}{dF} \gamma p'(\theta) \beta \int_{R_{i+1}}^{1} [1 - G(x)] dx \) with \( \frac{dR_{i+1}}{dF} = -1 \) from Proposition 1. Then, it remains to reason backward from \( i = T - 1 \) to \( i = 2 \) having in mind that \( \beta \leq 1 \), \( p'(\theta) \geq 0 \) and \( \frac{d\theta}{dF} \geq 0 \) (straightforward with \( H_i = F \)).

A given firing tax is thus found to reduce more (less) job destruction (creation) rate of older workers than younger ones. Otherwise stated, the potential employment gains related to \( F \) is larger for older workers.

To get further intuitions on this result, let also assume \( \beta \to 1 \). It is straightforward to see that in an infinite horizon economy à la MP, we would have no longer impact of the firing tax \( F \) on job destruction\(^{13}\): the value to prevent punishment today is equal to the job value loss induced by the expected firing taxes. In this extreme case, a constant firing tax is neutral. At the opposite, in our finite life time context, we still have \( \frac{dR_{i+1}}{dF} = -1 \) which implies \( \frac{dR_i}{dF} < 0 \; \forall i \). At the end of the working cycle, introducing a firing tax increases the present firing cost without any future consequences on the job value as the worker will be retired in the next period. In other words, the value to prevent punishment today is not canceled by the expected future loss. Indeed, in an infinite horizon, the present firing cost increases in the same proportion as in our life-cycle model, but the job value decreases, as the firm rationally expects the future cost of the firing tax. In some sense, retirement allows firms to avoid the firing tax, leading them to more labor hoarding of older workers. This suggests that evaluating employment protection in

\(^{13}\)To state this result, consider \( R_i = R_{i+1} \equiv R \) in (15). For \( \beta \to 1 \) and \( \gamma \to 0 \), we then have \( \frac{dR_i}{dF} = 0 \).
an infinite-lived agent context understates the potential employment gains implied by this policy.

We are even able to state that the impact of firing costs can be sizeable enough at the end of the life cycle to imply that older workers face a lower probability of job destruction than younger ones.

**Proposition 4.** There exists \( \hat{F} > 0 \) such that if \( F \geq \hat{F} \), then \( R_{T-1} \leq R_{i+1} \leq R_i \) \( \forall i \in [2, T-1] \).

**Proof.** Consider proposition 1 with \( z = 0 \), \( H_i = F \) \( \forall i \)\(^{14} \) and \( F_{i+1} = F_i \equiv F \), and let us define \( \Psi(y) \equiv (1 - \gamma p(\theta)) \int_y^1 [1 - G(x)] \, dx \), so that \( \Psi(y) < 0 \). By definition \( R_i = b - (1 - \beta)F - \Psi(R_{i+1}) \), so that \( R_i - R_{i+1} = \Psi(R_{i+2}) - \Psi(R_{i+1}) \). Accordingly, \( R_{T-2} \geq R_{T-1} \) is a sufficient condition to imply \( R_i \geq R_{i+1} \) \( \forall i \).

Then, \( R_{T-2} \geq R_{T-1} \iff F \geq (1 - \gamma p(\theta)) \int_{b-F}^1 [1 - G(x)] \, dx \) which implies that \( F \geq \int_{b-F}^1 [1 - G(x)] \, dx \) is a sufficient condition for \( R_{T-2} \geq R_{T-1} \), hence \( R_i \geq R_{i+1} \). Otherwise stated, from \( 0 \leq G(x) \leq 1 \), there exists a unique \( \hat{F} \) solving \( F = \int_{b-F}^1 [1 - G(x)] \, dx \), such that \( F \geq \hat{F} \) implies \( R_i \geq R_{i+1} \) \( \forall i \). \( \square \)

The intuition behind this result is the following. Without any policy instruments, older workers face a higher (lower) rate of job destruction (creations) because of their shorter horizon. In turn, the introduction of firing tax has larger impact on the probability of job destruction for older workers than for younger ones. Hence, allowing for sufficiently large firing taxes can reverse the age-dynamics of job destructions and job creations. More precisely, this is unambiguously the case whenever the value of the labor hoarding for a worker of age \( T - 2 \) (which is \( \int_{b-F}^1 [1 - G(x)] \, dx \)) is less than the firing cost \( F \). Then, since job destruction rate for a worker of age \( T - 2 \) turns out to be higher than that of a worker of age \( T - 1 \), this is the overall age-dynamics of job destructions which is reversed in the context of constant firing tax\(^{15} \).

To conclude at this stage, the properties 3 and 4 emphasize that the lay off of older workers is, in relative terms with respect to younger workers, very sensitive to the firing tax. Employment protection have age-differentiated impact. It may make unnecessary age-design employment protection if the only objective is to stimulate more the employment of older workers.

\(^{14}\)The same result holds for \( H_i = 0 \) \( \forall i \).

\(^{15}\)Indeed, job destruction at age \( T - 3 \) becomes higher than at age \( T - 2 \) because \( R_{T-2} \geq R_{T-1} \), and so on.
3.2 The impact of age-increasing firing taxes

It is obvious that not only the level of firing taxes, but also its shape with age play a key role on the age-dynamics of job flows. A quick look at proposition 1 shows that $F_i$ tends to push down $R_i$ by increasing the current cost of firing, while $F_{i+1}$ increases $R_i$ by reducing the value of labor-hoarding. This suggests that the shape of the actualized firing costs is crucial for the separation decisions. To go beyond this intuitive statement, let us define $F_{i+1} = (1 + \Delta F)F_i \forall i \in [2, T - 2]$ with $\Delta F \geq 0$ and still consider that hiring subsidies offset the perverse effect of firing tax on recruitment by assuming $H_i = F_i \forall i$.

Proposition 5. If $F_{i+1} = (1 + \Delta F)F_i \forall i \in [2, T - 2]$ and $H_i = F_i \forall i$, the impact of age-design employment protection is characterized by:

- $\frac{\partial R_{T-1}}{\partial F_{T-1}} < 0$
- If $\Delta F < \frac{1}{\beta} - 1$, then $\frac{\partial R_i}{\partial F_i} \leq 0 \forall i \in [2, T - 1]$
- If $\Delta F > \frac{1}{\beta} - 1$, then there exists a threshold age $\bar{i}$ such that $\frac{\partial R_i}{\partial F_i} \geq 0 \forall i \in [2, \bar{i}]$ and $\frac{\partial R_i}{\partial F_i} \leq 0 \forall i \in [\bar{i}, T - 1]$

Proof. See appendix C.1.

Proposition 5 first stresses that introducing a firing tax unambiguously decrease firings of the oldest workers. This is because, by definition, retirement allows to avoid this tax: this gives to firms incentives to wait one period instead of firing the worker at age $T - 1$ and being liable for the firing cost $F_{T-1}$. For all the other workers, the impact of firing taxes on job destruction rates depends on the age-dynamics of taxes. If firing taxes decrease with age or does not rise too fast ($\Delta F < \frac{1}{\beta} - 1$), they decrease job destruction rates for all ages. It is firms interest to reduce firings because the value to prevent punishment is higher than the expected loss induced by future taxes. Ultimately, firms avoid these costs by waiting for workers’ retirement. In some sense, it generalizes to all ages the idea that labor hoarding allows firms to avoid (at least partially) firing costs.

On the other hand, the growth rate of firing taxes with age can be so important that it increases job destruction rates for younger workers: the value to prevent punishment today is lower than the expected increase in future taxes, leading firm to fire early. For older workers, the proximity to

\[\text{Available upon request.}\]
retirement compensate for the impact of the expected growth in the firing tax.

Overall, Proposition 5 highlights that age-increasing firing tax have some perverse effects by increasing job destruction rates of younger workers. It is worth emphasizing that, these results can give theoretical supports to the empirical findings of Behaghel, Crépon and Sédillot [2008] who use microeconometric estimates to assess the French experimentation (since 1987) of higher firing tax for workers of 55 years old or more. The estimates mainly show that firings of people less than 55 years are increased. So does in our model whenever $\Delta F > \frac{1}{\beta} - 1$.

4 Optimal age-design employment protection

At this stage, we mainly argued that the potential employment gain for older workers of targeting higher firing taxes and hiring subsidies toward them can be large due to the short distance to retirement. However, such policy may have some perverse effects on younger generations. On the contrary, age-decreasing firing taxes lead to decrease job destructions for all workers.

It is then important to go beyond this positive approach by proposing a welfare analysis of the age-design employment protection. Is an age-increasing dynamics consistent with the optimal age-design employment protection?

4.1 An intergenerational externality

Traditionally, the equilibrium unemployment framework is known to generate congestion effects which take the decentralized equilibrium away from the efficient allocation. However, when the elasticity relative to vacancies in the matching function is equal to the bargaining power of firms (Hosios [1990] condition), social optimality can be reached. As demonstrated hereafter, this result no longer holds here, because there is a specific intergenerational externality.

We derive the optimal allocation by maximizing the steady-state output with respect to labor market tightness $\theta^*$ and reservation productivity for each age, $R_i^*$. The problem of the planner is stated as follows:

$$\max_{\{R_i^* \geq 0\}} \sum_{i=1}^{T-1} \left[ y_i + b u_i - \frac{c \theta^* P^*}{T - 1} \right]$$
where \( u^* = \sum_{i=1}^{T-1} u_i \) and subject to the unemployment dynamics and the output equation, respectively:

\[
\begin{align*}
    u_{i+1} &= G(R_{i+1}^*)(1 - u_i) + u_i \left( 1 - p(\theta^*)[1 - G(R_{i+1}^*)] \right) \\
    y_{i+1} &= u_i p(\theta^*) \int_{R_{i+1}^*}^{1} x dG(x) + (1 - u_i) \int_{R_{i+1}^*}^{1} x dG(x)
\end{align*}
\] (18) (19)

**Proposition 6.** Let \( \eta = 1 - \frac{\theta^* p(\theta^*)}{p(\theta^*)} \), the maximum value of steady-state output is reached when:

\[
\frac{c}{q(\theta^*)} = (1 - \eta) \sum_{i=1}^{T-1} \frac{u_i}{u} \left( \int_{R_{i+1}^*}^{1} [1 - G(x)] dx \right)
\] (20)

\[
R_i^* + \int_{R_{i+1}^*}^{1} [1 - G(x)] dx = b + \frac{\eta}{1 - \eta} c \theta^* \tau_i^* + c \theta^*(\tau_i^* - 1) \quad \forall i \in [2, T - 2]
\] (21)

\[
R_{T-1}^* = b - c \theta^*
\] (22)

where \( \tau_i^* \equiv \frac{\int_{R_{i+1}^*}^{1} [1 - G(x)] dx}{\sum_{i=1}^{T-1} \left( \frac{u_i}{u} \int_{R_{i+1}^*}^{1} [1 - G(x)] dx \right)}. \)

**Proof.** See Appendix C.2.

The equation (20) is similar to the equation (14) obtained in the decentralized equilibrium, on condition that the worker share of employment surplus (\( \gamma \)) is now replaced by the elasticity relative to the unemployment in the matching function (\( \eta(\theta^*) \)). The equation (21) shows how the planner shows the optimal allocation of the age \( i \) labor force: the expected profit from the marginal employed worker (the current product plus the option value for expected productivity shocks) must be equal to the social return of the search activity which corresponds to the allocation as an unemployed worker. At the equilibrium, the return of the unemployment is simply given by the reservation wage. For the social planner, the return on an additional age \( i \) unemployed worker is reduced by the cost of vacancy per age \( i \) unemployed worker, which is equal to \( c \theta^* \). The social value of the search activity is not symmetrical: because a young (old) worker increases (decreases) the average search value in the economy, the social value of the young unemployed worker is larger than that of the old unemployed worker. At the end of the life-cycle (\( i = T - 1 \)), the social return of a worker occupied in the search process is at its lowest value: the oldest workers can be contacted by a firm whereas the surplus associated to this match is nil. Indeed, the relative
surplus of an age $T - 1$ worker (the oldest workers) to the average of the employment surplus is equal to zero ($\tau^*_{T-1} = 0$): the return of the search is zero ($\frac{b}{1-\eta}c\theta^*\tau^*_{T-1} = 0$) and the size of the intergenerational externality takes its maximum value $c\theta^*$. Then, the option social value of unemployment for workers of age $T - 1$ turns out to be $b - c\theta^*$ (equation (22)).

**Proposition 7.** If $b > c\theta^*$, then the efficient allocation is characterized by $R^*_i \geq R^*_j$, so that $jd^*_i + 1 \geq jd^*_j$ and $j\theta^*_i \leq j\theta^*_j$. Otherwise, $R_i = R_{i+1} = R_{T-1} = 0 \forall i$.

**Proof.** First remark that $R^*_i$ can be re-stated as follows:

$$R^*_i = b - [1 - p(\theta^*)] \int_{R^*_{i+1}}^1 [1 - G(x)]dx - (1 - \eta)p(\theta^*) \sum_{i=1}^{T-1} \frac{u_i}{u} \left( \int_{R^*_{i+1}}^1 [1 - G(x)]dx \right)$$

Then, if $b > c\theta^*$ the proof is straightforward by solving backward this equation, and by noticing that $0 < [1 - p(\theta^*)] < 1$ and $(1 - \eta)p(\theta^*) \sum_{i=1}^{T-1} \frac{u_i}{u} \left( \int_{R^*_{i+1}}^1 [1 - G(x)]dx \right)$ is not age-dependent. In turn, if there does not exist $b > c\theta^*$, then $R_{T-1}$ is bounded by zero, and it unambiguously comes that $R_i = R_{i+1} = 0 \forall i$. □

Proposition 7 first emphasizes that higher (lower) job destruction (creation) rates for older workers is typically an efficient age-pattern of labor market flows, as in the decentralized equilibrium. Because of their shorter horizon, older workers must be more fired and less hired. Despite the shape is qualitatively the same, this does not mean however that the equilibrium job destruction and job creation rates are consistent with their efficient counterparts.\(^{17}\)

Importantly, this difference between private and social value of unemployment emphasizes the existence of inefficiencies. Contrary to the firms, the planner takes into account the impact of a particular unemployed worker of age $i$ on the search process. Compared to Pissarides [2000], our life-cycle framework introduces another externality, namely an intergenerational externality. Firms neither take into account that firings of older workers reduce the average value of a vacancy nor that firings of younger workers increase this average value.

\(^{17}\)But, it can be the case that search costs are so high ($b < c\theta^*$) that it implies no job destructions of older workers at the optimum, i.e. the planner never chooses to leave the oldest employed to enter the pool of unemployed because their congestion impact is too high. Then, so does also for younger workers who account for higher labor- hoarding, so that it could be the case that there are no job destructions at the optimum. This result is straightforward by noticing that the efficient productivity threshold can be restated as $R^*_i = b - c\theta^*[1 - p(\theta^*)] - \int_{R^*_{i+1}}^1 [1 - G(x)]dx$. 

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**Proposition 8.** The Hosios condition, \( \eta = \gamma \), does not achieve efficiency.

**Proof.** Straightforward by comparing the expression of \( R_i^* \) in Proposition 6 and \( R_i \) in Proposition 1 and considering \( \gamma = \eta \). \( \square \)

This proposition states clearly that the Hosios condition \( \gamma = \eta \) allows the private agents to internalize traditional search externalities in the decentralized equilibrium, but no longer the intergenerational externalities in the matching process. This last result suggest that age-specific labor market policies are needed to improve the outcome of the decentralized equilibrium. This gives some theoretical foundations to the age-specific firing taxes and hiring subsidies experimented in a large set of OECD countries.

### 4.2 The optimal age-dynamics of firing costs and hiring subsidies

Labor market policies designed by age may allow firms and workers to internalize the intergenerational externality. To focus on the impact of intergenerational externalities on the age-design of labor market policies, we assume throughout this section that \( \eta = \gamma \).

**Proposition 9.** Assuming \( \beta \to 1 \), \( z = 0 \) and \( \eta = \gamma \), an optimal age-sequence for firing taxes and hiring subsidies \( \{F_i^*, H_i^*\}_{i=1}^{T-1} \) solves:

\[
F_i^* - F_{i+1}^* = c\theta^* (1 - \tau_i^*) \quad \forall i \in [2, T - 2] \quad \text{and} \quad F_{T-1} = c\theta^* \\
H_i^* = F_i^* \quad \forall i \in [2, T - 1]
\]

where \( \{R_i^*\}_{i=2}^{T-1} \) and \( \theta^* \) are defined in Proposition 6.

**Proof.** Straightforward by comparing the propositions 1 and 6 when assuming \( \eta = \gamma \) and \( z = 0 \). \( \square \)

**Proposition 10.** There exists \( \tilde{i} \) defined by \( \tau_{\tilde{i}} = 1 \) such that \( F_{i+1}^* \geq F_i^* \quad \forall i \leq \tilde{i} \) and \( F_{i+1}^* \leq F_i^* \quad \forall i \geq \tilde{i} \).

**Proof.** Straightforward from Proposition 11 by recalling that \( \tau_1 > 1 \) and \( \tau_{T-1} = 0 \). \( \square \)

\(^{18}\text{Let us note that these policies will become useless if an equilibrium with directed search could be sustainable.}\)
Figure 1: Equilibrium vs. optimal age-dynamics of job destructions
Firings of the workers aged of more than \( \tilde{i} \) account for a negative externality by increasing the average search cost. Otherwise stated, there are too many firings of that type of workers in equilibrium (see Figure 1 for a graphical representation of this point). Then, it is optimal to implement an age-decreasing path of firing taxes for older workers: as emphasized earlier, the expectation of lower taxes in the future gives the right incentives for firms to keep workers (to postpone job destruction). On the contrary, for workers with age \( i \leq \tilde{i} \), who reduce the average search cost in the case of firings, an age-increasing dynamics of firings taxes turns out to be the optimal age-design of employment protection.

4.3 Second-best analysis

Globally, the optimal age-design employment protection would require to implement age-decreasing firing tax, at least for the older workers. Turning to a second-best perspective, we propose to examine how the optimal age-dynamics of firing taxes and hiring subsidies is affected by the introduction of unemployment benefits and worker’s bargaining power.

**Proposition 11.** Assuming \( \beta \to 1 \), \( z > 0 \) and \( \eta = \gamma \), an optimal age-sequence for firing taxes and hiring subsidies \( \{F_i^*, H_i^*\}_{i=1}^{T-1} \) solves:

\[
F_i^* - F_{i+1}^* = z + c\theta^* (1 - \tau_i^*) \quad \forall i \in [2, T-2] \quad \text{and} \quad F_T^* = z + c\theta^* \\
H_i^* = F_i^* \quad \forall i \in [2, T-1]
\]

where \( \{R_i^*\}_{i=2}^{T-1} \) and \( \theta^* \) are defined in Proposition 6.

**Proof.** Straightforward by comparing the propositions 1 and 6 when assuming \( \eta = \gamma \) and \( z > 0 \). \qed

It is straightforward to see that the existence of unemployment benefits accounts for a lower threshold age \( \hat{i} < \tilde{i} \) above which the firing tax is decreasing with age. Indeed, unemployment benefits require to tax firings (subsidy hirings) according to the flow \( z \) who increase wages and productivity thresholds, but this tax shall also incorporate the distortion effects of expected taxes in the future. In a symmetrical equilibrium (as if \( \tau_i = 1 \quad \forall i \)), this would imply that \( F_T^* = z \) and \( F_i^* = \sum_{j=i}^{T-1} \beta^{T-j-1} z \) which actually corresponds to the discounted sum of unemployment benefits. In our context with intergenerational externalities, the higher the unemployment benefits, the shorter the age-increasing path of firing taxes and hiring subsidies. Accordingly, jobs destructions turns out to be too high for some workers below \( \hat{i} \), despite firings of these workers account for positive externalities. Hence, to
reduce latter’s firings, it is required to introduce age-decreasing employment protection from a critical age \( i \) below \( i \). Ultimately, \( z \) could be large enough to imply a monotonously age-decreasing dynamics of optimal firing taxes and hiring subsidies.

Higher worker’s bargaining power qualitatively accounts for the same incidence on the policy design as implied by unemployment benefits.

**Proposition 12.** Assuming \( \beta \to 1 \), \( z = 0 \) and \( \eta > \gamma \), an optimal age-sequence for firing taxes and hiring subsidies \( \{F_i^*, H_i^*\}_{i=1}^{T-1} \) solves:

\[
F_i^* - F_{i+1}^* = c\theta^* \left( 1 - \tau_i^* + \tau_i^* \left( \frac{\gamma - \eta}{(1 - \eta)(1 - \gamma)} \right) \right) \quad \forall i \in [2, T - 2] \quad \text{and} \quad F_{T-1} = c\theta^* 
\]

where \( \{R_i^*\}_{i=2}^{T-1} \) and \( \theta^* \) are defined in Proposition 6.

The mechanism at work is however slightly different

- no future distortion effect
- by increasing productivity thresholds. It implies to implement age-decreasing employment protection from a lower age than the one obtained in the first best analysis.

Overall, this second-best analysis highlights the robustness of our results which call into question OECD practice of higher employment protection for older workers.

5 Conclusion

Globally, our theory of age-design employment protection preconizes of implementing age-decreasing firing taxes, at least for the older workers. This is at odds with all the experiences led in most OECD countries which display no or age-constant employment protection for workers aged of less than 50 years old on the one hand and an age-increasing level after this age on the other hand. This discrepancy between theory and practice is even larger in a second-best perspective when the distortions created by the existence of unemployment benefits are taken into account. This last point is not anecdotical as European countries which have implemented the highest employment protection for older workers also provide the most generous unemployment benefits. We then conclude that the existing experiences present strong perverse effects, both in terms of global employment and of social welfare. It is important to note that this reconsideration is not due to an opposition in terms of objectives: our framework is consistent with the idea that older
worker jobs must be protected. But reaching this objective implies to adopt age-decreasing firing taxes for older worker jobs. Finally, we show that it would be at least more efficient to return to an age-constant employment protection which has the advantage to preserve more the older worker jobs without distorting the job creations and destructions of younger workers.

References


A  Wage equations under Two-tier structure

The sharing rules can be written as:

\[-\gamma H_i - (1 - \gamma)U_i = \gamma [J^0_i(\epsilon) + W^0_i(\epsilon)] - W^0_i(\epsilon)\]
\[-\gamma F_i - (1 - \gamma)U_i = \gamma [J_i(\epsilon) + W_i(\epsilon)] - W_i(\epsilon)\]  \hspace{1cm} (23)

From value functions, it turns out to be that:

\[\gamma [J_i(\epsilon) + W_i(\epsilon)] - W_i(\epsilon) = \gamma \epsilon - w_i(\epsilon) + \gamma \beta \int_{R_i+1}^1 [J_{i+1}(x) + W_{i+1}(x)] dG(x)\]
\[-\beta \int_{R_i+1}^1 W_i(x) dG(x) - (1 - \gamma)\beta G(R_i+1)U_{i+1} - \gamma \beta G(R_i+1)F_{i+1}\]
\[= \gamma \epsilon - w_i(\epsilon) - (1 - \gamma)\beta U_{i+1} - \gamma \beta F_{i+1}\]  \hspace{1cm} (25)

Combining this with (24) yields:

\[w_i(\epsilon) = \gamma (\epsilon + F_i - \beta F_{i+1}) + (1 - \gamma) [U_i - \beta U_{i+1}]\]  \hspace{1cm} (26)

Similarly,

\[\gamma [J_i^0(\epsilon) + W^0_i(\epsilon)] - W^0_i(\epsilon) = \gamma \epsilon - w^0_i(\epsilon) - (1 - \gamma)\beta U_{i+1} - \gamma \beta F_{i+1}\]  \hspace{1cm} (27)

implies by combining with (23):

\[w_i^0(\epsilon) = \gamma (\epsilon + H_i - \beta F_{i+1}) + (1 - \gamma) (U_i - \beta U_{i+1})\]

Then, let us notice that the unemployed value solves in equilibrium:

\[U_i = b + \beta \left[ p(\theta) \int_{R_{i+1}}^1 \left(W^0_{i+1}(x) - U_{i+1}\right) dG(x) + U_{i+1} \right]\]
\[= b + \beta \left[ p(\theta) \frac{\gamma}{1 - \gamma} \int_{R_{i+1}}^1 \left(J^0_{i+1}(x) + H_{i+1}\right) dG(x) + U_{i+1} \right]\]
\[= b + \frac{\gamma}{1 - \gamma} \sum_{i=1}^{T-1} \left( \frac{w_u}{u} \int_{R_{i+1}}^1 \left(J^0_{i+1}(x) + H_{i+1}\right) dG(x) \right) + \beta U_{i+1}\]
\[= b + \frac{\gamma}{1 - \gamma} \sum_{i=1}^{T-1} \left( \frac{w_u}{u} \int_{R_{i+1}}^1 [1 - G(x)] dx \right) + \beta U_{i+1}\]  \hspace{1cm} (28)
where using \( p(\theta)/q(\theta) = \theta \) and \( \int_{R_{i+1}}^1 (J_{i+1}^0(x) + H_{i+1}) \, dG(x) = (1-\gamma) \int_{R_{i+1}}^1 [1-G(x)]dx \) by integrating by parts. Substituting for \( \mathcal{U}_i - \beta \mathcal{U}_{i+1} \) from this expression into \( w_i(\epsilon) \) and \( w_i^0(\epsilon) \) one gets (13) and (12).

\[ B \quad \text{Insider wage equilibrium} \]

Outsiders, once hired, have an _ex-post_ incentive to renege on the two-tier structure by demanding the insider wage. A two-tier wage structure might be unfeasible. At least, if \( F_i \geq H_i \), from (12) and (13), it comes that \( w_i^0(\epsilon) > w_i(\epsilon) \). In this section, as for instance Pissarides [2008], we look at the implications of a pure insider wage equilibrium, that is when the second tier wage sharing rule applies initially as well as subsequent renegotiations.

Otherwise stated, we now consider that \( w_i^0(\epsilon) = w_i(\epsilon) \) (hence \( R_i^0 = R_i \)), and the only relevant sharing rule is:

\[
\mathcal{W}_i(\epsilon) - \mathcal{U}_i = \gamma [J_i(\epsilon) + F_i + \mathcal{W}_i(\epsilon) - \mathcal{U}_i]
\]

In such circumstances, the unemployed value now solves:

\[
\mathcal{U}_i = b + \beta \left[ p(\theta) \int_{R_{i+1}}^1 (\mathcal{W}_{i+1}(x) - \mathcal{U}_{i+1}) \, dG(x) + \mathcal{U}_{i+1} \right]
\]

\[
= b + \beta \left[ p(\theta) \frac{\gamma}{1-\gamma} \int_{R_{i+1}}^1 (J_{i+1}(x) + F_{i+1}) \, dG(x) + \mathcal{U}_{i+1} \right]
\]

\[
= b + \beta \left[ p(\theta) \frac{\gamma}{1-\gamma} \int_{R_{i+1}}^1 (J_{i+1}(x) + H_{i+1}) \, dG(x) + \mathcal{U}_{i+1} \right]
\]

\[
+ \beta p(\theta) \frac{\gamma}{1-\gamma} \int_{R_{i+1}}^1 (F_{i+1} - H_{i+1}) \, dG(x)
\]

\[
= b + \frac{\gamma}{1-\gamma} c\theta r_i + \beta \mathcal{U}_{i+1} + \beta p(\theta) \frac{\gamma}{1-\gamma} [1 - G(R_{i+1})] (F_{i+1} - H_{i+1})
\]

Substituting for \( \mathcal{U}_i - \beta \mathcal{U}_{i+1} \) from this expression into \( w_i(\epsilon) \), one gets:

\[
w_i(\epsilon) = \gamma \left[ c + c\theta r_i + F_i - \beta F_{i+1} + \beta p(\theta) [1 - G(R_{i+1})] (F_{i+1} - H_{i+1}) \right] + (1-\gamma) (b + z)
\]

Then, the productivity threshold turns out to be defined by:

\[
b + z_i + \frac{\gamma}{1-\gamma} [c\theta r_i + \beta p(\theta) [1 - G(R_{i+1})] (F_{i+1} - H_{i+1})] =
\]

\[
R_i + \beta \int_{R_{i+1}}^1 [1 - G(x)]dx + F_i - \beta F_{i+1}
\]

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Then, if we let assume \( \gamma = \eta \) it comes that

\[
F^*_{i} - F^*_{i+1} = z + c\theta^* (1 - \tau^*_i) + p(\theta^*) \frac{\eta}{1 - \eta} [1 - G(R^*_{i+1})] \left( F^*_{i+1} - H^*_{i+1} \right)
\]

It is thus obvious that if hiring subsidies only aim at countering the negative impact of firing cost on recruitment policy \( (H^*_i = F^*_i) \), the age-design of firing taxes is leaved unaffected by the wage bargaining process, either two-tier wage structure or insider wage assumption. This is true for \( \gamma = \eta \), which implies that an optimal age-sequence for firing taxes and hiring subsidies \( \{F^*_i, H^*_i \}_{i=1}^{T-1} \) solves proposition 11.

In words, in the particular case where the labor market policy aims at internalizing intergenerational inefficiencies, since the optimal policy solves \( H^*_i = F^*_i \), the optimal two-tier wage structure collapse to the insider wage solution, i.e. \( w_i^0(\epsilon) = w_i(\epsilon) \).

C Proofs of propositions

C.1 The impact of \( \Delta_F \) on \( R_i \)

Let consider \( H_i = F_i \), which implies \( R_i^0 = R_i \), and

\[
R_i = b + z - F_i \left[ 1 - \beta(1 + \Delta_F) \right] - \beta(1 - \gamma p(\theta)) \int_{R_{i+1}}^{1} [1 - G(x)] dx
\]

Accordingly, it comes that

\[
dR_i = \beta(1 - \gamma p(\theta))[1 - G(R_{i+1})]dR_{i+1} - [1 - \beta(1 + \Delta_F)]dF_i \quad \forall i \leq T - 2
\]

Reasoning backward, starting with \( dR_{T-1} = -dF_{T-1} \), it is straightforward to see that \( \Delta_F < \frac{1}{\beta} - 1 \) leads to \( 1 - \beta(1 + \Delta_F) > 0 \) hence \( \frac{dR_i}{dF_i} \leq 0 \quad \forall i \leq T - 2 \).

In turn, for \( \Delta_F > \frac{1}{\beta} - 1, 1 - \beta(1 + \Delta_F) < 0 \), let remark that

\[
dR_{T-2} = -\beta(1 - \gamma p(\theta))[1 - G(R_{T-1})]dF_{T-1} + [\beta(1 + \Delta_F) - 1]dF_{T-2}
\]

\[
dR_{T-3} = \beta(1 - \gamma p(\theta))[1 - G(R_{T-2})] \{ -\beta(1 - \gamma p(\theta))[1 - G(R_{T-1})]dF_{T-1} + [\beta(1 + \Delta_F) - 1]dF_{T-2} \}
\]

\[
\ldots
\]

Hence, since \( \beta(1 - \gamma p(\theta))[1 - G(R_i)] < 1 \quad \forall i \), there exists a threshold age \( \tilde{i} \) such that \( \frac{dR_i}{dF_i} > 0 \quad \forall i \leq \tilde{i} \).
C.2 The efficient allocation

Let us denote $\lambda_i$ and $\mu_i$ the Lagrange multiplier associated with constraints (18) and (19); optimal decision rules with respect to $R_{i+1}$, $\theta$ and $u_i$, $y_i$ are respectively given by:

$$
\lambda_i = \mu_i R_{i+1}
$$

$$
\sum_{i=1}^{T-1} c \left( \frac{\sum_{i=1}^{T-1} u_i}{T-1} \right) = p'(\theta) \sum_{i=1}^{T-1} u_i \left( \mu_i \int_{R_{i+1}}^1 xdG(x) - \lambda_i [1 - G(R_{i+1})] \right)
$$

$$
\lambda_i = b - \frac{\sum_{i=1}^{T-1} c\theta}{T-1} + \lambda_i \left[ 1 - p(\theta) [1 - G(R_{i+1})] - G(R_{i+1}) \right] + \mu_i \left[ p(\theta) \int_{R_{i+1}}^1 xdG(x) - \int_{R_{i+1}}^1 xdG(x) \right]
$$

$$
\mu_i = 1
$$

Substituting for $\mu_i = 1$, hence $\lambda_i = R_{i+1}$, the remainder of the proof is straightforward with the definition $p'(\theta) = [1 - \eta(\theta)]q(\theta)$. 