Central Bank Lending and Inflation

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Abstract

Central banks have responded to the current financial crisis with an unprecedented program of lending to banks and other financial institutions. In some cases, this lending has lead to a substantial increase in the monetary base. Can such lending programs cause an increase in inflation? Is so, under what circumstances? I investigate these questions in a dynamic general equilibrium model with overlapping generations of agents. The model illustrates how unsterilized central bank loans of currency can improve welfare during a liquidity crisis. It also shows, however, how such lending can introduce less-desirable equilibria with higher inflation rates. Other forms of lending, including sterilized loans using central bank bonds, can capture the same benefits without introducing the higher-inflation equilibria.

The views expressed herein are those of the author and do not necessarily reflect those of the Federal Reserve Bank of New York or the Federal Reserve System.
1 Introduction

The current period of financial turmoil has witnessed unprecedented levels of lending by central banks to the private sector. The Federal Reserve, for example, has introduced a variety of new lending facilities, beginning with the Term Auction Facility in December 2007 and later including swap lines with foreign central banks and targeted programs for lending to banks and other financial institutions. It is important to understand the potential consequences of such lending for future inflation. Can central bank lending generate an increase in the rate of inflation? If so, do the operational details of the lending, such as whether or not it increases the monetary base, matter for its effect on the inflation rate?

I address these questions in a dynamic general equilibrium model in which money and central bank lending play explicit roles in improving the allocation of resources. In the absence of any lending by the central bank, the model has a unique equilibrium and the equilibrium inflation rate is equal to the growth rate of the money supply. When central bank lending is introduced, it can be either unsterilized, meaning that it increases the quantity of money in circulation, or sterilized through the sale of central bank bonds in order to leave the money supply unchanged. In both cases, there is an equilibrium where the lending policy improves the equilibrium allocation and leaves the rate of inflation unchanged. When the lending is unsterilized, however, there is also a continuum of other equilibria in which the rate of inflation is higher and accelerating over time. The real allocation of resources is less efficient in these equilibria. In this sense, central bank lending can potentially generate inflation, even in an environment where all loans are repaid and the central bank never experiences any losses. These high-inflation equilibria do not arise when the lending is sterilized using central bank bonds, indicating that the operational details of the lending programs may be critically important for determining their effects.

The model is a pure-exchange economy with overlapping generations of agents. Agents within a generation are heterogeneous, so that there is a role for intragenerational lending as well as intergenerational trade using money. Following Champ et al. [6], spatial separation and random relocation of agents generate a need for liquid assets to facilitate transactions. In particular, some agents face liquidity risk, that is, the possibility of needing to consume in a different location where...
illiquid assets (such as loans made in their home location) cannot be readily turned into consumption. To insure against idiosyncratic liquidity risk, these agents choose to pool their resources in a bank. Banks are subject to a reserve requirement specifying a minimum fraction of their assets which must be held in currency.

Two different environments are considered. In the first, representing “normal times,” privately-issued claims to future consumption, which are backed by consumption loans made by a bank, are verifiable and thus can be exchanged for current consumption in all locations and can circulate in equilibrium. In this sense, liquid assets can be created by the private sector when the demand for liquidity is high. There is no role for lending by the central bank in this environment. There is a unique equilibrium. The reserve requirement may or may not be binding in this equilibrium, depending on parameter values.\(^2\) The equilibrium inflation rate is equal to the growth rate of the money supply.

In the second environment, privately-issued claims cannot be verified and hence cannot be used by agents to purchase current consumption. This environment represents a situation that observers might call a *liquidity crisis*: assets which in other settings could be readily converted into consumption are now illiquid in the sense that they must be held to maturity in order to obtain their full value. In the absence of central bank intervention, a *liquidity premium* emerges in equilibrium: illiquid loans carry a higher rate of return than (liquid) money. As a result of this premium, the level of insurance provided by banks decreases and agents who need liquid assets suffer a loss of consumption relative to those who do not.

One policy that could mitigate the effects of such a crisis is central bank lending of currency.\(^3\) In particular, suppose the central bank offers loans of currency at a fixed nominal interest rate to any private bank, subject only to the constraint that the bank be able to repay the central bank in the following period using the proceeds of its matured consumption loans. In effect, the central bank can be viewed as taking now-illiquid consumption loans out of the hands of the private sector and replacing them with currency. This activity involves an expansion of the central bank’s balance sheet, as the newly-issued liabilities (currency) are matched by new assets in the form of claims against banks backed by consumption loans.

\(^2\) See Bhattacharya et al. [5] for a discussion of how reserve requirements can have important effects even when they do not bind in equilibrium.

\(^3\) Papers that study central bank lending in related environments include Freeman [7] - [8], Williamson [18], Smith and Weber [15], and Martin [11].
Under this lending policy, there is an equilibrium that yields the same consumption allocation as in the first environment, where all assets are liquid. In this sense, central bank lending can completely overcome the effects of a liquidity crisis in the environment studied here. The equilibrium rate of inflation is also the same as in the first environment. The stock of money in circulation is higher, reflecting the increased demand for liquid assets, but it remains constant over time. However, there is also a continuum of other equilibria. In all of these equilibria, the inflation rate is higher and is increasing over time. As the price level rises, banks’ demand for loans from the central bank rises in proportion. In effect, when agents expect future inflation under this policy, they correctly foresee that the stock of money in circulation will grow at a higher rate. This increase in the money supply, in turn, fulfills the original inflation expectations. The increasing rate of inflation in these equilibria drives down the real interest rate and distorts the allocation of resources in the economy.

An alternative policy is for the central bank to sterilize its lending through the sale of central bank bonds. A bond in the model is a liability of the central bank that can be redeemed for one unit of currency in the following period. The central bank redeems bonds in each location, which ensures that the bonds are completely liquid assets. The primary difference between bonds and currency in this setting is that only currency can be used by banks to meet reserve requirements. Under this policy, the central bank sells enough bonds so that the revenue it collects exactly offsets the amount loans it makes to banks. The net effect of the policy is to increase banks’ holding of liquid assets in the form of central bank bonds, while leaving the supply of money in circulation unchanged. This policy replicates the stationary equilibrium under the policy of unsterilized lending, without introducing the equilibria with higher inflation rates. In this sense, the model suggests that the way in which central bank lending programs are implemented, particularly how the lending affects the size of the monetary base, may play an important role in determining their effects.

Other policy alternatives are available and may be effective at preventing high-inflation equilibria. Instead of offering loans of currency at a fixed nominal interest rate, the central bank could set the interest rate it charges as a function of the current inflation rate. Such a policy can be designed in so that it replicates the effect of lending at a fixed nominal interest rate in the stationary

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4 This focus on the possibility of central bank policies generating undesirable equilibria follows Antinolfi et al. [1] and Antinolfi and Keister [2].

5 Equivalently, the central bank could directly loan bonds to banks rather than making its loans in currency. The Term Securities Lending Facility introduced by the Federal Reserve in March 2008 has this feature.
equilibrium, but without introducing any other, high-inflation equilibria. However, such policies rely on the central bank being able to react quickly and effectively to an increase in the inflation rate. In particular, agents must believe that the central bank will be willing to raise the interest rate it charges for loans, and decrease the real value lent, in response to an increase in inflation. Such policies may not be feasible in environments where the central bank is unable to commit to a future course of action.

The remainder of the paper is structured as follows. The next section presents the underlying environment and studies the properties of equilibrium in normal times, when all assets are liquid. Section 3 studies the case where private claims backed by consumption loans are illiquid, but the central bank does not engage in any lending. Section 4 studies two different lending policies, one using currency and the other using central bank bonds; this section contains the main results of the paper. Finally, Section 5 offers some concluding remarks.

2 The Model

This section presents the physical environment and the solutions to the optimization problems faced by consumers and by banks. It also presents and analysis of equilibrium for the case where all assets are liquid, which is taken to represent normal times.

2.1 The environment

The economy has an infinite sequence of overlapping generations of consumers, each of whom lives for two periods. There is a single, non-storable consumption good. In each period, a continuum of agents with unit mass is born at each of two identical locations. Half of these agents are “lenders” with endowments \((\omega_1, \omega_2) = (x, 0)\) and the other half are “borrowers” with endowments \((\omega_1, \omega_2) = (0, y)\). All consumers have preferences represented by \(u(c_1, c_2) = \ln(c_1) + \delta \ln(c_2)\).

It is assumed that \(y < \delta x\) holds, which implies that this is a Samuelson-case economy (see Gale [9]) and hence there is a potential role for money as a store of value. At \(t = 0\) there is a continuum of initial old agents with unit mass in each location.

The two locations are identical, but physically separate. At the beginning of a period, all agents receive their endowments. At this point, there is no movement between or communication across locations. There is a market in each location in which agents can exchange goods for money and, possibly, privately-issued claims. To simplify matters, I assume that all transactions must be
intermediated by a bank. There is a single, representative bank in each location. Young lenders deposit their savings in a bank, then young borrowers contact a bank and obtain a loan. The bank also trades with old agents in order to achieve its desired portfolio allocation between loans and money. Once this market closes, old agents consume and exit the economy. Young agents then have an opportunity to visit the other location, redeem any claims they may have against the bank there, and consume. After consuming, these agents return to their original location.

Next, a fraction $\pi$ of young lenders in each location is notified that they will be moved to the other location. Lenders who will be relocated are called “movers,” while the remaining lenders are called “non-movers.” Goods can never be transported between locations, but relocated agents can carry with them money and claims on the loans issued by their home bank. These agents have an opportunity to contact their bank to receive money and/or claims, then they are moved to the other location and the period ends.

Since there is a continuum of lenders, $\pi$ represents both the probability of relocation for each lender and the fraction of all lenders who move. Its value can be interpreted as the size of the aggregate demand for liquidity. Recall that relocated lenders are moved to the other location before the loans made by their bank have been repaid. An asset is considered liquid in this environment if it can be exchanged for consumption in the other location. Fiat money is universally accepted and, therefore, is always liquid. Claims issued by a bank, and backed by that bank’s portfolio of loans, may more or may not be liquid, depending on the structure of information. The present section studies the case where assets backed by loans are perfectly liquid, while Sections 3 and 4 examine situations where they are illiquid. It is assumed that

$$\pi > 1 - y/(\delta x)$$

holds, which implies that the liquidity needs of the economy are large relative to the potential store-of-value role for money.

Each member of the initial old generation is endowed with $M_0 > 0$ units of fiat currency. In subsequent periods, the stock of money – not including any loans by the central bank – grows at a constant rate

$$M_{t+1} = (1 + n) M_t,$$

This strong assumption is not necessary, although some restrictions on who can trade with whom are required. See the discussion in Champ, Smith, and Williamson [6].

This “clearing” opportunity is one of the features distinguishing the present model from others in the literature.
where $0 \leq n \leq (\delta x/y - 1)$. The latter inequality ensures, roughly speaking, that the growth rate of the money supply is small enough to keep the economy in the Samuelson case. Newly-minted money is used by the central bank to purchase goods; these purchases can be thought of as funding a public good that enters agents’ preferences in an additively-separable way. In Section 4, central bank loans of currency are introduced. For now, however, the evolution of the entire stock of currency is governed by equation (2).

### 2.2 Consumers

Borrowers choose their loan size $\ell_t$ taking the (gross) interest rate of $R_t$ as given. Their choice problem can be written as

$$\max_{\ell_t} \ln (\ell_t) + \delta \ln (y - R_t \ell_t).$$

which is easily solved for the demand for loans

$$\ell_t = \frac{y}{(1 + \delta) R_t}. \quad (3)$$

Lenders deposit all of their savings in a bank. The return they receive on this saving depends both on whether or not they move as well as on what fraction of all young lenders move. Specifically, the banking contract specifies a real return $r_t^{\text{m}} (\pi)$ if they move and $r_t^{\text{n}} (\pi)$ if they do not move. Lenders then choose the amount they save and deposit, denoted $d_t$, to solve

$$\max_{d_t} \ln (x - d_t) + \delta (\pi \ln [r_t^{\text{m}} d_t] + (1 - \pi) \ln [r_t^{\text{n}} d_t]).$$

The solution to this problem sets

$$d_t = d \equiv \frac{\delta x}{1 + \delta}. \quad (4)$$

Notice that the amount of saving is independent of the distribution of the rates of return. This result clearly depends on the assumptions of log utility and no old-age income for lenders, which imply that the income and substitution effects of a change in the rate of return exactly offset each other. These assumptions are useful for deriving analytical results below.

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8 The results would not change in any substantive way if the newly-minted money were instead given to young lenders as a lump-sum transfer.
2.3 Banks

Banks serve two important functions in this setup: they provide intermediation between borrowers and lenders and they provide lenders with insurance against the relocation shock. It is assumed that banks behave competitively in the sense that they (i) take the real return on assets as given and (ii) choose the returns \( r_m^t \) and \( r_n^t \) to maximize the expected utility of young lenders. A young lender deposits her entire savings \( d \) with a bank. Per unit of deposits, the bank acquires an amount \( \gamma_t \) of real money balances, and makes loans with a real value \( 1 - \gamma_t \). The feasibility constraints facing the bank in choosing these returns can be written in the following way. Let \( \alpha_{1,t} \) denote the fraction of the bank’s cash reserves that are given to movers and let \( \alpha_{2,t} \) denote the fraction of the returns from the bank’s consumption loans that are offered to movers through the issue of private claims. Let \( p_t \) denote the general price level at time \( t \); the return to holding money between time \( t \) and \( t+1 \) is then given by \( p_t/p_{t+1} \).

The return offered to movers must satisfy

\[
\pi r_m^t \leq \alpha_{1,t} \gamma_t \frac{p_t}{p_{t+1}} + \alpha_{2,t} (1 - \gamma_t) R_t
\]

for some \( 0 \leq \alpha_{1,t} \leq 1 \) and \( 0 \leq \alpha_{2,t} \leq 1 \). Note that this formulation of the constraint assumes that the claims issued by the bank are fully liquid in the sense that they can be sold at face value in the other location. In other words, there will never be a binding liquidity constraint in this setting because all of the assets owned by the bank are fully liquid. This situation is considered to represent “normal times” in the model. Later sections in the paper examine what happens when the bank’s loan portfolio is illiquid, which effectively imposes the constraint \( \alpha_{2,t} = 0 \).

The second feasibility constraint states that payments to non-movers cannot exceed the value of the bank’s remaining portfolio. This constraint can be written as

\[
(1 - \pi) r_n^t \leq (1 - \alpha_{1,t}) \gamma_t \frac{p_t}{p_{t+1}} + (1 - \alpha_{2,t}) (1 - \gamma_t) R_t.
\]

Banks also face a reserve requirement. This requirement states that the fraction of the bank’s assets held in cash must be at least equal to \( \mu > 0 \). As is discussed below, this requirement can be used by the central bank to influence the demand for real money balances and can potentially provide a nominal anchor. For simplicity, the value of \( \mu \) is assumed to be constant over time.

Banks maximize a typical young lender’s expected utility subject to these constraints. Given
(4), the bank’s problem is to choose \( r^m_t \) and \( r^n_t \) to maximize
\[
\ln \left( \frac{x}{1 + \delta} \right) + \delta \left( \pi \ln \left( r^m_t \frac{\delta x}{1 + \delta} \right) + (1 - \pi) \ln \left( r^n_t \frac{\delta x}{1 + \delta} \right) \right)
\]
subject to the constraints (5) and (6), which will hold with equality at an optimum, and the reserve requirement, which may or may not be binding. Substituting the feasibility constraints into this objective and dropping the constant terms yields the problem
\[
\max_{\alpha_{1,t}, \alpha_{2,t}} \pi \ln \left[ \alpha_{1,t} \gamma_t \frac{p_t}{p_{t+1}} + a_{2,t} (1 - \gamma_t) R_t \right] + (1 - \pi) \ln \left[ (1 - \alpha_{1,t}) \gamma_t \frac{p_t}{p_{t+1}} + (1 - \alpha_{2,t}) (1 - \gamma_t) R_t \right]
\]
subject to
\[
0 \leq \gamma_t \leq 1 \quad \text{and} \quad 0 \leq \alpha_{j,t} (\pi) \leq 1 \quad \text{for} \ j = 1, 2.
\]

One approach to solving this problem is to proceed backward, first choosing the optimal values of \( \alpha_{1,t} \) and \( \alpha_{2,t} \) taking \( \gamma_t \) as given, and then use the resulting decision rules when choosing the optimal \( \gamma_t \). It is fairly straightforward to show that there is an indeterminacy in the solution to the problem of choosing \( \alpha_{1,t} \) and \( \alpha_{2,t} \) to maximize (8). In order for the solution to be interior for either \( \alpha_{1,t} \) or \( \alpha_{2,t} \), it must be the case that the solution sets \( r^m_t = r^n_t \). In this case, many pairs of \( \alpha_{1,t} \) and \( \alpha_{2,t} \) will achieve this same outcome. For example, it is easy to see that the outcome \( r^m_t = r^n_t \) is achieved by setting
\[
\alpha_{1,t} = \alpha_{2,t} = \pi,
\]
and, hence, this is always a solution to the problem. The fact that the solution equates the consumption of movers and nonmovers is an indication that liquidity constraints are never binding in the current setup.

Now consider the bank’s portfolio choice problem. Because both money and loans are completely liquid, the bank will simply choose \( \gamma_t \) to maximize the total return on its portfolio, subject to the reserve requirement \( \gamma_t \geq \mu \). The solution sets
\[
\gamma_t \begin{cases} = \mu & \text{if } R_t > \end{cases} \begin{cases} \in [\mu, 1] \quad \text{if } R_t = \end{cases} \begin{cases} 1 \quad \text{if } R_t < \end{cases} \frac{p_t}{p_{t+1}}.
\]
2.4 Equilibrium in normal times

An equilibrium of this economy is characterized by the market clearing condition for real money balances in each location. Since the supply of real balances in period $t$ is equal to $M_t/p_t$ and the demand for real balances comes entirely from banks, market clearing requires

$$d_t - \ell_t = \frac{M_t}{p_t},$$

or, using (3) and (4),

$$1 - \frac{y}{\delta x R_t} = \gamma_t.$$  \hspace{1cm} (9)

Note that there cannot be an equilibrium with $\gamma_t = 1$ for any $t$, because this would imply $\ell_t = 0$ and hence an infinite value for $R_t$. In the region where $\gamma_t > \mu$, money and loans must yield the same return in equilibrium, that is, $R_t = p_t/p_{t+1}$ must hold. Using (2), we have the relationship

$$\frac{p_{t+1}}{p_t} = \frac{\gamma_t}{\gamma_{t+1}}(1 + n).$$  \hspace{1cm} (10)

Using this information, the market clearing condition (9) becomes

$$1 - \frac{y}{\delta x R_t} \frac{\gamma_t}{\gamma_{t+1}} (1 + n) = \gamma_t,$$

which can be solved for the equilibrium law of motion for $\gamma_t$ in the region where $\gamma > \mu$ holds,

$$\gamma_{t+1} = \frac{y}{\delta x} (1 + n) \frac{\gamma_t}{1 - \gamma_t}.  \hspace{1cm} (11)$$

Note that this equation has a unique stationary solution

$$\gamma^* \equiv 1 - (1 + n) \frac{y}{\delta x}.$$

When $\gamma_t = \mu$, the reserve requirement holds with equality and any value of $\gamma_{t+1}$ that generates the relationship $R_t \geq p_t/p_{t+1}$ is consistent with equilibrium. Using (9), it can be shown that $R_t \geq p_t/p_{t+1}$ holds if and only if

$$\gamma_{t+1} \leq \frac{y}{\delta x} (1 + n) \frac{\gamma_t}{1 - \gamma_t}.$$  

The equilibrium law of motion thus has a vertical segment at $\gamma_t = \mu$ between the horizontal axis and the curve given by (11).
Figure 1 depicts this law of motion. Panel (a) presents the case where $\gamma^* > \mu$ holds, while panel (b) gives the reverse. In both cases, there is a unique equilibrium, and that equilibrium is stationary. In panel (a), any value for the initial price level that leaves $\gamma_0$ at a value different from $\gamma^*$ will generate a trajectory that eventually violates either the constraint $\gamma_t \leq 1$ or the reserve requirement in finite time. In particular, note that the usual “hyperinflationary” equilibria associated with overlapping generations models do not exist here because the reserve requirement implies that there is always a positive demand for real money balances. Thus the presence of a reserve requirement affects the set of equilibria, even if the requirement does not bind in the unique equilibrium; this feature of the model is reminiscent of results in Bhattacharya et al. [5]. In panel (b), any value of $p_0$ that leaves $\gamma_0$ above $\mu$ will generate a trajectory that eventually violates the constraint $\gamma_t \leq 1$. In this case, the reserve requirement is binding in every period. Also note that in both cases, the gross inflation rate in equilibrium is equal to $(1 + n)$. This fact can be easily seen from (10) using the fact that $\gamma_t$ is constant in equilibrium.

The results of this section are summarized in the following proposition.

**Proposition 1** When all assets are perfectly liquid, there is a unique equilibrium. That equilibrium is stationary, with $\gamma_t = \max\{\gamma^*, \mu\}$ for all $t$, and the (gross) inflation rate is equal to $1 + n$.

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9 Recall from above that $\gamma_t = 1$ cannot be part of any equilibrium.
The remainder of the paper focuses on the case where $\mu < \gamma^*$ holds, so that the reserve requirement does not bind in equilibrium.

3 Liquidity Crises

In this section, I make one change to the environment studied above: claims to consumption based on loans made in one location are no longer recognizable or verifiable in the other location. One could imagine, for example, that agents can costlessly generate counterfeit claims. Recall that the timing assumptions above imply that relocated agents cannot directly redeem these claims. The opportunity to travel to the originating location and redeem claims occurs after old agents have exited the economy and, hence, is only available to young agents in a given period. In order for these claims to be useful to relocated agents, therefore, they must sell at a positive price in the market in the other location. If the claims can be costlessly counterfeited in unlimited quantities, however, they cannot sell at a positive price. In other words, in this section the loans made by banks, and any assets backed by those loans, are completely illiquid. These loans can only generate consumption for those agents who are able to hold them to maturity. I refer to the situation in which these assets are no longer liquid as a liquidity crisis.

3.1 Banks

In this new situation, each bank’s objective function is still given by (7). The feasibility constraints can still be written as (5) and (6), but with the added constraint

$$\alpha_{2,t} = 0.$$  \hfill (12)

In other words, the consumption of relocated agents is now equal to the value of the cash they carry with them; claims on consumption backed by the bank’s loans can no longer be used by these agents. The problem is again solved backward, first looking for the optimal choice of $\alpha_{1,t}$ for a given value of $\gamma_t$. The solution now sets

$$\alpha_{1,t} = \left\{ \frac{\pi \left( 1 + \frac{1}{\gamma_t} R_t^{\frac{p_{t+1}}{p_t}} \right) \gamma_t}{\pi R_t (1 - \pi) \frac{p_t}{p_{t+1}}} \right\} \text{ for } \gamma_t \left\{ \begin{array}{ll} \leq & \gamma' \\ > & \gamma' \end{array} \right\} \equiv \gamma'.$$  \hfill (13)

Note that when $\gamma_t > \gamma'$, the constraint $\alpha_{1,t} \leq 1$ is not binding; in fact, the constraint (12) is also slack in this case. In other words, when $\gamma_t$ is large enough, the liquidity constraint does not bind.
When \( \gamma_t \) is smaller, however, the constraint does bind.

Next, consider the problem of choosing \( \gamma_t \), using the optimal decision rule for \( \alpha_{1,t} \) above. The solution here depends critically on the rates of return offered by money and consumption loans. If money offers the higher return, choosing \( \gamma_t = 1 \) is clearly optimal because it both maximizes the return on the bank’s portfolio and ensures that the liquidity constraint is not binding. When the two returns are equal, the bank is indifferent between any values of \( \gamma_t \) that will make the liquidity constraint slack. Using (13) and that fact that \( \gamma' = \pi \) when \( R_t = p_t/p_{t+1} \), it follows that any \( \gamma_t \geq \pi \) will satisfy this criterion. Finally, when the return on money is lower than that on consumption loans, the bank will clearly not hold any currency beyond what it plans to give to movers; in other words, it will set \( \gamma_t \) in such a way that \( \alpha_{1,t} = 1 \) will be optimal. The objective in this case can be written as

\[
\pi \ln \left( \frac{\gamma_t}{\gamma_t} \right) + (1 - \pi) \ln \left( \frac{(1 - \gamma_t)}{1 - \pi} \right),
\]

which is solved by \( \gamma_t = \pi \). The optimal choice of \( \gamma_t \) can, therefore, be summarized as follows

\[
\gamma_t \begin{cases} 
\pi & \text{as } \frac{p_t}{p_{t+1}} = \begin{cases} < \\ = \\ > \end{cases} R_t \\
\in [\pi, 1] & \text{as } \frac{p_t}{p_{t+1}} = \begin{cases} < \\ = \\ > \end{cases} R_t. 
\end{cases}
\]

Together with (13), this expression summarizes the solution to the bank’s problem.

### 3.2 Equilibrium

The market clearing condition is again given by (9). Note that \( R_t < p_t/p_{t+1} \) cannot hold in equilibrium, because it would lead banks to hold only money, which, in turn, would imply that the interest rate \( R_t \) is infinite. If \( R_t = p_t/p_{t+1} \), then using (10) and (9) leads again to the equilibrium law of motion in (11). This equation thus gives the equilibrium law of motion in the present environment in the region \( \gamma_t \geq \pi \). Finally, note that

\[
\gamma_{t+1} \leq \frac{y}{\delta x} (1 + n) \frac{\gamma_t}{1 - \gamma_t},
\]

implies \( p_t/p_{t+1} < R_t \) holds and banks will set \( \gamma_t = \pi \). In other words, the equilibrium law of motion has a vertical segment at \( \gamma_t = \pi \), beginning at the curve in (11) and extending to the horizontal axis. Note that (1) implies that \( \pi \) is greater than \( \gamma^* \), as depicted in Figure 2. It is clear from the figure that there is again a unique equilibrium, and this equilibrium is stationary. Equation
(10) again implies that the equilibrium inflation rate equals $1 + n$. These results are summarized in the following proposition.

Proposition 2  When loans are illiquid, there is a unique equilibrium. This equilibrium is stationary, with $\gamma_t = \pi$ for all $t$, and the (gross) inflation rate is equal to $1 + n$.

It is interesting to compare the features of the unique equilibrium here with those of the equilibrium identified in Proposition 1. The equilibrium inflation rate is unchanged, but banks’ portfolio allocation is shifted toward holding more cash reserves and doing less lending than in the previous section.$^{10}$ Moreover, the fact that the equilibrium lies on the vertical segment of the equilibrium law of motion implies that $p_t / p_{t+1} > R_t$ holds. In other words, a liquidity premium emerges in this equilibrium, as banks earn a higher rate of return on illiquid loans than on liquid money holdings. This premium can be measured by the ratio of the returns received by nonmover and by movers

$$\frac{r^n}{r^m} = \frac{R_t}{(p_t / p_{t+1})} = \frac{1 + n + y}{1 - \pi \delta x}.$$  

Condition (1) implies that this ratio is greater than 1. These features – decreased real lending and a premium for liquid assets – characterize a liquidity crisis in this model. The next section explores central bank policies designed to mitigate the effects of such a crisis by lending to banks.

$^{10}$ This fact, in turn, implies that the initial price level is lower.
4 Lending and Inflation

This section examines central bank lending policies. The central bank does not own any real resources in this model, but it has the ability to print currency and other fiat objects (such as a central bank bonds) and to lend to banks and transact in the market using these objects. The central bank charges a gross nominal interest rate \( I \geq 1 \) on all types of loans.

The timing of central bank interventions in the economy is as follows. While the market is open in each location, banks have an opportunity to contact the central bank and obtain a loan. After the market closes and this borrowing opportunity has passed, reserve requirements are applied. Relocated agents then have an opportunity to withdraw from their bank, as above, before moving to the other location. The important feature of this timing is that money borrowed from the central bank can be given to relocating agents before they leave their original location.

In the following period, banks must repay the loan to the central bank with interest. This repayment takes place while the market is open in each period, so that a bank has the opportunity to collect on its consumption loans and use the proceeds to buy the currency and/or bonds needed to repay the loan. The central bank then destroys any amount of currency or bonds equal to the quantity of loans made, and it uses its interest earnings (if any) to purchase goods in the market. In this way, the stock of currency in circulation that is not borrowed from the central bank continues to follow the rule (2), as in the previous sections. That equation can now be interpreted has governing the behavior of nonborrowed reserves.

4.1 Lending in currency

Consider first the case where central bank lending takes place in currency. This lending is unsterilized in the sense that it results in an increase in the stock of currency in circulation. Let \( \delta_t \) denote the real value of the currency that a bank borrows from the central bank, per unit of deposits, at time \( t \). The bank’s feasibility constraints can then be written as

\[
\pi r_t^m \leq \alpha_{1,t} \gamma_t \frac{p_t}{p_{t+1}} + \alpha_{3,t} \delta_t \frac{p_t}{p_{t+1}}
\]

and

\[
(1 - \pi) r_t^m \leq \left[1 - \alpha_{1,t}\right] \gamma_t \frac{p_t}{p_{t+1}} + (1 - \gamma_t) R_t + \left[1 - \alpha_{3,t}\right] \delta_t \frac{p_t}{p_{t+1}} - \delta_t I \frac{p_t}{p_{t+1}},
\]

In other words, consistent with practice in the U.S. and elsewhere, money borrowed from the central bank can be applied toward a bank’s reserve requirement.
where \( \alpha_{3,t} \in [0,1] \) is the fraction of the currency borrowed from the central bank that is given to movers. The reserve requirement in this setting is

\[
\gamma_t + \delta_t \geq \mu. \tag{16}
\]

As in the previous sections, the problem is solved in steps, beginning with the optimal choice of the variables \( \alpha_{j,t} \) and \( \delta_t \) for a given value of \( \gamma_t \). It is shown below that the reserve requirement (16) is never binding at the solution to this problem. We can, therefore, solve the problem without imposing the constraint and then verify that the solution does indeed satisfy it. Substituting the feasibility constraints into the objective function and dropping constant terms yields

\[
\pi \ln \left[ \alpha_1 \gamma_t + \alpha_3 \delta_t \right] + (1 - \pi) \ln \left[ (1 - \alpha_1) \gamma_t \frac{p_t}{p_{t+1}} + (1 - \gamma_t) R_t + (1 - \alpha_3 - I) \delta_t \frac{p_t}{p_{t+1}} \right].
\]

The bank will choose \( \alpha_1, \alpha_3, \) and \( \delta_t \) to maximize this objective subject to the usual non-negativity constraints. When the reserve requirement does not bind, then \( \alpha_3 = 1 \) clearly holds, that is, the bank will never borrow money for the purpose of holding it between periods.\(^{12}\) Similar logic implies that if \( \alpha_1 \) is less than 1, then \( \delta_t \) must equal zero. In other words, the bank would never choose to borrow money if it were not giving all of its cash balances to movers.

We first look for a solution to this problem where \( \alpha_1 \) is interior. In this case, where \( \delta_t \) will be set to zero, the problem is identical to that in the previous section, where borrowing was not possible. The optimal choice of \( \alpha_1 \) is, therefore, again given by (13). Next, we ask when the solution of \( \delta_t \) is interior. This case entails \( \alpha_1 = 1 \) and the following first-order condition

\[
\frac{\pi}{\gamma_t + \delta_t} = \frac{(1 - \pi) I \frac{p_t}{p_{t+1}}}{(1 - \gamma_t) R_t - I \delta_t \frac{p_t}{p_{t+1}}},
\]

which implies that \( r_t^m = I r_t^m \), or that nonmovers consume more than movers by a factor of \( I \). The resulting value of \( \delta_t \) is given by

\[
\delta_t = \pi (1 - \gamma_t) \frac{R_t p_{t+1}}{I p_t} - (1 - \pi) \gamma_t,
\]

\(^{12}\) This statement is a matter of strict preference for the bank when \( I > 1 \). If \( I = 1 \), the bank is actually indifferent between borrowing any additional amount of money and holding it between periods, since it can simply repay the money (with no interest) in the following period. In this case \( \alpha_3 = 1 \) is a solution to the bank’s problem.
which applies in the region where this solution satisfies \( \delta_t \geq 0 \), or where

\[
\gamma_t \leq \frac{\pi R_t}{\pi R_t + (1 - \pi) \frac{p_t}{p_t^{t+1}}} \equiv \gamma''.
\]

Note that \( I > 1 \) implies \( \gamma'' < \gamma' \) holds. For values of \( \gamma_t \) between \( \gamma'' \) and \( \gamma' \), the constraints \( \alpha_{1,t} \leq 1 \) and \( \delta_t \geq 0 \) are both binding. To summarize, then, the optimal choices for \( \alpha_{1,t} \) and \( \delta_t \) in this case are given by

\[
\alpha_{1,t} = \left\{ \begin{array}{c} 1 \\
\frac{1}{\pi} \left( 1 + \frac{1}{\gamma_t} R_t \frac{p_t^{t+1}}{p_t} \right) \end{array} \right\}
\quad \text{and} \quad
\delta_t = \left\{ \begin{array}{c} \pi (1 - \gamma_t) \frac{R_t}{p_t} \frac{p_t^{t+1}}{p_t} - (1 - \pi) \gamma_t, \\
0 \\
0 \end{array} \right\}
\]

(17)

for \( \gamma_t \in \left\{ [0, \gamma''), [\gamma'', \gamma'), [\gamma', 1]\right\} \).

The next step in solving the bank’s problem is to choose the optimal level of \( \gamma_t \), taking into account the decision rules derived above. This choice will again depend critically on the relative returns offered by money and consumption loans. As in the previous section, if money offers a higher return the bank will clearly set \( \gamma_t = 1 \). If the returns on money and consumption loans are equal, the bank is indifferent between any values of \( \gamma_t \) in \([\pi, 1]\), since all imply that the liquidity constraint will not bind. When money is dominated in rate of return, the bank will not hold any currency beyond what it gives to movers. In other words, it will choose \( \gamma_t \) in such a way that \( \alpha_{1,t} = 1 \) will be optimal.

The new decision in this case is whether or not to borrow from the central bank and, if so, how much to borrow. When money is dominated in rate of return and \( \alpha_{1,t} \) is set to 1, the objective function can be written as

\[
\pi \ln [\gamma_t + \delta_t] + (1 - \pi) \ln \left[ (1 - \gamma_t) R_t - \delta_t \frac{p_t}{p_t^{t+1}} \right].
\]

This expression shows that the bank has two ways of transferring resources from non-movers to movers: it can hold non-borrowed currency \( \gamma_t \), or it can borrow \( \delta_t \) from the central bank. If \( \frac{I_t}{p_t^{t+1}} > R_t \), the first approach is more efficient and the bank will set \( \delta_t = 0 \). The first-order condition for non-borrowed money balances then requires \( \gamma_t = \pi \). If, on the other hand, \( \frac{I_t}{p_t^{t+1}} < R_t \) holds, the bank will set \( \gamma_t = 0 \) and only borrow money from the central bank, following the
decision rule in (17). When \(Ip_t/p_{t+1} = Rt\), the bank will be indifferent between any pairs \((\gamma_t, \delta_t)\) satisfying \(\gamma_t + \delta_t = \pi\). To summarize, then, the optimal choice of \(\gamma_t\) is given by

\[
\gamma_t = \begin{cases} 
0 & \text{as } p_t/p_{t+1} = 0 \\
\pi & \text{as } p_t/p_{t+1} = \pi \\
1 & \text{as } p_t/p_{t+1} = 1 
\end{cases}
\]

(18)

Turning to the analysis of equilibrium, the market clearing condition is again given by (9). As in previous sections, we cannot have \(p_t/p_{t+1} > Rt\) in any period in equilibrium. If \(p_t/p_{t+1} = Rt\), the same steps used above lead to the equilibrium law of motion (11). Using (18), we know that this law of motion applies in the range where \(\gamma_t \geq \pi\). If \(p_t/p_{t+1} = Rt/I\), the equilibrium law of motion becomes

\[
\gamma_{t+1} = \frac{y}{\delta x} \frac{1 + n}{I} \frac{\gamma_t}{1 - \gamma_t};
\]

(19)

this relationship applies in the region where \(\gamma_t \leq \pi\). Note that when \(\gamma_t = \pi\), any \(\gamma_{t+1}\) satisfying

\[
\frac{y}{\delta x} \frac{1 + n}{I} \frac{\gamma_t}{1 - \gamma_t} < \gamma_{t+1} < \frac{y}{\delta x} (1 + n) \frac{\gamma_t}{1 - \gamma_t}
\]

will lead to prices satisfying

\[
\frac{R_t}{I} < \frac{p_t}{p_{t+1}} < R_t.
\]

By (18), these prices are consistent with the choice of \(\gamma_t = \pi\) and, hence, the equilibrium law of motion has a vertical segment at \(\gamma_t = \pi\) connecting the curves defined by (11) and (19), as depicted in Figure 3.

The figure shows that there is again a stationary equilibrium. If the interest rate \(I\) charged by the central bank is small, this equilibrium will fall on the curve (19) as depicted in the figure. The level of real balances in this equilibrium is given by

\[
\gamma^{**} = 1 - \frac{1 + n}{I} \frac{y}{\delta x},
\]

and the level of borrowing by \(\delta = \pi - \gamma^{**}\). The inflation rate is again equal to \((1 + n)\). The liquidity premium in this equilibrium is given by

\[
\frac{r^m_t}{p_t/p_{t+1}} = \frac{R_t}{p_t/p_{t+1}} = I.
\]
For any $I < \frac{\mu}{\delta x} \frac{1+n}{1-\pi}$, therefore, central bank lending decreases the liquidity premium.\(^{13}\) It also decreases the real interest rate and moves the economy toward a more efficient allocation of resources. In the limit as the nominal interest rate charged by the central bank converges to 1, the liquidity premium disappears and the equilibrium allocation converges to that in the environment with no liquidity constraints.

![Figure 3: Central bank lending in currency](image)

Perhaps the most interesting aspect of the equilibrium law of motion depicted in Figure 3, however, is that equilibrium is no longer unique. There is now a continuum of equilibria in which the inflation rate is greater than $1+n$ and is accelerating over time. Consider any $p_0$ such that $M_0/p_0 < \gamma^{**}$ holds. This initial price level then generates a trajectory in which $\gamma_t$ converges monotonically to zero. These trajectories are consistent with equilibrium in every period. In particular, notice that the reserve requirement is always satisfied because at $\gamma_t$ falls, $\delta_t$ rises so that $\gamma_t + \delta_t = \pi$ holds in each period. The inflation rate in such an equilibrium is given by

$$\frac{P_{t+1}}{P_t} = \frac{\gamma_t}{\gamma_{t+1}} (1+n) = \frac{\delta x I}{y} (1 - \gamma_t).$$

For any $\gamma_t < \gamma^{**}$, this inflation rate is larger than $(1 + n)$. Moreover, the inflation rate rises over time as $\gamma_t$ falls along these trajectories. Because real borrowing from the central bank approaches $\delta_t = \pi$ as $t$ goes to infinity, nominal borrowing must grow at the rate of inflation in the limit.

\(^{13}\) For larger values of $I$, however, the stationary equilibrium will fall on the vertical part of the curve, with $\gamma_t = \pi$ for all $t$. In such cases, banks never borrow from the central bank and the stationary equilibrium is identical to the one studied in Section 3.
Hence, the total money supply (both borrowed and nonborrowed) grows at the rate of inflation in the limit, which is a form of the quantity theory of money.

The results for this section are summarized in the following proposition.

**Proposition 3** When the central bank loans currency at interest rate $I$, there is a continuum of equilibria. One equilibrium is stationary with $\gamma_t = \min [\gamma^{**}, \pi]$ for all $t$ and (gross) inflation rate $1 + n$. In all other equilibria, $\gamma_t$ is monotonically decreasing and the inflation rate is both greater than $1 + n$ and increasing over time.

These results show how a central bank lending policy can be useful in terms of improving the allocation of resources, but also has the potential to be destabilizing in the sense of allowing the expectation of higher inflation to become self-fulfilling. The next subsection examines an alternative lending policy that can capture the same benefits as lending in currency but without introducing the additional, less desirable equilibria.

### 4.2 Central bank bonds

Another possibility for the central bank is to lend claims that are liquid, but distinct from currency. Suppose that when a bank borrows from the central bank, it receives nominal bonds, where a bond issued in period $t$ is a claim on one dollar in period $t + 1$. The central bank lends the same types of bonds in both locations, and a bond can be redeemed in either location in the following period. It is assumed that these bonds are universally recognizable and cannot be counterfeited. These assumptions imply the bonds will necessarily be liquid assets in equilibrium.

The central bank charges a nominal interest rate $I$ on its loans on bonds, exactly as it did on loans of currency in the previous section. When a bank borrows a bond in period $t$, it must repay with a bond in period $t + 1$ plus $(I - 1)$ units of currency. There are several different ways of interpreting this policy, all of which are formally equivalent in the model presented here. One interpretation is that the central bank literally lends bonds instead of currency, as described above; this interpretation is similar to the Term Securities Lending Facility introduced by the Federal Reserve in March 2008. Alternatively, the central bank could make loans in currency (as in the previous section), but could engage in sterilizing open market operations, selling a quantity of bonds in the market in each period so that its receipt of currency on the bond sales exactly equals the amount of currency lent. The net result of such a program is that the quantity of money in the hands of the private sector is unchanged while the quantity of bonds has increased. This interpretation
matches the Federal Reserve’s handing of the Term Auction Facility (TAF) and other lending facilities during the first half of 2008.\textsuperscript{14} To economize on notation, I assume here that the central bank lends directly in bonds and does not engage in open market operations; both approaches lead to exactly the same conditions characterizing the set of equilibria.

The primary difference between bonds and currency in this model is that only currency can be used to meet reserve requirements. The statement of the bank’s problem is identical to the previous case, except that the reserve requirement is now

\[ \gamma_t \geq \mu. \]

The first step in solving this problem, choosing \( \alpha_{1,t} \) and \( \delta_t \) for a given value of \( \gamma_t \) is also unchanged and, thus, the solution (17) still applies. The optimal choice of \( \gamma_t \) is now given by

\[
\begin{align*}
\gamma_t = \begin{cases} 
\mu & \in [\mu, \pi] \\
\pi & \in [\pi, 1] \\
1 & \end{cases} \end{align*}
\]

as

\[
\frac{p_t}{p_{t+1}} = \begin{cases} 
< \frac{R_t}{I} & \in \left( \frac{R_t}{I}, \frac{R_t}{I} \right) \\
= \frac{R_t}{I} & \in \left[ \frac{R_t}{I}, R_t \right] \\
> R_t & \end{cases}
\]

reflecting the fact that the reserve requirement must be met with holdings of currency. The market clearing condition is again given by (9). Following the steps in the previous section shows that the law of motion depicted in Figure 3 still applies in the range where \( \gamma_t \geq \mu \). The new solution (20), however, implies that the law of motion now has a vertical segment at \( \gamma_t = \mu \), rather than continuing to the origin. The equilibrium law of motion for this case is presented in Figure 4.

As the figure indicates, the properties of the stationary equilibrium when the central bank loans bonds are exactly the same as when the loans take place in currency. The difference is that lending of bonds does not introduce other, nonstationary equilibria. These results are summarized in the following proposition.

**Proposition 4** When the central bank loans bonds at interest rate \( I \), there is a unique equilibrium. This equilibrium is stationary, with \( \gamma_t = \min [\gamma^*, \pi] \) for all \( t \), and the (gross) inflation rate is equal to \( 1 + n \).

It is worth emphasizing that the allocation of resources in the stationary equilibrium is exactly

\textsuperscript{14} Alternatively, the bonds used for sterilization could be issued by another agency, such as the Treasury department. Such an approach would resemble the U.S. Treasury Supplemental Financing Program (SFP) initiated in September 2008.
the same as when the central bank lends currency. The properties of this equilibrium depend only on the total quantity of liquid assets – both money and bonds – in circulation. The difference between the two policies appears if the inflation rate were to rise above $1 + n$. If this occurs, the real value of the stock of non-borrowed currency begins to decline. Such a path would remain consistent with equilibrium in both cases as long as the real value of this stock remains above $\mu$. Once it falls below $\mu$, however, the two policies have very different effects. If the central bank’s loans are made in currency, banks can continue to meet their reserve requirements using borrowed money. If the loans are made in bonds, or are made in currency but sterilized through the sale of bonds, the money market will not clear; banks’ demand for currency will be larger than the existing supply. Agents are able to forecast this event, of course, and hence the economy will never start down such a path when the lending is sterilized. The only equilibrium in this case is the stationary one, where the real value of the stock of non-borrowed currency is constant.

4.3 Other approaches

The analysis above assumes that the nominal interest rate charged by the central bank is fixed at $I$ and that banks can borrow in unlimited amounts at that interest rate. Antinolfi et al. [1] studied a related environment and looked at two alternative policies for lending currency: having the central bank charge a fixed real interest rate on its loans, so that the nominal rate increases when inflation rises, and placing a fixed, time-invariant limit on the amount of funds banks can borrow.
They showed how both of these policies are effective in eliminating nonstationary equilibria while preserving some or all of the benefits of central bank lending in the steady state.

The same types of policies would be effective in eliminating the equilibria with increasing inflation rates in the model studied here. In fact, any policy that systematically reduces the availability of funds for borrowing when the inflation rate rises, or increases the cost of these funds sufficiently, would have this effect. It is worth pointing out, however, that such approaches require commitment on the part of a central bank. At the time the loans are made in each period, the bank’s portfolio decision has already been made. At this point, the efficient response of the central bank is to lend at a zero nominal interest rate and in whatever amount is necessary to allow the bank to overcome the liquidity constraint. Policies that constrain the amount of lending or raise its prices in response to changes in the general price level are unlikely to be time consistent. The policy of lending in bonds studied above, however, is fully time consistent.

5 Concluding Remarks

The new lending programs introduced by central banks in response to the current financial crisis raise a variety of important economic issues. One interesting question is whether or not lending by a central bank should be expected to affect the rate of inflation. The present paper has examined this question using an overlapping-generations model in which spatial separation and random relocation of agents generates a need for liquid assets. A liquidity crisis in this environment is an event where privately-issued claims on consumption loans can no longer be used in exchange. The paper shows how a central bank can improve the allocation of resources in such a situation by lending liquid assets, either currency or central bank bonds, to private banks. Doing so is not necessarily inflationary in the sense that there exists an equilibrium in the model where the inflation rate is the same in every period as if the central bank did no lending. This result holds even though the lending program stays in place permanently.

However, if the central bank lending is unsterilized, so that it results in an increase in the monetary base, there exists a continuum of other equilibria in which the inflation rate is higher and increasing over time. This type of lending may generate an increase in the inflation rate because bank’s borrowing decisions are linked to the price level: an increase in inflation leads banks to

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15 Evaluating the effects of these programs in general is difficult; see, for example, Taylor and Williams [16] and McAndrews et al. [12].
borrow more in nominal terms, which results in more currency in circulation, potentially justifying the increase in inflation. This linkage disappears if the central bank lends bonds instead of currency. If the bonds are perfectly liquid, the benefits of increased liquidity can be captured without introducing additional equilibria.

The message of the paper is, therefore, a cautionary one. In times of liquidity crisis, central bank lending can play an important role in improving the allocation of resources. However, the overall effects of central bank lending may depend critically on the details of how it is implemented. Certain types of lending programs may leave the economy susceptible to self-fulfilling changes in agents’ inflation expectations, while others will not. Exploring these issues in a range of model environments and incorporating features such as private information and aggregate uncertainty is a promising area for further research.
REFERENCES


