Optimal Fiscal Policy over the Business Cycle Revisited

Martin Gervais\textsuperscript{1}  Alessandro Mennuni\textsuperscript{2}

\textsuperscript{1}Southampton and IFS
\textsuperscript{2}Southampton and EUI

Federal Reserve Bank of St Louis

January 2010
The Question

- How should fiscal policy be set over the business cycle?

- Can we rationalize a counter-cyclical fiscal policy in a stochastic neoclassical growth model?

- What are the statistical properties of fiscal policy instruments in the long run?
American Recovery and Reinvestment Act 2009

- Increase in spending (3.5% of GDP)
  - 1/4 of which for infrastructures (.85% of GDP)

- Tax cuts (2.0% of GDP)
  - Largely labor income taxes

- Financed by debt

Note: See Alesina and Ardagna (2009) for 90 other episodes
In the period of a negative productivity shock

1. Government Spending: exogenous (in their model)

2. Labor income taxes: depends on preferences

3. Capital income taxes: increase substantially

4. Debt: decreases
Main Features of the Model

1. Business cycle driven by technology shocks

2. Government infrastructures are an input (not chosen) into production

3. Investment becomes productive within the period

4. The government optimizes fiscal policy (taxes, debt and new infrastructures)

5. Complete and incomplete markets (without state contingent debt)
Main Findings

In the period of a negative shock
1. Infrastructure spending increases
2. Labor income taxes decrease
3. Capital income taxes decrease
4. State contingent debt *decreases!*
Main Findings

In the period of a negative shock
1. Infrastructure spending increases
2. Labor income taxes decrease
3. Capital income taxes decrease
4. State contingent debt decreases!

Without state contingent debt
1. Infrastructure spending increases
2. Labor income taxes decrease
3. Capital income taxes decrease
4. Debt increases
Related Literature

- Complete markets
  - Chari, Christiano, and Kehoe, 1994

- Incomplete markets without capital income taxes
  - Aiyagari, Marcet, Sargent, and Seppälä, 2002
  - Scott, 2007
  - Marcet and Scott, 2009

- Incomplete markets with pre-announced capital income taxes
  - Farhi, 2009
Household Problem

Preferences

\[
\sum_{t=0}^{\infty} \sum s^t \beta^t \pi(s^t) U\left(c(s^t), l(s^t)\right)
\]

Budget Constraint

\[
c(s^t) + k(s^t) + \sum_{s_{t+1}} q(s^t, s_{t+1}) b(s^t, s_{t+1})
\]

\[
= w(s^t) l(s^t) + r(s^t) k(s^t) + k(s^{t-1}) + b(s^t)
\]

Microfoundation
Euler Equation

- Conventional timing

\[ U_c(s^t) = \beta \sum_{s_{t+1}} \pi(s^{t+1} | s^t) U_c(s^{t+1})(1 + r(s^{t+1})) \]

- Our timing

\[ U_c(s^t)(1 - r(s^t)) = \beta \sum_{s_{t+1}} \pi(s^{t+1} | s^t) U_c(s^{t+1}) \]
Technology and Feasibility

- **Production**

\[ y(s^t) = A(s^t)k^g(s^t)\gamma k(s^t)^\alpha l(s^t)^{1-\alpha} \]

where \( \gamma + \alpha < 1 \)

- **Feasibility**

\[
\begin{align*}
    c(s^t) + c^g(s^t) + k(s^t) + k^g(s^t) \\
    = y(s^t) - \delta k(s^t) - \delta^g k^g(s^t) + k(s^{t-1}) + k^g(s^{t-1})
\end{align*}
\]
Fiscal Policy Instruments

- Proportional labor income tax: $\tau^w(s^t)$
- Proportional capital income tax: $\tau^k(s^t)$
- State-contingent debt: $b(s^t, s_{t+1})$
- Investment in infrastructures:
  \[ i^g(s^t) = k^g(s^t) - k^g(s^{t-1}) + \delta^g k^g(s^t) \]

Note: Complete tax system with no redundant instruments
Prices and Taxes

- **Before-tax prices**
  \[
  \hat{r}(s^t) = f_k(s^t) - \delta \\
  \hat{w}(s^t) = f_l(s^t)
  \]

- **After-tax prices**
  \[
  r(s^t) = \left[1 - \tau^k(s^t)\right] \hat{r}(s^t) \\
  w(s^t) = \left[1 - \tau^w(s^t)\right] \hat{w}(s^t)
  \]
Ramsey Problem

- Objective function

$$\max_{c,l,k,g(t,s^t)} \sum_{t,s^t} \beta^t \pi(s^t) U(c(s^t), l(s^t))$$

- Subject to feasibility and the implementability constraint

$$\sum_{t,s^t} \beta^t \pi(s^t) [U_c(s^t)c(s^t) + U_l(s^t)l(s^t)] = A_0$$

where $$A_0 = U_c(s_0)(k_{-1} + b_{-1})$$

Note: GBC holds
Optimal Policy: Government Infrastructures

Optimality requires that for any $s^t$

$$f_k(s^t) - \delta = f_{kg}(s^t) - \delta^g$$

- If $\delta^g = \delta$, then $k^g/k$ is constant
- If $\delta^g < \delta$, then $k^g/k$ is counter-cyclical
Optimal Policy: Labor Income Tax Rate

The tax rate on labor income is given by

$$\tau^w(s^t) = \frac{\lambda(H_l(s^t) - H_c(s^t))}{1 + \lambda + \lambda H_l(s^t)}$$

where

$$H_c(s^t) = \frac{U_{c,c}(s^t)c(s^t) + U_{c,l}(s^t)l(s^t)}{U_c(s^t)}$$

$$H_l(s^t) = \frac{U_{l,l}(s^t)l(s^t) + U_{l,c}(s^t)c(s^t)}{U_l(s^t)}$$

and $\lambda$ is the multiplier on the implementability constraint
If $U(c, l) = u(c) + v(l)$ and both $u$ and $v$ are CES (in consumption and labor)

then $\tau^w(s^t) = \tau^w(\tilde{s}^t)$
Optimal Policy: Labor Income Tax Rate

- If $U(c, l) = u(c) + v(l)$ and both $u$ and $v$ are CES (in consumption and labor)

  then $\tau^w(s^t) = \tau^w(\tilde{s}^t)$

- If $U(c, l) = (1 - \sigma)^{-1}c^{1-\sigma}(1 - l)^{\nu(1-\sigma)}$ and $l(s^t) > l(\tilde{s}^t)$

  then $\tau^w(s^t) > \tau^w(\tilde{s}^t)$ if and only if

  $$1 + \lambda(1 - \sigma)(1 + \nu) > 0$$
The tax rate on capital income determined by

\[
\frac{1 - r(s^t)}{1 - \hat{r}(s^t)} = \frac{\sum_{s_{t+1}} \pi(s^{t+1}|s^t)(1 + \lambda + \lambda H_c(s^t)) U_c(s^{t+1})}{\sum_{s_{t+1}} \pi(s^{t+1}|s^t)(1 + \lambda + \lambda H_c(s^{t+1})) U_c(s^{t+1})}
\]

Recall that

\[
H_c(s^t) = \frac{U_{c,c}(s^t)c(s^t) + U_{c,l}(s^t)l(s^t)}{U_c(s^t)}
\]
If $U(c, l) = u(c) + v(l)$ and $u$ is CES

then the capital income tax rate is zero from date 1 on
Optimal Policy: Capital Income Tax Rate

If \( U(c, l) = u(c) + v(l) \) and \( u \) is CES

then the capital income tax rate is zero from date 1 on

If \( U(c, l) = (1 - \sigma)^{-1} c^{1-\sigma} (1 - l)^{\nu(1-\sigma)} \) (\( \sigma > 1, \nu > 0 \)) and labor is pro-cyclical

then the capital income tax rate is ‘likely’ to be pro-cyclical

\[
H_c(s^t) = -\sigma - \nu(1 - \sigma) \frac{l(s^t)}{1-l(s^t)}
\]

is increasing in \( l \)
Numerical Simulation

• Preferences:

\[ U(c, l) = \frac{c^{1-\sigma} \eta (1 - l)^{\nu(1-\sigma)}}{1 - \sigma} \]

• Technology:

\[ y = Ak^{\gamma} k^\alpha l^{1-\alpha} \]

• Some parameters:
  - \( \gamma = .03 \)
  - \( \delta^g = 0.065 < 0.1 = \delta \)
  - \( \lambda \) such that \( b/y = .4 \)
In the period of a 2% negative shock:

- Government spending: 0.26 % of GDP
- Labor income tax revenue: -0.40 % of GDP
- Capital income tax revenue: -0.04 % of GDP
Simulation Results

In the period of a 2% negative shock:

- Government spending: 0.26 % of GDP
- Labor income tax revenue: -0.40 % of GDP
- Capital income tax revenue: -0.04 % of GDP
- Government debt: -4.32 % of GDP

⇒ New debt issued goes down in bad times!
Ruling out State Contingent Debt

- **Budget constraint with state contingent debt**

\[
c(s^t) + k(s^t) + \sum_{s_{t+1}} q(s^t, s_{t+1})b(s^t, s_{t+1})
\]

\[
= w(s^t)l(s^t) + r(s^t)k(s^t) + k(s^{t-1}) + b(s^t)
\]

- **Budget constraint without state contingent debt**

\[
c(s^t) + k(s^t) + q(s^t)b(s^t)
\]

\[
= w(s^t)l(s^t) + r(s^t)k(s^t) + k(s^{t-1}) + b(s^{t-1})
\]
Ruling out State Contingent Debt

- State by state implementability constraint

\[
c(s^t) + (k(s^t) + b(s^t))\beta \sum_{s_{t+1}} \pi(s^{t+1}|s^t) \frac{U_c(s^{t+1})}{U_c(s^t)} \\
= -\frac{U_l(s^t)}{U_c(s^t)} l(s^t) + k(s^{t-1}) + b(s^{t-1})
\]

\[
w(s^t) = -\frac{U_l(s^t)}{U_c(s^t)}
\]

\[
1 - r(s^t) = \beta \sum_{s_{t+1}} \pi(s^{t+1}|s^t) \frac{U_c(s^{t+1})}{U_c(s^t)}
\]
A Lagrangian – Marcet and Marimon

\[ L = \min_{\lambda(t,s^t), c, l, k, g, b(t,s^t)} \max_{c, l, k, g, b(t,s^t)} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t) \left\{ U(c(s^t), l(s^t)) \right. \\
\left. + \lambda(s^t) \left( c(s^t) + \frac{U_c(s^t)}{U(s^t)} l(s^t) - k(s^{t-1}) - b(s^{t-1}) \right) U_c(s^t) \right. \\
\left. + \lambda(s^{t-1}) (k(s^{t-1}) + b(s^{t-1})) U_c(s^t) \right\} \]

subject to

\[ c(s^t) + c^g(s^t) + k(s^t) + k^g(s^t) = y(s^t) - \delta k(s^t) - \delta^g k^g(s^t) + k(s^{t-1}) + k^g(s^{t-1}) \]
Evolution of the Multiplier $\lambda$

- Non-negative adjusted Martingale

\[
\lambda(s^t) = \frac{\sum_{s_{t+1}} \pi(s_{t+1}|s^t) U_c(s_{t+1}) \lambda(s_{t+1})}{\sum_{s_{t+1}} \pi(s_{t+1}|s^t) U_c(s_{t+1})}
\]

- Under a quasi-linear per-period utility (with ‘natural debt limits’): $\lambda \to 0$
Some (embarrassingly few) Results

- If $\delta^g < \delta$, then $k^g/k$ is counter-cyclical

- If $U(c, l) = c + v(l)$ then the capital income tax rate is zero

- If $U(c, l) = c + v(l)$ then the Ramsey allocation converges to a first best allocation
Numerical Simulation

Preferences:

\[ U(c, l) = \ln c + \eta \ln(1 - l) \]

Technology:

\[ y = Ak^\gamma k^\alpha l^{1-\alpha} \]

Some parameters:

- \( \gamma = 0 \) (Exogenous government spending)
- \( A \sim 2\)-state Markov matrix
Simulation: $\lambda$ and Debt

- Top graph shows the relationship between $\lambda$ and $\beta$ with values ranging from 0 to 3500.
- Bottom graph shows the relationship between $\beta$ and $\lambda$ with values ranging from 0 to 3500.

Gervais and Mennuni

Optimal Fiscal Policy
Simulation: $\tau^w$ and $\tau^k$
Quantitative Results with IM ($\rho = .5$)

In the period of a 2% negative shock:

- Government spending: 0.0% of GDP
- Labor income tax revenue: -1.02% of GDP
  - change in labor tax rate -0.004
- Capital income tax revenue: -0.21% of GDP
  - change in capital tax rate -1.26
- *Increase* in government debt: 1.17% of GDP
Quantitative Results with CM ($\rho = 0.5$)

In the period of a 2% negative shock:

- Government spending: 0.0% of GDP
- Labor income tax revenue: -0.69% of GDP
  - change in labor tax rate -.001
- Capital income tax revenue: 0.0% of GDP
  - change in capital tax rate 0.0

- Decrease in government debt: -0.86% of GDP
Conclusion

- Business cycle model with elastic capital-supply and endogenous government infrastructures
- Study complete and incomplete markets
- Provide a rationale for some kind of stimulus package within the neoclassical framework
- Plausible (and implementable) policy implications
- Found an ‘optimal’ amount of debt/gdp in the long run
Period $t$ composed of $n$ sub-periods

First sub-period budget constraint

$$c(s^t, 1) + k(s^t, 1) + \sum_{s_{t+1}} q(s^t, s_{t+1}) b(s^t, s_{t+1})$$

$$= w(s^t, 1) l(s^t, 1) + (1 + r(s^t, 1)) k(s^{t-1}) + b(s^t)$$

Sub-periods 2 to $n$

$$c(s^t, i) + k(s^t, i)$$

$$= w(s^t, i) l(s^t, i) + (1 + r(s^t, i)) k(s^t, i - 1), \; i = 2, ..., n$$
Summing up the sub-periods budget constraints

\[
\sum_{i=1}^{n} c(s^t, i) + k(s^t, n) + \sum_{s_{t+1}} q(s^t, s_{t+1})b(s^t, s_{t+1})
\]

\[
= \sum_{i=1}^{n} \left[ w(s^t, i)l(s^t, i) + r(s^t, i)k(s^t, i - 1) \right] + k(s^{t-1}) + b(s^t)
\]

Conventional assumption

\[
\sum_{i=1}^{n} \left[ (r(s^t, i))k(s^t, i - 1) \right] = r(s^t)k(s^{t-1})
\]

Opposite extreme

\[
\sum_{i=1}^{n} \left[ (r(s^t, i))k(s^t, i - 1) \right] = r(s^t)k(s^t)
\]