Trend inflation, nominal rigidities, and endogenous growth

Preliminary and incomplete. *

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Abstract
We study the implications of trend inflation for an economy’s long-run growth rate. To do so, we extend a New Keynesian model to allow for endogenous growth. The defining characteristic of the New Keynesian framework is that inflation and nominal price stickiness together induce relative price dispersion and thus reduce monopoly profits. When the framework is embedded in an endogenous growth model with expanding variety (Romer 1990), the impact on monopoly profits in turn reduces the return to innovation, and a link between inflation and the growth rate emerges. We explore this relationship, relate it to the empirical literature on the growth-inflation connection, and quantify its welfare implications. In particular, we show that the costs of positive trend inflation are substantial. Furthermore, we are currently attempting two extensions to environments where these costs are especially pronounced, namely (i) the case where nominal rigidities exist in the labour market and (ii) the case of endogenous growth à la Benhabib, Perli and Xie (1994), where new varieties can complement pre-existing ones. We believe that our focus on nominal price rigidity is novel, as the existing theoretical literature on the link between trend inflation and growth mostly focuses on the trade-off between money holdings and other forms of saving.

*The views expressed in this paper are those of the authors. No responsibility should be attributed to the Bank of Canada.
1 Introduction

Though central bankers throughout the developed world identify price stability as a priority for monetary policy, few use the term in its purest sense: rather than targeting a constant price level, most central banks aim to maintain inflation at a low, positive rate. For example, in Canada, an explicit commitment to keep annual CPI growth near two percent has guided monetary policy since 1993. It is natural to ask if this target is appropriate – that is, might some shift to a lower target be worthwhile? In this paper, we take a step toward answering this question. More specifically, we characterize the relationship between trend inflation and the economy’s steady-state growth rate using a model that features both endogenous growth and nominal rigidity. We show that the two-percent target is likely too high and that some reduction may have substantial, positive implications for output and welfare.

Our focus on endogenous growth follows from earlier work by Amano et al. (2007), who document a novel effect of trend inflation arising via its interaction with productivity growth in a New Keynesian environment. The authors show that deflation can offset the effects of wage rigidity by allowing real wages to rise in step with productivity gains despite nominal wages being fixed. Furthermore, since realistic parameterizations imply that wage rigidities have substantial welfare effects, the authors are led to conclude that deflation near the rate of growth is optimal. However, since productivity improves exogenously in Amano et al., it is natural to ask how robustly their results obtain when growth is endogenized, particularly in light of empirical studies by Fischer (1993), Ghosh and Phillips (1998), and Khan and Senhadji (2000) on the relationship between growth and inflation. As Lucas (1987) first argued, if policy has growth effects, then models abstracting from these effects can be misleading, since even small changes in welfare and output may become important after compounding over time.

To preview our model, it features endogenous growth through expanding variety, as seen in Romer (1990) and Barro and Sala-i-Martin (2004), among others. Final producers use a growing range of intermediate goods as inputs and buy each of these goods from a monopolistic competitor who owns the underlying patent. Patents are in turn generated by households through investment in an R&D technology. Our model also includes Taylor (1979) contracts in the market for intermediate goods. We believe that our focus on nominal price rigidity is novel, as the existing theoretical literature on the link between trend inflation and growth mostly focuses on the trade-off between money holdings and other forms of saving. For example, related work by Gomme (1993) subverts super-neutrality by introducing a cash-in-advance constraint, so inflation’s growth effects accrue by way of the incentive to hold money.

The fact that we abstract from any liquidity constraint should aid in understanding our most important finding of yet: even low, positive rates of trend inflation have substantial costs in steady state. Though inflation acts only via standard New Keynesian channels in our framework, novel effects arise through the growth rate. In particular, increases in mark-ups and price dispersion among monopolistic competitors tend to reduce monopoly profits, which discourages investment in new varieties and thus checks the rate of growth. At this point, Lucas’s logic kicks in: even modest changes in the growth rate have substantial welfare effects – in fact, when growth is in range of our two percent benchmark, preliminary findings show that reducing the rate by one basis point occasions a 0.5% drop in life-time consumption. We are currently attempting two extensions to environments where trend inflation’s costs are especially pronounced, namely (i) the case where nominal rigidities also exist in the labour market and (ii) the case of endogenous growth a la Benhabib, Perli and Xie (1994), where new varieties can complement pre-existing ones.

Apart from the extensive literature on the optimal rate of inflation, we also offer this paper
as a contribution to a related body of work concerning the long-run relationship between growth and inflation. Though many authors have studied this relationship empirically, relatively few have adapted frameworks for endogenous growth to model the connection theoretically, as in Gomme (1993). See Jones and Manuelli (1995), Wu and Zhang (1998), and Mino (1997) for some examples.

The paper is organised as follows. Section 2 presents the model. Section 3 solves the model. Section 4 presents preliminary results. Section 5 concludes.

2 The model

In this section we describe our model economy and the optimization problems solved by firms and households. The underlying framework arises from the endogenous growth literature of Romer (1987, 1990), and Rivera-Batiz and Romer (1991). As described by Barro and Sala-i-Martin (2004, chapter 6), in these model economies, the costs associated with developing new varieties of intermediary goods are by selling the exclusive rights to supply newly-developed varieties to firms operating within a monopolistic competition market structure. Growth occurs because the expanding variety of goods increases the marginal productivity of labour.

Our model embeds this growth framework within a New Keynesian model, in which the pricing decisions of monopolistic firms are affected by the presence of nominal rigidities. Specifically, we use an environment with Taylor-style price contracts, in which firms have to commit to a price for their good for a given number of periods, before getting the opportunity to reoptimize. In such an environment, positive trend inflation generates inefficient dispersion in relative prices. From an individual firm’s point of view, this means that the good it produces is priced suboptimally at any given time (relative to the perfectly flexible economy), decreasing monopoly profits. In turn, this link from trend inflation to monopoly profits can affect the incentive to develop new goods and thus the growth rate of the economy.

2.1 Final goods

The final good, \( Y_t \), is produced by assembling two broad categories of inputs, labor \( L_t \) and intermediates \( M_t \), according to a standard Cobb-Douglas production function, so we have

\[
Y_t = AL_t^{1-\alpha} M_t^{\alpha},
\]

(1)

where \( 1 - \alpha \) is the weight of labor in the production function and \( A \) is a scale factor for productivity. Note that contrary to a model with exogenous growth, the scale factor \( A \) does not grow with time.

In turn, the quantity of intermediates \( M_t \) used as input in (1) is itself a composite of differentiated intermediate goods aggregated using the standard Dixit-Stiglitz formula:

\[
M_t \equiv \left[ \int_0^{N_t} X_j^{\epsilon-1} \, dj \right]^\frac{1}{\epsilon}.
\]

(2)

where \( \epsilon \) measures the elasticity of substitution between different intermediate goods. At time \( t \), there are \( N_t \) such goods that have been developed and are used in the production of the composite \( M_t \). In this preliminary version of the paper, we follow Barro and Sala-i-Martin (2004, chapter 6) and assume that the elasticity of substitution \( \epsilon \) is such that \( \epsilon = 1/(1 - \alpha) \). This simplification is not chosen for its realism but because it simplifies the model and allows a transparent illustration.
of the mechanism by which trend inflation may affect growth in our setting.\footnote{Benhabib et al. (1994) discuss the case where \( \varepsilon \) and \( \alpha \) are independent from each other.} This simplification implies that \( \varepsilon / (\varepsilon - 1) = 1/\alpha \), which in turn allows to rewrite (1) as

\[
Y_t = AL_t^{1-\alpha} \int_0^{N_t} X_{jt}^\alpha dj. \tag{3}
\]

The final-good sector is competitive; profit maximization leads to the following input-demand functions for labor \( L_t \) and for each intermediate good \( j \):

\[
L_t = (1 - \alpha) \left( \frac{W_t}{P_t} \right)^{-1} Y_t; \tag{4}
\]

\[
X_{jt} = (\alpha A)^{1/j} \left( \frac{P_{jt}}{P_t} \right)^{1/j} L_t. \tag{5}
\]

Note that (5) represents the economy-wide demand for good \( j \) as a function of its relative price \( P_{jt}/P_t \) and of aggregate labor demand \( L_t \). Imposing the zero-profit condition in the final good sector provides the following expression for the final-good price index \( P_t \):

\[
P_t = \frac{W_t^{1-\alpha} \left( \int_0^{N_t} P_{jt}^{\alpha} dj \right)^{\alpha-1}}{A \alpha^{\alpha}(1 - \alpha)^{1-\alpha}}. \tag{6}
\]

### 2.2 Intermediate goods

The firm producing intermediate good \( j \) has access to a simple production function, which transforms one unit of final good into one unit of good \( j \).\footnote{This roundabout input-output structure follows that in Basu (1995).} Said otherwise, the marginal cost of producing the intermediate good \( j \) is simply the price of the final good, \( P_t \). We introduce nominal price stickiness into the model by assuming that producers of the intermediate goods set prices according to Taylor (1979) style staggered nominal contracts of fixed duration.\footnote{We assume Taylor-style contracts since fixed-duration nominal contracts have been found to be better suited for analyzing the welfare costs of inflation than the fixed-hazard rate specifications (such as Calvo, 1983) often used in the literature (see Ascani, 2004).} Specifically, firms set the price of their good for \( J \) quarters and, further, price setting decisions are staggered, so that every period, a fraction \( 1/J \) of the total number of firms \( N_t \) is resetting prices.

Current-period (nominal) profits are paid as dividends \( D_{jt} \) to the firm’s owners, where we have

\[
D_{jt} = (P_{jt} - P_t) X_{jt}. \tag{7}
\]

Each intermediate-good producing firm maximizes the discounted sum of future (real) dividends \( D_{jt}/P_t \). The relevant discount factor for dividends received at time \( t + k \) is \( \beta^k \lambda_{t+k}/\lambda_t \) (the intertemporal rate of substitution of households) because households own all firms.\footnote{As \( \lambda_t \) represents households’ marginal utility of income, \( \beta^k \lambda_{t+k}/\lambda_t \) measures their valuation of dividends received in period \( t + k \), in terms of the current period’s marginal utility of income.}
problem thus consists of choosing prices (every $J$ quarters) in order to solve the following problem:

$$\max_{P} \sum_{k=0}^{\infty} \left( \beta^k \frac{\lambda_{t+k}}{\lambda_t} \right) \left( \frac{P_{j,t+k}}{P_{t+k}} - 1 \right) X_{j,t+k}$$

with respect to the economy-wide demand for intermediate good $j$ as expressed by (5). After some algebra, the first-order condition for $P^*_{jt}$ (the price chosen by firms that are resetting in period $t$) yields the following expression:

$$P^*_{jt} = \frac{1}{\alpha} \sum_{\tau=0}^{J-1} \beta^\tau \lambda_{t+\tau} \left( \frac{P^*_{jt}}{P_{t+\tau}} \right)^{1/\alpha} L_{t+\tau}. \quad (9)$$

One important aspect of the analysis are the profits that the exclusive right to produce good $j$ generates for its owner. Over the course of one $J$-periods price contract, profits are measured by expression (8) evaluated at the optimum. Denoting these profits by $\Pi_t$, we have

$$\Pi_t = \sum_{\tau=0}^{J-1} \beta^\tau \lambda_{t+\tau} \left( \frac{P^*_{jt}}{P_{t+\tau}} \right)^{1/\alpha} L_{t+\tau} \left( \alpha A \right)^{-\frac{1}{\alpha}} \left( \frac{P^*_{jt}}{P_{t+\tau}} \right)^{\frac{1}{\alpha}} L_{t+\tau}, \quad (10)$$

where we have used (5) to express the economy-wide demand for good $j$ at the chosen price $P^*_{jt}$. Recall, however, that the firm that developed good $j$ receives exclusive rights to produce the good until the indefinite future. For innovators, the relevant profit figure thus results from an infinite sequence of $J$-periods price contracts with the total value of discounted profits obtained by the firm defined by

$$V_t = \Pi_t + (\beta^J \lambda_{t,J}/\lambda_t) \Pi_{t,J} + (\beta^{2J} \lambda_{t+2,J}/\lambda_t) \Pi_{t+2,J} + \ldots \quad (11)$$

### 2.3 Innovation

Following Romer (1990) and Barro and Sala-i-Martin (2004), we assume developing a new variety of intermediary good requires a resource cost of $\eta$ units of final goods. Further, there is free-entry in the development of new varieties of intermediate goods, so in equilibrium these activities make zero economic profits. This requires that in equilibrium, the cost of developing a new good and the profits derived from obtaining the exclusive right to produce and market this good should be equal. We therefore have the following equilibrium condition:

$$V_t = \eta. \quad (12)$$
2.4 Households

The representative household receives utility from consumption, $C_t$, and supplies $L_t$ units of labor exogenously. Expected lifetime utility is thus

$$\sum_{t=0}^{\infty} \beta^t \log C_t,$$

where $0 < \beta < 1$ is the discount factor.

Households own two types of assets: one-period non-contingent discount bonds and shares of firms producing intermediate goods. Their revenues thus consist of labor earnings $W_tL_t$, dividends from their ownership of firms and payments from bond holdings. These revenues must be sufficient to cover consumption as well as new bond purchases and investments in firms developing new goods, so that the following budget constraint is obeyed:

$$P_tC_t + \frac{B_t}{R_t} + P_tI_t = W_tL_t + n_t\tilde{D}_t + B_{t-1},$$

where $B_t$ represents nominal bond holdings, $R_t$ is the (gross) nominal interest rate, $I_t$ represents purchase of the shares of new firms, $n_t$ is the number of firms' shares that the household owns, and $\tilde{D}_t$ represents the average, per-share dividend paid by firms producing intermediate goods:

$$\tilde{D}_t = \frac{1}{N_t} \int_{0}^{N_t} D_{jt} \, dj.$$

Since the cost of developing a new variety of intermediate goods is equal to $\eta$ units of final goods, the following accumulation equation results, linking the purchases of shares in new firms to increases in the aggregate number of varieties $N_t$:

$$N_{t+1} = N_t + \frac{I_t}{\eta}$$

The first-order conditions for $C_t$ and $B_t$ are standard and yield

$$\lambda_t = C_t^{-1},$$

$$\frac{1}{R_t} = \beta \left[ \frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}} \right],$$

where $\pi_t$ represents the gross rate of change in the aggregate price index ($\pi_t \equiv P_t/P_{t-1}$) and $\lambda_t$ is the household's marginal utility of (real) income, that is, the Lagrange multiplier for equation (14). Since there are no financial frictions in this economy, the first-order condition for investment in the development of new goods $I_t$ is subsumed in the equilibrium relation (12) for innovating firms.

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5Household labor choices, under monopolistic competition and wage rigidity, will be added to the analysis in subsequent versions of the paper.

6The notation emphasizing the average per-share dividend from firms producing intermediate goods follows Cooley and Quadrini (1999).
2.5 Monetary Policy

Monetary policy consists of a constant, targeted rate of (steady-state) inflation $\pi$. This rate is to be consistent with equilibrium decisions of firms, so that in the (balanced growth) steady state of the economy, we have

$$\pi_t = \frac{P_t}{P_{t-1}} = \pi, \quad \forall t. \quad (17)$$

This monetary policy can be interpreted as implemented by a interest rate targeting rule or a money growth rule. Since we focus our analysis on the balanced-growth steady-state of the economy, either interpretation is valid.

7 To interpret monetary policy through a money-growth rule, we need to reserve a role for money in the model. This can be done, without changing any of our behavioral results, by reserving a role for money balances in utility, in a form separable relative to consumption.

3 Solution

3.1 Price dispersion

The extent of dispersion in relative prices present in the economy will be an important factor in understanding our results. Therefore, we define the following two measures of price dispersion:

$$\Delta_{pt} = \frac{1}{N_t} \int_0^{N_t} \left( \frac{P_{jt}}{P_t} \right)^{\frac{1}{\alpha - 1}} \text{d}j; \quad (18)$$

$$\tilde{\Delta}_{pt} = \frac{1}{N_t} \int_0^{N_t} \left( \frac{P_{jt}}{P_t} \right)^{\frac{\alpha}{\alpha - 1}} \text{d}j. \quad (19)$$

3.2 Equilibrium

The equilibrium to this economy consists of allocations and prices such that households, final-good producing firms and intermediate-good producing firms optimize, the monetary policy rule (17) is satisfied, and all markets clear.

We focus on cohort-symmetric equilibria in which all resetting, intermediate-good producing firms choose the same price $P^*_jt$ for the good they produce, so we can write $P^*_t$ to describe this choice. This implies that only $J$ different prices coexist in equilibrium at any time. It also implies that the firms within each price-setting cohort are characterized by identical demand for their product (so we can write $X_{\tau t}, \tau = 0, ..., J - 1$).

The market for intermediate goods clears when the demand arising from final good producers equals the quantity produced by each monopolist. Imposing this, using (5) and the definition of $\Delta_{pt}$ in (18) above allows us to compute the aggregate quantity of resource expanded in producing intermediate goods as
\[ X_t = \int_0^{N_t} x_{jt} \, dj = A^{1/\alpha} \alpha^{1/\alpha} L_t \Delta p_t N_t. \]  

(20)

Moreover, the total production of the final good must be allocated to consumption, to the production of intermediate goods, and to the development of new goods, as follows:

\[ Y_t = C_t + X_t + \eta (N_{t+1} - N_t). \]  

(21)

Finally, the quantity of shares held by households must equal the total number of firms (or varieties) in the economy:

\[ n_t = N_t. \]  

(22)

### 3.3 Data transformations

We conjecture that a balanced-growth path exists where aggregate output \( Y_t \), aggregate consumption \( C_t \), the real wage \( W_t/P_t \), and the aggregate quantity of intermediates produced \( X_t \) all grow at the same rate as the total number of intermediate-good varieties \( N_t \). To induce stationarity and allow us to compute a steady state, we thus define the following transformed variables:

\[ y_t \equiv \frac{Y_t}{N_t}; \quad c_t \equiv \frac{C_t}{N_t}; \quad \bar{w}_t \equiv \frac{W_t}{P_t} \frac{1}{N_t}; \quad x_t \equiv \frac{X_t}{N_t}; \quad g_t \equiv \frac{N_{t+1}}{N_t}; \]  

(23)

Moreover, the presence of positive trend inflation means that newly-set prices \( P_t^* \) are growing along the balanced-growth path. To induce stationarity and facilitate the computation of the deterministic steady state of the economy, we express the newly-set prices relative to the final good price index, as in

\[ p_t^* = \frac{P_t^*}{P_t}. \]  

(24)

These transformations allow us to rewrite the model’s equations as follows. Using (19) and equation (3), total output of the final good becomes

\[ y_t = A^{1/\alpha} \alpha^{1/\alpha} L_t \Delta_p N_t, \]  

(25)

whereas the input demand of final good producers in (4) and the aggregate quantity of intermediates used in the production process (20) become

\[ L_t = (1 - \alpha) \frac{y_t}{\bar{w}_t}, \]  

(26)

\[ x_t = A^{1/\alpha} \alpha^{1/\alpha} L_t \Delta_p N_t. \]  

(27)
Meanwhile the choice of $P_t^*$ expressed by (9) is rewritten as follows with stationary variables:

$$p_t^* = \frac{1}{\alpha} \sum_{\tau=0}^{J-1} \beta^\tau \lambda_{t+\tau} \left( \pi_{t,t+\tau} \right)^{\frac{1}{\alpha}} L_{t+\tau},$$  \hfill (28)

where $\pi_{t,t+\tau}$ denotes the gross increase of prices between periods $t$ and $t + \tau$ ($P_{t+\tau}/P_t$).

The two measures of relative-price dispersion in (18) and (19) can also be rewritten with stationary variables. Note that all along the interval $[0, N_t]$, firms are divided into price-setting cohorts. Cohort 0 just set its price (and chose $P_t^*$), cohort 1 chose its price 1 period ago and chose $P_{t-1}^*$, and so on until cohort $J - 1$, whose price is $P_{t-(J-1)}^*$. The price dispersion measure $\Delta_{pt}$ can thus be written as

$$\Delta_{pt} = \frac{1}{N_t} \left[ \int_0^{N_t/J} \left( \frac{P_t^*}{P_t} \right)^{\frac{1}{\alpha}} \, dj + \int_{N_t/J}^{2N_t/J} \left( \frac{P_{t-1}^*}{P_t} \right)^{\frac{1}{\alpha}} \, dj + \ldots + \int_{(J-1)N_t/J}^{N_t} \left( \frac{P_{t-(J-1)}^*}{P_t} \right)^{\frac{1}{\alpha}} \, dj \right] ,$$

$$\Delta_{pt} = \frac{1}{N_t} \left[ \frac{N_t}{J} \left( \frac{p_t^*}{\pi_{t,t}} \right)^{\frac{1}{\alpha}} + \frac{N_t}{J} \left( \frac{p_{t-1}^*}{\pi_{t-1,t}} \right)^{\frac{1}{\alpha}} + \ldots + \frac{N_t}{J} \left( \frac{p_{t-(J-1)}^*}{\pi_{t-(J-1),t}} \right)^{\frac{1}{\alpha}} \right] ,$$

$$\Delta_{pt} = \frac{1}{J} \sum_{\tau=0}^{J-1} \left( \frac{p_{t-\tau}^*}{\pi_{t-\tau,t}} \right)^{\frac{1}{\alpha}} ,$$  \hfill (29)

whereas an equivalent transformation yield the following for the measure of dispersion $\tilde{\Delta}_{pt}$:

$$\tilde{\Delta}_{pt} = \frac{1}{J} \sum_{\tau=0}^{J-1} \left( \frac{p_{t-\tau}^*}{\pi_{t-\tau,t}} \right)^{\frac{1}{\alpha}} .$$  \hfill (30)

The monopoly profits obtained by a firm during a $J$-period price-contract, expressed by (10), becomes the following in terms of stationary variables:

$$\Pi_t = (\alpha A)^{\frac{1}{\alpha}} \sum_{\tau=0}^{J-1} (\beta^\tau \lambda_{t+\tau}/\lambda_t) \left( \frac{p_{t-\tau}^*}{\pi_{t,t+\tau}} - 1 \right) \left( \frac{p_{t-\tau}^*}{\pi_{t-\tau,t}} \right)^{\frac{1}{\alpha}} L_{t+\tau} .$$  \hfill (31)

From the household problem, the first-order conditions (15) and (16) become

$$\bar{\lambda}_t = c_t^{-1} ;$$  \hfill (32)

$$\bar{\lambda}_t = \beta R_t \left[ \frac{\lambda_{t+1}}{p_{t+1}} \right] .$$  \hfill (33)

Finally, the resource constraint in (21) becomes

$$y_t = c_t + x_t + \eta (g_t - 1) .$$  \hfill (34)
3.4 Steady state

At the steady state, equation (33) implies that \( \bar{\lambda} = \beta \frac{R}{g} \bar{\lambda}/\pi \), or \( R = g \pi/\beta \). Further, this implies that the factor discounting future dividends, \( \beta^k \lambda_{t+k}/\lambda_t \), is equal to \( (\beta/g)^k \). This implies that the price-setting choice in (28) becomes:

\[
p^* = \frac{1}{\alpha} \sum_{\tau=0}^{J-1} (\beta/g)^\tau \pi^{1-\alpha} \sum_{\tau=0}^{J-1} (\beta/g)^\tau \pi^{1-\alpha} \cdot \tag{35}
\]

One interesting feature of (35) is that under zero trend inflation \( (\pi = 1) \), the expression simplifies to become

\[
p^* = \frac{1}{\alpha} \cdot \tag{36}
\]

which matches the expression in the flexible price economy of Barro and Sala-i-Martin (2004). In such an economy, \( p^* \) is independent of the growth rate \( g \). In the more general case with \( \pi > 1 \), (35) shows how the choice for \( p^* \) depends on trend inflation \( \pi \) and on the growth rate of the economy \( g \).

Next, the free-entry condition in the innovation sector (12) becomes the following in steady-state:

\[
\eta = \left( \frac{1}{1 - (\beta/g)} \right) A^{1-\alpha} \alpha^{1-\pi} L \sum_{\tau=0}^{J-1} (\beta/g)^\tau (p^*/\pi^{\tau-1} - 1) (p^*/\pi^{\tau-1})^{1-\alpha} \cdot \tag{37}
\]

Once again, notice that in the zero inflation economy, this expression simplifies greatly: using (36) and simple algebra leads to

\[
\eta = \frac{1-\alpha}{\alpha} A^{1-\alpha} L \alpha^{2-\alpha} \frac{1}{1 - \beta/g} \cdot \tag{38}
\]

which matches the equivalent equation in Barro and Sala-i-Martin (2004) and solves for the steady-state growth of the economy \( g \) without influence from the inflation rate \( \pi \).

In the general case with \( \pi > 1 \), (35) and (37) jointly determine the growth rate of the economy \( g \) and the price chosen by resetting firms \( p^* \) as a function of the model’s parameters and of the rate of steady-state inflation \( \pi \). The crucial feature of the link between inflation and growth is thus found in analyzing these two equations. Expression (37) shows that the price chosen by a resetting firm will see its real value eroded by inflation, affecting profits derived from producing this good. Since the real resource cost of developing new goods is fixed, this effect might decrease the growth rate of the economy. In turn, (35) shows how forward-looking firms, knowing this erosion in the relative price of their good will occur, modify their choice when resetting comes; further, the expression illustrates that the growth rate of the economy, by affecting the valuation of dividends obtained in the future, will affect this choice. Below, we calibrate the model and explore the quantitative relevance of this interaction between inflation, growth, and the price-setting choices of firms.

Once the growth rate and the choice of price-setting firms has been determined, equations (25), (26), (29), (30), (27), and (34) evaluated at steady state can be used to find solutions for \( y, \bar{w}, \Delta \).

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8Because Barro and Sala-i-Martin (2004) develop their model using continuous time and we use log utility, the exact formulations of the equations differ slightly. See eq. 6.24, page 296, Barro and Sala-i-Martin (2004).
\( \Delta_p, x \) and \( c \), as follows:

\[
y = A \frac{1}{1-\alpha} \alpha \frac{1}{1-\alpha} L \Delta_p; \tag{39}
\]

\[
\bar{w} = (1 - \alpha) \frac{y}{L}; \tag{40}
\]

\[
\Delta_p = \frac{1}{J} \sum_{\tau=0}^{J-1} \pi \tau^\alpha; \tag{41}
\]

\[
\bar{\Delta}_p = \frac{1}{J} \sum_{\tau=0}^{J-1} \pi \tau^\alpha; \tag{42}
\]

\[
x = A \frac{1}{1-\alpha} \alpha \frac{1}{1-\alpha} L \Delta_p \tag{43}
\]

\[
c = y - x - \eta(g - 1). \tag{44}
\]

Note from (39) that if trend inflation increases price distortion significantly, the increase in \( \bar{\Delta}_p \) will mean that inefficiencies in production reduce the steady-state output of the economy \( y \) for given values of other parameters. Further, (43) shows that a similar effect results in inefficiencies in producing intermediates. Turning to (44), one can see that competing effects will make the final effect on consumption uncertain: on the one hand, the inefficiencies created by price distortion reduce the aggregate quantity of output available for consumption, but a possible negative effect on growth actually increases available resources. A quantitative evaluation of the mechanism is therefore necessary to evaluate the net effects on consumption and thus on welfare.\(^9\)

### 3.5 Calibration

In order to compute the steady state of the economy, numerical values are assigned to parameters. On the production side, the requirement that the elasticity of substitution between different individual intermediate goods be equal to one over the labour share in production \( \epsilon = \frac{1}{1-\alpha} \) does not accord well with standard calibration procedures. Following Basu (1996) and Basu and Fernald (1997), the former is often calibrated to be 11, whereas the latter is one order of magnitude lower, around 1.5. In this version of the paper, we keep \( \epsilon = 11 \). Future versions of the paper will keep the two concepts separated, as in Benhabib et al. (1994). The length of price contracts \( J \) is set to 2, based on results reported in Bils and Klenow (2004). The innovation cost \( \eta \) is set such that the zero-inflation version of the economy exhibits an annualized rate of output growth of 2%. Finally, the discount factor \( \beta \) is such that the real interest rate in the economy along the balanced growth path is 5%.

\(^9\)As an example, Gomme (1993), using a different model of endogenous growth, found that trend inflation did indeed have an important negative effect on growth. However, the lower requirements for resources directed towards growth meant that consumption did not decrease significantly, so that the welfare implications of trend inflation were small.
4 Results

4.1 Trend inflation, price dispersion, and monopoly profits

Firms in this economy make constrained choices. Although they would like to re-optimize every period, the price they choose today will prevail for the length of the contract and they are thus unable to achieve their desired markups every period.

In an environment with positive trend inflation, the suboptimality of prices will have a specific pattern. In the early periods of the contract, relative prices will tend to be higher, resulting in lower demand but higher markup. As time progresses, the relative price will decrease, so that in the later stages of a contract, the relative price will be low, delivering greater demand but lower per unit markup.

To develop intuition about the effects that this erosion of relative prices has on monopoly profits and growth, Figure 1 plots (static) profits for a given intermediate-good producer from our model.\textsuperscript{10} The figure highlights a noticeable asymmetry around the optimal choice, which plays a key part in linking trend inflation to growth. The asymmetry arises from our assumption regarding the goods aggregator. In particular, we use an aggregator developed by Dixit and Stiglitz (1977) that admits non-linear demand curves for goods and, as an artifact, asymmetry in the profit functions. The degree of asymmetry is then controlled by the calibration. Owing to their inability to reoptimize, firms and households find themselves at different points along these profit functions over the course of their contracts: towards the right of the picture at earlier stages (with a high relative price) and, in later stages of the contract, to the left of the curve. The shape of the profit function thus determines pricing behaviour. Lower relative prices lead to high levels of demand for their good, but rapidly decreasing profits (left side of Figure 1). To avoid these periods of low relative prices and high demand for their output, firms attempt to increase their markup and displace the range of relative prices over the course of their contract shifts to the right of the optimum.\textsuperscript{11}

Since the mechanism described above (and depicted in Figure 1) depends on trend inflation eroding the relative value of the prices set by firms, one expects that it will become quantitatively more important as the rate of trend inflation increases. Figure 2 shows that this intuition is correct. The Figure graphs the relative price chosen by price setters $p^*$ as a function of trend inflation.\textsuperscript{12} This effect is quantitatively modest, but noticeable: the choice for $p^*$ increases by almost 1% as the economy moves from zero annual inflation to a still modest rate of 5%.

Next, Figure 3 shows that this effect of trend inflation translates into lower monopoly profits. The Figure evaluates the real profits in (10), obtained during a given price-contract cycle, for different rates of trend inflation.\textsuperscript{13} It shows that, as trend inflation increases and the asymmetry problem increases, the resulting high relative prices chosen by price-setters and the greater dispersion in relative prices during each price contract reduce profits. The fact that monopoly profits are lower in high trend-inflation economies means that, all things equal, the incentive to innovate is weaker.

\textsuperscript{10}More precisely, the figure depicts the objective functions in a one-period, unconstrained maximization problem for a monopolist whose demand and cost structures are similar to those facing firms in our economy.

\textsuperscript{11}See Amano et al. (2007) for a complete discussion of this asymmetry.

\textsuperscript{12}For ease of comparison, the value of $p^*$ arising at zero inflation as been scaled to 1.

\textsuperscript{13}The value of these profits at zero inflation have been scaled to 1 to facilitate comparisons.
4.2 Growth effects of trend inflation

Innovators must weight the benefit of developing a new good (the monopoly profits derived from exclusive rights to produce it) with the developing costs, represented by the parameter $\eta$. The discussion above has shown that, as trend inflation increases, monopoly profits decreases. One therefore expects that, as the profits from innovation decrease while the costs stay constant, the equilibrium growth rate itself will decrease.

Figure 4 verifies that this intuition is correct in our model. The figure depicts the growth rate of the economy (measured as a net, annualized rate) as a function of trend inflation. From a rate of 2% at the zero-inflation benchmark, the rate of growth decreases slightly as inflation grows. At the maximum rate considered, a rate of 5% inflation rate, the rate of growth of the economy is now 1.99%.

As forcefully argued in Lucas (1987), even small changes in growth can produce big welfare changes. Appendix A, detailing how we compute the welfare costs of inflation, shows that any change in growth is multiplied by a factor of roughly 100. In this context, it is not surprising that, even though the effect of trend inflation on growth is modest, Figure 5 shows that its welfare consequences are important. The figure reports the welfare costs of positive trend inflation, measured as the consumption equivalent that would make agents indifferent between the zero-inflation benchmark economy and the one with positive trend inflation. The figure illustrates that even small growth effects can have important welfare consequences, because these effects are capitalized over the infinite future. Specifically, even modest rates of trend inflation, say 5% annualized, are shown to have welfare costs equivalent to close to 0.5% of consumption. This figure is in the same order of magnitude of those arrived at by emphasizing the link between monetary distortions and growth (e.g. Gomme, 1993; Dotsey and Ireland, 1996; Love and Wen, 1999).

5 Conclusion

This paper combines a New Keynesian model with nominal rigidities and a monopolistically competitive production structure, with the endogenous growth model of Romer (1990). As trend inflation increases, the nominal rigidities imply that relative prices are increasingly dispersed and productive inefficiencies worsen. This reduces monopoly profits and decrease the incentive to innovate and create new goods, the main engine of long term growth in the Romer (1990) model. This link between inflation and growth, although modest, has important welfare consequences. As such the paper sheds light on a mechanism by which inflation imposes costs on an economy which has largely been ignored so far.

The model used in this version of the paper is meant to illustrate the mechanism, rather than provide a definitive quantitative estimate of its strength. Accordingly, future versions of the paper will explore this mechanism in more complete versions of the New Keynesian model, namely with wage rigidities and complementarities among varieties.
Literature cited


A Welfare Computations

Compute the expression for expected lifetime utility of the representative household in an economy with zero inflation:

\[ U = \sum_{t=0}^{\infty} \beta^t \log C_t^{\pi_0}. \]

(45)

This expression can be rewritten as the following once it is evaluated at the steady state:

\[ U = \sum_{t=0}^{\infty} \beta^t \log \left[ \gamma^{0} \pi_0^t c_t^{\pi_0} \right]. \]

where \( \gamma^{\pi_0} \) is the growth rate of the economy under zero inflation and \( c_t^{\pi_0} \) the (detrended) consumption in this economy.

Consider now computing lifetime utility for a non-zero positive rate of inflation \( \pi \), in which consumption occurring in that economy has been increased by a factor of \( x \):

\[ U = \sum_{t=0}^{\infty} \beta^t \log \left[ (1 + x) \gamma^{\pi} \pi c_t^{\pi} \right]. \]

where the subscript \( \pi \) indicates the dependence of that variable on the rate of trend inflation. The consumption equivalent measure of welfare changes between the two economies is simply the value of \( x \) which makes the two welfare measure equal. In other words, \( x \) represents the percentage increase in consumption which would make households indifferent between living in the (steady-state) economy with inflation \( \pi \) and the economy with the benchmark economy with zero inflation. Considering the two expressions, we can solve for \( x \) as follows:

\[ \log(1 + x) = \log(e^{\gamma^{\pi_0}/c^{\pi}}) + \beta/(1 - \beta) \log(\gamma^{0}/\gamma^{\pi}). \]

This expression makes clear that any changes in the growth rate between the two economies is multiplied by a factor of roughly 100 in the welfare computations.
Figure 1: Asymmetry in monopoly profits vs. relative price
Figure 2: Optimal choice of relative price vs. rate of trend inflation
Figure 3: Monopoly profits vs. rate of trend inflation

Profits during a Price–Contract Cycle
Figure 4: Growth rate vs. rate of trend inflation
Figure 5: Welfare costs of non-zero trend inflation