The Gold Standard and the French Great Depression

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February 14, 2010

Abstract

The analysis of the Great Depression through the lens of the gold standard theory has been mainly conducted from an international comparative perspective. As far as I know, there is no structural model-based studies of the link between the malfunctioning of the gold standard and the Great Depression. A such analysis would be helpful to assess the role of gold standard in the worldwide depression. Here, I intend to take step into this direction. I develop a gold standard DSGE model which aims to emphasize the working of the gold standard and the link of the latter with the real economy. The fluctuations of the variables of the artificial economy are driven by one real and three nominal shocks. The real shock is assumed to be a productivity shock in the final good sector. The nominal shocks are related to the working of the gold standard: a gold flow shock, a gold backing shock, and a money multiplier shock. The two lastest nominal shocks alter the ratio of the currency value of the monetary gold stock to the nominal money stock as in Bernanke (1995, Journal of Money, Credit, and Banking). I, then, analyze the steady state and dynamic properties of the model in the context of the French Great Depression.

Keywords: Gold Standard, Business Cycle model, Great Depression.

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1 Introduction

The Great Depression of the 1930s is seen as a vivid lesson for the ongoing global financial crisis. The causes and the transmission mechanisms of the Great Depression have attracted the interest of a considerable number of macroeconomists. For recent years, there is a body of literature, such as Cole and Ohanian (1999, 2004), Cole, Ohanian and Leung (2005), Bordo, Erecog and Evans (2000), and, Christiano, Motto and Rostagno (2003), assessing the ability of Dynamic Stochastic General Equilibrium (hereafter DSGE) models to replicate the economic crisis in the 1930s. However, few of them have shed light on the international dimension of the Great Slump, but as Eichengreen and Sachs (1985), Eichengreen (1992, 2004), Bernanke and James (1991), Bernanke and Carey (1996), and Bernanke (1995) argue, the worldwide nature of the economic crisis should not be ignored. It is crucial to analyze the Great Depression as a global phenomenon and not as a country specific one to gain a deeper understanding. According to those authors, the Depression was triggered off by a worldwide monetary contraction. The worldwide collapse of money supplies resulted from an inefficient management of the international monetary system that was at work during the interwar period, i.e. the gold standard. The current work intends to introduce the gold standard-based explanation of the worldwide Great Depression into the new body of literature. Basically, I propose a simple DSGE model economy in which the monetary policy is conducted consistently to the gold standard monetary system. In order to analyze the properties of the gold standard model, I take France as a case study. Thus, the current work aims to be a first step into the use of DSGE models as an evaluation tool of the gold standard-based explanation of the Great Depression.

Basically, the gold standard monetary system can be defined as follows. Countries which were committed to the gold standard, were supposed to define the prices of their domestic currencies in terms of a fixed weight of gold (Bordo 1981; Bordo et al. 2007). In other words, countries on the gold standard defined a fixed price of gold in terms of their domestic currencies. For example, during the interwar gold standard, the franc poincaré weighted 65 milligrams of gold 0.900 fine\(^1\), or equivalently 1 grams of gold fine was worth 16.92 francs poincaré\(^2\). In order to hold fixed the currency prices of gold, countries on the gold standard — actually their central banks — are assumed to stand ready to sell or purchase any quantity gold demanded or supplied at that price. The gold standard was an impure commodity money system\(^3\). By definition, in an impure commodity money system, the medium of exchange consists not only of the commodity money (e.g. gold coins) but also of claims to units of the commodity money (e.g. fiat money and fiduciary money). Each unit of those claims can be exchanged against gold at the fixed currency price of gold if desired. Countries on the gold standard substituted fiat and fiduciary money for gold coins as medium of exchange because the size of exchanges of goods and services had become high and because the production of gold was demanding in terms of resources and henceforth costly (Bordo 1981). The currency value of the additional instruments of payment — fiat money and fiduciary money — could exceed the currency value of gold. Nevertheless, the ratio of the fiat and fiduciary monies to gold should remain stable over time in order to ensure that the maintenance of the gold standard (Bordo 1981). Since the gold standard was an international monetary system, gold was allowed to move freely from one country to another. More specifically, countries on the gold standard were forbidden to prevent gold from flowing away.

The central banks of the countries involved in the gold standard, where they existed, were supposed to conduct their monetary policies in agreement with the so called rules of the game in order to ensure the continued good management of the gold standard. In particular, a central banks had to use its nominal interest rate so that the effect of the ongoing gold flow — in or out of the country — on the domestic money supply was strengthened. For example, the central bank of a country experiencing a gold outflow (inflow) had to raise (lower) its nominal interest rate so that the total money supply was further reduced (increased). By raising (lowering) the nominal interest rate, the central bank led the commercial banks to decline (raise) their credit supply (that is fiduciary money supply).

The gold standard monetary system took two forms during the last two centuries — the 19\(^{th}\) and 20\(^{th}\) centuries. The first form of the gold standard, called the classical gold standard, was the one

\(^1\)See the article 2 of the French monetary law of June 25, 1928. This monetary law has been also published in the Federal Reserve Bulletin of August 1928 (see Federal Reserve Board 1928).

\(^2\)The movements in the French price of gold during the interwar period are reported in Annuaire Statistique (1966).

\(^3\)This expression is borrowed from McCallum (1989, p 250).
described above. It operated over the 1821-1914 period, that is from the end of the Napoleonic Wars to the onset of the World War 1. In the beginning of that period only few countries were on the classical gold standard, the main country being England. The classical gold standard were outspread to a large part of the world between 1880 and 1914 (Bordo 1981; Bordo et al. 2007). The classical gold standard were suspended at the beginning of the World War 1 in order to allow countries involved into the worldwide military conflict to face the war related expenditures. The gold standard were restored during the interwar period under a new form following the recommendations of the Genoa Conference that took place in 1922 (Nurkse 1944). The main recommendation stated that the money issued by central banks could be backed not only by gold — as it was during the classical gold standard period — but also by foreign exchanges. Those foreign exchanges — specifically sterling and dollars — were convertible into gold. Thus the legal reserves of the central banks consisted of gold and foreign exchange4. By defining the central banks reserves in terms of both gold and foreign exchange rather than in terms of only gold, the participants of the Genoa Conference expected to be able to deal with a possible shortage of the world gold stock, in the future. The shortage of world gold stock might be caused by a reduction in the world gold production or an increase of the world money demand (Nurkse 1944). The gold standard that took place during the interwar period was called the gold exchange standard.

Several authors, such as Choudhri and Kochin (1980), Eichengreen and Sachs (1985), Bernanke and James (1991), and Bernanke (1995), provided some evidences that the internation Great Depression took its roots in the gold standard. Choudhri and Kochin (1980) showed that four small European economies — Belgium, Italy, Netherlands and Poland — which were committed to the gold standard, experienced large drops in output and prices between 1928 and 1932. Instead, Spain, which remained outside the gold standard during all the interwar period, was almost not affected by the economic crisis as its european neighbors; in 1932, the Spanish output and prices were close to their respective 1928 level. In addition, Choudhri and Kochin (1980) showed that three Scandinavian countries — Denmark, Finland and Norway — which were in the gold standard in the beginning the Great Depression then moved out5 from it 1931 along with the United Kingdom, experienced less severe collapses in output and prices than Belgium, Italy, Netherland and Poland did during the 1928-1932 period. Eichengreen and Sachs (1985), using a sample of ten European countries — Belgium, Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Sweden, and United Kingdom —, showed that the countries which had left the gold standard got their industrial productions back to their 1929 levels quickly after they had depreciated their currencies. For example, the industrial productions in Finland, Denmark, United Kingdom, Norway and Sweden — those countries abandoned the gold standard in 1931 — were well above their 1929 levels in 1935. In constrast, countries which were still in the gold standard in the end of 1935 — France and Netherlands which abandoned the gold standard in 1936 — had their industrial production below their 1929 levels in 1935. Similarly, Bernanke and James (1991), using a sample of 24 countries, found that on average, the industrial production growth rates were higher in countries which had left the gold standard in 1931 during the 1932-1935 period than those in countries which were still in the gold standard in 1935. Additionally, Bernanke and James (1991) showed that all the countries which were in the gold standard on the onset of the Great Depression suffered from a severe deflation in the early 1930s. In 1933, the deflationary movement of prices slowed down or stopped in countries which moved away from the gold standard in 1931. Afterwards, the price level slightly increased in those countries. On the contrary, the price levels continued to drop after 1933 in countries which remained committed to the gold standard until 1936. Bernanke and James (1991) also compared the money supply behaviors of the two groups of countries — the countries which abandoned the gold standard in 1931 on the one hand, the countries which remained in the gold standard until 1936 on the other hand — between 1930 and 1935. Before 1932, the two groups of countries experienced contractions in money supplies. The countries which left the gold standard in 1931, started to raise their money supply in 1933. The money supplies decreased during all the 1930-1935 period in countries which remained in the gold standard until 1936. From this result, Bernanke and James (1991) pointed out that the commitment to the gold standard made it difficult for a country to raise its money supply and henceforth to restrain the deflationary movement of prices. In other words, by abandoning the gold standard, a country

4The United Kingdom and the United States hold their reserves only in gold during the interwar period.
5In general in this literature, represented here by Choudhri and Kochin (1980), Eichengreen and Sachs (1985), Bernanke and James (1991), and Bernanke (1995), a country is considered as abandoning the gold standard if either it devalues its currency, impose exchange controls or officially abandons the gold parity of its currency.
could more easily conduct an expansionary monetary policy\(^6\). These evidences led those authors to put forward the idea that several monetary shocks, transmitted from one country to another through the gold standard monetary system, had caused prices and output to collapse in countries on the gold standard.

As stressed by Bernanke and Mihov (2000), the proponents of the gold standard-based explanation of the Great Depression have suggested several sources of the collapse of national money supplies during the 1930s: contractionary monetary policy through the sterilization of gold — offsetting the effect of gold inflow on the money supply — inefficiency of the distribution or maldistribution of the world gold among the countries involved in the gold standard; the conversion of foreign exchange into gold by the central banks; the collapse of the money multiplier caused by successive banking crises; and the scarcity of gold at the world level. The three first factors of the collapse of the money supplies mainly resulted from the inefficiency or absence of management of the gold standard during the interwar period\(^7\). Bernanke (1995) and, Bernanke and Mihov (2000) suggest to decompose, for several countries, the identity linking the money supply (specifically the monetary aggregate M1) of a country committed to the gold standard to the gold stock held by the central bank of that country into several ratios in order to identify quantitatively the main contributors of the collapse of money supplies among the contractionary mechanisms proposed above. These ratios are assumed to reflect the banking crises (through the money multiplier ratio or the ratio of M1 to monetary base), the sterilization of gold inflow (through the gold backing ratio or the ratio of international reserves — gold plus foreign exchange — to monetary base), and the conversion of foreign exchange (through the ratio of the international reserves to the gold stock). From those studies, it result that initially, period prior to 1931, the money supplies have fallen mainly because the countries which have attracted massively gold — mainly the United States, Germany and France — have sterilized those gold inflows — reflected by an increase of the gold backing ratio: the countries from where gold have flown away toward the United States, Germany and France have no choice than to restrict their money supplies in order to keep safe their respective parity to gold. For the period post 1931, the banking crises have been the main culprit of the collapse of the money supplies — reflected by a decrease of the money multiplier\(^8\). Bernanke and Mihov (2000) have also shown that the maldistribution of the world gold stock has worsened after 1931, becoming an important factor of the monetary contractions\(^9\).

Nonetheless, the presented evidences of a possible link between the gold standard and the drop of domestic money supplies leave open the question of the channel through which the monetary contractions have led domestic outputs to fall during the 1930s. Most of proponents of the gold standard-base explanation of the Great Depression claim that the fall in the global money supply has been transmitted to the real part of the world economy, mainly through nominal wage stickiness. Basically, the sticky wage story of the Great Depression can be summarized as follows. Consequently to a monetary contraction, prices go down. Production costs should decrease also in order to maintain the level of production. Thus, when nominal wages do not respond quickly to the decrease in prices (due to labor market imperfections), firms lay off and produce less. This results in an increase in unemployment and a drop in output. Eichengreen and Sachs (1985) and Bernanke and Carey (1996) have provided some evidences for the sticky wage story. Eichengreen and Sachs (1985), using a sample of 10 European countries as said above, have shown that in 1935, countries which have left the gold standard earlier have displayed high industrial production growth rates and low real

\(^6\)Bordo, Choudhri and Schwartz (2002) claim that the proposition stating that the commitment to the gold standard makes it difficult for a country to conduct an expansionary monetary policy during the Great Depression holds only for small open economies with little gold reserves. They claim that the U.S., which was a large economy and detained large gold reserves, could have raised its money supply before 1933 without putting in danger its commitment to the gold standard.

\(^7\)If during the prewar period, the gold standard system was well managed, this was not the case during the interwar period. England was the main financial place before World War 1, a position which was weakened after the war. By consequence, after the majority of developed countries was back on the gold standard, the position of financial leader was free but no country was ready to fill it. Moreover, because of the war related questions of reparations and debts, the relationships between countries, such as France and Germany, were difficult.

\(^8\)Those findings are also pointed out by Hamilton (1987).

\(^9\)Bernanke and Mihov (2000) define an index of the maldistribution of gold as being “approximately the covariance between the country economic weights and log gold shares.” The economic weight of a country corresponds to the ratio of its current GDP to its 1928 GDP.
wages comparatively to the levels that those two variables have taken in 1929. In contrast, in 1935, countries which have remained committed to the gold standard until its full dissolution in 1936, such as France, have low industrial production growth rates and high real wages in comparison with the levels that those two variables have taken in 1929. Bernanke and Carey (1996), using a sample of 22 countries, have found similar results to those of Eichengreen and Sachs (1985). In addition, Bernanke and Carey (1996) have reported that in countries where industrial productions have strongly fallen over the 1931-1936 period — countries which have been on the gold standard until the break down of that monetary system — the nominal wages have adjusted slowly to movements in prices. The debt-deflation hypothesis of Fisher (1933) is also considered as a possible channel of transmission of the monetary contractions on real economies. The debt-deflation story can be stated as follows. The collapse of money supplies cause prices to decline. The deflation raises the real value of the debts contracted by firms. It results that the net worth positions of those firms becomes so weak that they cannot get access to new credits. Worst, the increase of the real value of their debts might lead them to lay off or go bankrupt. It follows that investment and employment decline. So does output (Bernanke and James 1991; Bernanke 1995).

The analysis of the gold standard-based explanation of the Great Depression has been mostly treated in a comparative international perspective10. In particular, the gold standard approach of the international Great Depression has received little theoretical attention. Of course, several models about the working of the classical gold standard have been developed and analyzed, such as those of Barro (1979), Baxley and Summers (1988), Goodfriend (1988)11. Barro (1979) develops a dynamic model to emphasize how the price level is determined under the gold standard. Barro (1979) recalls that under the gold standard the money supply is defined upon the existing quantity of gold and the prices of the latter is set fixed by the central bank (or any monetary authority). In other words, gold is a commodity money which price is maintained fixed12. It follows that the determination of the price level — price of other goods than gold — is equivalent to the determination of the relative price of the commodity money — the ratio of the fixed price of gold to the price level. Hence, the relative price of gold is determined by the equilibrium on the gold market, which in turn corresponds to the equality between the total demand for gold stock — gold demand for monetary and non-monetary uses — and the supply of gold stock. The model of Barro (1979) presents the following features: the gold standard model economy is defined as a close economy; the real part of the gold standard economy, represented by the output (or real income), is treated as exogenous; gold is not only held for monetary purpose but also for non-monetary uses; the expected inflation rate is assumed to be nil in the steady state; only the non-monetary gold stock depreciates over time (at a constant rate); and the supply of new gold is defined as a negative function of the relative price of gold. Barro (1979) uses its model to analyze the effects of a technical progress in the gold sector, a change in output — which can be interpreted as a change of productivity in the output sector —, and a change in the ratio of monetary gold stock to nominal money stock — considered as the gold backing ratio in the model — on the price level and gold stocks. The analysis is conducted with the use of a phase diagram. An increase of the productivity in the gold production — such as a discovery of a new gold source — leads the price level, monetary gold stock and non-monetary gold stock to increase. In other words, following an increase of gold production, the gold standard economy moves to a new steady state where the levels of price level and gold stocks are higher. Barro (1979) shows that an increase in output leads the gold standard economy towards a new steady state characterized by a lower price level and higher non-monetary gold stock. However, the net effect of an increase of output on the monetary gold stock is generally ambiguous. From this, Barro (1979) argues that since technical progresses are expected to materialized more often in the output sector than in the gold sector, deflationary episodes are likely to happen under the gold standard monetary system. Then, Barro (1979) evaluates whether lowering the gold backing ratio can dampen the deflationary forces of the gold standard13. Barro (1979) finds that even though the decrease of the gold backing ratio leads the price level to increase, this effect is only transitory. Indeed, the decline of the gold backing ratio leads the gold standard economy to new steady state where only the level of monetary gold has

10The expression is borrowed from Bernanke and Carey (1996).
11There are also the models of Dowd and Sampson (1993) and Chappell and Dowd (1997).
12For a discussion on the commodity money of see Friedman (1951).
13According to Barro (1979), this evaluation is based on the idea that the creation of paper gold, representing gold promises, would allow shield countries committed to the gold standard against the deflationary forces of the gold standard.
changed — it takes a lower value. From this, Barro (1979) concludes that lowering the gold backing ratio cannot weaken the deflationary forces of the gold standard.

Bardsky and Summers (1988) provides some evidence that the Gibson paradox — the positive correlation between the price level and nominal interest rates — can be observed over the period during which the gold standard monetary system has prevailed. Hence, those authors develop a gold standard model in order to demonstrate that the Gibson paradox is closely related to the working of the gold standard. The model of Bardsky and Summers (1988) is similar to the one developed by Barro (1979). However, while the real money demand depends on the expected inflation in the Barro (1979)'s model, the real money demand is function of the nominal interest rate in the Bardsky and Summers (1988)'s model.

Goodfriend (1988) uses an intertemporal, rational expectations, asset pricing model to study the monetary policies conducted by central banks under the classical gold standard. Goodfriend (1988) bases its gold standard analysis on an asset pricing model because it provides a rational expectation-based framework for the private valuation of money and gold stocks, that is the pricing of those two assets. Goodfriend (1988) analyzes how a central bank can affect the pricing of money and gold stocks behaviors, through money and gold policy rules, so that the money price of gold is maintained fixed. The Goodfriend (1988)'s model represents a closed economy which consists of two types of agents: an infinitely-lived representative household and a central bank (or a government). In this economy there are four assets: capital stock, money stock, gold stock and bond stock. The household gets satisfaction from consuming goods (other than gold and capital) and holding real money and gold stock for non-monetary purposes. The household chooses sequences for consumption and assets so that it maximizes its expected intertemporal utility function subject to its budget constraint. The production of consumption good is treated as exogenous in the model. Indeed, there is no labor in the economy and the capital stock is assumed to be fixed to unity in the equilibrium. Hence, the production of the consumption good is driven by an exogenous stochastic productivity shock. The economy is endowed once for all with a fixed stock of gold. Each period, this total gold stock is exchanged between the household and the central bank. The latter holds gold for monetary purposes. The central bank is assumed to conduct two policies in terms of gold and money: a pure gold policy which consists of buying/selling gold from the household and a pure monetary policy which consists of supply or destruction of money. Both policies are financially supported through the purchases or sales of bonds to the household. Goodfriend (1988) shows formally how the central bank can use either of its two policies to ensure the fixity of the money price of gold, which in turn is defined as the ratio of the real price of gold — that is consumption price of gold — to real price of money — that is consumption price money or inverse of the price level. This author demonstrates that if the central bank commits itself not only to ensure the fixity of the money price of gold but also to keep the ratio of monetary gold to money constant over time, it cannot additionally conduct policies affecting the price level or the nominal interest rate behavior in the economy. However, according to Goodfriend (1988), the central bank can both satisfy the gold standard rule on the fixity of the money price of gold and influences the behavior of the price level or the one of the nominal interest rate if it relaxes temporally the constraint on the monetary gold-money ratio — the monetary gold-money ratio is allowed to fluctuate around a narrowed range.

As it can be noted, those models offer a good description of the working of the gold standard in terms of money, price and interest rate. However, the implications of the working of the gold standard on the real part of the economy are not well emphasized. Thus, they are not very useful if one wants to analyze the Great Depression through the lens of the gold standard theory. More recently, Bordo et al. (2007) have developed a DSGE model representing a closed gold standard economy. In this model, the final good production sector and the gold production sector are both explicitly defined. The productions of the final good and gold require the use of labor and capital as inputs. The produced gold stock can be held for three different purposes in this economy: utility purpose — holding gold yields utility to the household —, monetary purpose, and production purpose — gold is used as capital in both final good and gold sectors. In this economy, the household needs money

\[ \text{Basically, the central bank stands ready to exchange gold for money or money for gold at a fixed money price of gold.} \]

\[ \text{The price level and nominal interest rate policies considered in Goodfriend (1988) are price level smoothing and nominal interest rate smoothing policies.} \]

\[ \text{Bordo et al. (2007) use a DSGE model to study and compare the dynamic behaviors of the price level under different monetary systems — for example the gold standard monetary system and flat monetary system.} \]
because it allows to decrease the time spent into transaction activities. The economy includes also a central bank which controls the money supply through an interest rate rule. Still, this model seems not suitable for a formal analysis of the gold standard-based explanation of the Great Depression. The Bordo et al. (2007) model seems not to explicit the link between the monetary gold stock and the money supply as for example in Barro (1979) and Goodfriend (1988)\textsuperscript{17}. Hence, it would be difficult to reconcile this model with the works of Bernanke and its coauthors.

Accordingly to the above discussion on gold standard models, I propose to develop a new DSGE model which would emphasize the working of the gold standard and its link to the real economy. I essentially rely on the works of barro (1979) and Goodfriend (1988) when I define the nominal part of the gold standard artificial economy. The model is basically a monetary business cycle model with a commodity money. In particular, it incorporates the following features. The gold standard model represents a close economy in which there are five competitive markets — final good market, labor market, gold market, money market and bond market — and four assets — gold, money, bonds, and physical capital. The gold standard artificial economy is populated by three types of agents: households, final good producers, and a central bank. The supply of new gold is treated as exogenous and in terms of flows rather than in terms of production. Note that the supply of new gold can be either positive (inflow of gold) or negative (outflow of gold). In the economy, gold has two functions: the households gets satisfaction from holding money (non-monetary gold stock) and the central bank bases its money supply decision on the value of its gold stock (monetary gold stock). The central bank is committed to purchase of sell any quantity of gold in exchange of money that households asks for at a fixed price: the currency price of gold is assumed to be constant over time. Households hold money in order to reduce the transactions costs. Indeed, those transactions costs introduce a distortive wedge in the optimal decisions of the households. In particular, the transaction cost wedge affects the real part of economy as a negative preference shock to consumption. The physical capital stock and the non-monetary gold stock are assumed to depreciate over time at constant rates. In the steady state the inflation rate is assumed to be zero so that there are no growths in the gold and money stocks. The artificial gold standard economy can be perturbed through two exogenous stochastic shocks: the productivity shock in the final good sector and the gold flow shock. In addition, the central bank can affect the state of the economy through two monetary policies. Specifically, the monetary authority can modify the gold-money ratio through the money multiplier and the gold backing ratio (defined as in Bernanke 1995 and Bernanke and Mihov 2000)\textsuperscript{18}. This gold standard model includes two channels of transmission through which the perturbations in any nominal shock — gold flow shock, gold backing shock, and money multiplier shock — is transmitted to the real part of the economy: the transaction cost wedge and the optimal condition on the non-monetary gold stock. The transaction cost wedge represents a bridge between the nominal and the real parts of the artificial economy because it is endogenously linked to the gross nominal interest rate. The optimal condition on the non-monetary gold stock can be interpreted also as a bridge between the real and the nominal part of the economy because this optimal condition states that any change in the working of the gold standard — changes in the nominal part of the economy — would affect the consumption decision of the households.

Then, I evaluate, through simulation exercises, whether the gold standard model is useful for the study of the Great Depression under the gold standard focus. I concentrate this work on the French experience of the Great Depression over the 1929-1936 period. France is an interesting case study because this country has experienced a deep and long-lasting depression during the 1930s and has been one the lastest country to abandon the gold standard monetary system. France has legally returned to the gold standard in 1928 with the approval of the monetary law of June 1928. The french monetary law has defined the price of the franc in terms of a fixed weight of gold\textsuperscript{19}. It has restored the convertibility of the franc in gold\textsuperscript{20}. It has imposed a legal minimum for the ratio of

\textsuperscript{17}The link between the money supply and the monetary gold stock might exist in the Bordo et al. (2007)’s model, but I do not manage to identify it so far.

\textsuperscript{18}Actually, the central bank cannot control, at least directly, the money multiplier of the economy. In the current framework, since the commercial banks are not modelized, I treat the money multiplier as a monetary policy instrument of the central bank.

\textsuperscript{19}Article 2: “The franc, the French monetary unit, shall contain 65.5 milligrams of gold 0.900 fine [...]”. See Federal Reserve Board (1928).

\textsuperscript{20}Article 3: “The Bank of France shall guarantee the convertibility in gold of its notes to bearer and at sight [...]”. See Federal Reserve Board (1928).
monetary gold stock to the monetary base: the gold reserves of the Bank of France are required to represent at least 35 percent of the monetary base. The monetary law has also forbidden the Bank of France purchasing additional foreign exchanges. France has left the gold standard with the devaluation its currency in September 1936. In the literature, the monetary policies that France has undertaken during the interwar period are often considered as responsible, at least partly, for the malfunctioning of the gold standard and hence for the worldwide Great Depression. In particular, France is blamed for attracting too much gold to the detriment of other countries and to sterilized the gold inflows. The massive gold flows into France and their sterilization would have exacerbated the decline of money supplies and prices elsewhere (Nurkse 1944; Eichengreen 1986; Bordo and MacDonald 2003). This leads me to wonder whether those French monetary policies are also responsible for the depression that the country has experienced in the 1930s. The developed gold standard model might be helpful to answer this question.

Before confronting the predictions of the gold standard model to the French historical data, I examine the dynamic properties of the model through the impulse response functions to the real and nominal shocks. The results of the exercise can be summarized as follows. A transitory increase of the gold flow shock leads the monetary and non-monetary gold stocks, nominal money and the price level to increase. The gold flow shock displays no liquidity effect as the increase of the money supply is not followed by a decline of the nominal interest rate. The effects of the gold flow shock on the nominal part of the economy are strong and highly persistent. The increase of the gold flow shock does affect the real economy. In particular, it causes a recession as output, consumption, labor and investment decline. One reason of this is that the increase of the nominal interest rate raises the transaction cost wedge, which in turn discourages consumption. However, the effects of the gold flow shock on the real economy are relatively small. A non-permanent decrease of the gold backing ratio leads the money stock and the price level to increase. The monetary gold stock declines while the non-monetary gold stock rises when the gold backing is lowered. Contrary to gold flow shock, the gold backing shock displays a liquidity effect. The effects of the gold backing shock on the nominal part of the economy are weaker and less persistent than those of the gold flow shock. Lowering the gold backing ratio leads the real part of the economy towards an expansion as output, consumption, investment and labor increase. The expansionary effect of the decrease of the gold backing shock is partly due to the following mechanism: because of the liquidity effect, the transaction cost wedge declines and henceforth stimulates the households’ consumption. Nevertheless, the effects of the gold backing shock on the real part of the economy are very weak. I find that the effects of a transitory increase of the money multiplier on the economy — both real and nominal parts of the economy — are qualitatively similar to those of a transitory drop in the gold backing ratio. However, there are some quantitative differences between the effects of a decrease of the gold backing ratio on the economy and those of an increase of the money multiplier, notably in terms of persistence. Not surprisingly, the transitory increase of the productivity shock has an expansionary effect on the real part of the economy. Besides, raising the productivity in the final good sector leads the price level, nominal interest rate, the monetary gold stock and the money stock to decline. The non-monetary gold stock increases following the productivity improvement in the final good sector.

In order to confront the predictions of the model to the French historical data, I construct time series for the different shocks considered in the artificial economy. I find that the historical measures of the productivity shock, gold flow shock, gold backing shock and money multiplier shock deviate strongly from their respective 1929 level during the 1930s. Then, I feed the historical measures of the real and nominal shocks, one at a time and in combination, into the gold standard model to simulate the latter. The simulation results can be summarized as follows. I find that the nominal part of the artificial gold standard economy is mainly affected by the historical nominal shocks — gold flow shock, gold backing shock and money multiplier — while the real part of that artificial economy is mainly affected by the historical productivity shock. However, with all the historical shocks taken together, the gold standard model is not able to replicate the French Great Depression. Hence, in its state, the model suffers from two important problems. First, the channels of transmission of the nominal shocks to the real part of the economy are too weak to allow the model to replicate an

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21Article 4: “The Bank of France shall maintain a metallic reserve in gold bullion and gold coin equal at the minimum of 35 per cent of the combined amount of its notes in circulation and its liabilities on current account [...]”. See Federal Reserve Board (1928).

22The United States faces the same critics. According to Eichengreen (1986), more than 60 percent of the world monetary gold stock are in the hands of France and the United States in 1931.
historical event such as the Great Depression. Second, the working of the gold standard in France during the 1930s seems not to be well captured by the model. Indeed, for example, the gold standard model predicts that the historical nominal shocks cause the nominal money stock and the price level to decline more than what it is observed in the historical data. As well, in the artificial gold standard economy, the historical nominal shocks lead the nominal interest rate to follow a different pattern to the one followed by the actual nominal interest rate.

Hence, the proposed gold standard model is too stylized to be used for an evaluation of the gold standard-based explanation of the Great Depression in general and in France in particular. The model needs to be improved in several aspects. For example, in order to reinforce the non-neutrality of the nominal shocks in the gold standard model, one can introduce nominal wage rigidity in the model. Besides, the model can be extended to a small open economy framework in order to better capture the international dimension of the gold standard.

The remainder of the paper is organized as follows. Section 2 presents in details the gold standard model. Section 2 describes the construction of the French historical data, including the measures of the real and nominal shocks. Section 4 outlines the method used to solve the model, discusses the calibration procedure, and analyzes the steady state properties of the model. Section 5 presents the quantitative analysis of the model to the French Great Depression. Section 6 concludes.

2 A Basic Gold Standard Model

This section describes the gold standard model and its equilibrium.

2.1 The artificial economy

The artificial economy consists of three sectors — households, final good producers and a central bank — and of five competitive markets — final good market, labor market, credit (bonds) market, money market and gold market. In the following, I discuss each sector and the gold market in details. Since the members of each sector are in large number and identical, I focus my attention on the behavior of a representative member of each sector. Thus, all the variables of the model are expressed in per capita terms. Besides, I assume that there is no growth in the economy.

2.1.1 Central bank

In the classical gold standard system, the medium of exchange, namely money, is partially or fully defined as claims to units of the commodity money, that is monetary gold. Those claims can take the form of bank notes and commercial bank deposits. The value of the money stock can be larger than the value of the monetary gold stock, up to a ceiling value. I assume that the monetary gold stock can be held only by the monetary authority of the economy, that is the central bank. The main goal of the central bank, under the gold standard system, is to defend the convertibility of its currency into gold at a fixed price. To do so, the central bank is committed to buy or sell any quantity of gold in exchange of money that private agents ask for, at a fixed price. Formally, the money supply rule, under the gold standard system, is stated as follows (see Barro, 1979):

\[ M_t^s = \frac{1}{\mu_t} P_G G_{m,t} \quad 0 \leq \mu_t \leq 1 \]  

where \( M_t^s \) denotes the nominal money supply — money aggregate M1 —, \( G_{m,t} \) the monetary gold stock and \( P_G \) the fixed price of gold. The variable \( \mu_t \) indicates how much the money supply is tightening up on the monetary gold stock. The monetary gold stock is assumed not to depreciate over time.

Following Bernanke (1995), I decompose the money-monetory gold ratio, measured by \( \frac{1}{\mu_t} \), in two terms:

\[ \frac{1}{\mu_t} = \frac{M_t^s}{M_t^b} \frac{M_t^b}{G_{m,t}} \]  

\(^{23}\)When the gold standard has been rebuilt after the World War I, the public has been asked to bring their monetary gold stock (coins and bullions) to central banks in order to deal with the scarcity of the money commodity.
where $M_t^b$ denotes the monetary base. The first term in equation (2), the ratio of money supply to monetary base, is called the money multiplier and is assumed to be larger than one. Indeed, in economies with a fractional-reserve banking system the total amount of money supply — money aggregate M1 — exceeds the amount of monetary base. According to Bernanke (1995), the money multiplier is an indicator of the degree of development of the financial system of an economy: Closer to unity the money multiplier is, less the financial system is developed.

The second term of equation (2), the ratio of monetary base to monetary gold, is called the inverse of gold backing ratio or called the coverage ratio. The inverse of gold backing ratio is reglemented by the public authority. In particular, the monetary base-monetary gold ratio is bounded by a ceiling value. For example, in France, following the monetary law of June 1928, the value of the monetary gold stock of the Bank of France must be larger or equal to 35 percent of the amount of its monetary base. Nonetheless, as noted by Bernanke (1995), the coverage ratio is not bounded by a floor.

Hence, the gold standard money supply rule, presented in equation (1), can be restated as follows

$$M_t^g = \frac{1}{\mu_{1,t}} \frac{1}{\mu_{2,t}} P_G G_{m,t} \quad 0 \leq \mu_{1,t}, \mu_{2,t} \leq 1$$

with

$$\frac{1}{\mu_{1,t}} = \frac{M_t^g}{M_t^b}, \quad \frac{1}{\mu_{2,t}} = \frac{M_t^b}{G_{m,t}}$$

In equation (3), $\mu_{1,t}$ denotes the inverse of the money multiplier and $\mu_{2,t}$ the gold backing ratio. Since, the financial-banking sector is not explicitly introduced in the model, variables $\mu_{1,t}$ and $\mu_{2,t}$ are taken exogenously. The exogenous stochastic process followed by $\mu_{1,t}$ and $\mu_{2,t}$, are described below.

### 2.1.2 Households

The representative household gets satisfaction from having a consumption good, having leisure and holding a gold stock for non-monetary purposes. Households care about non-monetary gold stock because they reap a benefit from the latter in the form of, for example, jewelry, ornamentation or safe asset. Several authors, such as Goodfriend (1988), Velde and Weber (2000), and Bordo et al. (2007), make the assumption that non-monetary gold stock yields satisfaction to households. Gold is introduced in the model for non-monetary uses because, together with gold production, it represents a major causal factor of the monetary gold stock. Barro (1979) emphasizes this point, referring to Irving Fisher. In addition, following Barro (1979), the non-monetary gold stock is assumed to depreciate over time at a constant rate. According to McCulmm (1989), the depreciation of non-monetary gold stock measures the “avoidable losses” to which the total gold stock is subject. Leisure is measured as the share of the representative household’s total time endowment, normalized to unity, spent in non-market activities.

Thus, the household’s preferences are stated over consumption, denoted $C_t$, non-monetary gold stock, denoted $G_{c,t}$, and leisure time, denoted $1 - h_t$ where $h_t$ is labor supply. Those preferences are described by an expected intertemporal utility function of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t U (C_t - bC_{t-1}, G_{c,t}, 1 - h_t) \quad 0 \leq b < 1$$

where $E_0$ denotes the expectation operator conditional on the period-$t$ information set, $\beta$ is a subjective discount factor and $U (\cdot)$ is a period-$t$ utility measure. Note that the period-$t$ information set of the representative household includes the realizations of all endogenous variables of the economy.

---

24Bernanke (1995) considers a more general decomposition of the money-monetary gold ratio:

$$\frac{1}{\mu_t} = \frac{M_t^g}{M_t^b} \frac{RES_t}{G_{m,t}}$$

where $RES_t$ is the international reserves of the Central Bank, including monetary gold and foreign assets.

25As stressed by Fisher, [...] the two key determinants of the monetary gold stock in a dynamic context are gold production and the extent to which gold is held for non-monetary purposes.” Quoted from Barro (1979).
as well as the exogenous shocks, through period \( t \). Note that the parameter \( b \) introduces internal habit formation in consumption when it is strictly positive. Several authors, among them Fuhrer (2000), Christiano et al. (2001), argue that habit formation in consumption is helpful to understand the monetary propagation mechanism for the post-war period. Specifically, those authors find that a positive money supply shock causes a decrease in the real interest rate and a hump-shaped increase in consumption. According to those authors, introducing internal habit formation in consumption allows monetary models to replicate those facts. Indeed, in absence of habit formation in consumption, the optimal choices of households lead the current consumption to be larger than the (expected) next period consumption when the real interest rate decreases. With internal habit formation in consumption, households care about the variations in consumption rather than about the level of consumption. Therefore, in that case, the optimal choices of households state that the current change in consumption is higher than the next period change in consumption when the real interest rate falls. It follows that the response of consumption to a money supply shock is hump-shaped. Furthermore, as it will be shown below, habit formation in consumption allows investment to be procyclical following an increase of the gold flow shock or the inverse of money multiplier shock even though the latter are not highly persistent. Finally, the utility function, \( U(C_t - bC_{t-1}, G_{c,t}, 1 - h_t) \), satisfies the following standard properties

\[
U_{1,t} \equiv \frac{\partial U(C_t - bC_{t-1}, G_{c,t}, 1 - h_t)}{\partial (C_t - bC_{t-1})} > 0, \quad U_{11,t} \equiv \frac{\partial^2 U(C_t - bC_{t-1}, G_{c,t}, 1 - h_t)}{\partial (C_t - bC_{t-1})^2} < 0
\]

\[
U_{2,t} \equiv \frac{\partial U(C_t - bC_{t-1}, G_{c,t}, 1 - h_t)}{\partial G_{c,t}} > 0, \quad U_{22,t} \equiv \frac{\partial^2 U(C_t - bC_{t-1}, G_{c,t}, 1 - h_t)}{\partial G_{c,t}^2} < 0
\]

\[
U_{3,t} \equiv \frac{\partial U(C_t - bC_{t-1}, G_{c,t}, 1 - h_t)}{\partial (1 - h_t)} > 0, \quad U_{33,t} \equiv \frac{\partial^2 U(C_t - bC_{t-1}, G_{c,t}, 1 - h_t)}{\partial (1 - h_t)^2} < 0
\]

\[
U_{12,t} \equiv \frac{\partial^2 U(C_t - bC_{t-1}, G_{c,t}, 1 - h_t)}{\partial (C_t - bC_{t-1}) \partial G_{c,t}} = U_{21,t} \leq 0, \quad U_{13,t} \equiv \frac{\partial^2 U(C_t - bC_{t-1}, G_{c,t}, 1 - h_t)}{\partial (C_t - bC_{t-1}) \partial (1 - h_t)} = U_{31,t} \leq 0
\]

\[
U_{23,t} \equiv \frac{\partial^2 U(C_t - bC_{t-1}, G_{c,t}, 1 - h_t)}{\partial G_{c,t} \partial G_{c,t}} = U_{32,t} \leq 0
\]

Hence, the utility function is strictly increasing in its three arguments, and strictly concave. Additionally, the preferences are allowed to be non-separable over consumption, non-monetary gold stock and leisure.

In the beginning of each period, the representative household is endowed with the new gold flow, \( g_t \), which value is given by \( P_G G_t \). This new flow of gold is added to the net of depreciation non-monetary gold stock of the household. Then, the representative household decides how much of the newly accumulated non-monetary gold stock is going to be exchanged against money with the central bank, at the fixed price \( P_G \), accordingly to the money supply rule (3) and how much is going to be used for its own satisfaction. Therefore, variable \( G_{c,t} \) represents the stock of non-monetary gold once the gold-money trade between the household and the central bank is realized. For the sake of simplicity, I assume that the costs of minting gold to species — that is costs of converting gold to monetary gold — are negligible.

Households demand money because it facilitates the access to the consumption good. Indeed, households face transactions costs which are inherent to the purchase of the consumption good. Those transactions costs increase proportionally to the volume of the consumption good. Households can reduce the transactions costs by holding money. In other words, the transactions costs rise with the consumption-based velocity of money. The latter, denoted \( v_t \), is formally defined as follows:

\[
v_t = \frac{P_t C_t}{M^d_t} \quad (5)
\]

where \( P_t \) is the price level, \( M^d_t \) denotes the nominal money stock demanded by the representative household.

Following Schmitt-Grohé and Uribe (2004, 2006), the transaction cost function, denoted \( L(v_t) \), is assumed to satisfy the following properties:
(i) \( L(v_t) \) is non-negative and twice continuously differentiable;

(ii) there exists a satiation level of consumption-based velocity of money, denoted \( \bar{v} > 0 \), such that 
\[
L(v) = L'(v) = 0;
\]

(iii) \( (v - \bar{v})L'(v) > 0 \) for \( v \neq \bar{v} \);

(iv) \( 2L'(v) + vL''(v) > 0 \) for \( v \geq \bar{v} \).

Assumption (ii) ensures that a zero nominal interest rate — Friedman rule — does not mean that the demand of money is infinite. Besides, this assumption makes the cost of transactions as well as the implied distortions fade away when the nominal interest rate takes a value of zero. Assumption (iii) states that the equilibrium consumption-based money velocity is always larger than the satiation level \( \bar{v} \). Assumption (iv) guarantees that the demand of money falls when the nominal interest rate rises.\(^2\)

Moreover, each period, households can get an other asset, that is a full set of nominal state-contingent bonds, at cost \( B_t \). The nominal pay-off for bonds is given by \( q_{t-1}B_{t-1} \), where \( q_t \) represents the gross nominal interest rate.

The representative household also purchases an investment good, denoted \( I_t \), at price \( P_t \) and uses it to accumulate its stock of capital, \( K_t \), as follows,

\[
K_t = (1 - \delta_k) K_{t-1} + I_t
\]

where \( \delta_k \) is the capital depreciation rate. The representative household rents its stock of capital to the final good producers at the rental price \( R_t \).

Each period, the representative household supplies its labor force to the representative final good producer and receives, in counterpart from the latter, a wage bill of total amount \( W_th_t \), where \( W_t \) denotes the nominal hourly wage rate.

In addition, households are assumed to own firms which produce the final good. Therefore, households receive the profits realized by the producers.

The intratemporal budget constraint of the representative household is given by

\[
P_tC_t(1 + L(v_t)) + P_GG_{c,t} + P_tI_t + B_t + M^d_t + T_t \leq W_t h_t + R_t K_{t-1} + q_{t-1}B_{t-1} + M^d_{t-1} + (1 - \delta_g) P_GG_{c,t-1} + \Phi_t + P_Gg^s_t + TR_t
\]

\( \Phi_t \) denotes the profits realized by the final good producers. \( \delta_g \) indicates the non-monetary gold depreciation rate. \( T_t \) represents a lump-sum transfer from households to the central bank inherent to the gold-money trade. \( TR_t \) is a lump-sum transfer from the central bank to households. The purposes of the two transfers, \( T_t \) and \( TR_t \), are discussed below. The left-hand side of the budget constraint describes the uses of the representative household wealth: Purchases of consumption good, investment good and state-contingent bonds, holdings of non-monetary gold and money stocks, payment of the transactions costs and payment of the lump-sum transfer to the central bank. The right-hand side describes the sources of wealth: Labor and capital incomes, profits from final good producers, interests on bonds acquired in the previous period, money stock and net of depreciation non-monetary gold stock brought from the previous period, new flow of gold, and the receipt of lump-sum transfer from the central bank.

The representative household is supposed to choose sequences for \( C_t, G_{c,t}, I_t, h_t, M^d_t, K_t \) and \( B_t \) which maximize its expected intertemporal utility function (4), subject to its intratemporal budget constraint (7), the capital accumulation law (6) and the expression of the consumption-based velocity of money (5), and taking price level, \( P_t \), price of gold \( P_G \), nominal wage, \( W_t \), nominal capital rental price, \( R_t \), gross nominal interest rate on bonds, \( q_t \), firms profits, \( \Phi_t \), newly produced gold, \( g^s_t \), lump-sum transfer, \( T_t \), and the initial conditions, \( C_{-1}, G_{c,-1}, K_{-1}, M^d_{-1}, B_{-1} \), as given.

\(^2\)For more details on this specification of the transaction cost function, the reader is referred to Schmitt-Grohé and Uribe (2004, 2006)
2.1.3 Final good producers

The representative firm makes use of a constant return to scale technology to produce its final good,

\[ Y_t \leq z_t F(K_{t-1}, h_t) \]  

(8)

The factor inputs used by this technology are capital, \( K_{t-1} \), and labor, \( h_t \). \( Y_t \) denotes the quantity of the final good, or output, produced by the representative firm. \( z_t \) represents a stochastic neutral productivity shock to the final good production technology. Hereafter, I refer to \( z_t \) as the productivity shock. It is determined exogenously by a stochastic process, which I specify below. The production function \( F(\cdot) \) is assumed to satisfy the following standard properties:

\[
F_{1,t} \equiv \frac{\partial F(K_{t-1}, h_t)}{\partial K_{t-1}} > 0, \quad F_{11,t} \equiv \frac{\partial^2 F(K_{t-1}, h_t)}{\partial K_{t-1}^2} < 0 \\
F_{2,t} \equiv \frac{\partial F(K_{t-1}, h_t)}{\partial h_t} > 0, \quad F_{22,t} \equiv \frac{\partial^2 F(K_{t-1}, h_t)}{\partial h_t^2} < 0
\]

Hence, the production function is strictly increasing in its two arguments and is strictly concave.

The representative firm makes profits

\[ \Phi_t = P_t Y_t - W_t h_t - R_t K_{t-1} \]  

(9)

by selling its output, \( Y_t \), at the market price, \( P_t \), and paying its labor and capital inputs, \( K_{t-1} \) and \( h_t \), at their respective market price, \( W_t \) and \( R_t \).

The representative firm chooses the quantities of capital and labor inputs, \( K_{t-1} \) and \( h_t \), to maximize its profits, (9), subject to the technology of production constant, (8), taking the price level, \( P_t \), the nominal rental rate of capital, \( R_t \), and the nominal hourly wage, \( W_t \), as given.

2.1.4 Gold market

In this model, the gold mining sector is not defined. Instead and as a simplification, I consider the gold supply in terms of gold flow between the economy and a no identified place outside the economy. In addition, the gold flows are taken exogenously by the agents of the economy. In particular, each period, a new flow of gold is exogenously shipped to (from) households from (to) a place outside the economy. Therefore, the demand for new gold is constrained by the supply side.

In this economy, gold has two functions: (i) Gold is used by households for non-monetary purpose, \( G_{c,t} \); (ii) the central bank bases its monetary policy on its gold reserves, \( G_{m,t} \). Thus, the total demand for gold in the economy, denoted \( G_{t}^d \), is given by the sum of non-monetary gold stock and monetary gold stock:

\[ G_t^d = G_{m,t} + G_{c,t} \]  

(10)

It follows that the total demand for new gold, denoted \( g_t^d \), is given by

\[ g_t^d = G_t^d - G_{t-1}^d + \delta g_{c,t-1} \]

(11)

The supply of new gold, denoted \( g_t^s \), is assumed to follow an exogenous stochastic process that I specify the form below.

Note that the supply of new gold, \( g_t^s \), can be either positive or negative. In the former case, there is a gold inflow in the economy while in the latter case there is an outflow of gold from the economy.

2.1.5 Exogenous shocks

The artificial economy is subject to four exogenous stochastic shocks. These are the productivity shock, \( z_t \), the inverse of money multiplier shock, \( \mu_{1,t} \), the gold backing shock, \( \mu_{2,t} \), and the gold flow shock, \( \eta_t^s \).

I assume that the exogenous variables \( z_t, \mu_{1,t}, \mu_{2,t} \) and \( \eta_t^s \) are orthogonal to each other and follow autoregressive processes of order one. The later are defined such that:

\[ \ln z_t = (1 - \rho_z) \ln z + \rho_z \ln z_{t-1} + \varepsilon_{z,t} \]  

(12)
where \( \varepsilon_{z,t} \sim i.i.d. (0, \sigma_z^2) \) and \( z_{-1} \) is given.

\[
\ln \mu_{1,t} = (1 - \rho_{\mu_1}) \ln \mu_1 + \rho_{\mu_1} \ln \mu_{1,t-1} + \varepsilon_{\mu_1,t}
\]

where \( \varepsilon_{\mu_1,t} \sim i.i.d. (0, \sigma_{\mu_1}^2) \) and \( \mu_{1,-1} \) is given.

\[
\ln \mu_{2,t} = (1 - \rho_{\mu_2}) \ln \mu_2 + \rho_{\mu_2} \ln \mu_{2,t-1} + \varepsilon_{\mu_2,t}
\]

where \( \varepsilon_{\mu_2,t} \sim i.i.d. (0, \sigma_{\mu_2}^2) \) and \( \mu_{2,-1} \) is given.

\[
g_t^* = (1 - \rho_{g^*}) g^* + \rho_{g^*} g_{t-1}^* + \varepsilon_{g^*,t}
\]

where \( \varepsilon_{g^*,t} \sim i.i.d. (0, \sigma_{g^*}^2) \) and \( g_{t-1}^* \) is given. The variables written without time subscript denote the steady state levels.

The productivity shock, \( z_t \), measures the efficiency with which the factor inputs are used in the final good sector. The inverse of the money multiplier shock, \( \mu_{1,t} \), measures the degree of development of the financial-banking system of the economy. In other words, an increase in the money multiplier — a decrease of \( \mu_{1,t} \) — means that the economy is able to provide a larger volume of instruments of payment (in addition to bank notes) to the economic agents. The gold backing shock, \( \mu_{2,t} \), represents the policy instrument of the central bank in terms of money supply. By rising the gold backing ratio, the central bank would conduct a restrictive monetary policy: Following a positive shock to the gold backing ratio, the size of the issued money is reduced. Hereafter, I consider the inverse of the money multiplier shock, \( \mu_{1,t} \), and the gold backing shock, \( \mu_{2,t} \), as being two money supply shocks. The gold flow shock, \( g_t^* \), indicates whether the economy is subject to an entry of new gold or an exit of existing gold.

### 2.2 Solving the agents’ optimization problems

The agents’ optimization problems stated above focus only on interior solutions, that is all the quantities of endogenous variables are supposed to be strictly positive. In this subsection, I present the solutions of these problems and the associated market clearing conditions.

#### 2.2.1 Household’s optimization problem

After combining the household’s budget constraint (7), the capital accumulation law (6) and the expression of consumption-based velocity of money (5), the representative household’s optimization problem can be stated as follows

\[
\max_{C_t, G_{c,t}, h_t, M_t^d, B_{t-1}, K_t} E_0 \sum_{t=0}^{\infty} \beta^t U \left( C_t - bC_{t-1}, G_{c,t}, 1 - h_t \right)
\]

subject to

\[
P_t C_t \left( 1 + L \left( \frac{P_t C_t}{M_t^d} \right) \right) + P_t G_{c,t} + P_t [K_t - (1 - \delta_k) K_{t-1}] + B_t + M_t^d + T_t = W_t h_t + R_{t-1} K_{t-1} + q_{t-1} B_{t-1} + M_{t-1}^d + (1 - \delta_g) P_G G_{c,t-1} + \Phi_t + P_G g_{t-1}^* + T R_t
\]

The Lagrangian associated to the representative household’s optimization problem is given by

\[
\mathcal{L}^H = \sum_{t=0}^{\infty} \beta^t \left\{ U \left( C_t - bC_{t-1}, G_{c,t}, 1 - h_t \right) + \lambda_t \left[ W_t h_t + R_{t-1} K_{t-1} + q_{t-1} B_{t-1} + M_{t-1}^d + (1 - \delta_g) P_G G_{c,t-1} + \Phi_t + P_G g_{t-1}^* + T R_t - P_t C_t \left( 1 + L \left( \frac{P_t C_t}{M_t^d} \right) \right) - P_t G_{c,t} - P_t [K_t - (1 - \delta_k) K_{t-1}] - B_t - M_t^d - T_t \right]\}
\]
where \( \frac{\lambda_t}{P_t} \) is the Langrange multiplier associated to the budget constraint. The first order conditions of (16) with respect to \( C_t, G_{c,t}, h_t, M^d_t, B_t \) and \( K_t \), in that order, are

\[
\lambda_t [1 + L(v_t) + v_t L'(v_t)] = U_{1,t} - b\beta E_t (U_{1,t+1})
\]

\[\lambda_t \frac{P_G}{P_t} = U_{2,t} + \beta (1 - \delta_t) E_t \left( \lambda_{t+1} \frac{P_G}{P_{t+1}} \right)
\]

\[U_{3,t} = \lambda_t \frac{W_t}{P_t}
\]

\[
\lambda_t = \lambda_t v_t^2 L'(v_t) + \beta E_t \left( \lambda_{t+1} \frac{P_t}{P_{t+1}} \right)
\]

\[
\lambda_t = q_t \beta E_t \left( \lambda_{t+1} \frac{P_t}{P_{t+1}} \right)
\]

\[
\lambda_t = \beta E_t \left( \lambda_{t+1} \left[ \frac{R_t}{P_{t+1}} + 1 - \delta_t \right] \right)
\]

The consumption-based velocity of money definition, (5), the capital accumulation law, (6), and the household’s intertemporal budget constraint, (7), holding with equality, complete the set of household’s optimization problem first order conditions. There are also transversality conditions which can be ignored in regards to the purpose of the study.

The optimality condition (17) states that the expected marginal utility of current consumption, \( U_{1,t} - b\beta E_t (U_{1,t+1}) \), must be equal to the opportunity cost of the additional consumption in period \( t \), \( \lambda_t [1 + L(v_t) + v_t L'(v_t)] \). From that equation, note that the transactions costs put a wedge between the expected marginal utility of consumption and the marginal utility of wealth, \( \lambda_t \). According to the properties of the transaction cast function, presented above, this wedge vanishes if the consumption-based velocity of money is at its satiation level.

The optimality condition (18) states that the marginal benefit of an additional unit of non-monetary gold in period \( t \) must be equal to the opportunity cost of holding that additional unit of non-monetary gold, in terms of expected marginal utility of current consumption, \( \lambda_t \frac{P_G}{P_t} \), in period \( t \). The marginal benefit of an additional unit of non-monetary gold comes from, on the one hand, the marginal utility brought by the non-monetary gold stock, \( U_{2,t} \), on the other hand, the present expected value of that additional unit of non-monetary gold — net of depreciation — brought to the household’s resources in the next period, measured in terms of expected marginal utility of future consumption, \( \beta (1 - \delta_t) E_t \left( \lambda_{t+1} \frac{P_G}{P_{t+1}} \right) \).

The optimality condition (19) states that the marginal utility of current leisure, \( U_{3,t} \), has to be set equal to the opportunity cost of that additional leisure at time \( t \), evaluated in terms of expected marginal utility of current consumption, \( \lambda_t \frac{W_t}{P_t} \).

The optimality condition (20) states that the marginal benefit of an additional unit of money in period \( t \) must be equal to the opportunity cost of holding that additional unit of money, in terms of expected marginal utility of current consumption, \( \lambda_t \), in period \( t \). The marginal benefit of an additional unit of money consists of two elements. First, holding additional money reduces the transactions costs. This first marginal benefit, measured in terms of expected marginal utility of current consumption, is given by \( \lambda_t v_t^2 L'(v_t) \). Second, holding additional money brings resources to the household’s budget in the future. This second marginal benefit, measured in terms of expected marginal utility of future consumption, is evaluated in the current period by \( \beta E_t \left( \lambda_{t+1} \frac{P_t}{P_{t+1}} \right) \).

The optimality condition (21) states that the present expected value of a marginal unit of bond brought to the household’s resources in the next period, measured in terms of expected marginal utility of future consumption, \( q_t \beta E_t \left( \lambda_{t+1} \frac{P_t}{P_{t+1}} \right) \), must equal the opportunity cost of holding that marginal unit of bond in the current period, evaluated in terms of expected marginal utility of current consumption, \( \lambda_t \).
The optimality condition (22) states that the expected value of future capital returns, in terms of expected marginal utility of future consumption, \( \beta E_t \left( \lambda_{t+1} \left[ \frac{\rho_{t+1}}{P_{t+1}} + 1 - \delta_t \right] \right) \), has to be set equal to the opportunity cost of holding that additional capital in the current period, in terms of expected marginal utility of current consumption, \( \lambda_t \).

In the following, I denote \( \kappa_t \) the wedge introduced by the transactions costs:

\[
\kappa_t = 1 + L (v_t) + v_t L' (v_t)
\] (23)

According to the properties of the transaction cost function, the wedge, \( \kappa_t \), increases with the consumption-based velocity of money. This wedge resembles to a consumption tax rate in the way it distorts the representative household’s optimization problem. The frictions that transactions costs introduce in the economy represent a transmission channel of the gold flow shocks and the two money supply shocks to the real part of the economy. Indeed, as it will be shown later, the transaction cost wedge is endogenously driven by the gross nominal interest rate, \( q_t \).

Combining equations (17) and (18) leads to

\[
\frac{U_{1,t} - b \beta E_t (U_{1,t+1})}{\kappa_t} \frac{P_G}{P_t} - U_{2,t} = \beta (1 - \delta_t) E_t \left( \frac{U_{1,t+1} - b \beta E_t (U_{1,t+2})}{\kappa_{t+1}} \right) \frac{P_G}{P_{t+1}}
\] (24)

Hence, equation (24) defines a non-monetary gold pricing function. Note that the pricing of non-monetary gold is expressed in terms of consumption good: The key price variable in equation (24) is \( \frac{P_G}{P_t} \), that is the relative price of gold — or real price of gold. Goodfriend (1988) derives a similar non-monetary gold pricing function. Besides, the transactions costs alter the optimal non-monetary gold demand decision.

Combining equations (17) and (19) leads to

\[
\frac{U_{3,t}}{U_{1,t} - b \beta E_t (U_{1,t+1})} \kappa_t = \frac{W_t}{P_t}
\] (25)

From equation (25), one can observe that the existence of transactions costs prevents the expected marginal rate of substitution of consumption for leisure, \( \frac{U_{3,t}}{U_{1,t} - b \beta E_t (U_{1,t+1})} \), from being equal to the real wage, \( \frac{W_t}{P_t} \). Specifically, the transactions costs alter the optimal behavior of households in terms of consumption and labor supply. Hence, given the real wage and — if preferences are non-separable — the non-monetary gold stock, a higher consumption-based velocity of money (higher wedge) leads households to consume less and to reduce their labor supply.

Combining equations (17) and (20) leads to

\[
v_t^2 L' (v_t) = 1 - \beta E_t \left( \frac{U_{1,t+1} - b \beta U_{1,t+2}}{U_{1,t} - b \beta U_{1,t+1}} \right) \frac{\kappa_t}{\kappa_{t+1}} \frac{P_t}{P_{t+1}}
\] (26)

Equation (26) represents a pricing equation for money. Note that the transactions costs distort the pricing equation for money.

Combining equations (17) and (21) leads to

\[
\frac{1}{q_t} = \beta E_t \left( \frac{U_{1,t+1} - b \beta U_{1,t+2}}{U_{1,t} - b \beta U_{1,t+1}} \right) \frac{\kappa_t}{\kappa_{t+1}} \frac{P_t}{P_{t+1}}
\] (27)

Equation (27) represents a pricing equation for the nominal bonds. This equation states that the gross nominal interest rate, \( q_t \), is increasing with the expected next period inflation, \( E_t \left( \frac{P_t}{P_{t+1}} \right) \), and increasing with the expected marginal rate of substitution of future consumption for current consumption, \( E_t \left( \frac{U_{1,t+1} - b \beta U_{1,t+1}}{U_{1,t+1} - b \beta U_{1,t+2}} \right) \). In other words, the gross real interest rate, \( q_t E_t \left( \frac{P_t}{P_{t+1}} \right) \), is increasing with the expected marginal rate of substitution of future consumption for current consumption. In addition, one can note that the transactions costs distort the pricing equation for nominal bonds.
Combining equations (17) and (21) leads to

\[ 1 = \beta E_t \left( \begin{bmatrix} U_{1,t+1} - b\beta U_{1,t+2} \\ U_{t} - b\beta U_{t+1} \end{bmatrix} \begin{bmatrix} \kappa_t \\ \kappa_{t+1} \end{bmatrix} \begin{bmatrix} R_{t+1} \\ P_{t+1} \end{bmatrix} + (1 - \delta_k) \right) \] (28)

From equation (28), one can note that the transactions costs introduce an intertemporal wedge, \( E_t \left( \frac{\kappa_t}{\kappa_{t+1}} \right) \), between the expected marginal rate of substitution of future consumption for current consumption, \( E_t \left( \frac{U_{1,t+1} - b\beta U_{1,t+2}}{U_{t+1} - b\beta U_{t+2}} \right) \), and the real interest rate, \( E_t \left( \frac{R_{t+1}}{P_{t+1}} + 1 - \delta_k \right) \).

From the previous Euler equations (24), (26), (27) and (28), one can observe that the weight that the representative household puts on the present comparatively to the future is higher when it determines optimally the sequence of non-monetary gold stock than when it determines optimally the sequences of money, bond and capital stocks. The household behaves so because the non-monetary gold depreciates over time\(^{27}\). While it uses the subjective discount factor, \( \beta \), when it determines the optimal sequence of money, bonds and capital, it discounts the future with \( \beta (1 - \delta_g) \) when it chooses the optimal sequence of non-monetary gold stock. Hence, higher the non-monetary gold depreciation rate is, lower the value of non-monetary gold becomes in the future comparatively to its current value.

The household’s optimal decision on the non-monetary gold holding happens to be an additional channel through which the gold flow shock and the two money supply shocks are transmitted to the real part of the economy. Indeed, as shown in equation (24), the household’s optimal decisions in terms of consumption and non-monetary gold are linked to each other. It is worth to note that if preferences are non-separable in consumption, leisure and non-monetary gold, then labor and non-monetary gold also feature in the Euler equations (24), (26), (27) and (28). Therefore, the non-separability hypothesis on the household’s utility function may reinforce this channel.

Besides, from combining (26) and (27), one can get the following expression

\[ 1 - \frac{1}{q_t} = v_t^2 L'(v_t) \] (29)

The left-hand side of (29), defining the opportunity cost of holding money instead of bonds, is an increasing function of the gross nominal interest rate \( q_t \). The right-hand side of (29) is an increasing function of the consumption-based velocity of money, \( v_t \), according to the assumptions made on the transaction cost function \( L (\cdot) \). One can interpret this equation as a liquidity preference function which is decreasing with gross nominal interest rate and has an elasticity in consumption of one (see Schmitt-Grohé and Uribe, 2006).

Furthermore, rearranging terms in equation (24),

\[ 1 = \frac{P_t}{P_G} \frac{U_{2,t}}{U_{1,t} - b\beta E_t (U_{1,t+1})} \kappa_t + (1 - \delta_g) \beta E_t \left( \frac{U_{1,t+1} - b\beta E_t (U_{1,t+2})}{U_{t+1} - b\beta E_t (U_{1,t+1})} \frac{P_G}{P_t} \frac{P_t}{P_{t+1} \frac{P_t}{P_G}} \right) \]

and using equation (27), one gets

\[ 1 = \frac{P_t}{P_G} \frac{U_{2,t}}{U_{1,t} - b\beta E_t (U_{1,t+1})} \kappa_t + (1 - \delta_g) \frac{1}{q_t} \]

\[ \Leftrightarrow \frac{P_G}{P_t} \left( 1 - \frac{1 - \delta_g}{q_t} \right) = \kappa_t \frac{U_{2,t}}{U_{1,t} - b\beta E_t (U_{1,t+1})} \] (30)

Hence, equation (30) shows that the expected marginal rate of substitution of consumption for non-monetary gold, \( \frac{U_{2,t}}{U_{1,t} - b\beta E_t (U_{1,t+1})} \), depends not only on the real price of gold, \( \frac{P_G}{P_t} \), but also on the gross nominal interest rate, \( q_t \). In particular, the expected marginal rate of substitution of consumption for

\(^{27}\)The capital stock also depreciates over time. However, the capital depreciation rate is used to define the real return on capital rather than to discount the future. Indeed, capital is remunerated, which is not the case of non-monetary gold.
non-monetary gold is increasing with the real price of gold and the gross nominal interest rate. Note that the transactions costs alter the consumption-non-monetary gold margin through the wedge, $\kappa_t$.

I have stressed that the transaction cost wedge and the optimal decision on non-monetary gold holding represent two channels of transmission of the gold flow shock and the two money supply shocks to the real part of the artificial economy. I discuss these two channels of transmission more in details when I analyse the dynamic properties of the gold standard model.

2.2.2 Final good producer’s problem

The representative final good producer’s problem can be stated as follows,

$$\max_{h_t, K_{t-1}} P_t Y_t - R_t K_{t-1} - W_t h_t$$

subject to $Y_t \leq z_t F(K_{t-1}, h_t)$ \hspace{1cm} (31)

The first order conditions of (31) with respect to $h_t$ and $K_{t-1}$, in that order, are

$$\frac{W_t}{P_t} = z_t F_{2,t}$$ \hspace{1cm} (32)

$$\frac{R_t}{P_t} = z_t F_{1,t}$$ \hspace{1cm} (33)

Hence, equation (32) states that firms hire workers until the point where the marginal product of labor, $z_t F_{2,t}$, is equal to the real wage, $\frac{W_t}{P_t}$. As well, equation (33) states that firms use capital services until the point where the marginal product of capital, $z_t F_{1,t}$, is equal to the real rental price of capital, $\frac{R_t}{P_t}$.

The technology constraint, (8), holding with equality, completes the set of final good producer’s problem first order conditions:

$$Y_t = z_t F(K_{t-1}, h_t)$$ \hspace{1cm} (34)

2.2.3 Market clearing conditions

Here, I discuss the market clearing conditions in the gold market, money market, bond market and final good market.

2.2.3.1 Market clearing condition in the gold market

In the equilibrium, the gold market must satisfy the following equation

$$g^d_t = g^s_t$$ \hspace{1cm} (35)

Hence, in the equilibrium, the gold flow demand, $g^d_t$, is supposed to be equal to the gold flow supply, $g^s_t$.

Using equation (11), the previous gold market equilibrium condition can be restated as an accumulation equation

$$G^d_t - C^d_{t-1} = g^s_t - \delta_y G_{c,t-1}$$ \hspace{1cm} (36)

The left-hand side of the gold market clearing equation represents the change of the total gold stock demand, while the right-hand side represents the flow of new gold net of the non-monetary gold depreciation. Besides, equation (36) states that, in any period $t$, the total gold stock increases (decreases) if, at the equilibrium relative price of gold, $\frac{P_t}{P_{t-1}}$, the new gold flow, $g^s_t$, is larger (lower) than the depreciation of the non-monetary gold stock, $\delta_y G_{c,t-1}$.
2.2.4 Functional forms

I use the following functional form for the households' preferences:

\[
U_i = \frac{\phi_i}{G_i} \left( C_i - \frac{C_i - C_i^*}{1 - \phi_i} \right) \left( 1 - \frac{1}{1 - \phi_i} \right)
\]

Thus, output is absorbed by consumption and investment. The transactions costs are related to households because they are considered as a private cost and not as a social cost.

\[
Y = C + I
\]

As said above, the gold trade between the representative household and the central bank leads to a lumpsum transfer from the former to the latter. In particular, this transfer is supposed to cover the difference between the value of the monetary unit in the former and the latter. More precisely, following Kim and Subramanian (2006), I assume that the central bank pays back the transactions costs to households in a lumpsum way: \( TR = P_i (C_i^* - C_i) \). Therefore, the resource constraint of the economy is given by the household's budget constraint (7).

\[
U_i = \frac{\phi_i}{G_i} \left( C_i - \frac{C_i - C_i^*}{1 - \phi_i} \right) \left( 1 - \frac{1}{1 - \phi_i} \right)
\]

\[
\phi_i \in (0, 1)
\]

2.2.3 Market clearing conditions in the final good market

The resource constraint of the economy, defining how the output is absorbed in the equilibrium, is derived from the households' budget constraint (7). Hence, combining equations (9), (10), (36) with the household's budget constraint (7), one gets:

\[
PC(1 + L_i) + P_i + T_i = P_i \left( G_{m,i} - G_{m,i-1} \right) - (M_t - M_{ti}) + TR_i
\]

Where \( t = 1, \ldots, T \).

2.2.3.1 Market clearing conditions in the money market and the bond market

Hence, equation (37) states simply that, in the equilibrium, money supply and money demand must coincide. In turn, equation (38) states that the representative household does not hold bonds in the equilibrium, since the bond market is complete.

\[
B_t = B_{t-1} = 0
\]

\[
M_t = M_t
\]
where $\Omega_t = (C_t - bC_{t-1})^{\phi_1} (1 - h_t)^{\phi_2} G_{c_t}^{\phi_3}$.

I assume that the production function used by final good producers is a Cobb-Douglas of the form:

$$F (K_{t-1}, h_t) = K_{t-1}^\theta h_t^{1-\theta}$$

where $0 < \theta < 1$.

It follows from the definition of the production technology of final good producers that the partial derivatives of the production function with respect to its arguments are

$$F_{1,t} = \theta K_{t-1}^{(\theta-1)} h_{t}^{1-\theta}$$
$$= \theta \frac{F (K_{t-1}, h_t)}{K_{t-1}}$$

$$F_{2,t} = (1 - \theta) K_{t-1}^\theta h_t^{-\theta}$$
$$= (1 - \theta) \frac{F (K_{t-1}, h_t)}{h_t}$$

Following Schmitt-Grohé and Uribe (2004), I assume the following functional form for the transactions costs

$$L (v_t) = \gamma_1 v_t + \gamma_2 \frac{1}{v_t} - 2\sqrt{\gamma_1 \gamma_2}$$

Thus, some useful expressions with the derivative of the transaction cost function are given below

$$L' (v_t) = \gamma_1 - \gamma_2 \frac{1}{v_t^2}$$
$$v_t L' (v_t) = \gamma_1 v_t - \gamma_2 \frac{1}{v_t}$$
$$v_t^2 L' (v_t) = \gamma_1 v_t^2 - \gamma_2$$

It is worth to note the satiation level of consumption-based velocity of money $\not{\sqrt{}}$ is given by

$$L' (\not{\sqrt{}}) = 0$$
$$\Leftrightarrow \gamma_1 - \gamma_2 \frac{1}{\not{\sqrt{}}}^2 = 0$$
$$\Leftrightarrow \frac{1}{\not{\sqrt{}}}^2 = \frac{\gamma_1}{\gamma_2}$$
$$\Leftrightarrow \not{\sqrt{}} = \left(\frac{\gamma_2}{\gamma_1}\right)^\frac{1}{2}$$

### 2.3 Equilibrium

The stationary non-linear equilibrium equations of the gold standard economy are (3), (5), (6), (10), (23), (24), (25), (26), (28), (29), (32), (33), (34), (36), (37) and (39).

Below, I restate the stationary non-linear equilibrium equations using the functional forms defined above. Before doing so, I proceed to the following change of variables:

$$w_t = \frac{W_t}{P_t}, \quad r_t = \frac{R_t}{P_t}, \quad P_{g,t} = \frac{P_{g,t}}{P_t}, \quad m_t = \frac{M_t}{P_t} \quad (40)$$

which denotes, respectively, real wage, real rental price of capital, real price of gold and real money balances. Indeed, wage and rental price of capital are always expressed in real terms in the first order conditions of households and firms. Besides, it is convenient for the analysis of the gold standard model to focus on the real price of gold rather than on the price level. Therefore, it is also convenient
to express money in real terms in the equilibrium equations\textsuperscript{29}. Anyway, since the price of gold is assumed fixed, it is easy to come back to definition of the price level.

The money supply rule, (3), is restated as follows

$$m_t = \frac{1}{\mu_{1,t}} \frac{1}{\mu_{2,t}} p_{g,t} G_{m,t}$$

(41)

The consumption-based velocity of money, (5), is restated as follows

$$v_t = \frac{C_t}{m_t}$$

(42)

The capital accumulation law is

$$K_t = (1 - \delta_k) K_{t-1} + I_t$$

(6)

The total gold stock demand is

$$G_t^d = G_{c,t} + G_{m,t}$$

(10)

The transaction cost wedge, (23), is restated as follows

$$\kappa_t = 1 + 2\gamma_1 v_t - 2\sqrt{\gamma_1 \gamma_2}$$

(43)

The non-monetary gold pricing equation, (24), is restated as follows

$$\frac{p_{g,t}}{\kappa_t} \left[ \frac{[1 - h_t]^{\phi_2} G_{c,t}^{\phi_3}}{(C_t - bC_{t-1})^{1 - \phi_1(1 - \phi_4)}} - b\beta E_t \left( \frac{[1 - h_{t+1}]^{\phi_2} G_{c,t+1}^{\phi_3}}{(C_{t+1} - bC_t)^{1 - \phi_1(1 - \phi_4)}} \right) \right] - \frac{\phi_2}{\phi_1} \frac{[C_t - bC_{t-1}]^{\phi_1} (1 - h_t)^{\phi_4}}{G_{c,t}^{\phi_3(1 - \phi_4)}} = (1 - \delta_g) \beta E_t \left( \frac{p_{g,t+1}}{\kappa_{t+1}} \right) \times \left[ \frac{[1 - h_{t+1}]^{\phi_2} G_{c,t+1}^{\phi_3}}{(C_{t+1} - bC_{t+1})^{1 - \phi_1(1 - \phi_4)}} - b\beta \frac{[1 - h_{t+2}]^{\phi_2} G_{c,t+2}^{\phi_3}}{(C_{t+2} - bC_{t+1})^{1 - \phi_1(1 - \phi_4)}} \right]$$

(44)

The optimal labor supply condition, (25), is restated as follows

$$\frac{\phi_2}{\phi_1} \frac{[C_t - bC_{t-1}]^{\phi_1} G_{c,t}^{\phi_3}}{(1 - h_t)^{1 - \phi_2(1 - \phi_4)}} \left[ \frac{[1 - h_t]^{\phi_2} G_{c,t}^{\phi_3}}{(C_t - bC_{t-1})^{1 - \phi_1(1 - \phi_4)}} - b\beta E_t \left( \frac{[1 - h_{t+1}]^{\phi_2} G_{c,t+1}^{\phi_3}}{(C_{t+1} - bC_t)^{1 - \phi_1(1 - \phi_4)}} \right) \right]^{-1} \frac{w_t}{\kappa_t}$$

(45)

The Euler equation for money demand, (26), is restated as follows

$$\gamma_1 v_t^2 - \gamma_2 = 1 - \beta E_t \left( \frac{\kappa_t}{\kappa_{t+1}} \frac{p_{g,t+1}}{p_{g,t}} \left[ \frac{[1 - h_t]^{\phi_2} G_{c,t}^{\phi_3}}{(C_t - bC_{t-1})^{1 - \phi_1(1 - \phi_4)}} - b\beta \frac{[1 - h_{t+1}]^{\phi_2} G_{c,t+1}^{\phi_3}}{(C_{t+1} - bC_t)^{1 - \phi_1(1 - \phi_4)}} \right]^{-1} \times \left[ \frac{[1 - h_{t+1}]^{\phi_2} G_{c,t+1}^{\phi_3}}{(C_{t+1} - bC_{t+1})^{1 - \phi_1(1 - \phi_4)}} - b\beta \frac{[1 - h_{t+2}]^{\phi_2} G_{c,t+2}^{\phi_3}}{(C_{t+2} - bC_{t+1})^{1 - \phi_1(1 - \phi_4)}} \right] \right)$$

(46)

\textsuperscript{29}For example, if the price level is expressed only relatively to the gold price, money must be stated in real terms to make the money supply rule, (3), consistent.
The optimal intertemporal consumption decision, (28), is restated as follows

\[
\frac{1}{\kappa_t}\left[ \left( 1 - h_t \right)^{\phi_2} G_{c,t}^{\phi_3} \right]^{1 - \phi_4} - b \beta E_t \left( \left( 1 - h_{t+1} \right)^{\phi_2} G_{c,t+1}^{\phi_3} \right) = b \beta E_t \left( \left( 1 - h_{t+2} \right)^{\phi_2} G_{c,t+2}^{\phi_3} \right)^{1 - \phi_4} \right] \left( r_{t+1} + 1 - \delta_k \right)
\]

(47)

The liquidity preference function, (29), is restated as follows

\[
\gamma_1 v_t^2 - \gamma_2 = 1 - \frac{1}{q_t}
\]

(48)

The optimal labor demand condition, (32), is restated as follows

\[
w_t = (1 - \theta) \frac{Y_t}{h_t}
\]

(49)

The optimal capital demand condition, (33), is restated as follows

\[
r_t = \theta \frac{Y_t}{K_{t-1}}
\]

(50)

The production technology constraint is

\[
Y_t = z_t K_{t-1}^\theta h_t^{1-\theta}
\]

(51)

The total gold stock accumulation law is

\[
G_{t}^d = G_{t-1}^d - \delta_y G_{c,t-1} + g_t^s
\]

(36)

The resource constraint of the economy is

\[
Y_t = C_t + I_t
\]

(39)

In addition, I define the real interest rate equilibrium equation

\[
\bar{r}_t = q_t E_t \left( \frac{p_{g,t+1}}{p_{g,t}} \right)
\]

(52)

Besides, I recapitulate the exogenous stochastic processes

\[
\ln z_t = (1 - \rho_z) \ln z + \rho_z \ln z_{t-1} + \varepsilon_{z,t}
\]

(12)

\[
\ln \mu_{1,t} = (1 - \rho_{\mu_1}) \ln \mu_1 + \rho_{\mu_1} \ln \mu_{1,t-1} + \varepsilon_{\mu_{1,t}}
\]

(13)

\[
\ln \mu_{2,t} = (1 - \rho_{\mu_2}) \ln \mu_2 + \rho_{\mu_2} \ln \mu_{2,t-1} + \varepsilon_{\mu_{2,t}}
\]

(14)

\[
g_t^s = (1 - \rho_{g^s}) g^s + \rho_{g^s} g_{t-1}^s + \varepsilon_{g^s,t}
\]

(15)

Hence, a stationary competitive equilibrium is a set of processes

\[
\{ Y_t, C_t, I_t, K_t, h_t, w_t, G_{c,t}, G_{m,t}, G_{t}^d, m_t, r_t, q_t, \bar{r}_t, p_{g,t}, v_t, \kappa_t \}
\]

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satisfying the following non stationary equilibrium equations (41), (42), (6), (10), (43), (44), (45), (46), (47), (48), (49), (50), (51), (36), (39) and (52), given the exogenous stochastic processes for \( z_t, \mu_{1,t}, \mu_{2,t} \) and \( g_t^2 \), (12), (13), (14) and (15), and initial conditions \( K_{-1}, C_{-1}, G_{c,-1}, G_{d,-1} \).

If the preferences over consumption, non-monetary gold and leisure are separable, \( \phi_4 = 1 \), and the preferences over consumption are time separable, \( b = 0 \), then equilibrium equations (44), (45), (46) and (47), in that order, take the following simpler forms:

\[
\frac{p_{g,t}}{\kappa_tC_t} - \frac{\phi_3}{\phi_1 G_{c,t}} = (1 - \delta_g) \beta E_t \left( \frac{p_{g,t+1}}{\kappa_{t+1}C_{t+1}} \right) \quad (44')
\]

\[
\frac{\phi_2 C_t}{\phi_1 1 - h_t} = w_t \quad (45')
\]

\[
\gamma_1 v_t^2 - \gamma_2 = 1 - \beta E_t \left( \frac{C_t}{C_{t+1}^\kappa_t} \frac{p_{g,t+1}}{p_{g,t}} \right) \quad (46')
\]

\[
\frac{1}{C_t^\kappa_t} = \beta E_t \left( \frac{1}{C_{t+1}^\kappa_{t+1}} \left[ r_{t+1} + 1 - \delta_k \right] \right) \quad (47')
\]

The expressions of the remaining equilibrium equations, (41), (42), (6), (10), (43), (48), (49), (50), (51), (36), (39) and (52), are not modified when \( \phi_4 = 1 \) and \( b = 0 \).

It is worth to note from the equilibrium equations (44), (45), (46) and (47) that the transaction cost wedge, \( \kappa_t \), affects the economy as a negative preference shock to consumption would do. It is easier to observe this in the case where \( \phi_4 = 1 \) and \( b = 0 \). Several authors, such as Baxter and King (1991), have studied the effect of preference shocks on the business cycle. Thus, in Baxter and King (1991), the equilibrium condition on labor supply and the Euler equation on capital are similar to the equilibrium equations (45') and (47'). In particular, the transaction cost wedge manifests itself as a negative preference shock in (45') and (47'). Hence, the transaction cost wedge creates an urge to slow down consumption.

As shown in equation (48), the consumption-based velocity of money, \( v_t \), fluctuates directly with the gross nominal interest rate, \( q_t \). As a result, the fluctuations in the transaction cost wedge, \( \kappa_t \), are driven, endogenously, by the gross nominal interest rate.

From the expression of the liquidity preference function (48) and the expression of the consumption-based velocity of money (42), I can derive the expression of the real money demand function. Specifically, I plug (42) into (48), then I get the expression of money demand function

\[
\left( \frac{C_t}{m_t} \right)^2 = \frac{\gamma_2}{\gamma_1} + \frac{1}{\gamma_1} q_t - \frac{1}{\gamma_1} q_t
\]

\[
\Leftrightarrow m_t = C_t \left( \frac{\gamma_2}{\gamma_1} + \frac{1}{\gamma_1} q_t - \frac{1}{\gamma_1} q_t \right)^{-\frac{1}{2}} \quad (53)
\]

Hence, real money balances are increasing with consumption and decreasing with nominal interest rate.

Besides, writing the money demand expression in logarithm,

\[
\ln m_t = \ln C_t - \frac{1}{2} \ln \left( \frac{\gamma_2}{\gamma_1} + \frac{1}{\gamma_1} - \frac{1}{\gamma_1} q_t \right)
\]

one can note that the elasticity of money demand with respect to consumption is equal to unity

\[
\frac{\partial \ln m_t}{\partial \ln C_t} = 1
\]

23
The optimal condition on the marginal rate of substitution of consumption for non-monetary gold, \( (30) \), can be restated as follows

\[
p_{g,t} \left( 1 - \frac{1 - \delta_g}{q_t} \right) = \frac{\phi_3 \Omega_t^{1-\phi_4}}{\phi_1} \frac{1}{G_{c,t}} - \frac{1}{C_{t-1} - bC_{t-1}} - b\beta E_t \left( \Omega_{t+1}^{1-\phi_4} \right)
\]

To analyze how consumption and non-monetary gold stock interact, I focus, here, on the simple case where preferences are separable, \( \phi_4 = 1 \), and display no habit formation in consumption, \( b = 0 \). Therefore, equation (54) becomes

\[
p_{g,t} \left( 1 - \frac{1 - \delta_g}{q_t} \right) = \frac{\phi_3 C_t}{\phi_1 G_{c,t}}
\]

Then, I log-linearize (54’) in the neighborhood of the steady state:

\[
\dot{C}_t - \left( \dot{p}_{g,t} + \dot{G}_{c,t} \right) + \kappa_t = \omega \dot{q}_t
\]

\[
\Leftrightarrow \dot{C}_t - \left( \dot{p}_{g,t} + \dot{G}_{c,t} \right) = \varphi \dot{q}_t
\]

where \( \omega = \frac{\beta(1-\delta_g)}{1-\beta(1-\delta_g)} \), \( \varphi = \omega - \frac{\beta}{\omega} \), and \( \dot{x}_t = \ln \left( \frac{x_t}{x_{t-1}} \right) \). The move from equation (55) to equation (56) is done because the transaction cost wedge is linked only to the gross nominal interest rate (actually through the consumption-based velocity of money, as shown in equilibrium equations (43) and (48)). Thus, both equations (55) and (56) point out how the nominal shocks — the gold flow shock and the two money supply shocks — are propagated to the real part of the economy. The mechanism of transmission works through the transaction cost wedge, on the one hand, the non-monetary gold stock optimal choice, on the other hand. Written that way, equations (55) and (56), refer to the optimal condition on the marginal rate of substitution of consumption, \( C_t \), for real non-monetary gold, \( p_{g,t} G_{c,t} \), rather than on the marginal rate of substitution of consumption, \( C_t \), for non-monetary gold. Hence, in equilibrium, the marginal rate of substitution of consumption for real non-monetary gold must be equal to a linear function of the nominal interest rate. However, the transactions costs introduce a wedge, \( \kappa_t \), in that optimal condition. Clearly, equation (55) is more convenient than equation (56) if one wants to highlight the role of the transaction cost wedge in the optimal condition on the marginal rate of substitution of consumption for real non-monetary gold. Under reasonable calibration, the coefficients \( \omega \) and \( \varphi \) are more likely to be strictly positive. From both equations (55) and (56) one can note that the effects of the nominal shocks on consumption can be ambiguous. Indeed, the responses of consumption to the nominal shocks depend on how the gross nominal interest rate, the real price of gold, the non-monetary gold stock and the transaction cost wedge interact between themselves. Assuming that \( \omega, \varphi > 0 \) (and hence \( \omega > \varphi \)), the transactions costs come to weaken the positive effect of the gross nominal interest rate on consumption.

Besides, from equation (56), one can note that for any given consumption, \( C_t \), if following a positive nominal shock — an increase of gold flow shock, \( g_t \), or a decrease of money supply shocks, \( \mu_{i,t} \) — the real non-monetary gold stock, \( p_{g,t} G_{c,t} \), increases, the gross nominal interest rate, \( q_t \), has to decline. I call the negative effect of the nominal shocks on the gross nominal interest rate, the \textit{non-monetary gold liquidity effect}. Indeed, it resembles the standard \textit{liquidity effect}\(^{31}\). The liquidity effect and the non-monetary gold liquidity effect are interrelated: When the latter arises, the former shows up too.

As well, equations (55) and (56) point out how the real shock — the productivity shock — is transmitted to the nominal part of the economy.

The equilibrium equation (52) represents the Fisher equation which relates the real and nominal interest rates to the expected inflation\(^{32}\). The real interest rate, \( \tilde{r}_t \), is linked to the capital gross real rate of return as follows: \( \tilde{r}_t = E_t \left( r_{t+1} + 1 - \delta_b \right) \).

\(^{30}\)The non-separability of the preferences and habit formation in consumption hypotheses do not alter the main points of the following analysis.

\(^{31}\)In the literature, one calls liquidity effect, the negative effect of money supply on the gross nominal interest rate. The liquidity effect takes place when both the nominal money and real money increase.

\(^{32}\)Here, the gross inflation rate is defined with the real price of gold rather than with the price level.
3 The Data

The aim of this work is to evaluate the ability of the gold standard model to replicate the fluctuations of French key macroeconomic variables over the 1929-1936 period. Therefore, this section presents the construction of the historical French data on output, consumption, investment, capital, labor, real wage, nominal money, real money, monetary gold, real price of gold, gross nominal interest rate and gross real interest rate. In addition, it discusses the construction of the four shocks considered in the model: Productivity shock, inverse of money multiplier shock, gold backing shock and gold flow shock. All those data — historical and shocks — are constructed consistently to the model.

3.1 Raw data

Table 1 provides a description of the French raw data used to construct the historical data and shocks. Thus, in the current analysis, I use different sources for the data: The data collected by Villa (1993)\(^\text{33}\), Saint-Marc (1983), Banque de France\(^\text{34}\), INSEE and, Jones and Obstfeld (2001). Note that the raw data are mainly available at the annual frequency\(^\text{35}\). Therefore, I conduct the analysis on the basis that the model period is the year.

Before going through the construction of the historical data and shocks, I need to ensure that all the raw data, but hourly nominal wage in firms, money market rate and weekly average hours worked per worker, are defined are measured in a same unit, i.e. in billions. In particular, the raw data data defined in thousands — series dhe, emp and pop — must be divided by 1000000. The raw data defined in units — series Billets au porteur en circulation, Compte courant du Trésor, Compte courant de la Caisse Autonome d’Amortissement, Comptes courants et comptes de dépôts de fonds, Dispositions et autres engagements à vue and Encaisse Or — must be divided by 100000000.

Even though the historical data might display a determinist trend, I do not remove it. Indeed, for convenience, I have assumed that the gold standard model exhibits no growth: No deterministic growth rate has been introduced in the model. Nevertheless, this point is not crucial since I am interested in a very short window of the historical time span, that is the period going from 1929 to 1936. Bordo et al. (2001) argue that introducing a deterministic trend term in a model might not be appropriate if that model intends to explain an historical event such the Great Depression. As a consequence, removing a deterministic trend from the historical data might not be justified neither. Those authors claims if there is a deterministic trend to remove from data that cover the Great Depression period, it may be a negative one instead of a positive one.

3.2 Historical data

In this subsection, I present and discuss the construction of annual historical French data on output, consumption, investment, capital, labor, real wage, nominal money, real money, monetary gold, real price of gold, nominal interest rate and real interest rate. The historical data set covers the interwar period.

3.2.1 Output and its components

The historical measure of consumption is assumed to be equal to the real household consumption time series (series czm). In turn, the historical measure of investment is obtained by adding together the real household investment time series (series izon) and the real firm investment time series (series ize). Consistently to the baseline and learning model, the historical measure of output is got by taking the sum of the historical measures of consumption and investment.

Because output variable and its components — consumption and investment — are defined in per capita terms in the gold standard model, their respective empirical measures are divided by the population measure (series pop). Then, I deflate the per capita measures of output, consumption

\(^{33}\)The Villa (1993)’s data are freely available in the website of the CEPII: www.cepii.fr.

\(^{34}\)The Banque de France ara available in its website: www.banque-france.fr

\(^{35}\)As far as I know, there are no French macroeconomic data, except for money, monetary gold and interest rate variables, available at a higher frequency.
Table 1: French Raw Data

<table>
<thead>
<tr>
<th>Name</th>
<th>Source</th>
<th>Coverage period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor market and population</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weekly average hours worked per worker</td>
<td>Villa P., 1993 (dh)</td>
<td>Annual 1919-1939 and 1946-1985</td>
</tr>
<tr>
<td>Hourly nominal wage in firms (prudhommes)</td>
<td>Villa P., 1993 (shpe)</td>
<td>Annual 1913 and 1920-1939</td>
</tr>
<tr>
<td>National income account</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real household consumption (in billion franc of 1938)</td>
<td>Villa P., 1993 (cem)</td>
<td>Annual 1804-1939</td>
</tr>
<tr>
<td>Real firm investment (in billion franc of 1938)</td>
<td>Villa P., 1993 (ie)</td>
<td>Annual 1804-1939</td>
</tr>
<tr>
<td>Nominal household consumption (in billion franc)</td>
<td>Villa P., 1993 (en)</td>
<td>Annual 1804-1939</td>
</tr>
<tr>
<td>Nominal household investment (in billion franc)</td>
<td>Villa P., 1993 (im)</td>
<td>Annual 1804-1985</td>
</tr>
<tr>
<td>Nominal firm investment (in billion franc)</td>
<td>Villa P., 1993 (ie)</td>
<td>Annual 1804-1985</td>
</tr>
<tr>
<td>Money, gold and interest rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Notes in circulation (in franc)</td>
<td>Banque de France (Billets au porteur en circulation)</td>
<td>Weekly 14/04/1808-21/03/1974</td>
</tr>
<tr>
<td>Current account of the Treasury (in franc)</td>
<td>Banque de France (Compte courant du Trésor)</td>
<td>Weekly 14/04/1808-21/03/1974</td>
</tr>
<tr>
<td>Current account of the autonomous sinking fund (in franc)</td>
<td>Banque de France (Compte courant de la Caisse Autonome d’Amortissement)</td>
<td>Weekly 28/06/1928-29/12/1949</td>
</tr>
<tr>
<td>Current account and deposits (in franc)</td>
<td>Banque de France</td>
<td>Weekly 28/06/1928-29/12/1949</td>
</tr>
<tr>
<td>Other liabilities on current account (in franc)</td>
<td>Banque de France</td>
<td>Weekly 28/06/1928-29/12/1949</td>
</tr>
<tr>
<td>Gold reserve (coln and bullion) (in franc)</td>
<td>Banque de France</td>
<td>Weekly 14/04/1808-21/03/1974</td>
</tr>
</tbody>
</table>
and investment by the 1929 value of the per capita measure of output. As a consequence, output, for example, is evaluated to unity in 1929. By doing so, I ensure that the historical data on output, consumption and investment are defined in the same scale as the data generated by the model.

The empirical measures of output and its components cover the 1919-1939 period.

3.2.2 Labor

I construct the empirical measure of labor as follows. First, I compute the total number of hours worked each year of the interwar period, using data on total employment (series emp) and weekly average number of hours worked per worker (series dh), and assuming that a year consists in 52 weeks. Second, I express the total number of hours worked in per capita terms using data on population (series pop). Finally, I divide the per capita total number of hours worked by the total time an individual is endowed each year, on average, in order to get an empirical measure of labor consistent with the model. In particular I assume that a household can allocate 4992 hours per year between market and non-market activities.

The empirical measure of labor covers the 1919-1939 period.

3.2.3 Capital

In the reported raw data there is a series of firms’ real capital stock (series kze). However, the constructed data on investment includes not only the investment made by firms but also those realized by households. Therefore, the raw data on capital may not reflect correctly all the investment done in the French economy during the interwar period. Thus, I construct a new series for capital using the capital accumulation law defined in the gold standard model (equations (6)) and the constructed investment series. To generate a series of capital with the accumulation law equation, I still need an initial value for capital and to assign a value to the capital depreciation rate, \( \delta_k \). To do so, first I divide the raw data on firms’ capital (series kze) by the population series (series pop). Then, I divide the per capita series of firms’ capital by the 1929’s value of per capita output. Second, I calibrate the capital depreciation rate using the steady state expression of the capital accumulation law, together with the per capita firms’ capital stock, denoted \( k^i_t \), and the empirical measure of investment, denoted \( i^d_t \),

\[
\delta_k = \frac{\bar{i}_t}{\bar{k}^i_t}
\]

where \( \bar{i}_t \) denotes the sample mean of the investment-firm’s capital ratio, \( i^d_t / k^i_t \), over the 1896-1939 period. Thus, \( \delta_k \) is evaluated to 0.0752. Third, I generate a new capital series over the 1896-1939 period, using the capital accumulation law, a capital depreciation rate of 7.5 percent and the 1896’s firms capital value as an initial value. Finally, I extract the 1919-1939 sample from the new capital time series.

3.2.4 Interest rate, wage and real price of gold

I take the nominal market rate (series txxm) as a measure of the nominal interest rate. In particular, I construct a gross nominal interest rate time series, consistently to the model, over the 1919-1939 period as follows

\[
q^d_t = 1 + \frac{mmr_t}{100} \quad \forall t = 1919, \ldots, 1939
\]

where \( q^d_t \) denotes the empirical measure of nominal interest rate and \( mmr_t \) corresponds to the raw series txxm.

I need a measure of price to in order to construct the series of real wage and gross real interest rate. To get a measure of price, I calculate the output deflator, that is the ratio of nominal output to real output. I get a measure of nominal output in the same way as I calculate the real output, using data on nominal household consumption (series cm), nominal household investment (series im) and nominal firm investment (series i.e). Then, the obtained output deflator is redefined such that it takes the value of 100 in 1929. Besides, as output, it covers the 1919-1939 period.
Thus, the historical measure of real wage is computed as the ratio of the hourly nominal wage in firms series (series whpe) to the output deflator series. The constructed series of real wage covers the 1920-1939 period.

In turn, I calculate the historical measure of gross real interest rate, over the 1919-1939 period, using the following formula

\[ \hat{r}_t^d = \frac{q_t^d}{\pi_t^d}, \quad \pi_{t+1}^d = \frac{p_{t+1}^d}{p_t^d}, \quad \forall \ t = 1919, \ldots, 1939 \]

where \( \hat{r}_t^d \) denotes the real interest rate and \( p_t^d \) the output deflator.

Recall that under the gold standard monetary system, the price of gold is assumed to be fixed. As a simplification and without loss of generality, I normalize the price of gold to unity. It follows that the real price of gold is defined as the inverse of the price level. Therefore, I construct an empirical measure of real price of gold by taking the inverse of the output deflator: \( p_{t+1}^d = \frac{1}{p_t^d} \). Since the price of gold has been maintained fixed between June 1928 and September 1936 in France\(^{36} \), the constructed real price of gold series covers the 1928-1936 period.

### 3.2.5 Monetary gold and money

Note that since I normalize the fixed price of gold to unity, the monetary gold stock is evaluated in nominal terms. The time series of nominal monetary gold stock owned by the Banque de France (Encaisse Dr) is provided by that institution. Being available at a weekly frequency, I need to convert the data on monetary gold to annual frequency. Since monetary gold is a stock data, I do the conversion as follows

\[ G_{m,t}^{ad} = G_{m,t}^{wd}, \quad \forall \ t = 1898, \ldots, 1939 \]

where \( G_{m,t}^{ad} \) denotes the annual monetary gold stock series and \( G_{m,t}^{wd} \) the weekly monetary gold stock series. The time index \( \tau_t \) indicates the last week of the year \( t \). In words, for example, the 1920 value of the annual monetary gold stock series is equal to the last week of december 1920 value of the weekly monetary gold stock. Besides, note that France experiences a devaluation in 1928. In particular, the monetary law of June 1928 brings a new definition of the French currency: The franc poincaré. Hence, 1 franc germinal (old currency definition) is equal to 4.925 franc poincaré. Therefore, I reevaluate the monetary gold stock series in franc poincaré using that conversion rate. Note that the monetary gold stock series needs to be converted only for the 1898-1927 period:

\[ G_{m,t}^{d,f} = 4.925 \times G_{m,t}^{d,f}, \quad \forall \ t = 1898, \ldots, 1927 \]

where \( G_{m,t}^{d,f} \) denotes the value of monetary gold stock in franc poincaré and \( G_{m,t}^{d,f} \) the value of monetary gold stock in franc germinal.

In the gold standard model, the nominal money variable is identified as being the monetary aggregate M1. Thus, I construct the monetary aggregate M1 by taking the sum of notes in circulation issued by the Banque de France (Billets au porteur en circulation) and total sight deposits (Dépôts à vue). I exclude metallic coins from the definition of the monetary aggregate M1. Indeed, in France, during the gold standard period (classical gold standard period and gold-exchange standard period), a large part of the metallic coins are in gold and silver (France has been in a bimetallism monetary system before joining the gold standard monetary system). According to Saint-Marc (1983), coins in copper or bronze happen to be the main metallic coins in France only after 1936, that is once the country left the gold standard monetary system.

The time series of notes in circulation is provided by the Banque de France. As Saint-Marc (1983) emphasizes it, the historical data on notes in circulation are of good quality since the Banque de France has always causiously kept its note issues in book-record. Because the data on notes in circulation are reported at a weekly frequency, I have to convert them to annual frequency. To do so, I proceed as for the monetary gold data.

\(^{36}\)In France, the price of gold is set to 19.96 francs poincaré over the period going from June 1928 to September 1936.
A time series of the French sight deposits, over the period 1807-1952, is provided by Saint-Marc (1983). Those data include the sight deposits in the Bank of France, the sight deposits in all commercial banks and the postal current accounts.

According to Saint-Marc (1983), the value of sight deposits in the Bank of France is correctly evaluated over the full period since, as said above, that institution kept its banking activities in book-record. The postal current accounts exist only since 1920. Nonetheless, the value of the postal current accounts is well measured since the institution managing those assets has always kept its activities in book-record too.

However, there are no good or official evaluation of the value of commercial banks’ sight deposits for the full period. In particular, Saint-Marc (1983) tells that the regulation of the banking activity is regimented only since the World War 2 in France. The first evaluations of the French banking activity start in 1900 and the official recordings in 1942. Therefore, Saint-Marc (1983) proposes her own evaluation of the sight deposits of commercial banks. To do so, the author proceeds methodically by subperiods. Since, the current work focuses only on the interwar period, I describe briefly her method for the post 1900 (included) period. Saint-Marc (1983) uses the measure of total deposits constructed by the INSEE\(^3\) for the period going from 1900 to 1950. Subtracting the time series of the Bank of France’s sight deposits from the INSEE’s time series, the author deduces the value of commercial banks’ sight deposits over the post-1900 period. The author claims that the INSEE’s data have to be taken with caution because they have been overevaluated.

In addition, I construct a time series of the real money variable. To do so, I simply divide the time series of monetary aggregate M1 by the measure of price level, \(p_t\).

Finally, the constructed data on nominal money, real money and nominal monetary gold stock are divided by the population series then scaled by the 1929s’ value of per capita output. The final empirical measures of nominal money, real money and nominal monetary gold stock cover the 1919-1939 period.

3.2.6 Summary

To summarize this subsection, Figures 1 and 2 display the annual historical data constructed above, over the 1928-1936 period. The historical data are plotted in log-deviation from their respective 1929 level.

![Graphs](image.png)

**Figure 1:** French historical data, 1929 = 0 (a).

\(^3\)The Institut National de la Statistique et des Études Économiques is the French national institute on statistics and economic studies.
From Figure 1, one can observe that output and investment fall dramatically from 1930 onwards. Consumption starts to decrease one year earlier than output and investment. However, the former experiences a milder collapse than the latter. The capital stock starts to decline when investment moves below its 1929 level. Nonetheless, since investment rises strongly before 1930, capital remains above its 1929 level until 1936. Labor declines significantly between 1929 and 1932. Then it remains at a low level until 1936. Real wage rises strongly over all the 1928-1936 period.

![Monetary gold, Nominal money, Nominal interest rate, Real price of gold, Real money, Real interest rate graphs](image)

Figure 2: French historical data, 1929 = 0 (b).

As it can be noted from Figures 2, the monetary gold stock increases strongly between 1928 and 1932. Then, it declines until 1936. Nonetheless, the monetary gold stock remains above its 1929 level between 1929 and 1936. Nominal money also rises between 1928 and 1931. It is worth to note that the nominal money increase is milder than the monetary gold stock one. From 1931 to 1935, nominal money declines significantly. It moves below its 1929 level between 1934 and 1935. Besides, the real price of gold drops between 1928 and 1930. After what it rises until 1936. Note that the real price of gold moves above its 1929 level as soon as the nominal money starts to fall. The gross nominal interest rate collapses between 1929 and 1931. Then, it moves upwards again, reaching back its 1929 level in 1935. The 1930-1935 inflation in the real price of gold drives upwards the gross real interest rate during that period. As well, the real money rises between 1929 and 1936.

### 3.3 Shocks

In this subsection, I present and discuss the construction of the four shocks considered in the model over the interwar period.

#### 3.3.1 Productivity shocks

Here, I describe and discuss the construction of a historical sequence of productivity shocks. They are defined as being movements in output which are not explained by movements in factor inputs.

Note that the productivity shocks are not directly observable in the macroeconomic data. Nevertheless, they can be measured conditionally to the form of the macroeconomic production function. In particular, a measure of technology shocks, $\zeta^d_t$, associated to a constant return to scale Cobb-Douglas production function with two inputs — capital and labor — can be generated, for the interwar period, as follows

$$\zeta^d_t = \frac{y_t^d}{(k_{t-1}^d)^{\sigma} (h_t^d)^{1-\sigma}} \quad \forall, t = 1920, \cdots, 1939$$

(57)
where $y^d_t$, $k^d_t$ and $h^d_t$ denote the historical measures of, respectively, output, capital and labor.

### 3.3.2 Gold flow shocks

Here, I describe and discuss the construction of a historical sequence of gold flow shocks. They are defined as being the entry/exit flows of gold in/out of the economy.

The INSEE, in its Annuaire Statistique (1966), reports the net exports of gold in France for the interwar period, evaluated in franc poincaré. However, there are missing observations in 1925 and 1926. Jones and Obstfeld (2001) propose also a measure of net gold exports from 1850 to 1938. Those authors splice together different data on gold exports over that period. In particular, their data between 1930 and 1938 are taken from Annuaire Statistique (1966). Thus, I fill the missing observations of the net gold export series of Annuaire Statistique (1966) with those collected by Jones and Obstfeld (2001). Note that the 1925 and 1926’s values of net gold exports, taken from Jones and Obstfeld (2001), are converted in franc poincaré in order to make them consistent with the data of Annuaire Statistique (1966). Note that there are some differences between the Annuaire Statistique (1966)’s data and those collected by Jones and Obstfeld (2001) for the 1920-1930 period. Here, to construct a time series of gold flow shocks, I favor the data from Annuaire Statistique (1966) rather than those from Jones and Obstfeld (2001) because the former is directly originated from France.

In the gold standard model, the gold flow variable can be interpreted as a net gold inflow. Therefore, I construct the gold flow shock series by taking the minus net gold export time series over the 1920-1938 period. In addition, the constructed gold flow shock series is divided by the population series then scaled by the 1929’s value of per capita output.

### 3.3.3 Money supply shocks

Here, I describe and discuss the construction of historical sequences of the two money supply shocks considered in the gold standard model: The inverse of money multiplier shocks and the gold backing shocks. The former represents the ratio of monetary base to monetary aggregate M1. The latter represents the ratio of monetary gold to monetary base. Therefore, I need a measure of the monetary base.

The monetary base is defined as being the sum of notes in circulation and the Banque de France liabilities on current account. Specifically, the Banque de France liabilities on current account consist in the current account of the Treasury (Compte courant du Trésor), the current account of the autonomous sinking fund (Compte courant de la Caisse Autonome d’Amortissement), current account and deposits (Dispositions et autres engagements à vue), and other liabilities on current account (Dispositions et autres engagements à vue). Note that this definition of the Banque de France liabilities on current account is essentially based on the monetary law of June 1928 and particularly on the new balance sheet of the Banque de France. Indeed, from Table 1, one can observe that only the data on the current account of Treasury are available before 1928. According to Morant (1951), the current account of the autonomous sinking fund has been introduced in 1926. However, in the website of the Banque de France the data on the current account of the autonomous sinking fund are available only starting from mid-1928. As a consequence, for the period pre-1928 (excluded), I construct the monetary base time series by taking the sum of notes in circulation and current account of the Treasury. Note that the obtained measure of monetary base is, at this stage, defined at a weekly frequency since the raw data used to compute it are displayed at this frequency. Hence, I convert that measure of monetary base to annual frequency following the same procedure as for the monetary gold data.

The constructed time series of monetary base is divided by the population series then scaled by the 1929’s value of per capita output in order to make it consistent with the time series on monetary gold and monetary aggregate M1.

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38 For the period going from 1915 to 1930, Jones and Obstfeld (2001) take their data from those collected by the League of Nations.
3.3.4 Summary

To summarize this subsection, Figure 3 shows the realizations of the shocks constructed above, over the 1928-1936 period. The realizations of the technology shock and those of the money supply shocks — inverse of money multiplier shocks and gold backing shocks — are plotted in log-deviation from their respective 1929 level. Instead, the realizations of the gold flow shock are reported in deviation from their 1929 level because they can take negative values.

![Figure 3: French historical shocks, 1929–100.](image)

From Figure 3, one can observe that the productivity shock follows the same pattern as output. In particular, it starts to collapse in 1930 where it is above its 1929 level. The productivity shock drop lasts until the end of the 1928–1936 period. The inverse of money multiplier shock increases during all the gold-exchange standard period. As well, the gold backing shock rises between 1928 and 1934. After what, it declines but still remains above its 1929 level until the end of the period. The starred line in the subplot represents the log-deviation of the gold backing shock from that the 35 percent rule on the monetary gold-monetary base ratio. Clearly the policy rule is respected over all the period. Note that the gold backing shock increase happens to be larger than the inverse of money multiplier shock increase. It is worth to note that, taking together, the movements in the two money supply shocks explain the relative weakness of the nominal money increase comparatively to the monetary gold rise.

As it can observed in Figure 4, the French economy experiences a gold (net) inflow between 1928 and 1933. The net imports of gold start to decline in 1932. From 1933 onwards, gold moves away from the country. It is worth to note that as gold flows out of France, the monetary gold reserve of the Banque de France declines.

Bernanke and Mihov (2000) decompose the identity that links the nominal money stock of a country to its monetary gold stock into several ratios. They do that for several countries that are in the gold standard during the interwar period. For France, they find that the money multiplier decreases strongly between 1928 and 1936 while the gold backing ratio remains relatively stable during that same period. Thus, my results contrast with the ones of those authors: I find that the gold backing shock is significantly more volatile than the inverse of money multiplier shock between 1928 and 1936. The difference between my measure of the gold backing ratio and the one of Bernanke and Mihov (2000) is mainly due to the fact these two measures of the gold backing ratio are not identically defined. Specifically, those authors define the gold backing ratio as the ratio of the monetary base to the international reserves of the central bank. The international reserves consist of the foreign exchange and monetary gold held by the central bank. In the construction of my measure of the gold backing ratio, I do not include the foreign assets held by the central bank.
Indeed, even though the French central bank is allowed to hold foreign exchange, they are not taken into account in the definition of the legal minimum coverage ratio. Specifically, the Bank of France is legally required to ensure that its gold stock exceeds or is equal to 35 percent of its sight liabilities (monetary base). An other source of difference can come from the construction of the monetary base. In Bernanke and Mihov (2000), the monetary base is defined as the sum of notes in circulation and bank reserves. However, they do not explicitly precise of what consist the bank reserves in the case of France. Nevertheless, my measure of the gold backing ratio is consistent with the one constructed by Mouré (1991, Table 2.1, col. Reserve ratio, pp. 55-56).

My measure of the money multiplier and the one of Bernanke and Mihov (2000) follow the same definition: The money multiplier corresponds to the ratio of money aggregate M1 to monetary base. The difference between the two measures of the money multiplier can come from two sources. As said above, I might not have constructed the monetary base as have done those authors. Furthermore, my measure of the monetary aggregate M1 might be different from the one of Bernanke and Mihov (2000). Indeed, those authors define the nominal money stock as being equal to the notes in circulation plus the total commercial bank deposits. I have also included the private deposits held by the Bank of France in the definition of M1. Indeed, during the interwar period, the French central bank is also involved into commercial banking activities.

4 Solution Method, Calibration and Steady State

In this section, I present the solution method used to derive the decision rules of the gold standard model. Then, I discuss the calibration of the model’s parameters and define the steady state of the artificial economy.

4.1 Solution method

The system of equilibrium equations of the gold standard model are log-linearized in the neighborhood of the deterministic steady state. Then, the obtained log-linear system of equilibrium equations are solved using the method developed by Klein (2000). Nonetheless, in order to apply the solution method of Klein (2000), I need to assign values to the parameters of the model and define the steady state of the economy. Appendix A reports in details the log-linearization of the system of equilibrium equations.
4.2 Calibration and steady state

The model contains 11 structural parameters, gathered in the set

\[ \Theta = \{ \beta, \delta_k, \delta_g, \phi_1, \phi_2, \phi_3, \phi_4, \theta, \gamma_1, \gamma_2, b \} \]

8 exogenous stochastic process parameters, gathered in the set

\[ \Delta = \{ \rho_z, \rho_{\mu_1}, \rho_{\mu_2}, \rho_{g^\prime}, \sigma_z, \sigma_{\mu_1}, \sigma_{\mu_2}, \sigma_{g^\prime} \} \]

and 20 variables, which steady state levels are gathered in the set

\[ \Xi = \{ Y, C, I, K, h, w, \bar{r}, p_g, G_c, G_m, G^d, m, v, \kappa, z, \mu_1, \mu_2, g^s \} \]

In order to assign a value to each parameter and steady state variable of the model, I solve a system formed by the deterministic steady-state expressions of the equilibrium equations and 23 additional constraints. When I solve that system, I assume that the time unit in the model is the year.

The deterministic steady state expressions of the equilibrium equations are

\[ m = \frac{1}{\mu_1 \mu_2} p_g G_m \]  
(58)

\[ v = \frac{C}{m} \]  
(59)

\[ K = (1 - \delta_{k,1}) K + I \]  
(60)

\[ G^d = G_c + G_m \]  
(61)

\[ \kappa = 1 + 2 \gamma_1 v - 2\sqrt{\gamma_1 \gamma_2} \]  
(62)

\[ \frac{p_g}{\kappa} \left[ \frac{(1 - h)^{\phi_2} G_c^{\phi_3} (1 - \phi_4)}{(C - bC)^{1 - \phi_1 (1 - \phi_4)}} - b\beta \frac{(1 - h)^{\phi_2} G_c^{\phi_3} (1 - \phi_4)}{(C - bC)^{1 - \phi_1 (1 - \phi_4)}} \right] - \frac{\phi_3}{\phi_1} \frac{(C - bC)^{\phi_1} (1 - h)^{\phi_2} G_c^{\phi_3} (1 - \phi_4)}{G_c^{1 - \phi_1 (1 - \phi_4)}} = \phi_2 \frac{(C - bC)^{\phi_1} G_c^{\phi_3} (1 - \phi_4)}{(1 - h)^{1 - \phi_2 (1 - \phi_4)}} \left[ \frac{(1 - h)^{\phi_2} G_c^{\phi_3} (1 - \phi_4)}{(C - bC)^{1 - \phi_1 (1 - \phi_4)}} - b\beta \frac{(1 - h)^{\phi_2} G_c^{\phi_3} (1 - \phi_4)}{(C - bC)^{1 - \phi_1 (1 - \phi_4)}} \right]^{-1} = \frac{w}{\kappa} \]  
(63)

\[ \gamma_1 v^2 - \gamma_2 = 1 - \beta \frac{\kappa p_g}{\kappa p_g} \left[ \frac{(1 - h)^{\phi_2} G_c^{\phi_3} (1 - \phi_4)}{(C - bC)^{1 - \phi_1 (1 - \phi_4)}} - b\beta \frac{(1 - h)^{\phi_2} G_c^{\phi_3} (1 - \phi_4)}{(C - bC)^{1 - \phi_1 (1 - \phi_4)}} \right]^{-1} \times \left[ \frac{(1 - h)^{\phi_2} G_c^{\phi_3} (1 - \phi_4)}{(C - bC)^{1 - \phi_1 (1 - \phi_4)}} - b\beta \frac{(1 - h)^{\phi_2} G_c^{\phi_3} (1 - \phi_4)}{(C - bC)^{1 - \phi_1 (1 - \phi_4)}} \right] \]  
(65)
\[
\frac{1}{\kappa} \left[ \frac{(1 - h)^{\phi_2} G_{c}^{\phi_3}}{(C - bC)^{1 - \phi_1(1 - \phi_4)}} \right]^{1 - \phi_4} - b\beta \left[ \frac{(1 - h)^{\phi_2} G_{c}^{\phi_3}}{(C - bC)^{1 - \phi_1(1 - \phi_4)}} \right]^{1 - \phi_4} = \beta \left[ \frac{(1 - h)^{\phi_2} G_{c}^{\phi_3}}{(C - bC)^{1 - \phi_1(1 - \phi_4)}} \right]^{1 - \phi_4} - b\beta \left[ \frac{(1 - h)^{\phi_2} G_{c}^{\phi_3}}{(C - bC)^{1 - \phi_1(1 - \phi_4)}} \right]^{1 - \phi_4} \left[ \frac{r + 1 - \delta_k}{\kappa} \right]
\]

(66)

\[
\gamma_1 v^2 - \gamma_2 = 1 - \frac{1}{q}
\]

(67)

\[
w = (1 - \theta) \frac{Y}{h}
\]

(68)

\[
r = \frac{\theta}{K}
\]

(69)

\[
Y = z K^\theta h^{1 - \theta}
\]

(70)

\[
G^d = G^d - \delta_y G_c + g^s
\]

(71)

\[
Y = C + I
\]

(72)

\[
\bar{r} = q \left( \frac{p_g}{p_g} \right)
\]

(73)

The additional constraints are put on the exogenous stochastic process parameters included in \( \Delta \), a subset of structural parameters

\[
\Theta = \{ \delta_k, \delta_y, \phi_1, \phi_2, \phi_3, \phi_4, \theta, b, \gamma_1, \gamma_2 \}
\]

and on a subset of steady state variable levels

\[
\Xi = \{ h, q, z, \mu_1, \mu_2, g^s \}
\]

I present and discuss the restrictions below.

I rely partly on the constructed historical data and partly on the business cycle literature to define the additional restrictions on parameters and steady state levels of variables. The capital share parameter, \( \theta \), is set to 0.34 as in Beaudry and Portier (2002). Those authors find that 66 percent of the French GDP goes to labor in the interwar period.

I restrict the sum of the preference parameters for consumption, non-monetary gold and leisure to be equal to unity: \( \phi_1 + \phi_2 + \phi_3 = 1 \). In addition, the preference parameter for gold parameter, \( \phi_3 \) is set to 0.002. This assumption implies that households put relatively a smaller weight on non-monetary gold than on consumption and leisure. Two alternative values for the utility function curvature parameter, \( \phi_4 \), are considered. The first value, \( \phi_4 = 1 \), corresponds to the case where the household’s preferences are separable in consumption, leisure and non-monetary gold. The second value, \( \phi_4 = 2 \), corresponds to the case where the household’s preferences are non-separable in consumption, leisure and non-monetary gold. As well, the habit formation in consumption parameter, \( b \), can take two alternative values. First, I consider the case where consumption is not subject to habit formation: \( b = 0 \). Second, I consider the case where consumption exhibits habit persistence: \( b = 0.65 \).

\[ ^{39} \text{For the non-separable preferences case, I assign a value (strictly) larger than 1 to } \phi_4 \text{ rather than a value (strictly) lower than 1 in order to ensure the concavity of the utility function.} \]
value for the habit formation parameter, \( b \), is quite standard in the business cycle literature (see, for example, Schmitt-Grohé and Uribe, 2005).

The capital depreciation rate, \( \delta_k \), has been calibrated above, when the construction of an historical measure of capital stock is presented. Specifically, I have set \( \delta_k = 0.0752 \). As in Bordo et al. (2007), the non-monetary gold stock is assumed to depreciate annually at the constant rate of \( \delta_g = 0.02 \).

I estimate the liquidity preference function (48), by the ordinary least square estimation procedure (OLS hereafter), to identify the transaction cost parameters, \( \gamma_1 \) and \( \gamma_2 \), using the annual data on nominal interest rate, \( q^d_t \), consumption, \( c^d_t \) and real money, \( m^d_t \). The results of the estimation, performed over the 1920-1934 period are given by

\[
\left( \frac{c^d_t}{m^d_t} \right)^2 = \frac{1.3391 + 80.6615(t^d_t - 1)}{(2.2074) (5.8569)} \quad \forall \ t = 1920, \ldots, 1934
\]

(74)

\[
R^2 = 0.70
\]

where the numbers in parentheses are the t-statistics. The estimates are significantly different from zero according to the t-statistics. Besides, the adjusted coefficient of determination, \( R^2 \), is large enough. Thus, the transaction cost parameter \( \gamma_1 \) is given by the inverse of the slope the regression, that is \( \gamma_1 = 0.0124 \). The transaction cost parameter \( \gamma_2 \) is equal to the intercept of the regression times \( \gamma_1 \); \( \gamma_2 = 0.0166 \). Nonetheless, those estimation results have to be taken with cautions since the sample size is small. The choice of the sample, over which the estimation is done, is motivated by technical reasons. Indeed, the estimation results based on larger samples — still with the interwar period — become statistically not significant.

I restrict the steady state value of the nominal interest rate to be equal to its 1928’s historical value. Indeed, 1928 can be considered as a “normal” year in France because in June of that year, the Poincaré’s government carries out a monetary law and introduces a new definition of the French currency, the franc poincaré, putting an end to a high inflation period. Besides, that year comes just before the onset of the worldwide Great Depression. Specifically, the steady state values of the nominal interest rate is \( q = q^d_{1928} = 1.0353 \). The steady state value of labor, \( h \), is set to the sample mean of its historical measure over the interwar period: \( h = \overline{h^d_t} \), where the upperbar stands for the sample mean over the 1919-1939 period.

The parameters of the exogenous stochastic processes, gathered in \( \Delta \), are estimated by OLS, using data on productivity shock, \( z^d_t \), inverse of money multiplier shock, \( \mu^d_t \), gold backing shock, \( \mu^d_t \), and gold flow shock, \( g^d_t \). Because the evaluation of the gold standard model, with regard to the French Great Depression, will be done in terms of (log-)deviation from the 1929’s levels, the estimations are based on the autoregressive processes of the exogenous variables restated in (log-)deviation from the 1929’s levels. The steady state values of the exogenous variables, \( z, \mu_1, \mu_2 \) and \( g \), are set equal to the sample mean of their respective empirical measure.

The estimation of the productivity shock autoregressive process of order 1 over the 1921-1939 period, leads to the following results

\[
\begin{align*}
\ln z^d_t - \ln z^d_{t-1} &= 0.6557 (\ln z^d_{t-1} - \ln z^d_{t-2}) + \hat{\varepsilon}_{z,t} \\
& \tau = 1929, \forall \ t = 1922, \ldots, 1939 \\
\sigma_z &= 0.0443, \quad R^2 = 0.3575, \quad DW = 1.9583
\end{align*}
\]

(75)

The t-statistic of the autoregressive parameter’s estimator appears in parenthesis. \( \hat{\varepsilon}_{z,t} \) represents the estimation residual. Hence, the t-statistic being larger than 1.96 in absolute value, the autoregressive parameter, \( \rho_z \), is significantly different from zero. Besides, the Durbin-Watson statistic test (DW), being relative close to 2, suggests that the residuals are not autocorrelated. However, the adjusted coefficient of determination, \( R^2 \), is small. The quality of estimation does not get better by increasing the number of lags in the autoregressive process. Those estimation results have to be taken with cautions since the sample size is small. I set the steady state level of productivity shock equal to

\[\delta = 0.005.\]

40More precisely, in their model, Bordo et al. (2007) assume that gold is also used as a capital in the production sectors — gold sector and goods sector. Those authors assume that the gold capital depreciate over time at a quarterly rate of 0.005.
the sample mean of its historical measure over the estimation sample: \( z = \frac{\bar{d}}{\bar{t}} = 1.8716 \) where the upperbar stands for the sample mean over the 1921-1939 period.

The estimation of the inverse of money multiplier shock autoregressive process of order 1, over the 1926-1939 period, leads to the following results

\[
\ln \mu_{1,t}^d - \ln \mu_{1,t}^d = 0.7904 \left( \ln \mu_{1,t-1}^d - \ln \mu_{1,t}^d \right) + \hat{\varepsilon}_{\mu_1,t} \quad \tau = 1929, \quad \forall t = 1927, \ldots, 1939
\]

\[
\sigma_{\mu_1} = 0.0601, \quad \hat{R}^2 = 0.6066, \quad DW = 1.9286
\]

(76)

The t-statistic of the autoregressive parameter’s estimator appears in parenthesis. \( \hat{\varepsilon}_{\mu_1,t} \) represents the estimation residual. Hence, the t-statistic being larger than 1.96 in absolute value, the autoregressive parameter, \( \rho_{\mu_1} \), is statistically significant. Besides, the Durbin-Watson statistic test, being relatively close to 2, indicates that the residuals are not autocorrelated. The adjusted coefficient of determination, \( \hat{R}^2 \), is not too small. Note that I do not estimate the exogenous stochastic process of \( \mu_1 \) over a larger sample — still within the interwar period — because otherwise the results with regard to the t-statistic and DW become poorer. In addition, increasing the number of lags in the autoregressive process does not improve the quality of the statistical inference. Those estimation results need to be taken cautiously because the sample is quite short. I set the steady state level of the inverse of money multiplier shock equal to the sample mean of its historical measure over the estimation sample: \( \mu_1 = \bar{\mu}_{1,t}^d = 0.5974 \) where the upperbar stands for the sample mean over the 1926-1939 period.

Recall that the gold backing shock represents a monetary policy instrument reflecting the commitment of an economy to the gold standard system. Therefore, the exogenous stochastic process of \( \mu_{2,t} \) must be defined only for the period during which France has been in the gold-exchange standard system, that is for the 1928-1936 period. The estimation of the gold backing shock autoregressive process of order 1, over the 1928-1936 period, leads to the following results

\[
\ln \mu_{2,t}^d - \ln \mu_{2,t}^d = 0.9065 \left( \ln \mu_{2,t-1}^d - \ln \mu_{2,t}^d \right) + \hat{\varepsilon}_{\mu_2,t} \quad \tau = 1929, \quad \forall t = 1929, \ldots, 1936
\]

\[
\sigma_{\mu_1} = 0.1627, \quad \hat{R}^2 = 0.3379, \quad DW = 0.5185
\]

(77)

The t-statistic of the autoregressive parameter’s estimator appears in parenthesis. \( \hat{\varepsilon}_{\mu_2,t} \) represents the estimation residual. According to the standard error of the residuals, \( \sigma_{\mu_2} \), the uncertainty is quite high for the realizations of \( \mu_{2,t} \). Hence, the t-statistic being larger than 1.96 in absolute value, the autoregressive parameter, \( \rho_{\mu_2} \), is statistically significant. However, the Durbin-Watson statistic test, being very small, indicates that the residuals are autocorrelated. Besides, the adjusted coefficient of determination, \( \hat{R}^2 \), is small too. Adding further lags to the stochastic process does not improve the quality of the statistical inference. Those estimation results are not satisfactory because the sample is very short. I set the steady state level of the gold backing shock equal to the sample mean of its historical measure over the estimation sample: \( \mu_2 = \bar{\mu}_{2,t}^d = 0.6268 \) where the upperbar stands for the sample mean over the 1928-1936 period.

The estimation of the gold flow shock autoregressive process of order 1, over the 1920-1937 period, leads to the following results

\[
g_{t}^{sd} - g_{\tau}^{sd} = 0.8225 \left( g_{t-1}^{sd} - g_{\tau}^{sd} \right) + \hat{\varepsilon}_{g^{sd},t} \quad \tau = 1929, \quad \forall t = 1921, \ldots, 1937
\]

\[
\sigma_{g^{sd}} = 0.0177, \quad \hat{R}^2 = 0.4141, \quad DW = 1.7322
\]

(78)

The t-statistic of the autoregressive parameter’s estimator appears in parenthesis. \( \hat{\varepsilon}_{g^{sd},t} \) represents the estimation residual. Hence, the t-statistic being larger than 1.96 in absolute value, the autoregressive parameter, \( \rho_{g^{sd}} \), is statistically significant. Besides, the Durbin-Watson statistic test, being relatively close to 2, indicates that the residuals are not autocorrelated or weakly. The adjusted coefficient of determination, \( \hat{R}^2 \), is quite small. Increasing the number of lags in the autoregressive process does not improve the quality of the statistical inference. Nonetheless, those estimation results ought to be taken cautiously because the sample is quite short. I set the steady state level of the gold flow shock
equal to the sample mean of its historical measure over the estimation sample: \( g^* = \bar{g}^{sd} = 0.0022 \)
where the upperbar stands for the sample mean over the 1920-1937 period. Note that the steady state value of gold flow shock is positive: In the steady state, there is a (constant) gold inflow.

The additional restrictions are summarized in Table 2.

<table>
<thead>
<tr>
<th>Table 2: Additional restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital share parameter</td>
</tr>
<tr>
<td>Degree of habit formation in consumption parameter</td>
</tr>
<tr>
<td>Preference for non-monetary gold parameter</td>
</tr>
<tr>
<td>Sum of preference parameters</td>
</tr>
<tr>
<td>Utility function curvature parameter</td>
</tr>
<tr>
<td>Non-monetary gold depreciation rate parameter</td>
</tr>
<tr>
<td>Capital depreciation rate parameter</td>
</tr>
<tr>
<td>Transaction cost parameter</td>
</tr>
<tr>
<td>Transaction cost parameter</td>
</tr>
<tr>
<td>Steady state nominal interest rate</td>
</tr>
<tr>
<td>Steady state labor demand</td>
</tr>
<tr>
<td>Steady state technology shocks</td>
</tr>
<tr>
<td>Steady state gold flow shocks</td>
</tr>
<tr>
<td>Steady state gold backing shocks</td>
</tr>
<tr>
<td>Steady state inverse of money multiplier shocks</td>
</tr>
<tr>
<td>Productivity shock stochastic process autoregressive parameter</td>
</tr>
<tr>
<td>Technology shock stochastic process standard deviation parameter</td>
</tr>
<tr>
<td>Inverse of money multiplier shock stochastic process autoregressive parameter</td>
</tr>
<tr>
<td>Inverse of money multiplier shock stochastic process standard deviation parameter</td>
</tr>
<tr>
<td>Gold backing shock stochastic process autoregressive parameter</td>
</tr>
<tr>
<td>Gold backing shock stochastic process standard deviation parameter</td>
</tr>
<tr>
<td>Gold flow shock stochastic process autoregressive parameter</td>
</tr>
<tr>
<td>Gold flow shock stochastic process standard deviation parameter</td>
</tr>
</tbody>
</table>

Note that at this stage, the values of the following parameters

\[ \{\gamma_1, \gamma_2, \phi_3, b, \delta_k, \delta_g, \theta, \rho_z, \rho_{\mu_1}, \rho_{\mu_2}, \rho_{g^*}, \sigma_z, \sigma_{\mu_1}, \sigma_{\mu_2}, \sigma_{g^*}\} \]

and steady state variable levels

\[ \{q, h, z, \mu_1, \mu_2, g^*\} \]

are known.
I use the deterministic steady state equations and the additional restrictions to evaluate the remaining parameters and steady state levels of the model variables

\[ \{ Y, C, I, K, w, r, \bar{r}, G_c, G_m, G^d, m, p_g, v, \kappa, \beta, \phi_1, \phi_2 \} \]

Combining equations (65) and (67), I get the discount factor

\[ \beta = \frac{1}{q} \]

From equation (66) I get the real rental price of capital

\[ r = \frac{1}{\beta} - 1 + \delta_k \]

From equation (67) I get the steady state value of the consumption-based velocity of money

\[ v = \left( \frac{\gamma_2}{\gamma_1} + \frac{1}{\gamma_1} - \frac{1}{q} \right)^{\frac{1}{2}} \]

From equation (62) I get the steady state value of the transaction cost wedge

\[ \kappa = 1 + 2\gamma_1 v - 2\sqrt{\gamma_1 \gamma_2} \]

Combining equations (69) and (70), I get the steady state value of capital

\[ K = \left( \frac{r}{\theta} \right)^{\frac{1}{1-\gamma_1}} z^{\frac{1}{1-\gamma_1}} h \]

From equation (69) I get the steady state value of output

\[ Y = \frac{r}{\theta} K \]

From equation (60) I get the steady state value of investment

\[ I = \delta_k K \]

From equation (72) I get the steady state value of consumption

\[ C = Y - I \]

From equation (68) I get the steady state value of real wage

\[ w = (1 - \theta) \frac{Y}{h} \]

From equation (59) I get the steady state value of real money

\[ m = \frac{C}{v} \]

From equation (71) I get the steady state value of non-monetary gold stock

\[ G_c = \frac{g^s}{\delta_g} \]

39
To calculate the value of the preference parameters, $\phi_1$ and $\phi_2$, first I express the latter in function of the former using equation (64)

$$\phi_2 = \frac{w (1 - h) 1 - b \beta}{C \kappa} \phi_1$$

(*

Second, I use (*) and the assumption that $\sum_{i=1}^{3} \phi_i = 1$, to get the value of $\phi_1$

$$\phi_1 = [1 - \phi_3] \left[ 1 + \frac{w (1 - h) 1 - b \beta}{C \kappa} \right]^{-1}$$

Third, I deduce the value of $\phi_2$ from (*).

From equation (63) I get the steady state value of the real price of gold

$$p_g = \frac{\phi_3 \ k \ 1 - b}{\phi_1 \ g_c \ 1 - b \beta} (1 - (1 - \delta_g \beta))^{-1}$$

From equation (58) I get the steady state value of the monetary gold stock

$$G_m = \mu_1 \mu_2 \frac{m}{p_g}$$

From equation (61) I get the steady state value of total gold stock

$$G^d = G_c + G_m$$

Note that the steady state of the gold standard model is defined independently to the utility function curvature parameter, $\phi_4$, value. However, different values for the habit formation in consumption parameter, $b$, lead to different steady state levels for the real price of gold and monetary gold. Since for any alternative values of $b$, the steady state level of labor is restricted to take one and only one possible value, the values of the preference parameters $\phi_1$ and $\phi_2$ are conditional on the value taken by $b$. Nonetheless, the differences are not large.

On can compare the steady state implications of the gold standard model, in terms of ratios, with the corresponding historical quantities. This exercise allows to assess the accuracy of the calibration. The results of this exercise are reported in Table 3. The first column of Table 3 describes several ratios under consideration. The second and third columns display the steady state values, implied by the model, of those ratios, when $b = 0$ and $b = 0.65$ respectively. The fourth columns reports the sample means of the ratios, calculated using the historical data, over the interwar period. The last column reports the sample means of the ratios, calculated using the historical data, over the gold-exchange standard period. Hence, according to Table 3, globally, the model reproduces quite well, in the steady state, the properties of the historical data over the interwar and gold-exchange standard periods. Therefore, the calibration of the model, presented above, seems acceptable.

In appendix A, the calibration of the model parameters are summarized in Table 7.

4.3 Analysis of the steady state

Two properties of the steady state of the gold standard economy are worth to be emphasized. The inflation is implicitly set to zero in the steady state. Indeed, the real interest rate is set equal to the nominal interest rate in the steady state: $\bar{r} = q$. Besides, the steady state level of gold flow shock is just sufficient to cover the depreciation of the non-monetary gold stock: $q^* = \delta_g \ g_c$. Indeed, by assumption, the total gold stock, $G^d$, is neither increasing nor decreasing in the steady state. It follows that the relative price of gold, $p_g$, is constant in the steady state. So is the price level\footnote{The fact that the relative price of gold is constant in the steady state implies that all the gold stocks and nominal money are constant too.}. Clearly, these two properties of the steady state are interrelated.
Table 3: Steady state properties of the model vs historical properties of the French data

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Model, $b = 0$</th>
<th>Model, $b = 0.65$</th>
<th>France 1919-1939</th>
<th>France 1928-1936</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y/Y$</td>
<td>0.7686</td>
<td>0.7686</td>
<td>0.8216</td>
<td>0.8106</td>
</tr>
<tr>
<td>$K/Y$</td>
<td>0.2314</td>
<td>0.2314</td>
<td>0.1784</td>
<td>0.1894</td>
</tr>
<tr>
<td>$m/Y$</td>
<td>0.3801</td>
<td>0.3801</td>
<td>0.4202</td>
<td>0.4537</td>
</tr>
<tr>
<td>$K/Y$</td>
<td>0.3251</td>
<td>0.3251</td>
<td>0.4521</td>
<td>0.4398</td>
</tr>
<tr>
<td>$F/K$</td>
<td>0.0752</td>
<td>0.0752</td>
<td>0.0814</td>
<td>0.0845</td>
</tr>
<tr>
<td>$L/K$</td>
<td>4.6115</td>
<td>4.6115</td>
<td>3.7839</td>
<td>4.1188</td>
</tr>
<tr>
<td>$G^m$</td>
<td>2.0222</td>
<td>2.0222</td>
<td>2.0172</td>
<td>1.8074</td>
</tr>
<tr>
<td>$T$</td>
<td>0.1294</td>
<td>0.1315</td>
<td>0.1245</td>
<td>0.1619</td>
</tr>
</tbody>
</table>

Now, I compare the steady states of the gold standard model assuming different values of the gold flow shock, the gold backing shock or the productivity shock.\textsuperscript{42} First, I study the impact of a higher steady state gold flow shock on the steady state of the gold standard economy. Specifically, I double the steady state value of gold flow shock, the other exogenous shocks being unchanged: $g^y = 0.0044$. Then, I analyse the effects of a lower steady state gold backing shock on the steady state of the gold standard economy. In particular, I decrease the steady state value of the gold backing to its bottom bound, the other exogenous shocks being unchanged: $\mu_2 = 0.35$. Finally, I analyse the effects of a 25 percent higher steady state productivity shock on the steady state of the gold standard economy: $z' = 2.34$. In those exercises, the structural parameters of the model are kept unchanged. Only the endogenous steady state variables are allowed to change.

Here, I consider the case where consumption is not subject to habit formation. The results of the exercises are reported in Table 4. The first column displays the steady state variables. The second column shows how the steady state variables are computed analytically. The last three columns reports the numerical values of the steady state variables in the initial state, with a higher gold flow shock and with a lower gold backing shock, respectively.

According to Table 4, the steady state levels of nominal interest rate, real rental rate of capital, real interest rate, consumption-based velocity of money, transaction cost wedge and labor — $q, r, \bar{r}, v, \kappa$ and $h$ — are defined independently to the values of the exogenous variables, $z, \mu_1, \mu_2$ and $g^y$. The steady state levels of output, capital, investment, consumption, real money and real wage — $Y, K, I, C, m$ and $w$ — are subject (positively) only to the productivity shock. The steady state level of non-monetary gold, $G_n$, can be affected only by the gold flow shock. Otherwise, the gold stock variables (and henceforth nominal money and real price of gold) would grow in the steady state. The steady state level of the real price of gold, $p_q$, depends positively on the productivity shock (through consumption) and negatively on the gold flow shock (through non-monetary gold). The steady state level of the monetary gold stock, $G_m$, can be affected by the two money supply shocks and gold flow shock. In particular, $G_m$ increases with the two money supply shocks, $\mu_1$ and $\mu_2$. As well, the steady state level of the monetary gold stock rises with the gold flow shock (through the real price of gold). In a first glance, the steady state value of the monetary gold stock seems to be subject the productivity shock too. Indeed, an increase of $z$ would be transmitted to $G_m$, positively through real money, $m$, and negatively through the real price of gold, $p_q$. The computation of the derivative of the expression defining $G_m$ with respect to $z$ show that one effect compensate for the other one. The final effect of the productivity shock to the steady state level of monetary gold stock is nil. Being defined as the sum of monetary and non-monetary gold stocks, the steady state level of the total gold stock, $G^y$, is subject to the money supply shocks and gold flow shock. Table 4 reports also the steady state level of nominal money, $M$. Hence, being equal to the ratio of real money to real price of gold, $M$ is subject only to the gold flow shock (positively through real price of gold). As for the monetary gold stock, the productivity shock has no effects on nominal money. To summarize,

\textsuperscript{42}In a steady state analysis of the gold standard model, the inverse of the money multiplier shock plays a same role as the gold backing shock.
Table 4: Changes in the steady state, with \( b = 0 \)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Expression</th>
<th>Initial steady state</th>
<th>With ( g'' &gt; g' ) ( g' = 0.0044 )</th>
<th>With ( \mu_2' &lt; \mu_2 ) ( \mu_2' = 0.35 )</th>
<th>With ( z' &gt; z ) ( z' = 2.34 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q )</td>
<td>( \frac{1}{\beta} )</td>
<td>1.0353</td>
<td>1.0353</td>
<td>1.0353</td>
<td>1.0353</td>
</tr>
<tr>
<td>( r )</td>
<td>( \frac{1}{\beta} - 1 + \delta_k )</td>
<td>0.1105</td>
<td>0.1105</td>
<td>0.1105</td>
<td>0.1105</td>
</tr>
<tr>
<td>( \bar{r} )</td>
<td>( \frac{1}{\beta} )</td>
<td>1.0353</td>
<td>1.0353</td>
<td>1.0353</td>
<td>1.0353</td>
</tr>
<tr>
<td>( v )</td>
<td>( \left( \frac{2g}{\gamma_1} + \frac{1}{\gamma_1} \right)^{\frac{1}{2}} )</td>
<td>2.0222</td>
<td>2.0222</td>
<td>2.0222</td>
<td>2.0222</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>1 + 2( \gamma_1 )v - 2( \sqrt{\gamma_1 \gamma_2} )</td>
<td>1.0214</td>
<td>1.0214</td>
<td>1.0214</td>
<td>1.0214</td>
</tr>
<tr>
<td>( h )</td>
<td>( \left[ 1 + \frac{\gamma_2}{\gamma_1} \frac{1 - \beta_3}{1 - \beta_2} \frac{\kappa}{\gamma_2} \left( \frac{1 - \beta_3}{1 - \beta_2} + \delta_k \right) \right]^{-1} )</td>
<td>0.2341</td>
<td>0.2341</td>
<td>0.2341</td>
<td>0.2341</td>
</tr>
<tr>
<td>( Y )</td>
<td>( z^{\frac{1}{1-\delta}} \left[ \frac{1}{\bar{r}} \left( \frac{1}{\beta} - 1 + \delta_k \right) \right]^{\frac{\theta}{\vartheta - 1}} h )</td>
<td>1.0794</td>
<td>1.0794</td>
<td>1.0794</td>
<td>1.5136</td>
</tr>
<tr>
<td>( K )</td>
<td>( \theta \frac{Y}{\bar{r} - 1 + \delta_k} )</td>
<td>3.3202</td>
<td>3.3202</td>
<td>3.3202</td>
<td>4.6559</td>
</tr>
<tr>
<td>( I )</td>
<td>( \delta_k K )</td>
<td>0.2498</td>
<td>0.2498</td>
<td>0.2498</td>
<td>0.3503</td>
</tr>
<tr>
<td>( C )</td>
<td>( Y - I )</td>
<td>0.8296</td>
<td>0.8296</td>
<td>0.8296</td>
<td>1.1633</td>
</tr>
<tr>
<td>( m )</td>
<td>( \frac{C}{\bar{r}} )</td>
<td>0.4102</td>
<td>0.4102</td>
<td>0.4102</td>
<td>0.5753</td>
</tr>
<tr>
<td>( w )</td>
<td>( (1 - \theta) \frac{Y}{h} )</td>
<td>3.0436</td>
<td>3.0436</td>
<td>3.0436</td>
<td>4.2679</td>
</tr>
<tr>
<td>( G_c )</td>
<td>( \frac{g'}{\delta_k} )</td>
<td>0.1085</td>
<td>0.2169</td>
<td>0.1085</td>
<td>0.1085</td>
</tr>
<tr>
<td>( p_g )</td>
<td>( \frac{\phi_3 C_\phi (1-b)}{\phi_1 G_c (1-b) \beta (1-(1-\delta_2) \beta)} )</td>
<td>1.0995</td>
<td>0.5498</td>
<td>1.0995</td>
<td>1.5418</td>
</tr>
<tr>
<td>( G_m )</td>
<td>( \mu_1 \mu_2 \frac{m}{p_g} )</td>
<td>0.1397</td>
<td>0.2794</td>
<td>0.0780</td>
<td>0.1397</td>
</tr>
<tr>
<td>( G_d )</td>
<td>( G_c + G_m )</td>
<td>0.2482</td>
<td>0.4969</td>
<td>0.1865</td>
<td>0.2482</td>
</tr>
<tr>
<td>( M )</td>
<td>( \frac{m}{p_g} )</td>
<td>0.3731</td>
<td>0.7462</td>
<td>0.3731</td>
<td>0.3731</td>
</tr>
</tbody>
</table>
all the real variables are invariant across different values of $\mu_1$, $\mu_2$ or $g^*$ in the steady state because neither nominal money nor gold stock grow. In turn, all the nominal variables (including gold stock variables) but the real price of gold are defined independently to the productivity shock in the steady state.

Considering only the nominal variables, this analytical steady state analysis of the gold standard model is consistent to the findings of Barro (1979) and McCallum (1989). Those authors analysis the working of the gold standard — in particular the determination of the price level under the gold standard — taking the real part of the economy as exogenously given. They assume that expected inflation is (correctly) anticipated to be nil in the steady state. In addition, the gold flow (or gold production) is an endogenous variable in their models. Specifically, the gold production is assumed to be an increasing function of the real price of gold and (exogenously) subject to technical progress. For the sake of simplification, they omit the nominal interest rate from their analysis. The former author studies the working of the gold standard using a phase diagram while the latter author does a comparative static analysis of the gold standard. Barro (1979) shows that following an upward shift of the gold supply function, $g^*$, (because of technical progress in gold production) the economy moves to a new steady state where the monetary and non-monetary gold stocks are higher and the real price of gold is lower. Since the steady state level of monetary gold stock is larger, the nominal money happens to be greater in the new steady state too, as shown by the gold standard money supply rule.

In addition, Barro (1979) points out that following a decrease in the gold backing ratio, $\mu_2$, the economy shifts to a new steady state characterised by a higher monetary gold stock. The levels of the real price of gold and non-monetary gold stock remain unchanged in the new steady state. In Barro (1979), the steady state level of the non-monetary gold stock is altered by the change in the gold backing ratio because the real price of gold is defined independently to $\mu_2$. Indeed, in Barro (1979), the gold flow is function of the real price of gold, so is the non-monetary gold stock in the steady state. Thus, the reason of the neutrality of change in the gold backing ratio with respect the non-monetary gold in Barro (1979) has to be contrasted to the reason of that neutrality in the current work. Barro (1979) finds that the steady state level of nominal money remains unchanged following a change in the gold backing ratio because the increase in the steady state monetary gold compensates for the drop in $\mu_2$. This explanation of the fixity of the nominal money holds in the present work too.

Barro (1979) also considers the impact of an increase in real income (or real output) — which is equivalent to an increase in the productivity shock in the current work — on the steady state of the gold standard economy. The author shows that raising the value of real income leads the real price of gold to take a higher value in the steady state. Besides, the Barro (1979)'s model predicts that an increase of real income implies a higher steady state value for the non-monetary gold stock. However, Barro (1979) claims that the effect of an increase in real income on the monetary gold stock is in general not clear. The results obtained in the current study contrast with those obtained by Barro (1979) because I have assumed that the gold flow is exogenous whereas Barro (1979) has defined it as being endogenous and function to the real price of gold. In other words, in Barro (1979), the increase of the steady state level of the real price of gold, induced by the change in real income, drives up the steady state level of the gold flow. It follows that the steady state level of the non-monetary gold stock is raised too. Instead, in the present work, the gold flow shock, being exogenously given and orthogonal to the other shocks, remains fixed. Therefore, the steady state level of the non-monetary gold stock is not affected by the increase in the productivity shock.

According to four last columns of Table 4, the numerical exercises confirm the analytical analysis. The steady state levels of nominal interest rate, real rental rate of capital, real interest rate, consumption-based velocity of money, transaction cost wedge and labor are invariant to changes in the steady state values of any shocks. In the steady state, the levels of output, capital, investment, consumption, real money and real wage become higher when the productivity shock takes a larger value. However, those real variables are invariant to changes in the steady state values of the money supply shocks or gold flow shock. Nonetheless, with a higher gold flow shock, the monetary and non-monetary gold stocks increase. So do the total gold stock as well as the nominal money stock.

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43Technical progress in gold production in Barro (1979) is equivalent to a positive gold flow shock in the current model.

44Barro (1979) argues that the non-monetary gold stock rises if and only if the sum of the price elasticities of the gold production function and non-monetary gold demand function are larger than unity in absolute value.
However, the real price of gold becomes lower. When the gold backing shock is lowered to its minimum value, the monetary gold stock becomes smaller. The non-monetary gold stock and real price of gold are not modified. It follows that the total gold stock decreases and the nominal money remains unchanged. In the steady state, all the nominal variables but real price of gold are not affected by a change in the productivity shock level.

As shown in Table 5, the results of the steady state analysis are not affected when habit formation in consumption is introduced in the model.

5 Quantitative Analysis

In this section, first I examine the dynamic properties of the gold standard model through the impulse response functions, then I confront the predictions of this model for several key macroaggregate variables to the corresponding historical data.

5.1 Impulse Response Functions

Here, I analyse the impulse responses functions (IRFs hereafter) of the gold standard model’s variables to the gold flow shock, the money supply shocks and to the productivity shock. They measure quantitatively the effects of a single (positive or negative) 1 percent innovation to each exogenous shock, in terms of log-deviation from the steady state, on key model variables over $T$ periods after the impact.

In the discussion of the IRFs of the gold standard model’s variables to each exogenous shock, I consider three versions of the model. Specifically, the first alternative model — labelled baseline model — corresponds to a gold standard economy with separable preferences and without habit formation in consumption ($\phi_4 = 1$, $b = 0$). The second alternative model — labelled habit model — corresponds to a gold standard economy with separable preferences and habit formation in consumption ($\phi_4 = 1$, $b = 0.65$). The last alternative model — labelled complete model — corresponds to a gold standard economy with non-separable preferences and habit formation in consumption ($\phi_4 = 2$, $b = 0.65$).

The plots of the French historical data have shown that output and investment move in the same directions during the 1929-1936 period. However, an innovation to either productivity shock, gold flow shock, gold backing shock or the inverse of money multiplier shock might generate, through the transaction cost wedge, countercyclical investment in the baseline model. Recall that the transaction cost wedge acts as a negative preference shock. Baxter and King (1991) and Wen (2006) point out that a standard RBC model can generate, through preference shocks, countercyclical investment unless the preference shock is highly persistent. Agreed, this would not be too much troublesome if all the shocks taken together would not generate countercyclical investment. However, as it will be shown, investment is more likely to be countercyclical following an innovation to the gold flow shock or to either money supply shock than following an innovation to the productivity shock. Thus, I need to ensure that changes in the gold flow shock and money supply shocks do not lead output and investment to move in opposite directions, at least initially, in order to assess the ability of the gold standard model, through the gold flow shock and the money supply shocks, to explain the French Great Depression. Wen (2006) demonstrates that the introduction of either habit formation in consumption or non-separable preferences into the standard RBC model can reduce the necessary degree of preference shock persistence in order to ensure the procyclicality of investment. Hence, I also simulate the habit and complete models in order to check whether the findings of Wen (2006) are still relevant to the gold standard model or not. Simply put, I find that, given the calibration of the gold standard model, investment and output are ensured to move in opposite directions following a change in any of the shocks, at least initially, in both habit and complete models.

5.1.1 Impulse response functions to the gold flow shock

Figures 5 and 6 display the impulse responses of the baseline and habit models to a single positive innovation to the gold flow shock. The IRFs of the baseline model’s variables are represented with starred lines while the IRFs of the habit model are represented with circle lines. In period 0, the gold flow shock is assumed to increase by 1 percent.
### Table 5: Changes in the steady state, with $b = 0.65$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Expression</th>
<th>Initial steady state</th>
<th>With $g'' &gt; g'$</th>
<th>With $\mu'_2 &lt; \mu_2$</th>
<th>With $z' &gt; z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>$\frac{1}{\beta}$</td>
<td>1.0353</td>
<td>1.0353</td>
<td>1.0353</td>
<td>1.0353</td>
</tr>
<tr>
<td>$r$</td>
<td>$\frac{1}{\beta} - 1 + \delta_k$</td>
<td>0.1105</td>
<td>0.1105</td>
<td>0.1105</td>
<td>0.1105</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>$\frac{1}{\beta}$</td>
<td>1.0353</td>
<td>1.0353</td>
<td>1.0353</td>
<td>1.0353</td>
</tr>
<tr>
<td>$v$</td>
<td>$\left( \frac{\gamma_2}{\gamma_1} + \frac{1}{\gamma_1} \frac{q-1}{q} \right)^{\frac{1}{2}}$</td>
<td>2.0222</td>
<td>2.0222</td>
<td>2.0222</td>
<td>2.0222</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>$1 + 2\gamma_1 v - 2\sqrt{\gamma_1 \gamma_2}$</td>
<td>1.0214</td>
<td>1.0214</td>
<td>1.0214</td>
<td>1.0214</td>
</tr>
<tr>
<td>$h$</td>
<td>$\left[ 1 + \frac{\frac{\gamma_2}{\gamma_1} - 1}{\frac{\gamma_2}{\gamma_1} - 1 + \delta_k} \kappa \left( \frac{\gamma_2}{\gamma_1} - 1 + \delta_k \right) \right]^{-1}$</td>
<td>0.2341</td>
<td>0.2341</td>
<td>0.2341</td>
<td>0.2341</td>
</tr>
<tr>
<td>$Y$</td>
<td>$z^{\frac{1}{\delta_k}} \left[ \frac{1}{\beta} \left( \frac{1}{\beta} - 1 + \delta_k \right) \right]^{\frac{\theta}{\beta-1}} h$</td>
<td>1.0794</td>
<td>1.0794</td>
<td>1.0794</td>
<td>1.5136</td>
</tr>
<tr>
<td>$K$</td>
<td>$\frac{\theta Y}{\beta - 1 + \delta_k}$</td>
<td>3.3202</td>
<td>3.3202</td>
<td>3.3202</td>
<td>4.6559</td>
</tr>
<tr>
<td>$I$</td>
<td>$\delta_k K$</td>
<td>0.2498</td>
<td>0.2498</td>
<td>0.2498</td>
<td>0.3503</td>
</tr>
<tr>
<td>$C$</td>
<td>$Y - I$</td>
<td>0.8296</td>
<td>0.8296</td>
<td>0.8296</td>
<td>1.1633</td>
</tr>
<tr>
<td>$m$</td>
<td>$\frac{C}{v}$</td>
<td>0.4102</td>
<td>0.4102</td>
<td>0.4102</td>
<td>0.5753</td>
</tr>
<tr>
<td>$w$</td>
<td>$(1 - \theta) \frac{Y}{h}$</td>
<td>3.0436</td>
<td>3.0436</td>
<td>3.0436</td>
<td>4.2679</td>
</tr>
<tr>
<td>$G_c$</td>
<td>$\frac{g_s'}{\delta_s}$</td>
<td>0.1085</td>
<td>0.2169</td>
<td>0.1085</td>
<td>0.1085</td>
</tr>
<tr>
<td>$p_g$</td>
<td>$\frac{\phi_2 C_c(1-b)}{\phi_1 G_c(1-b)(1-\delta_g)\beta}$</td>
<td>1.0820</td>
<td>0.5410</td>
<td>1.0820</td>
<td>1.5173</td>
</tr>
<tr>
<td>$G_m$</td>
<td>$\mu_1 \mu_2 \frac{m}{p_g}$</td>
<td>0.1420</td>
<td>0.2839</td>
<td>0.0793</td>
<td>0.1420</td>
</tr>
<tr>
<td>$G^d$</td>
<td>$G_c + G_m$</td>
<td>0.2504</td>
<td>0.5009</td>
<td>0.1817</td>
<td>0.2504</td>
</tr>
<tr>
<td>$M$</td>
<td>$\frac{m}{p_g}$</td>
<td>0.3791</td>
<td>0.7583</td>
<td>0.3791</td>
<td>0.3791</td>
</tr>
</tbody>
</table>
Figure 5: Selected impulse response functions to a 1 percent innovation to the gold flow shock (a).

Figure 6: Selected impulse response functions to a 1 percent innovation to the gold flow shock (b).
First, I focus my attention on the dynamic behavior of non-monetary gold stock, monetary gold stock, total gold stock, real price of gold, gross nominal interest rate, nominal money and real money. As shown in Figure 5, the baseline and habit models display very similar IRFs for those variables. Hence, the existence of habit formation in consumption does not alter the impulse responses of those variables to the single innovation to the gold flow shock. In what follows, I consider indifferently the baseline model or the habit model when I discuss the IRFs of non-monetary gold stock, monetary gold stock, total gold stock, real price of gold, nominal interest rate, nominal money and real money.

Thus, following a positive innovation to the gold flow shock, the inflow of new gold, \( g_t^* \), becomes larger than the depreciation of the non-monetary gold stock brought from the previous period, \( \delta_{Gt}G_{t-1} \). Therefore, the existing total gold stock of the economy increases. Hence, gold becomes relatively cheaper. Indeed, as shown in Figure 5, on impact, the total gold stock, \( G_t^* \), rises and the real price of gold (the price level) moves down (up). Since the price level is perfectly flexible, the fall of the real price of gold is quick. Because the gold flow shock is positively serially correlated (\( \rho_{Gt} > 0 \)), households expect that the innovation to that shock, once it is observed, is going to drive the real price of gold above its steady state level in the subsequent periods. It follows that, under full flexibility of the price level assumption, the household reduces its demand for real money. In addition, note that the gross nominal interest rate responds positively to the gold flow shock. As illustrated in Figure 5, the gross nominal interest rate, \( q_t \), rises on impact, consistently to the fall in real money demand. As a result, the equilibrium real money, \( m_t \), decreases too (see Figure 5).

How the increase in the total existing gold stock is allocated between monetary and non-monetary uses? As shown in Figure 5, the model predicts that both non-monetary and monetary gold stocks respond positively to the gold flow shock. Since the real price of gold declines, the value of gold decreases relatively to the consumption good. As a result, the representative household increases its demand for non-monetary gold stock, \( G_{nt} \). On impact, the real price of gold collapses more than the real money does. In other words, the household increases its demand of nominal money. Therefore, the household has to bring additional gold to the central bank to satisfy its demand of nominal money, the two money supply shocks being unchanged. As a result, the monetary gold stock, \( G_{nt} \), and nominal money, \( M_t \), increase. It is worth to note that by holding gold for non-monetary purpose, the household prevents the new gold inflow from being fully captured by the central bank. As a consequence, on impact, the monetary gold stock does not deviate from its steady state level as much as the gold flow shock does. Thus, less money is injected in the economy when the household enjoys holding gold for non-monetary use comparatively to what would happen if gold brought no satisfaction to the household. Hence, the gold flow shock generates a smaller price level rise when the household gets satisfaction from holding gold for non-monetary purpose comparatively to what would happen if gold yields no utility to the household.

As one can observe from Figure 5, the increases of nominal money and non-monetary gold are not followed by a fall in the gross nominal interest rate: The positive innovation to the gold flow shock does not generate a liquidity effect or a non-monetary gold liquidity effect. Therefore, the real non-monetary gold stock can be decreasing though the non-monetary gold stock is increasing. As a matter of fact the real non-monetary gold stock does decrease since the real price of gold deviates more strongly from its steady state level on impact than the non-monetary gold does.

Note that the impulse responses of the gold stock variables, real price of gold variable and nominal money variable are hump-shaped\(^{45}\). Even though the non-monetary gold stock reaches a higher maximum than the monetary gold stock, the latter moves up higher than the former on impact. In addition, those variables deviate strongly from their respective steady state level and move back very slowly. Indeed, 100 years following the single innovation to the gold flow shock, \( G_{ct}, G_{nt}, G_t^d, p_t \) and \( M_t \) are still far from their respective steady state level. According to Friedman (1951), the movements in gold stocks are large and the adjustment mechanism is slow because the gold flow represents a small fraction of the existing gold stock: “Because current output of the currency commodity is generally a small fraction of the existing stock, deviations from equilibrium can be substantial; and a relatively long time may be required to correct them\(^{46}\)”. As a matter of fact, those results are sensitive to the calibration of the non-monetary gold depreciation rate, \( \delta_p \). In particular, higher the non-monetary gold depreciation rate is, higher the increase of non-monetary gold stock is on impact. For example, with \( \delta_p = 0.1 \), the non-monetary gold stock deviates from its steady state level substantially more

\(^{45}\) Nonetheless, the impulse response of the real price of gold is not sluggish: It jumps below its steady state level strongly on impact.

\(^{46}\) The quotation is from Friedman (1951), p. 205
than the monetary gold stock does on impact. In addition, the latter reaches a higher maximum than the former. Instead, with a non-monetary gold depreciation rate almost nil, say $\delta_g = 10^{-9}$, the non-monetary gold stock responds negatively to the innovation to the gold flow shock on impact, then moves above its steady state level. However, the monetary gold stock increases on impact. Note that this observation is consistent with what I have said about the optimal behavior of the representative household with respect to the non-monetary gold. Precisely, I have stressed that the household discounts the future with $(1 - \delta_g) \beta$ rather than with $\beta$ when it determines its optimal sequence of non-monetary gold stock. Hence, higher $\delta_g$ is, lower the value of non-monetary gold is in the future comparatively to its current value. In other words, when the non-monetary gold depreciation rate is larger, the household chooses to accumulate more non-monetary gold stock in the period during which the innovation to the gold flow shock is realized than in the subsequent periods. Furthermore, the speed at which the the gold stock variables, real price of gold variable and nominal money variable come back to their respective steady state level, once they deviate from it, depends on $\delta_g$ too. Specifically, higher the non-monetary gold depreciation rate is, quicker the adjustment mechanism is. As an illustration to these comments, I report in Appendix B the impulse response functions of non-monetary gold and monetary gold stocks to an innovation to the gold flow shock for the cases where $\delta_g = 0.1$ and $\delta_g = 10^{-9}$.

Second, I focus my attention on the real sphere of the economy. In the gold standard model, the gold flow shock is considered as a nominal shock because it affects the money supply of the economy through the optimal household’s decision on non-monetary gold holding and henceforth through the monetary gold stock. This leads to the following question: How an innovation to the gold flow shock is transmitted to the real part of the economy? The gold standard model includes two channels through which the innovation to the gold flow shock is transmitted to the real part of the economy. Specifically, these two channels are the transaction cost wedge, $\kappa_t$, and the optimal condition on the non-monetary gold stock, $G_{c,t}$. The transaction cost wedge represents a bridge between the nominal and the real parts of the economy because it is endogenously linked to the gross nominal interest rate. Indeed, the transaction cost wedge is a (positive) function of the consumption-based velocity of money (see equation (43)) which in turn comoves (positively) with the gross nominal interest rate (see equation (48)). As mentioned above, the transaction cost wedge alters the optimal decisions of the representative household in terms of consumption, labor supply and capital that is the household’s optimization problem first order conditions (17), (25) and (28). The optimal condition on non-monetary gold stock represents also a bridge between the nominal and the real part of the gold standard economy. This second channel is made clearer by the optimal condition of the (expected) marginal rate of substitution of consumption for (real) non-monetary gold. As said above, the interaction between non-monetary gold stock, real price of price and gross nominal interest rate affects consumption through the optimal condition (56).

As shown in Figure 6, in the baseline model, consumption, $C_t$, responds negatively to the increase of the gold flow shock on impact. Indeed, the increase of the gross nominal interest rate, $\gamma_t$, raises the transaction cost wedge, $\kappa_t$, as reported in Figure 6. The rise of transactions costs leads the household to decrease its demand for consumption good. Furthermore, the optimal condition (56) states that if the gross nominal interest rate moves up, the marginal rate of substitution of consumption for real non-monetary gold must increase too. Specifically, this optimal condition states that the decrease of real non-monetary gold stock, the increase of nominal interest rate and hence the rise of transaction cost wedge following a positive innovation to the gold flow shock, make it optimal for the household to decrease its consumption. I cannot disentangle the effect of the transaction cost wedge from the effect of the optimal condition on non-monetary gold stock because those effects are linked to each other as shown in equation (55). However, I can evaluate the importance of the two channels through a reduction of the variability of the transaction cost wedge. Precisely, I weaken the link of the transaction cost wedge to the consumption-based velocity of money by assigning smaller values to $\gamma_1$ and $\gamma_2$. For example, I set $\gamma_1$ and $\gamma_2$ to $10^{-9}$. The IRF of the baseline model’s consumption with this alternative calibration of the transaction cost parameters are reported in the

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47 See the comments on the first order conditions of the household’s problem.

48 See the comments on the optimal condition on the marginal rate of substitution of consumption for real non-monetary gold.

49 Note that this alternative calibration is used only for a comparative analysis. Indeed, this alternative calibration of the transaction cost function parameters leads the consumption-based velocity of money to take an unreasonable high value in the steady state.
left window of Figure 750. Thus, consumption still responds negatively to the innovation to the gold flow shock on impact. However, quantitatively, this response of consumption is significantly weaker comparatively to the response of consumption in the case where the transactions costs are more sensitive to the consumption-based velocity of money. Hence, it seems that more the transaction cost wedge is sensitive to the consumption-based velocity of money, stronger the two channels are.

![Graph showing impulse response functions](image)

Figure 7: Selected impulse response functions to a 1 percent innovation to the gold flow shock (c).

In the baseline model, the increase of the transaction cost wedge, following the innovation to the gold flow shock, lowers the opportunity cost of leisure in terms of the marginal utility of consumption, making it optimal for the household to take more leisure. Consequently, labor and output collapse on impact (see Figure 6). However, as shown in Figure 6, the baseline model predicts that investment responds positively on impact to the rise of the gold flow shock. As said above, the intertemporal transaction cost wedge, $\frac{\kappa}{\kappa + 1}$, alters the optimal condition on the intertemporal marginal rate of substitution of consumption, (28). In particular, that intertemporal wedge prevents the expected marginal rate of substitution of future consumption for current consumption from being equal to the gross real interest rate in the equilibrium. The baseline model predicts that the transaction wedge wedge rises on impact, then decreases toward its steady state level. Hence, the intertemporal transaction cost wedge moves above its steady state level when the innovation to the gold flow shock is realized. Thus, the rise of the intertemporal transaction cost wedge may have a crowding in effect on investment. Indeed, the upward shift of the intertemporal transaction cost wedge may lead consumption to decrease more than output does, at least initially. The household would decrease that much its consumption by postponing it. By doing so, the household increases its savings, which take the form capital accumulation. Therefore, investment increases. The increase of the capital stock,

50The impulse responses of the other variables of the baseline model with small transactions costs to the innovation to the gold flow shock are reported in Figures 42 and 43 in appendix B. Note that the IRFs of the baseline model are qualitatively similar to the IRFs of the baseline model with small transactions costs. Nonetheless, output, consumption, investment, labor, capital, real wage and gross real interest rate are less affected by the gold flow increase in the baseline model with small transaction cost than in the baseline model. On the contrary, gold stocks, money, real price of gold and gross nominal interest rate are more volatile in the baseline model with small transactions costs than in the baseline model. The pick responses of those variables are higher in the baseline model with small transactions costs than in the baseline model. Those variables converge to their respective steady state level more quickly in the baseline model with small transactions costs than in the baseline model. Thus, it seems that the transaction cost wedge dampens down the effect of the gold flow shock on gold stocks, money, real price of gold and nominal interest rate.
together with the low level of future labor comparatively to its steady state level, leads the gross real interest rate — the capital gross rate of return — to fall. The crowding in effect is more likely to happen if the innovation to the gold flow shock has not a persistent effect on the transaction cost wedge. Lower the interval of time, in which the transaction cost wedge is above its steady state level, is, larger the frictions induced by the transactions costs are, and more likely to happen the crowding in effect is. One way to step down the distortions introduced by the transactions costs and henceforth to generate a decreasing investment, is to raise the persistence of the gold flow shock. Indeed, increasing the persistence of the gold flow shock leads to rise the interval of time during which the gross nominal interest rate and henceforth the transaction cost wedge are above their respective steady state level. Baxter and King (1991) and Wen (2006) show that the preference shock to consumption can lead investment to be crowded out if this preference shock is not sufficiently persistent: The preference shock generates countercyclical investment. As said above, the transaction cost wedge alters the equilibrium of an economy — the real part of the economy — as a negative preference shock to consumption does. Thus, the crowding out effect described by those authors is similar to the crowding in effect discussed in the current work.

In order to illustrate the discussion on the crowding in effect of the (intertemporal) transaction cost wedge\(^{51}\) on investment, the IRFs of the baseline model’s variables to an innovation to the gold flow shock are computed with a highly persistent gold flow shock. Specifically, the autoregressive parameter of the gold flow shock, \(\rho_g\), set to 0.95. The right window of Figure 7 shows that the initial upward shift of the intertemporal transaction cost wedge is milder when the gold flow shock is highly persistent comparatively to the case where \(\rho_g\) is set to its estimated value. Consequently, consumption increases back smoothly toward its steady state level. In particular, as displayed in Figure 8, the interval of time in which consumption is below its steady state level is longer when \(\rho_g = 0.95\) than when \(\rho_g = 0.82\), the latter being the estimated value of the gold flow shock autoregressive parameter. It follows that the household decrease its savings, at least initially. Therefore, investment collapses in response to the rise of the gold flow shock. The gross real interest rate is still decreasing on impact when the gold flow shock is highly persistent because the marginal productivity of capital is lowered by a smaller level of future labor. The initial increase of capital does not lead the gross real interest rate to increase because the former is not enough large to counteract the effect of future labor. Indeed, capital responses to the rise of the gold flow shock is hump-shaped. In other words, capital reacts only gradually to the upward shift of the gold flow shock\(^{52}\). Note that later in the impulse responses, investment moves above its steady state level while output and consumption are still below their respective steady state level. To fully eliminate the crowding in effect of the transaction cost wedge on investment, the value of the parameter \(\rho_g\) has to be larger than 0.995.

This solution is not convenient since I have initially calibrated the autoregressive parameter of the gold flow shock exogenous stochastic process according to the French data. Below, I follow an other strategy to ensure that investment decreases on impact as output does.

Recall that in the gold standard model, the real wage is equal to the marginal productivity of labor in the equilibrium. Since the production function is assumed to be a Cobb-Douglas, the log-deviation of the real wage from its steady state level corresponds to the log-deviation of the average labor productivity from its steady state level. From Figure 6, one can observe that on impact labor deviates from its steady state level more strongly than output does. It follows that the average labor productivity increases. So the real wage does (see Figure 6).

As shown in Figure 6, one period after the realisation of the innovation to the gold flow shock, output, consumption, investment, labor and real wage move back to their respective steady state level in the baseline model. Later in the impulse responses, output, consumption and labor move above their respective steady state level for some periods. The impulse response of capital to the innovation to the gold flow shock is hump-shaped in the baseline model. In particular, capital continues to increase after the impact for several periods. Then, it decreases toward its steady state

\(^{51}\)One can assume that the crowding in/out effect on investment is generated by either the transaction cost wedge or the intertemporal wedge. Thus, in what follows, I consider indifferently that the crowding in/out effect is created by the transaction cost wedge or the by the intertemporal transaction cost wedge.

\(^{52}\)The impulse responses of the other variables — gold stocks, real price of gold, nominal money, real money, consumption-based velocity of money, real wage and rental rate of capital — to the innovation to the gold flow shock, in the baseline model with highly persistent gold flow shock, are reported in Figure 44 in Appendix B. Note that all those variables display more persistent and volatile responses in the baseline model with highly persistent gold flow shock than in the baseline model.
level. Approximately forty years after the impact, the real wage moves (weakly) below its steady state level. Figure 5 shows that the gross real interest rate starts to increase in the period following the realization of the innovation to the gold flow shock toward its steady state level. It is worth to note that the rise of the gold flow shock has a relatively small effect on output, consumption, investment, labor, capital, real wage and gross real interest rate. As well, those variables converge to their respective steady state level more quickly than the gold stocks, real price of gold, real money, nominal money and gross nominal interest rate variables do.

As said above, there is another way to get investment decreasing following an innovation to the gold flow shock without increasing the persistence of that shock. Specifically, as shown in Figure 6, habit formation in consumption has the effect of making investment decreases on impact. Indeed, the innovation to the gold flow shock generates a smoother and more persistent consumption drop. From Figure 6, one can observe that the impulse response of consumption is inversely hump-shaped when it is subject to habit formation. Hence, consumption continues to decrease one period after the innovation to the gold flow shock is realized. As a consequence, the household finds optimal to decrease its current investment in order to face the expected fall in its subsequent consumption. Even though the transaction cost wedge behaves similarly in the two alternative models, it has not a crowding in effect on investment in the habit model. The habit formation in consumption weakens the crowding in effect, at least initially. Indeed, the habit formation in consumption leads the transaction cost wedge to have a more persistent effect on consumption. It follows that the household decreases its savings and henceforth the investment, at least initially. The gross real interest rate drops on impact in the habit model because the marginal productivity of capital is decreased by a lower level of future labor. Indeed, in the second period, labor is still below its steady state level. As in the baseline model with highly persistent gold flow shock, the initial increase of capital is not enough large to counteract the effect of the future labor on the gross real interest rate. Moreover, note that the trough response of consumption is less deeper in the habit model than in the baseline model. Wen (2006) claims that habit formation in consumption leads a transitory preference shock to consumption to have persistent effect on consumption. Consequently, investment increases following a positive innovation to the preference shock to consumption. The persistent effect of the transitory consumption shock is endogenously generated by the habit formation in consumption. Thus, in the current work, the same mechanism is at work: The habit formation in consumption increases endogenously the persistent effect of the transaction cost wedge on consumption so that investment increases initially. Output and particularly labor deviate more strongly from their respective steady state on impact in the
baseline model than in the model with habit. Consequently, the real wage increases more in the baseline model than in the habit model. Figure 5 shows that on impact the gross real interest rate declines more strongly in the baseline model than in the habit model.\footnote{This means that there must be differences between the impulse responses of the gross nominal interest rate and/or the real price of gold across the alternative models. However, I have said above that they were similar when I have discussed the nominal part of the economy. Indeed, for example, the differences between the impulse response of the real price of gold in the baseline model and the one in the habit model are negligible when only the nominal part of the economy is considered. Since any change in the gold flow shock has little effects on the real part of the economy, those differences become noticeable in the real part of the economy.}

Thus, the existence of habit formation in consumption leads investment to decrease on impact — as output and consumption do. However, the initial drop in investment is very weak. Indeed, one period after the impact, investment moves above its steady state level while output and consumption are still below their respective steady state level. Thus, in addition to habit formation in consumption, I assume that preferences over consumption, leisure and non-monetary gold stock are non-separable in order to get investment decreases for a longer period following an innovation to the gold flow shock. According to Wen (2006), consumption and leisure are more likely move in opposite directions when consumption and leisure are better substitute. Thus, following an increase of the gold flow shock, the household would be more willing to decrease its labor supply when the preferences are non-separable. Consequently, that the elasticity of output supply would become higher in the complete model — gold standard model featuring habit formation in consumption and non-separable preferences — than in the habit model and the effect of the intertemporal transaction cost wedge on investment would be weakened. In other words, output would experience a deeper decline in the complete model than in the habit model, implying a decrease of investment.\footnote{Wen (2006) explains why non-separable preferences over consumption and leisure can lead investment to behave procyclically, in the case where preferences in consumption are time-separable. The argument seems to hold in the case where preferences in consumption are time-separable.} Indeed, Figure 9 shows that the complete model predicts that a rise in the gold flow shock leads investment to decrease on impact and to remain below its steady state level for one period after the realisation of the innovation. The initial decrease of investment happens to be deeper in the complete model than in the habit model. Indeed, on impact, labor and hence output experience a deeper collapse in the complete model than in the habit model.\footnote{The effects of habit formation in consumption and non-separable preferences on the behavior of investment are not totally satisfactory. Indeed, as shown in Figure 9, even tough investment decreases on impact as output does, one cannot conclude that investment is procyclical.} On impact, the consumption drop is a little less severe in the complete model than in the habit model. On impact, output, consumption and labor decline more in the baseline model than in the complete model. Comparing Figure 6 with Figure 9, one can observe that on impact, the real wage jumps higher in the complete model than in the habit model. The effect of the increase of the gold flow shock on the real wage happens to be less persistent in the complete model than in the habit model. The initial increase of the real wage is weaker in the complete model than in the baseline model. As reported in Figures 5 and 10, the gold stocks, money, gross nominal interest rate and real price of gold respond similarly to the rise of the gold flow shock across the three alternative versions of the gold standard model. However, on impact, the gross real interest rate decreases more in the complete model than in the habit model. The innovation to the gold flow shock affects the gross real interest rate less persistently in the complete model than in the habit model. The initial drop of the gross real interest rate is milder in the complete model than in the baseline model.

5.1.2 Impulse response functions to the money supply shocks

In the previous subsubsection, I have found that an increase of the gold flow shock leads the money supply to rise. There are other ways to make the money supply increases endogenously. In particular, a decrease of either the gold backing shock or the inverse of money multiplier shock causes the money supply to increase. Thus, here, I study first the effect of a decrease of the gold backing shock on the gold standard economy, then the effect of a decline of the inverse of the money multiplier shock on the gold standard economy.
Figure 9: Selected impulse response functions to a 1 percent innovation to the gold flow shock: Complete model (a).

Figure 10: Selected impulse response functions to a 1 percent innovation to the gold flow shock: Complete model (b).
5.1.2.1 Impulse response functions to the gold backing shock

Figures 11 and 12 report the impulse responses of the baseline and habit models to a single negative innovation to the gold backing shock. The IRFs of the baseline model’s variables are represented with starred lines whereas the IRFs of the habit model are represented with circle lines. In period 0, the gold backing shock is assumed to decrease by 1 percent.

![Impulse response functions](image)

Figure 11: Selected impulse response functions to a 1 percent innovation to the gold backing shock (a).

First, I analyze the dynamic behavior of non-monetary gold stock, monetary gold stock, total gold stock, real price of gold, gross nominal interest rate, nominal money and real money. From Figure 11, one can note that the baseline and habit models generate, through the gold backing shock, very similar IRFs for those variables. Consequently, the existence of habit formation in consumption does not affect the impulse responses of those variables to a single negative innovation to the gold backing shock. In the following, I consider indifferently the baseline model or the habit model when I comment the IRFs of non-monetary gold stock, monetary gold stock, total gold stock, real price of gold, gross nominal interest rate, nominal money and real money.

By lowering the gold backing ratio, the central bank lets the household knows that it is willing to supply more money in exchange of a same amount of monetary gold stock. As a result, the value of gold declines. In other words, the innovation to the gold backing shock leads the real price of gold (the price level) to fall (to rise) on impact, as shown in Figure 11. Therefore, the household goes to the central bank to buy gold for non-monetary use. It follows, that the household increases its non-monetary gold holding and the central bank decreases its monetary gold stock. Indeed, from Figure 11, one can observe that the decrease of the gold backing shock causes the non-monetary gold stock to move up on impact and the monetary gold stock to fall on impact. Nonetheless, the gold standard model predicts that the decline of monetary gold is smaller in size than the decrease of the gold backing ratio. Therefore, the central bank raises its money supply: As reported in Figure 11, the nominal money increases on impact, consistently to the increase of the nominal money supply. As shown in Figure 11, the total gold stock existing in the economy remains at its steady state level on impact: In the current case, the gold flow shock sticks to its steady state level so that on impact it is just sufficient to cover the depreciation of the non-monetary gold stock brought from the previous period.

Figure 11 shows that the increases in the nominal money and the non-monetary gold stock are followed by a decline of the gross nominal interest rate. Hence, the (negative) innovation to the gold backing shock generates a liquidity effect and a non-monetary gold liquidity effect. The liquidity
effect makes it optimal for the household to increase its demand of real money. Indeed, the gold standard model predicts that the real money jumps in response to a decrease in the gold backing shock, as displayed in Figure 11. As well, the non-monetary gold liquidity effect leads the real non-monetary gold stock to increase on impact: As one can note from Figure 11, the (log-)deviation of non-monetary gold from its steady state level is larger than the (log-)deviation of the real price of gold from its steady state level.

From Figure 11, one can observe that non-monetary gold stock, nominal money and real money reach their peak response to the (negative) innovation to the gold backing shock on impact. The monetary gold stock, real price of gold and gross nominal interest rate reach their trough response to fall of the gold backing shock on impact too. Then, non-monetary gold stock, monetary gold stock, total gold stock, real price of gold, gross nominal interest rate, nominal money and real money converge to their respective steady state level. Nonetheless, the decrease of gold backing shock has a relatively persistent effect on those variables. In particular, it takes approximately 25 years for nominal money to reach back its steady state level. After that, it even moves (weakly) below its steady state level. Real money returns to its steady state level approximately 45 years after the impact. In turn, it takes around 30 years for non-monetary gold stock and gross nominal interest rate to come back to their respective steady state level. 50 years after the innovation to the gold backing shock hit the economy, the monetary gold stock is still below its steady state level. The real price of gold is the first variable, among those considered here, to converge to its steady state level, that is around 20 years after the impact. Then, it moves (weakly) above its steady state level. From Figure 11, one can note the total gold stock starts to decrease one period after the impact for approximately 30 years. As said above the total gold stock increases (decreases) if the inflow of gold is larger (lower) than the depreciation of the non-monetary gold stock brought from the previous period. In the current case, the non-monetary gold stock is above its steady state level for approximately 30 years after the impact while the gold flow shock remains at its steady state level for all that period. As a consequence, the gold inflow happens to be not sufficient to cover the depreciation of non-monetary gold stock during all that period. Thus, the total gold stock decreases. Thus, the total gold stock starts to move back to its steady state level one period after the non-monetary gold stock returns to its steady state level.

Figure 12: Selected impulse response functions to a 1 percent innovation to the gold backing shock (b).

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56The adjustment mechanism becomes quicker if the depreciation rate of non-monetary gold stock is raised. The explanation of that is the same as the one provided when the IRFs to gold flow shock are discussed.
Second, I focus my attention on the real part of the economy. In the gold standard model, the gold backing shock can be interpreted as a nominal shock since it influences the money supply of the economy through the optimal household’s decision on non-monetary gold stock and henceforth through the monetary gold stock. As for the gold flow shock, a change in the gold backing ratio is transmitted to the real part of the economy through the transaction cost wedge and the optimal condition on the non-monetary gold stock.

As shown in Figure 12, the baseline model predicts that the (negative) innovation to the gold backing shock leads consumption to rise on impact. Because of the liquidity effect implied by the decrease in the gold backing ratio, the transaction cost wedge declines on impact: The decrease of of the gold backing ratio pushes the gross nominal interest rate down, which in turn leads the transaction cost wedge to decrease. The initial fall of transactions costs makes it optimal for the household to increase its demand for consumption good. Moreover, the optimal condition (56) states that if the gross nominal interest rate drops, the marginal rate of substitution of consumption for real non-monetary gold must decrease too. The non-monetary gold liquidity effect leads the gross nominal interest rate and the real non-monetary gold to move in opposite directions. Taking into account the collapse of the transaction cost wedge, the increase of the real non-monetary gold stock is sufficiently larger in size than the decline of the nominal interest rate to make it optimal for the household to increase its consumption.

In the baseline model, the decrease of the transaction cost wedge, that follows the drop in the gold backing ratio, leads the opportunity cost of leisure, in terms of the marginal utility of consumption, to rise. Thus, it becomes profitable for the household to reduce its leisure. It follows that labor and output increase on impact, as shown in Figure 12. The left window of Figure 13 reports that lowering the gold backing ratio causes investment to increase too in the baseline model. Note that the transaction cost wedge moves back to its steady state level after its initial drop. It follows that the intertemporal transaction cost wedge falls below its steady state level when the innovation to the gold backing shock is realized. The decline of the intertemporal transaction cost wedge does not have a crowding out effect on investment, at least initially. The gold backing shock is sufficiently persistent so that the frictions induced by the transactions costs are small. The interval of time in which the transaction cost wedge is below its steady state level is sufficiently large so that the effect of the transaction cost wedge on consumption is persistent, at least initially. The household finds it optimal to increase initially its savings in order to ensure that its future consumption is still sufficiently high. By postponing its consumption, the household accumulates more capital. Therefore, investment increases. However, the response of investment to the decline of the gold backing shock is very small. Hence, higher persistence of the gold backing shock is required to increase to volatility of investment.

As shown in Figure 12, the gross real interest rate increases on impact: Since future labor is still well above its steady state level, the marginal productivity of capital is shifted upwards. The initial increase of capital is too small to push the marginal productivity of capital down. Figure 12 reports that the decrease of the gold backing shock leads the average labor productivity to decrease because labor increases more than output does on impact. As a result, the real wage decreases when the gold backing ratio is lowered.

Figure 12 shows that output, consumption, investment, labor and real wage go back to their respective steady state level one period after the innovation to the gold backing shock is realized in the baseline model. Output, consumption, labor and real wage return to their respective steady state level approximately 45 years after the impact. Investment returns to its steady state level quickly, that is approximately 6 years after the impact. Then it moves below its steady state level. Later in the impulse responses, output, consumption and labor drop below their respective steady state level. The impulse response of capital to the downward shift of the gold backing ratio is hump-shaped. Capital continues to increase for four years after the impact. After which it decreases towards its steady state level. Later in the impulse response, as investment is decreasing below its steady state level, it falls below its steady state level. As shown in Figure 11, one period after the impact, the gross real interest rate decreases back toward its steady state level. It gets back to its steady state level approximately 30 years after the realization of the innovation to the gold backing shock. It is worth to note that the innovation to the gold backing shock has relatively little effect on output, consumption, investment, labor, capital, real wage and gross real interest rate. Globally, those variables converge to their respective steady state as slowly as the gold stocks, real price of gold, nominal money, real money and gross nominal interest rate do.

As mentioned above, the response of investment to the (negative) innovation to the gold backing shock is positive but is relatively weak. Figures 12 and 13 show that habit formation in consumption
amplifies the effect of a decrease of the gold backing ratio on investment. Indeed, consumption continues to increase for few periods after the impact. Thus, expecting consumption increase in the subsequent periods, the household chooses to rise its savings through capital accumulation. Consequently, investment increases more strongly on impact in the habit model than in the baseline model. As said above, when the household is subject to habit formation in consumption, the effects of the transaction cost wedge on consumption become more persistent so that the household chooses to increase its consumption over several periods rather than over one single period. The initial response of consumption to the decrease of the gold backing shock is smaller in the habit model than in the baseline model. As well, the pick response of the consumption is smaller in the habit model than in the baseline model. Interestingly, as one can observe in Figure 12, the habit model generates a hump-shaped impulse response of output to the innovation to the gold backing shock. Indeed, output continues to increase one period after the impact. As for consumption, the initial increase of output is smaller in the habit model than in the baseline model. When the innovation to the gold backing shock is realized, labor increases in the habit model but less strongly than in the baseline model, which is consistent with the impulse response of output in the habit model. The real wage declines in the habit model when the gold backing ratio is lowered, as in the baseline model. Nonetheless, the initial drop of the real wage is deeper in the baseline model than in the habit model. The gross real interest rate moves above its steady state level when the innovation to the gold backing shock is realized. However, the initial increase of the gross real interest rate is milder in the habit model than in the baseline model.

I have also computed the IRFs of the complete model — gold standard model featuring habit formation in consumption and non-separable preferences — to a (negative) innovation to the gold backing shock. Those IRFs are reported in Figures 14 and 15.

As said above, according to Wen (2006), consumption and leisure are more likely to move in opposite direction if the preferences over consumption and leisure are non-separable. Thus, following a decrease of the gold backing ratio, increasing labor supply should be less costly to the household when the preferences are non-separable. Consequently, output would jump higher on impact in the complete model than in the habit model, implying a higher initial increase in investment too. Comparing Figure 15 with Figure 11, one can note that on impact, investment increases higher in the complete model than in the habit model. Additionally, investment remains above its steady state level after the impact for a longer time in the complete model than in the habit model. Indeed, the initial increases of labor and hencefore output are stronger in the complete model than in the

Figure 13: Impulse response functions of investment to a 1 percent innovation to the inverse of money multiplier shock: Baseline model vs habit model.
Figure 14: Selected impulse response functions to a 1 percent innovation to the gold backing shock: Complete model (a).

Figure 15: Selected impulse response functions to a 1 percent innovation to the gold backing shock: Complete model (b).
habit model. Furthermore, when the innovation to the gold backing shock is realized, consumption increases less strongly in the complete model than in the habit model. Besides, the increases of output, consumption, investment and labor are milder in the complete model than in the baseline model. As shown in Figures 12 and 14, the initial real wage drop is deeper in the complete model than in the habit model. Nonetheless, the larger initial collapse of the real wage is generated by the baseline model. Comparing Figure 15 with Figure 11, one can note that the gold stocks, real price of gold, nominal money, real money, gross nominal interest rate respond to the innovation to the gold backing shock similarly across the three alternative versions of the gold standard model. On impact, the gross real interest rate increases in the complete model as much as in the habit model. However, the initial jump of the gross real interest rate is higher in the baseline model than in the complete model. The innovation to the gold backing shock has a less persistent effect on the gross real interest rate in the complete model than in the habit and baseline models.

If an increase of the gold flow shock or a decrease of the gold backing shock leads the central bank to raise the money supply, those two shocks do not have the same effects on the economy. When the increase of the money supply results from a drop in the gold backing ratio, a liquidity effect comes out: The increase of the money supply is followed by a drop in the gross nominal interest rate. Instead, when the money supply is shifted upwards because of an increase of the gold inflow, the liquidity effect does not happen. When there is a liquidity effect, the transaction cost wedge is shifted upwards and henceforth affects the economy as a positive preference shock to consumption does. The transaction cost wedge causes an urge to consume. If the liquidity effect does not emerge, the transaction cost wedge moves below its steady state level and therefore leads the household to decrease its consumption. The transaction cost wedge behaves as a negative preference shock to consumption. Consequently, lowering the gold backing ratio has an expansionary effect on the economy whereas increasing the inflow of gold has a recessionary effect on the economy. The differences between the effects of an increase of the gold flow shock on the economy and the effects of a decrease of the gold backing shock on the economy show up also in quantitative terms. Specifically, the effects caused the positive innovation to the gold flow shock on the economy are larger and more persistent than the effects caused by a negative innovation to the gold backing shock on the economy. The fact that a decrease of the gold backing shock generates a liquidity effect while an increase of the gold flow shock does not is puzzling because the market conditions are not modified. Here, I put forward the following possible explanation. Expecting that the interval of time in which the real price of gold is below its steady state level would be larger following a positive innovation to the gold flow shock than following a negative innovation to the gold backing shock, the household is more willing to increase its real money demand in response to the latter shock than in response to the former. Therefore, the gross nominal interest rate would have to drop in order to ensure that the real money demand increased following the decline of the gold backing ratio. The gross nominal interest rate would not have to decline following an increase of the inflow of gold since the real money demand decreased. Agreed, these comments hold for an artificial economy where the prices — real price of gold, nominal wage and gross nominal interest rate — are determined in competitive markets. The only frictions that appear in this economy are introduced by the transactions costs. Thus, within this framework, it seems that if the public authorities want to drive the economy on a expansionary phase of the cycle through an increase of the money supply, it would be better to lower the gold backing ratio rather than to import more gold in the economy. However, the monetary policy consisting in lowering the gold backing ratio has to be conducted cautiously. Indeed, if the gold backing ratio is set to a very low level, the additional quantity of non-monetary gold stock that the household chooses to hold, may be larger than the total quantity of monetary gold held by the central bank. Consequently, the monetary authority would not be able to stand ready to sell or buy gold at the same fixed price of gold, \( P_G \). The central bank may have to raise the price of gold. This would lead the economy to leave the gold standard system. In the current state of the model, I cannot find the level of the gold backing ratio, \( \mu_2 \), under which, the fixity of the price gold, \( P_G \), cannot be held. To do so, I should have defined explicitly and separately the price level and the real price of gold so that the condition on the fixity of the price of gold would have been, somehow, endogeneized. In the current work, I assume that this condition is always fulfilled\(^{57}\).

\(^{57}\)The reader interested about this question is refered to Goodfriend (1988).
5.1.2.2 Impulse response functions to the inverse of money supply shock

In the gold standard model, the inverse of money multiplier shock and the gold backing shock are expected, by definition, to have similar effects on the economy, at least qualitatively. Obviously, if the degree of persistence of the inverse of money multiplier shock is different from the degree of persistence of the gold backing shock, those two shocks will affect differently the economy from a quantitative perspective. As a matter of fact, according to the calibration of the autoregressive parameters of the exogenous stochastic processes, the inverse of money multiplier shock is less persistent than the gold backing shock. This difference in terms of persistence might lead the two money supply shocks to have different impacts on the economy quantitatively as well as qualitatively. Indeed, if the degree of persistence of the inverse of money multiplier shock was not enough high, it would generate a crowding out effect on investment through the transaction cost wedge. Thus, in what follows, I will mainly focus my attention on the differences between the effects of the two money supply shocks on the economy. The economic mechanisms behind the impulse responses of the model’s variables to a negative innovation to the inverse of money multiplier shock are the same as those behind the impulse responses of the model’s variables to a negative innovation to gold backing shock. The usefulness of the distinction between the gold backing shock and the inverse of the money multiplier shock may not be clear at this stage of the analysis. Actually, this distinction will make more sense when the gold standard model is confronted to the historical data.

Figures 16 and 17 report the impulse responses of the baseline model and model with habit to a single negative innovation to the inverse of money multiplier shock. The IRFs of the baseline model’s variables are represented with starred lines while the IRFs of the model with habit are represented with circle lines. In period 0, the inverse of money multiplier shock is assumed to decrease by 1 percent.

![Graphs showing impulse responses](image)

Figure 16: Selected impulse response functions to a 1 percent innovation to the inverse of money multiplier shock (a).

From Figure 16, one can note that the baseline and habit models generate, through the inverse of money multiplier shock, very similar IRFs for non-monetary gold stock, monetary gold stock, total gold stock, real price of gold, gross nominal interest rate, nominal money and real money. Therefore, the existence of habit formation in consumption does not affect the impulse responses of those variables to a single negative innovation to the inverse money multiplier shock. This finding has been also obtained with a negative innovation to the gold backing shock. Thus, I consider indifferently the baseline model or the habit model when I comment the IRFs of non-monetary gold stock, monetary gold stock, total gold stock, real price of gold, gross nominal interest rate, nominal money and real

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percent deviation

percent deviation

percent deviation

percent deviation

percent deviation

Real interest rate

Real money

Inverse of money multiplier
money. The signs of the impulse responses of gold stocks, real price of gold, gross nominal interest rate, nominal money and real money to the innovation to the inverse of the money multiplier shock are identical to signs of the impulse responses of those variables to the innovation to the gold backing shock. Nonetheless, there are some differences between the impulse responses of those variables to a decrease of the inverse of money multiplier and the impulse responses of those variables to a drop in the gold backing ratio. Not surprisingly, the innovation to the inverse of money multiplier shock has a less persistent effect on gold stocks, real price of gold, nominal money, real money and gross nominal interest rate than the innovation to the gold backing shock has. On impact, the deviations of non-monetary gold stock, monetary gold stock, total gold stock, gross nominal interest rate, and real money from their respective steady state level are slightly larger following a decline of the inverse of money multiplier than following a drop in the gold backing ratio. Instead, the drop of the real price of gold is significantly deeper when it is the gold backing ratio which is lowered rather than the inverse of the money multiplier. As well, on impact, the jump of nominal money above its steady state level is slightly higher following an downward shift of the gold backing ratio than following a decrease of the inverse of money multiplier.

Now, I focus my attention on the real part of the economy. In the artificial gold standard economy, the inverse of the money multiplier shock can be defined as a nominal shock because it determines, indirectly, the money supply of that economy, as the gold backing shock does. As for the gold flow shock and the gold backing shock, a change in the inverse of money multiplier shock is transmitted to the real part of the economy through the transaction cost wedge and the optimal condition on the non-monetary gold stock. Considering only the baseline model predictions, Figures 17 and 12 show that the signs of the impulse responses of output, consumption, labor, real wage and transaction cost wedge to the negative innovation to the inverse of the money multiplier shock are identical to signs of the impulse responses of those same variables to the negative innovation to the gold backing shock. However, investment and hence capital drop on impact following a decrease of the inverse of money multiplier whereas those two variables jump above their respective steady state when the gold backing ratio is lowered. Hence, in the baseline model, the decrease of the inverse of money multiplier leads the transaction cost wedge to have a crowding out effect on investment. The inverse of money multiplier shock is not sufficiently persistent to rule out the crowding out effect of the transaction cost wedge on investment. Specifically, the innovation to the inverse of money multiplier shock has not enough persistent effect on the transaction cost wedge so that the distortions introduced by the transaction cost wedge are large. The initial drop of the transaction cost wedge is larger following

Figure 17: Selected impulse response functions to a 1 percent innovation to the inverse of money multiplier shock (b).
a decrease of the inverse of money multiplier than following a decline of the gold backing ratio: The decrease of the inverse of money multiplier leads to a significantly stronger urge to consume than a drop of the gold backing ratio does. Consequently, the initial increase of consumption is significantly higher following a decrease of the inverse of money multiplier shock than following a drop in the gold backing ratio. As a result, investment declines when the inverse of money multiplier is lowered. As well, the innovation to the inverse of money multiplier shock has a less persistent effect on consumption than the innovation to the gold backing shock has. In addition, the impulse responses of labor and output to the innovation to the inverse of money multiplier shock are less persistent than the impulse responses of those two variables to the innovation to the gold backing shock. The initial jumps of labor and output above their respective steady state level are slightly higher following a decrease of the gold backing ratio than following a drop of the inverse of money multiplier. Globally, according to Figures 17, 12, 16 and 11, the impulse responses of the real wage and the gross real interest rate to the innovation to the inverse of money multiplier shock are similar to the impulse responses of those same variables to the innovation to the gold backing shock.

As shown in Figure 17, introducing habit formation in consumption in the baseline model allows to weaken the crowding out effect of the transaction cost wedge on investment. Indeed, the habit model predicts that investment moves above its steady state level when the inverse money multiplier shock is decreased, at least initially. The crowding out effect of the transaction cost wedge on investment is even more dampened down when the gold standard model features both habit formation in consumption and non-separable preferences. Indeed, from Figures 18 and 17 one can observe that the initial increase of investment following a decline of the inverse of money multiplier shock is higher in the complete model than in the habit model. As well, the interval of time in which investment is above its steady state level is slightly larger in the complete model than in the habit model. In both habit and complete models, the impulse response of consumption to the innovation to the inverse of money multiplier shock is hump-shaped, as the impulse response of consumption to the innovation to the gold backing shock is. Nonetheless, in both habit and complete models, the impulse response of consumption is higher following a drop of the gold backing ratio than following a decrease of the inverse of money multiplier. Additionally, in both habit and complete models, the impulse response of consumption takes place later when the gold backing ratio is lowered than when the inverse of money multiplier is decreased. Output continues to increase one period after the realization of the innovation to the gold backing shock in the habit and complete model whereas, it already decreases back to its steady state level one period after the realization of the innovation to the inverse of money multiplier shock in the habit and complete model. As well, considering both habit and complete models, the decrease of the inverse of money multiplier has less persistent effects on output and labor than the drop of the gold backing ratio does. The initial increases of labor and real wage are higher following a decrease of the gold backing ratio than following a drop of the inverse of money multiplier in both habit and complete models.

Comparing Figure 19 with Figure 16, one can observe that the gold stocks, real price of gold, nominal money, real money, gross nominal interest rate respond to the negative innovation to the inverse of money multiplier shock similarly across the three alternative versions of the gold standard model. This finding is identical to the one obtained with the inverse of money multiplier shock. However, in all three versions of the gold standard model, the decrease of the inverse of money multiplier shock has less persistent effects on those variables than the drop of the gold backing shock. Furthermore, considering baseline, habit and complete models, the initial increase of the gross real interest rate is higher following a drop of the gold backing ratio than following a decline of the inverse of the money multiplier.

5.1.3 Impulse response functions to the productivity shock

Now, I study the effects of an increase of productivity on the artificial gold standard economy. Figures 21 and 20 display the impulse responses of the baseline model and model with habit to a single positive innovation to the productivity shock. The IRFs of the baseline model’s variables are represented with starred lines while the IRFs of the model with habit are represented with circle lines. In period 0, the productivity shock is assumed to increase by 1 percent.

First, I consider the effects of an increase of productivity on the baseline model. Figure 20 shows that the innovation to the productivity shock does not affect the non-monetary gold stock, monetary gold stock, total gold stock, nominal money and the gross nominal interest rate. However,
Figure 18: Selected impulse response functions to a 1 percent innovation to the inverse of money multiplier shock: Complete model (a).

Figure 19: Selected impulse response functions to a 1 percent innovation to the inverse of money multiplier shock: Complete model (b).
Figure 20: Selected impulse response functions to a 1 percent innovation to the productivity shocks (a).

Figure 21: Selected impulse response functions to a 1 percent innovation to the productivity shocks (b).
as reported in Figures 21 and 20, the other variables—real price of gold, gross real interest rate, output, consumption, investment, labor, capital, real wage and real rental rate of capital—move away from their respective steady state level following an increase of the productivity shock. The transaction cost wedge does not distort the economy following the increase of the productivity shock when the baseline model is considered. The orthogonality of the gold stocks, nominal money and gross nominal interest rate to the productivity shock is due to the log and time-separability specification of the preferences in the baseline model. Nevertheless, it is still useful to analyze the effect of a rise of productivity on the baseline model.

The increase of the productivity shock raises the real wage, that is the marginal productivity of labor. Acknowledging that the current wage is higher than the expected subsequent real wages, the household is more willing to increase its labor supply when the innovation to the productivity shock is realized. Indeed, labor increases on impact in the baseline model, as shown in Figure 21. The increases of both productivity shock and labor lead output to move above its steady state level on impact. As a result, the log-deviation output from its steady state level is larger than the log-deviation of labor from its steady state level, consistently to the increase of the real wage. This can be observed in Figure 21. The baseline model predicts that the additional output is essentially absorbed by investment. Indeed, Figure 21 reports that the 1.6 percent rise of output is used to increase investment by 5.6 percent. This allows the household to have a higher consumption level comparatively to its steady state level, in the current period as well as in the future periods. As shown in Figure 21, the impulse response of consumption to the innovation to the productivity shock is hump-shaped. In particular, consumption rises mildly on impact then continues to increase for two periods. After which it decreases back to its steady state level. One period after the impact the productivity shock and labor are still above their respective steady state level so that the marginal productivity of capital is increased. The capital stock moves above its steady state level on impact since investment is increasing. However, the initial jump of capital is not enough high to offset the effects of future productivity shock and labor on the marginal product of capital. Therefore, the gross real interest rate increases on impact, as reported in Figure 20. The initial increase of the gross real interest rate is consistent with the hump-shaped impulse response of consumption to the increase of the productivity shock. The gross real interest rate moves below its steady state level when the consumption reaches its peak response.

As reported in Figure 21, output, labor and investment decrease back to their respective steady state level one period after the impact. Those three variables reach back quickly their respective steady state level. In particular, output, labor and investment return to their respective steady state level 30, 4 and 5 periods after the impact, respectively. Little capital stock is accumulated through investment so that the interval of time in which output remains above its steady state level is relatively short. Consumption returns to its steady state level later than output does: Consumption reaches back its steady state level approximately 40 periods after the impact. Indeed, the household chooses to smooth its consumption behavior in order to keep its consumption above its steady state level for a period of time as long as possible. Note that labor drops below its steady state level after having returned to its steady state level. In particular, labor moves below its steady state level when capital starts to decline back to its steady state level. Thus, few periods after the impact, the household chooses to enjoy higher consumption and leisure levels comparatively to their respective steady state level by taking advantage of the capital stock, previously accumulated. Investment also moves below its steady state level after having reached back its steady state level in order to keep consumption above its steady state level. The real wage declines back to its steady state level one period after the impact. Nonetheless, the interval of time in which it remains above its steady state level is relatively large: The real wage returns to its steady state level approximately 40 periods after the realization of the innovation to the productivity shock. Indeed, labor decreases back to its steady state level

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58 If the gross nominal interest rate does not respond to the innovation to the productivity shock, the consumption-based velocity of money and the transaction cost wedge do not move away from their respective steady state level for all the period following an increase of productivity, consistently to equations (48) and (43).

59 It is not easy to have a formal proof of this statement. However, I have simulated a monetary model with a transaction cost function and log-separable preferences. The household is not subject to habit formation in consumption. There is no motive for the household to hold non-monetary gold stock and the money supply is determined independently of the monetary gold stock of the central bank. In particular the money supply is exogenously defined by a stochastic process. I have found that the gross nominal interest rate does not respond to an innovation to the productivity shock.
quicker than output does so that the marginal productivity of labor is kept above its steady state level for a relatively long time. The drop of labor below its steady state level has also maintained the marginal productivity of labor above its steady state level. One reason why the productivity shock has not a strong persistent effect on output, labor and investment is that the productivity shock is itself weakly persistent. Figure 20 shows that the productivity shock returns to its steady state level approximately 15 periods after the impact. The lack of a strong internal propagation mechanism in the baseline model also explains why output, labor and investment go back to their respective steady state level so quickly\footnote{Those findings are similar to those obtained by King and Rebelo (1999) with the standard RBC model.}

As said above, the increase of the productivity shock affects also the real price of gold and real money in the baseline model. More precisely, the real price of gold and the real money respond positively to the increase of the productivity shock. One way to understand why the innovation to the productivity shock raises the real price of gold, is to sketch out the working of economy is a standard aggregate supply-aggregate demand diagram, where the price level is represented in the y-axis and the aggregate supply and demand levels are represented in the x-axis. Thus, the aggregate supply curve is upward sloping while the aggregate demand is downward sloping in this diagram. The increase of the productivity shock shifts the aggregate supply curve rightward so that the aggregate supply increases. This leads to a downward movement along the aggregate demand curve so that the price level drops. Hence, the increase of productivity shock causes an increase of output and a decline of the price level. Since the price of gold is assumed fixed, the decrease of the price level leads the real price of gold to move above its steady state level, as shown in Figure 20. The initial increase of real money is driven only by the movement in the real price of gold since the nominal money does not respond to the change in the productivity shock. From Figures 21 and 20, one can note that the IRFs of the real price of gold and real money are identical to the IRF of consumption in the baseline model. Indeed, if the gross nominal interest rate is set to its steady state level for every periods, the decision rules of output, consumption, investment, capital, labor, real wage, real rental rate of capital, real money, real price of gold and gross real interest rate can be derived from the equilibrium equations (42), (6), (39), (45'), (46'), (47'), (49), (50), (51) and (52)\footnote{If the gross nominal interest rate is set to its steady state level for every periods, the consumption-based velocity of money and the transaction cost wedge are fixed to their respective steady state level too, consistently to equations (48) and (43).}. The log-linearization of this block of equilibrium equations shows that the gross growth rate of consumption, $\frac{c_{t+1}}{c_t}$, and the gross rate of real money, $\frac{m_{t+1}}{m_t}$, would deviate from their respective steady state level as much as the gross growth rate of the real price of gold, $\frac{p_{gt+1}}{p_{gt}}$, would do following a change in the productivity shock\footnote{Specifically, the log-linearized form of equation (46') is given by 

$$E_t \left( \tilde{C}_t - \tilde{C}_{t+1} + \tilde{p}_{gt,t+1} - \tilde{p}_{gt,t} \right) = 0$$

since $\kappa_t$ is assumed to be fixed to its steady state level. Hence, the log-deviation of the gross growth rate of consumption from its steady state level is equal to the log-deviation of the gross rate of the real price of gold from its steady state level. Besides, according to (42), $\tilde{C}_t = m_t$ since $\kappa_t = v$ by assumption. Therefore, the gross growth rate of real money from its steady state level is equal to the log-deviation of the gross growth rate of consumption from its steady state level and henceforth to the log-deviation of the real price of gold from its steady state level.}.

It would follow that $\tilde{C}_t = \tilde{m}_t = \tilde{p}_{gt}$ since, the nominal money stock and the gross nominal interest rate, do not respond to the innovation to the productivity shock.

Recall that the productivity shock does not affect the gold stocks, nominal money and gross nominal interest rate in the baseline model. Thus, I modify the specification of the utility function in order to allow the productivity shock to be fully transmitted to the nominal part of the economy. In particular, I introduce habit formation in consumption in the utility function. Figure 20 shows that non-monetary gold stock, monetary gold stock, total gold stock, nominal money and gross nominal interest rate respond to the change in the productivity shock in the habit model. Before going through the analysis of the IRFs of non-monetary gold stock, monetary gold stock, total gold stock, nominal money, real money, real price of gold and gross nominal interest rate in the habit model, I study how the existence of habit formation in consumption alters the effects of the increase of the productivity shock on the real part of the gold standard economy.

As shown in Figure 21, the hump-shaped impulse response of consumption to the innovation to
the productivity shock is more pronounced in the habit model than in the baseline model. Thus, the household substitutes more future consumption for current consumption when it is subject to habit formation in consumption so that the initial increase of consumption is lower in the habit model than in the baseline model. Consequently, on impact, investment jumps higher in the habit model than it does in the baseline model. This allows the household to continue to increase its consumption over four periods after the impact in the habit model. Nonetheless, the interval of time in which consumption is above its steady state level is approximately identical in both models. After the impact, investment behaves similarly in both models. The initial responses of output and labor to the innovation to the productivity shock are slightly lower in the habit model than in the baseline model. On the contrary, on impact, the real wage increases slightly higher in the habit model than in the baseline model. However, after the impact, the IRFs of output, labor and real wage are similar across the two versions of the gold standard model. In the habit model, the transaction cost wedge moves away from its steady state level following a change in the productivity shock, as shown in Figure 21. Precisely, the transaction cost wedge declines on impact. Then, it increases back to its steady state level. Later in the impulse response, the transaction cost wedge moves slightly above its steady state level. Nonetheless, the initial decrease of the transaction cost is relatively weak so that it does not alter significantly the behavior of the real variables such as consumption and investment. It might have played a secondary role in the increase of consumption. The impulse response of the gross real interest rate follows the same pattern in both models. There are two minor differences between the IRF of the gross real interest rate in the habit model and the IRF of the gross real interest rate in the baseline model. Specifically, the initial increase of the gross real interest rate is slightly lower in the habit model than in the baseline model. Later in the impulse response, the gross real interest rate moves below its steady state level in both models. However, the gross real interest rate drop below its steady state level is slightly deeper in the habit model than in the baseline model. Thus, introducing the habit formation in consumption in the baseline model does not lead the productivity shock to affect the real part of the economy more persistently.

Now, I focus my attention on the nominal part of the economy. As shown in Figure 20, the impulse response of the real price of gold to the innovation to the productivity shock follows a pattern in the habit model which is similar to one followed in the baseline model. The real price of gold jumps above its steady state level following an increase of the productivity shock in the habit and baseline models. Nonetheless, the initial increase of the real price of gold is higher in habit model than in the baseline model. The real price of gold continues to increase after the impact for two periods in both models. The pick response of the real price of gold is higher in the habit model than in the baseline model. Then, the real price of gold declines towards its steady state level in the habit model as it does in the baseline model. The habit model predicts that the gross nominal interest rate responds negatively to the innovation to the productivity shock. The intuition is that the increase of the productivity shock leads the gross nominal interest rate to decline so that the transactions costs are lowered. As a result, consumption is stimulated. However, the initial decrease of the gross nominal interest rate is relatively weak. One period after the impact, the gross nominal interest rate increases back to its steady state level. Later in the impulse response, it mildly moves above its steady state level. The decline of the gross nominal interest rate leads the household to raise its real money demand. Indeed, as shown in Figure 20, the real money increases on impact. As in the baseline model, the impulse response of real money is hump-shaped in the habit model. However, the initial real money increase is lower in the habit model than in the baseline model. Furthermore, real money continues to increase after the impact in the habit model as in the baseline model. Nonetheless, the increase of real money lasts one period more in the habit model than in the baseline model.

The increase of the productivity shock is transmitted to the gold sector of the economy through the optimal condition on the non-monetary gold stock. Specifically, the optimal condition on the marginal rate of substitution of consumption for non-monetary gold, (54), also defines an optimal demand function for non-monetary gold\(^{63}\). The demand function for non-monetary gold stock is implicitly given by equation (54). In the habit model, this demand function for non-monetary gold

\[ \hat{\zeta}_{c,t} = \hat{\zeta}_t - \hat{\rho}_{q,t} + \varphi \hat{\theta}_t \]

\(^{63}\)The demand function for non-monetary gold in the baseline model has been implicitly defined when the optimal condition on the marginal rate of substitution of consumption for non-monetary gold is discussed. This demand function, in its log-linearized form, is given by equation (56), that is
takes the following form

\[
G_{ct} = \frac{\phi_3}{\phi_1} \frac{\kappa_t}{p_{g,t}} \left(1 - \frac{1 - \delta_t}{q_t}\right)^{-1} \left[\frac{1}{C_t - bC_{t-1}} - b\beta E_t \left(\frac{1}{C_{t+1} - bC_t}\right)\right]^{-1}
\]

This equation states that given the real price of gold, the gross nominal interest rate and the transaction cost wedge, the demand for non-monetary gold increases if the current change in consumption is higher than the (discounted) future change in consumption. The non-monetary gold stock is also increasing with the gross nominal interest rate. However, it is decreasing with the real price of gold and the transaction cost wedge. Since the impulse response of consumption is hump-shaped, the increase of consumption would lead the non-monetary gold stock. The decline of the transaction cost wedge would raise the demand for non-monetary gold. Instead, the increase of the real price of gold and the decrease of the gross nominal interest rate would push down the demand for non-monetary gold stock. According to Figure 20, the net result is an increase of the non-monetary gold stock. Since no additional gold is shipped in the economy, the household has to buy gold for non-monetary use to the central bank. Consequently, monetary gold and nominal money decline, as shown in Figure 20. After the impact, the gold stocks and nominal money go back toward their respective steady state level. Note that later in the impulse responses, non-monetary gold stock moves below its steady state level while monetary gold moves above its steady state level.

The IRFs of the gold standard model’s variables to the innovation to the productivity shock are also computed in the case where the model features non-separable preferences and habit formation in consumption. Those IRFs are displayed in Figures 22 and 23.

![Figure 22: Selected impulse response functions to a 1 percent innovation to the productivity shock: Complete model (a).](image)

Output, consumption, investment, labor and real wage follow similar patterns across the three versions of the gold standard model. Nonetheless, the initial increases of output and labor are lower in the complete model than in the two other models. After the impact, the IRFs of output and labor in the complete model become close to the IRFs of those same variables in the baseline and habit models. On impact, investment increases slightly lower in the complete model than in the habit model. However, on impact, it increases in the complete model almost as much as it does in the baseline model. In the subsequent periods, investment behaves similarly across the three models. The initial increase of consumption is lower in the complete model than in the baseline model. The initial (log-)deviation of consumption from its steady state level is almost of the same size in the
Figure 23: Selected impulse response functions to a 1 percent innovation to the productivity shock: Complete model (b).

complete model as in the habit model. Though, interval of time in which consumption continues to increase after the impact is one period shorter in the complete model than in the habit model. In other words, consumption continues to increase after the impact in the complete model for as long as it does in the baseline model. The pick response of consumption is slightly lower in the complete model than in the habit model. Therefore, the largest (log-)deviation consumption from its steady state level is significantly smaller in the complete model than in the baseline model. The highest initial response of the real wage to the increase of the productivity shock is generated in the complete model. Nevertheless, after the impact, the real wage behaves similarly in all three models. From Figures 20 and 23 shows that the impulse response of the gross real interest rate in the complete model is similar to the impulse response of that variable in the habit model. Thus, introducing the habit formation in consumption and the non-separability of preferences in the baseline model does not lead the productivity shock to have more persistent effect on the real part of the economy.

The comparison of Figure 20 with Figure 23 shows that the real price of gold displays a hump-shaped impulse response to the innovation to the productivity shock in the complete model as in the habit and baseline models. The expansion part of the IRF of the real price of gold in the complete model is almost identical to the one in the habit model. However, after its pick response, the real price of gold declines back towards its steady state level more slowly in the complete model than in the two other models. Real money also displays a hump-shaped impulse response to the increase of the productivity shock in the complete model as in the habit and baseline models. The initial increase of real money is as large in the complete model as in the habit model. The pick response of real money happens to be slightly higher in the complete model than in the habit model. Nonetheless, the period at which real money reaches its highest response is the same in both complete and habit models. Thus, the initial increase of real money is lower in the complete model than in the baseline model. The pick response of real money takes place one period later in the complete model than in the baseline model. However, it is lower in the complete model than in the baseline model. After the pick response, the decrease of real money towards its steady state level is slightly less steep in the complete model than in the baseline and habit models. The gross nominal interest rate responds negatively to the increase of the productivity shock in the complete model as in the habit model. However, there are some notable differences between the IRF of gross nominal interest rate generated by the complete model and the one generated by the habit model. The initial decrease of the gross nominal interest rate is deeper in the habit model than in the complete model. Besides, while the gross nominal interest rate continues to decrease for one period after the impact in the complete ...
model, it is already increasing back towards its steady state level. Hence, the gross nominal interest rate moves back to its steady state level more gradually in the complete model than in the habit model. As shown in Figures 20 and 23, when the innovation the productivity shock is realized, the non-monetary gold stock increases slightly more in the complete model than in the habit model. In both complete and habit models, the non-monetary gold stock declines back towards its steady state level one period after the impact. However, it takes 11 periods after the impact to the non-monetary gold stock for getting back to its steady state level in the complete model whereas the non-monetary gold stock returns to its steady state level only 6 periods after the impact in the habit model. As in the habit model, monetary gold and henceforth nominal money fall in the complete model following an increase of the productivity shock. Then, they increase back towards their respective steady state level. However, on impact, monetary gold stock and nominal money decline slightly deeper in the complete model than in the habit model. Furthermore, the productivity shock has a more persistent effect on monetary gold and nominal money in the complete model than it has in the habit model. Indeed, those two variables reach back their respective steady state level later in the complete model than in the habit model.

### 5.1.4 Theoretical moments of the gold standard model

To close the discussion of the dynamic properties of the gold standard model, Table 6 reports selected second moments for the 3 versions of that model — baseline, habit and complete models. In particular, Table 6 displays the standard deviations (s.d.) of several key variables and the correlations of output with the other key variables. As it can be observed from Table 6, I consider two cases. In the first case (top part of Table 6), the fluctuations in the model’s variables are driven by all the shocks — productivity, gold flow, gold backing, and inverse of money multiplier shocks. In the second case (bottom part of Table 6), the fluctuations in the model’s variables are driven by only the nominal shocks — gold flow, gold backing and inverse of money multiplier shocks.

Above, I have shown that output and investment have collapsed strongly during the 1929-1936 period in France, meaning that investment has behaved procyclically (with output) over that period. The gold standard model is intended to be used for an evaluation of the gold standard-based explanation of the French Great Depression. Hence, the model, through all the nominal shocks, should be able to generate procyclical investment. Thus, if one focuses only on the effects of the nominal shocks on the gold standard artificial economy, one would consider the complete model rather than the baseline and habit models. Indeed, the baseline model displays a weak correlation between output and investment. Though introducing habit formation in consumption in the baseline model increases the correlation between output and investment, the value of this correlation generated from the habit model is still low. One has to introduce habit formation in consumption and non-separable preferences in the baseline model to get the nominal shocks generate procyclical investment in the gold standard artificial economy. Indeed, the nominal shocks driven complete model displays a high correlation between output and investment. This confirms what have been said on the subject when the analysis of the IRFs has been conducted.

### 5.2 The gold standard and the French Great Depression

Before analyzing the French Great Depression through the gold standard model, it is worth to have again in mind some facts about that historical event. As soon as France returned to the gold standard, that is in 1928, an important flow of gold moved towards that country. Specifically, the French net import of gold increased significantly between 1928 and 1932. In the meantime, the Bank of France raised substantially its gold reserves. Despite of the large accumulation of monetary gold, the nominal stock of money increased only mildly between 1928 and 1932. Indeed, the Bank of France raised its gold backing ratio well above the 35 percent legal minimum during that period. As well, the money multiplier (inverse of money multiplier shock) was decreasing (increasing) between 1929 and 1932, meaning that the French commercial banks were experiencing some difficulties. Nonetheless, the increase of the gold backing ratio accounts more than the decrease of the money multiplier does for the gap between nominal money and monetary gold. As the nominal stock of money was increasing, the real price of gold — that is the inverse of the price level — dropped between 1928 and 1930. Besides, the Bank of France lowered its gross nominal interest rate in the beginning of the 1930s. From 1932 until France left the gold standard, gold moved away from the country.
Table 6: Theoretical moments of the gold standard model

<table>
<thead>
<tr>
<th>Variables $u_t$</th>
<th>Real + nominal shocks driven economy</th>
<th>Nominal shocks driven economy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Models</td>
<td>Models</td>
</tr>
<tr>
<td></td>
<td>Baseline</td>
<td>Habit</td>
</tr>
<tr>
<td>$y_t$</td>
<td>s.d. 1.0000</td>
<td>corr ($u_t, y_t$) 0.0973</td>
</tr>
<tr>
<td>$c_t$</td>
<td>s.d. 0.8023</td>
<td>corr ($u_t, y_t$) 0.6715</td>
</tr>
<tr>
<td>$x_t$</td>
<td>s.d. 0.9194</td>
<td>corr ($u_t, y_t$) 0.9154</td>
</tr>
<tr>
<td>$h_t$</td>
<td>s.d. 0.8343</td>
<td>corr ($u_t, y_t$) 0.7483</td>
</tr>
<tr>
<td>$m_t$</td>
<td>s.d. 0.2127</td>
<td>corr ($u_t, y_t$) 0.1985</td>
</tr>
<tr>
<td>$q_t$</td>
<td>s.d. -0.0732</td>
<td>corr ($u_t, y_t$) -0.0944</td>
</tr>
<tr>
<td>$w_t$</td>
<td>s.d. 0.9082</td>
<td>corr ($u_t, y_t$) 0.9179</td>
</tr>
<tr>
<td>$p_{g,t}$</td>
<td>s.d. 2.7705</td>
<td>corr ($u_t, y_t$) -0.0509</td>
</tr>
<tr>
<td>$G_{m,t}$</td>
<td>s.d. 2.9398</td>
<td>corr ($u_t, y_t$) 0.0680</td>
</tr>
<tr>
<td>$G_{c,t}$</td>
<td>s.d. 3.0784</td>
<td>corr ($u_t, y_t$) 0.0825</td>
</tr>
<tr>
<td>$k_t$</td>
<td>s.d. 0.7291</td>
<td>corr ($u_t, y_t$) 0.7416</td>
</tr>
<tr>
<td>$\bar{r}_t$</td>
<td>s.d. 0.1272</td>
<td>corr ($u_t, y_t$) -0.0463</td>
</tr>
</tbody>
</table>

|                 | Complete                              | Complete                     |
| $y_t$           | s.d. 1.0000                           | corr ($u_t, y_t$) 0.0962      |
| $c_t$           | s.d. 0.8023                           | corr ($u_t, y_t$) 0.6715      |
| $x_t$           | s.d. 0.9194                           | corr ($u_t, y_t$) 0.9154      |
| $h_t$           | s.d. 0.8343                           | corr ($u_t, y_t$) 0.7483      |
| $m_t$           | s.d. 0.2127                           | corr ($u_t, y_t$) 0.1985      |
| $q_t$           | s.d. -0.0732                          | corr ($u_t, y_t$) -0.0944     |
| $w_t$           | s.d. 0.9082                           | corr ($u_t, y_t$) 0.9179      |
| $p_{g,t}$       | s.d. 2.7705                           | corr ($u_t, y_t$) -0.0509     |
| $G_{m,t}$       | s.d. 2.9398                           | corr ($u_t, y_t$) 0.0680      |
| $G_{c,t}$       | s.d. 3.0784                           | corr ($u_t, y_t$) 0.0825      |
| $k_t$           | s.d. 0.7291                           | corr ($u_t, y_t$) 0.7416      |
| $\bar{r}_t$    | s.d. 0.1272                           | corr ($u_t, y_t$) -0.0463     |
Indeed, after 1932, the French net import of gold decreased substantially. After 1934, France even became a net exporter of gold. Hence, the gold reserves of the Bank of France declined during that period, though the monetary gold stock was by 1936 still higher than its 1929 level. The nominal stock of money also collapsed between 1932 and 1936. In 1935, it slightly dropped below its 1929 level, though the Bank of France lowered the gold backing ratio between 1934 and 1936. The decline of the gold backing was not large enough to offset the effect of the monetary gold decrease on the nominal money since the gold backing ratio was still above the 35 percent legal minimum in 1936. Note that the money multiplier continued to rise after 1932, though the increase was still relatively weak. Between 1931 and 1936 the French economy was in deflation as the real price of gold increased above its 1929 level. In the meanwhile, the French central bank raised its gross nominal interest rate so that it was slightly above its 1929 level in 1936. The fluctuations of the real price of gold and nominal money in the 1930s lead real money to increase between 1929 and 1932, then to stabilize around its 1932 level until 1936. During all those years, the real part of the French economy was experiencing a depression as output, consumption, investment, labor and gross real interest rate were strongly declining. The decline of the hours worked was followed by an increase of the real wage.

5.2.1 Methodology

This subsection aims at to answer to the question whether the perturbations that have hit the nominal part of the French economy in the 1930s also account for the depression that the real part of the French economy has undergone. Put it simply, this subsection aims at to answer to the question whether the commitment of France to the gold standard has put the country into a depression. The gold standard related perturbations are the gold flow shock, the gold backing shock and the inverse of money multiplier shock. In order to control the role of the nominal shocks in the French Great Depression, I also consider a real shock, that is the productivity shock, in this study. Basically, the productivity shock is an alternative to the nominal shocks for the understanding of the French Great Depression. The complete (gold standard) model, which dynamic properties has been discussed above, is put to use to evaluate the roles of the nominal and real shocks in the French Great Depression.

This analysis is performed in four steps. First, I use the constructed sequences of innovations to the gold flow shock, gold backing shock and inverse of money multiplier shock one at a time as period-by-period impulses to the gold standard model — complete model — to generate artificial time series for the key variables — output, consumption, investment, labor, gross real interest rate, real wage, nominal money, real money, monetary gold, real price of gold and gross nominal interest rate. Hence, for each variable, $s_t$, 3 artificial time series are created, $s_{j,t}$, $j = 1, 2, 3$. $s_{j,t}$ measures the effects of the historical shock $j$ on variable $s$. By doing so, I can isolate the effects of each historical nominal shock on the French economy during the 1930s. I assess the effects of each of the historical nominal shock on the French economy by comparing the model’s realizations of the key variables with the historical data.$^{64}$ Second, I feed all the constructed sequences of innovations to the gold flow shock, gold backing shock and inverse of money multiplier shock into the complete model (hereafter the gold standard model) in order to generate artificial time series for the key variables. In this second step only one artificial time series is generated for each variable. The generated artificial time series $s_{nt,t}$ measures the effects of all the historical nominal shocks, taken together, on variable $s_t$. This exercise aims at to assess whether the gold standard is the main cause of the French Great Depression. Specifically, if the gold standard model’s predictions are close to the historical data, then one can conclude that the commitment to the gold standard is a good avenue for understanding the French Great Depression. Third, I analyze the effects of the alternative shock, that is the productivity shock, on the French economy. Thus, I feed the constructed sequence of innovations to the productivity shock into the gold standard model in order to generate artificial time series for the key variables. The generated artificial time series $s_{pt,t}$ measures the effects of the historical productivity shock on variable $s_t$. As in the first step, I evaluate the effects of the historical productivity shock on the French economy by comparing the artificial time series of the key variables with the historical data. Note that these exercises — steps 1 to 3 — are performed under the implicit assumption that the gold standard model offers a reasonable representation of how the French economy has worked in the 1930s. However, this assumption has not been verified.

$^{64}$Several researchers have followed this methodology in their investigations into historical events such as the Great Depression: Bordo et al. (2000), Cole and Ohanian (1999) and Weder (2006) among others.
yet. I do this in the fourth step. Specifically, I feed all the historical sequences of innovations to the gold flow shock, gold backing shock, inverse of money multiplier shock and productivity into the gold standard model in order to generate artificial time series for the key variables. Then, I compare the simulated series of the key variables to their empirical counterparts. If the simulated time series do match with the historical data, one can conclude that the gold standard model is an acceptable representation of how the French economy has worked during the interwar gold standard period. If it is not the case, this would not mean that the simulation exercises, performed in steps 1 to 3, are useless. They might help the researcher to find in which dimensions the gold standard model needs to be improved.

5.2.2 Empirical results

This subsection presents and discusses the results of the application of the gold standard model to the French Great Depression.

5.2.2.1 The Effects of the gold flow shock

Assuming that the gold standard model offers a reasonable representation of how the French economy has worked in the 1930s, I use that model and the constructed sequence of innovations to the gold flow shock in order to evaluate the role of that shock in the observed fluctuations of key macroeconomic variables over the 1929-1936 period.

![Figure 24: Simulated data vs French historical data: Gold flow shock effects, 1929 = 0 (a).](image-url)

Figure 24 displays the effects of the historical gold flow shock on the nominal part of the gold standard artificial economy. As shown in Figure 24, the simulated variables — represented in starred lines — are also confronted to their respective historical measure — represented in solid line. The gold standard model predicts that the 1929-1932 increase of the gold flow shock leads the monetary gold stock to move above its steady state level. However, the simulated monetary gold stock does not increase as much as the actual monetary gold stock does between 1929 and 1932. In particular, in 1932, the gold standard, through the gold flow shock, explains almost 40 percent of the monetary gold stock increase. Note that the Bank of France has also increased its gold reserves during that period by converting a large fraction of its foreign exchanges in gold. In the gold standard model, additional monetary gold can come from the new gold inflow and/or the gold stock held by the household. Thus, the additional monetary gold coming from the conversion of the Bank of France’s
foreign exchanges are not captured by the model. This might explain, at least partly, why the gold standard model generates a monetary gold stock increase that is milder than what is observed in the historical data. Indeed, the amount of foreign exchange accumulated by the Bank of France between 1926 and 1928 is large. According to Nurkse (1944), the Bank of France holds more than half of the total world's stock of foreign exchanges by the end of 1928. The Bank of France starts to liquidate its foreign exchanges for the benefit of its gold reserves when England leaves the gold standard, that is in September 1931. The amounts of foreign exchanges converted into gold increases dramatically during 1932. By 1933, the stock foreign exchange held by the Bank of France is almost fully liquidated (see Mouré, 1991, 2002, and Nurkse, 1944). From 1932 to 1936, the simulated monetary gold stock drops strongly. By 1936, the model's monetary gold stock is 65.5 percent below its steady state level. However, as said above, the observed decline of the monetary gold stock is not such large since the historical monetary gold stock is still above its 1929 level in 1936. The gold standard model predicts that the outflow of gold leads the monetary gold stock to decline more than the non-monetary gold stock does, as shown in Figure 25. The household is less willing to let its non-monetary gold stock decrease than to let the gold reserve of the central bank decline. This might explain why the monetary gold stock falls so much in the artificial economy following an outflow of gold.

Figure 25: Simulated monetary gold vs simulated non-monetary gold: Gold flow shock effects, 1929 = 0.

Since the productivity shock and the money supply shock are set to their respective steady state, the gold standard model predicts that the historical sequence of innovations to the gold flow shock

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65 In August 1926 — so before the country returned officially to the gold standard — Poincaré passed a law — the law of 7 August 1926 — which allowed the Bank of France to buy foreign exchange and gold at market price. Between 1926 and 1928, the bank of France bought a large amount of foreign exchange and gold in the market in order to ensure that the value of the French currency would not appreciate above its 1926 value. The volume of foreign exchange purchased was significantly greater than the acquired stock of gold. With the monetary law of June 1928, the Bank of France was forbidden to buy further foreign exchange after 1928. See Nurkse (1944) and Mouré (1991, 2002) for more details.

66 According to Mouré (1991, 2002), the French authorities are not keen to let the Bank of France hold foreign assets. They believe that the virtue of the pre-war gold standard cannot be found into the gold-exchange standard. They claim that France cannot return to the pre-war gold standard system as long as the Bank of France continues to hold foreign assets. The Bank of France suffers important losses from its foreign exchange held in pounds when England devalues its currency in September 1931. Thus, the French authorities fear to get further losses from holding foreign exchanges. This leads the Bank of France to liquidate its foreign assets.
leads nominal money to behaves as the monetary gold stock does over the 1929-1936 period. Figure 24 shows that the simulated nominal money fluctuates relatively closely to the historical measure of nominal money between 1929 and 1933. Afterward, the simulated nominal money collapses strongly below its steady state while the actual nominal money fluctuates closely to its 1929 level. In the current simulation exercise the observed gap between nominal money and monetary gold is not captured by the gold standard model because both nominal supply shocks are shut down.

As reported in Figure 24, the gold standard model predicts that the observed movements in the gold flow shock cause large fluctuations in the real price of gold. Specifically, the simulated real price of gold drops by 40.6 percent below its steady state level between 1929 and 1932. Then, the real price of gold rises substantially until 1936, where it is 121 percent above its steady state level. Even though the simulated real price of gold and the actual real price of gold follow qualitatively similar patterns, the volatilities of both series have to be contrasted. The model predicts that if the French economy is perturbed only by the gold flow shock, it would experience a severer inflation between 1929 and 1932 than what is observed in the historical data. Afterward, the gold standard model predicts that the French economy would undergo a stronger deflation until 1936 comparatively to what it is reported by the historical data.

As one can note from Figure 24, the gold standard model, through the historical gold flow shock, fails to replicate the observed fluctuations in the gross nominal interest rate over the 1929-1936 period. The simulated gross nominal interest rate and historical gross nominal interest rate move in opposite directions during the 1930s. Besides, the model’s gross nominal interest rate is significantly more volatile than the actual gross nominal interest rate. This result is not surprising. The historical data report that in the beginning of the decade, the increase of the nominal money is followed by a decline of the gross nominal interest. However, I have shown above that the gold flow shock does not generate a liquidity effect in the gold standard model. Note that the apparent liquidity effect is most probably due to international factors rather than to domestic factors. According to Mouré (1991), the Bank of France lowers its nominal interest rate in mid-1930 then in early 1931 in response to the drops of the Bank of England’s nominal interest rate. By doing so, the French central bank prevents the differential between the French nominal interest rate and the English nominal interest rate from being exacerbated. The Bank of France believes that an increase of the interest rate gap would encourage gold flows towards France, which in turn would cause an increase of money supply and hence an increase of the price level.

Figure 24 also compares the gold standard model’s real money to the historical measure of real money over the 1929-1936 period. According to the historical data, real money increases until 1932 where it is 21 percent above its 1929 level. Then, it remains close to its 1932 level until 1936. The simulated real money follows a different pattern. In the gold standard artificial economy, the volatility of the real price of gold is larger than the volatility of nominal money. Consequently, the simulated real money follows the same pattern as the simulated real price of gold. The simulated real money drops below its steady state level by 19 percent between 1929 and 1931. Then, real money rises dramatically until 1936, where it is 55.5 percent above its steady state level.

Figures 26 and 27 display the effects of the historical gold flow shock on the real part of the gold standard artificial economy. As shown in Figures 26 and 27, the simulated variables — represented in the left windows and in starred lines — are also confronted to their respective historical measure — represented in the right windows and in solid line. Clearly, the channels through which the gold flow shock is transmitted to the real part of the gold standard artificial economy do not have strong amplification mechanism. The effects of the historical measure of the gold flow shock on output, consumption, investment, labor, real wage and gross real interest rate are very weak in the gold standard model. Specifically, the fluctuations of the simulated output, consumption, investment, labor, real wage and gross real interest rate are quantitatively insignificant comparatively to what it is observed in the historical data. Nonetheless, it is worth to comment the qualitative effects of the historical gold flow shock on those real variables. The gold standard model predicts the historical sequence of innovations to the gold flow shocks leads output, consumption, labor and gross real interest rate to decline slightly between 1929 and 1932. Then, they increase and move above their respective steady state level. The simulated investment also decreases in the beginning of the decade. However, the predicted investment decline lasts one year less than the predicted output decrease does. The simulated investment increases from 1931 to 1936 where it is 0.5 percent above its steady state level. The simulated real wage moves above its steady state level between 1929 and 1932. Afterward, it decreases and drops below its steady state level. Those results are not surprising. Indeed, the analysis of the impulse response functions of the gold standard model has shown that an
Figure 26: Simulated data vs French historical data: Gold flow shock effects, 1929 = 0 (b).

Figure 27: Simulated data vs French historical data: Gold flow shock effects, 1929 = 0 (c).
increase of the gold flow shock has a weak and negative effect on output, consumption, investment, labor and gross real interest rate. This analysis also has demonstrated that an increase of the gold flow shock leads the real wage to shift upwards. Thus, the predictions of the gold standard model, through the historical gold flow shock, in terms of output, consumption, investment, labor, real wage and gross real interest rate, have to be constrained with what the historical data tell also from a qualitative point of view. The historical gold flow shock causes the real artificial gold standard economy to enter into a (mild) recession in the beginning of the decade as it is observed in the historical data. However, through the historical gold flow shock, the real gold standard artificial economy counterfactually recovers quickly afterwards. Indeed, the simulated output, consumption, investment and labor are above their respective steady state level after 1932 whereas the historical output, consumption, investment and labor continue to decline until 1936. Besides, the historical gold flow shock does not lead the real wage to increase over all the 1929-1936 period in the artificial gold standard model as it is observed in the historical data. As well, the simulated gross real interest rate and the actual gross real interest rate follow different patterns between 1929 and 1936.

Thus, the gold standard model, through the historical gold flow shock, cannot fully explain the observed fluctuations in the nominal part of the French economy during the 1930s. Besides, the gold standard model does not include an amplification mechanism which would increase the magnitude of the effect of the gold flow shock on the real part of the economy. This leads me to wonder whether the historical measures of the two money supply shocks together with the historical measure of the gold flow shock can improve the prediction power of the gold standard model, at least with respect to the nominal part of the French economy. However, it is worth to study effects of the historical measures of the money supply shocks on the artificial gold standard model before answering to this question.

5.2.2.2 The Effects of the money supply shocks

Assuming that the gold standard model offers an acceptable representation of how the French economy has worked during the inter-war gold standard period, I use that model and the constructed data on the nominal shocks in order to evaluate separately the role of the gold backing shock and the role of the inverse of money multiplier shock in the observed fluctuations of key macroeconomic variables over the 1929-1936 period.

![Graphs showing simulated vs historical data](image-url)

Figure 28: Simulated data vs French historical data: Money supply shock effects, 1929 = 0 (a).

Figure 28 displays the effects of the historical gold backing shock and the effects of the historical inverse of money multiplier shock on the nominal part of the gold standard artificial economy. As
shown in Figure 28, the simulated variables — the effects of the gold backing shock are represented in circle lines while the effects of the inverse of money multiplier shock are represented in starred lines — are also confronted to their respective historical measure — represented in solid line. The gold standard model predicts that the increase of the historical gold backing shock that takes place between 1929 and 1934 leads the nominal stock of money to drop deeply below its steady state level over that period. The simulated nominal money is 23.7 percent below its steady state in 1934. From 1934 onwards, as the gold backing shock is decreasing, the predicted nominal money increases back towards its steady state level. However, in 1936 the simulated nominal money is still below its steady state level. As well, the gold standard model predicts that the increase of the inverse of money multiplier shock that takes place between 1929 and 1936 causes nominal money to decrease over all that period. Nonetheless, the effects of the gold backing shock on nominal money are larger than the effects of the inverse of money multiplier shock on nominal money. This is not surprising since the historical measure of the gold backing shock is more volatile than the inverse of money multiplier shock. Thus, the two money supply shocks cannot explain the movements of the nominal stock of money that are observed in the historical data. However, they might explain why the historical nominal money does not increase as much as the historical monetary gold stock does in the beginning of the 1930s.

The gold standard model, through the historical gold backing shock, predicts that the monetary gold stock increases between 1929 and 1934, then declines until 1936. Thus, the simulated monetary gold stock follows a pattern similar to the one followed the actual monetary gold stock over the 1929-1936 period. However, the movements of the historical monetary gold stock are significantly larger than the movements of the simulated monetary gold stock. The increase of the historical inverse of money multiplier shock leads the gold standard model to generate an increasing monetary gold stock over the 1929-1936 period. The increase of the simulated monetary gold stock, driven by the inverse of money multiplier shock, is however weak. The simulated monetary gold stock increases above its steady state level because the household chooses to reduce its non-monetary gold stock by selling gold to the central bank following the increase of either the gold backing shock or the inverse of money multiplier shock as shown in Figure 29.

Figure 29: Simulated monetary gold vs simulated non-monetary gold: Money supply shock effects, 1929 = 0.

The gold standard model predicts that the observed fluctuations of the gold backing shock cause the real price of gold to rise between 1929 and 1934, then to decline until 1936. Nonetheless, the simulated real price of gold, driven by the gold backing shock, is still above its steady state level in 1936. In the gold standard artificial economy, the historical inverse of money multiplier shock
leads the real price of gold to increase over the 1929-1936 period. The effects of the historical gold backing shock on the real price of gold are significantly larger than the effects of the inverse of money multiplier shock on the real price of gold. Thus, the money supply shocks cannot explain the decline of the historical real price of gold that takes place between 1929 and 1930. However, they — particularly the gold backing shock — might account for a non-negligible fraction of the increase of the historical real price of gold that starts after 1930.

As shown in Figure 28, in the gold standard artificial economy, the historical gold backing shock leads the gross nominal interest rate to rise between 1929 and 1934 then to decline. The decrease of the actual gold backing shock is not sufficiently to push the gross nominal interest rate to its steady state level in 1936. In the model, the gross nominal interest rate is predicted to be still above its steady state level by the time France leaves the gold standard. Hence, the gold backing shock driven gross nominal interest rate and the historical gross nominal interest rate move in opposite directions over the 1929-1936 period. The gold standard model, through the historical inverse of money multiplier shock, predicts that the gross nominal interest rate mildly increases between 1929 and 1936. Thus, the gold standard model through the two money supply shocks cannot explain the observed movement in the gross nominal interest rate. Indeed, as said above, the fluctuations of the historical gross nominal interest rate are most probably driven by international factors which are are captured by the model.

Figure 28 reports that in the gold standard artificial economy, the historical gold backing shock causes real money to drop between 1929 and 1934 by 13.33 percent. Then, the gold backing shock driven real money increases weakly until 1936. At this date the simulated real money is still below its steady state level by 5 percent. As well, the gold standard model, through the historical inverse of money multiplier shock, generates a decreasing real money over the 1929-1936 period. However, the decline of the inverse money multiplier shock driven real money is milder than the gold backing shock driven real money. Hence, the two money supply shock cannot account for the increase of the actual real money over the 1929-1936 period.

Figure 30: Simulated data vs French historical data: Money supply shock effects, 1929 = 0 (b).

Figures 30 and 31 display the effects of the historical gold backing shock and the effects of the historical inverse of money multiplier shock on the real part of the gold standard artificial economy. As one can note from Figures 26 and 27, the simulated variables — represented in the left windows — are also confronted to their respective historical measure — represented in the right windows. As for the gold flow shock, the channels through which the money supply shocks are transmitted to the real part of the gold standard artificial economy do not have a strong amplification mechanism. Indeed, the effects of the historical gold backing shock as well as those of the historical inverse of
money multiplier shock on output, consumption, investment, labor, real wage and gross real interest rate are very small in the gold standard model. Precisely, the movements of the simulated output, consumption, investment, labor, real wage and gross real interest rate are quantitatively insignificant comparatively to what it is observed in the historical data. Nevertheless, it is worth to discuss the qualitative effects of the historical gold backing shock and those of the historical inverse of money multiplier shock on those real variables. The gold standard model predicts that the historical gold backing shock leads output and labor to decline from 1929 until 1934. Then, the gold backing shock driven output and labor weakly increase back towards their respective steady state level. By 1936, those simulated variables are not returned into their respective steady state level. In the gold standard artificial economy, the historical gold backing shock causes consumption to collapse from 1929 to 1935. Then, the gold backing shock driven consumption initiates a mild increase. The gold standard model, through the historical gold backing shock, predicts that investment declines between 1929 and 1932, then increases back towards its steady state level. The gold backing shock driven investment move eventually above its steady state level. In the gold standard artificial economy, the real wage is driven upwards by the historical gold backing shock over the 1929-1932 period. Afterward, the gold backing shock driven real wage declines slowly. This simulated variable is still above its steady state level by 1936. The model predicts that the historical gold backing shock leads the gross real interest rate to drop between 1929 and 1932. The gold backing shock driven gross real interest rate remains at a low level during the next two years, then it increases back towards its steady state level. However, in 1936 the simulated gross real wage is still lower than its steady state level. The historical inverse of money multiplier shock also causes output, consumption, investment, labor and gross real interest rate to decrease over the 1929-1936 period in the gold standard artificial economy. In addition, the gold standard model, through the historical inverse of money multiplier shock, predicts that the real wage fluctuates above its steady state level between 1929 and 1936. However, the effects of the historical inverse of money multiplier shock on output, consumption, investment, labor, real wage and gross real interest rate are significantly smaller than the effects of the historical gold backing shock on those real variables.

Thus, the predictions of the gold standard model, through the historical gold backing shock, in terms of output, consumption, investment, labor, real wage and gross real interest rate, have to be constrained with what the historical data tell also from a qualitative point of view. The historical gold backing shock causes output, consumption and labor to decline in the artificial gold standard economy during the first half of the 1930s, as it is observed in the historical data. However, the (weak) recovery those simulated variables initiate in 1934 is not observable in the historical data. Indeed, the actual
output, consumption continue to decrease after 1934. The gold backing shock driven investment starts to decline one year before than the actual investment. Besides, the historical investment is not returned to its 1929 level by 1936 as the simulated investment is. The historical investment collapses between 1930 and 1936. The historical gold backing shock leads the real wage to fluctuate above its steady state level over all the 1929-1936 period in the artificial gold standard model as it is observed in the historical data. Nonetheless, the actual real wage increases during all the 1929-1936 period which is not the case of the simulated real wage as shown above. Moreover, the gold backing driven gross real interest rate and the actual gross real interest rate move in opposite directions between 1929 and 1936.

From a qualitative point of view, the predictions of the gold standard model, through the historical inverse of money multiplier shock, in terms of output, consumption, investment, labor and real wage, are globally consistent with what the historical data tell. However, the inverse of money multiplier gross real interest rate and the actual gross real interest rate follows different patterns over the 1929-1936 period.

5.2.2.3 The nominal shocks and the French Great Depression

The analysis of the effects of each of the historical nominal shocks — gold flow shock, gold backing shock and inverse of money multiplier shock — on the key variables of the gold standard model, suggests that taken together those nominal shocks might explain, within the framework of the gold standard model, the observed fluctuations in the nominal part of the French economy over the 1929-1936 period, at least partially. Precisely, the historical gold flow, gold backing and inverse of money multiplier shocks are likely to cause the observed fluctuations in monetary gold, real price of gold, nominal money and real money over the period 1929-1936 in the gold standard artificial economy. However, it seems that the gold standard model, through the historical nominal shocks, cannot generate a series for the gross nominal interest which would resemble to actual series for the gross nominal interest rate. Besides, the study of the effects of each of the historical nominal shock on the key variables of the gold standard model suggests that, within the framework of the gold standard model, none of the historical nominal shocks virtually can affect the real part of the French economy over the 1929-1936 period. Below, I simulate the gold standard model by feeding all the nominal shocks into it in order to assess whether those intuitions are verified or not.

![Graphs showing nominal shocks and monetary variables](image)

Figure 32: Simulated data vs French historical data: Nominal shock effects, 1929 = 0 (a).

First, I evaluate whether the gold standard model, through all the historical nominal shocks, can replicate the observed fluctuations in monetary gold, nominal money, real money, real price of gold...
and gross nominal interest rate over the 1929-1936 period. The results of the simulation are reported in Figure 32, where the simulated series are represented in starred line and the historical series are represented in solid line. The gold standard model predicts that the historical nominal shocks lead the monetary gold stock to rise between 1929 and 1932. The gold standard model, through the historical nominal shock, can explain 77.6 percent of the actual increase of the monetary gold stock over the 1929-1932 period. As said above, the French central bank has also raised its gold reserves between 1929 and 1932 by converting its foreign exchange. This might explain why the simulated monetary gold stock does not increase as much as the historical monetary gold stock over that period since the model does not include foreign assets nor include the possibility to turn them into gold. From 1932 onwards, the gap between the historical monetary gold and the simulated monetary gold increases. Indeed, the nominal shocks driven monetary gold stock drops deeply than the actual monetary gold stock as the former moves below its steady state level eventually. By 1936, the simulated monetary gold stock is 48.1 percent below its steady state level while the historical monetary gold stock is 35.4 above its 1929 level.

In the gold standard artificial economy, the historical nominal money shocks cause nominal money to fluctuate closely to its steady state level between 1929 and 1932. Then, the nominal shocks driven nominal money collapses strongly below its steady state level. The historical nominal money increases between 1929 and 1931. Afterward, the actual nominal money declines. It slightly drops below its 1929 level by 1935. Thus, the behavior of the simulated nominal money constrains with the one of the actual nominal money over the 1929-1936 period. This contrast mirrors the one of the simulated monetary gold stock with the historical monetary gold stock. Indeed, the observed gap between monetary gold and nominal money is replicated in the gold standard artificial economy since the two historical money supply shocks are fed into the model.

As shown in Figure 32, the gold standard model, through the historical nominal shocks, generates a highly volatile real price of gold. In particular, the pattern followed by the nominal shocks driven real price of gold is very similar — qualitatively as much as quantitatively — to the one followed by the gold flow shock driven real price of gold over the 1929-1936 period. Hence, the effects of the historical gold flow shocks on the real price of gold highly dominate those of the historical money supply shocks on the real price of gold. Even though globally the nominal shocks driven real price of gold behaves qualitatively as the actual real price of gold does over the 1929-1936 period, the former deviates from its 1929 level substantially more than the latter does.

As expected, the gold standard model, through the historical nominal shocks, fails to replicate the observed fluctuations of the gross nominal interest rate between 1929 and 1936. In particular, the simulated gross nominal interest rate and the actual gross nominal interest rate move in opposite directions during that period. Besides, the simulated gross nominal interest rate is more volatile than the historical gross nominal interest rate. Figure 32 shows also the effects of the historical nominal shocks on real money in the 1930s. Hence, the gold standard model, through the historical nominal shocks, fails to predict the observed movements in real money. While the historical real money increases between 1929 and 1932, the simulated real money drops below its steady state level. After 1932, the actual real money fluctuates closely to its 1932 level. The simulated real money moves above its steady state level only after 1933. However, the simulated real money becomes higher than the actual real money by 1936.

Second, I evaluate whether the gold standard model, through all the historical nominal shocks, can replicate the observed fluctuations in output, consumption, investment, labor, real wage and gross real interest rate over the 1929-1936 period. The results of the simulation are reported in Figures 33 and 34, where the simulated series are represented in the left windows and the historical series are represented in the right windows. As expected, the historical nominal shocks have quantitatively almost no effects on the real part of the economy. Nonetheless, it is worth to comment briefly the qualitative effects of the nominal shocks on output, consumption, investment, labor, real wage and gross real interest rate. The gold standard model predicts the historical nominal shocks cause output, consumption, labor and gross real interest rate to decline slightly between 1929 and 1932. Then, they increase back towards their respective steady state level. They move above their respective steady state level by 1935. The nominal shocks driven investment also declines in the beginning of the 1930s. However, the predicted investment decrease lasts one year less than the predicted output decrease does. The simulated investment increases from 1931 to 1936. It moves above its steady state level by 1933. The simulated real wage fluctuates above its steady state level between 1929 and 1934. After what, it drops below its steady state level. Thus, from 1932 onwards, the gold standard model, through the historical nominal shocks, predicts counterfactually a recovery period for the French
Figure 33: Simulated data vs French historical data: Nominal shocks effects, 1929 = 0 (b).

Figure 34: Simulated data vs French historical data: Nominal shocks effects, 1929 = 0 (c).
economy. Indeed, the historical data tell that the French economy undergoes a depression until 1936: The actual output, consumption, investment and labor decline during the 1930s. Besides, in the gold standard artificial economy, the historical nominal shocks do not cause the real wage to increase above its steady state level during all the 1929-1936 period as it is observed in the historical data. The nominal shock driven gross nominal interest rate and the historical gross nominal interest rate follow different patterns between 1929 and 1936.

Thus, the gold standard model, through the nominal shocks, fails the replicate the French Great Depression. This is mainly due to the fact that the channels through which the nominal shocks are transmitted to the real economy have virtually no amplification mechanism. Furthermore, in the gold standard artificial economy, when a large quantity of gold is moving away, the household behaves so that the non-monetary gold stock is less reduced than the monetary gold stock. Consequently, the gold standard model predicts that after 1932 the historical nominal shocks cause the monetary gold stock to collapse more deeply than what it is observed in the historical data. This leads to counterfactual predictions of nominal money after 1932. The gold standard model, through the historical nominal shocks, fails to track the observed movements in the gross nominal interest rate because the fluctuations of the actual gross nominal interest rate are more likely due to international factors than to domestic factors.

5.2.2.4 The effects of the productivity shock

Assuming that the gold standard model offers an acceptable representation of how the French economy has worked during the inter-war gold standard period, I use that model and the constructed data on the productivity shocks in order to assess whether the productivity shock is a better alternative to the nominal shocks for explaining the French Great Depression.

Figure 35: Simulated data vs French historical data: Productivity shock effects, 1929 = 0 (a).

Figure 35 displays the effects of the historical productivity shock on the real part of the gold standard artificial economy. As shown in Figure 35, the simulated variables — represented in starred line — are also confronted to their respective historical measure — represented in solid line. The gold standard model, through the historical productivity shock, performs well in tracking the observed fluctuations in output over the 1929-1936 period. Still, the predicted collapse of output is slightly less deeper than the actual decrease of output. Indeed, by 1936, the simulated output is 13.7 percent below its steady state level while the historical output is 16.5 percent below its 1929 level. The gold standard model predicts that the historical productivity shock leads consumption to drop below
its steady state in 1932. This contrasts with what the historical data tell. Indeed, the historical consumption falls below its 1929 level in 1930. Globally, during the 1930s the simulated consumption declines almost as much as the actual consumption does. In 1936 the predicted consumption is 5.2 percent below its steady state level while the historical consumption is 6.4 percent below its 1929 level. However, the decrease of the historical consumption is erratic whereas the decline of the simulated consumption is rather regular. Consistently to the what it is observed in the data, the historical productivity shock causes investment to increase between 1929 and 1930 then to collapse dramatically until 1936 in the gold standard artificial economy. Note that between 1929 and 1931, the predicted investment and the actual investment move very closely to each other. After 1931, the decrease of the historical investment becomes quicker than the decline of the simulated investment. Thus, in 1936, the predicted investment is 41.8 percent below its steady state level whereas the actual investment is 60.8 percent below its 1929 level. In the gold standard artificial economy, the historical productivity shock leads labor to fluctuate above its steady state level between 1929 and 1930, then to decline below its steady state level until 1936. However the decrease of the predicted labor happens one year later than in the historical data. Besides, the decline of the simulated labor is not as severe as the the fall of the historical labor is. Indeed, in 1936 the simulated labor is 4.2 percent below its steady state level whereas the actual labor is 16.5 percent below its 1929 level. The gold standard model, through the historical productivity shock, fails to replicate the observed movement in the real wage and the gross real interest rate between 1929 and 1936. The historical productivity shock cause counterfactually the real wage to decrease between 1930 and 1936. Indeed, the historical wage rises over the 1929-1936 period. In turn, the historical productivity shock has almost no effects on the gross real interest rate in the gold standard artificial economy between 1929 and 1936. This contrasts with what the data tell: The actual gross real interest rate increases between 1929 and 1935 then declines towards its 1929 level.

Figure 36: Simulated data vs French historical data: Productivity shock effects, 1929 = 0 (b).

Figures 36 and 37 show the effects of the historical productivity shock on the nominal part of the gold standard artificial economy. As one can note from Figures 36 and 37, the simulated variables — represented in the left windows — are also confronted to their respective historical measure — represented in the right windows. The results of the simulation show that the channels through which the productivity shocks are transmitted to the nominal part of the gold standard artificial economy do not include a strong amplification mechanism. Indeed, the effects of the historical productivity shock on monetary gold, nominal money and gross nominal interest rate are small in the gold standard artificial economy. However, the effects of the historical productivity shock on the real price of gold and real money are quantitatively significant for the historical event under study.
Figure 37: Simulated data vs French historical data: Productivity shock effects, 1929 = 0 (c).

As shown in Figure 36, in the gold standard artificial economy, the historical productivity shock causes the real price of gold to move above its steady state in 1930 then to decline below its steady state level until 1936. The simulated real price of gold is 8.2 percent below its steady state level by 1936. Hence, the simulated real price of gold and the actual real price of gold move in opposite directions during the 1929-1936 period. The gold standard model, through the historical productivity shock, predicts that the real money fluctuates above its steady state level between 1929 and 1931, then drops below its steady state level until 1936. The behavior of the productivity shock driven real money contrasts with the the behavior of the historical real money. Indeed, the latter increases above its 1929 level between 1929 and 1936.

Even though the historical productivity shock affects only a little the monetary gold stock, the nominal money and the gross nominal interest rate over the 1929-1936 period in the artificial gold standard economy, it is worth to examine the qualitative effects of that shock on those variables. The gold standard model, through the historical productivity shock, predicts that monetary gold, nominal money and gross nominal interest rate drop below their respective steady state level between 1929 and 1930, then increase above their respective steady state level until 1936. Thus, the historical productivity shock does not lead nominal money, monetary gold and gross nominal interest rate to fluctuate consistently to what is observed in the historical data neither quantitatively nor qualitatively.

Thus, within the gold standard artificial economy framework, the historical productivity shock appears as an acceptable alternative to the historical nominal shocks for the understanding of what happens in the real part of the French economy in the 1930s. However, the historical productivity shock seems not appropriate to explain the fluctuations that takes place in the nominal part of the French economy during that period.

5.2.2.5 Evaluation of the gold standard model

The evaluation of the effects of each of the nominal shocks and of the productivity shock on the key variables of the gold standard model has been performed under the assumption that the model provides a reasonable representation of how the French economy has worked during the interwar gold standard period. Here, I assess whether this assumption is relevant or not.

The analysis of the effects of each of the nominal shocks and of the productivity shock on the key macroeconomic variables suggests that the nominal part of the artificial gold standard economy is mainly affected by the historical nominal shocks while the real part of that artificial economy is
mainly affected by the historical productivity shock. Thus, considering the results of the simulations performed above, it seems that the gold standard model, through the historical gold flow, gold backing, inverse of money multiplier and productivity shocks, taken together, is not a good representation of how the French economy has worked during the Great Depression. This suggestion is confirmed by Figures 38 and 39 where I report the combined effects of all the nominal shocks and the productivity shocks on the key variables of the gold standard model.

![Figure 38: Simulated data vs French historical data: Effects of all the shocks, 1929 = 0 (a).](image)

Even though the gold standard model does not offer a good description of how the French economy has worked in the interwar gold standard period, the analysis of the effects of each of the historical shocks on the artificial gold standard economy provides some indications on how to improve the model’s predictions. Below, I discuss briefly two possible avenues to improve the gold standard model’s ability to replicate the French Great Depression.

The two channels of transmission of the nominal shocks to the real part of an economy — the transaction cost wedge and the optimal condition on non-monetary gold — are not sufficient to lead those shocks to account for the French Great Depression. Most of the proponents of the gold standard explanation of the worldwide Great Depression — Eichengreen and Sachs (1985) Bernanke (1995) and, Bernanke and Carey (1996) among others — claim that the non-neutrality of the fall in the global money supply is more likely due to nominal wage stickiness. The role of the nominal wage stickiness in the transmission of the money supply collapses to real economies can be stated as follows: Following a monetary contraction, prices go down. Production costs should decrease in order to maintain the level of production. Thus, when nominal wages do not respond quickly to the decrease in prices (due to labor market imperfections), firms lay off and produce less. This results in an increase of unemployment and a drop in output. As shown in the French historical data, nominal money and the price level (inverse of the real price of gold) decline from 1931. In the meanwhile, the French real wage rises. This means that if the nominal wage decreases in the 1930s, its drop is less severe than the price level’s one. In other words, the adjustment of the nominal wage to the money supply decline happens to be slow in France during the 1930s. Hence, the sticky nominal wage hypothesis seems to be an interesting avenue for the understanding of the transmission of the nominal shocks to the real part of the French economy.

In the gold standard artificial economy, neither of the historical nominal shocks is able to lead the gross nominal interest rate follows a pattern similar to the one followed by the historical gross nominal interest rate. As said above, the fluctuations of the gross nominal interest rate are more likely driven by non-domestic factors. Thus, the gold standard model should be extended to a small open economy in order to better capture the observed fluctuations of the gross nominal
interest rate. Indeed, France can be considered as a small open economy during that time. This could be done by following the standard literature on small open real business cycle models such that Schmitt-Grohé and Uribe (2003). An other way to deal with that problem might be to add a mechanism in the model so that the gold flow shock induces a liquidity effect. Indeed, the historical data show that the nominal money and the gross nominal interest rate fluctuate in opposite directions between 1929 and 1936. In particular, the nominal increases in the beginning of the decade while the gross nominal interest rate drops. Agreed, the two money supply shocks generate a liquidity effect. However, only the historical gold flow shock pushes the nominal money upwards in the beginning of the 1930s. Introducing price stickiness in the good market might lead the gold flow shock to generate a liquidity effect. Nonetheless, the price stickiness hypothesis would bring a new difficulty in the construction of the model as in the gold standard literature the price level is determined in the gold market which is assumed competitive. In order to ensure that the determination of the price level in the good market coincides with the determination of the price level in the gold market, one should differentiate the definition of the price level from the one of the real (relative) price of gold in the model as in Goodfriend (1988). In the current state of the gold standard model, this differentiation is not explicitly done since the only price variable that is considered in the model is the real price of gold.

6 Conclusion

The analysis of the Great Depression through the lens of the gold standard theory has been mainly conducted from an international comparative perspective. As far as I know there is no structural model-based studies of the link between the malfunctioning of the gold standard and the Great Depression. A such analysis would be helpful to assess the role of gold standard in the worldwide depression. In this work, I have intended to take step into this direction. In particular, I have developed a gold standard DSGE model which aims to emphasize the working of the gold standard and the link of the latter with the real part of an economy. The fluctuations of the aggregate variables of the artificial economy are driven by one real and three nominal shocks. The real shock is assumed to be a productivity shock in the final good sector. The nominal shocks are all related to the working of the gold standard: a gold flow shock, a gold backing shock, and a money multiplier shock. The two lastest nominal shocks alter the ratio of the currency value of the monetary gold stock to the nominal money stock. The gold standard model is then put to use to study, through simulation exercises,
the French Great Depression over the 1929-1936 period. The results of the simulation exercises can be summarized as follows. The historical measures of the nominal shocks affect significantly the nominal part of the artificial gold standard economy and very little the real part of that economy. The historical productivity shock affects significantly the real part of the artificial economy and weakly the nominal part of that economy. Nonetheless, with all the historical shocks taken together, the gold standard model is not able to replicate the French Great Depression. Thus, the gold standard model, in its state, suffers from two deficiencies. First, the channels of transmission of the nominal shocks to the real part of the economy are strong enough to allow the model to replicate an historical event such as the Great Depression. Second, the functioning of the gold standard in France during the 1930s seems not to be fully captured by the model. Even though this work lets the question of the structural model-based evaluation of the gold standard-based explanation of the Great Depression still fully open, it can be seen as a starting point for future researches.

A  Solving the Gold Standard Model

A.1 Calibration

Table 7 displays the values of the model parameters.

A.2 Log-linearization

In order to log-linearize the non-linear stationary dynamic equations, we express all the variables in logarithmic deviation (log-deviation hereafter) from their respective steady-state level. We denote \( \hat{x}_t \) the log-deviation of variable \( x_t \). Formally,

\[ \hat{x}_t = \ln \left( \frac{x_t}{x} \right) \]

Thus, according to this definition,

\[ x_t = x \exp (\hat{x}_t) \]

If \( \hat{x}_t \) is close enough to zero, then

\[ x_t = x \exp (\hat{x}_t) \approx x (1 + \hat{x}_t) \]

In the system of the non-linear stationary dynamic equations, there is a variable, namely the gold flow shock, \( g_t \), which can have negative realizations. Therefore, this exogenous variables cannot be expressed in log-deviation from its steady state level. Consequently, \( g_t \) is expressed rather in deviation from its steady state level, with the following notation \( \hat{g}_t = g_t - g^* \).

The log-linearized expression of equation (45) is computed as follows:

First, I restate equation (45)

\[
\frac{\phi_2}{\phi_1} \left[ (C_t - bC_{t-1})^\phi_1 G_{c,t}^{\phi_2} \right]^{1 - \phi_3} \zeta_t^{-1} = \frac{w_t}{\kappa_t} \\
(45')
\]

where

\[
\zeta_t = \frac{\left[ (1 - h_t)^{\phi_2} G_{c,t}^{\phi_1} \right]^{1 - \phi_4}}{(C_t - bC_{t-1})^{1 - \phi_4}} - b\beta E_t \left[ \frac{\left[ (1 - h_{t+1})^{\phi_2} G_{c,t+1}^{\phi_1} \right]^{1 - \phi_4}}{(C_{t+1} - bC_t)^{1 - \phi_4}} \right] \\
(79)
\]

Second, I log-linearize (45')

\[
\frac{\phi_2}{\phi_1} \left[ (C - bC_t)^\phi_1 G_{c,t}^{\phi_2} \right]^{1 - \phi_4} \zeta_t^{-1} \left( 1 + \frac{(1 - \phi_4) \phi_1}{1 - b} \hat{C}_t - \frac{(1 - \phi_4) \phi_1 b}{1 - b} \hat{C}_{t-1} + \phi_3 (1 - \phi_4) \hat{G}_{c,t} \right) + \frac{(1 - \phi_2 \phi_4)}{1 - h} \hat{h}_t - \hat{\zeta}_t = \frac{w}{\kappa} (1 + \hat{w}_t - \hat{\kappa}_t)
\]

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Table 7: Calibration

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<th>Parameter</th>
<th>Model with $b = 0$</th>
<th>Model with $b = 0.65$</th>
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<td>Discount factor</td>
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\begin{align*}
&\to \frac{(1 - \phi_4) \phi_1}{1 - b} \hat{C}_t - \frac{(1 - \phi_4) \phi_1 b}{1 - b} \hat{C}_{t-1} + \phi_3 (1 - \phi_4) \hat{G}_{c,t} + \frac{(1 - \phi_2 (1 - \phi_4))}{1 - h} \hat{h}_t - \hat{\zeta}_t = \hat{w}_t - \hat{\kappa}_t \quad (45^*), \end{align*}
\]
and (79)
\[
\begin{align*}
\zeta &+ \hat{\zeta}_t = \frac{\left[ (1 - h) \phi_2 G_{c,t}^{\phi_2} \right]^{1 - \phi_4}}{(C - bC)^{1 - \phi_4}} \left[ 1 - (1 - \phi_4) \phi_2 \frac{h}{1 - h} \hat{h}_t + (1 - \phi_4) \phi_3 \hat{G}_{c,t} - \frac{1 - (1 - \phi_4) \phi_1}{1 - b} \hat{C}_t + \
&+ \frac{1 - (1 - \phi_4) \phi_1 b}{1 - b} \hat{C}_{t-1} \right] - b \beta \left[ \frac{(1 - h)^{\phi_2} G_{c,t}^{\phi_2}}{(C - bC)^{1 - \phi_1 (1 - \phi_4)}} \right] E_t \left[ 1 - (1 - \phi_4) \phi_2 \frac{h}{1 - h} \hat{h}_{t+1} + (1 - \phi_4) \phi_3 \hat{G}_{c,t+1} - \
&- \frac{1 - (1 - \phi_4) \phi_1}{1 - b} \hat{C}_{t+1} + \frac{1 - (1 - \phi_4) \phi_1 b}{1 - b} \hat{C}_t \right]
\end{align*}
\]
\[
\begin{align*}
&\zeta = -\frac{(1 - \phi_4) \phi_2}{1 - b \beta} \frac{h}{1 - h} \hat{h}_t + \frac{1 - (1 - \phi_4) \phi_3}{1 - b \beta} \hat{G}_{c,t} - \left[ \frac{1 - (1 - \phi_4) \phi_1}{1 - b \beta (1 - b)} + \frac{1 - (1 - \phi_4) \phi_1 b^2 \beta}{(1 - b \beta) (1 - b)} \right] \hat{C}_t + \
&+ \frac{1 - (1 - \phi_4) \phi_1 b}{(1 - b \beta) (1 - b)} \hat{C}_{t-1} + \frac{(1 - \phi_4) \phi_2}{1 - b \beta} \frac{h}{1 - h} b \beta E_t \left( \hat{h}_{t+1} \right) - \frac{(1 - \phi_4) \phi_3}{1 - b \beta} b \beta E_t \left( \hat{G}_{c,t+1} \right) + \
&+ \frac{1 - (1 - \phi_4) \phi_1 b}{1 - b \beta (1 - b)} b \beta E_t \left( \hat{C}_{t+1} \right)
\end{align*}
\]
\[
(80)
\]
Then, combining equations (45*) and (80), I get the log-linearized expression of (45)
\[
\begin{align*}
b \beta (1 - \phi_4) \phi_2 \frac{h}{1 - h} E_t \left( \hat{h}_{t+1} \right) - \frac{b \beta (1 - \phi_4) \phi_3}{1 - b \beta} E_t \left( \hat{G}_{c,t+1} \right) + \frac{b \beta}{(1 - b \beta) (1 - b)} \left[ \frac{1 - (1 - \phi_4) \phi_1}{1 - b \beta} b \beta E_t \left( \hat{C}_{t+1} \right) + \
&+ \frac{(1 - \phi_4) \phi_1}{1 - b \beta} \frac{h}{1 - h} \hat{h}_t - \hat{\zeta}_t - \frac{1 - (1 - \phi_4) \phi_1 b}{1 - b \beta} \frac{1}{1 - b} \hat{C}_{t-1} \right] \zeta_t = \frac{1 - (1 - \phi_4) \phi_3}{1 - b \beta} \frac{(C_t - bC_{t-1})^{\phi_3 (1 - \phi_4)}^{1 - \phi_4}}{G_{c,t}^{1 - \phi_4 (1 - \phi_4)}} (1 - \delta_g) \beta E_t \left( \frac{p_{g,t+1}}{k_{t+1}} \zeta_{t+1} \right) \quad (44')
\end{align*}
\]
Second, I log-linearize equation (44')
\[
\begin{align*}
&\frac{p_g}{k} \zeta \left( 1 + \hat{p}_{g,t} - \hat{\kappa}_t + \hat{\zeta}_t \right) - \frac{\phi_3}{\phi_1} \frac{(C - bC)^{\phi_1 (1 - h)^{\phi_2}}^{1 - \phi_4}}{G_{c,t}^{1 - \phi_4 (1 - \phi_4)}} \left[ 1 + (1 - \phi_4) \phi_1 \left( \frac{1}{1 - b} \hat{C}_t - \frac{b}{1 - b} \hat{C}_{t-1} \right) \right] - \
&- (1 - \phi_4) \phi_2 \frac{h}{1 - h} \hat{h}_t + [1 - \phi_3 (1 - \phi_4)] \hat{G}_{c,t} + (1 - \delta_g) \beta \frac{p_{g}}{k} \zeta E_t \left( \hat{p}_{g,t+1} - \hat{\kappa}_{t+1} + \hat{\zeta}_{t+1} \right) = 0
\end{align*}
\]
\[
\begin{align*}
&\approx \hat{p}_{g,t} - \hat{\kappa}_t + \hat{\zeta}_t - \frac{\phi_3}{\phi_1} \frac{(C - bC)^{\phi_1 (1 - h)^{\phi_2}}^{1 - \phi_4}}{G_{c,t}^{1 - \phi_4 (1 - \phi_4)}} \left( \frac{(1 - \phi_4) \phi_1 \hat{C}_t - [(1 - \phi_4) \phi_1] b \hat{C}_{t-1}}{1 - b} \right) - \
&- (1 - \phi_4) \phi_2 \frac{h}{1 - h} \hat{h}_t + [1 - \phi_3 (1 - \phi_4)] \hat{G}_{c,t} = (1 - \delta_g) \beta E_t \left( \hat{p}_{g,t+1} - \hat{\kappa}_{t+1} + \hat{\zeta}_{t+1} \right)
\end{align*}
\]
\[
\begin{align*}
\left\langle \hat{p}_{g,t} - \hat{\kappa}_t + \hat{\zeta}_t - \frac{[1 - (1 - \delta_g) \beta] (1 - \phi_4) \phi_1}{1 - b} \hat{C}_t + \frac{[1 - (1 - \delta_g) \beta] [(1 - \phi_4) \phi_1] b}{1 - b} C_{t-1} + \\
+ [1 - (1 - \delta_g) \beta] (1 - \phi_4) \phi_2 \frac{h}{1 - h} \hat{h}_t + [1 - (1 - \delta_g) \beta] [1 - \phi_3 (1 - \phi_4)] \hat{G}_{c,t} =
\end{align*}
\]

\[\text{(44"')}\]

noting that
\[
\zeta = \left[\frac{(1 - h)^{\phi_2} G_c^{\phi_3}}{(C - bC)^{1 - \phi_1 (1 - \phi_4)} (1 - b\beta)}\right]^{1 - \phi_4}
\]

\[
\kappa \phi_3 \left[\frac{(C - bC)^{\phi_1 (1 - h)^{\phi_2}}}{G_c^{1 - \phi_3 (1 - \phi_4)}}\right]^{1 - \phi_4} = 1 - (1 - \delta_g) \beta
\]

Then, combining equations (44"') and (80), I get the log-linearized expression of (44)

\[
\begin{align*}
(1 - \delta_g) \beta E_t (\hat{p}_{g,t+1} - (1 - \delta_g) \beta E_t (\hat{\kappa}_{t+1}) - \frac{\phi_2 h}{1 - h} \frac{(1 - \phi_4) (1 - \delta_g) \beta}{1 - b\beta} + \frac{\phi_2 h}{1 - h} \frac{(1 - \phi_4) b\beta}{1 - b\beta} E_t (\hat{h}_{t+1}) + \\
+ \left[\frac{(1 - \phi_4) (1 - \delta_g) \beta \phi_3}{1 - b\beta} + \frac{(1 - \phi_4) b\beta \phi_3}{1 - b\beta}\right] E_t \left(\hat{G}_{c,t+1}\right) - \left[\frac{(1 - \delta_g) \beta [1 - \phi_1 (1 - \phi_4)]}{(1 - b\beta) (1 - h)} + \\
+ \frac{(1 - \delta_g) [1 - \phi_1 (1 - \phi_4)] b^2 \beta^2}{(1 - b\beta) (1 - b)} + \frac{[1 - \phi_1 (1 - \phi_4)] b\beta}{(1 - b\beta) (1 - h)} \right] E_t \left(\hat{C}_{t+1}\right) + \left[\frac{(1 - \delta_g) b^2 \beta (1 - \phi_4) \phi_2}{1 - b\beta} E_t \left(\hat{h}_{t+2}\right) - \\
- \frac{\phi_3 (1 - \phi_4) b^2 \beta (1 - \delta_g) E_t \left(\hat{G}_{c,t+2}\right) + [1 - \phi_1 (1 - \phi_4)] b\beta^2 (1 - \delta_g) E_t \left(\hat{C}_{t+2}\right) = \hat{p}_{g,t} - \hat{\kappa}_t - \\
- \left[\frac{(1 - \phi_4) \phi_1 [1 - (1 - \delta_g) \beta]}{1 - b} + \frac{1 - \phi_1 (1 - \phi_4)}{(1 - b\beta) (1 - b)} + \frac{[1 - \phi_1 (1 - \phi_4)] b^2 \beta}{(1 - b\beta) (1 - h)} + \\
+ \frac{[1 - \phi_1 (1 - \phi_4)] (1 - \delta_g) b\beta}{(1 - b\beta) (1 - h)} \right] \hat{C}_t + \left[\frac{(1 - \phi_4) \phi_2 [1 - (1 - \delta_g) \beta] h}{1 - h} - \frac{(1 - \phi_4) \phi_2 h}{(1 - b\beta) (1 - h)} \right] \hat{h}_t + \\
+ \left[\frac{1 - (1 - \delta_g) \beta [1 - \phi_1 (1 - \phi_4)] + (1 - \phi_4) \phi_3}{(1 - b\beta) (1 - b)} \right] \hat{G}_{c,t} + \left[\frac{\phi_1 (1 - \phi_4) b [1 - (1 - \delta_g) \beta]}{1 - b} + \\
+ \frac{[1 - \phi_4 (1 - \phi_4)] b}{(1 - b\beta) (1 - h)} \right] \hat{C}_{t-1}
\end{align*}
\]
\[
\begin{align*}
\L & (1 - \delta_g) \beta E_t (\hat{p}_{g,t+1}) - (1 - \delta_g) \beta E_t (\hat{\kappa}_{t+1}) - \frac{\phi_2 h}{1 - h} \frac{(1 - \phi_4) \beta [1 - \delta_g + b]}{1 - b\beta} E_t (\hat{h}_{t+1}) + \\
& + \frac{(1 - \phi_4) \beta (1 - \delta_g + b)}{1 - b\beta} E_t \left( \hat{G}_{ct+1} \right) + \frac{1}{1 - b} \left[ \frac{(1 - \phi_4) (1 - \delta_g + b) \beta \phi_1}{1 - b\beta} + \frac{b\beta^2 (1 - \phi_4) (1 - \delta_g) \phi_1 \beta}{1 - b\beta} \right] E_t (\hat{h}_{t+2}) - \\
& - \frac{\beta [b + (1 - b^2 \beta) (1 - \delta_g)]}{1 - b\beta} E_t (\hat{C}_{t+1}) + \frac{1 - \delta_g \beta b^2 (1 - \delta_g) (1 - \phi_4) \phi_2}{1 - b\beta} \frac{h}{1 - h} E_t (\hat{h}_{t+2}) - \\
& - \phi_3 (1 - \phi_4) b\beta^2 (1 - \delta_g) \frac{E_t \left( \hat{G}_{ct+2} \right)}{1 - b\beta} + \frac{1}{(1 - b\beta) (1 - b)} \left[ \frac{1}{(1 - \delta_g) - \phi_1} \right] E_t (\hat{C}_{t+2}) = \\
= \hat{p}_{g,t} - \hat{\kappa}_t - \frac{(1 - \phi_4) \phi_2 h}{1 - h} \left[ \frac{b\beta}{1 - b\beta} + (1 - \delta_g) \beta \right] \hat{h}_t + \\
& + \left[ - \frac{1 + b^2 \beta (1 - \delta_g) \beta}{(1 - b\beta) (1 - b)} + \frac{\beta (1 - \phi_4) (1 - \delta_g + b)}{1 - b\beta} \phi_3 \frac{1 - \delta_g \beta b^2 (1 - \delta_g) (1 - \phi_4) \phi_2}{1 - b\beta} \frac{h}{1 - h} \frac{1 - b}{1 - b} + \frac{1}{(1 - b\beta) (1 - b)} \left[ \frac{1}{(1 - \delta_g) - \phi_1} \right] E_t (\hat{C}_{t+2}) + \\
& + \left[ b \frac{1}{1 - b\beta} - (1 - \phi_4) \frac{b\beta}{1 - b\beta} + (1 - \delta_g) \beta \phi_1 \right] \hat{C}_{t-1}
\end{align*}
\]

The log-linearized expression of equation (46) is computed as follows:

First, I restate equation (46)

\[
\gamma_1 v_t^2 - \gamma_2 = 1 - \beta E_t \left( \frac{\hat{\kappa}_{t+1} \hat{p}_{g,t+1} \hat{\zeta}_{t+1}}{\hat{p}_{g,t} \hat{\zeta}_t} \right)
\]

Second, I loglinearize equation (46)\('

\[
\gamma_1 v_t^2 (1 + 2 \hat{v}_t) - \gamma_2 = 1 - \beta \frac{\kappa P_{g,t} \hat{\zeta}}{\kappa P_{g,t} \hat{\zeta}} E_t \left( 1 + \hat{\kappa}_t - \hat{\kappa}_{t+1} + \hat{p}_{g,t+1} - \hat{p}_{g,t} + \hat{\zeta}_t + \hat{\zeta}_{t+1} \right)
\]

\[
\L 2\gamma_1 v_t^2 \hat{v}_t + \hat{\kappa}_t - \hat{p}_{g,t} - \hat{\zeta}_t = E_t \left( \hat{\kappa}_{t+1} - \hat{p}_{g,t+1} - \hat{\zeta}_{t+1} \right)
\]

Then, combining equations (46") and (80), I get the log-linearized expression of (46)

\[
E_t (\hat{\kappa}_{t+1}) - E_t (\hat{p}_{g,t+1}) + \left[ \frac{(1 - \phi_4) \phi_2}{1 - b\beta} \frac{h}{1 - h} + \frac{b\beta (1 - \phi_4) \phi_2}{1 - b\beta} \frac{h}{1 - h} \right] E_t (\hat{h}_{t+1}) - \\
- \left[ \frac{(1 - \phi_4) \phi_3}{1 - b\beta} + \frac{(1 - \phi_4) \phi_2 b\beta}{1 - b\beta} \right] E_t \left( \hat{G}_{ct+1} \right) + \left[ \frac{1 - \phi_4}{(1 - \phi_4) \phi_3} \phi_1 \frac{h}{1 - h} \right] E_t \left( \hat{h}_{t+2} \right) + \frac{b\beta (1 - \phi_4) \phi_3}{1 - b\beta} E_t \left( \hat{G}_{ct+2} \right) - \\
+ \left[ \frac{(1 - \phi_4) \phi_1}{(1 - \phi_4) \phi_3} \phi_3 \phi_1 b\beta \frac{h}{1 - h} \right] E_t \left( \hat{C}_{t+1} \right) - \frac{b\beta (1 - \phi_4) \phi_2}{1 - b\beta} \frac{h}{1 - h} E_t (\hat{h}_{t+2}) + \frac{b\beta (1 - \phi_4) \phi_3}{1 - b\beta} E_t \left( \hat{G}_{ct+2} \right) - \\
- \left[ \frac{(1 - \phi_4) \phi_1}{(1 - \phi_4) \phi_3} \phi_3 \phi_1 b\beta \frac{h}{1 - h} \right] E_t \left( \hat{C}_{t+2} \right) = \frac{2\gamma_1 v^2}{\beta} \hat{v}_t + \hat{\kappa}_t - \hat{p}_{g,t} + \frac{(1 - \phi_4) \phi_2}{1 - b\beta} \frac{h}{1 - h} \hat{h}_t - \frac{(1 - \phi_4) \phi_3}{1 - b\beta} \hat{C}_{ct+1} + \\
+ \left[ \frac{(1 - \phi_4) \phi_1}{(1 - \phi_4) \phi_3} \phi_3 \phi_1 b\beta \frac{h}{1 - h} \right] E_t \left( \hat{C}_{t+2} \right) - \frac{b\beta (1 - \phi_4) \phi_3}{1 - b\beta} E_t \left( \hat{C}_{ct+2} \right) - \frac{b\beta (1 - \phi_4) \phi_3}{1 - b\beta} E_t \left( \hat{C}_{ct+2} \right) - \\
+ \left[ \frac{(1 - \phi_4) \phi_1}{(1 - \phi_4) \phi_3} \phi_3 \phi_1 b\beta \frac{h}{1 - h} \right] E_t \left( \hat{C}_{t+2} \right) - \frac{b\beta (1 - \phi_4) \phi_3}{1 - b\beta} E_t \left( \hat{C}_{ct+2} \right) - \\
\]
\[
E_t(\hat{\kappa}_{t+1}) - E_t(\hat{\rho}_{g,t+1}) + \frac{1 + b \beta}{1 - b \beta} (1 - \phi_4) \phi_2 \frac{h}{1 - h} E_t \left( \hat{h}_{t+1} \right) + \frac{1 + b \beta (1 - \phi_4) \phi_3}{1 - b \beta} E_t \left( \hat{G}_{c,t+1} \right) + \frac{1}{1 - b} \left[ b \beta (1 - \phi_4) \phi_3 + \frac{h}{1 - h} E_t \left( \hat{h}_{t+2} \right) \right]
\]

(83)

The log-linearized expression of equation (47) is computed as follows:
First, I restate equation (47)
\[
\frac{\zeta_t}{\kappa_t} = \beta E_t \left( \frac{r_{t+1} + 1 - \delta_k}{\kappa_{t+1}} \right)
\]

(47')

Second, I log-linearize equation (47')
\[
\frac{\zeta}{\kappa} \left( 1 + \hat{\zeta}_t - \hat{\kappa}_t \right) = \beta \frac{r + 1 - \delta_k}{\kappa} E_t \left( 1 + \hat{\zeta}_{t+1} - \hat{\kappa}_{t+1} + \frac{r}{r + 1 - \delta_k} \hat{r}_{t+1} \right)
\]

(47'')

Then, combining equations (47'') and (80), I get the log-linearized expression of (47)
\[
\frac{r}{r + 1 - \delta_k} E_t \left( \hat{r}_{t+1} \right) - E_t (\hat{\kappa}_{t+1}) - \left[ \frac{(1 - \phi_4) \phi_2 h}{1 - b \beta} + \frac{b \beta (1 - \phi_4) \phi_2 h}{1 - b \beta} \right] E_t \left( \hat{h}_{t+1} \right) + \frac{1}{1 - b} \left[ (1 - \phi_4) \phi_3 + \frac{(1 - \phi_4) \phi_3 b \beta}{1 - b \beta} \right] E_t \left( \hat{G}_{c,t+1} \right) - \left[ \frac{1 - (1 - \phi_4) \phi_4}{1 - b \beta} \frac{h}{1 - h} \frac{E_t \left( \hat{h}_{t+2} \right)}{1 - b \beta} \right] E_t \left( \hat{G}_{c,t+2} \right) + \frac{1}{1 - b \beta} \left[ (1 - \phi_4) \phi_3 \frac{h}{1 - h} \hat{r}_t + (1 - \phi_4) \phi_3 \hat{G}_{c,t} - \hat{\kappa}_t \right] - \frac{1}{1 - b \beta} \left[ (1 - \phi_4) \phi_4 \frac{h}{1 - h} \hat{r}_t + (1 - \phi_4) \phi_4 (1 - \phi_4) \phi_4 \frac{b \beta}{1 - b \beta} \right] \hat{C}_t + \frac{1}{1 - b \beta} \left[ (1 - \phi_4) \phi_4 \frac{b \beta}{1 - b \beta} \hat{C}_{t-1} \right]
\]

(84)
The loglinearized expression of equation (41) is

\[ m (1 + \hat{m}_t) = \frac{1}{\mu_1 \mu_2} p_g G_m \left( 1 + \hat{p}_{g,t} + \hat{G}_{m,t} - \hat{\mu}_{1,t} - \hat{\mu}_{2,t} \right) \]

\[ \Leftrightarrow \hat{m}_t = \hat{p}_{g,t} + \hat{G}_{m,t} - \hat{\mu}_{1,t} - \hat{\mu}_{2,t} \]  
(85)

The loglinearized expression of equation (42), is

\[ v (1 + \hat{v}_t) = \frac{C}{m} \left( 1 + \hat{C}_t - \hat{m}_t \right) \]

\[ \Leftrightarrow \hat{v}_t = \hat{C}_t - \hat{m}_t \]  
(86)

The loglinearized expression of equation (6) is

\[ K \left( 1 + \hat{K}_t \right) = \left( 1 - \delta_k \right) K \left( 1 + \hat{K}_{t-1} \right) + I \left( 1 + \hat{I}_t \right) \]

\[ \Leftrightarrow \hat{K}_t = \left( 1 - \delta_k \right) \hat{K}_{t-1} + \delta_k \hat{I}_t \]  
(87)

The loglinearized expression of equation (10) is

\[ G^d \left( 1 + \hat{G}^d_t \right) = G_c \left( 1 + \hat{G}_{c,t} \right) + G_m \left( 1 + \hat{G}_{m,t} \right) \]

\[ \Leftrightarrow G^d \hat{G}^d_t = G_c \hat{G}_{c,t} + G_m \hat{G}_{m,t} \]  
(88)

The loglinearized expression of equation (23) is

\[ \kappa \left( 1 + \hat{\kappa}_t \right) = 1 + 2\gamma_1 v \left( 1 + \hat{v}_t \right) - 2\sqrt{\gamma_1 \gamma_2} \]

\[ \Leftrightarrow \hat{\kappa}_t = \frac{2\gamma_1 v}{\kappa} \hat{v}_t \]  
(89)

The loglinearized expression of equation (48), is

\[ \gamma_1 v^2 (1 + 2\hat{v}_t) - \gamma_2 = 1 - \frac{1}{q} (1 - \hat{q}_t) \]

\[ \Leftrightarrow \hat{q}_t = 2\gamma_1 v^2 q \hat{v}_t \]  
(90)

The loglinearized expression of equation (49) is

\[ w \left( 1 + \hat{w}_t \right) = (1 - \theta) Y \frac{d}{h} \left( 1 + \hat{Y}_t - \hat{h}_t \right) \]

\[ \Leftrightarrow \hat{w}_t = \hat{Y}_t - \hat{h}_t \]  
(91)

The loglinearized expression of equation (50) is

\[ r \left( 1 + \hat{r}_t \right) = \theta Y \frac{d}{K} \left( 1 + \hat{Y}_t - \hat{K}_{t-1} \right) \]

\[ \Leftrightarrow \hat{r}_t = \hat{Y}_t - \hat{K}_{t-1} \]  
(92)

The loglinearized expression of equation (51) is

\[ Y \left( 1 + \hat{Y}_t \right) = z K^\theta h^{1-\theta} \left( 1 + \hat{z}_t + \theta \hat{K}_{t-1} + (1 - \theta) \hat{h}_t \right) \]
The loglinearized expression of equation (36) is
\[ G^d \left( 1 + \hat{G}^d_t \right) = G^d \left( 1 + \hat{G}^d_{t-1} \right) - \delta_g G_c \left( 1 + \hat{G}_{c,t-1} \right) + g^s (1 + \hat{g}^s_t) \]
\[ \Leftrightarrow G^d \hat{C}^d_t = G^d \hat{C}^d_{t-1} - g^s \hat{G}_{c,t-1} + \hat{g}^s_t \]  \hspace{1cm} (94)

The loglinearized expression of equation (39) is
\[ Y \left( 1 + \hat{Y}_t \right) = C \left( 1 + \hat{C}_t \right) + I \left( 1 + \hat{I}_t \right) \]
\[ \Leftrightarrow Y \hat{Y}_t = C \hat{C}_t + I \hat{I}_t \]  \hspace{1cm} (95)

The loglinearized expression of equation (52) is
\[ \bar{r} \left( 1 + \hat{r}_t \right) = \frac{q p_\mu}{p_g} (1 + \hat{q}_t + E_t (\hat{p}_{g,t+1} - \hat{p}_{g,t})) \]
\[ \Leftrightarrow E_t (\hat{p}_{g,t+1}) = \hat{r}_t - \hat{q}_t + \hat{p}_{g,t} \]  \hspace{1cm} (96)

Consistently, the exogenous stochastic processes are restated as follows
\[ \hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_{z,t} \]  \hspace{1cm} (97)
\[ \hat{\mu}_{1,t} = \rho_{\mu_1} \hat{\mu}_{1,t-1} + \varepsilon_{\mu_1,t} \]  \hspace{1cm} (98)
\[ \hat{\mu}_{2,t} = \rho_{\mu_2} \hat{\mu}_{2,t-1} + \varepsilon_{\mu_2,t} \]  \hspace{1cm} (99)
\[ \hat{g}^s_t = \rho_g \hat{g}^s_{t-1} + \varepsilon_{g^s,t} \]  \hspace{1cm} (100)

The system of log-linearized stationary equilibrium equations, (81)-(96), can be restated, in a matrix form, as a linear stochastic difference equation of order 3. Indeed, the difference between the highest lead (2) and the highest lag (1) is of order 3. However, the method for solving rational expectation models considered in the current work — Klein (2000) method — is suitable for first order linear stochastic difference equations. In order to obtain a system of first order (log-)linear stochastic difference equations, it is convenient to use auxiliary variables. Therefore, one needs to define the following auxiliary variables:

\[ \hat{C}_{t+1}^+ := F \hat{C}_{t+1} = \hat{C}_{t+2} \]
\[ \hat{h}_{t+1}^+ := F \hat{h}_{t+1} = \hat{h}_{t+2} \]
\[ \hat{G}_{c,t+1}^+ := F \hat{G}_{c,t+1} = \hat{G}_{c,t+2} \]
\[ \hat{K}_t^- := L \hat{K}_t = \hat{K}_{t-1} \]
\[ \hat{C}_t^- := L \hat{C}_t = \hat{C}_{t-1} \]
\[ \hat{G}_{t}^- := L \hat{G}_t = \hat{G}_{t-1} \]
\[ \hat{G}_{c,t}^- := L \hat{G}_{c,t} = \hat{G}_{c,t-1} \]

where \( F \) and \( L \) are the lead and lag operator, respectively.
By substituting \( \hat{C}_{t+1}^+, \hat{h}_{t+1}^+, \hat{G}_{c,t+1}^+, \hat{K}_t^-, \hat{\bar{C}}_t^-, \hat{\bar{G}}_{c,t}^d \) and \( \hat{\bar{G}}_{c,t}^d \) for \( \hat{C}_{t+2}, \hat{h}_{t+2}, \hat{G}_{c,t+2}, \hat{K}_{t-1}, \hat{\bar{C}}_{t-1}, \hat{\bar{G}}_{c,t-1}^d \) and \( \hat{\bar{G}}_{c,t-1}^d \) respectively, in the system of log-linearized stationary equilibrium equations,

\[
\frac{b\beta (1 - \phi_4) \phi_2}{1 - b\beta} \frac{h}{1 - h} E_t \left( \hat{h}_{t+1} \right) - \frac{b\beta (1 - \phi_4) \phi_3}{1 - b\beta} E_t \left( \hat{G}_{c,t+1}^+ \right) + \frac{b\beta (1 - (1 - \phi_4) \phi_1)}{(1 - b\beta) (1 - b)} E_t \left( \hat{C}_{t+1}^+ \right) = \\
= \left[ \frac{(1 - \phi_4) \phi_1}{1 - b} + \frac{(1 - (1 - \phi_4) \phi_1)}{(1 - b\beta) (1 - b)} + \frac{1 - (1 - \phi_4) \phi_1}{b\beta} \right] h \frac{\hat{h}_t}{1 - h} + \frac{\hat{\bar{C}}_t}{1 - b} - \frac{1 - (1 - \phi_4) \phi_1}{1 - b\beta} \hat{C}_t - \frac{1}{1 - b} \hat{\bar{G}}_{c,t}^d + (101)
\]

\[
(1 - \delta_g) \beta E_t \left( \hat{p}_{g,t+1} \right) - (1 - \delta_g) \beta E_t \left( \hat{\bar{C}}_{t+1} \right) - \frac{\phi_2 h}{1 - h} \frac{(1 - \phi_4) \beta [1 - \delta_g + b]}{1 - b\beta} E_t \left( \hat{h}_{t+1} \right) + \\
+ \frac{(1 - \phi_4) \beta (1 - \delta_g + b)}{1 - b\beta} E_t \left( \hat{G}_{c,t+1}^+ \right) + \frac{1 - (1 - \phi_4) (1 - \delta_g + b) \beta \phi_1}{1 - b\beta} + \frac{b\beta^2 (1 - \phi_4) (1 - \delta_g) \phi_1 b}{1 - b\beta} \\
+ \frac{\beta b [1 + (1 - b\beta) (1 - \delta_g)]}{1 - b\beta} E_t \left( \hat{C}_{t+1} \right) + \frac{1 - (1 - \delta_g) b\beta^2 (1 - \phi_4) \phi_2}{1 - b\beta} \frac{h}{1 - h} E_t \left( \hat{h}_{t+1} \right) - \\
- \frac{\phi_3 (1 - \phi_4) b\beta^2 (1 - \delta_g)}{1 - b\beta} E_t \left( \hat{G}_{c,t+1}^+ \right) + \frac{b\beta^2 (1 - \delta_g) (1 - \phi_4)}{1 - b\beta} \frac{1}{1 - (1 - \phi_4)} - \frac{\phi_3}{1 - b\beta} E_t \left( \hat{C}_{t+1}^+ \right) = \\
= \hat{p}_{g,t} - \frac{(1 - \phi_4) \phi_2 h}{1 - h} \left[ \frac{b\beta}{1 - b\beta} + (1 - \delta_g) \beta \right] \hat{h}_t + \\
+ \frac{1 + b\beta^2 (1 - \delta_g) \beta}{1 - b\beta} \phi_1 \frac{1 - (1 - \phi_4) (1 - \delta_g + b)}{1 - b\beta} \frac{h}{1 - h} + \frac{1 - (1 - \delta_g) \beta (1 - \phi_4)}{1 - b\beta} \frac{b\beta}{1 - b\beta} + (1 - \delta_g) \beta \phi_3 \hat{G}_{c,t}^d + \\
+ \left[ \frac{b}{1 - b\beta} - (1 - \phi_4) \phi_1 \frac{b\beta}{1 - b\beta} + (1 - \delta_g) \beta \phi_3 \right] \hat{\bar{C}}_t + (102)
\]

\[
E_t \left( \hat{\bar{C}}_{t+1} \right) - E_t \left( \hat{p}_{g,t+1} \right) + \frac{(1 + b\beta) (1 - \phi_4) \phi_2}{1 - b\beta} \frac{h}{1 - h} E_t \left( \hat{h}_{t+1} \right) - \frac{(1 + b\beta) (1 - \phi_4) \phi_3}{1 - b\beta} E_t \left( \hat{G}_{c,t+1}^+ \right) + \\
+ \frac{1}{1 - b} \left[ \frac{1 + b\beta^2 + b\beta}{1 - b\beta} - \frac{(1 + b\beta) (1 - \phi_4)}{1 - b\beta} \phi_1 - \frac{(1 - \phi_4) b\beta}{1 - b\beta} \phi_1 \right] E_t \left( \hat{C}_{t+1} \right) + \frac{b\beta (1 - \phi_4) \phi_3}{1 - b\beta} E_t \left( \hat{G}_{c,t+1}^+ \right) - \\
- \frac{b\beta (1 - \phi_4) \phi_2}{1 - b\beta} \frac{h}{1 - h} E_t \left( \hat{h}_{t+1} \right) + \frac{(1 - \phi_4) b\beta}{1 - b\beta} \phi_1 - \frac{1}{1 - (1 - \phi_4)} \right) E_t \left( \hat{C}_{t+1}^+ \right) = \frac{2\gamma t^2}{\beta} \hat{\bar{C}}_t + \hat{\bar{G}}_{c,t+1}^d + \\
+ \frac{(1 - \phi_4) \phi_2}{1 - b\beta} \frac{h}{1 - h} \hat{h}_t - \frac{(1 - \phi_4) \phi_3}{1 - b\beta} \hat{G}_{c,t}^+ + \frac{1}{1 - b} \left[ \frac{1 + b\beta^2 + b\beta}{1 - b\beta} - \frac{(1 + b\beta) (1 - \phi_4)}{1 - b\beta} \phi_1 b - \frac{(1 - \phi_4) \phi_1 b}{1 - b\beta} \right] \hat{\bar{C}}_t - \\
\frac{[1 - (1 - \phi_4) \phi_1 b]}{1 - b\beta} \hat{C}_t^d = (103)
\]
\[
\frac{r}{r + 1 - \delta_k} E_t \left( \hat{r}_{t+1} \right) - E_t \left( \hat{\kappa}_{t+1} \right) - \frac{(1 + b\beta) (1 - \phi_4) \phi_2}{1 - b\beta} \frac{h}{1 - h} E_t \left( \hat{h}_{t+1} \right) + \frac{(1 + b\beta) (1 - \phi_4) \phi_3}{1 - b\beta} E_t \left( \hat{G}_{c,t+1} \right) - \frac{1}{1 - b} \left[ \frac{1 + b^2\beta + b\beta}{1 - b\beta} \phi_1 + \frac{1 - (1 - \phi_4) b\beta}{1 - b\beta} \phi_1 b \right] E_t \left( \hat{C}_{t+1} \right) - \frac{b\beta (1 - \phi_4) \phi_3}{1 - b\beta} E_t \left( \hat{G}_{c,t+1}^+ \right) + \frac{b\beta (1 - \phi_4) \phi_2}{1 - b\beta} \frac{h}{1 - h} E_t \left( \hat{h}_{t+1}^+ \right) - \frac{(1 - \phi_4) b\beta}{1 - b\beta} \left( \phi_1 - \frac{1}{1 - (1 - \phi_4)} \right) E_t \left( \hat{C}_{t+1}^+ \right) = -\frac{(1 - \phi_4) \phi_2}{1 - b\beta} \frac{h}{1 - h} \hat{h}_{t+1}^+ + \frac{1 - (1 - \phi_4) \phi_1}{1 - b\beta} b \hat{C}_{t+1}^-
\]
(104)

\[
\hat{K}_t = (1 - \delta_k) \hat{K}_t^- + \delta_k \hat{I}_t
\]
(105)

\[
\hat{r}_t = \hat{Y}_t - \hat{K}_t^-
\]
(106)

\[
\hat{Y}_t = \hat{z}_t + \theta \hat{K}_t^- + (1 - \theta) \hat{h}_t
\]
(107)

\[
G^d \hat{c}_t^d = G^d \hat{c}_t^{d-} - g^s \hat{G}_{c,t} + \tilde{g}_t^s
\]
(108)

and adding the following equations

\[
E_t \hat{K}_{t+1}^- = \hat{K}_t
\]
(109)

\[
E_t \hat{G}_{t+1}^{d-} = \hat{c}_t^d
\]
(110)

\[
E_t \hat{G}_{c,t+1}^- = \hat{G}_{c,t}
\]
(111)

\[
E_t \hat{C}_{t+1}^- = \hat{C}_t
\]
(112)

\[
E_t \hat{h}_{t+1} = \hat{h}_t^+
\]
(113)

\[
E_t \hat{G}_{c,t+1} = \hat{G}_{c,t}^+
\]
(114)

\[
E_t \hat{C}_{t+1} = \hat{C}_t^+
\]
(115)

one get the required system of first order (log-)linear stochastic difference equations.

### A.3 Setting up the system in a matrix form

The system of first order (log-)linear stochastic difference equations is given by

\[
\dot{q}_t - 2\gamma_1 v^2 q \dot{v}_t = 0
\]
(S1)

\[
\delta_k \dot{I}_t = \dot{K}_t - (1 - \delta_k) \dot{K}_t^-
\]
(S2)

\[
\dot{v}_t + \dot{m}_t - \dot{C}_t = 0
\]
(S3)

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\[ \dot{Y}_t - \hat{h}_t = \dot{w}_t \]  
\[ \dot{Y}_t - \hat{r}_t = \hat{K}_t^{-} \]  
\[ \dot{Y}_t - (1 - \theta) \hat{h}_t = \theta \hat{K}_t^{-} + \hat{z}_t \]  
\[ \hat{m}_t - \hat{G}_{m,t} = \hat{p}_{g,t} - \hat{\mu}_{1,t} - \hat{\mu}_{2,t} \]  
\[ Y \dot{Y}_t - C \hat{C}_t - I \hat{I}_t = 0 \]  
\[ G^d \hat{G}^d_{t} - G_m \hat{G}_{m,t} = G_c \hat{G}_{c,t} \]  
\[ G^d \hat{G}^d_{t} = G^d \hat{G}^d_{t} - g^* \hat{G}_{c,t} + \tilde{g}_t \]

\[
(1 - \delta_g) \beta E_t (\hat{p}_{g,t+1}) - (1 - \delta_g) \beta \frac{2\gamma_1 v}{\kappa} E_t (\hat{v}_{t+1}) - \frac{1}{1 - h} \chi_1 E_t \left( \hat{h}_{t+1} \right) + \chi_1 \phi_3 E_t \left( \hat{G}_{c,t+1} \right) + \\
+ \frac{1}{1 - b} \left[ \chi_1 \phi_1 + \chi_2 \phi_1 b - \chi_3 \right] E_t \left( \hat{C}_{t+1} \right) + \chi_2 \phi_2 \frac{1}{1 - h} E_t \left( \hat{h}_{t+1}^+ \right) + \phi_3 \chi_2 E_t \left( \hat{G}_{c,t+1}^+ \right) + \\
+ \left[ - \frac{1}{1 - \phi_4} - \phi_1 \right] \frac{\chi_2}{1 - b} E_t \left( \hat{C}_{t+1}^+ \right) = \hat{p}_{g,t} - \frac{2\gamma_1 v}{\kappa} \hat{v}_t - \chi_4 \frac{\phi_2 h}{1 - h} \hat{h}_t + \\
+ \left[ - \chi_5 + \chi_1 \frac{b \phi_1}{1 - b} + \chi_4 \frac{\phi_1}{1 - b} \right] \hat{C}_t + \left[ 1 - (1 - \delta_g) \beta + \chi_4 \phi_3 \right] \hat{G}_{c,t} + \frac{b}{1 - b} \left[ \frac{1}{1 - b \beta} - \phi_1 \chi_4 \right] \hat{C}_t
\]

\[
\Psi_1 \phi_2 \frac{h}{1 - h} E_t \left( \hat{h}_{t+1} \right) - \Psi_1 \phi_3 E_t \left( \hat{G}_{c,t+1} \right) + \frac{1}{1 - b} \left[ \frac{1}{1 - \phi_4} - \phi_1 \right] E_t \left( \hat{C}_{t+1} \right) = \\
= \frac{1}{1 - b} \left[ 1 + b^2 \beta - \Psi_1 \phi_1 (1 + b) \right] \hat{C}_t - \phi_3 \Psi_1 \Phi_{1} \hat{G}_{c,t} + \left[ 1 + \phi_2 \Psi_1 \right] \frac{h}{1 - h} \hat{h}_t + \\
+ \frac{2\gamma_1 v}{\kappa} \hat{v}_t - \hat{w}_t + \left[ \Psi_1 \phi_1 - \frac{1}{1 - b \beta} \right] \frac{b}{1 - b} \hat{C}_t
\]

\[
\frac{2\gamma_1 v}{\kappa} E_t (\hat{v}_{t+1}) - E_t (\hat{p}_{g,t+1}) + \Psi_2 \phi_2 \frac{h}{1 - h} E_t \left( \hat{h}_{t+1} \right) - \Psi_2 \phi_3 E_t \left( \hat{G}_{c,t+1} \right) + \\
+ \frac{1}{1 - b} \left[ \Psi_3 - \Psi_2 \phi_1 - \Psi_1 \phi_1 b \right] E_t \left( \hat{C}_{t+1} \right) + \Psi_1 \phi_3 E_t \left( \hat{G}_{c,t+1}^+ \right) - \Psi_1 \phi_2 \frac{h}{1 - h} E_t \left( \hat{h}_{t+1}^+ \right) + \\
+ \frac{1}{1 - b} \left[ \phi_1 - \frac{1}{1 - \phi_4} \right] E_t \left( \hat{C}_{t+1}^+ \right) = 2\gamma_1 v \left( \frac{v}{\beta} + \frac{1}{\kappa} \right) \hat{v}_t - \hat{p}_{g,t} + \frac{1}{1 - b \beta} \frac{h}{1 - h} \hat{h}_t - \\
- \frac{1}{1 - b \beta} \hat{G}_{c,t} + \frac{1}{1 - b} \left[ 1 + b^2 \beta + b - \frac{1}{1 - b \beta} \right] \phi_1 \hat{C}_t + \left[ (1 - \phi_4) \phi_1 - 1 \right] \frac{1}{1 - (b - 1 - b \beta)} \hat{C}_t
\]

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\[
\begin{align*}
\frac{r}{r+1-\delta_k} E_t(\hat{r}_{t+1}) - \frac{2\gamma_1 v}{\kappa} E_t(\hat{v}_{t+1}) - \Psi_2 \phi_2 \frac{h}{1-h} E_t(\hat{h}_{t+1}) + \Psi_2 \phi_3 E_t(\hat{G}_{c,t+1}) + \\
\frac{1}{1-b} [\Psi_2 \phi_1 - \Psi_3 + \Psi_1 \phi_1 b] E_t(\hat{C}_{t+1}) - \Psi_1 \phi_3 E_t(\hat{G}_{c,t+1}) + \\
\Psi_1 \phi_2 \frac{h}{1-h} E_t(\hat{h}_{t+1}) + \frac{1}{1-b} \left[ 1 - \frac{1 - \phi_4 - \phi_1}{1 - \phi_4} \right] E_t(\hat{C}_{t+1}) = -\frac{(1 - \phi_4) \phi_2}{1 - b \beta} \frac{h}{1-h} \hat{h}_{t+1} \\
+ \frac{(1 - \phi_4) \phi_3}{1 - b \beta} \hat{G}_{c,t} - \frac{2\gamma_1 v}{\kappa} \hat{v}_{t+1} + \frac{1}{1-b} \left[ -1 + (1 + b^2 \beta + b) \Psi_2 \phi_1 b + (1 - \phi_4) \phi_1 \right] \hat{C}_{t+1} + \\
\frac{[1 - (1 - \phi_4) \phi_1] b}{(1-b \beta) (1-b)} \hat{C}_{t+1}^{-}
\end{align*}
\]

\[E_t(\hat{p}_{g,t+1}) = \hat{r}_t - \hat{q}_t + \hat{p}_{g,t} \tag{D5}\]

\[E_t \hat{K}_{t-1}^{-} = \hat{K}_t \tag{D6}\]

\[E_t \hat{G}_{t+1}^{d-} = \hat{G}_t^d \tag{D7}\]

\[E_t \hat{G}_{c,t+1}^{d-} = \hat{G}_{c,t} \tag{D8}\]

\[E_t \hat{C}_{t+1}^{-} = \hat{C}_t \tag{D9}\]

\[E_t \hat{h}_{t+1} = \hat{h}_{t+1} \tag{D10}\]

\[E_t \hat{G}_{c,t+1} = \hat{G}_{c,t}^{+} \tag{D11}\]

\[E_t \hat{C}_{t+1} = \hat{C}_t^{+} \tag{D12}\]

where

\[
\begin{align*}
\chi_1 &= \beta \frac{(1 - \phi_4) (1 - \delta_g + b)}{1 - b \beta} \\
\chi_2 &= \frac{b \beta^2 (1 - \phi_4) (1 - \delta_g)}{1 - b \beta} \\
\chi_3 &= \frac{b \beta [b + (1 + b^2 \beta) (1 - \delta_g)]}{1 - b \beta} \\
\chi_4 &= (1 - \phi_4) \left[ \frac{b \beta}{1 - b \beta} + (1 - \delta_g) \beta \right] \\
\chi_5 &= \frac{1 + b^2 \beta + (1 - \delta_g) b \beta}{(1 - b \beta) (1 - b)} \\
\Psi_1 &= \frac{(1 - \phi_4) b \beta}{1 - b \beta} \\
\Psi_2 &= \frac{(1 - \phi_4) (1 + b \beta)}{1 - b \beta} \\
\Psi_3 &= \frac{1 + b^2 \beta + b \beta}{1 - b \beta}
\end{align*}
\]
Note that I have eliminated variable \( \hat{\kappa}_t \) and equation (89) from the system of log-linearized equilibrium equations.

Besides, the stochastic processes of the exogenous variables are given by

\[
\begin{align*}
\hat{z}_t &= \rho_z \hat{z}_{t-1} + \varepsilon_{z,t} \\
\hat{\mu}_{1,t} &= \rho_{\mu_1} \hat{\mu}_{1,t-1} + \varepsilon_{\mu_1,t} \\
\hat{\mu}_{2,t} &= \rho_{\mu_2} \hat{\mu}_{2,t-1} + \varepsilon_{\mu_2,t} \\
\hat{g}^*_{t} &= \rho_g \hat{g}^*_{t-1} + \varepsilon_{g^*,t}
\end{align*}
\]

(E1) (E2) (E3) (E4)

The systems of equilibrium equations and exogenous stochastic processes can be set up in the following matrix form:

\[
\begin{align*}
N_y Y_t &= N_x X_t + N_z Z_t \\
M_{xp} E_t X_{t+1} + M_{yy} E_t Y_{t+1} + M_{zp} E_t Z_{t+1} &= M_x X_t + M_y Y_t + M_z Z_t \\
Z_t &= P_z Z_{t-1} + \epsilon_t \quad \epsilon_t \sim \text{i.i.d.} (0_4, \Omega)
\end{align*}
\]

Matrix equation (116) gathers all the static log-linear equilibrium equations. Matrix equation (117) concerns all the dynamic log-linear equilibrium equations. The stochastic exogenous processes are put in matrix equation (118). Hence, matrix equation (116) is built according to the static equations (S1)-(S10). Equations (D1)-(D12) form the matrix equation (117). The stochastic exogenous processes (E1)-(E4) define the matrix equation (118).

Before giving explicit expressions to matrices \( N_y, N_x, N_z, M_{xp}, M_{yy}, M_{zp}, M_x, M_y, M_z, P_z \) and \( \Omega \) I have to define the vectors of variables \( Y_t, X_t, Z_t \) and \( \epsilon_t \):

\[
Y_t = \begin{pmatrix}
\hat{Y}_t \\
\hat{C}_t \\
\hat{I}_t \\
\hat{h}_{t} \\
\hat{\hat{g}}_t \\
\hat{m}_t \\
\hat{\hat{v}}_t \\
\hat{G}_{m,t} \\
\hat{G}_{d,t} \\
\hat{r}_t
\end{pmatrix}
\quad X_t = \begin{pmatrix}
\hat{K}_t \\
\hat{G}_{d,t} \\
\hat{G}_{c,t} \\
\hat{C}_t \\
\hat{\hat{K}}_t \\
\hat{\hat{w}}_t \\
\hat{\hat{g}}_{t} \\
\hat{\hat{v}}_t \\
\hat{\hat{G}}_{c,t} \\
\hat{\hat{G}}_{d,t} \\
\hat{\hat{r}}_t \\
\hat{\hat{C}}_t
\end{pmatrix}
\quad Z_t = \begin{pmatrix}
\hat{\hat{z}}_t \\
\hat{\hat{\mu}}_{1,t} \\
\hat{\hat{\mu}}_{2,t} \\
\hat{\hat{g}}^*_{t}
\end{pmatrix}
\quad \epsilon_t = \begin{pmatrix}
\varepsilon_{z,t} \\
\varepsilon_{\mu_1,t} \\
\varepsilon_{\mu_2,t} \\
\varepsilon_{g^*,t}
\end{pmatrix}
\]

\( Y_t \) is a \((10 \times 1)\) vector which elements are the control variables. \( Z_t \) is a \((4 \times 1)\) vector which elements are the exogenous variables. \( \epsilon_t \) is a \((4 \times 1)\) vector which elements are the perturbations to the exogenous variables. \( X_t \) is a \((12 \times 1)\) vector of state variables. The latter can be partitioned in two vectors as follows,

\[
X_t = \begin{pmatrix}
X_t^h \\
X_t^f
\end{pmatrix}
\]

where vector \( X_t^h \) contains the predetermined state variables and vector \( X_t^f \) contains the jump state variables. In the gold standard model, \( \hat{K}_t, \hat{G}_{d,t}^+, \hat{G}_{c,t}^+ \) and \( \hat{C}_t^+ \) are considered as predetermined state variables, and \( \hat{K}_t, \hat{\hat{w}}_t, \hat{\hat{g}}_{t}, \hat{\hat{v}}_t, \hat{\hat{G}}_{c,t}^+ \) and \( \hat{\hat{C}}_t^+ \) as jump state variables.

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Thus, the matrices appearing in (116) are defined as follows:

\[
N_y = \begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 0 & -\frac{2\gamma_1 v^3}{3} & 0 & 0 & 0 \\
0 & 0 & \delta_k & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
1 & 0 & 0 & \theta - 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\
Y & -C & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -G_m & G^d & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & G^d & 0
\end{pmatrix}
\]

\[
N_x = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta_k - 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & G^d & -G^s & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

\[
N_z = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

\[
M_{yp} = \begin{pmatrix}
0 & \frac{\phi_1 c^2 - \phi_1 b c^2 - \phi_2 x^2}{x^{\frac{1}{3}}} & 0 & -\frac{\phi_2 h}{x^{\frac{1}{3}}} & 0 & 0 & -\frac{(1-\delta_k)2\gamma_1 v^3}{3} & 0 & 0 & 0 \\
0 & \frac{\psi_1(1-b)}{x^{\frac{1}{3}}} & 0 & \frac{\psi_2 h}{x^{\frac{1}{3}}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\psi_3 - \psi_2 \phi_1 - \phi_1 b \psi_3}{x^{\frac{1}{3}}} & 0 & \frac{\psi_2 h}{x^{\frac{1}{3}}} & 0 & 0 & \frac{2\gamma_1 v^3}{3} & 0 & 0 & 0 \\
0 & -\frac{1-b}{x^{\frac{1}{3}}} - \psi_2 \phi_1 - \phi_1 b \psi_1 & 0 & -\frac{\psi_2 h}{x^{\frac{1}{3}}} & 0 & \frac{\gamma_1 v^3}{3} & 0 & \frac{r}{r+1-\delta_k} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]
\[
M_{zp} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & (1 - \delta_g) \beta & 0 & \chi_1 \phi_3 & \chi_2 \frac{\phi_2 h}{1 - h} & -\chi_2 \phi_3 & \frac{\chi_2}{1 - \delta_g} \phi_3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & -\Psi_1 \phi_3 & -\Psi_1 \phi_2 h & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

\[
M_{zp} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

\[
M_y = \begin{pmatrix}
0 & \chi_1 \frac{\phi_1 h}{1 - b} + \chi_4 \frac{\phi_1}{1 - b} - \chi_5 & 0 & 0 & 0 & -\chi_4 \frac{\phi_2 h}{1 - h} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]
\[
M_x = \begin{pmatrix}
0 & 0 & 0 & \frac{b(1-\phi_1\chi_4)}{1-b} & 0 & 0 & 1 & 0 & 1 - (1 - \delta_g) \beta + \chi_4 \phi_3 & 0 & 0 \\
0 & 0 & 0 & -\frac{b(1-\phi_1\psi_1)}{1-b} & 0 & -1 & 0 & 0 & -\psi_1 \phi_3 & 0 & 0 \\
0 & 0 & 0 & \frac{b(1-\phi_4)}{1-b}(1-b) & 0 & 0 & 0 & 0 & 0 & -\frac{1-\phi_3}{1-b} \phi_3 & 0 & 0 \\
0 & 0 & 0 & \frac{b(1-\phi_4)(1-b)}{1-b}(1-b) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

\[
P_z = \begin{pmatrix}
\rho_z & 0 & 0 & 0 \\
0 & \rho_{\mu_1} & 0 & 0 \\
0 & 0 & \rho_{\mu_2} & 0 \\
0 & 0 & 0 & \rho_{\mu^*}
\end{pmatrix}
\]

\[
\Omega = \begin{pmatrix}
\sigma_z^2 & 0 & 0 & 0 \\
0 & \sigma_{\mu_1}^2 & 0 & 0 \\
0 & 0 & \sigma_{\mu_2}^2 & 0 \\
0 & 0 & 0 & \sigma_{\mu^*}^2
\end{pmatrix}
\]

Notice that matrix \( N_p \) should be square and invertible. Moreover, both \( M_{zp} \) and \( M_x \) should be square matrices. The matrix governing the exogenous processes, \( P_z \), should have its eigenvalues lying inside the unit circle.

Hence, the method developed by Klein (2000) is used to solve the matrix form equations (116)-(118).

**B Additional Figures**

![Figure 40: Impulse response functions to a 1 percent innovation to the gold flow shocks: Non-monetary gold stock vs monetary gold stock with \( \delta_g = 0.1 \).](image)
Figure 41: Impulse response functions to a 1 percent innovation to the gold flow shocks: Nonmonetary gold stock vs monetary gold stock with $\delta_g = 10^{-9}$.

Figure 42: Impulse response functions to a 1 percent innovation to the gold flow shocks: Baseline model with small transactions costs (a).
Figure 43: Impulse response functions to a 1 percent innovation to the gold flow shocks: Baseline model with small transactions costs (b).

Figure 44: Impulse response functions to a 1 percent innovation to the gold flow shocks: Baseline model vs baseline model with highly persistent gold flow shock (b).
References


