A Model of Indivisible Commodity Money with Minting and Melting

Angela Redish
Warren Weber

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Introduction

Focus: Early (800 - 1500) commodity money systems (coins)

Commodity money: Money that is

- Intrinsically useful, or
- Redeemable on demand into something that is
Introduction

Stylized facts about evolution of commodity money systems:

1. Started with a single silver coin
   - Initially same size across countries
   - Over time, debased in some countries, not debased in others

2. Next added a second silver coin
   - If original silver coin small, then new silver coin larger
   - If original silver coin large, then new silver coin smaller

3. Much larger gold coin added even later
Introduction

- **Purpose**: Build model of commodity money that provides a rationale for such an evolution

- **Features we incorporate in the model**:
  1. Coins have limited divisibility (coins cannot be too small)
  2. Limited number of types of coins
  3. No fixed exchange rate between coins of different metals
Outline

- Documentation of stylized facts
- Model
- Numerical analysis
Stylized facts

- Silver pennies the same size until 1100
  - Carolingian penny 96% fine; 1.7 gms;
  - English penny 92.5% fine; 1.5 gms.
Stylized facts

- By the 12th century, the silver content of the penny varied across Europe

<table>
<thead>
<tr>
<th></th>
<th>Year</th>
<th>Weight</th>
<th>Fineness</th>
<th>Fine weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>England</td>
<td>1160</td>
<td>1.46 gms</td>
<td>.925</td>
<td>1.35 gms</td>
</tr>
<tr>
<td>Cologne</td>
<td>1160</td>
<td>1.4 gms</td>
<td>.925</td>
<td>1.30 gms</td>
</tr>
<tr>
<td>Paris</td>
<td>1160</td>
<td>1.28-.85 gms</td>
<td>&lt; .5</td>
<td>&lt; .60 gms</td>
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<tr>
<td>Melgueil</td>
<td>1125</td>
<td>1.1 gms</td>
<td>.42</td>
<td>0.46 gms</td>
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<tr>
<td>Lucca</td>
<td>1160</td>
<td>0.6 gms</td>
<td>0.6</td>
<td>0.35 gms</td>
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<tr>
<td>Barcelona</td>
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<td>0.66 gms</td>
<td>0.2</td>
<td>0.13 gms</td>
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<tr>
<td>Venice</td>
<td>1160</td>
<td></td>
<td></td>
<td>0.05 gms</td>
</tr>
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</table>
## Stylized facts

- Gold coins introduced later

<table>
<thead>
<tr>
<th></th>
<th>Coin</th>
<th>Metal</th>
<th>Fine weight</th>
<th>Value in unit of account</th>
<th>Silver equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Venice</strong></td>
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<tr>
<td>1170</td>
<td>Denarius</td>
<td>Silver</td>
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<td>1d</td>
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<td>1194</td>
<td>Grossus</td>
<td>Silver</td>
<td>2.1 gms</td>
<td>d</td>
<td></td>
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<tr>
<td>1284</td>
<td>Ducat</td>
<td>Gold</td>
<td>3.5 gms</td>
<td></td>
<td>50gms</td>
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<td><strong>France</strong></td>
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<td></td>
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<td></td>
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<tr>
<td>1200</td>
<td>Denier</td>
<td>Silver</td>
<td>0.36 gms</td>
<td>1d</td>
<td></td>
</tr>
<tr>
<td>1266</td>
<td>Gros tournois</td>
<td>Silver</td>
<td>4.2 gms</td>
<td>12d</td>
<td></td>
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<tr>
<td>1290</td>
<td>Royale</td>
<td>Gold</td>
<td>6.97 gms</td>
<td>300d</td>
<td>100 gms</td>
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<tr>
<td><strong>England</strong></td>
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<td></td>
<td></td>
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<tr>
<td>1200</td>
<td>Penny</td>
<td>Silver</td>
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<td>1d</td>
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<td>Farthing</td>
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<td>.25d</td>
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<td>103 gms</td>
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<td>1351</td>
<td>Groat</td>
<td>Silver</td>
<td>5.18 gms</td>
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</table>
Environment

- Time discrete and infinite
- One nonstorable, perfectly divisible consumption good
- Two storable metals in **fixed supply**
  - silver – supply \((m^s)\)
  - gold – supply \((m^g)\)
Silver and gold can be held as coins or jewelry (bullion)

Silver and gold coins are indivisible, but can be minted or melted

Possibly two silver coins

- first (original) silver coin contains $b_s$ ounces of silver
- second silver coin (if exists) contains $\eta b_s$ ounces of silver
  - $\eta \in \{2, 3, \ldots\}$
  - restriction on $\eta$ consistent with actual coinage

Only a single gold coin
- contains $b_g$ ounces of gold
Environment

- Agents hold
  - $s_1$ silver coins of first type
  - $s_2$ silver coins of second type
  - $j^s$ units silver jewelry
  - $g$ gold coins
  - $j^g$ units gold jewelry

⇒ Only coins can be used in trade

⇒ Only jewelry yields utility (similar to Velde-Weber)
Agents

- [0, 1] continuum, infinitely-lived
- Preferences:
  \[ u(q_{n+1}) - q_n + \mu(b_s j^s, b_g j^g) - \gamma(s_1 + s_2 + g) \]
  - \( c \) consumption; \( q \) production
  - \( u(0) = 0, u' > 0, u'(0) = \infty, u'' < 0 \)
  - \( \gamma \) utility cost of holding a coin

- Maximize expected discounted (\( \beta \)) lifetime utility
- Prob \( \frac{1}{2} \) can either consume or produce, but not both, in each period
- \( \theta \) prob of a single coincidence match
Trade

Each period has two subperiods

1. First subperiod: decentralized trade in bilateral matches
   - Preference assumption rules out double coincidence matches

2. Second subperiod: agents can alter
   - coin/coin mix
   - coin/jewelry mix (minting or melting)
Trade

Information:

- past trading histories private (no monitoring or commitment technology)
  - rules out gift-giving equilibrium

- agents are anonymous
  - rules out credit
Trade

- Single coincidence matches: potential consumer makes TIOLI offer \((q, p_1^s, p_2^s, p^g)\)

  \[
  q \in \mathbb{R}_+ \text{ quantity of production demanded}
  \]

  \[
  p_i^s \in \mathbb{Z} \text{ quantity of silver coins of type } i \text{ offered}
  \]

  \[
  p^g \in \mathbb{Z} \text{ quantity of gold coins offered}
  \]

  \[
  p_i^s, p^g < 0 \text{ interpreted as making change}
  \]

- Let \(\Lambda\) be set of all feasible TIOLI offers

- Lotteries not permitted
After trade agents make portfolio adjustment \((z, z_1^s, z_2^s, z^g)\)

- \(z \in \mathbb{Z}\) quantity of silver coins of type 2 exchanged for coins of type 1
- \(z_i^s \in \mathbb{Z}\) quantity of silver coins of type \(i\) minted \((> 0)\) or melted \((< 0)\)
- \(z^g \in \mathbb{Z}\) quantity of gold coins minted \((> 0)\) or melted \((< 0)\)
Model: Value functions

- Expected value of holding $y_t = (s_1,t, s_2,t, g_t, j_t^s, j_t^g)$ beginning second subperiod

$$v_t(y_t) = \max_{z_t, z_{1,t}, z_{2,t}, z_t} \beta w_{t+1}(s_1,t + \eta z_t + z_{1,t}^s, s_2,t - z_t + z_{2,t}^s,$$

$$j_t^s - z_{1,t}^s - \eta z_{2,t}^s, g_t + z_{t}^g, j_t^g - z_{t}^g)$$

$$- S(j_t^s, z_{1,t}^s, z_{2,t}^s, j_t^g, z_{t}^g) - \phi z_t$$

$S(j_t^s, z_{1,t}^s, z_{2,t}^s, j_t^g, z_{t}^g)$ seigniorage tax on minting

$\phi$ cost of converting a silver coin of type 2 into $\eta$ coins of type 1

- Note: Coins minted must be of same metal as brought to mint
Model: Value functions

- Expected value of holding $y_t$ beginning of first subperiod

$$w_t(y_t) = \frac{\theta}{2} \sum_{\tilde{y}_t} \pi_t(\tilde{y}_t) \max[u(q_t)$$

$$+ v_t(s_{1,t} - p_{1,t}^s, s_{2,t} - p_{2,t}^s, j_t^s, g_t - p_g^t, j_t^s, j_t^g)]$$

$$(1 - \frac{\theta}{2})v_t(y_t) + \mu(b_s j_t^s, b_g j_t^g) - \gamma(s_{1,t} + s_{2,t} + g_t)$$

$$\pi_t(y_t) = \text{fraction of agents with } y_t \text{ beginning first subperiod}$$

$\tilde{y}$ denotes seller
Model: Asset distributions

- Define $\lambda_b(k_1^s, k_2^s, k^g; y_t, \tilde{y}_t)$ probability buyer with $y_t$ meeting seller with $\tilde{y}_t$ leaves with $s_1 = k_1^s$, $s_2 = k_2^s$, $g = k^g$

- $\lambda_s(k_1^s, k_2^s, k^g; y_t, \tilde{y}_t)$ similar for seller

- Post-trade (pre-mint/melt) asset distribution is

$$\tilde{\pi}_t(k_1^s, k_2^s, k^g, j^s, j^g) = \frac{\theta}{2} \sum_{y_t, \tilde{y}_t} \pi_t(y_t)\pi_t(\tilde{y}_t)[\lambda_b(\cdot) + \lambda_s(\cdot)]$$

$$+ (1 - \frac{\theta}{2})\pi_t(k_1^s, k_2^s, k^g, j^s, j^g)$$
Model: Asset distributions

- Define $\delta(k_1^s, k_2^s, k^g, h^s, h^g; y_t)$ to be probability agent with $y_t$ has $s_1 = k_1^s, s_2 = k_2^s, g = k^g, j^s = h^s, j^g = h^g$ after second subperiod.

- Pre-trade next period asset distribution is

$$\pi_{t+1}(k_1^s, k_2^s, k^g, h^s, h^g) = \sum_{y_t} \tilde{\pi}_t(y_t) \delta(k_1^s, k_2^s, k^g, h^s, h^g; y_t)$$
Steady state symmetric equilibrium:

Value functions $w, v$; asset holdings $\pi$ and $\tilde{\pi}$; quantities $p^s_1, p^s_2, p^g, z, z^s_1, z^s_2, z^g, q$ that satisfy

1. Bellman equations
2. asset transitions
3. market clearing
Results

- Numerical – analytic results not possible

- Assume:

\[
\theta = \frac{2}{3} \\
\beta = 0.9 \\
m_s = 0.1 \\
m_g = 0.01 \\
\sigma_s = 0.04 \\
\sigma_g = 0.02 \\
\gamma = 0.001 \\
\phi = 0.0001 \\
u(q) = q^{1/4} \\
\mu(b_s j_s, b_g j_g) = 0.05(b_s j_s)^{1/2} + 0.1581(b_g j_g)^{1/2} \\
S(j_t^s, z_{1,t}^s, z_{2,t}^s, j_t^g, z_t^g) = \\
\max\{\mu(b_s j_t^s, b_g j_t^g) - \mu[b_s j_t^s - b_s \sigma_s(z_{1,t}^s + \eta z_{2,t}^s), b_g j_t^g - \sigma_g b_g z_t^g], 0\}\
\]
Results – Single Silver Coin

- For large coins, welfare increases as $m_s$ increases
- Opposite true for small coins

![Graph showing the relationship between welfare and silver coin size for different values of $m_s$.]
Results – Single Silver Coin

- Welfare increases as trading opportunities increase

![Graph showing welfare increasing with silver coin size for different values of theta.](image)

- Theta = 1/3
- Theta = 1/10

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Results – Single Silver Coin

- Are model’s results consistent with historical changes in single silver coin size?
Single Coin: Silver or Gold?

- Ex ante welfare by silver coin size

![Graph showing the relationship between welfare and silver coin size with a peak at a certain size.]

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Single Coin: Silver or Gold?

- **Ex ante welfare by gold coin size**

![Graph showing welfare vs gold coin size]

- Max welfare at a specific gold coin size.
Aside Interesting finding:

- $\frac{m_s}{b_s} = \frac{m_g}{b_g}$ where $\hat{}$ denotes optimal

Intuition: Given preferences for output and jewelry and $\gamma$, there is some optimal amount of jewelry that (on average) agents want to trade for output

Aside: we might use the $q$ from different $m_s$ results as evidence for this intuition