Financial Frictions and Export Dynamics

David Kohn, Fernando Leibovici and Michal Szkup

*New York University*

Abstract

This paper studies the relationship between firm export dynamics and financial constraints in a small open economy model. Robust findings in the literature show that new exporters begin by exporting very small quantities, with most of them exiting the foreign market soon after. However, those that survive expand very rapidly and are much less likely to exit. As reported by Ruhl and Willis (2008a), standard trade models with heterogeneous firms cannot replicate these facts. We add borrowing constraints to an otherwise standard model and calibrate it using microdata on export dynamics. We find that financial constraints can largely account for the decreasing hazard rate and the increasing export volume of new exporters. We then provide empirical evidence supporting this mechanism and study its implications for understanding the effects of a large devaluation on aggregate exports.

Introduction

New exporters (i) start by exporting small volumes but grow very rapidly, conditional on continuing exporting, and (ii) face very low probability of continuing exporting, which increases rapidly over time. These facts on new exporter dynamics have been robustly documented in the literature by Eaton et al (2008), Eaton et al (2009), and Ruhl and Willis (2008), yet they remain
a puzzle through the lens of standard trade models. We argue that financial frictions faced by exporters can largely account for these facts.

Trade models with heterogeneous firms, in the spirit of Melitz (2003) and Eaton and Kortum (2002), have had great success in explaining the cross-sectional pattern of entry and exit, and its low frequency movements. However, their potential to explain the dynamics of new exporters is very limited. Ruhl and Willis (2008) extend Melitz (2003) to allow for stochastic idiosyncratic productivity shocks in the context of a small open economy and evaluate its implications for new exporter dynamics. They find that the model stands in stark contrast with the data: (i) export volumes are largely constant over time, and (ii) the probability of continuing exporting decreases over time.

In this paper, we argue that financial frictions faced by exporters can largely account for the stylized facts on new exporters dynamics. To do so, we study a small-open economy with heterogeneous firms subject to financial constraints. In our model, in order to produce, a firm has to pay up-front its wage bill, which can be financed via internal or external funds. In contrast to standard models, however, firms face a collateral constraint which limits the amount of funds they can obtain in any given period. This implies that firms with low internal funds have to operate below its unconstrained optimum level. This in turn affects the firm decisions along both the extensive and intensive margins. We calibrate this model to match key firm-level facts and find that financial frictions can largely explain the stylized facts on new exporter dynamics.

In our economy, new exporters enter the foreign market with low internal funds which, due to the financial constraint, prevents them from operating at their optimal scale. As they stay in the export market, they are able to
relax the financial constraint by accumulating internal funds, which in turn allows them to expand their production. This allows us to capture the first of the two stylized facts reported above. At the same time, given that new exporters hold less internal funds, their small scale makes exporting less profitable. Therefore, relatively small negative productivity shocks are enough to make it unprofitable and lead them to stop exporting. However, if they continue exporting, they are able to increase their scale by accumulating internal funds. This increases the profitability of exports, making them less likely to exit. This allows us to capture the second stylized fact.

The empirical evidence on the importance of financial frictions on exporters is ample. Minetti and Zhu (2010) use Italian data to show that firms that are credit rationed are less likely to export and, if they export, they are likely to export less. In a similar spirit, Bellone et al. (2010) reports a negative relationship between firms’ financial health and both its export status and export intensity. Finally, Suwantaradon (2008) and Wang (2010), using World Bank Enterprise Survey, find evidence of the importance of financial constraints for export decisions. Moreover, Wang (2010) reports that the probability of exporting and the export volume increase with age, which is consistent with the hypothesis that firms need to accumulate enough collateral before they can borrow enough funds to find it profitable to export.¹

Alternative explanations have been proposed to account for the stylized facts on new exporter dynamics. Eaton et al. (2009) point to the role of search frictions to explain the small and increasing export volumes upon entry. They

¹ See also Eggers and Kesina (2010), Berman and Hericourt (2010) and Muuls (2008). On the other hand Greenaway et al. (2007) find little evidence of importance of financial on firms’ decision to export.
argue that the low but increasing probability of continuing being an exporter arises from initial uncertainty about the idiosyncratic profitability of being an exporter. Arkolakis (2010) points to the role of marketing costs and customer capital to explain the small and increasing export volumes. In contrast to these alternative explanations, the nature of our mechanism allows us to use readily available firm-level data to discipline its implications, for instance, on domestic sales, exports, and cash flows.

This paper contributes also to a growing theoretical literature on the role of financial frictions for trade. The first papers to study the effects of financial constraints on export decisions were Chaney (2005) and Manova (2010). This paper differs from them in many dimensions. Most importantly, our model focuses on the dynamics of the new exporters while these papers focused solely on the entry decisions.

More closely related to ours are Wang (2010) and Suwantaradon (2008). Wang (2010) develops a model based on Cooley, Marimon and Quadrini (2004) and investigates how financial constraints impact firm’s exporting behavior and how this effect evolves over time. However, his model is unable to speak about new exporter dynamics since he abstracts from exit decisions by assuming that new exporters continue exporting forever. Suwantaradon (2008) introduces financial constraints into a Melitz (2003) model and explores their effects on aggregate export dynamics, but does not study the dynamics of new exporters. Moreover, in contrast to ours, these papers do not to evaluate the quantitative importance of financial frictions for explaining the export dynamics observed in the data.
1 Model

We consider a small-open economy, partial equilibrium model where both real wage and the demand schedule faced by firms are taken as exogenous. We abstract from the general equilibrium effects since our focus is on the decisions made by plants in response to productivity shocks and the way in which financial constraints affect these decisions.

We assume there is a unit mass of monopolistically competitive firms, each producing a differentiated good. A firm chooses how much to produce in the domestic market and whether to enter an export market. If it decides to exports it chooses also the volume of foreign sales. A firm that enters export market has to pay a fixed cost of exporting $F$ and a sunk cost $S$, while a firm that is a continuing exporter pays only the fixed cost. Both costs are measure in terms of labor units. The sunk cost is supposed to capture costs involved in finding a trading partner, setting up distributional channels and initial advertising to make consumers aware of the good.

We assume that each firm produces according to the following production function

\[ y_i = z_i L_i \quad i \in [0, 1] \]

where $L_i$ is the labor input and $z_i$ is firm’s idiosyncratic productivity. We assume that $\ln z_i$ follows a time invariant $AR(1)$ process

\[ \ln z_{i,t} = \rho \ln z_{i,t-1} + \varepsilon_{i,t} \]

where $\varepsilon_{i,t} \sim N(0, \sigma \varepsilon)$. 
Firms face exogenous domestic and foreign CES demand schedules:

\[ q^d_i = \kappa^d_i \left( \frac{p^d_i}{P^d} \right)^{-\sigma^d} Q^d \]
\[ q^e_i = \kappa^e_i \left( \frac{p^e_i}{P^e} \right)^{-\sigma^e} Q^e \]

where \( p^d_i \) is a price charged by a firm \( i \) in the domestic market, \( p^e_i \) is a price charged in the foreign market, \( P^d \) is the aggregate price level in home country, \( P^e \) is the aggregate price level in foreign country, \( Q^d \) is the aggregate demand in the domestic market, \( Q^e \) is the aggregate demand in the foreign market and finally \( \kappa^d_i \) and \( \kappa^e_i \) are domestic and foreign demand shocks. Both aggregate price level and aggregate demand are taken as exogenous. Both \( \ln \kappa^d_i \) and \( \ln \kappa^e_i \) are assumed to follow an \( AR(1) \) process

\[ \ln \kappa^d_{i,t,i} = \rho^d \ln \kappa^d_{i,t-1,i} + \nu^d_{i,t} \]
\[ \ln \kappa^e_{i,t,i} = \rho^e \ln \kappa^e_{i,t-1,i} + \nu^e_{i,t} \]

where \( \nu^d_{i,t} \sim N(0, \sigma^d) \) and \( \nu^e_{i,t} \sim N(0, \sigma^e) \).

We assume that firms take the real wage as given and they have to pay the wage bill at the beginning of period before the production occurs. Therefore, in order to hire labor, firms have to depend on internal and external finance.

Finally, we introduce financial frictions in the economy. More precisely, we require all firms to pay for labour and costs of exporting at the beginning of the period before the production takes places. Moreover, we assume that firms can only borrow up to a multiple \( \lambda \) of their assets. [to do: add intuition]

For now, we assume that each firm faces separate collateral constraints in the domestic and foreign markets.\(^2\)

\(^2\) This assumption is easy to relax.
We proceed by describing firm’s static and dynamic problems.

1.1 Static Problem

Each period a firm maximizes static profits which are the sum of the profits from sales in domestic and foreign markets (if the firm is an exporter). Note that due to constant returns to scale technology, we can separate firms static problem into domestic and foreign problem. The static problem of a firm with productivity $z$, assets $a$ and facing a demand shock $\kappa_d$ in the domestic market is:

$$
\pi_d(a, z, \kappa_d) = \max_{p_d} p_d q_d - \frac{w}{z} q_d \\
\text{s.t.} \\
q_d = z L_d \\
wL_d \leq \lambda_d a \\
q_d = \kappa_d \left( \frac{p_d^d}{P_d} \right)^{-\sigma_d} Q_d
$$

while firm’s static problem in the foreign market, given that a firm decided to export, is

$$
\pi_e(a, z, \kappa_e) = \max_{p_e} p_e y_e - \frac{w}{z} q_e \\
\text{s.t.} \\
q_e = z L_e \\
wL_e + w TC \leq \lambda_e a \\
q_e = \kappa_e \left( \frac{p_e^e}{P_e} \right)^{-\sigma_e} Q_e
$$

where

$$
TC = \begin{cases} 
F & \text{if a firm is a continuing exporter} \\
F + S & \text{if a firm is a new exporter}
\end{cases}
$$

Note that in order to produce in a foreign market, a firm has to cover costs
associated with exporting (fixed costs of being exporter and sunk cost in the case of new exporters).

1.2 Dynamic problem

Having described the static problem faced by a firm we move now to describe the dynamic problem. Let \( v^0 \) be the value function of a firm that did not export last period and is deciding whether to start exporting today, \( v^1 \) be the value function of a firm that participated in the foreign market last period and is deciding whether to keep producing for the foreign market and \( v^d \) be the value function for a firm that have chosen to produce only in the domestic market this period. The relevant individual state variables for an individual firm are: productivity level, \( z \), the amount of assets the firm has, \( a \), and the domestic and foreign demand shocks \( \kappa_d \) and \( \kappa_e \). The aggregate state variables are \( P_d, Q_d, P_e \) and \( Q_e \); the aggregate price levels and aggregate demands at home and abroad.\(^3\)

The value function for a firm that did not export last period is given by

\[
v^0 (a, z, \kappa_d, \kappa_e) = \max_{\{0,1\}} \left\{ v^d (a, z, \kappa_d, \kappa_e), \max_{a'} \frac{c^{1-\gamma}}{1-\gamma} + \beta E \left[ v^1 (a', z', \kappa'_d, \kappa'_e) \right] \right\}
\]

s.t
\[
c = (1 + r)a - a' + \pi^d (a, z, \kappa_d, \kappa_e) + \pi^e (a, z, \kappa_d, \kappa_e) - wF - wS
\]

where the expectation is taken over the future values of productivity and demand shocks.

\(^3\) To simplify the notation we will omit aggregate state variables when writing firm’s dynamic problem.
A firm that did not export last period decides at the beginning of a current period if it wants to keep producing only for the domestic market or become an exporter. In the former case it gets \( v^d(a, z, \kappa_d, \kappa_e) \), the value of producing only for home market. In the latter case a firm pays a fixed and sunk cost of exporting and earn profits both from home and foreign markets. It then decides how much to pay out in terms of dividend, \( c \), and how much assets to bring into the next period, \( a' \).

The value function for a firm that exported last period is given by

\[
v^1(a, z, \kappa_d, \kappa_e) = \max_{\{0, 1\}} \left\{ v^d(a, z, \kappa_d, \kappa_e), \max_{a'} \frac{c^{1-\gamma}}{1-\gamma} + \beta E \left[ v^1(a', z', \kappa'_d, \kappa'_e) \right] \right\}
\]

subject to

\[
c = (1 + r) a - a' + \pi^d(a, z, \kappa_d, \kappa_e) + \pi^e(a, z, \kappa_d, \kappa_e) - wF
\]

Again, the expectation is taken over future values of productivity and demand shocks. Note that the only difference between the continuing exporter and the potential starter is the fact that the former doesn’t have to pay a sunk cost \( wS \) if it decides to export this period.

Finally the value function for a firm that has already decided to produce only domestically is given by

\[
v^0(a, z, \kappa_d) = \max_{a'} \frac{c^{1-\gamma}}{1-\gamma} + \beta E \left[ v^0(a', z', \kappa'_d, \kappa'_e) \right]
\]

subject to

\[
c = (1 + r) a - a' + \pi^d(a, z, \kappa_d, \kappa_e)
\]

More precisely, a firm that produces only domestically, chooses how much assets to carry into the next period and how much dividends to pay out. Next period it will again face the decision whether to produce only domestically or
produce both in home and foreign markets.

2 Main Mechanism

We now describe the main mechanism of the model and explain how financial frictions allow us to capture qualitatively the stylized facts of the new exporters dynamics. In what follows we abstract from demand shocks introduced above by setting $\kappa_d = \kappa_f = 1$.

2.1 Frictionless Economy

In the absence of financial frictions the only variable determining export entry and exit decisions is firm’s productivity level. The structure of the problem implies that there is a productivity threshold level, $\bar{z}$, such that a firm with productivity $z \geq \bar{z}$ finds it profitable to export. On the other hand, the presence of sunk cost implies that there is another productivity threshold level $\underline{z}$ such that if $z < \underline{z}$ a firm finds it optimal to stop exporting.

![Figure 1](image_url)
Our frictionless economy implies that the export intensity and export volume of the new exporters adjust immediately to its optimal level and stays constant conditional on staying in the export market. Moreover, the model predicts that the hazard rate, that is the probability of exiting export market, is increasing over time.

These two finding are in stark contrast to the data. This failure of a standard models to replicate the stylized facts of new exporter dynamics have been reported first by Ruhl and Wills (2008). Our model without financial frictions is very similar to the setup considered in their paper and hence it is not surprising we reach the same conclusions.

2.2 Economy with Financial Frictions.

In the presence of financial constraints the amount of internal funds available to a firm becomes important. Recall that we assumed that each firm has to finance both its wage bill and export costs in advance and can borrow only up to a multiple $\lambda$ of its assets. Hence each firm faces a borrowing constraint. Moreover, we assumed that firms face separate borrowing constraints in domestic and foreign markets when raising funds to cover the costs of production and exporting.

First, we investigate how the financial constraints affect firms’ domestic production. Recall that the static problem in the domestic market is:
\[
\pi_d(a, z) = \max_{p_d} p_d q_d - \frac{w}{z} q_d
\]
\[
s.t.
\]
\[
q_d = z L_d
\]
\[
w L_d \leq \lambda_d a_d
\]
\[
q_d = \left( \frac{p_d^s}{P_d} \right)^{-\sigma_d} Q_d
\]

The above problem can be re-written as

\[
\pi_d(a, z) = \max_{p_d} p_d \left( \frac{P_d}{p_d} \right)^{-\sigma_d} Q_d - \frac{w}{z} \left( \frac{p_d}{P_d} \right)^{-\sigma_d} \kappa_d Q_d
\]
\[
s.t.
\]
\[
p_d \geq \left[ \frac{Q_d w}{\lambda_d a z} \right]^\frac{1}{\sigma_d} P_d
\]

Solving for the optimal price \( p_d^*(a, z, \kappa_d) \) we get

\[
p_d^*(a, z, \kappa_i) = \begin{cases} 
\frac{\sigma}{\sigma - 1} \frac{w}{z} & \text{if } p_d \geq \left[ \frac{Q_d w}{\lambda_d a z} \right]^\frac{1}{\sigma_d} P_d \\
\left[ \frac{Q_d w}{\lambda_d a z} \right]^\frac{1}{\sigma_d} P_d & \text{otherwise}
\end{cases}
\]

while the optimal quantity \( q_d^*(a, z, \kappa_d) \) is given by

\[
q_d^*(a, z, \kappa_i) = \begin{cases} 
\left( \frac{\sigma_d}{\sigma_d - 1} \frac{w}{z} \right)^{-\sigma_d} P_d^2 Q_d & \text{if } p_d \geq \left[ \frac{Q_d w}{\lambda_d a z} \right]^\frac{1}{\sigma_d} P_d \\
\frac{\lambda_d a z}{w} & \text{otherwise}
\end{cases}
\]

So we see that in the presence of financial frictions, a firm that is financially constrained is forced to produce less than it would in the absence of the borrowing constraints and it chooses to charge a higher price.

It follows that the profits \( \pi_d(a, z, \kappa_d) \) are given by
\[
\pi_d^*(a, z, \kappa_d) = \begin{cases}
\frac{1}{\sigma_d} \left( \frac{\sigma_d}{\sigma_d - 1} \right) \frac{w}{z} P_d^{\sigma_d} Q_d & \text{if } p_d > \left[ \frac{Q_d w}{\lambda_d a z} \right]^{\frac{1}{\sigma_d}} P_d \\
\lambda_d a \left[ \left( \frac{w}{z} \right)^{\frac{1}{\sigma_d} - \frac{1}{\sigma_d}} - \frac{1}{\lambda_d a} \right]^{\frac{1}{\sigma_d}} (Q_d)^{\frac{1}{\sigma_d}} P_d - 1 & \text{otherwise}
\end{cases}
\]

A financially constrained firm earns lower profits than an identical firm that is not constrained.

To sum up, we see that financial frictions affect the domestic production and prices by limiting the production of constraint firms and hence lead to lower profits in the domestic market for these firms.

Consider now a firm that is deciding whether to start exporting. In order to enter the foreign market this firm has to pay \( w (S + F) \) to cover both the sunk cost associated with entering a foreign market and a fixed cost of exporting. Firms with insufficient funds, i.e. firms with assets \( a \leq \frac{w(S + F)}{\lambda} \), are unable to pay these costs and hence are unable to enter the foreign market. Similarly a firm that exported last period but has assets lower than \( \frac{wF}{\lambda} \) is forced to leave a foreign market. Note, that these conditions on assets are independent of the productivity level of a given firm. This is in contrast with the frictionless economy where only productivity level determines whether a firm enters or exits the export market.

Suppose now that a firm has sufficient funds to pay the entry costs. We now derive expressions for quantity, price and profits for such firm taking its productivity level, \( z \) and assets level, \( a \) as given. Recall that the static problem faced by a firm in a foreign market is
\[ \pi_e(a, z) = \max_{p_e} \frac{p_e y_e - \frac{w}{z} q_e}{s.t.} \]

\[ q_e = zL_e \]

\[ wL_e + wTC \leq \lambda_e a \]

\[ q_e = \left( \frac{p^e}{P^e} \right)^{-\sigma_e} Q_e \]

Solving the above problem we obtain:

\[ q^*_e(a, z) = \begin{cases} 
\left( \frac{\sigma_e \frac{w}{z}}{\sigma_e - 1} \right)^{-\sigma_e} P^e Q_e & \text{if } p_e > \left[ \frac{Q_w w}{(\lambda_e a - wTC)z} \right]^{\frac{1}{\sigma_e}} P_e \\
\frac{(\lambda_e a - wTC)z}{w} & \text{otherwise}
\end{cases} \]

and

\[ p^*_e(a, z) = \begin{cases} 
\frac{\sigma_e \frac{w}{z}}{\sigma_e - 1} P^e & \text{if } p_e > \left[ \frac{Q_w w}{(\lambda_e a - wTC)z} \right]^{\frac{1}{\sigma_e}} P_e \\
\left[ \frac{Q_w w}{(\lambda_e a - wTC)z} \right]^{\frac{1}{\sigma_e}} P_e & \text{otherwise}
\end{cases} \]

Again, we see that financially constrained firms produce below their optimal level and are forced to charge a higher price than they would in the absence of the borrowing constraint. The profits from exporting are given by

\[ \pi^*_e(a, z) = \begin{cases} 
\frac{1}{\sigma_e} \left( \frac{\sigma_e \frac{w}{z}}{\sigma_e - 1} \right)^{1-\sigma_e} P^e Q_e & \text{if } p_e > \left[ \frac{Q_w w}{(\lambda_e a - wTC)z} \right]^{\frac{1}{\sigma_e}} P_e \\
(\lambda_e a - wTC) \left[ \frac{Q_w w}{w(\lambda_e a - wTC)z} \right]^{\frac{1}{\sigma_e}} P^e - 1 & \text{otherwise}
\end{cases} \]

We conclude again that financially constrained firms earn lower static profits in foreign market than identical firms that are not financially constrained. Moreover, a financially constrained firm exports less than it would in the absence of financial frictions. Note that holding everything else constant, the
quantity exported by a financially constrained firm is increasing in $a$. However, once a firm becomes financially unconstrained assets have no impact on export volume.

Figure 2

Similarly we see that the price charged by a financially constrained firm is decreasing in $a$ and if the firm is not financially constrained assets have no impact on the price charged by a firm. The same is true for the domestically charged prices.

Before analyzing decision to enter/exit foreign market it is important to explain carefully the way the financial constraints affect firms’ sales. Consider a firm with assets $a$ and productivity $z$ and, for simplicity, assume that it produces only in the domestic market. In the absence of financial frictions such firm would charge a price $p^o = \frac{w}{\sigma - \frac{1}{z}}$ and produce $q^o = \left(\frac{\sigma d}{\sigma d - 1} \frac{w}{z}\right)^{-\sigma d} P_d^o Q_d$ units of good. To do so it would require $L = \left(\frac{\sigma d s}{\sigma d - 1} \frac{w}{z}\right)^{-\sigma d} P_d^o Q_d$ units of labour.

Our timing assumption implies that in order to produce $q^o$ this firm has to
pay the wage bill equal to \( \left( \frac{\sigma_d w}{\sigma_d - 1 \frac{a}{z}} \right)^{-\sigma_d} P_d^\sigma Q_d w \) at the beginning of period before the production takes place. In the frictionless economy this doesn’t pose a problem to a firm. It simply borrows the required amount at the beginning of period and once the production took place it pays it back. In the economy with financial frictions this is not the case. The funds available to a firm at the beginning of the period are limited, namely, the firm can only pay up to \( \lambda a \) of expenditures at the beginning of the period. If firm’s productivity is relatively high and its assets are relatively low it cannot hire as much labor as it would otherwise and is forced to limit its production and charge higher prices. Since the firm can’t produce at its optimum the profits are lower.

What needs to be emphasized is the interaction between the assets and productivity in this simple framework. Namely, the more productive a firm is the more assets it needs in order to be unconstrained. Let \( \pi(z) \) be a level of assets such that a firm with productivity \( z \) and assets \( a > \pi(z) \) is financially unconstrained. The above discussion implies that \( \pi(z) \) is increasing in \( z \). This is easily verified analytically.

We now consider firm’s decision to enter export market. A firm will become exporter if and only if

\[
\frac{[(1 + r) a - a' + \pi^d(a, z) + \pi^e(a, z) - wF - wS]^{1-\gamma}}{1 - \gamma} + \beta E \left[ v^1(a', z') \right] \\
\geq \frac{[(1 + r) a - a' + \pi^d(a, z)]^{1-\gamma}}{1 - \gamma} + \beta E \left[ v^0(a', z') \right]
\]

that is, if the value of becoming exporter with assets \( a \) and productivity \( z \) is greater or equal to the value of producing only domestically. Similarly, a firm will find it optimal to continue exporting if
From above conditions we can see that financial constraints affect both decision problems through its effect on \( \pi^e (a, z, \kappa_d, \kappa_e) \). As it was emphasized above a firm that is financially constrained earns lower profits than it would otherwise and hence its incentives to enter/exit are distorted. More precisely, consider a firm that is deciding whether to enter or not the foreign market. Such firm cares only about the present discounted streams of dividends and hence it compares the value of becoming exporter to the value of producing only for domestic market. Exporting is beneficial since it increases profits but at the same time it is costly because of the presence of fixed and sunk costs. At the same time, profits stream from exporting depends both on the productivity of the firm and, as it was explained above, on the level of assets the firm has. A very productive firm with productivity \( z > \bar{z} \) (i.e. a firm that would have become exporter in the frictionless economy) may find it optimal not to enter if its assets are low because it would not be able to produce enough to make exporting profitable. However, holding assets fixed, the more productive the firm is, the more incentives it has to become exporter because it can produce larger quantity with the same assets even if it is constrained. Hence for each assets level \( a \) there exists productivity level \( \bar{z}(a) \) such that firm with assets \( a \) will only find it optimal to enter a foreign market if \( z > \bar{z}(a) \). This is shown
2.3 The role of financial frictions:

We now ask how financial constraints affect the productivity threshold for exporting. For simplicity we assume away sunk costs of export and keep only fixed costs of exporting. In this environment, a firm becomes exporter only if

$$\pi^e(a, z) \geq wF$$

We fix $a$ and solve for $z(a)$ such that a firm will find it optimal to become exporter if $z \geq z(a)$. Note that a firm is indifferent between exporting and non-exporting if

$$\pi^e(a, z) = wF$$

Denote by $z(a)$ a level of productivity for which the above equation holds, i.e.

$$\pi^e(a, z(a)) = wF$$
We now solve above equation for $z(a)$. Suppose first that a firm is financially unconstrained. Then:

$$
\pi^e(a, z) = \frac{1}{\sigma_e} \left( \frac{\sigma_e}{\sigma_e - 1} \frac{w}{z} \right)^{1-\sigma_e} P_e^{\sigma_e} Q_e
$$

So, the productivity threshold $z(a)$ is a solution to the equation

$$
\frac{1}{\sigma_e} \left( \frac{\sigma_e}{\sigma_e - 1} \frac{w}{z^u(a)} \right)^{1-\sigma_e} P_e^{\sigma_e} Q_e = wF
$$

$$
\frac{\sigma_e}{\sigma_e - 1} \frac{w}{z^u(a)} = \left[ \sigma_e \frac{wF}{P_e^{\sigma_e} Q_e} \right]^{\frac{1}{1-\sigma_e}}
$$

$$
z^u(a) = w \frac{\sigma_e}{\sigma_e - 1} \left[ \sigma_e \frac{wF}{P_e^{\sigma_e} Q_e} \right]^{\frac{1}{1-\sigma_e}}
$$

Hence we see that if a firm is financially unconstrained, the assets do not affect its decision to enter and the productivity threshold for becoming an exporter is the same as the productivity threshold in the frictionless economy.

Consider now a financially constrained firm with assets $a$. Such firm is indifferent between entering and not entering only if

$$
(\lambda_e a - wF) \left[ \frac{\pi^c(a)}{w} \left( \frac{Q_e w}{(\lambda_e a - wF) \pi^c(a)} \right) \right]^{\frac{1}{\sigma_e}} P_e - 1 = wF
$$

$$
\left[ \frac{\pi^c(a)}{w} \left( \frac{Q_e w}{(\lambda_e a - wF) \pi^c(a)} \right) \right]^{\frac{1}{\sigma_e}} P_e = \frac{wF}{(\lambda_e a - wF)}
$$

$$
z^c(a) = w \frac{\lambda_e a}{P_e (\lambda_e a - wF)} \left( \frac{Q_e w}{(\lambda_e a - wF)} \right)^{-\frac{1}{\sigma_e}}
$$

We can simplify the above equation to get:
\( \bar{z}^c(a)^{\frac{\sigma_e - 1}{\sigma_e}} = (\lambda_e a - wF)^{-\frac{\sigma_e - 1}{\sigma_e}} \left( \frac{1}{Q_e w} \right)^{\frac{1}{\sigma_e}} \frac{w \lambda_e a}{P_e} \)

or

\( \bar{z}^c(a) = \frac{1}{(\lambda_e a - wF)^{\frac{1}{\sigma_e - 1}}} \left( \frac{1}{Q_e w} \right)^{\frac{1}{\sigma_e - 1}} \left( \frac{w \lambda_e a}{P_e} \right)^{\frac{\sigma_e}{\sigma_e - 1}} \)

The elasticity of \( \bar{z}^c(a) \) with respect to \( a \) is equal to

\[
\frac{\partial \log \bar{z}^c(a)}{\partial \log a} = \frac{\sigma_e}{\sigma_e - 1} - \frac{\lambda_e}{(\lambda_e a - wF)}
\]

Hence, the higher the assets the lower the productivity threshold for exporting given that a firm is financially constrained.

It is also important to derive the productivity threshold for a firm to be financially constrained in a domestic and foreign markets. Recall that the more productive firm, the more output it wants to produce and hence the more assets it needs to finance the production. Let \( \bar{z}^d(a) \) be a productivity threshold such that if \( z > \bar{z}^d(a) \) then a firm is financially constrained at home and \( \bar{z}^e(a) \) be a productivity threshold such that if \( z > \bar{z}^e(a) \) then a firm is financially constrained abroad. For any given level of assets we can calculate this threshold using the expression for the optimal price. Recall that if a firm is financially constrained in the foreign market it charges a price

\[
p_d = \left[ \frac{Q_d w}{\lambda_d a z} \right]^{\frac{1}{\sigma_d}} P_d
\]
and if it is not financially constrained it charges a price

\[ p_d = \frac{\sigma_d w}{\sigma_d - 1} \]

Hence we can find \( z^d(a) \) by solving the following equation for \( z \):

\[
\frac{\sigma_d}{\sigma_d - 1} z^d(a) = \left[ \frac{Q_d w}{\lambda_d a z^d(a)} \right]^\frac{1}{\sigma_d} P_d
\]

\[
z^d(a)^{-1 + \frac{1}{\sigma_d}} = \frac{\sigma_d - 1}{\sigma_d} \frac{1}{w} \left[ \frac{Q_d w}{\lambda_d a} \right]^\frac{1}{\sigma_d} P_d
\]

\[
z^d(a) = \frac{\sigma_d - 1}{\sigma_d} \frac{1}{w} \left[ \frac{Q_d w}{\lambda_d a} \right]^\frac{1}{\sigma_d} P_d
\]

So,

\[
z^d(a) = \left( \frac{\sigma_d - 1}{\sigma_d} \right)^\frac{\sigma_d}{\sigma_d - 1} w \left[ \frac{Q_d}{\lambda_d a} \right]^\frac{1}{\sigma_d} P_d^\frac{\sigma_d}{\sigma_d - 1}
\]

or

\[
z^d(a) = w \left( \frac{\sigma_d}{\sigma_d - 1} \right)^\frac{\sigma_d}{\sigma_d - 1} \left( \frac{\lambda_d a}{Q_d P_d^\sigma_e} \right)^\frac{1}{\sigma_d - 1}
\]

Similarly,

\[
z^e(a) = w \left( \frac{\sigma_e}{\sigma_e - 1} \right)^\frac{\sigma_e}{\sigma_e - 1} \left( \frac{\lambda_e a - wF}{Q_e P_e^\sigma_e} \right)^\frac{1}{\sigma_e - 1}
\]

The elasticity of \( z^d(a) \) to a change in assets is given by

\[
\frac{\partial \log z^d(a)}{\partial \log a} = \frac{1}{\sigma_d - 1}
\]

while the elasticity of \( z^e(a) \) to a change in assets is given by

\[
\frac{\partial \log z^e(a)}{\partial \log a} = \frac{1}{\sigma_e - 1} \frac{\lambda_e a}{\lambda_e a - wF}
\]
We are now in a position to solve for the assets level such that if \( a \geq a^* \) financial constraints have no impact on the entry/exit decisions. From the above discussion, we know that financial constraint does not distort entry/exit decision if \( z^e(a) \geq z^c(a) \). Then \( a^* \) is simply defined as an asset level that solves the following equation:

\[
z^e(a^*) = z^c(a^*)
\]

Substituting for \( z^e(a) \) and \( z^c(a) \) yields:

\[
 w \left( \frac{\sigma_e}{\sigma_e - 1} \right) \left( \frac{\lambda_e a^* - wF}{Q e P_e \sigma_e} \right)^{1/\sigma_e - 1} = \frac{w}{\left( \frac{\lambda_e a^* - wF}{P e Q_e \sigma_e} \right)} \left( \frac{\sigma_e}{\sigma_e - 1} \right) \left( \frac{1}{Q e P_e \sigma_e} \right)^{1/\sigma_e - 1} (\lambda_e a^* - wF)^{1/\sigma_e - 1} = \left( \frac{1}{P e Q_e \sigma_e} \right)^{1/\sigma_e - 1} (\lambda_e a^*)^{1/\sigma_e - 1} \\
\left( \frac{\sigma_e}{\sigma_e - 1} \right) \left( \frac{\lambda_e a^* - wF}{Q e P_e \sigma_e} \right)^{1/\sigma_e - 1} = \left( \frac{\sigma_e}{\sigma_e - 1} \right) \left( \frac{\lambda_e a^* - wF}{Q e P_e \sigma_e} \right)^{1/\sigma_e - 1} = \left( \frac{\sigma_e}{\sigma_e - 1} \right) \left( \frac{\lambda_e a^* - wF}{Q e P_e \sigma_e} \right)^{1/\sigma_e - 1} = \left( \frac{\sigma_e}{\sigma_e - 1} \right) \left( \frac{\lambda_e a^* - wF}{Q e P_e \sigma_e} \right)^{1/\sigma_e - 1} = \lambda_e a^*
\]

Rearranging the last expression we obtain

\[
a^* = \frac{\sigma_e wF}{\lambda_e}
\]

Hence, when \( a \geq \frac{\sigma_e wF}{\lambda_e} \) then the firm’s assets do not affect the decision to enter/exit. The higher the fixed cost \( F \), wage rate \( w \) and elasticity of substitution in the foreign market \( \sigma_e \), the larger the impact of financial constraints on the entry and exit decision. However, it is important to remember that as long as \( z > z^e(a) \) financial constraint will still have impact on the intensive margin of export and will limit the export volume.

\[\text{Note that higher } \sigma_e, w \text{ or } F \text{ corresponds to lower static profits from foreign market.}\]
3 Numerical Estimations

We now parametrize the model so that it replicates the cross-sectional data and then we ask if our model can replicate the stylized facts concerning new exporter dynamics. The next subsection sketches an algorithm we use to solve the model numerically. Then we describe our parameterization and discuss the results of simulations. Finally, we compare the results of our model with and without financial frictions.

3.1 Algorithm

Below we list the steps of our algorithm:

1. Construct first-order Markov Chains to approximate idiosyncratic shock processes;
2. Construct grid over state space \( S = \{(a, z, \kappa_d, \kappa_f)\} \);
3. For every \( s \in S \), compute the solution to the firm’s static problem
4. Solve firm’s dynamic problem, given the solution to the firm’s static problem, using value function iteration
5. Simulate a panel of \( N \) firms over \( T \) periods.

3.2 Calibration

We now describe our baseline calibration. First we describe the calibration of the parameters affecting firms’ static problem. We set the multiplier in the domestic market borrowing constraint, \( \lambda_d \) equal to 5, and the multiplier in the foreign market borrowing constraint, \( \lambda_e \) equal to 1. We also normalize wage
$w$ to be 1. We assume that the foreign and domestic demand schedule are identical and set $P_d = P_e = 1$, $Q_d = Q_e = 50$ and $\sigma_d = \sigma_e = 5$. We assume away sunk cost by setting $S = 0$ and we set a fixed cost $F = 2.5$.

Now we describe the parameters of the firm’s dynamic problem and the shock processes. We set the discount rate, $\beta$ to be equal 0.85, risk aversion parameter, $\gamma$ equal to 2 and risk-free interest rate, $r$ equal to 0.02. Regarding the $AR(1)$ process for $\ln z$ we set the $\rho = 0$ and $\sigma_z = 0.08$, where $\sigma_z$ is the variance of the innovations in the process for $\ln z$.

All the parameters used for the baseline calibration for convenience are reported in the table below.

<table>
<thead>
<tr>
<th>Static Problem</th>
<th>Dynamic Problem</th>
<th>Grid for assets</th>
<th>Productivity shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$ = 1</td>
<td>$\beta$ = 0.85</td>
<td>$n_a$ = 100</td>
<td>$n = 70$</td>
</tr>
<tr>
<td>$\lambda_d$ = 5</td>
<td>$\gamma$ = 2</td>
<td>$r_u$ = 25.5</td>
<td>$c = 3$</td>
</tr>
<tr>
<td>$\lambda_f$ = 1</td>
<td>$r$ = 0.02</td>
<td>$a0$ = 0.4</td>
<td>$\sigma = 0.08$</td>
</tr>
<tr>
<td>$Q_d$ = 50</td>
<td></td>
<td></td>
<td>$\sigma_z = 0.08$</td>
</tr>
<tr>
<td>$P_d$ = 1</td>
<td></td>
<td></td>
<td>$\bar{c} = 1$</td>
</tr>
<tr>
<td>$Q_f$ = 50</td>
<td></td>
<td></td>
<td>$\rho = 0$</td>
</tr>
<tr>
<td>$P_f$ = 1</td>
<td>$F$ = 2.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S$ = 0</td>
<td>$\sigma_d$ = 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_f$ = 5</td>
<td>$\sigma_e$ = 5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1
3.3 Results

Following Ruhl and Willis we check overall performance of our model using five moments of the data: the average export intensity (average export sales ratio), the ratio of average employment of exporter to the average employment of the non-exporter (size premium), the fraction of firms that export (share of exporters), the fraction of firms that start exporting each year (starter rate) and the fraction of firms that stop exporting each year (stopper rate). Table 1 reports the moments from simulating our economy populated by 20000 firms for 200 periods with and without financial frictions.

<table>
<thead>
<tr>
<th></th>
<th>RW (2008) data</th>
<th>Constrained</th>
<th>Unconstrained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exports/Total sales</td>
<td>0.126</td>
<td>0.36</td>
<td>0.50</td>
</tr>
<tr>
<td>Size premium</td>
<td>2.79</td>
<td>1.73</td>
<td>2</td>
</tr>
<tr>
<td>Share of exporters</td>
<td>0.25</td>
<td>0.36</td>
<td>0.93</td>
</tr>
<tr>
<td>Share of starters</td>
<td>0.04</td>
<td>0.07</td>
<td>0.93</td>
</tr>
<tr>
<td>Share of stoppers</td>
<td>0.13</td>
<td>0.12</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Table 2

We see that the model without financial frictions does a poor job in matching the data. Introducing financial frictions improves the fit of the model drastically by bringing four out of the five studied moments closer to the values observed in the data. The only dimension in which the model with financial frictions does worse is the size premium.

We now check whether our model can replicate the new exporters dynamics. Figure 4 shows the hazard rate of exiting foreign market and the export to
total sales ratio. As reported by previous authors, the standard model ("unconstrained") can’t replicate these facts. It predicts that the hazard rate is virtually constant and same is true regarding export intensity. On the other hand, a model with financial frictions can replicate both facts. Firstly, the hazard rate is decreasing and with a sharp decrease taking place in the first couple of periods. Secondly, the ratio of export sales to total sales is increasing over time conditional on the firm staying in the export market.

Figure 5

The next figure shows the evolution of a the share of constrained exporters for a cohort that began exporting at time $t$ and the average assets held by the firms in this cohort. We see that the share of constrained firms in a cohort decreases over time while the average assets of the firms in the cohort keep raising. Note also that in the model with financial frictions firms hold on average higher assets that in the frictionless economy. This shouldn’t be surprising given that in the presence of borrowing constraints firms have to hold assets to finance their production, a saving motive that is absent in the economy without financial frictions.

Figure 5 shows the exit/entry policy function of the firms as a function of
assets, $a$ and productivity, $z$. The blue area denotes the pair $(z, a)$ such that firms find it optimal to export while the white area denotes the combinations of assets and productivity for which firms produce only domestically. Note that for low productivity levels firms never enter regardless of their asset position. The same is true for the low levels of assets. However as the productivity increases firms find it optimal to enter if they have sufficiently high level of assets. Importantly, even a very productive firm may find it unoptimal to enter (or be forced to exit) if its assets are low.

This figure also helps to explain why our model with the above parameterization can reproduce a decreasing hazard rate. In the simulations, firms that enter hold very few assets and hence even a small negative shock to their productivity forces them to stop exporting. However, those firms that are lucky and continue exporting accumulate assets and hence move away from the white area. The further away they are the lower the probability that a negative productivity shock will make them exit the foreign market. That is why our model implies a decreasing hazard rate.

Figure 6
3.4 Persistent Productivity Shocks

In the above simulations we assumed that the productivity is \textit{i.i.d}. We now allow the productivity to be persistent and set $\rho = 0.8$. It turns out that even in the absence of financial frictions the hazard rate is decreasing. However, financial frictions act as an amplification mechanism making exit rate higher for each period and implying steeper decrease of the exit rate in the first couple of periods.

4 Further Work

5 Conclusions

Acknowledgements

This is the Acknowledgements section. This section is placed at the end of the article, just before the references.

References


[16] Ruhl, K. and J. Willis, 2008a, "New Exporter Dynamics", NYU Stern working paper

