Figure 1 documents a strong negative relation in the United States between wealth (household net worth, from the Federal Reserve Flow of Funds, as a fraction of GDP) and aggregate volatility, measured as standard deviation of real GDP growth rate. Periods when net worth is high, reflecting high prices for housing and/or stocks, tend to be periods of low volatility in aggregate output, employment and consumption. Conversely, periods in which asset values are low tend to be periods of high macroeconomic volatility. The 1970s and the late 2000s were periods of low asset values and high volatility. The early 1960s and the Great Moderation of the 1980s and 1990s were periods of high asset values and low volatility.
Motivated by these facts, we develop a simple theoretical model that links asset values to the size of business cycle fluctuations. Our set-up is a micro-founded dynamic equilibrium model that contains elements of a traditional Keynesian framework in which economic fluctuations are driven by fluctuations in household optimism or pessimism. The novel feature is that the range of the fluctuations due to “animal spirits” (and hence the level of volatility) depends crucially on the value of wealth in the economy. When wealth is high ”animal spirits” do not cause fluctuations, the economy has a unique equilibrium and it behaves like a neoclassical one, and demand management is typically not effective as a stabilization policy. When wealth is low the economy is vulnerable to confidence shocks, high volatility is possible due to a multiplicity of equilibria and demand management is an effective policy instrument.

To fix ideas we first study a closed economy with fixed wealth. Competitive firms operate a linear technology that transforms labor into consumption good. Competition among firms insures that the wage is equal to the (constant) marginal product of labor. Households supply their labor inelastically, and hence are willing to work at any wage, but labor markets do not necessarily clear due to a search friction which does not allow any individual firm to attract unemployed workers by offering a low wage: this friction is important in our set-up as it generates equilibrium unemployment. A large family of infinitely lived agents in each period, given their wealth and their expectations about unemployment risk, place consumption orders. Firms then hire sufficient workers to satisfy those orders. After placing orders each agent employment opportunities are allocated randomly across agents. Unemployed agents must finance their consumption orders either through savings or through expensive external borrowing.

Our first result is to show that this environment allows for multiple equilibria in which households can collectively co-ordinate on a range of expectations about unemployment, each of which turns out to be self-fulfilling. In particular, the model typically has two steady states.

Steady states equilibria of the economy are described in figure 2. In the figure the straight negatively sloped line represents the resource constraint of the economy, showing that when unemployment $u$ is high, output is low and consumption $c$ must be low. The curves $S(a)$ and $S(a')$ represent the steady state Euler equations for consumers for two levels of wealth $a$ and $a'$ with $a > a'$. The curve is also negatively sloped as it reflects the fact that the higher the expected unemployment, the lower the consumption demand as consumption entails the additional borrowing cost paid by the unemployed members. Notice that when wealth is lower (the curve $S(a')$ for each level of unemployment consumers set lower consumption as with lower wealth the borrowing costs are larger.
In the optimistic steady state, households expect low unemployment, are not too concerned about credit constraints, and set consumption demand high. Facing high demand, firms employ a large fraction of workers, and the expectation of low unemployment is rationalized. This optimistic equilibrium is represented by point 1 in figure 2.

In the pessimistic steady state (point 2 in figure 2), households expect high unemployment. Because they do not want to commit to high consumption given high idiosyncratic unemployment risk and costly credit, they set consumption demand low. Facing low demand, firms hire few workers, and unemployment is in fact high, as expected.

In the dynamic version of the model it turns out that all points on the resource constraint between 1 and its intersection with the x axis (the shaded area in the figure) are possible (non steady state) sunspot equilibria. So even in absence of shocks this economy can display fluctuations.

A key feature of the model is that precautionary savings on the part of households offers a way to self-insure against unemployment risk. The less wealth a household has, the more reliant the household will be on costly credit in the event of unemployment. Thus the lower is household wealth, the more sensitive is consumption demand to the expected unemployment rate. This
increased sensitivity of demand to expectations increases the range of unemployment rates that can be supported in a rational expectations equilibrium. To see this consider the extreme case in which households have no wealth and they are maximally pessimistic (i.e. they expect 100% unemployment) they will set consumption to 0, and so 100% unemployment can be an equilibrium. But if households have positive wealth they will still order positive consumption even if they expect 100% unemployment, firms must then hire a positive fraction of workers to fill these orders and so 100% unemployment is not an equilibrium. In the figure observe that when wealth falls from \( a \) to \( a' \) the range of possible equilibria (all the points on the resource constraint between \( 1' \) and the \( x \) axis) is expanded.

This means that in times when the price of assets (and net worth) is low the economy is potentially subject to large fluctuations in economic activity driven by fluctuations in households “animal-spirits”, while in times when the price of asset is high, the economy becomes less sensitive to these sunspot-like shocks. Thus the model suggests an explanation for why wealth and volatility are so strongly positively correlated at the aggregate level.

To make things concrete, the great moderation was a time in which US house and stock prices were very high by historical standards relative to US GDP. We argue that high household wealth levels in this period meant that the economy was robust in the sense that it was not subject to large recessions induced by declines in confidence. However, the sharp declines in house and stock prices between mid 2007 and mid 2009 left the economy fragile, in the sense that demand became much more sensitive to expectations. Thus in this period a loss of consumer confidence - perhaps triggered by the initial fall in asset prices - had a much larger impact on household demand and ultimately output and employment. Our model does not offer a positive theory of asset price dynamics, but given the process for asset prices, agent’s savings decisions are optimal. Of course, fluctuations in consumer confidence are only one source of business cycles, and over a longer history economic cycles in the United States likely have a number of causes above and beyond fluctuations in animal spirits. However, we find a confidence-type shock quite appealing as one force underlying the Great Recession, in part because there is little evidence of a negative shock to labor productivity being operative over this period. We are not the first to argue for a link between asset values and volatility, but our mechanism reverses the usual direction of causation. Others (see Lettau et al. 2008) have pointed out that higher aggregate risk should drive up the risk premium on risky assets relative to safe assets. Lower prices for risky assets like housing and equity then just reflect higher expected future returns on these assets. In our model, asset prices are the primitive, and the level of asset prices determines the possible range of equilibrium output.
fluctuations, i.e. volatility. To take a step towards explaining fluctuations in asset prices, we extend the model to endogenize prices. In this version of the model asset prices reflect both the value of claims to capital income, and the liquidity value of being able to finance consumption out of savings in the event of unemployment. Because this liquidity value is tied to the level of unemployment, we find that asset prices themselves are indeterminate, and like the unemployment rate, fluctuate in response to changes in expectations. Interestingly, while greater pessimism raises unemployment, it does not necessarily reduce asset prices: the negative effect on asset valuation coming from lower dividends can be offset by the additional positive value of assets as a source of liquidity. The model has policy implications. It indicates that one way to reduce economic volatility is to undertake policies that increase asset values: recent large scale asset purchases by the Federal Reserve may be interpreted in this light. An alternative policy measure that would have similar effects is to increase the generosity of unemployment benefits. This would also serve to make unemployment less painful, and to thereby make demand less sensitive to the expected unemployment rate. A final policy implication is that, when wealth is low, expectation of a strongly counter-cyclical government spending policy (like the 2009 stimulus plan) would also make demand less sensitive to pessimistic expectations and thus would reduce economic volatility. This is consistent with the evidence presented by Corsetti et al. (2010) which suggests that the only time in which the fiscal multiplier is large is during episodes of financial crises, i.e. episodes in which wealth is depressed.

1 Model

There is a continuum of identical households. Each representative household has a continuum of members. There is a single consumption good, produced by a continuum of identical competitive firms using labor. The mass of individuals per household, the mass of households, and the mass of firms are all normalized to one. Thus we can envision a representative firm interacting with mass one members of a representative household. The consumption good is not storable, and its price is normalized to one in each period. There is a fixed supply of durable objects we call assets, whose mass we also normalize to one. The economy is closed.

In each period $t$ agents wake up, observe productivity $z_t$, the price of one unit of the asset relative to one unit of consumption, $p_t$, and (possibly) a sunspot $v_t$. The current period state $s_t$ is summarized by the triplet $(p_t, z_t, (v_t))$. Let $s^t$ be the history up to $t$. Given $s^t$ households form an expectation – which is validated ex post – about the current period unemployment rate $u(s^t)$. They also assign probabilities to future states $s_{t+j}, j \geq 1$. We assume that $s_t$ follows a first order Markov
process: thus $s_t$ contains sufficient information to forecast $s_{t+1}$. We assume that all households form the same expectations.

Given $s^t$, households send out members to buy consumption and to look for jobs. Employment opportunities will be randomly allocated across individuals. Thus, the optimal strategy is to send each member out with the same order $c(s^t)$ and an equal fraction $a(s^{t-1})$ of the assets the household carries in the period. The fraction $1 - u(s^t)$ of household members who find a job are paid a wage $w(s^t)$ and use wage income and asset holdings to clear their orders. The fraction $u(s^t)$ who are unemployed pay for as much of their consumption order $c(s^t)$ as they can given assets on hand, and borrow to pay the rest at a penalty rate. These penalties are rebated to the households as lump-sum transfers we denote by $T(s^t)$. At the end of the period the household reforms and pools resources, which determines the value of wealth carried into the next period $a(s^t)$.

Preferences for the household (exploiting the fact that each household member enjoys the same consumption level) are given by

$$
\sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t) u(c(s^t))
$$

The household budget constraint has the form:

$$
c(s^t) + p(s^t) (a(s^t) - a(s^{t-1})) \leq (1 - u(s^t)) w(s^t) - \frac{\psi}{2z} u(s^t) \min \left\{ \left( p(s^t) a(s^{t-1}) - c(s^t) \right), 0 \right\}^2 + T(s^t)
$$

The left hand side of the budget constraint captures consumption and the cost of net asset purchases. The first term on the right hand side if household earnings, while the second is the cost of penalties for unemployed workers who use credit to pay for consumption. Note that this cost is quadratic and only applies to the fraction $u(s^t)$ of workers who are unemployed, and only if the household set the consumption order above the value of wealth. Note that $a(s^{t-1})$ was effectively chosen in the previous period. In the current period, given aggregate variables $u(s^t)$, $w(s^t)$, $p(s^t)$ and $T(s^t)$, the choice for $c(s^t)$ implicitly defines the quantity of wealth carried into the next period $a(s^t)$. 
1.1 Household’s problem

Let $\mu(s^t)$ be the multiplier on the budget constraint at history $s^t$. The first order conditions with respect to $c(s^t)$ and $a(s^t)$ are:

\[
\beta^t u_c(s^t) + \mu(s^t) \left[ -1 + \frac{\psi}{z} u(s^t) \min \{(p(s^t)a(s^{t-1}) - c(s^t)), 0]\right] = 0 \tag{1}
\]

\[
-\mu(s^t)p(s^t) + \sum_{s^{t+1}} \pi(s^{t+1}|s^t)\mu(s^{t+1}) \left[ p(s^{t+1}) - \frac{\psi}{z} u(s^{t+1})p(s^{t+1}) \min \{(p(s^{t+1})a(s^t) - c(s^{t+1})), 0\} \right] = 0 \tag{2}
\]

From (1)

\[
\mu(s^t) = \frac{\beta^t u_c(s^t)}{1 - \frac{\psi}{z} u(s^t) \min \{(p(s^t)a(s^{t-1}) - c(s^t)), 0\}}
\]

Substituting into (2)

\[
\frac{u_c(s^t)}{1 - \frac{\psi}{z} u(s^t) \min \{(p(s^t)a(s^{t-1}) - c(s^t)), 0\}} p(s^t) = \sum_{s^{t+1}} \pi(s^{t+1}|s^t)\beta u_c(s^{t+1})p(s^{t+1})
\]

This looks like a standard inter-temporal first order condition for a consumption-savings problem, except the denominator of the left hand side indicates an additional motivation for saving: saving one additional unit of the asset is really cheaper than the price $p(s^t)$ because reducing current consumption reduces the expected penalty cost of borrowing.

1.2 Production and Labor Markets

Each representative firm produces according the following linear technology:

\[
y(s^t) = z(s^t)n(s^t)
\]

where $n(s^t)$ is the mass of workers employed by the representative firm. In equilibrium $u(s^t) = 1 - n(s^t)$. We now describe how equilibrium employment is determined.

Firms and workers meet in a decentralized labor market. Matching is random in the sense that the contact rate is independent of the wage offered (i.e. the labor market is not Walrasian, nor is it characterized by search directed). The matching technology is Leontief: the number of matches formed is the minumum of the number of individuals looking for work and the number of vacancies.

Households first observe productivity $z(s^t)$ and then give workers instructions about what wages to accept if they are matched with a firm, ie they specify a reservation wage. As long as the firm
anticipates being able to sell the output produced, it will agree to employ a worker if and only if the worker announces a reservation wage that is less than or equal to $z(s^t)$: if the worker demands a wage greater than $z(s^t)$, the firm will make a loss. Understanding the firms’ incentives, a representative household will optimally assign its members a reservation wage $w(s^t) = z(s^t)$. Note that each household is itself small, so the household has no power to influence any aggregate variables by choosing a different reservation wage: in particular, a lower reservation wage does not increase the probability that a given household member will find a job.

Firms take as given the wage $w(s^t)$ and the price at which they can sell output (normalized to one), and decides how many workers to hire. Each representative firm faces the same demand $c(s^t)$. At $w(s^t) = z(s^t)$, firms make zero profit on each worker, and are happy to employ sufficient workers to satisfy demand: $n(s^t) = c(s^t)/z(s^t)$. Hiring more than this number would entail a loss, because output in excess of demand $c(s^t)$ can neither be sold nor stored.\(^1\)

Thus in this environment demand $c(s^t)$ determines employment $n(s^t)$ and unemployment $u(s^t) = 1 - n(s^t)$. This is why, given knowledge of the production function and the representative household’s own orders $c(s^t)$, the household can perfectly forecast $u(s^t)$. Note also that if orders fall short of potential output, i.e., if $c(s^t) < z(s^t)$, then labor supply will exceed labor demand, in the sense that all measure 1 of workers per household are willing to work at wage $w(s^t) = z(s^t)$, while employment is determined by labor demand $n(s^t) = c(s^t)/z(s^t) < 1$.

### 1.3 Equilibrium

An equilibrium in this model is a process for $s_t = (p_t, z_t, v_t)$ and associated decision rules and prices $n(s_t), u(s_t), w(s_t), c(s_t), a(s_t), T(s_t)$ that satisfy, for all $t$ and for all $s_t$:

1. 

$$w(s_t) = z(s_t)$$

2. 

$$n(s_t) = 1 - u(s_t)$$

3. 

$$a(s_t) = 1$$

\(^1\)Note that charging a price different than one would also not be profitable: a higher price would mean no customers, while a lower price would mean making a loss on each unit sold.
4. 
\[ c(s_t) = z(s_t)(1 - u(s_t)) \]

5. 
\[ T(s_t) = \psi u(s_t) \min \{ (p(s^t) - c(s^t)), 0 \}^2 \]

6. 
\[ \frac{u_c(s_t)}{1 - \frac{\psi}{z} u(s_t) \min \{ (p(s_t) - c(s_t)), 0 \}} p(s^t) = \beta \sum_{s_{t+1}} \pi(s_{t+1}|s_t) u_c(s_{t+1}) p(s_{t+1}) \]

We will say the model has a unique equilibrium (multiple equilibria) if there is (is not) a unique set of values for \((n, u, w, c, a, T)\) for a given pair \(p\) and \(z\) and any value for \(v\).

1.4 Steady States

We begin by characterizing steady states for the model. A steady state is defined by a constant \((p, z, v)\) and an associated constant unemployment rate \(u\) that satisfies
\[ \frac{1}{1 - \psi u \min \{ \frac{p}{z} - (1 - u), 0 \}} = \beta \]

This is simply the steady state version of the household’s inter-temporal first order condition, using the resource constraint to substitute out consumption.

Note immediately that if \(\beta < 1\), then the model has no steady state for \(p \geq z\). Thus we will focus on the case \(p < z\). Then the set of steady states is defined by the solutions to the quadratic equation:
\[ \frac{1}{\beta} - 1 = -\psi u \frac{p}{z} + \psi u (1 - u) \]
or
\[ \psi u^2 + (\psi \frac{p}{z} - \psi)(1 - u) u + \left( \frac{1}{\beta} - 1 \right) = 0 \]

There are at most two interior solutions for \(u\), which we can solve for in closed form.

Let \(\rho = \frac{1}{\beta} - 1\). Then
\[ u^2 + \left( \frac{p - z}{z} \right) u + \frac{\rho}{\psi} = 0 \]

It is immediately clear that the steady state solutions for \(u\) depend only on the ratios \(p/z\) and \(\rho/\psi\). Let \(q = p/z\). Using the usual formula for quadratic equations, the solutions are
\[ u = \frac{1}{2} \left( (q - 1) \pm \sqrt{(q - 1)^2 - 4 \frac{\rho}{\psi}} \right) \]
We can also compute the range of values for $p/z$ such that there exists a steady state (solutions for $u$ above that are real and between 0 and 1).

Steady state demand is the value for $c_d$ that solves

$$\frac{1}{1 - \frac{\psi}{z} u (p - c_d)} = \beta$$

so

$$c_d = \rho \frac{z}{\psi u} + p$$

Thus steady state demand is positively related to $z$ and $\rho$ (more impatience $\rightarrow$ more demand), inversely related to $\psi$ and $u$, and linearly related to $p$.

Steady state supply is

$$c_s = z (1 - u)$$

We can now compute the maximum value for the price to output ration, $p/z$ for which a steady state exists (for higher values, demand will always exceed supply)

Excess demand is

$$c_d - c_s = \rho \frac{z}{\psi u} + p - z (1 - u)$$

Thus function is convex in $u$ (the second derivative with respect to $u$ is $2 \rho \frac{z}{\psi} u^{-3}$). Thus excess demand is minimized when the first derivative is zero.

This is at

$$\frac{\rho}{\psi} u^{-2} = z$$

$$u = \sqrt{\frac{\rho}{\psi}}$$

Thus the maximum value for $\frac{p}{z}$ at which there exists a steady state is

$$\frac{p}{z} = \left(1 - \sqrt{\frac{\rho}{\psi}}\right) - \frac{\rho}{\psi \sqrt{\frac{p}{\psi}}}$$

$$= 1 - 2 \sqrt{\frac{\rho}{\psi}}$$

1.5 Non-stochastic Dynamics

Next, we look at dynamics, assuming constant $p$ and $z$, for arbitrary initial $u$, and characterize the set of values for initial $u$ for which there exist equilibria.
Dynamics will depend on the form of the utility function. Suppose

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

Then the dynamics of the system are given by

$$\frac{c_t^{\gamma}}{1 - \frac{\psi}{z} u_t (p - c_t)} = \beta c_{t+1}^{\gamma}$$

$$c_t = z(1 - u_t)$$

Using the second equation to substitute out for $c_t$ gives

$$\frac{(1 - u_t)^{-\gamma}}{1 - \psi u_t \left( \frac{p}{z} - (1 - u_t) \right)} = \beta (1 - u_{t+1})^{-\gamma}$$

Let $q = p/z$

$$u' = 1 - \left( \frac{(1 + \rho) (1 - u)^{-\gamma}}{1 - \psi u (q - (1 - u))} \right)^{-\frac{1}{\gamma}}$$

Suppose we pick an initial $u_0$. We can then compute directly the implied dynamics for $u_t$ and $c_t$

Suppose we set

$\gamma = 2$

$\rho = 0.1$

$\psi = 1$

The maximum value for $q = p/z$ such that a steady state exists is given by $y = 1 - 2\sqrt{\frac{p}{\psi}} = 0.36754$

We need to pick a value for $q$ below this threshold.

$q = 0.3$

For this calibration the two steady state unemployment rates are

$$u_L = \frac{1}{2} \left( -(q - 1) - \sqrt{(q - 1)^2 - 4\frac{\rho}{\psi}} \right) = 0.2$$

$$u_H = \frac{1}{2} \left( -(q - 1) + \sqrt{(q - 1)^2 - 4\frac{\rho}{\psi}} \right) = 0.5$$

Figure 3 shows steady state demand and supply as a function of the steady state unemployment rate for this calibration.
The maximum value for $\theta = \pi = \zeta$ such that a steady state exists is given by $\varphi = 1 - 2q = 0.367$.

We need to pick a value for $\theta$ below this threshold.

For this calibration the two steady state unemployment rates are:

$\nu^\# = \frac{1}{2}\mu - (\theta - 1) - r(\theta - 1)^2 - 4q = 0.2$

$\nu^\# = \frac{1}{2}\mu - (\theta - 1) + r(\theta - 1)^2 - 4q = 0.5$

This picture shows steady state supply and demand as a function of the steady state unemployment rate for this calibration.

This picture indicates that, for this calibration, the low unemployment steady state is a sink: for any initial $\nu_0 \in (0, 0.5)$ the unemployment rate will converge to $\nu^\# = 0.2$. For $\nu_0 = 0.5$ the unemployment rate will stay at $0.5$. For $\nu_0 > 0.5 \nu \to 1$. However, these paths do not constitute equilibria, since along these paths $\chi \to 0$. But it can never be optimal for the household to reduce consumption to zero when it holds positive wealth. In fact these paths for $\chi$ violate the transversality condition, and thus they are not optimal.

2.6 Stochastic Dynamics

Suppose now that households perceive that $\nu^\#$ follows a stochastic process, driven by sunspot fluctuations in $\varphi^\#$. The equation defining the law of motion for $\nu^\#$ is:

$$(1 - \nu^\#) - \theta \nu^\#(\theta - (1 - \nu^\#)) = \eta^\#(1 - \nu^\# + 1) - \varphi^\#

Now suppose $\nu^\# + 1 = \eta^\#[\nu^\#] + 1 \sim \eta(0, \theta)$

So $\eta^\#(1 - \nu^\# + 1) - \varphi^\# = \eta^\#(1 - \eta^\#[\nu^\#] + 1) - \varphi^\#$

The variable $\eta^\#$

Suppose we linearize the equation defining the equilibrium law of motion for unemployment:

Figure 3: Steady state supply and demand

Figure 4: Unemployment dynamics

Figure 4 shows the change in the unemployment rate for different initial unemployment rates.

This picture indicates that, for this calibration, the low unemployment steady state is a sink:
for any initial $u_0 \in (0,0.5)$ the unemployment rate will converge to $u_L = 0.2$. For $u_0 = 0.5$, the unemployment rate will stay at 0.5. For $u_0 > 0.5$, $u_t \to 1$. However, these paths do not constitute equilibria, since along these paths $c_t \to 0$. But it can never be optimal for the household to reduce consumption to zero when it holds positive wealth. In fact these paths for $c_t$ violate the transversality condition, and thus they are not optimal.

References


