House Prices Booms and Current Account Deficits∗

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Abstract

One of the most striking features of the period before the Great Recession of 2007-2009 is the strong positive correlation between house price appreciation and current account deficits in countries that have subsequently experienced the highest degree of financial turmoil. A progressive relaxation of credit constraints can rationalize this empirical observation. Lower collateral requirements facilitate access to external funding and drive up house prices. Households increase their leverage borrowing from the rest of the world so that the current account turns negative. Several pieces of evidence support this view. The paper further compares this mechanism with the role of monetary policy, the exchange rate regime and foreign saving shocks in accounting for the evidence.

∗Thanks to Daniel Herbst for excellent research assistance. The views expressed in this paper do not necessarily reflect the position of the Federal Reserve Bank of New York or of the Federal Reserve System.
1 Introduction

“...[C]ountries in which current accounts worsened...had greater house price appreciation over this period [2001Q4-2006Q3]. ... This simple relationship requires more interpretation before any strong conclusions about causality can be drawn...”

Speech by Chairman Ben S. Bernanke
Annual Meeting of the American Economic Association
Atlanta, GA – January 3, 2010

During the five years before the eruption of the recent financial crisis, two of the most discussed indicators of U.S. imbalances were the soaring house prices and the widening current account deficits. Figure 1 shows that the correlation between house price dynamics and current account balances is in fact a robust global phenomenon affecting both advanced and emerging market economies.1 Interestingly, countries like Iceland, Ireland and Spain that witnessed house prices booms and substantial external deficits experienced among the

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1Bernanke (2010) plots the cumulative change between 2001Q4 and 2006Q4 in current account balances and house prices for advanced economies. The August 2007 ECB Monthly Bulletin features a similar figure for the period 1997-2005. Figure 1 extends the sample to include emerging market economies such as China, which play a key role in financing the U.S. current account deficit.
highest degrees of financial turmoil during the crisis.\(^2\)

This paper argues that a progressive relaxation of borrowing constraints can generate a strong negative correlation between house prices and current account. Lower collateral requirements facilitate access to external funding and drive up house prices. Households increase their leverage borrowing from the rest of the world so that the current account turns negative.

The analysis relies on a two-country model with tradable consumption goods and housing.\(^3\) The expected value of housing represents the collateral for households' debt. This endogenous borrowing constraint is buffeted by a time-varying parameter which constitutes the key shock in the model. This parameter controls the loan-to-value threshold. An increase in this threshold, for given value of the collateral, leads households to lever up and demand more housing, hence driving up house prices. To the extent that the relaxation of credit constraints affects the whole economy, the increase in domestic borrowing must be financed from abroad, thus generating a current account deficit.

Few empirical studies discuss the relation between house prices and external imbalances. Ahearne et al. (2005) document the co-movement between house price dynamics and current account balance since 1970. Aizenman and Jinjarak (2009) provide a precise estimate of the relation between the two variables: a one standard deviation increase in lagged current account deficits is associated with a 10% appreciation of real estate prices.\(^4\) Fratzscher, Juvenal and Sarno (2010) adopt the opposite perspective: according to their estimates, together with equity market shocks, house price shocks account for up to 32% of the movements in the U.S. trade balance over a 20-quarter horizon.

Three recent papers investigate the connection between house prices and the current account balance in quantitative general equilibrium frameworks. The mechanism that generates the negative correlation between house prices and current account balance in this work is similar to one of the shocks in Punzi (2006), who considers a richer model with residential investment and preference heterogeneity also within countries. Shocks to the loan-to-value ratio provide a rationale for increased housing demand, which is the more basic type of shocks investigated in Gete (2010).\(^5\) These demand-driven explanations contrast with the idea that differentials in expected output growth lead to house price booms and current account deficits, as in Kole and Martin (2009).

In a closed economy setting, Favilukis, Ludvigson and Van Nieuwerburgh (2011) develop a rich two-sector model with heterogenous households and idiosyncratic risk. Together with the reduction in transaction costs for housing and business cycle factors, the relaxation of borrowing constraints accounts for the observed increase in the observed price-to-rent ratio. Iacoviello and Neri (2010) estimate a DSGE model with housing and find that slow

\(^2\)Similar dynamics for capital inflows and real estate prices occurred before the Asian crisis in the late 1990s (see Obstfeld and Rogoff (2010) and the references therein).
\(^3\)The model essentially interprets the non-tradable sector in Ferrero, Gertler and Svensson (2010) as housing.
\(^4\)Kole and Martin (2009) find similar results.
\(^5\)Midrigan and Philippon (2010) use credit constraint shocks in an island economy to match the distribution of house prices across U.S. counties.
technological progress in the housing sector explain the long run upward trend in U.S. house prices. Housing preference and technology shocks account for about 50% of the variance of housing investment and prices at business cycle frequencies.

Starting with Kiyotaki and Moore (1997), several authors have highlighted the role of endogenous borrowing constraint as source of amplification of other types of shocks. Monacelli (2009) shows that borrowing constraints can reconcile the empirical evidence on the response of durable and non-durable spending to monetary shocks with the dynamics of an otherwise standard New Keynesian model. In a small open economy, Iacoviello and Minetti (2003) relate the strength of the impact of monetary policy shocks on house prices to the degree of financial liberalization. Recently, Eggertsson and Krugman (2010) argue that a tightening of borrowing constraints (relative to pre-crisis levels) can lead to a substantial drop in aggregate demand and potentially create depression-like scenarios.

The view that reductions in collateral requirements have generated increases in house prices and consequently external imbalances is obviously not uncontroversial. Glaeser, Gottlieb and Gyourko (2010) find no evidence that changes in approval rates or loan-to-value levels explain a large part of the increase in house price in the U.S. between 1996 and 2006. By their own admission, however, their empirical estimates suffer from an endogeneity problem and the quality of the data in the empirical analysis may also be questionable.

The paper presents several pieces of evidence supporting the role financial liberalizations in generating the increase in house prices. Not surprisingly, several alternative explanations exist. Taylor (2008) argues that loose monetary policy both in the U.S. and abroad is at the heart of the problem. Developments in mortgage markets and in the securitization process only contributed to worsen the problem. Extending the model to include nominal rigidities and a role for active monetary policy allows for a quantitative evaluation of the relative importance of this explanation vis-a-vis the role of shocks to the loan-to-value ratio. Importantly, this extension allows for studying the role of foreign exchange rate pegs to the U.S. dollar – a popular rationale for the recent U.S. current account deficits which has not been much discussed in relation to the house price boom.

The “saving glut” hypothesis (Bernanke, 2005) is another potential explanation of the negative correlation between house prices and current account balances. Building on their earlier work, Caballero, Fahri and Gourinchas (2008) argue that global demand for liquid assets generated capital flows from the rest of the world toward the U.S. where asset prices, and hence house prices too, took off. The paper evaluates the quantitative appeal of this competing story by including preference shocks that make the rest of the world more patient relative to the U.S.

The rest of the paper proceeds as follows. The next section provides some evidence on

\footnote{Sterk (2010), however, argues that this result depends crucially on assuming nominal rigidities in the durable sector.}

\footnote{The “Bretton Woods II” story (see for instance Dooley, Folkerts-Landau and Garber, 2008) focuses on the interplay between managed exchange rate regimes in Asian countries and U.S. current account deficits while abstracting to a large extent from house price dynamics.}
2 Evidence on the Relaxation of Collateral Constraints

The key shock that generates a house price boom and a contemporaneous current account deficit in the model below is a reduction in the parameter that measures the loan-to-value requirement. At a broad level, lower collateral requirements capture easier access to credit for housing finance. In this sense, the growth in subprime lending is a first piece of evidence about the relaxation of credit constraints at the household level.

Nichols, Pennington-Cross and Yezer (2005) note that the share of subprime lending in the U.S. mortgage market grew from 0.74% to almost 9% during the 1990s. After slowing down in 2001 (7%) and 2002 (1%), subprime origination soared from 8% in 2002 to 20% in 2005 and 2006. In the same period, Alt-A mortgages and home equity loans also gained popularity. Table 1 shows that by 2006 the higher risk segment accounted for 48% of securitized origination (equivalent to 34% of the dollar volume).8

Table 2 presents more direct evidence on the evolution of loan-to-value ratios, broken down by type of mortgages (Prime, Alt A and Subprime) and rate (Fixed and Adjustable). For the prime segment, the average loan-to-value ratio increased by 10 percentage points between 2002 and 2006, for both the fixed- and adjustable-rate types. In both categories, the fraction of mortgages with loan-to-value ratios higher than 80% increased from less than 5% in 2002 to about 25% in 2006. Alt A fixed-rate mortgages featured a similar increase,

<table>
<thead>
<tr>
<th>Year</th>
<th>FHA/VA</th>
<th>Conv/Conf</th>
<th>Jumbo</th>
<th>Subprime</th>
<th>Alt A</th>
<th>HEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>8</td>
<td>57</td>
<td>20</td>
<td>7</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>2002</td>
<td>7</td>
<td>63</td>
<td>21</td>
<td>1</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>2003</td>
<td>6</td>
<td>62</td>
<td>16</td>
<td>8</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>2004</td>
<td>4</td>
<td>41</td>
<td>17</td>
<td>18</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>2005</td>
<td>3</td>
<td>35</td>
<td>18</td>
<td>20</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>2006</td>
<td>3</td>
<td>33</td>
<td>16</td>
<td>20</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>2007</td>
<td>4</td>
<td>48</td>
<td>14</td>
<td>8</td>
<td>11</td>
<td>15</td>
</tr>
</tbody>
</table>


The third section presents the general model and then focuses on the special case without nominal rigidities. The fourth section discusses the calibration and the basic quantitative experiment. The fifth section focus on nominal rigidities and addresses the quantitative importance of alternative explanations. Finally, the sixth section concludes.

8Mian and Sufi (2010) show that home equity loans are responsible for a significant fraction of the increase in U.S. household leverage between 2002 and 2006.
Table 2: Evolution of loan-to-value ratios (in %). Source: Abraham, Pavlov and Wachter (2008). CLTV stands for combined (i.e. first and second mortgage) loan-to-value ratio. CTLV > 80% refers to the fraction of combined loan-to-value ratios larger than 80%.

<table>
<thead>
<tr>
<th>Year</th>
<th>Fixed-Rate</th>
<th>Adjustable-Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CLTV</td>
<td>CLTV &gt; 80%</td>
</tr>
<tr>
<td>2002</td>
<td>65.4</td>
<td>3.0</td>
</tr>
<tr>
<td>2003</td>
<td>63.8</td>
<td>4.4</td>
</tr>
<tr>
<td>Prime 2004</td>
<td>67.4</td>
<td>7.0</td>
</tr>
<tr>
<td>2005</td>
<td>70.9</td>
<td>13.4</td>
</tr>
<tr>
<td>2006</td>
<td>74.5</td>
<td>23.1</td>
</tr>
<tr>
<td>2002</td>
<td>74.7</td>
<td>22.0</td>
</tr>
<tr>
<td>2003</td>
<td>71.5</td>
<td>21.4</td>
</tr>
<tr>
<td>Alt A 2004</td>
<td>75.3</td>
<td>29.5</td>
</tr>
<tr>
<td>2005</td>
<td>76.2</td>
<td>31.3</td>
</tr>
<tr>
<td>2006</td>
<td>79.4</td>
<td>39.6</td>
</tr>
<tr>
<td>2002</td>
<td>77.3</td>
<td>38.0</td>
</tr>
<tr>
<td>2003</td>
<td>78.0</td>
<td>41.7</td>
</tr>
<tr>
<td>Subprime 2004</td>
<td>77.7</td>
<td>41.2</td>
</tr>
<tr>
<td>2005</td>
<td>78.7</td>
<td>44.5</td>
</tr>
<tr>
<td>2006</td>
<td>78.7</td>
<td>44.6</td>
</tr>
</tbody>
</table>

with loan-to-value ratios higher than 80% roughly doubling between 2002 and 2006. Over the same period, the increase in loan-to-value ratios for Alt A adjustable-rate mortgages was smaller (of the order of 5 percentage points) but loan-to-value ratios higher than 80% more than doubled, reaching 55% in 2006. For the subprime segment the increase in loan-to-value ratio was also of the order of 5 percentage points and Subprime loan-to-value ratios higher than 80% soared too, especially for the adjustable-rate type.

The bottom line is that financial innovation, supported by the securitization process, provided greater access to mortgage finance for a broader pool of households, both at the extensive (higher share of subprime mortgages) and intensive (lower loan-to-value requirements) margin.9

2.1 International Evidence

Direct evidence on the relaxation of households’ borrowing constraints for countries other than the U.S. is much more scattered. The European Mortgage Federation provides some information on housing finance in Europe although data on loan-to-value ratios are generally not available. One notable exception is Iceland, where loan-to-value ratios increased from

9 Although not explicitly modeled here, the reduction in housing transaction costs provides further evidence in support of the process of liberalization in real estate financing. See Favilukis, Ludvigson and Van Nieuwerburgh (2011) for details.
Table 3: Residential mortgage debt in % of GDP. Source: European Mortgage Federation Hypostat, 2008.

<table>
<thead>
<tr>
<th></th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>21.7</td>
<td>22.6</td>
<td>24.2</td>
<td>26.0</td>
<td>29.3</td>
<td>32.2</td>
</tr>
<tr>
<td>Germany</td>
<td>53.1</td>
<td>53.2</td>
<td>53.5</td>
<td>52.4</td>
<td>51.9</td>
<td>51.3</td>
</tr>
<tr>
<td>Greece</td>
<td>11.8</td>
<td>14.8</td>
<td>17.2</td>
<td>20.2</td>
<td>25.1</td>
<td>29.3</td>
</tr>
<tr>
<td>Iceland</td>
<td>59.4</td>
<td>61.1</td>
<td>66.2</td>
<td>71.0</td>
<td>80.8</td>
<td>75.5</td>
</tr>
<tr>
<td>Ireland</td>
<td>32.8</td>
<td>36.3</td>
<td>42.7</td>
<td>52.2</td>
<td>61.4</td>
<td>70.1</td>
</tr>
<tr>
<td>Spain</td>
<td>32.5</td>
<td>35.9</td>
<td>40.0</td>
<td>45.7</td>
<td>52.3</td>
<td>58.1</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>58.9</td>
<td>63.9</td>
<td>69.3</td>
<td>74.1</td>
<td>78.4</td>
<td>83.1</td>
</tr>
<tr>
<td>United States</td>
<td>60.5</td>
<td>66.1</td>
<td>71.1</td>
<td>76.1</td>
<td>81.1</td>
<td>84.8</td>
</tr>
</tbody>
</table>

65% to 90% in 2003 before going back to 80% in 2006. Interestingly, and perhaps not surprisingly, Iceland experienced a 60% increased in real house prices between 2001 and 2006, together with one of the largest deteriorations of the current account (more than 20%) among Western economies.

More indirect evidence, however, points in the direction of a large boom in housing finance in several European countries. Table 3 reports residential mortgage debt as fraction of GDP for a selected group of countries over the period 2001-2006. Iceland, the U.S. and the U.K. featured a similar pattern with mortgage debt growing from about 60 to about 80% of GDP or more. Countries like Spain and Ireland started from lower levels (approximately 30%) but roughly doubled their shares. Mortgage finance in Greece accounted for a small fraction of GDP (12%) in 2001 but reached about 30%, close to the level of France, where mortgage finance increased a more moderate 10% over the sample period.

All these examples of significant growth in mortgage debt contrast with the case of Germany, where the share of GDP remained roughly constant just above 50%. The increase in mortgage finance relative to GDP was small also in Japan, from 25% in 1990 to 36% in 2006 (IMF World Economic Outlook, 2008). Finally, while on the uprise from essentially zero in 1998, mortgage debt was still a small 10% GDP in China as of 2004 (Jain-Chandra and Chamon, 2010).

To summarize, credit market liberalizations have greatly stimulated housing finance. The evidence is quite clear for the U.S. and is at least suggestive for several other countries that have experienced contemporaneous house prices booms and current account deficits. Conversely, countries where the process of financial innovation has been less abrupt have experienced a much lower degree of house price appreciation and often current account surpluses. The next section develops a model in which the relaxation of borrowing constraints plays a key role to account for these facts.
3 An Open Economy Model with Borrowing Constraints

Time is discrete and indexed by $t$. The world consists of two countries, Home and Foreign, of equal size. In each country, a continuum of measure one of firms produce a final tradable good using a labor aggregate as the only factor of production. The representative household in each country comprises a continuum of measure one of workers who supply differentiated labor inputs and consume a composite of the tradable goods produced in each country as well as housing services, which are assumed to be proportional to the fixed housing stock. The value of housing represents the collateral needed to obtain private credit. Goods and labor markets are imperfectly competitive. Goods prices and wages are set on a staggered basis.

Household’s Preferences and Constraints

This section presents the household objective and constraints from the perspective of the Home country. An asterisk * denotes foreign variables when relevant.

The representative household maximizes

$$U_t = E_t \left\{ \sum_{s=0}^{\infty} \beta^s e^{\varsigma_t} \left[ \frac{X_t^{1-\sigma}}{1-\sigma} - \frac{1}{1+\nu} \int_0^1 L_t(i)^{1+\nu} di \right] \right\}.$$  \hspace{1cm} (1)

Per-period utility depends positively on the consumption index $X_t$ and negatively on hours worked by each member of the representative household $L_t(i)$. The parameter $\sigma > 0$ is the coefficient of relative risk aversion while $\nu > 0$ is the inverse Frisch elasticity of supply of a specific labor input. The intertemporal preference shock $\varsigma_t$ follows an AR(1) process with persistence $\rho_{\varsigma} \in (0, 1)$ and standard deviation $\sigma_{\varsigma}$.

The index $X_t$ combines consumption of goods $C_t$ and housing services $H_t$ with constant elasticity of substitution $\varepsilon > 0$

$$X_t \equiv \left[ \bar{\eta} C_t^{\varepsilon-1} + (1 - \bar{\eta}) H_t^{\varepsilon-1} \right]^{\frac{1}{\varepsilon-1}},$$  \hspace{1cm} (2)

where $\bar{\eta} \in (0, 1)$ represents the share of goods in total consumption.

The tradable bundle $C_t$ combines consumption of goods produced in the Home ($C_{ht}$) and Foreign ($C_{ft}$) country with constant elasticity of substitution $\gamma > 0$

$$C_t \equiv \left[ \alpha \frac{1}{\gamma} C_{ht}^{\gamma-1} + (1 - \alpha) \frac{1}{\gamma} C_{ft}^{\gamma-1} \right]^{\frac{1}{\gamma-1}},$$  \hspace{1cm} (3)

where $\alpha \in [0, 1)$ is the share of domestic tradable goods.\(^{10}\)

\(^{10}\)If $\alpha > 0.5$, preferences for tradable goods exhibit home bias. The Foreign tradable bundle places a weight $\alpha$ on consumption of Foreign tradable goods.
The budget constraint for the representative household is

\[ P_{ht}C_{ht} + P_{ft}C_{ft} + Q_tH_t - B_t \leq \int_0^1 W_t(i) L_t(i) di + \mathcal{P}_t + Q_tH_{t-1} + T_t - (1 + i_t-1)B_{t-1}, \]  

(4)

where \( P_j \) is the Home price of good \( j = \{h, f\} \), \( Q_t \) is the price of housing, \( W_t(i) \) is the nominal wage for the specific labor input supplied by the \( i^{th} \) household member, \( \mathcal{P}_t \) are profits from ownership of the firms, \( T_t \) are lump-sum transfers and \( i_t \) is the net nominal interest rate on an internationally-traded one-period risk-free debt instrument \( B_t \), denominated in the Home currency.

Household’s members perfectly pool their consumption risk within each country. The representative household can smooth consumption intertemporally by borrowing and lending in international financial markets, subject to a collateral constraint in the form of housing wealth

\[ (1 + i_t)B_t \leq \Theta_t E_t(Q_{t+1}H_t), \]  

(5)

where the borrowing constraint parameter \( \Theta_t \) is an exogenous shock with mean \( \Theta \) and support over the unit interval. The idea behind the borrowing constraint is that the Foreign household can only recover a fraction \( \Theta_t \) of the collateral in case of default, possibly due to various costs associated with the bankruptcy process.\(^{11}\)

**Firms and Production**

Final goods producing firms pack intermediate goods according to a constant return technology with elasticity of substitution \( \phi_p > 1 \)

\[ Y_{ht} = \left[ \int_0^1 Y_t(i) \frac{\phi_p - 1}{\phi_p} \frac{di}{\phi_p - 1} \right]^{\phi_p - 1}. \]  

(6)

All intermediate goods producing firms have access to the same constant return technology which uses a labor aggregate \( L_t \) as the only factor of production

\[ Y_t(h) = AL_t, \]  

(7)

where \( A \) is a constant productivity factor.

The labor aggregate combines differentiated labor inputs with an elasticity of substitution \( \phi_w > 1 \)

\[ L_t = \left[ \int_0^1 L_t(i) \frac{\phi_w - 1}{\phi_w} \frac{di}{\phi_w - 1} \right]^{\phi_w - 1}. \]  

(8)

\(^{11}\)See, for instance, Kiyotaki and Moore (1997) or Kocherlakota (2000).
Finally, the stock of housing (land) is assumed to be fixed

\[ H_t = H. \]  \hspace{1cm} (9)

**Monetary Policy**

The central bank sets the short-term nominal interest rate in response to deviations of inflation and output from their targets

\[
(1 + i_t) = (1 + i_{t-1})^{\rho_i} \left( 1 + i \right) \left( \frac{\Pi_t}{\Pi_t} \right)^{\phi_R} \left( \frac{Y_{ht}}{Y_{ht}} \right)^{\phi_y} e^{\varepsilon_t},
\]  \hspace{1cm} (10)

where \( \rho_i \) is the degree of interest rate smoothing, \( \Pi_t \equiv P_t/P_{t-1} \) is the inflation rate of goods prices \( P_t, \Pi_t \) and \( Y_{ht} \) are the targets for inflation and output respectively and \( \varepsilon_t \) is an i.i.d. normal innovation to the interest rate rule with mean zero and standard deviation \( \sigma_i \).

### 3.1 Equilibrium with Perfect Competition and Flexible Prices/Wages

Consider first a version of the model in which workers and firms have no market power \( (\phi_k \to \infty) \) and prices and wages are flexible \( (\zeta_k \to 0) \) for \( k = \{p, w\} \). Further suppose that the monetary authority in each country stabilizes overall inflation \( \Pi_t \equiv P_t/P_{t-1} \) at zero so that the nominal interest rate coincides with the real interest rate \( (R_t \equiv 1 + i_t \) and \( R^*_t \equiv 1 + i^*_t \)).

In the absence of market power and nominal rigidities, the real wage equals the marginal disutility of labor in units of marginal utility of consumption

\[
\frac{W_t}{P_t} = \frac{L^\nu_t}{1/C_t}.
\]  \hspace{1cm} (11)

On the firm side, optimality implies

\[
P_{ht} = \frac{W_t}{A}.
\]  \hspace{1cm} (12)

Therefore, equilibrium in the labor market requires

\[
\frac{P_{ht}}{P_t} = \frac{AL^\nu_t}{1/C_t},
\]  \hspace{1cm} (13)

or, using the production function

\[
\frac{P_{ht}}{P_t} = A^{1-\nu} Y_{ht}^\nu C_t,
\]  \hspace{1cm} (14)

where the last equation makes use of the production function to eliminate labor. Similarly,
in the Foreign country

\[ \frac{P^*_t}{P^*_t} = A^{1-\nu}Y^*_{ft}C^*_t, \]  

(15)

The relative prices of the Home and Foreign tradable goods depend on the terms of trade

\[ \mathcal{T}_t \equiv \frac{P^{*}_t}{P^{*}_t} = \frac{P^{*}_t}{P^{*}_t} \]

(16)

and

\[ \frac{P^*_t}{P^*_t} = \left[ \alpha + (1 - \alpha)T^{-1-\gamma}_{t} \right]^{\frac{1}{1+\gamma}} \]  

and

\[ \frac{P^*_t}{P^*_t} = \left[ \alpha + (1 - \alpha)T^{-1-\gamma}_{t} \right]^{\frac{1}{1+\gamma}} \]  

(17)

The labor market equilibrium pins down Home and Foreign output of tradable goods as a function of the terms of trade and Home and Foreign consumption respectively

\[ \left[ \alpha + (1 - \alpha)T^{-1-\gamma}_{t} \right]^{\frac{1}{1+\gamma}} = A^{1-\nu}Y^*_{ht}C^*_t, \]  

(18)

and

\[ \left[ \alpha + (1 - \alpha)T^{-1-\gamma}_{t} \right]^{\frac{1}{1+\gamma}} = A^{1-\nu}Y^*_{ft}C^*_t, \]  

(19)

Next, the goods market equilibrium pins down Home and Foreign consumption as a function of the terms of trade and the real exchange rate

\[ Y_{ht} = \left[ \alpha + (1 - \alpha)T^{-1-\gamma}_{t} \right]^{\frac{1}{1+\gamma}} \left[ \alpha C^*_t + (1 - \alpha)S^*_t C^*_t \right]. \]  

(20)

The Foreign country counterpart of the last equation is

\[ Y_{ft} = \left[ \alpha + (1 - \alpha)T^{-1-\gamma}_{t} \right]^{\frac{1}{1+\gamma}} \left[ (1 - \alpha)S^*_t C^*_t + \alpha C^*_t \right]. \]  

(21)

Real house prices are

\[ Q_t = \eta \left( \frac{H}{C_t} \right)^{-\frac{1}{\epsilon}} + \beta \mathbb{E}_t \left[ \left( \frac{X_{t+1}}{X_t} \right)^{-\frac{1}{\epsilon}} \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\epsilon}} Q_{t+1} \right] + \Xi_t \Theta_t E_t (Q_{t+1}). \]  

(22)

The Foreign counterpart of equation (22) is

\[ Q^*_t = \eta \left( \frac{H^*}{C^*_t} \right)^{-\frac{1}{\epsilon}} + \beta^* \mathbb{E}_t \left[ \left( \frac{X^*_{t+1}}{X^*_t} \right)^{-\frac{1}{\epsilon}} \left( \frac{C^*_{t+1}}{C^*_t} \right)^{-\frac{1}{\epsilon}} Q^*_{t+1} \right]. \]  

(23)

Differently from the Home economy, the borrowing constraint never binds in the Foreign
country, therefore \( \Xi_t^* = 0 \) at all times.

The borrowing constraint (5) pins down the stock of internationally-traded real debt \( B_t \equiv B_t/P_t \)

\[
R_t B_t = \Theta t E_t (Q_{t+1} H).
\] (24)

The shadow price of the borrowing constraint is

\[
R_t \Xi_t = 1 - \beta R_t E_t \left[ \left( \frac{X^*_{t+1}}{X^*_t} \right)^{\frac{1-\sigma}{\gamma}} \left( \frac{C^*_{t+1}}{C^*_t} \right)^{-\frac{\gamma}{2}} \right].
\] (25)

The Euler equation for the Foreign country pins down the real return in international financial markets

\[
1 = \beta^* R_t E_t \left[ \left( \frac{X^*_{t+1}}{X^*_t} \right)^{\frac{1-\sigma}{\gamma}} \left( \frac{C^*_{t+1}}{C^*_t} \right)^{-\frac{\gamma}{2}} \frac{S_t}{S_{t+1}} \right].
\] (26)

No arbitrage (residually) pins down the real return in the Foreign country

\[
R_t^* E_t \left[ \left( \frac{X^*_{t+1}}{X^*_t} \right)^{\frac{1-\sigma}{\gamma}} \left( \frac{C^*_{t+1}}{C^*_t} \right)^{-\frac{\gamma}{2}} \frac{S_t}{S_{t+1}} \right] = R_t E_t \left[ \left( \frac{X^*_{t+1}}{X^*_t} \right)^{\frac{1-\sigma}{\gamma}} \left( \frac{C^*_{t+1}}{C^*_t} \right)^{-\frac{\gamma}{2}} \frac{S_t}{S_{t+1}} \right].
\] (27)

The law of motion of foreign debt (from the resource constraint) pins down the terms of trade

\[
-B_t = -R_{t-1} B_{t-1} + \left[ \alpha + (1 - \alpha) T_t^{1-\gamma} \right]^{-\frac{1}{1-\gamma}} Y_{ht} - C_t.
\] (28)

Finally, the world resource constraint pins down the real exchange rate

\[
\left[ \alpha + (1 - \alpha) T_t^{1-\gamma} \right]^{-\frac{1}{1-\gamma}} Y_{ht} + \left[ \alpha + (1 - \alpha) T_t^{-(1-\gamma)} \right]^{-\frac{1}{1-\gamma}} Y_{ft} = C_t + S_t C^*_t.
\] (29)

3.1.1 Steady State

If the borrowing constraint is not binding in either country, the model admits a symmetric steady state in which all relative prices (those of tradable goods, the terms of trade and the real exchange rate) are equal to one and foreign debt is zero. In this steady state, each country is in autarchy and the level of productivity pins down output (and hence consumption). House prices are equal to the present discounted value of the marginal utility of housing services while the real return is equal to the inverse of the discount factor. The unattractive feature of a perfectly symmetric steady state is that, up to a linear approximation, borrowing constraints are irrelevant for house prices dynamics, as it can be seen from equation (22).

Considering an asymmetric steady state is one way to resuscitate a role for borrowing constraints in affecting house prices dynamics while retaining the tractability of a linear ap-
Even with identical preferences and technologies, simply imposing that one country’s borrowing constraint is binding is enough to generate an asymmetric steady state. However, assuming a different degree of patience across countries provides a more fundamental reason why one country’s steady state may be binding. This assumption ($\beta^* > \beta$) will be maintained throughout, implying that the Foreign country is a net saver in international financial markets. Differences in the steady state level of productivity can still be used to obtain a steady state in which relative prices are equal to one.

4 Quantitative Results

This section discusses the calibration of the parameters and presents the central quantitative experiment of the paper – a relaxation of the borrowing constraint parameter $\Theta$.

4.1 Calibration

Two parameters are fairly standard in the international macroeconomics literature. The domestic share of tradable consumption $\alpha$ is set to 0.7. The elasticity of substitution between Home and Foreign tradable goods $\gamma$ equals 2.

The parameter $\eta$ is chosen so that in steady state the housing share of total expenditure corresponds to 20%. A loan-to-value ratio ratio of 70% pins down the steady state value of $\Theta$.

The inverse Frisch elasticity of labor supply is assumed to be equal to 2. The level of Home productivity $A$ is fixed to one while the Foreign level $A^*$ is adjusted so that the steady state terms of trade and real exchange rate are equal to one.

The Foreign discount factor is calibrated so that the steady state return on international assets is 4% ($\beta^* = 0.99$). As mentioned in the previous section, the Home discount factor is assumed to be lower ($\beta = 0.98$) so that the Home country is a net international borrower.

Finally, the preference and borrowing constraint shocks follow AR(1) processes with persistence equal to 0.99 and i.i.d. innovations $\sim \mathcal{N}(0, 1)$.

4.2 Financial Innovation, House Price Booms and External Imbalances

The model is approximated up to the first order about the asymmetric steady state described above. The appendix lists the log-linear system of equations that characterize the flexible price equilibrium.

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12 An alternative would be to use non-linear methods for solving models with occasionally binding constraints (e.g. Christiano and Fisher, 1999)

13 A very common assumption in macroeconomics is to set the inverse Frisch elasticity equal to 1. This choice, however, would prevent steady state productivity to play any role in determining relative prices, which is instead useful here to fix the steady state terms of trade and real exchange rate to one.
The main experiment consists of shocking the collateral constraint parameter $\Theta$. In particular, financial innovation corresponds to a higher loan-to-value ratio. Households can borrow a higher fraction of the expected value of their house. To illustrate the mechanism and its limitations, this section focuses on the response of the endogenous variables to a 1% innovation in $\Theta$.

Figure 2 plots the response of house prices (top-left), the current account balance (top-right) and consumption (bottom-left) for the Home country, as well as the world real interest rate (bottom-right), defined as the weighted sum of the Home and Foreign interest rate.

At a very basic level, the model captures the negative correlation between house prices and current account balance. In response to the shock, house prices persistently increase while the current account worsens, at least for a couple of periods, before slightly overshooting its long run average of zero. A more realistic sequence of shocks approximating a progressive relaxation of collateral constraints (the process of financial liberalization) is likely to generate a run-up of house prices and a deterioration of the external balance more in line with what observed in the data.

The intuition for the result is simple. For given house prices, a relaxation of borrowing
constraints induces households to lever up and demand more housing. Home households finance their higher leverage borrowing from abroad. As foreign debt increases, the current account turns negative, hence generating the negative correlation consistent with the empirical evidence.

Figure 2, however, also highlights some limitations of the basic model with flexible prices. First, in response to the shock, Home consumption drops, although the movement is almost negligible. In the data, consumption grew strongly during the house prices boom of the first half of the 2000s. The counterfactual movement of consumption partly depends on the preference assumption. If tradable consumption and housing were true complements, the shock would likely also generate an increase in consumption. Flexible prices, however, exacerbate this adjustment. In response to the shock, the terms of trade instantaneously adjust as to make Home goods more expensive. Because of home bias, domestic households disproportionately suffer from the adverse movement in relative prices.

A second issue with the basic model with flexible prices is that the drop in the world real interest rate is driven by the drop in the Foreign real rate. The Home real interest rate, however, is the relevant variable to compare with the data, which show a persistent decline of real rates on a global scale.

The next section reactivates price and wage rigidities with the objective of addressing these issues and evaluating the quantitative importance of some popular competing explanations.

5 Competing Explanations

This section evaluates the performance of the model with nominal price and wage rigidities with two purposes. The first question is whether nominal rigidities are helpful in mitigating the instantaneous adjustment in relative prices occurring after the shock to the collateral constraint. Second, the presence of nominal rigidities allows for the investigation of the role of monetary policy and foreign exchange rate pegs in amplifying the relaxation of loan-to-value requirements.

5.1 Equilibrium with Nominal Rigidity

The only difference compared to the previous section is the determination of the supply side of the economy.

The wage determination process yields a non-linear wage Philips curve, which combines the optimal choice of household members who reset their wage in any given period and their mass with the aggregate wage index

\[
1 - \zeta w \Pi_{wt}^{\phi_w - 1} \frac{1 - \phi_w (1 - v)}{1 - \phi_w} = K_{wt} \frac{F_{wt}}{F_{wt}}.
\]
According to expression (30), wage inflation \( \Pi_{wt} \equiv W_t/W_{t-1} \) is a non-linear function of the present discounted value of the marginal disutility of labor \( K_{wt} \)

\[
K_{wt} = \frac{\phi_w}{\phi_w - 1} e^{\kappa t} L_t^\nu + \beta \zeta_w E_t \left[ (\Pi_{wt+1})^{\phi_w} K_{wt+1} \right] \quad (31)
\]

and of the present discounted value of the real wage in units of marginal utility of consumption \( F_{wt} \)

\[
F_{wt} = \tilde{\eta} e^{\kappa t} X_t^{\frac{1}{2} - \sigma} C_t^{-\frac{1}{2}} \frac{W_t L_t}{P_t} + \beta \zeta_w E_t \left[ (\Pi_{wt+1})^{\phi_w - 1} F_{wt+1} \right] . \quad (32)
\]

Price setting decisions yield a non-linear price Phillips curve, which combine the optimal choice of firms who reset their price in any given period and their mass with the price index for domestic tradable goods

\[
\left[ \frac{1 - \zeta_p \Pi_{ht}^{\phi_p - 1}}{1 - \zeta_p} \right] \frac{1}{1 - \phi_p} = \frac{K_{pt}}{F_{pt}} . \quad (33)
\]

According to expression (33), inflation in the domestic tradable good sector \( \Pi_{ht} \equiv P_{ht}/P_{ht-1} \) is a non-linear function of the present discounted value of real marginal costs \( K_{pt} \)

\[
K_{pt} = \frac{\phi_p}{\phi_p - 1} e^{\kappa t} X_t^{\frac{1}{2} - \sigma} C_t^{-\frac{1}{2}} \frac{W_t Y_{ht}}{P_t} + \beta \zeta_p E_t \left[ (\Pi_{ht+1})^{\phi_p} K_{pt+1} \right] \quad (34)
\]

and of the present discounted value of real marginal revenues

\[
F_{pt} = e^{\kappa t} X_t^{\frac{1}{2} - \sigma} C_t^{-\frac{1}{2}} \frac{P_{ht} Y_{ht}}{P_t} + \beta \zeta_p E_t \left[ (\Pi_{ht+1})^{\phi_p - 1} F_{pt+1} \right] . \quad (35)
\]

1. Loose monetary policy
2. Foreign pegs
3. Global saving glut hypothesis

### 6 Conclusions

TBW

### References


A Optimality Conditions for Households and Firms

This section presents the optimality conditions for households and firms of the Home country. Given the assumption of a representative household in each country, borrowing and lending occurs in equilibrium only at the international level. In what follows, the borrowing constraint is always assumed to bind for the Home economy and never for the Foreign economy.
Cost Minimization

Expenditure minimization determines the allocation of total consumption between Home and Foreign tradable goods as a function of their relative prices and total demand

\[ C_{ht} = \alpha \left( \frac{P_{ht}}{P_t} \right)^{-\gamma} C_t \quad \text{and} \quad C_{ft} = (1 - \alpha) \left( \frac{P_{ft}}{P_t} \right)^{-\gamma} C_t, \tag{36} \]

where the price of the aggregate consumption bundle \( P_t \) is

\[ P_t = \left[ \alpha P_{ht}^{1-\gamma} + (1 - \alpha) P_{ft}^{1-\gamma} \right]^\frac{1}{1-\gamma}. \tag{37} \]

Expenditure minimization also implies

\[ P_{ht} C_{ht} + P_{ft} C_{ft} = P_t C_t. \tag{38} \]

Final goods producers are perfectly competitive. Their cost minimization problem generates the demand for intermediate goods according to

\[ Y_t(h) = \left[ \frac{P_t(h)}{P_{ht}} \right]^{-\phi_p} Y_{ht}, \tag{39} \]

where the price index of the tradable bundle \( P_{ht} \) is

\[ P_{ht} = \left[ \int_0^1 P_t(h)^{1-\phi_p} dh \right]^\frac{1}{1-\phi_p}. \tag{40} \]

Intermediate goods producing firms minimize labor costs and demand the generic labor input \( L_t(i) \) according to

\[ L_t(i) = \left[ \frac{W_t(i)}{W_t} \right]^{-\phi_w} L_t, \tag{41} \]

where \( W_t \) is the aggregate wage index

\[ W_t = \left[ \int_0^1 W_t(i)^{1-\phi_w} di \right]^\frac{1}{1-\phi_w}. \tag{42} \]

Utility Maximization

The representative household maximizes utility (1) subject to the budget constraint (4) and the borrowing constraint (5). Let \( \lambda_t \) and \( \lambda_t \Xi_t \) be the lagrange multipliers on the two constraints. Workers operate in monopolistic competition taking the demand for their generic labor input as given. Therefore, equation (41) becomes an additional constraint for the household problem.
The first order condition for consumption is
\[ \tilde{\eta}e^{\alpha X_t^{\frac{1}{2} - \sigma} C_t^{\frac{1}{2}}} - \lambda_t P_t = 0. \] (43)

The first order condition for housing services is
\[ (1 - \tilde{\eta})e^{\alpha X_t^{\frac{1}{2} - \sigma} H_t^{\frac{1}{2}}} - \lambda_t Q_t + \beta E_t(\lambda_{t+1} Q_{t+1}) + \lambda_t \Xi_t \Theta_t E_t(Q_{t+1}) = 0. \] (44)

The first order condition for debt is
\[ \lambda_t - \beta (1 + i_t) E_t(\lambda_{t+1}) - \lambda_t \Xi_t (1 + i_t) = 0. \] (45)

Wages are set on a staggered basis (Calvo, 1983). The probability of not being able to adjust the wage is \( \zeta_w \). The optimality condition for a worker who is able to adjust the wage at time \( t \) is
\[ E_t \left\{ \sum_{s=0}^{\infty} (\beta \zeta_w)^s L_{t+s}(i) \left[ \lambda_{t+s} W_t(i) - \frac{\phi_w}{\phi_w - 1} e^{\alpha L_{t+s}(i)} \right] \right\} = 0. \] (46)

Using the labor demand equation (12) and the expression for the marginal utility of consumption (43) into the previous expression yields
\[ E_t \left\{ \sum_{s=0}^{\infty} (\beta \zeta_w)^s e^{\alpha L_{t+s}} \left[ \tilde{\eta} X_{t+s}^{\frac{1}{2} - \sigma} C_{t+s}^{\frac{1}{2}} \left( \frac{W_t(i)}{W_{t+s}} \right)^{-\phi_w} \right] \left( \frac{W_t(i)}{P_t} \right)^{-\phi_u} L_{t+s}^{\nu} \right\} = 0. \] (47)

Equation (47) can be rearranged as to express the relative wage of type \( i \) as a function of the ratio between the present discounted value of the marginal disutility of labor and the present discounted value of the real wage in units of marginal utility of consumption
\[ \left[ \frac{W_t(i)}{W_t} \right]^{1-\phi_u(1-\nu)} = \frac{K_{wt}}{F_{wt}}. \] (48)

The terms on the right-hand side of the last expression can be written recursively as
\[ K_{wt} = \frac{\phi_w}{\phi_w - 1} e^{\alpha L_{t+1}^\nu} + \beta \zeta_w E_t \left[ (\Pi_{wt+1})^{\phi_u} K_{wt+1} \right] \] (49)

and
\[ F_{wt} = \tilde{\eta} e^{\alpha X_t^{\frac{1}{2} - \sigma} C_t^{\frac{1}{2}}} W_t L_t P_t + \beta \zeta_w E_t \left[ (\Pi_{wt+1})^{\phi_u - 1} F_{wt+1} \right], \] (50)

where \( \Pi_{wt} \equiv W_t/W_{t-1} \) represents wage inflation. Expressions (48)-(50) show that the optimal choice of household members who optimally reset their wage in any given period is a function of aggregate variables only. Finally, the aggregate wage index (42) can be rewritten
as to link the relative wage of type \(i\) to wage inflation

\[
\zeta_w (\Pi_{wt})^{\phi_w} - 1 + (1 - \zeta_w) \left[ \frac{W_t(i)}{W_t} \right]^{1-\phi_w} = 1. \tag{51}
\]

Using the first order condition for consumption (43), the first order conditions for housing services (44) becomes

\[
Q_t = \eta \left( \frac{H_t}{C_t} \right)^{-\frac{1}{\epsilon}} + \beta \mathbb{E}_t \left[ e^{\sigma_t - \sigma - \frac{1}{\epsilon}} \left( \frac{X_{t+1}}{C_t} \right)^{\frac{1}{\epsilon}} \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\epsilon}} \right] Q_{t+1} + \Xi_t \Theta_t \mathbb{E}_t (\Pi_{t+1} Q_{t+1}), \tag{52}
\]

where \(Q_t \equiv Q_t/P_t\) defines real house prices and \(\eta \equiv \bar{\eta}^{-1} - 1\). Equation (52) consists of a standard part, according to which real house prices are equal to the marginal utility of housing services in units of marginal utility of consumption plus expected discounted future house prices, and a second part which measures the contribution of the borrowing constraint via the shadow price \(\Xi_t\).

Similarly, using again the first order condition for consumption (43), the first order condition for debt (45) becomes

\[
(1 + i_t) \Xi_t = 1 - \beta (1 + i_t) \mathbb{E}_t \left[ e^{\sigma_t - \sigma - \frac{1}{\epsilon}} \left( \frac{X_{t+1}}{C_t} \right)^{\frac{1}{\epsilon}} \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\epsilon}} \right] \left( \frac{1}{\Pi_{t+1}} \right). \tag{53}
\]

Equation (53) shows that the shadow price \(\Xi_t\) represents a wedge in the standard consumption Euler equation due to the borrowing constraint.

**No Arbitrage**

The representative household in the Foreign country solves the same maximization problem with one substantial difference. While the Foreign representative household can purchase Home debt, Foreign debt only circulates domestically. No arbitrage then implies the consumption-based uncovered interest parity condition

\[
\mathbb{E}_t \left\{ e^{\sigma_t - \sigma} \left( \frac{X_{t}^*}{X_t} \right)^{\frac{1}{\epsilon}} \left( \frac{C_{t}^*}{C_t} \right)^{-\frac{1}{\epsilon}} \frac{1}{\Pi_{t+1}} \right\} = (1 + i_t^*) - (1 + i_t) \mathbb{E}_t \frac{\xi_t}{\xi_{t+1}} = 0. \tag{54}
\]

Because of the representative household assumption, Foreign debt is in zero net supply in equilibrium. Additionally, the Foreign country is assumed to be a net saver in international financial markets so that the Foreign borrowing constraint to never bind \((\Xi_t^* = 0, \forall t)\).

**Profit Maximization**

Intermediate goods producers set prices on a staggered basis taking the demand for their product (39) as given and subject to the technology constraint (7). The probability of not being able to adjust the price is \(\zeta_p\). The optimality condition for a firm able to adjust its
price at time t is\(^{14}\)

\[
E_t \left\{ \sum_{s=0}^{\infty} (\beta \zeta_p)^s \lambda_{t+s} Y_{t+s}(h) \left[ P_t(h) - \left( \frac{\phi_p}{\phi_p - 1} \right) \frac{W_{t+s}}{A} \right] \right\} = 0. \tag{55}
\]

As for the wage, using the demand for intermediate goods (39) and the expression for the marginal utility of consumption (43) into the previous expression yields

\[
E_t \left\{ \sum_{s=0}^{\infty} (\beta \zeta_p)^s e^{a_t + \eta_t} X_{t+s}^{\frac{1}{2} - \sigma} C_{t+s}^{-\frac{1}{2}} \left[ \frac{P_t(h)}{P_{ht+s}} \right]^{-\phi_p} \frac{Y_{ht+s}}{P_{ht+s}} \left[ P_t(h) - \left( \frac{\phi_p}{\phi_p - 1} \right) \frac{W_{t+s}}{A} \right] \right\} = 0. \tag{56}
\]

In this case, equation (56) can be rearranged as to express the relative price of variety \(h\) as a function of the ratio between the present discounted value of the real marginal cost and the present discounted value of the real marginal revenues

\[
\left[ \frac{P_t(h)}{P_{ht}} \right] = \frac{K_{pt}}{F_{pt}}. \tag{57}
\]

The terms on the right-hand side of the last expression can be written recursively as

\[
K_{pt} = \frac{\phi_p}{\phi_p - 1} e^{a_t} X_t^{\frac{1}{2} - \sigma} C_t^{-\frac{1}{2}} \frac{W_t Y_{ht}}{AP_t} + \beta \zeta_p E_t \left[ (\Pi_{ht+1})^{\phi_p - 1} K_{pt+1} \right] \tag{58}
\]

and

\[
F_{pt} = e^{a_t} X_t^{\frac{1}{2} - \sigma} C_t^{-\frac{1}{2}} \frac{P_{ht} Y_{ht}}{P_t} + \beta \zeta_p E_t \left[ (\Pi_{ht+1})^{\phi_p - 1} F_{pt+1} \right]. \tag{59}
\]

Expressions (57)-(59) show that the optimal choice of firms who reset their price in any given period is a function of aggregate variables only. Finally, the aggregate price index (40) can be rewritten as to link the relative price of variety \(h\) to price inflation

\[
\zeta_p (\Pi_{ht})^{\phi_p - 1} + (1 - \zeta_p) \left[ \frac{P_t(h)}{P_{ht}} \right]^{1-\phi_p} = 1, \tag{60}
\]

where \(\Pi_{ht} = P_{ht}/P_{ht-1}\) represents domestic inflation.

**Market Clearing**

The law of one price holds for tradable goods

\[
P_{ht} = E_t P_{ht}^*, \tag{61}
\]

where \(E_t\) is nominal exchange rate (the price of the Foreign currency in terms of the Home

\[\text{22}\]
currency). Home bias, however, implies that purchasing power parity does not hold (i.e. $P_t \neq E_t P^*_t$).

Final goods producing firms sell their products in the Home and Foreign market. Market clearing requires

$$Y_{ht} = C_{ht} + C^*_{ht} = \alpha \left( \frac{P_{ht}}{P_t} \right)^{-\gamma} C_t + 1 - \alpha \right) \left( \frac{P^*_{ht}}{P^*_t} \right)^{-\gamma} C^*_t, \quad (62)$$

where the second part of (62) uses (36) and its Foreign country counterpart. The Home tradable goods market equilibrium (62) can alternatively be expressed in terms of the Home relative price and the real exchange rate $S_t \equiv E_t P^*_t / P_t$

$$Y_{ht} = \left( \frac{P_{ht}}{P_t} \right)^{-\gamma} \left[ \alpha C_t + (1 - \alpha) S^*_t C^*_t \right]. \quad (63)$$

As mentioned, the housing stock is fixed in both countries

$$H_t = H \quad \text{and} \quad H^*_t = H^*. \quad (64)$$

Market clearing for financial assets requires

$$B_t + B^*_t = 0, \quad (65)$$

where $B^*_t$ is the Foreign country stock of international debt.

**B Steady States**

This appendix presents first a symmetric steady state in which both countries are in autarchy and then illustrates the modifications to obtain the asymmetric steady state considered in the paper. In both cases, the governments in each country use a subsidy to eliminate monopolistic distortions so that the steady state allocation is efficient.

**B.1 Symmetric**

Assume that all parameters, including the discount factor as well as the steady state level of productivity and stock of housing, are equal across countries. Then, there exists a steady state in which relative prices of tradable goods are equal to one and so are the terms of trade ($T_t = 1$) from equations (16) and (17). Given these results, the definition of the real exchange rate requires

$$S = \frac{E P^*}{P} = \frac{E P^*_f}{P_h} \frac{P_h P^*}{P_P f} = 1 \quad (66)$$

23
Combining the labor market equilibrium conditions and the goods market equilibrium conditions yields

\[ Y_h = C^{1-\frac{1}{\beta}} = \alpha C + (1 - \alpha)C^* \quad \text{and} \quad Y_f = C^{1-\frac{1}{\beta}} = (1 - \alpha)C + \alpha C^*, \quad (67) \]

where the underlying assumption is that \( A = A^* = 1 \). The last two expressions imply that each country is always in autarchy \((Y_h = C = C^* = Y_f = 1)\). From the resource constraint, external debt is zero \((B = 0)\).

The borrowing constraint is slack \((\Xi = \Xi^* = 0)\) in both countries. House prices only depend on the present discounted value of the marginal utility of housing

\[ Q = Q^* = \frac{\eta H^{\frac{1}{\beta}}}{1 - \beta}. \quad (68) \]

Finally, the real interest rate is equal to the inverse of the discount factor

\[ R = R^* = \frac{1}{\beta^*}. \quad (69) \]

### B.2 Asymmetric Steady State

To build an asymmetric steady state in which country H is a net borrower but relative prices, terms of trade and real exchange rate are still equal to one, start with the assumption that the Home country representative household is relative more impatient \((\beta < \beta^*)\). Assume that the borrowing constraint is binding for country H but not for country F \((\Xi > 0 \text{ and } \Xi^* = 0)\).

Equations \((67)\) still characterize the equilibrium in the goods and labor market in each country. As before, those two equations pin down \( C \) and \( C^* \) (and, consequently, \( Y_h \) and \( Y_f \)) as a function of productivity only. However, now relax the assumption \( A = A^* = 1 \), so that relative prices are still equal to one, although \( Y_h \neq C \) and \( Y_f \neq C^* \). Furthermore, in general, \( Y_h \neq Y_f \) and \( C \neq C^* \).

No arbitrage still implies

\[ R = R^* = \frac{1}{\beta^*}. \quad (70) \]

Since the borrowing constraint is binding for country H, debt is equal to

\[ B = \Theta \beta^* QH. \quad (71) \]

The house price equation yields

\[ Q = \frac{\eta (H/C)^{\frac{1}{\beta}}}{1 - \beta - \Xi \Theta}. \quad (72) \]

Holding consumption constant, higher loan-to-value ratios increase house prices and debt,
both directly and indirectly. In the Foreign country, the borrowing constraint is not binding, thus house prices are

\[
Q^* = \frac{\eta C^*}{1 - \beta^*}.
\] (73)

Note that \( H \) and \( H^* \) only matter for the steady state level of house prices but not for the determination of the other endogenous variables. For instance, the ratio between the housing stocks in the two countries can be chosen so that the steady state house prices are the same.

C Log-Linear Approximation of the Model

Unless otherwise noted, for any given variable \( Z_t \) define \( z_t \equiv \log(\frac{Z_t}{Z}) \simeq \frac{Z_t - Z}{Z} \), where \( Z \) is the steady state of \( Z_t \).

C.1 Flexible Prices and Wages

A first order approximation to the relative price equations (16) and (17) around the asymmetric steady state described above is

\[
p_{ht} = -(1 - \alpha)\tau_t \quad \text{and} \quad p^*_{ft} = (1 - \alpha)\tau_t. \tag{74}
\]

A first order approximation to labor market equilibrium conditions (18) and (19) is

\[
p_{ht} = \nu y_{ht} + c_t \quad \text{and} \quad p^*_{ft} = \nu y_{ft} + c^*_t. \tag{75}
\]

Equilibrium in goods markets can be approximated as

\[
y_{ht} = -\gamma p_{ht} + s_H [\alpha c_t + (1 - \alpha)c^*_R(\gamma s_t + c^*_t)] \tag{76}
\]

and

\[
y_{ft} = -\gamma p^*_{ft} + s_F [(1 - \alpha)c_R(c_t - \gamma s_t) + \alpha c^*_t]. \tag{77}
\]

where \( s_i \equiv C_i/Y_i \) is the steady state consumption share of output in country \( i = \{H,F\} \) and \( c_R \equiv C/C^* \) is relative consumption across countries.

Next, the approximation of the house price equations (22) yields

\[
q_t = (1 - \beta - \Xi \Theta)c_t + \beta \left[ \left( \frac{1}{\xi} - \sigma \right) (E_{t}x_{t+1} - x_t) - \frac{1}{\xi} (E_{t}c_{t+1} - c_t) \right] + \Xi \Theta (\xi_t + \theta_t) + (\beta + \Xi \Theta) E_{t}(q_{t+1}). \tag{78}
\]

The lagrange multiplier on the borrowing constraint char introduces a wedge in the Home
country Euler equation. A first order approximation of equation (25) gives

$$r_t + \beta R \left( \left( \frac{1}{\varepsilon} - \sigma \right) (\mathcal{E}_t x_{t+1} - x_t) - \frac{1}{\varepsilon} (\mathcal{E}_t c_{t+1} - c_t) \right) + (1 - \beta R) \xi_t = 0. \quad (79)$$

In the Foreign country, the slack borrowing constraint implies that equation (23) becomes

$$q^*_t = (1 - \beta^*) x^*_t + \beta^* \left( \left( \frac{1}{\varepsilon} - \sigma \right) (\mathcal{E}_t x^*_{t+1} - x^*_t) - \frac{1}{\varepsilon} (\mathcal{E}_t c^*_{t+1} - c^*_t) \right) + \beta^* \mathcal{E}_t(q^*_{t+1}). \quad (80)$$

The approximation of the borrowing constraint (24) is

$$r_t + b_t = \theta_t + \mathcal{E}_t(q_{t+1}). \quad (81)$$

A first order approximation to country F Euler equation (26) gives

$$r_t + \left( \frac{1}{\varepsilon} - \sigma \right) (\mathcal{E}_t x^*_{t+1} - x^*_t) - \frac{1}{\varepsilon} (\mathcal{E}_t c^*_{t+1} - c^*_t) + s_t - \mathcal{E}_t(s_{t+1}) = 0. \quad (82)$$

Up to the first order, the no-arbitrage relation (27) can be written as

$$r_t = r^*_t - s_t + \mathcal{E}_t(s_{t+1}). \quad (83)$$

The log-linear approximation of the index (2) for the Home and Foreign country gives

$$x_t = \left( \frac{1}{1 + \eta} \right) \left( \frac{C}{X} \right)^{\frac{\eta+1}{\eta}} c_t \quad \text{and} \quad x^*_t = \left( \frac{1}{1 + \eta} \right) \left( \frac{C^*}{X^*} \right)^{\frac{\eta+1}{\eta}} c^*_t \quad (84)$$

The dynamics of debt (28) can be approximated as

$$-b_t = -R(rt_{-1} + b_{t-1}) + b^{-1}_y(p_{ht} + y_{ht} - s_H c_t), \quad (85)$$

where $b_y \equiv B/Y_h$ is the steady state ratio between net foreign debt and GDP for the Home country.

Finally, up to a first order approximation, the world resource constraint (29) gives

$$p_{ht} + y_{ht} + \frac{S_H}{C_{RSF}}(p^*_f + y_f) = s_H c_t + \frac{S_H}{C_R}(s_t + c^*_t). \quad (86)$$

C.2 Nominal Rigidities

A first order approximation of the relative wage equation (48) gives

$$[1 - \phi_w(1 - \nu)]\omega_t(i) = k_{wt} - f_{wt}, \quad (87)$$

where $\omega_t(i)$ is the log-deviation of the relative wage of type $i$ from steady state (in which $W(i) = W \Rightarrow \omega(i) = 1$). A first order approximation of the present discounted value of the
marginal disutility of labor (49) and the real wage in units of marginal utility of consumption
(50) yields

$$k_{wt} = (1 - \beta \zeta_w) \nu \ell_t + \beta \zeta_w \mathbb{E}_t (\phi_w \nu \pi_{wt+1} + k_{wt+1})$$

(88)

and

$$f_{wt} = (1 - \beta \zeta_w) \left[ w_t + \left( \frac{1}{\varepsilon} - \sigma \right) x_t - \frac{1}{\varepsilon} \ell_t + \ell_t \right] + \beta \zeta_w \mathbb{E}_t [(\phi_w - 1) \pi_{wt+1} + f_{wt+1}],$$

(89)

where $w_t \equiv \log[(W_t/P_t)/(W/P)]$ stands for the log-deviation of the real wage from its steady state value. A first order approximation of the wage index equation (51) is

$$\omega_t(i) = \frac{\zeta_w}{1 - \zeta_w} \pi_{wt}. \quad (90)$$

Combining the last four expressions gives a standard forward looking wage Phillips curve

$$\pi_{wt} = \kappa_w \left[ (\nu - 1) \ell_t - w - \left( \frac{1}{\varepsilon} - \sigma \right) x_t + \frac{1}{\varepsilon} \ell_t \right] + \beta \mathbb{E}_t (\pi_{wt+1}), \quad (91)$$

where $\kappa_w \equiv (1 - \beta \zeta_w)(1 - \zeta_w)/\{\zeta_w[1 - \phi_w(1 - \nu)]\}$. For prices, first order approximation of equation (57) gives

$$p_t(h) = k_{pt} - f_{pt}, \quad (92)$$

where $p_t(h)$ is the log-deviation of the relative price of variety $h$ from steady state (in which $P(h) = P_h \Rightarrow p(h) = 1$). A first order approximation of the present discounted value of marginal costs (58) and marginal revenues (59) yields

$$k_{pt} = (1 - \beta \zeta_p) \left[ \left( \frac{1}{\varepsilon} - \sigma \right) x_t - \frac{1}{\varepsilon} \ell_t + w_t + y_{ht} \right] + \beta \zeta_p \mathbb{E}_t (\phi_p \pi_{ht+1} + k_{pt+1})$$

(93)

and

$$f_{pt} = (1 - \beta \zeta_p) \left[ \left( \frac{1}{\varepsilon} - \sigma \right) x_t - \frac{1}{\varepsilon} \ell_t + p_{ht} + y_{ht} \right] + \beta \zeta_p \mathbb{E}_t [(\phi_p - 1) \pi_{ht+1} + f_{pt+1}] \quad (94)$$

A first order approximation of the price index equation (60) is

$$p_t(h) = \frac{\zeta_p}{1 - \zeta_p} \pi_{ht}. \quad (95)$$

Combining the last four expressions gives a standard forward looking price Phillips curve

$$\pi_{ht} = \kappa_p (w_t - p_{ht}) + \beta \mathbb{E}_t (\pi_{ht+1}), \quad (96)$$

where $\kappa_p \equiv (1 - \beta \zeta_p)(1 - \zeta_p)/\zeta_p$. 

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