Optimal Fertility Distribution

Alice Schoonbroodt

February 2, 2011

Summary

In a risk-sharing dynastic model, Hosseini, Jones, and Shourideh (2009) find that fertility should be negatively related to ability. However, they only consider the case where ability is i.i.d. across generations and restrict attention to the case where children cost only time (i.e. cost proportional to productivity). These notes ask if this result may be mitigated or even overturned in various settings, if there is persistence in intergenerational ability and/or some of the costs of children are in terms of goods (i.e. not proportional to productivity). It turns out that in Barro-Becker type models (i.e. where parents care about the utility from children) if children only cost time, the negative ability-fertility relationship remains even if shocks are perfectly persistent. If children cost only goods, then the optimal relationship is positive with persistence and flat with i.i.d. shocks. So, in the simplest case, it is a mix of the relative sizes of goods and time costs together with the degree of persistence which determine what the optimal fertility distribution looks like.

Further, we analyze the case where the combination of goods and time costs to raise children can be chosen. If both inputs in childcare are essential, somewhat substitutable and somewhat complementary, and if there is some intergenerational persistence in ability, we find that the optimal fertility distribution is U-shaped. Low ability people have a comparative advantage by using mostly time to raise many children, while high ability types have a comparative advantage in producing many high ability children using mostly goods. The middle ability types don’t have these advantages and therefore have the fewest children.
Adding a quality choice for children (and a production function where parental ability enters) is the object of current investigations. This partly endogenizes intergenerational persistence in ability.

More precisely, we find:

1. If (exogenous) costs of children comprise a (fixed) goods component and a (fixed) time component, there exists a level of intergenerational persistence, \( p^* \), such that fertility is decreasing for less persistence than \( p^* \), while fertility is increasing for more persistence than \( p^* \).

2. If (exogenous) costs of children can be produced using goods or time

   (a) with some substitutability and some complementarity then

   i. the planner wants high ability people to use more goods than time per child;
   ii. low ability people can cheaply produce children using lots of time;
   iii. however, the higher persistence, the lower quality children are produced by low ability parents;
   iv. the higher persistence, the higher quality children are produced by high ability parents;
   v. however, children of high ability types are expensive because some time is needed to raise them.

   These effects result in a U-shaped fertility-ability relationship. Types in the “middle” have neither a cost nor a quality advantage and are therefore the ones with the lowest fertility. The types with lowest fertility.

   As persistence increases, the lowest fertility type shifts towards lower ability types. That is the cheap time cost effect in (b) becomes less and less important. A wider range sees a positive relationship between ability and fertility due to the effect in (d).

   (b) with perfect complementarity, then the result boils down to 1.

   (c) with perfect substitutability, then the result in 2. comes out but the relationship is V-shaped, instead with low types using only time and high types only goods, i.e. substitution effect only, then income effect only.

   2
3. The next change is bigger than the difference between 1 and 2. This time, the costs of children actually are chosen because they represent quality that may enter utility directly or indirectly because it increases productivity of children (currently effect not there). Parental productivity may be important in quality production (currently not in production function).

Ultimately, we plan to decentralize the planner’s solution and compare the predicted optimal family and education policies to those currently in place in different countries.

References