In this paper we quantitatively investigate the implications of a model of consumption risk sharing where infinitely-lived households are subject to exogenous idiosyncratic shocks to their earnings, and where the realization of these shocks are private information. Our theoretical contribution relative to the existing literature is to allow for persistence in earnings. This creates some formidable obstacles to recursive computation of the optimal contract. If the earnings process follows a Markov chain whose state space has \( N \) elements, the state space for the optimal dynamic contracting problem has \( N \) continuous dimensions and is a nontrivial subset \( W \subset \mathbb{R}^N \). Such a problem is prohibitively complex, first because of the dimension of the dimensionality of the state space and, second, because it is difficult to solve for \( W \) itself.

Our approach is to develop a discrete shock analogue to a first-order approach, interpreting a “local” deviation in the discrete context as a one-step deviation. This approach is based on the conjecture that if one-step deviations from truth-telling are not optimal then no deviations are optimal and truth-telling is optimal. That conjecture can be verified numerically ex post. As it turns out, our approach is able to resolve both of the obstacles in the recursive formulation. First, the state space now has only two continuous dimensions. Second, we show that if the period utility is bounded above and the Markov chain exhibits the monotone likelihood ratio property, then the state space is a convex cone in \( \mathbb{R}^2 \), and we provide a complete characterization of the state space. These findings enable us to solve numerically for the optimal contract, and allow us to confront the implications of the model with micro data.

For the empirical part of the paper, we plan to use panel data on consumption and income. The requisite data was not available for the United States until fairly recently. However, thanks to the work of Blundell, Pistaferri and Preston, there now exists a data set with both income and consumption; this is made possible by imputing consumption into the Panel Study of Income Dynamics (PSID) using empirical relationships estimated on data from the Consumer Expenditure Survey.
(CEX). We are therefore able to obtain covariances between consumption and income, which enables us to measure things like the marginal propensity to consume out of permanent and transitory shocks and compare the model’s implications with what we find in the data.