Public Sector Employment in an Equilibrium Search and Matching Model (Work in Progress)

Jim Albrecht,\textsuperscript{1} Lucas Navarro,\textsuperscript{2} and Susan Vroman\textsuperscript{3}

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\textsuperscript{1}Georgetown University and IZA
\textsuperscript{2}ILADES, Universidad Alberto Hurtado, Santiago, Chile
\textsuperscript{3}Georgetown University and IZA
Introduction

- The public sector accounts for a substantial fraction of employment in both developed and developing countries.

- There is a public-sector wage premium in many countries both in the raw data and after controlling for observables and endogenous sector choice.

- Wages in the public sector tend to be more compressed than in the private sector. The public-sector premium is higher at lower quantiles, and there is a negative public-sector premium at higher quantiles in some countries.
Questions

- How do the private- and public-sector labor markets interact?

- What types of workers tend to work in the public sector and in the private sector?

- How do the size of the public sector and the way that wages are set in that sector affect the overall unemployment rate and the distributions of productivity and wages?
A natural approach to these questions is to incorporate public-sector employment into an equilibrium search and matching model. Surprisingly, there are few papers that do this.


Quadrini and Trigari (ScanJE 2007) – Pissarides (2000) model in which private-sector productivity varies stochastically over time.

Neither paper (nor any others that we know about) allows for worker heterogeneity.
Model - Basic Ingredients

- Worker heterogeneity \(- Y \sim F(y) \) interpreted as human capital \(- \) from ANV (EJ 2009)

- Match-specific productivity \(- X \sim G_s(x | y), s = \{p, g\} \) with a first-order stochastic dominance assumption, i.e., 
  \( y' > y \Rightarrow G_s(x | y') < G_s(x | y) \). A similar idea (with a 2-point distribution for \( Y \)) can be found in Dolado, Jansen and Jimeno (BEJournal Macro 2007)

- The public sector posts \( v_g \) vacancies (exogenous). We formulate the hiring decision as a threshold rule, \( x \geq \zeta_g(y) \), and assume that wages are set according to an exogenous rule, \( w_g(x, y) \).
Model - Assumptions

- Random search – only by the unemployed, no on-the-job search

- The rate at which unemployed workers meet prospective employers is given by an exogenous contact function, $m(\theta)$, where $\theta = (v_p + v_g)/u$. The fraction of those contacts that are with private-sector vacancies is $\phi = v_p/(v_p + v_g)$. The rate at which prospective employers meet job seekers is $m(\theta)/\theta$

- Labor market tightness is endogenous – there is a free-entry condition for private-sector vacancy creation
Model - Assumptions

- Not every contact leads to a match. We derive a reservation acceptance threshold for private-sector jobs for type-$y$ workers. Public sector matches form only if $x \geq \xi_g(y)$

- Private-sector wages are determined by Nash bargaining; public-sector wages are given by $w_g(x, y)$

- Steady-state analysis
Value Functions - Workers

- $U(y)$, $N_p(x, y)$, and $N_g(x, y)$ are the values for a worker of type $y$ associated with unemployment and with private-sector and public-sector jobs with match-specific productivity $x$.

$$rU(y) = z + \phi m(\theta) E \max[N_p(x, y) - U(y), 0] + (1 - \phi) m(\theta) E \max[N_g(x, y) - U(y), 0]$$

$$rN_p(x, y) = w_p(x, y) + \delta_p(U(y) - N_p(x, y))$$

$$rN_g(x, y) = w_g(x, y) + \delta_g(U(y) - N_g(x, y))$$
$J(x, y)$ is the value associated with a private-sector job filled by a worker of type $y$ with match-specific productivity $x$. $V$ is the value of posting a private-sector vacancy.

$$rJ(x, y) = x - w_p(x, y) + \delta_p(V - J(x, y))$$

$$rV = -c + \frac{m(\theta)}{\theta} \mathbb{E} \max[J(x, y) - V, 0]$$

The expectation is taken with respect to the joint distribution of $(X, Y)$ across the unemployed.
Free entry $\implies V = 0$ in steady state. With $V = 0$, the Nash bargaining solution with exogenous worker share $\beta$ gives

$$w_p(x, y) = \beta x + (1 - \beta)rU(y)$$

Again, $w_g(x, y)$ is exogenous.
Reservation Thresholds

- \( R_p(y) \) is the value of \( x \) such that \( N_p(x, y) = U(y) \). Using \( N_p(R_p(y), y) = U(y) \), the Nash bargaining solution implies \( R_p(y) = rU(y) \).

- The reservation threshold for a public-sector job is \( R_g(y) = \xi_g(y) \). This is equivalent to assuming that \( N_g(\xi_g(y), y) \geq U(y) \).
More on Reservation Thresholds

To derive a recursion for $R_p(y)$, we rewrite our expression for $rU(y)$. This gives

$$R_p(y) = z + \phi m(\theta) \frac{\beta}{r + \delta_p} \int_{R_p(y)}^{\bar{x}} (1 - G_p(x|y)) dx$$

$$+ (1 - \phi) m(\theta) \frac{1}{r + \delta_g} \int_{\xi_g(y)}^{\bar{x}} (w_g(x, y) - R_p(y)) dG_g(x|y)$$

Given $\theta$ and $\phi$, the reservation thresholds are uniquely determined.
Free Entry

- The free-entry condition for private-sector vacancy creation is

\[ c = \frac{m(\theta)}{\theta} E \max[J(x, y), 0] \]

Let \( f_u(y) \) be the density of \( y \) across the unemployed. Substituting for \( J(x, y) \) gives

\[ c = \frac{m(\theta)}{\theta} \left( \frac{1 - \beta}{r + \delta_p} \right) \int \int_{\overline{y}} \overline{x} (1 - G_p(x|y)) dx f_u(y) dy \]
Steady-State Conditions

- By Bayes Law

\[ f_u(y) = \frac{u(y)f(y)}{u} \]

where \( u(y) \) is the type-\( y \) unemployment rate and
\( u = \int u(y)f(y)dy \) is the overall unemployment rate.

- The type-specific unemployment rates are derived from steady-state conditions – the flow of type-\( y \) workers from unemployment to private-sector employment must equal the flow in the reverse direction and similarly for transitions between unemployment and public-sector employment.

\[ u(y) = \frac{\delta_g \delta_p}{\delta_g \delta_p + \delta_g \phi m(\theta)(1 - G_p(R_p(y)|y)) + \delta_p(1 - \phi)m(\theta)(1 - G_g(R_g(y)|y))} \]
A Final Unknown

The final unknown is $\phi$, the fraction of vacancies that are posted by private-sector firms. Since $v_p + v_g = \theta u$,

$$\phi = \frac{v_p}{v_p + v_g} \implies \phi = \frac{\theta u - v_g}{\theta u}$$
Equilibrium

- An equilibrium is a function, \( R_p(y) \), and a pair of scalars, \( \theta \) and \( \phi \), that satisfy the relevant equations (private-sector matches are formed iff there is positive surplus, free-entry, steady-state).

- Equilibrium always exists. This can be seen from the free-entry condition:

\[
c = \frac{m(\theta)}{\theta} \left( \frac{1 - \beta}{r + \delta_p} \right) \int_{\bar{y}}^{\bar{x}} \int_{\underline{y}}^{\underline{x}} (1 - G_p(x|y)) dx f_u(y) dy
\]

- The RHS of this equation can be written as a function of \( \theta \) alone. It is continuous in \( \theta \), converges to \( \infty \) as \( \theta \to 0 \), and converges to zero as \( \theta \to \infty \). Once we solve for \( \theta \), we can recover the other equilibrium objects.

- Note that we do not claim uniqueness.
Wage Distributions

Let $H_s(w)$ denote the distribution function of wages paid in sector $s$. We can develop expressions for $H_p(w)$ and $H_g(w)$ as follows. Consider first the distribution of private-sector wages across workers of type $y$, say $H_p(w|y)$.

$$H_p(w|y) = 0 \text{ for } w < w_p(R_p(y), y).$$

For $w < w_p(R_p(y), y)$, we have

$$H_p(w|y) = P[w_p(R_p(y), y) \leq w_p(X, y) \leq w|y]$$
$$= P[R_p(y) \leq \beta X + (1 - \beta)R_p(y) \leq w|y]$$
$$= P[R_p(y) \leq X \leq \frac{w - (1 - \beta)R_p(y)}{\beta}|y]$$
$$= G_p \left( \frac{w - (1 - \beta)R_p(y)}{\beta} | y \right) - G_p \left( R_p(y) | y \right)$$
The unconditional distribution of private-sector wages is

\[ H_p(w) = \int_{\bar{y}}^{y} H_p(w|y)f_p(y)dy, \]

where

\[ f_p(y) = \frac{n_p(y)f(y)}{n_p} \]

is the density of \( Y \) across workers employed in the private sector. The private-sector employment rate for type-\( y \) workers, \( n_p(y) \), is derived from steady-state conditions.

The same approach can be used to find the distribution of public-sector wages.
What’s Next?

- Specify functional forms for $F(y)$ and $G_s(x|y)$
  
  $Y \sim \text{unif}[0, 1]$  
  $X \sim \text{unif}[y, y+1]$  
  $\ln X \sim N(y, \sigma^2)$

- Specify functional forms for $\zeta_g(y)$ and $w_g(x, y)$
  
  $\zeta_g(y) = \alpha + R_p(y)$
  $w_g(x, y) = \gamma x + (1 - \gamma) rU(y) = \gamma x + (1 - \gamma) R_p(y)$

- Calibrate

- Run numerical policy experiments – change $v_g$, $\zeta_g(y)$ and $w_g(x, y)$
Assume $Y \sim \text{unif}[0, 1]$ and $X \sim \text{unif}[y, y + 1]$.

Assumed parameter values:

- $r = 0.012$
- $z = 0.3$
- $c = 0.3$
- $\beta = 0.75$
- $m(\theta) = 3\theta^{0.25}$
- $\delta_p = 0.1$
- $\delta_g = 0.05$
- $v_g = 0.0125$
- $\alpha = 0$
- $\gamma = 0.5$

Results: $\theta = 0.95$, $u = 0.094$, $\phi = 0.86$