The Interplay Between Different Types of Unsecured Credit and Amplification of Consumer Default*

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Abstract

We analyze, theoretically and quantitatively, the interactions between different forms of unsecured credit and their implications for default behavior of young U.S. households. One type of credit mimics credit cards in the U.S. and the default option resembles a bankruptcy filing under Chapter 7 and the other type mimics student loans in the U.S. and the default option resembles Chapter 13 of the U.S. Bankruptcy Code. In the credit card market financial intermediary offers a menu of credit limits and interest rates based on individual credit scores. Scores evolve based on past borrowing and repayment behavior. In the student loan market default has no effect on credit scores. The government sets the interest rate and chooses a wage garnishment to pay for the cost associated with default. We prove the existence of a steady-state equilibrium and characterize the circumstances under which a household defaults on each of these loans depending on household characteristics as well as on the financial arrangements in both markets. Our model is consistent with the main facts regarding borrowing and default on both forms of unsecured credit for young U.S. households. We show that there are important *cross-market* effects: financial arrangements in one market non-trivially affect default in the other market. We plan to use the model to quantify the effects of increased college debt burdens and more severe credit card terms on the increase in default rates in recent years and to conduct policy analysis regarding loan terms and bankruptcy arrangements in both markets.

JEL Codes: D91; I22; G19;

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1 Introduction

We propose a theory about the interactions between unsecured credit markets with different financial arrangements and their impact on consumer default. Our theory is motivated in part by some facts regarding differences in market arrangements in student loan and credit card markets in the U.S., and in particular, the roles played by credit scores and bankruptcy arrangements. As we argue in this paper, this interaction between different arrangements for credit card and student loans induces significant trade-offs in default behavior in the two markets. Yet previous research analyzed these two markets separately with the main focus being on credit card debt. This paper attempts to bridge this gap. Our results are not specific to this interpretation, however, and speak to consumer default in any environments that feature differences in financial market arrangements and thus induce a trade-off in default incentives for consumers that participate in these markets.[1]

The financial and legal environment surrounding the U.S. credit cards and student loan markets is characterized by the following features.

1) Student loans are not secured by any tangible asset, so there might be some similarities with credit card markets, but unlike credit card loans, guaranteed student loans are uniquely risky, since the eligibility conditions are very different. Loans are based on financial need, not on credit ratings, and are subsidized by the government. Agents are eligible to borrow up to the full college cost minus expected family contributions.

2) In contrast, for credit card loans, lenders use credit scores (FICO) as a proxy for the risk of default. FICO scores are based on a large set of information about borrowers past credit history: payment history (35%), amount of outstanding debt (30%), length of credit history (15%), new credit/recent credit inquiries (10%), and types of credit used (10%).[2]

3) For credit cards, lenders impose loan terms (credit limits and interest rates) that vary significantly across individuals of different credit scores. The menu of interest rates is designed to capture the default risks across individuals. In general, in unsecured credit markets the feedback of any bankruptcy law into the interest rate is exactly how the default is paid for.

4) However, the interest rate on student loans does not incorporate the risk that some borrowers might exercise the option to default. The interest rate is fixed by the government. Several default penalties implemented in the student loan program such as wage garnishments upon default might bear part of the default risk.

5) Default on credit card debt triggers a drop in credit scores and thus worse loan terms in the

[1]For instance, another application of this theory may be international consumer credit markets (credit card loans in the U.S. and euro area that feature differences in eligibility conditions and bankruptcy rules).

[2]Source: http://www.myfico.com/CreditEducation/WhatsInYourScore.aspx. Also for a mapping between FICO scores and delinquency rates see Chatterjee, Corbae, and Rios-Rull (2010).

future. As mentioned, borrowing and repayment behavior in this market (even if default does not occur) induce an evolution of credit scores and consequently result in changes in future loan terms. At the same time default in the student loan market has no effect on credit scores.

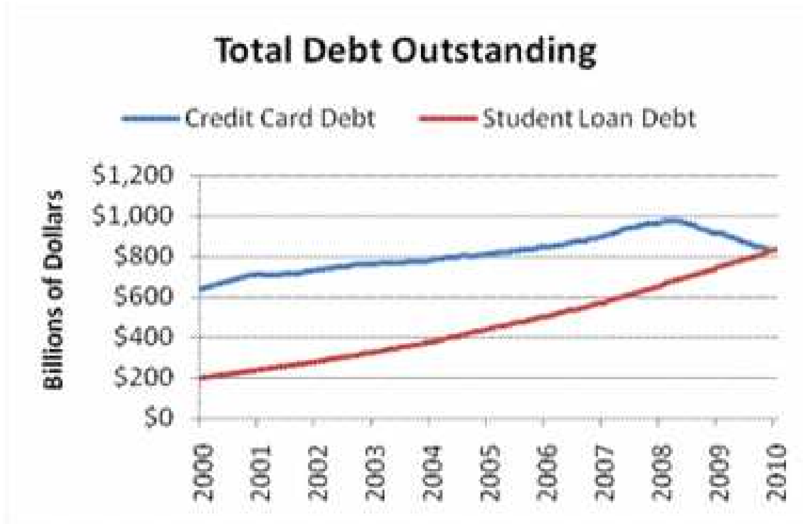
6) Finally, individuals can file for bankruptcy for credit cards under Chapter 7 of the U.S. Bankruptcy Code, which implies permanent discharge of net debt (liabilities minus assets above exemption levels). In contrast, individuals can file for bankruptcy for student loans only under Chapter 13, which does not allow for discharge and implies a fixed term repayment schedule.

These financial arrangements induce different repayment and default incentives for the two types of loans as they assume different insurance mechanisms. For example, consider an individual who experiences a large negative shock and may prefer to default on at least one type of loan. Depending on which loan she defaults on, she may discharge her credit card debt and/or delay repayments on her student loans. On the one hand, the cost associated with the negative impact on credit scores following default on credit card debt could be quite large given significant limitations on future borrowing and consumption smoothing capabilities. On the other hand, non-dischargeability of student loans and wage garnishments may seem quite restrictive in the immediate future. However, this bankruptcy arrangement has no severe implications for future smoothing capabilities. If the individual hit by the negative shock also has high levels of college and credit card debt, restricted market participation is more costly and so she will prefer to default on student loans. These interactions tilt the incentives to default on one type of loan versus the other and are suggestive of amplifying default for student loans. There is no existing work embedding this trade-off in default incentives into a quantitative dynamic model. Also, with several exceptions, there is no theoretical framework that accounts for the importance of credit scores in consumer credit.

Yet, understanding the interaction between these financial arrangements is important in the light of the recent trends in borrowing and default behavior in these two markets. As of June 2010, total student loan debt passed total credit card debt for the first time (see Figure 1). According to the Federal Reserve releases, U.S. households owe $826.5 billion in revolving credit (98% of revolving credit is credit card debt), down from $958 billion at the end of 2008. At the same time they owe $829.785 billion in student loans — both federal and private.[3] Currently 70% of individuals who enroll in college take out student loans (College Board, 2008). The majority of these individuals also have credit card debt (76%), according to our findings from the Survey of Consumer Finance (SCF). These trends are alarming considering the large risks that young borrowers face: data show that job outcome prospects of college going students have worsen. The

[3]Partially, this is a story about paying down credit card debt. At the same time the growth rate in college debt has reached records high in the past several years given the dramatic increase in the net price of college (by 43 and 27 percent for public and private four-year institutions between 1997-98 and 2007-08).

Figure 1:



Source: Federal Reserve's G19 Consumer Credit

college dropout rate has increased dramatically in the past decade (from 38% to 50% for the cohorts that enroll in college in 1995 and 2003 respectively)[4] In addition, the unemployment rate among young workers with college education has jumped up significantly during the Great recession: 8% of young college graduates and 14.1% of young workers with some college are unemployed in 2010 (Bureau of Labor Statistics). At the same time, credit card utilization ratios have risen and credit terms on credit card accounts have worsen adversely affecting households's capability to diversify risk. For young borrowers this is particularly problematic, because even modest balances have more of an impact than the same balance belonging to a consumer who has a much older or robust credit history.[5]

The combination of high income risks, high indebtedness and worse financial terms implies that borrowers are more likely to default on at least one of their loans. Indeed young U.S. households (of which a large percentage have both college and credit card debt) now have the second highest rate of bankruptcy (just after those aged 35 to 44) and the rate among 25- to 34-year-olds increased between 1991 and 2001, indicating that this generation is more likely to file bankruptcy as young adults than were young boomers at the same age.[6]. A couple of questions arise immediately: Which default option do young borrowers find more attractive and why? Is the current

[4]We define the dropout rate as the fraction of students who enroll in college and do not obtain a degree 6 years after they enroll. Numbers are based on the BPS 1995 and 2003 data.

[5]This is because of the principle of revolving utilization as well as the weight given in credit scoring to the age of a consumer's credit file. These borrowers have younger credit reports and fewer accounts, which implies that they are likely be scored in a "thin file" or "young file" score card.

[6]Source:http://www.creditcards.com/

environment conducive to higher default incentives in one of these two markets and does this fact have implications for transferring risk between the two markets?

Data show that student loans have a higher default rate than credit cards or any other loan, including car loans and home loans.[7] The national default rates on student loans increased for three consecutive years for the first time since 1990 and reached records high in the past decade. According to the Department of Education the two-year basis cohort default rate (CDR) was 7.1% in 2008, the highest rate since 1998 and up from 4.5% in 2005.[8] Based on an analysis of the Presidents FY2011 budget, in FY2009 the total defaulted loans outstanding are around $45 billion. The default rate for credit card debt and the total amount outstanding from defaulted credit card debt has not increased at the same pace.

In order to address the proposed issues we develop a general equilibrium economy that mimics features of student and credit card loans. Infinitely lived agents differ in college debt and income levels as well as a credit risk index (score), which determines the loan terms agents face on their credit card accounts (limits and interest rates). Agents face uncertainty in earnings and may save/borrow and as in practice, borrowing terms are individual specific. In equilibrium our model delivers that an agent with a high credit index will receive a lower interest rate and face a more relaxed credit limit on her credit card account relative to an agent with a lower credit index. Central to the model is the decision of young households to repay or default on their credit card and student loans. Consequences to default for student and credit card loans differ in several important ways: for student loans they include a wage garnishment and for credit cards they deliver an evolution of the credit score, which is updated periodically as a result of agent's borrowing and repayment behavior. In turn, the update of the agent's credit score induces changes in the agent's credit card terms. As in practice, good repayment behavior and a low utilization ratio (credit card balance to limit ratio) imply a higher credit score and better credit terms in the future.

In the theoretical part of the paper, we first characterize the default behavior for both markets. We determine the set of debt and income levels for which borrowers default on student loans, on credit card debt or on both types of loans and show how these sets vary with households characteristics as well as with the financial arrangements in the two markets. Our theory explains facts related to borrowing and default behavior of young U.S. households (presented in Section 2):

1. The incentive to default on student loans increases in college debt burden (debt-to-income

[7] According to a survey conducted by the FRB New York, the national student loan delinquency rate 60+ days in 2010 is 10.4 percent compared to only 5.6 percent for the mortgage delinquency rate 90+ days, 1.9 percent for bank card delinquency rate and 1.3 percent for auto loans delinquency rate.

[8] The 2-year CDR is computed as the percentage of borrowers who enter repayment in a fiscal year and default by the end of the next fiscal year.

ratio), i.e. default on student loans is more likely to occur for individuals with low levels of earnings and high levels of college debt. This result was first pointed out in a numerical framework in Ionescu (2008) and is supported by empirical evidence in Dynarki (1994) and our findings from the SCF.

2. Similarly, the incentive to default on credit card debt increases in credit card debt. This result is consistent with findings in Chatterjee et. al. (2007) who also show that the likelihood of default increases in the size of the loan.

3. In addition, a household with a bad credit risk index is more likely to default on credit card debt. There is ample empirical evidence for this fact which is usually assumed in the literature (see Chatterjee et. al. (2010)). Our research provides theoretical support for this approach.

Our main theoretical contribution consists in demonstrating the existence of *cross-market effects* and their implications for default behavior. This contribution is three fold:

1) A borrower with high enough college debt burden and high enough credit card utilization ratio defaults for sure and will choose to default in the student loan market. This result innovates by showing that while a high college debt burden is necessary to induce default on student loans, this effect is amplified by high indebtedness in the credit card market. This arises from the differences in bankruptcy arrangements in the two markets: the financially constrained borrower finds it optimally to default on student loans (even though she cannot discharge her debt) in order to be able to access the credit card market at relatively better terms. Since there is no effect on her credit score from defaulting on student loans the borrower with high credit card utilization ratio and high college debt to income ratio prefers the default penalty in the student loan market over restricted credit card market participation.

2) Tight credit conditions in the credit card market adversely affect default on both credit card debt and on student loans. This result is particularly interesting given the recent adjustments in the credit card terms by card companies as a response to the financial crisis (details are provided in Section 2). Young U.S. households who face worse financial arrangements for credit card accounts will have a higher incentive to default on both student loans and credit card debt. This fact corroborated with high levels of credit card debt and a large increase in college debt burdens imply an amplification of default for student loans but not for credit card debt (this is a direct implication from the previous result). These two results together explain the default patterns in the two markets given the recent trends in borrowing behavior (details are presented in Section 2).

3) Finally, our theory delivers an interesting result that cross-market effects are not symmet-

ric. Specifically, in contrast to finding #2 above, we show that severe consequences to default on student loans adversely affect default on credit card debt and *reduce* default on student loans. Again the differences in bankruptcy arrangements between the two markets drive this asymmetry in cross-market effects. Defaulting on student loans triggers a one time wage garnishment whereas defaulting on credit card debt triggers limited market participation for several periods. Consequently, relatively more severe default consequences for student loans work directly as a force towards reducing default for student loans and increasing credit card defaults (everything else constant) whereas more severe consequences to default on credit cards (as driven by tighter loan terms for any credit score) restrict consumption smoothing possibilities over long term and thus work towards increasing default in both loans. An observable implication of this result is a sharp decline in default rates for student loans and an increase in default rates for credit card debt for young borrowers following a policy change in bankruptcy rules for student loans in the early 1990s.

In the quantitative part of our paper, we parametrize the model to match statistics regarding college debt, credit card debt, income, credit card limits and interest rates of young borrowers with student loans aged 20-26 as delivered by the SCF 2004 and use the model to quantify the effects of the changes in college debt burdens, in credit card utilization ratios and in credit card terms on default rates. We plan to explore the policy implications of our theory, in particular we study loan repayment policies contingent on terms in the other market and income contingent repayments with partial dischargeability.

1.1 Related literature

Our paper is related to two strands of existing literature: default within credit card markets and within student loans. The first strand relates to research by Athreya, Tam, and Young (2009), Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), Chatterjee, Corbae, and Rios-Rull (2010), and Livshits, MacGee, and Tertilt (2007). The two first studies explicitly model a menu of credit levels and interest rates offered by credit suppliers with the focus on default under Chapter 7 within the credit card market. Livshits, MacGee, and Tertilt (2007) quantitatively compare liquidation in the U.S. to reorganization in Germany in a life-cycle model with incomplete markets, earnings and expense uncertainty.

In the student loan literature there are several papers closely related to the current study including research by Ionescu (2010), Ionescu and Simpson (2010) and Lochner and Monge (2010). These papers incorporate the option to default on student loans when analyzing various government policies. Out of these studies, the only one that accounts for the role of credit scores is

Ionescu and Simpson (2010) who recognize the importance of scores in the context of the private market of student loans. The model, however, is silent with respect to the role of scores for credit cards or for the allocation of consumer credit, the study being restricted to the analysis of the student loan market. Ionescu (2010) is the only study that models both dischargeability and non-dischargeability of loans for young U.S. households in the U.S., but only in the context of the student loan market. Furthermore, as in Livshits, MacGee, and Tertilt (2007), Ionescu (2010) studies various bankruptcy rules in distinct environments that mimic different periods in the student loan program (in Livshits, MacGee, and Tertilt (2007) in different countries) rather than modeling them as alternative insurance mechanisms available to borrowers.[9]

Our paper builds on this body of work and improves on the modeling of the outside insurance options available to borrowers with student loans and credit card debt. On a methodological level, our paper is more related to Chatterjee, Corbae, Nakajima, and Rios-Rull (2007). As in their paper, we model a menu of prices for credit card loans based on the individual risk of default. In their paper the probability of default is linked to the size of the loan. We take a step further in this direction. First, we model the menu of prices based on individual credit score as a proxy for the default probability. The score incorporates not only the borrowing behavior of the individual (current size of the loan), but also the individual past repayment and borrowing behavior. In this direction our study is in the spirit of recent work by Chatterjee, Corbae, and Rios-Rull (2010) who provide a theory that explores the importance of credit scores for consumer credit based on a limited information environment. Furthermore, we endogenize both credit limits and interest rates on credit card debt, and allow them to respond to changes in default incentives created by alternative bankruptcy arrangements in the two markets.

To this end, the novelty of our work consists in providing a theory about interactions between credit markets with different financial arrangements and their role in amplifying consumer default. In this respect our paper is related to Chatterjee, Corbae, and Rios-Rull (2008) who provide a theory of unsecured credit based on the interaction between unsecured credit and insurance markets.[10] In related empirical work, Edelberg (2006) studies the evolution of credit card and student loan markets and finds that there has been an increase in the cross-sectional variance of interest rates charged to consumers which is largely due to movements in credit card loans: the premium spread for credit card loans more than doubled, but education loan and other consumer loan premiums are statistically unchanged.

[9] The modeling of alternative bankruptcy rules and induced trade-offs in default decisions poses obvious technical challenges which will be addressed in this paper.

[10] In Chatterjee, Corbae, and Rios-Rull (2008) there is private information about a person's type. The type signaling incentive in insurance markets may change households' incentives to default in credit markets. This approach is motivated by the role of credit scores in unsecured credit and auto insurance markets.

The paper is organized as follows. In Section 2, we describe several important facts about student loans and credit card terms that motivate the current study. We develop the model and present the theoretical results in Section 3. We calibrate the economy to match important features of the markets for student and credit card loans and present quantitative results in Section 4. Section 5 concludes.

2 Facts

Findings documented in this section are based on the SCF data for young borrowers aged 20-26 years old who have some college education (with or without having a college degree) and who took out student loans to finance their college education. They are no longer enrolled in college and they need to repay their student loans. We construct these samples in the SCF 2004 and SCF 2007 for the findings presented in this section. The appendix presents an extended analysis using five more SCF surveys, starting from 1989. Table 1 provides details on facts related to college debt burdens documented in 2-4 and Table 2 provides details for the facts related to credit cards documented in 5-8. Details on the trends in the past 20 years which are documented in 11 are provided in the Appendix. Facts 9 and 10 are based on previous findings documented in Athreya (2002), Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), Dynarsky (1994) and Ionescu (2008).

1. The national two-year basis cohort default rates on student loans increased from 4.5% in 2005 to 7.1% in 2008 (according to the statistics from the Department of Education).

2. College debt borrowed increased by almost 35% during this period of time.

3. Income of young individuals has not kept pace: it only increased by 28.7% according to the SCF data. The unemployment rate of young borrowers increased from 3.76% (2004) to 6.06% (2007).

4. Consequently, college debt burdens (debt-to-income ratios) increased from 0.98 (2004) to 1.15 (2007).

5. Young borrowers with student loans use credit cards at very high rates:
   around 72% of young U.S. households in both years in the SCF have at least a credit card and 76% of those who are credit card users have positive balances.

6. Overall 92% of young consumers with education loans have net negative assets (computed as total unsecured debt minus deposits in CDs, transaction accounts, checking accounts and bond savings).

Table 1: College debt burdens for borrowers with student loans aged 20-26 years old

Year	2004	2007
Amt. college debt borrowed	16467.74/15000 (16968.41)	22204.85/20000 (19283.8)
Amt. college debt outstanding	15607.82/9500 (19857.65)	19920.9/20000 (20712.3)
Income	31029.32/27000 (19128.63)	39944.24/39500 (26287.2)
Debt-to-income ratio	0.9677/0.37 (2.2591)	1.151/0.57 (3.404)
Unemployment rate	3.76%	6.06%

Note: the first number represents the mean, the second the median and the number in parenthesis the standard deviation.

7. Terms on credit card accounts of young borrowers worsen:
   credit limits decline from $14,014 in 2004 to $10,949 in 2007 and the average interest rate increased from 9.9% to 13% during this period. Additionally, the credit card utilization ratio increased from 0.48 to 0.74.

8. Credit card debt burden increased from 2004 (at 0.11) to 2007 (at 0.13) and a higher fraction of young borrowers hardly repay on their credit card debt in 2007 relative to 2004.

9. High college debt burdens increase the likelihood of default for student loans (see Dynarski (1994) and Ionescu (2008) for details).

10. High credit card burdens increase the likelihood of default on credit card debt (see Athreya et. al. (2008), Chatterjee et. al. (2008)).

11. Trends in default behavior, college debt-to-income ratios and credit terms differ across 1990-2005, 2005-2008 such that default rates for student loans decline before 2005 and increase afterwards. At the same time college debt-to-income ratios increase before 2005, however at a slower pace than after 2005, whereas terms on credit cards improve before 2005 and deteriorate after 2005.

These facts seem to suggest that changes in college debt burdens alone cannot explain the recent increase in default rates on student loans. The changes in the credit card market may play a key role in accounting for this trend. The terms on credit cards greatly affect the young household's capability to smooth consumption and pay their education loans.

Furthermore, in the past three years, credit card providers have levied some of the largest increases in interest rates, fees and minimum payments. Credit card issuers cite the recent economic turmoil to explain the changes. They argue that they are responding to record defaults and new regulations (see the Credit Card Act of 2009) by raising fees and interest rates to help the banks

Table 2: Credit cards usage and terms for borrowers with student loans aged 20-26 years old

Variable	2004	2007
Percentage with credit card	69.9%	72.7%
Percentage with credit card balance	78.5%	75%
Percentage with net negative assets	92.5%	90.9%
Credit card rate	10.6/9.9 (5.547)	12.56/13 (4.86)
Credit card limit	14014.36/6000 (22499.5)	10949/7000 (9546.97)
Credit card balance	3318.76/1300 (4837.8)	4155.58/925 (6258.1)
Credit card utilization ratio	0.479/0.42 (0.3296)	0.735/0.64(0.664)
Credit card debt burden	0.1069/0.036 (0.1543)	0.1269/0.02 (0.1758)
Perc. hardly repay credit card	40.3%	43.48%

Note: the first number represents the mean, the second the median and the number in parenthesis the standard deviation.

absorb losses and maintain profit margins. For instance, JPMorgan Chase, the biggest credit card provider, has levied some of the largest increases. The bank raised the minimum payment on outstanding balances from 2% to 5% for some customers, raised its balance-transfer fee from 3% to 5% – the highest rate among the large consumer banks and also changed its United Mileage Plus Visa Signature card from a single 13.24% rate to a range of 13.24% to 19.24%, meaning most cardholders are likely to qualify for those costlier rates (June 30 Bloomberg article). Citigroup has reportedly raised rates on outstanding balances nearly 3 percentage points to an average of 24% for 13 million to 15 million cardholders (July 1 2009 Financial Times article). Retailers have also become stingier with credit. For instance, American Express has taken the most heat over slashing credit limits. Nearly half of its portfolio underwent a major overhaul that included cutting limits by a half or more. Chase decreased credit lines or closed accounts in 2008 totaling $129 billion. Most credit card issuers enforced fees, and reduced credit lines on their in-store cards (examples include Home Depot, Target).

3 Model

3.1 Legal environment

Consumers who participate in the student loan and credit cards markets, namely young U.S. households with student loans, are small, risk-averse, price takers. They differ in levels of college debt and income, as well as credit risk indexes. They are endowed with a line of credit, which they may use for transactions and consumption smoothing. They choose to repay on their student loans as well as on their credit cards. Their repayment behavior in each of these two markets have very different consequences on their characteristics and future decisions.

3.1.1 Credit cards

Bankruptcy for credit cards in the model resembles Chapter 7 "total liquidation" bankruptcy. Additionally, the model captures the fact that credit card issuers use consumer credit risk indexes (FICO scores which represent the most widely used form of credit risk when taking out loans) to assess the likelihood that any single borrower will default. Loan prices and credit limits imposed by credit card issuers are set to account for the individual default risk and are tailored to each credit account.

Consider a household that starts the period with some credit card debt, b_t and a credit risk index f_t. Depending on the household decision to declare bankruptcy as well as on the household borrowing behavior, the following things happen:

1. If a household files for bankruptcy, $\lambda_b = 1$ (and she can do so irrespective of current income or past consumption), then the household unsecured debt is discharged and liabilities are set to 0.

2. The household cannot save during the period when default occurs, which is a simple way of modeling that the U.S. bankruptcy law does not permit those invoking bankruptcy to simultaneously accumulate assets.

3. Household credit risk index for the following period, f_{t+1} is set to the lowest possible value, f_1. Thus a household who defaults on credit in period t starts period $t+1$ with $f_{t+1} = f_1$.

4. A household who starts the period with credit risk index f_1 cannot borrow, but is allowed to save. The risk index can be improved later on and defaulters may be able to borrow at future dates (but at more severe credit terms).

5. In contrast, a household who starts the period with credit risk index $f_t \neq f_1$ is allowed to borrow and save according to individual credit terms: credit limits assigned to household by credit lenders vary with credit risk indexes. Furthermore, loan prices also depend on credit indexes. This latter feature is important to allow for capturing default risk pricing in equilibrium.

6. In the case where the household does not file for bankruptcy, there are consequences on credit risk indexes, which evolve according to $f_{t+1} = g(f_t, b_t, b_{t+1} | \lambda_b = 0)$: the credit index in period $t+1$ depends on the credit index in period t, as well as on repayment behavior in period t, the asset position that the household begins period t with and chooses for period $t+1$.

This formulation captures the idea that there is more restricted market participation for borrowers who have defaulted in the credit card market relative to borrowers who have not. It also implies more stringent credit terms for consumers who take on more credit card debt, i.e. precisely the type of borrowers who are more constrained in their capability to repay their loans. These features are consistent with the fact that credit card issuers reward good repayment behavior and penalize bad repayment behavior. Default is reported to credit agencies and FICO scores are severely damaged. In addition, creditors also take into account borrowing behavior when they determine credit risk indexes. In particular, a high credit utilization ratio signals a high risk borrower and thus the amount borrowed each period together with credit limits play an important role in updating credit risk indexes of borrowers. In the computation of FICO scores, payment history is assigned the highest weight (35 percent), followed by the amount of outstanding debt (30 percent). Finally, defaulters are not in autarky, which is consistent with evidence. In U.S. consumer credit markets, households retain a storage technology after bankruptcy, namely, the ability to save. Furthermore, even the highest-risk borrowers who have defaulted in the past several years are still able to obtain credit at worse terms.

3.1.2 Student loans

Bankruptcy for student loans in the model resembles Chapter 13 "reorganization" bankruptcy, which requires reorganization and repayment of defaulted loans. Under the current Federal Loan Program students who participate cannot discharge on their student loans [11]Consequently default on student loans in the model at period t (denoted by $\gamma_d = 1$) simply means a delay in repayment that triggers the following consequences:

1. There is no debt repayment in period t. However, the college debt is not discharged. The defaulter must repay it in period $t + 1$.

2. The defaulter is not allowed to borrow or save in period t, which is line with the fact that credit bureaus are notified when default occurs and thus access to the credit card market is restricted. Also, as before, this feature captures the fact that the U.S. bankruptcy law does not permit those invoking bankruptcy to simultaneously accumulate assets.

3. There are no consequences on the household credit risk index from defaulting on student

[11] Borrowers are considered in default if they do not make any payments within 270 days in the case of a loan repayable in monthly installments or 330 days in the case of a loan repayable in less frequent installments. Loan forgiveness is very limited. It is granted only in the case constant payments are made for 25 years or in the case where repayment causes undue hardship, but as a practical matter, it is very difficult to demonstrate undue hardship unless the defaulter is physically unable to work.

loans. This assumption is justified by the fact that in practice immediate repayment and rehabilitation of the defaulted loan will result in deleting the default status reported by the loan holder to the national credit bureaus. Most of the defaulters choose to take this path given severe penalties upon repeating default.

4. A fraction μ of the defaulter's wages are garnished in period t. Once the defaulter rehabilitates her student loan, the wage garnishment is interrupted. This penalty captures the default risk for student loans in the model.[12]

5. The household begins next period with a record of default on student loans. Let $h_t \in H = \{0, 1\}$ denote the default flag for a household in period t, where 1 indicates in period t a record of default in the previous period and $h = 0$ denotes the absence of such record. Thus a household who defaults in period t starts period $t + 1$ with $h_{t+1} = 1$.

6. A household that begins period t with a record of default must pay the full outstanding debt which consists in the debt owed in period t, d_t plus the debt defaulted in the previous period adjusted for interest, $d_t(1 + r_g)$. In period $t + 1$ the household starts with $h_{t+1} = 0$ and a new round of loan payment due, d_t.[13]

3.2 Preferences and endowments

At any point in time the economy is composed of a continuum of infinitely lived households with unit mass.[14] Agents differ in three dimensions: student debt levels, $d \in D = [d_{\min}, d_{\max}]$, income levels, $y \in Y = [y_{\min}, y_{\max}]$, and credit scores, $f \in F = \{f_1, ..., f_{N_F}\}$ that measure their credit risk as perceived by credit card lenders. There is a constant probability $(1 - \rho)$ that households will die at the end of each period. Households that do not survive are replaced by newborns who have not defaulted on student loans ($h = 0$), have zero assets ($b = 0$) and with labor income, college debt and credit risk drawn independently from the probability measure space $(Y \times D \times F, \mathcal{B}(Y \times D \times F), \psi)$ where $\mathcal{B}(\cdot)$ denotes the Borel sigma algebra and ψ denotes the joint probability measure. Surviving households independently draw their labor income at time t from a stochastic process.

[12]This penalty can be as high as 15% of defaulter's wages. In addition, consequences include seizure of federal tax refunds, possible hold on transcripts and ineligibility for future student loans.

[13]The household cannot default the following period after default occurs. In practice, less than 1% of borrowers repeat default given that the U.S. government seizes tax refunds in the case when the defaulter does not rehabilitate her loan soon after default occurs. This penalty is severe enough to induce immediate repayment after default (and at a larger amount).

[14]The use of infinitely lived households is justified by the fact that I focus on the cohort default rate for young borrowers, which means that age distributions are not crucial for analyzing default rates in the current study. The use of a continuum of households is natural, given the size of the credit market.

The amount that the household needs to pay on her student loan is the same. Credit indexes evolve endogenously given households' decisions in the previous period. Household characteristics are then defined on the measurable space $(Y \times D, \mathcal{B}(Y \times D))$. The transition function is given by an i.i.d. process $\Phi(y_t)$.

Assumption 1. *We assume that* $y - d \geq 0$ *for any* $d \in D$ *and* $y \in Y$.

This assumption insures that even for the worst possible realization of income the amount owed on student loans each period is not that large to exceed the per period income.[15]

The preferences of the households are given by the expected value of a discounted lifetime utility consists of:

$$E_0 \sum_{t=0}^{\infty} (\rho\beta)^t U(c_t) \tag{1}$$

where c_t represents the consumption of the agent during period t and $\beta \in (0,1)$ is the discount factor and $\rho \in (0,1)$ the survival probability.

Assumption 2. *The utility function* $U(\cdot)$ *is increasing, concave and twice differentiable. It also satisfies Inada condition:* $\lim_{c \to 0^+} U(c) = -\infty$.

3.3 Markets Arrangements

There are several similarities as well as important differences between the credit card market and the market for student loans.

3.3.1 Credit cards

The market for privately issued unsecured credit in the U.S. is characterized by a large, competitive market place where price-taking lenders issue credit through the purchase of securities backed by repayments from those who borrow. These transactions are intermediated principally by credit card issuers. Given a default option and consequences on the credit risk index from default behavior as well as borrowing behavior, the market arrangement departs from the conventional modeling of borrowing and lending. As in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) our model correctly handles the competitive pricing of default risk, a risk that will vary with household characteristics.[16] In this dimension, our model departs from Chatterjee, Corbae, Nakajima, and

[15] This assumption is made for expositional purposes and it is not crucial for the results. In fact, all the results go through if this assumption is relaxed. Details are provided in the Appendix.

[16] Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) handles the competitive pricing of default risk by expanding the "asset space" and treating unsecured loans of different sizes for different types of households (of different characteristics) as distinct financial assets.

Rios-Rull (2007) in several important ways: the default risk is summed up in the household credit index, which encapsulates both repayment and borrowing behavior. Competitive default pricing is achieved through letting prices vary with credit indexes. Also, credit limits vary with household credit indexes. These modeling features are novel in the literature and are meant to capture that fact that in practice the price of the loan depends on various elements such as past repayment behavior, utilization ratio (which includes current loan size), which are summed-up in credit scores. Unsecured credit card lenders use these credit scores as a signal for household credit risks and thus their probability of default. They tailor loan prices to individual credit scores, not only to individual loan sizes. In addition, lenders adjust other elements of credit terms beyond loan prices, such as credit limits, which also depend on individual risks.[17]

A household with credit index f_t can borrow or save by purchasing a single one-period pure discount bond with a face value in a finite set $B(f_t) \subset \mathbb{R}$. The set $B(f_t)$ contains 0 and positive and negative elements. The largest element of $B(f_t)$ is $b_{\max}$, which is the same for all households and the smallest element is $b_{\min}(f_t)$ where $b_{\min}(f_t^{j+1}) < b_{\min}(f_t^j)$ and $b_{\min}(f_t^1) = 0$, which means that individuals with relatively high credit risk indexes are assigned higher credit limits than individuals with relatively low credit indexes. Individuals with the lowest index (which is a result of defaulting in the previous period) cannot borrow. Denote the maximal credit set $B = B(f_n)$ to be the set of loans available to the household with the best credit risk index. Let N_B be the cardinality of this set.[18]

A purchase of a discount bond in period t with a nonnegative face value b_{t+1} means that the household has entered into a contract where it will receive $b_{t+1} \geq 0$ units of the consumption good in period $t+1$. The purchase of a discount bond with a negative face value b_{t+1} and credit index f_t means that the household receives $q_{t,f_{t+1}}(-b_{t+1})$ units of the period-t consumption good and promises to deliver, conditional on not declaring bankruptcy, $-b_{t+1} > 0$ units of the consumption good in period $t+1$; if it declares bankruptcy, the household delivers nothing. The total number of credit indexes is N_f where N_f denotes the cardinality of F. Let the entire set of N_f prices in period t be denoted by the vector $q_t \in R^{N_f}$. We restrict q_t to lie in a compact set $Q \equiv [0, q_{\max}]^{N_f}$ where $0 < q_{\max} < 1$.

In the economy, f_{t+1} evolves endogenously and it is based on the borrower's current credit index,

[17] A similar approach is taken in Chatterjee, Corbae, and Rios-Rull (2010) where credit limits and loan pricing arise from the optimal response of private lenders to limited information about the agent's credit risk.

[18] Note that households are liquidity constrained in the model. The existence of such constraints in credit card markets has been documented by Gross and Souleles (2000). Furthermore, these limits vary across borrowers with different credit risk. Thus the lender responds to default risk by adjusting individual credit limits, among other things. However, overall credit availability has not decreased along with bankruptcy rates over the past several years before the crisis and so aggregate response of credit supply to changing default has not been that large (see Athreya 2002).

f_t, current asset position, b_t, new asset position, b_{t+1}, and current repayment decision, which takes the form $\gamma_{b,t} = 1$, if default occurs and $\gamma_{b,t} = 0$ if it does not. Thus, $f_{t+1} = g(f_t, \lambda_{b,t}, b_t, b_{t+1})$.

$$\begin{aligned}
\text{Case 1: } g(f_t, 1, b_t, b_{t+1}) &= f_1 \\
\text{Case 2: } g(f_t, 0, b_t, b_{t+1}) &= f_t - \Delta \text{ if } b_{t+1} < 0 \text{ and } b_{t+1} < b_t \\
\text{Case 3: } g(f_t, 0, b_t, b_{t+1}) &= f_t + \Delta \text{ if } b_t < 0 \text{ and } b_t < b_{t+1} \\
\text{Case 4: } g(f_t, 0, b_t, b_{t+1}) &= f_t \text{ if } b_t, b_{t+1} \geq 0 \text{ or } b_t = b_{t+1} < 0
\end{aligned} \quad (2)$$

where Δ is the difference between two adjacent credit risk indexes. When the agent enters the current period with credit card loans ($b_t < 0$), default plays an important role. In the case where the borrower defaults on credit card payments ($\gamma_b = 1$), she is severely punished, which means she will receive the worst possible credit index in the future, f_1 (case 1). In the case the borrower does not default ($\gamma_b = 0$), credit indexes are determined by the credit card utilization ratio: credit terms deteriorate if she accumulates more debt, which in turn implies a higher utilization ratio in the current period (case 2) and credit indexes improve if the agent saves or borrows relatively less and thus this ratio declines in the current period (case 3). In the case the household borrows the same amount, credit scores are unchanged (case 4). When the agent enters the current period with no credit card loans ($b_t \geq 0$), default is irrelevant; if she decides to borrow during the current period, her utilization ratio increases, and thus credit scores deteriorate (case 2). If she decides to save in this period ($b_{t+1} \geq 0$), credit terms remain unchanged (case 4). Finally, in the cases where there is no debt or savings in the current period and/or for the future period credit scores are unchanged (case 4).

3.3.2 Student loans

Student loans represent a rather different from unsecured credit. First, loans are primarily provided by the government (either direct or indirect and guaranteed through the FSLP), and do not share the features of a competitive market[19] Unlike for credit cards, the interest rate on student loans is fixed by the government (r_g) and does not reflect the risk of default in the student loan market.[20] However, the penalties for default capture some of this risk. In particular, the wage garnishment is adjusted to cover default. More generally, loan terms are based on financial need, not on credit

[19]Recently, students started to use pure private student loans, not guaranteed by the government. This new market is a hybrid between government loans and credit cards featuring characteristics of both markets. However, concerns about the national default rates are specific to student loans in the government program, default rates for pure private loans being of much lower magnitudes (for details see Ionescu and Simpson (2010)).

[20]After the Higher Education Reconciliation Act of 2005 was passed, the interest rate has been set to 6.8%. Before 2005 the rate was based on the 91-day Treasury-bill rate.

risk. Secondly, taking out student loans is a decision made during college years. Once they are out of college, households need to repay their loans in equal rounds over a determined period of time subject to the fixed interest rate. Thus the amount of student loan owed each period, d, is fixed given the total college debt, loan duration and the interest rate. I model college loan bound households that are out of school and need to repay d per period; there is no borrowing decision for student loans.[21]

Agents decide to repay on their student loans ($\gamma_d \in \{0, 1\}$). In the case they default ($\gamma_d = 1$), a wage garnishment is imposed. Interest accumulates on their college loans and they need to repay this amount during the following period after default occurs. Recall that in the following period the defaulter receives a student loan default flag, $h = 1$ and in the case default does not occur then the household starts the next period with $h = 0$.

We define the state space of credit characteristics of the households by $\mathcal{S} = B \times F \times H$ to represent the asset position, the credit risk index and student loan default flag. Let $N_{\mathcal{S}} = N_B \times N_F \times 2$ be the cardinality of this set.

To this end, an important note is that the assumption that all debt that young borrowers access is unsecured is made for a specific purpose and is not restrictive. The model is designed to represent the section of households who have student loans and credit card debt. As argued, these borrowers rely on credit cards to smooth consumption and have little or no collateral debt.

3.4 Decision problems

The timing of events in any period are: (i) idiosyncratic shocks, y_t are drawn for survivors and newborns and a credit index and college debt is drawn for newborns; (ii) households default/repay on both credit card and student loans and borrowing/savings decisions are made; also consumption takes place and the college debt and credit index for the next period is determined. I focus on steady state equilibria where $q_t = q$.

3.4.1 Households

I will present the households' decision problem in a recursive formulation where any period t variable x_t is denoted by x and its period $t + 1$ value by x'.

Each period, given their college debt, d, current income, y, credit index, f, and beginning-of-period assets, b, households must choose consumption, c and asset holdings to carry forward into the next period, b'. In addition, agents may decide to repay/default on their student loans, $\gamma_d \in \{0, 1\}$

[21] While returning to school and borrowing another round of loans is a possibility, this decision is beyond the scope of the paper.

and credit card loans, $\gamma_b \in \{0, 1\}$. As described before, these decisions have different consequences: while default on student loans implies a wage garnishment μ and no effect on credit terms, default on credit card payments deteriorates credit terms and have no effect on income. Agents repayment and borrowing behavior produces a law of motion for credit risk index $f^{'} = g(f, \lambda_b, b, b^{'})$.

The household's current budget correspondence, $B_{b,f,h,\lambda_b,\lambda_d}(d, y; q)$ depends on the exogenously given income, y, college debt, d, credit index, f, beginning of period asset position, b, and student loan default record, h. It also depends on default/repayment decisions in both student loan and credit card markets, λ_d and λ_b as well as on the prices in the credit card market, q.

1. If a household with income y has a good student loan record, $h = 0$ and does not exercise its default option in either market, $\lambda_d = 0$ and $\lambda_b = 0$, then

$$B_{b,f,0,0,0}(d, y; q) = \{c \in R_+; b^{'} \in B(f); f^{'} \in F; c + q_{f'} b^{'} \leq y + b - d; f^{'} = g(f, b, b^{'})\}$$

In this case both loans are paid off and the household's credit index improves or worsens depending on its new asset position, $b^{'}$ relative to the previous position, b.

2. If a household with income y and credit card debt, $b < 0$ has a good student loan record, $h = 0$ and exercises its default option in the credit card market only, $\lambda_d = 0$ and $\lambda_b = 1$, then

$$B_{b,f,0,1,0}(d, y; q) = \{c \in R_+; b^{'} = 0; f^{'} = f_1; c \leq y - d\}$$

In this case, there is no repayment on credit card debt; however the household pays the student loan debt. Consequently, the household cannot borrow or accumulate assets and the credit index is severely damaged (to the lowest possible value).

3. If a household with income y has a good student loan record, $h = 0$ and exercises its default option in the student loan market only, $\lambda_d = 1$ and $\lambda_b = 0$, then

$$B_{b,f,0,0,1}(d, y; q) = \{c \in R_+; b^{'} = 0; f^{'} = f; c \leq y + b\}$$

There is no repayment on student loans. But the household pays back the credit card debt (if net liabilities, $b < 0$). As before, the household cannot borrow or accumulate assets. However, there are no consequences on the credit risk index, $f^{'} = f$. If they were, then the individuals will see an improvement in the credit index, given that $b^{'}$ has to be 0 (case in Equation).

4. If a household with income y and credit card debt, $b < 0$ has a good student loan record, $h = 0$ and exercises its default option in both markets, $\lambda_d = 1$ and $\lambda_b = 1$, then

$$B_{b,f,0,1,1}(d, y; q) = \{c \in R_+; b^{'} = 0; f^{'} = f_1; c \leq y\}$$

The household does not pay back any loan. Thus the household cannot borrow or accumulate assets and the credit index is severely damaged (to the lowest possible value).

5. If a household with income y has a bad student loan record, $h = 1$ then the household does not exercise its default option in either market, $\lambda_d = 0$ and $\lambda_b = 0$

$$B_{0,f,1,0,0}(d, y; q) = \{c \in R_+; b^{'} \in B(f); f^{'} \in F; c + q_{f'}b^{'} \leq y(1-\gamma) - d(2 + r_g); f^{'} = g(f, 0, b^{'})\}$$

This case is similar to case 1 where the household pays off both loans. However, the household has a bad student loan record and thus pays back the amount of college debt owed this period plus the amount that was not paid in the previous period and was not discharged. Interest rate r_g accumulates to outstanding debt. The household is not allowed to repeat default by choice. Also since there is no credit card debt there is no default option available for credit cards.

There are several important observations: 1) we account for the fact that the budget constraint may be empty; in particular if the household is deep in debt, earnings are low, new loans are expensive, then the household may not be able to afford non-negative consumption. The implication of this is that involuntary default may occur. This situation applies to cases 1, 2 and 5; 2) Repeated default (and thus discharge) on credit card debt may occur, in line with Chatterjee et. al. (2008); and 3) Repeated default on student loans occurs on a limited basis (i.e. when $B_{0,f,1,0,0}(d, y; q) = \emptyset$) and it is followed by partial dischargeability, which is in line with the data.

Assumption 3. *We assume that consuming y_{min} today and starting with zero assets, $b = 0$ and a bad credit record, $f = f_1$ and student loan default record, $h = 1$ with more debt to repay on student loans and garnished wage (i.e. the worst utility with a feasible action) gives a better utility than consuming zero today and starting next period with maximum savings, b_{max} and a good credit record, $f \neq f_1$ and student loan default record, $h = 0$ (i.e. the best utility with an unfeasible action).*

Let $v(d, y; q)(b, f, h)$ denote the expected lifetime utility of a household that starts with asset b, credit index f, student loan debt d and default record h, has earnings y and faces prices q. Then v is in the set $\mathcal{V}$ of all continuous functions $v : D \times Y \times Q \to \mathbb{R}^{N_S}$. The household's optimization problem can be described in terms of an operator $(Tv)(d, y; q)(b, f, h)$ which yields the maximum lifetime utility achievable if the household's future lifetime utility is assessed according to a given function $v(d, y; q)(b, f, h)$.

Definition 1. For $v \in \mathcal{V}$, let $(Tv)(bd, y; q)(b, f, h)$ be defined as follows:

1. For $h = 0$ and $b \geq 0$

$$
\begin{aligned}
(Tv)(d,y;q)(b,f,0) \;&= \max \;\Bigg\{ \max_{c,b' \in B_{b,f,0,0,0}} U(c) + \beta\rho \int v_{b',f',0}(d,y^{'};q)\Phi(dy^{'}), \\
&\quad U(y+b) + \beta\rho \int v_{0,f,1}(d(2+r_g),y^{'};q)\Phi(dy^{'}) \Bigg\}
\end{aligned}
$$

where $f' = g(f,b,b')$.

2. For $h = 0$, $B_{b,f,0,0,0}(d,y;q) \neq \emptyset$, $B_{b,f,0,0,1}(d,y;q) \neq \emptyset$ and $b < 0$

$$
\begin{aligned}
(Tv)(d,y;q)(b,f,0) \;&= \max \;\Bigg\{ \max_{c,b' \in B_{b,f,0,0,0}} U(c) + \beta\rho \int v_{b',f',0}(d,y^{'};q)\Phi(dy^{'}), \\
&\quad U(y+b) + \beta\rho \int v_{0,f,1}(d(2+r_g),y^{'};q)\Phi(dy^{'}) \\
&\quad U(y-d) + \beta\rho \int v_{0,f_1,0}(d,y^{'};q)\Phi(dy^{'}) \\
&\quad U(y) + \beta\rho \int v_{0,f_1,1}(d(2+r_g),y^{'};q)\Phi(dy^{'}) \Bigg\}
\end{aligned}
$$

where $f' = g(f,b,b')$.

3. For $h = 0$, $B_{b,f,0,0,0}(d,y;q) \neq \emptyset$, $B_{b,f,0,0,1}(d,y;q) = \emptyset$ and $b < 0$

$$
\begin{aligned}
(Tv)(d,y;q)(b,f,0) \;&= \max \;\Bigg\{ \max_{c,b' \in B_{b,f,0,0,0}} U(c) + \beta\rho \int v_{b',f',0}(d,y^{'};q)\Phi(dy^{'}), \\
&\quad U(y-d) + \beta\rho \int v_{0,f_1,0}(d,y^{'};q)\Phi(dy^{'}) \\
&\quad U(y) + \beta\rho \int v_{0,f_1,1}(d(2+r_g),y^{'};q)\Phi(dy^{'}) \Bigg\}
\end{aligned}
$$

4. For $h = 0$, $B_{b,f,0,0,0}(d,y;q) = \emptyset$, $B_{b,f,0,0,1}(d,y;q) \neq \emptyset$ and $b < 0$

$$
\begin{aligned}
(Tv)(d,y;q)(b,f,0) \;&= \max \;\Bigg\{ U(y+b) + \beta\rho \int v_{0,f,1}(d(2+r_g),y^{'};q)\Phi(dy^{'}) \\
&\quad U(y-d) + \beta\rho \int v_{0,f_1,0}(d,y^{'};q)\Phi(dy^{'}) \\
&\quad U(y) + \beta\rho \int v_{0,f_1,1}(d(2+r_g),y^{'};q)\Phi(dy^{'}) \Bigg\}
\end{aligned}
$$

where $f' = g(f,b,b')$.

5. For $h = 0$, $B_{b,f,0,0,0}(d, y; q) = \emptyset$ and $B_{b,f,0,0,1}(d, y; q) = \emptyset$ and $b < 0$

$$\begin{aligned}(Tv)(d, y; q)(b, f, 0) \quad &= \max \quad \Big\{U(y-d) + \beta\rho \int v_{0,f_1,0}(d, y^{'}; q)\Phi(dy^{'}) \\ &\qquad U(y) + \beta\rho \int v_{0,f_1,1}(d(2+r_g), y^{'}; q)\Phi(dy^{'})\Big\}\end{aligned}$$

6. For $h = 1$, $B_{0,f,1,0,0}(d, y; q) \neq \emptyset$

$$(Tv)(d, y; q)(b, f, 1) \quad = \quad \max_{c,b' \in B_{b,f,1,0,0}} U(c) + \beta\rho \int v_{b',f',0}(d, y^{'}; q)\Phi(dy^{'})$$

where $f' = g(f, 0, b')$.

7. For $h = 1$, $B_{0,f,1,0,0}(d, y; q) = \emptyset$

$$(Tv)(d, y, ; q)(b, f, 1) \quad = \quad U(y(1-\gamma)) + \beta\rho \int v_{b_{min}(f),f',0}(d, y^{'}; q)\Phi(dy^{'})$$

where $f' = g(f, 0, b_{min}(f))$.

The first part of this definition says that a household with a good student loan default record and no credit card debt, may choose to default only on student loans since default on credit card is not an option. The budget set conditional on not defaulting on both loans is non-empty in this case given Assumption 1. In the case the household chooses to repay student loans she may also choose borrowing and savings. In the case she decides to default there is no choice on assets position. In the case where both default and no default options deliver the same utility the household may choose either. The second part says that if the household with a good student loan default record has credit card debt in addition to student loans, she can choose one of the following: repay on both loans, default on student loans, default on credit card loan or default on both loans. For all these cases to occur we need to have that the budget sets conditional on not defaulting on both loans and conditional on defaulting on student loans are non-empty. In the case at least one of them is empty, as in parts 3-5 of the definition, then automatically the attached option (default on both loans and/or default on student loans, respectively) is not available. For instance, in part three, the household may choose between not defaulting on either loan, defaulting on credit card debt or defaulting on both types of loans. In cases 4 and 5 involuntarily default occurs since not defaulting is not an option. However, the household may choose the optimal default option: default on student loans, default on credit cards or default on both (case 4); default on student loans only is not an option in case 5.

The last two parts represent cases for a household with a bad student loan default record. In these last cases, default on student loans is not an option. Also since there is no credit card debt given default in the previous period there is also no choice to default on credit card. Thus, the household simply solves a consumption/savings decision in part 6. In part 7, however, the budget set conditional on not defaulting on either loan is empty and involuntarily default occurs. Here we assume that the households pays off part of her student loan, up to the amount that she is allowed to borrow in the credit card market, $b_{min}(f)$. The remaining amount is discharged. This feature captures the fact that a very small proportion of households partially discharge their student loan debt.[22]

We next proceed as follows: we provide a first set of results which contains the existence and uniqueness of the household's problem and the existence of the invariant distribution. The second set of results contains the characterization of both default decisions in terms of households characteristics and market arrangements. We prove the existence of cross-market effects: financial arrangements in one market affect default behavior in the other market. The last set of results contains the existence of the equilibrium and the characterization of prices. All the proves are provided in the Appendix.

Existence and uniqueness of a recursive solution to the household's problem

Theorem 1. *There exists a unique $v^* \in V$ such that $v^* = Tv^*$ and*

1. *v^* is increasing in y, b and f and decreasing in d.*

2. *The optimal policy correspondence implied by Tv^* is compact-valued, upper hemi-continuous.*

3. *Default is strictly preferable to zero consumption and optimal consumption is always positive*

Since Tv^* is a compact-valued upper-hemi continuous correspondence by Theorem 7.6 in Stockey, Lucas, and Prescott (Measurable Selection Theorem) there are measurable policy functions, $c^*(d, y, ; q)(b, f, h)$, $b^*(d, y; q)(b, f, h)$, $f^*(d, y; q)(b, f, h)$, $\lambda_b^*(d, y; q)(b, f, h)$ and $\lambda_d^*(d, y; q)(b, f, h)$. These measurable functions together with the function $g(\cdot)$ determine a transition matrix for f, f', namely $F^*_{y,d,h,q} : B \times F \times F \to [0, 1]$:

$$F^*_{y,d,h,q}(b, f, f' = f_1) = \begin{cases} 1 & \text{if } \lambda_b^* = 1 \\ 0 & \text{otherwise} \end{cases}$$

[22]In practice dischargeability is limited to undue hardships and occurs in less than 1% of the default cases.

$$F^*_{y,d,h,q}(b, f, f' = f_i)_{i=2,\ldots,n} = \begin{cases} 0 & \text{if } \lambda^*_b = 1 \\ 0 & \text{if } |f - f_i| > \Delta \\ 1 & \text{if } f_i = f \text{ and } b \geq 0, b^* \geq 0 \text{ or } b = b^* < 0, \lambda^*_b = 0 \\ 1 & \text{if } f_i = f + \Delta \text{ and } b < 0, b < b^*, \lambda^*_b = 0 \\ 1 & \text{if } f_i = f - \Delta \text{ and } b^* < 0, b > b^*, \lambda^*_b = 0 \\ 0 & \text{otherwise} \end{cases}$$

Also the policy functions imply a transition matrix for the student loan default record, $H^*_{y,d,b,f,q} : H \times H \to [0,1]$ which gives the student loan record for the next period, h':

$$H^*_{y,d,b,f,q}(h' = 1) = \begin{cases} 1 \text{ if} & \lambda^*_d = 1 \\ 0 \text{ if} & \lambda^*_d = 0 \end{cases}$$

$$H^*_{y,d,b,f,q}(h' = 0) = \begin{cases} 1 \text{ if} & \lambda^*_d = 0 \\ 0 \text{ if} & \lambda^*_d = 1 \end{cases}$$

Existence of invariant distribution

Let $X = Y \times D \times F \times B \times H$ be the space of household characteristics. Then the transition function for the surviving households' state variable $TS^*_q : X \times \mathcal{B}(X) \to [0,1]$ is given by

$$TS^*_q(y, d, f, b, h, Z) = \int_{Z_y \times Z_d \times Z_f \times Z_h} \mathbf{1}_{\{b^* \in Z_b\}} F^*_q(y, d, b, h, f, df') H^*_q(y, d, f, b, h, dh') \Phi(dy')$$

where $Z = Z_y \times Z_d \times Z_f \times Z_b \times Z_h$ and $\mathbf{1}$ is the indicator function. The households that die are replaced with newborns. The transition function for the newborn's initial conditions, $TN^*_q : X \times \mathcal{B}(X) \to [0,1]$ is given by

$$TN^*_q(y, d, f, b, h, Z) = \int_{Z_y \times Z_d \times Z_f} \mathbf{1}_{\{(b',h')=(0,0)\}} \Psi(dy', dd', df')$$

Combining the two transitions we can define the transition function for the economy, ${T^*}_q : X \times \mathcal{B}(X) \to [0,1]$ by

$${T^*}_q(y, d, f, b, h, Z) = \rho TS_q(y, d, f, b, h, Z) + (1 - \rho) TN_q(y, d, f, b, h, Z)$$

Given the transition function T_q^*, we can describe the evolution of the distribution of households μ across their state variables (y, d, f, b, h) for any given prices q. Specifically, let $\mathcal{M}(x)$ be the space of probability measures on X. Define the operator $\Gamma_q : \mathcal{M}(x) \to \mathcal{M}(x)$:

$$(\Gamma_q \mu)(Z) = \int T_q^*((y, d, f, b, h), Z) d\mu(y, d, f, b, h).$$

Theorem 2. *For any $q \in Q$ and any measurable selection from the optimal policy correspondence there exists a unique $\mu_q \in \mathcal{M}(x)$ such that $\Gamma_q \mu_q = \mu_q$.*

3.4.2 Characterization of the default decisions and the interplay between the two markets

We establish results on how the default decision in each market is determined by college debt burdens (college debt-to-income ratios). We first determine the set for which default occurs for student loans (including the worst form of default with partial dischargeability), the set for which default occurs for credit card debt, as well as the set for which default occurs for both of these two loans. Let $D^{SL}_{b,f,1}(q)$ be the set for which involuntarily default on student loans and partial dischargeability occurs. This set is defined as combinations of earnings, y and student loan amount, d for which $B_{b,f,1,0,0}(b, d, y; q) = \emptyset$ in the case $h = 1$. For $h = 0$ let $D^{SL}_{b,f,0}(d; q)$ be the set of earnings for which the value from defaulting on student loans exceeds the value of not defaulting on student loans. Similarly, let $D^{CC}_{b,f,0}(d; q)$ be the set of earnings for which the value from defaulting on credit card debt exceeds the value of not defaulting on credit card debt in the case $h = 0$. Finally, let $D^{Both}_{b,f,0}(d; q)$ be the set of earnings for which default on both types of loans occurs.

Theorem 3 characterizes the sets when default on student loans occurs (voluntarily or nonvoluntarily). Theorem 4 characterizes the sets when default occurs on credit card debt and Theorem 5 presents the set for which default occurs for both types of loans.

Theorem 3. *Let $q \in Q$, $f \in F$, $b \in B(f)$. If $h = 1$ and the set $D^{SL}_{b,f,1}(q)$ is nonempty, then $D^{SL}_{b,f,1}(q)$ is closed and convex. In particular the sets $D^{SL}_{b,f,1}(d; q)$* are closed intervals for all d. If $h = 0$ and *the set $D^{SL}_{b,f,0}(d; q)$ is nonempty,* then $D^{SL}_{b,f,0}(d; q)$ is a closed interval for all d.

Theorem 4. *Let $q \in Q$, $(b, f, 0) \in \mathcal{S}$.* If $D^{CC}_{b,f,0}(d; q)$ is nonempty then it is a closed interval for all d.

Theorem 5. *Let $q \in Q$, $(b, f, 0) \in \mathcal{S}$. If the set $D^{Both}_{b,f,0}(d; q)$* is nonempty then it i*s a closed interval* for all d.

Next we determine how the set of default on credit card debt varies with the credit card debt and with the credit score of the individual. Specifically, Theorem 6 shows that the set of default on credit card debt expands with the amount of debt for credit cards. This result is not new. It was first demonstrated in Chatterjee et. a. (2007). In addition, we show that this default set is expands when the credit score declines, which imply that individuals of worse credit scores have are more likely to default on credit card debt (Theorem 7). This reasoning is assumed in the literature. Our result provides theoretical support for this approach.

Theorem 6. *For any price $q \in Q$, $f \in F$ and $h \in H$, the set $D^{CC}_{b,f,h}(q)$ expand when b decreases, for all $d \in D$.*

Theorem 7. *For any price $q \in Q$, $b \in B$ and $h \in H$, the sets $D^{CC}_{b,f,h}(d;q)$ and $D^{Both}_{b,f,h}(d;q)$ shrink when f increases, for all $d \in D$.*

Since the novel feature in this paper is the interaction between different types of markets and its effects on default decisions, we show how the default decision in each market varies not only with the loan amount in the respective market, but also with the loan amount in the other market. Theorem 8 demonstrates that for a high enough college debt burden (college debt relative to income) a borrower who defaults will choose to default on student loans rather than on credit card debt. In addition, if the borrower's credit card utilization ratio is also high a borrower defaults for sure, and hence will default in the student loan market.

Theorem 8. *For any price q, there is $\varepsilon_1 > 0$ such that if $y - d < \varepsilon_1$ then a defaulter will choose to default on student loans rather then on credit card debt. Furthermore, we can find $\varepsilon_2 > 0$ such that if $b - q_{f'} b_{min}(f) < \varepsilon_2$, where $f' = g(f, b, b_{min}(f))$, then the agent defaults.*

The intuition behind this result is that with very high college debt relative to earnings, consumption is very small in the case the agent does not default at all or defaults only on credit card debt. Therefore the agent chooses to default on student loans only. In addition, if the borrower's credit card debt utilization ratio is also very high (i.e. the amount of debt the borrower brings into the period is high relative to the amount of debt she/he is allowed to borrow in the current period) then the borrower will default for sure since the option of no defaulting at all delivers almost 0 consumption. This result clearly illustrates the fact that when borrowers find themselves in financial hardship and have to default they will always choose to default on student loans. The financial arrangements in the two markets, and in particular the difference in bankruptcy rules and default consequences between the two types of credit certainly play an important role in shifting these default incentives for these types of borrowers. Next we analyze these issues in more detail:

we first present the financial intermediary and the government's decision problems and then we study how the two market arrangements impact default behavior for both types of loans.

3.4.3 Financial intermediaries

The (representative) financial intermediary has access to an international credit market where it can borrow or lend at the risk-free interest rate $r \geq 0$. The intermediary operates in a competitive market and takes prices as given and the chooses the set of probabilities $\xi_{f_t}(b_t)$ on $B(f_t)$. These represent the weight assigned to each loan size $b_t \in B(f_t)$ for individuals with credit score f_t. So the intermediary picks $\xi_{f_t}(b_t)$ for all type (f_t, b_t) contracts for each t to maximize the present discounted value of current and future cash flows $\sum_{t=0}^{\infty}(1+r)^{-t}\pi_t$, given that $\xi_{f-1} = 0$. The period t cash flow is given by

$$\pi_t = \rho \sum_{f_{t-1}\in F} \sum_{b_t \in B(f_{t-1})} (1 - p^b_{f_{t-1}})\xi_{f_{t-1}}(b_t)(-b_t) - \sum_{f_t \in F} \sum_{b_{t+1}\in B(f_t)} \xi_{f_t}(b_{t+1})(-b_{t+1})q_{f_t} \tag{3}$$

where $p^b_{f_t}$ is the probability that a contract of type f_t where $b_t < 0$ experiences default; if $b_t > 0$, automatically $p^b_{f_t} = 0$. These calculations take into account the survival probability ρ.

If a solution to the financial intermediary's problem exists, then optimization implies $q_{f_{t+1}} \leq \frac{\rho}{(1+r)}(1 - p^b_{f_t})$ if $b_{t+1} < 0$ and $q_{f_{t+1}} \geq \frac{\rho}{(1+r)}$ if $b_{t+1} \geq 0$. If any optimal $\xi_{f_t}(b_t)$ are nonzero the associate conditions hold with equality. Our problem is a natural extension of the one described in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) in that we consider weights that depend on the loan size (as in their paper). In addition, weights in our model depend on credit scores as well. Our approach will automatically deliver credit limits assigned to individuals of credit risk f_t, i.e. the smallest element, $b_{\min}(f_t)$ in $B(f_t)$. In equilibrium we obtain that $b_{\min}(f^1_t) = 0$ and $b_{\min}(f^{j+1}_t) < b_{\min}(f^j_t)$, consistent with empirical observations.

3.4.4 Government

The only purpose for the government in this model is to operate the student loan program. The government needs to collect all student loans provided for college education. The cost to the government is the total amount of college loans plus the interest rate subsidized in college. Denote by D this loan price. We compute the per period payment on student loans, d as the coupon payment of a student loan with its face value equals to its price (a debt instrument priced at par) and infinite maturity (console). Thus the coupon rate equals its yield rate, r_g. In practice, this represents the government interest rate on student loans. When no default occurs the present value of coupon payments from all borrowers (revenue) is equal to the price of all the loans made (cost), i.e. the government balances its budget.

$$\int D d\mu_t = \int d \sum_{t=0}^{\infty} (1+r_g)^{-t} d\mu_t$$

where μ_t is the distribution of households over $B \times F \times \{0,1\} \times D \times Y$ at time t.

However, since default is a possibility, government's budget constraint may not hold. In this case the government revenue from a household in state b with credit index f, income y and college debt d is given by $(1-p^d)d + p^d\gamma y$. The government will choose the default penalty, γ to recover the losses incurred when default for student loans arises. The budget constraint is then given by

$$\int d \sum_{t=0}^{\infty} (1+r_g)^{-t} d\mu_t = \int \sum_{t=0}^{\infty} (1+r)_g^{-t} [(1-p^d)d + p^d \gamma y] d\mu_t$$

The penalty γ is chosen such that the budget constraint balances.[23] We turn now to the definition of equilibrium in the model.

3.4.5 Cross-effects of market arrangements on default decisions

In this section we show how default incentives for both types of loans vary with credit card market tightness.

Definition 2. We say that economy E_1 features a relatively tighter credit card market compared to economy E_2 if for any $f \in F$, $b_{\min}^{E_1}(f) \geq b_{\min}^{E_2}(f)$ where $b_{\min}^{E_i}(f)$ is the smallest element of $B(f)$ in economy E_i, $i = 1, 2$.

Our definition assumes that terms on loans are relatively more severe for any credit score in the sense that the support of loans in the credit card market shrinks for each credit score. This means that the maximum amount each individual is eligible to borrow decreases in a tighter economy. This analysis is motivated by the recent changes in credit card terms as presented in Section 2. We also discuss how default incentives for both types of loans are affected by the severity of default consequences on government student loans, i.e. an increase in wage garnishments following default.

Theorem 9. *The incentive to default on any type of loan or on both increases when the market tightens.*

[23] Note that the additional collection that the government gets from defaulters ($d(1+r_g)$ in the following period after default occurs) is justified to cover additional costs incurred when default occurs, which are associated with attorney and collection fees. We abstract from including both the additional source of revenue and cost in the budget constraint, as the two terms simply cancel each other out, assumption which is in line with the data.

Theorem 10. *The incentive to default on student loans (credit card debt) decreases (increases) in γ.*

The intuition behind Theorem 9 is that when credit availability shrinks, the credit card market becomes less functional. It restricts individuals' capabilities of smoothing out consumption. Consequently they will be more inclined to use any of the available default options for this purpose. On the other hand, stricter rules in the student loan market limit the role of student loan default as an insurance mechanism but have no bearing on market tightness. Therefore individuals will shift their default incentives towards the credit card market.

Our theory produces several facts consistent with the reality (presented in Section 2):

1. First, the incentive to default on student loans increases in college debt burden (debt-to-income ratio), i.e. default on student loans is more likely to occur for individuals with low levels of earnings and high levels of college debt. This result was first pointed out in a numerical framework in Ionescu (2008).

2. Similarly, the incentive to default on credit card debt increases in credit card debt. This result is consistent with findings in Chatterjee et. al. (2007) who also show that the likelihood of default increases in the size of the loan.

3. In addition, a household with a bad credit risk index is more likely to default on credit card debt. This result is consistent with empirical findings and it is usually assumed in the literature. Our research is the first to provide theoretical support for this approach.

In addition, our theory confirms our conjectures regarding cross-market effects:

1. A borrower with high enough college debt burden and high enough credit card utilization ratio defaults for sure and will choose to default in the student loan market.

2. Tight credit conditions in the credit card market adversely affect default on credit card debt and on student loans.

These last two results innovate by showing that while a high college debt burden is necessary to induce default on student loans, this effect is amplified by high indebtedness in the credit card market and/or worse financial arrangements for credit card accounts. Finally, our theory delivers an interesting result that market cross-effects are not symmetric. Specifically, in contrast to finding #2 above, we show that severe consequences to default on student loans adversely affect default on credit card debt while reducing default on student loans.

We next define and characterize the equilibrium in the economy.

3.5 Steady-state equilibrium

In this section we define a steady state equilibrium, prove its existence and characterize the properties of the price schedule for individuals with different credit risk indexes.

Definition 3. A steady-state competitive equilibrium is a set of strictly positive price r^* and a non-negative price vector $q^* = (q^*_{f'})_{f' \in F}$, non-negative credit card loan default frequency vector $p^{b*} = (p^{b*}_f)_{f \in F}$, a non-negative student loan frequency p^{d*}, non-negative default penalty, γ^*, a vector of non-trivial credit card loan measures, $\xi^* = (\xi_f)_{f \in F}$, decision rules $b'^*(y, d, f, b, h, q)$, $\lambda^{b*}(y, d, f, b, h, q)$, $\lambda^{d*}(y, d, f, b, h, q)$, $c^*(y, d, f, b, h, q)$ and a probability measure μ^* such that:

1. $b'^*(y, d, f, b, h, q)$, $\lambda^{b*}(y, d, f, b, h, q)$, $\lambda^{d*}(y, d, f, b, h, q)$, $c^*(y, d, f, b, h, q)$ solve the household's optimization problem;

2. ξ^* solves the intermediary's optimization problem;

3. $p^{b*}_f = \int \lambda^{b*}(y, d, f, b, h) d\mu$ for $b' < 0$ and $p^{b*}_f = 0$ for $b' \geq 0$ (intermediary consistency);

4. γ^* solves the government's budget constraint;

5. $p^{d*} = \int \lambda^{d*}(b, f, h, d, y, q) d\mu$ (government consistency);

6. $\sum_{b' \in B(f)} \int \mathbf{1}_{\{b'^*(b,f,h,d,y,q^*) = b'\}} \mu^*(dy, dd, f, db, dh) = \sum_{b' \in B(f)} \xi^*_f(b') \; \forall (f) \in F$ (each credit score market clears);

7. $\pi^* = 0$ (profits are zero);

8. $\mu^* = \mu_{q^*}$where $\mu_{q^*} = \Gamma_{q^*} \mu_{q^*}$ (μ^* is an invariant probability measure).

The computation of equilibrium in incomplete markets models has been made standard by a series of papers including (Aiyagari, 1994) and (Huggett, 1993) and have been extensively used in recent papers with (Chatterjee, Corbae, Nakajima, and Rios-Rull, 2007) the most related to the current study. The dimensionality of the state vector, the non-trivial zero profit condition and market clearing conditions which include a menu of loan prices and limits for households of different credit scores, the condition for the government balancing budget as well as the interaction between the two types of credit make computation more involved than previous work.

3.5.1 Existence of equilibrium and characterization

Theorem 11. Existence *A steady-state competitive equilibrium exists.*

In equilibrium the credit card loan price vector has the property that all possible face-value loans (household deposits) bear the risk-free rate and negative face-value loans (household borrowings) bear a rate that reflects the risk-free rate and a premium that accounts for the default probability, results which are in line with those in (Chatterjee, Corbae, Nakajima, and Rios-Rull, 2007). In our setup, however, the default probability depends on the credit score of the individual (and not only on the size of the loan) and thus loan prices will depend on credit scores. This result is delivered by the free entry condition of the financial intermediary which implies that cross-subsidization across loans made to individuals of different credit scores is not possible. Each credit score market clears in equilibrium and it is not possible for intermediary to charge more than the cost of funds for individuals with very low risk in order to offset losses on loans made to high risk individuals. Positive profits in some contracts would offset the losses in others, and so intermediaries could enter the market for those profitable loans. We turn now to characterizing the equilibrium price schedule and credit limits.

Theorem 12. Characterization of equilibrium prices *In any steady-state equilibrium the following is true:*

1. *For any $b^{'} \geq 0$, $q^{*}_{f'} = \rho/(1+r)$ for all f.*
2. *If the grid of F is sufficiently fine, there is $\overline{f} > f_1$ such that $q^{*}_{\overline{f}} = \rho/(1+r)$ for any $b^{'} \in B(\overline{f})$.*
3. *If the set of income levels for which the household is indifferent between defaulting on credit card debt and any other available option is of measure zero, then $f_1 < f_2$ implies $q^{*}_{f_1} > q^{*}_{f_2}$.*
4. *There is $\underline{f}$ such that $q^{*}_{\underline{f}} = 0$.*

Theorem 12 demonstrates that firms charge the risk-free interest rate on deposits (property 1) and to any loan size made to individuals with credit scores $\overline{f}$, the highest possible score (property 2). At this high credit score, the cost from defaulting is large and so the individual never finds it optimal to default on credit card debt. Given the competitive nature of our environment, firms will charge the risk-free rate for any loans made to individuals with this credit score. Property 3 shows that individuals with better credit scores are assigned higher loan prices, which implies that interest rates are lower for individuals with better scores. The last property says that individuals with the lowest credit score, $\underline{f}$, face zero prices on their loans. At zero price, financial intermediaries are indifferent as to how many loans to offer since they expect no payoff in the next period at no cost (zero price). At the same time, individuals taking out these loans buy nothing in the current period but start the following period with liabilities. Consequently, households are better off by choosing $b^{'} = 0$ so there is no demand for zero priced loans. A direct implication of this result is that in equilibrium individuals with the lowest credit score will be assigned a zero credit limit on their loans. This result is presented in 13. In addition, we show that the equilibrium delivers credit limits for each group of credit scores which are decreasing in credit scores, i.e. individuals with lower credit scores will be assigned tighter credit limits.

Theorem 13. Characterization of equilibrium credit limits *In any steady-state equilibrium,*

1. *For $\underline{f}$ such that $q^*_{\underline{f}} = 0$, we get that $b_{min}(\underline{f}) = 0$.*
2. *If the set of income levels for which the household is indifferent between defaulting on credit card debt and any other available option is of measure zero, then $f_1 < f_2$ implies $b_{min}(f_1) > b_{min}(f_2)$.*

4 Quantitative analysis

In progress.

5 Conclusion

We propose a theory of unsecured credit and the risk of default based on interactions between different forms of unsecured credit and their implications for default incentives. Different financial market arrangements and in particular, different bankruptcy rules alter incentives to default conducing to, in some circumstances, amplification of the default behavior. Our theory is motivated by facts related to borrowing and repayment behavior of young U.S. households with college and credit card debt, and in particular by recent (alarming) trends in the default rates.

We develop a general equilibrium economy that mimics features of student and credit card loans. In particular, our model accounts for 1) the legal and financial arrangement differences between the two types of loans and 2) the importance of credit scores in consumer credit in the U.S. Our theory explains borrowing and default behavior of young U.S. households: the incentive to default on student loans increases in college debt burdens and the incentive to default on credit card debt increases in credit card debt. In addition, the likelihood to default on credit card debt declines with the credit score of the individual.

Our theory reveals important cross-market effects: a borrower with high enough college debt burden and high enough credit card utilization ratio defaults for sure and will choose to default in the student loan market. We find that cross-market effects are not symmetric. Specifically, we show that tight credit conditions in the credit card market adversely affect default on credit card debt and on student loans, whereas severe consequences to default on student loans adversely affect default on credit card debt while reducing default on student loans.

Our paper innovates in demonstrating how financial arrangements in one market may amplify default behavior in another (related) market: while a high college debt burden is necessary to induce default on student loans, our results imply that this effect is amplified by high indebtedness in the credit card market and/or worse financial arrangements for credit card accounts.

We parametrize the model to match statistics regarding college debt, income, credit card limits and interest rates of young borrowers with student loans aged 20-26 as delivered by the SCF 2004. We plan to use the model to quantify the effects of the changes in college debt burdens and in credit card terms on default rates for student loans and to explore the policy implications of our theory. Specifically, we plan to study loan repayment policies contingent on terms in the other market and income contingent repayments with partial dischargeability.

References

Aiyagari, S. (1994): "Uninsured idiosyncratic risk and aggregate saving," *The Quarterly Journal of Economics*, 109(3), 659–684.

Athreya, K. (2002): "Welfare implications of the Bankruptcy Reform Act of 1999," *Journal of Monetary Economics*, 49(8), 1567–1595.

Athreya, K., X. Tam, and E. Young (2009): "Unsecured credit markets are not insurance markets," *Journal of Monetary Economics*, 56(1), 83–103.

Chatterjee, S., D. Corbae, M. Nakajima, and J. Rios-Rull (2007): "A Quantitative theory of Unsecured Consumer Credit With Risk of Default," *Econometrica*, 75(6), 1525–1589.

Chatterjee, S., D. Corbae, and J. Rios-Rull (2008): "A finite-life private-information theory of unsecured consumer debt," *Journal of Economic Theory*, 142(1), 149–177.

——— (2010): "Credit scoring and competitive pricing of default risk," Working paper.

Dynarsky, M. (1994): "Who defaults on student loans? Findings for the National Postsecondary Student Aid Study," *Economic of Education Review*, 13(1), 55–68.

Edelberg, W. (2006): "Risk-based pricing of interest rates for consumer loans," *Journal of Monetary Economics*, 53(8), 2283–2298.

Huggett, M. (1993): "The risk-free rate in heterogeneous-agent incomplete-insurance economies," *Journal of economic Dynamics and Control*, 17(5-6), 953–969.

Ionescu, F. (2008): "Consolidation of Student Loan Repayments and Default Rates Across Income Groups," *The B.E. Journal of Macroeconomics*, 8(1), Topics.

——— (2010): "Risky Human Capital and Alternative Bankruptcy Regimes for Student Loans," Working paper.

Ionescu, F., and N. Simpson (2010): "Credit Scores and College Investment," Working paper.

Livshits, I., J. MacGee, and M. Tertilt (2007): "Consumer bankruptcy: A fresh start," *The American Economic Review*, 97(1), 402–418.

Lochner, L., and N. Monge (2010): "The Nature of Credit Constraints and Human Capital," *American Economic Review*, Forthcoming.

A Appendix

Proofs of theorems

A.1 Proofs of Theorems 1 and 2

Let $c_{\min} = y_{\min} + b_{\min} - b_{\max} - d_{\max}(2 + r_g)$ and $c_{\max} = y_{\max} + b_{\max}$. Then, if c is the consumption in any of the cases in the definition of T, we have that

$$U(c_{\min}) \leq U(c) \leq U(c_{\max}),$$

because U is increasing. Recall that $\mathcal{S} = B \times F \times H$ is a finite set and let $N_{\mathcal{S}}$ be the cardinality of $\mathcal{S}$.

Definition A1. Define $\mathcal{V}$ to be the set of continuous functions $v : D \times Y \times Q \to \mathbb{R}^{N_{\mathcal{S}}}$ such that

1. For all $(b, f, h) \in \mathcal{S}$ and $(d, y, q) \in D \times Y \times Q$

$$U(c_{\min})/(1 - \beta\rho) \leq v(d, y, q)(b, f, h) \leq U(c_{\max})/(1 - \beta\rho). \tag{4}$$

2. v is increasing in b, f and y.

3. v is decreasing in d.

Let $\big(C(D \times Y \times Q; \mathbb{R}^{N_S}), \|\cdot\|\big)$ denote the space of continuous functions $v : D \times Y \times Q \to \mathbb{R}^{N_S}$ endowed with the supremum norm

$$\|v\| = \max_{(d,y,q)} \|v(d, y, q)\|,$$

where the norm of a vector $w = (w(b, f, h))_{(b,f,h)\in S} \in \mathbb{R}^{N_S}$ is

$$\|w\| = \max_{(b,f,h)\in S} |w(b, f, h)|.$$

Then $\mathcal{V}$ is a subset of $C(D \times Y \times Q; \mathbb{R}^{N_S})$. Define also $C(D \times Y \times Q \times \mathcal{S})$ to be the set of continuous real valued functions $v : D \times Y \times Q \times \mathcal{S} \to \mathbb{R}$ with the norm

$$\|v\| = \max_{(d,y,q,b,f,h)} |v(d, y, q, b, f, h)|.$$

In the first lemma we show that the two spaces of functions that we defined above are interchangeable.

Lemma A1. *The map* $V : C(D \times Y \times Q; \mathbb{R}^{N_S}) \to C(D \times Y \times Q \times \mathcal{S})$ *defined by*

$$V(v)(d, y, q, b, f, h) = v(d, y, q)(b, f, h)$$

is a surjective isomorphism.

Proof. We prove first that if $v \in C(D \times Y \times Q; \mathbb{R}^{N_S})$ then $V(v)$ is continuous. Let $(d_n, y_n, q_n, b_n, f_n, h_n)_{n\in\mathbb{N}}$ be a sequence that converges to (d, y, q, b, f, h) and let $\varepsilon > 0$. Since $\mathcal{S}$ is a finite set it follows that there is some $N_1 \geq 1$ such that $b_n = b$, $f_n = f$, and $h_n = h$ for all $n \geq N_1$. Since v is continuous then there is $N_2 \geq 1$ such that if $n \geq N_2$ then

$$\|v(d_n, y_n, q_n) - v(d, y, q)\| < \varepsilon.$$

Thus $|v(d_n, y_n, q_n)(b, f, h) - v(d, y, q)(b, f, h)| < \varepsilon$ for all $n \geq N := \max\{N_1, N_2\}$. Therefore

$$|V(v)(d_n, y_n, q_n, b_n, f_n, h_n) - V(v)(d, y, q, b, f, h)| < \varepsilon \text{ for all } n \geq N$$

and $V(v)$ is continuous. It is clear from the definition of the norms that $\|V(v)\| = \|v\|$ for all $v \in C(D \times Y \times Q; \mathbb{R}^{N_S})$. Thus V is an isomorphism. Finally, if $w \in C(D \times Y \times Q \times \mathcal{S})$ then one can define $v \in C(D \times Y \times Q; \mathbb{R}^{N_S})$ by

$$v(d, y, q)(b, f, h) = w(d, y, q, b, f, h).$$

Then $T(v) = w$ and T is surjective. □

In the following we are going to tacitly view $\mathcal{V}$ either as a subset of $C(D \times Y \times Q; \mathbb{R}^{N_S})$ or as a subset of $C(D \times Y \times Q \times \mathcal{S})$ via $V(\mathcal{V})$. For example, we are going to prove in the following lemma that $(\mathcal{V}, \|\cdot\|)$ is a complete metric space by showing that it('s image under V) is a closed subspace of $C(D \times Y \times Q \times \mathcal{S})$, which is a complete metric space.

Lemma A2. $(\mathcal{V}, \|\cdot\|)$ *is a complete metric space.*

Proof. As mentioned above, we are going to show that $\mathcal{V}$ is a closed subspace of $C(D \times Y \times Q \times \mathcal{S})$. Notice first that $\mathcal{V}$ is nonempty because any constant function that satisfies (4) is in $\mathcal{V}$. Let now $\{v_n\}_{n\in\mathbb{N}}$ be a sequence of functions in $\mathcal{V}$ that converge to a function v. Then, since $C(D \times Y \times Q \times \mathcal{S})$ is complete, it follows that v is continuous. Since inequalities are preserved by taking limits it follows immediately that v satisfies the conditions of Definition A1, because each v_n satisfies those conditions. Therefore $v \in \mathcal{V}$ and, thus, $(\mathcal{V}, \|\cdot\|)$ is a closed subspace of $C(D \times Y \times Q \times \mathcal{S})$ and, hence, a complete metric space. □

Lemma A3. *The operator T defined on $C(D \times Y \times Q; \mathbb{R}^{N_S})$ maps $\mathcal{V}$ into $\mathcal{V}$ and its restriction to $\mathcal{V}$ is a contraction with factor $\beta\rho$.*

Proof. We will show first that if $v \in \mathcal{V}$ then $Tv \in \mathcal{V}$. Since $v \in \mathcal{V}$ we have that

$$\frac{U(c_{\min})}{1-\beta\gamma} \leq v(d, y', q)(b', f', h') \leq \frac{U(c_{\max})}{1-\beta\gamma}$$

for all $(d, y', q) \in D \times Y \times Q$ and $(b', f', h') \in \mathcal{S}$. Integrating with respect to y' we obtain that

$$\frac{U(c_{\min})}{1-\beta\gamma} \leq \int v_{(b',f',h')}(d, y'; q)\Phi(dy') \leq \frac{U(c_{\max})}{1-\beta\rho},$$

because $\int \Phi(y, dy') = 1$. Since $U(c_{\min}) \leq U(c) \leq U(c_{\max})$ for all c appearing in the definition of T, it follows that

$$U(c) + \beta\rho \int v_{(b',f',h')}(d, y'; q)\Phi(dy') \leq U(c_{\max}) + \frac{\beta\rho U(c_{\max})}{1-\beta\rho} = \frac{U(c_{\max})}{1-\beta\rho},$$

and, similarly

$$\frac{U(c_{\min})}{1-\beta\rho} \leq U(c) + \beta\rho \int v_{(b',f',h')}(d,y';q)\Phi(dy').$$

Thus the condition (4) of Definition A1 is satisfied. To prove that Tv is increasing in b, f and y, and decreasing in d, note that the sets $B_{b,f,h,\lambda_b,\lambda_d}(d,y,;q)$ are increasing with respect to b, f and y, and decreasing with respect to d. These facts coupled with the same properties for v (which are preserved by the integration with respect to y') imply that Tv satisfies the remaining conditions from Definition A1, with the exception of the continuity, which we prove next.

Since D and Q are compact spaces, it follows by a simple $\varepsilon - \delta$ argument that the integral is continuous with respect to d and q as well. Since $U(\cdot)$ is continuous with respect to c and c is continuous with respect to d and y, it follows that $T(v)$ is continuous.

Finally we prove that T is a contraction with factor $\beta\rho$ by showing that T satisfies Blackwell's conditions. For simplicity, we are going to view $\mathcal{V}$ one more time as a subset of $C(D\times Y\times Q\times\mathcal{S})$. Let $v,w\in\mathcal{V}$ such that $v(d,y,q,b,f,h)\leq w(d,y,q,b,f,h)$ for all $(d,y,q,b,f,h)\in D\times Y\times Q\times\mathcal{S}$. Then

$$\beta\rho\int v_{(b',f',h')}(d,y';q)\Phi(dy') \leq \beta\rho\int w_{(b',f',h')}(d,y';q)\Phi(dy')$$

for all (d,y,q,b',f',h'). This implies that $Tv\leq Tw$. Next, if $v\in\mathcal{V}$ and a is a constant it follows that

$$\beta\rho\int\big(v_{(b',f',h')}(d,y';q)+a\big)\Phi(dy') = \beta\rho\int v_{(b',f',h')}(d,y';q)\Phi(dy') + \beta\rho a.$$

Thus $T(v+a) = Tv + \beta\rho a$. Therefore T is a contraction with factor $\beta\rho$. □

Theorem 1. *There exists a unique $v^*\in\mathcal{V}$ such that $v^* = Tv^*$ and*

1. *v^* is increasing in y, b and f, and decreasing in d,*

2. *the optimal policy correspondence implied by Tv^* is compact-valued, upper hemi-continuous,*

3. *default is strictly preferable to zero consumption and optimal consumption is always positive.*

Proof. The first part follows from Definition A1 and Lemmas A2 and A3. The third part follows from our assumptions on U. So we need only to prove the second part of the theorem. The optimal policy correspondence is

$$\Xi_{(d,y,q)(b,f,h)} = \big\{(c,b',f',\lambda_d,\lambda_b,h')\in B_{(b,f,h)}(d,y,q) \text{ that attains } v^*_{(b,f,h)}(d,y,q)\big\}.$$

For simplicity of our notation we will write $x=(d,y,q,b,f,h)$. For a fixed x we need to show that if Ξ_x is nonempty then it is compact. First notice that

$$\Xi_x \subset [c_{\min}, c_{\max}] \times B \times F \times \{0,1\} \times \{0,1\} \times \{0,1\}$$

and, thus, it is a bounded set. We need to prove that it is closed. Let $\{(c_n, b'_n, f'_n, \lambda_d^n, \lambda_b^n, h'_n)\}_{n\in\mathbb{N}}$ be a sequence in Ξ_x that converges to some

$$(c, b', f', \lambda_d, \lambda_b, h') \in [c_{\min}, c_{\max}] \times B \times F \times \{0,1\} \times \{0,1\} \times \{0,1\}.$$

Since B, F, and $\{0,1\}$ are finite sets it follows that there is some $N \geq 1$ such that $b'_n = b'$, $f'_n = f'$, $\lambda_d^n = \lambda_d$, $\lambda_b^n = \lambda_b$, and $h'_n = h'$ for all $n \geq N$. In particular we have that $f' = g(f, h, b, b')$. Moreover if we let

$$\phi(c) = U(c) + \beta\rho \int v_{(b',f',h')}(d, y'; q)\Phi(dy')$$

then ϕ is continuous. Since $\phi(c_n) = v^*_{(b,f,h)}(d, y; q)$ for all $n \geq 1$ it follows that

$$\phi(c) = \lim_{n\to\infty} \phi(c_n) = v^*_{(b,f,h)}(d, y; q).$$

Thus $(c, b', f', \lambda_d, \lambda_b, h') \in \Xi_x$ and Ξ_x is a closed and, hence, compact set.

To prove that Ξ is upper hemi-continuous consider $x \in D \times Y \times Q \times S$, $x = (d, y, q, b, f, h)$ and let $\{x_n\} \in D \times Y \times Q \times \mathcal{S}$, $x_n = (d_n, y_n, q_n, b_n, f_n, h_n)$ a sequence that converges to x. Since B, F, and $\{0,1\}$ are finite sets it follows that there is $N \geq 1$ such that if $n \geq N$ then $b_n = b$, $f_n = f$, and $h_n = h$. Let $y_n = (c_n, b'_n, f'_n, \lambda_d^n, \lambda_b^n, h') \in \Xi_{x_n}$ for all $n \geq N$. We need to find a convergent subsequence of $\{y_n\}$ whose limit point is in Ξ_x. Since B, F, and $\{0,1\}$ are finite sets we can find a subsequence $\{y_{n_k}\}$ such that $b'_{n_k} = b'$, $f'_{n_k} = f'$, $\lambda_d^{n_k} = \lambda_d$, $\lambda_b^{n_k} = \lambda_b$, and $h'_{n_k} = h'$ for some $b' \in B$, $f' \in F$, $\lambda_d, \lambda_b, h \in \{0,1\}$. It follows then that $f' = g(f, h, b, b')$, since $f'_n = g(f_n, h_n, b_n, b'_n)$ for all $n \geq N$. Since $\{c_{n_k}\} \subset [c_{\min}, c_{\max}]$ which is a compact interval, there must be a convergent subsequence, which we still label c_{n_k} for simplicity. Let $c = \lim_{k\to\infty} c_{n_k}$ and let $y_{n_k} = (c_{n_k}, b', f', \lambda_d, \lambda_b, h)$ for all k. Then $\{y_{n_k}\}$ is a subsequence of $\{y_n\}$ such that

$$\lim_{k\to\infty} y_{n_k} = y := (c, b', f', \lambda_d, \lambda_b, h).$$

Moreover, since

$$\phi(c_{n_k}) = v^*_{(b,f,h)}(d_{n_k}, y_{n_k}; q_{n_k}) \text{ for all } k$$

and since ϕ and v^* are continuous functions it follows that

$$\phi(c) = \lim_{k\to\infty} \phi(c_{n_k}) = \lim_{k\to\infty} v^*_{(b,f,h)}(d_{n_k}, y_{n_k}; q_{n_k}) = v^*_{(b,f,h)}(d, y; q).$$

Thus $y \in \Xi_x$ and Ξ is an upper hemi-continuous correspondence. □

Theorem 2. *For any $q \in Q$ and any measurable selection from the optimal policy correspondence there exists a unique $\mu_q \in \mathcal{M}(x)$ such that $\Gamma_q \mu_q = \mu_q$.*

Proof. The Measurable Selection Theorem implies that there exists an optimal policy rule that is measurable in $X \times \mathcal{B}(X)$ and, thus, T^*_q is well defined. We show first that T^*_q satisfies Doblin condition. It suffices to prove that TN^*_q satisfies Doblin condition (see Exercise 11.4g of Stockey, Lucas, Prescott (1989)). If we let $\varphi(Z) = TN^*_q(y, d, f, b, h, Z)$ for any $(y, d, f, b, h) \in X$ it follows that if $\varepsilon < 1/2$ and $TN^*_q(y, d, f, b, h, Z) < \varepsilon$ then $1 - \varepsilon > 1/2$ and

$$\varphi(Z) = TN^*_q(y, d, f, b, h, Z) < \varepsilon < \frac{1}{2} < 1 - \varepsilon.$$

Next, notice that if $\varphi(Z) > 0$ then $TN^*_q(y, d, f, b, h, Z) > 0$ and, thus,

$$T^*_q(y, d, f, b, h, Z) = TS^*_q(y, d, f, b, h, Z) + TN^*_q(y, d, f, b, h, Z) > 0.$$

Then Theorem 11.10 of Stockey, Lucas, Prescott (1989) implies the conclusion of the theorem. □

A.2 Proofs of Theorems 3 through 8

Let $(b, f, h) \in \mathcal{S}$ and $q \in Q$ be fixed. Before proving the theorem we will introduce some notation which will ease the writing of our proofs. For $y \in Y$ and $q \in Q$ we write

$$\psi_{nodef}(c, b', f', y, d) = U(c) + \beta\rho \int v_{b',f',0}(d, y'; q)\Phi(dy')$$

if $(c, b', f') \in B_{b,f,0,0,0}(d, y; q)$,

$$\psi_{sl}(y, d) = U(y + b) + \beta\rho \int v_{0,f,1}(d(2 + r_g), y'; q)\Phi(dy'),$$

$$\psi_{cc}(y, d) = U(y - d) + \beta\rho \int v_{0,f_1,0}(d, y'; q)\Phi(dy'),$$

and

$$\psi_{both}(y,d) = U(y) + \beta\rho \int v_{0,f_1,1}(d(2+r_g), y'; q)\Phi(dy').$$

Note that these functions are continuous in y and d. Also, these functions depend on b, f, and q. Later on it will be useful to remember that ψ_{nodef} and ψ_{sl} increase with f and b, while ψ_{cc} and ψ_{both} remain unchanged when we vary f and b.

Theorem 3. *Let $q \in Q$, $f \in F$, $b \in B(f)$. If $h = 1$ and the set $D^{SL}_{b,f,1}(q)$ is nonempty, then $D^{SL}_{b,f,1}(q)$ is closed and convex. In particular the sets $D^{SL}_{b,f,1}(d;q)$* are closed intervals for all d. If $h = 0$ and *the set $D^{SL}_{b,f,0}(d;q)$ is nonempty,* then $D^{SL}_{b,f,0}(d;q)$ is a closed interval for all d.

Proof. If $h = 1$ then $D^{SL}_{b,f,1}(q)$ is the combinations of earnings y and student loan amount d for which $B_{b,f,1,0,0}(b,d,y;q) = \emptyset$. Then they satisfy the inequality $y(1-\gamma) - d(2+r_g) - q_f b_{\min}(f) \leq 0$. Thus $D^{SL}_{b,f,1}(q)$ is a closed. Moreover, if (y_1,d_1) and (y_2,d_2) are elements in $D^{SL}_{b,f,1}(q)$ then if $(y,d) = t(y_1,d_1) + (1-t)(y_2,d_2)$ with $t \in (0,1)$ it follows easily that

$$y(1-\gamma) - d(2+r_g) - q_f b_{\min}(f) \leq 0$$

and, thus, $(y,d) \in D^{SL}_{b,f,1}(q)$. So $D^{SL}_{b,f,1}(q)$ is convex.

Let now $h = 0$. Let $d \in D$ be fixed and assume first that $b \geq 0$. Let y_1 and y_2 with $y_1 < y_2$ be in $D^{SL}_{b,f,0}(d;q)$. Therefore

$$\psi_{sl}(y_i,d) \geq \psi_{nodef}(c_i^*, b_i^*, f_i^*, y_i, d), \tag{5}$$

where $(c_i^*, b_i^*, f_i^*) \in B_{b,f,0,0,0}(d, y_i; q)$ are the optimal choices for the maximization problems, $i = 1,2$. Let $y \in (y_1,y_2)$ and assume, by contradiction, that $y \notin D^{SL}_{b,f,0h}(d;q)$. Then

$$\psi_{sl}(y,d) < \psi_{nodef}(c^*, b^*, f^*, y, d), \tag{6}$$

where $(c^*, b^*, f^*) \in B_{b,f,0,0,0}(d,y;q)$ is the optimal choice for the maximization problem. Let $\overline{c}_1 = c^* - (y - y_1)$. If $\overline{c}_1 \leq 0$ then $\overline{c}_1 < y_1 + b$ and thus

$$c^* = \overline{c}_1 + (y - y_1) < y_1 + b + (y - y_1) = y + b. \tag{7}$$

If $\overline{c}_1 > 0$ we have that $(\overline{c}_1, b^*, f^*) \in B_{b,f,0,0,0}(d, y_1; q)$ and, thus,

$$\psi_{nodef}(c_1^*, b_1^*, f_1^*, y_1, d) \geq \psi_{nodef}(\overline{c}, b^*, f^*, y_1, d).$$

Combining this inequality with (5) we obtain that

$$U(y_1+b)+\beta\rho\int v_{0,f,1}(d(2+r_g),y';q)\Phi(dy')\geq U(\overline{c}_1)+\beta\rho\int v_{b^*,f^*,0}(d,y';q)\Phi(dy'). \tag{8}$$

Subtracting (8) from (6) we have that

$$U(y+b)-U(y_1+b)<U(c^*)-U(\overline{c}_1).$$

Since $(y+b)-(y_1+b)=y-y_1=c^*-\overline{c}_1$ and U is strictly concave it follows that $c^*<y+b$.

Consider now $\overline{c}_2=c^*+(y_2-y)$. Then $(\overline{c}_2,b^*,f^*)\in B_{b,f,0,0,0}(d,y_2;q)$ and thus

$$U(c_2^*)+\beta\rho\int v_{b_2^*,f_2^*,0}(d,y';q)\Phi(dy')\geq U(\overline{c}_2)+\beta\rho\int v_{b^*,f^*,0}(d,y';q)\Phi(dy'). \tag{9}$$

Using inequalities (5), (6), and (9) we obtain that

$$U(y_2+b)-U(y+b)>U(\overline{c}_2)-U(c^*).$$

Thus $c^*>y+b$, and we obtain a contradiction with $c^*<y+b$. Therefore $y\in D^{SL}_{b,f,0}(d;q)$ and, thus, $D^{SL}_{b,f,0}(d;q)$ is an interval. Since it is also a closed set it follows that it is a closed interval.

Assume now that $b<0$. The set $D^{SL}_{b,f,0}(d;q)$ is closed because ψ_{sl}, ψ_{both}, ψ_{cc}, and ψ_{nodef} are continuous with respect to y. Assume that $D^{SL}_{b,f,0}(d;q)$ is nonempty, and let $y_1,y_2\in D^{SL}_{b,f,h}(d,q)$ such that $y_1<y_2$ and let $y\in(y_1,y_2)$. We have the following three cases: $B_{b,f,0,0,0}(d,y_i;q)=\emptyset$, $i=1,2$, or $B_{b,f,0,0,0}(d,y_i;q)\neq\emptyset$, $i=1,2$, or $B_{b,f,0,0,0}(d,y_1;q)=\emptyset$ and $B_{b,f,0,0,0}(d,y_2;q)\neq\emptyset$. In the first two cases it follows that $y\in D^{SL}_{b,f,0}(d;q)$ via a straightforward modification of our proof of the similar statement for $D^{SL}_{b,f,0}(d;q)$ when $b\geq 0$. So we are going to prove next the last case. Since $y_i\in D^{SL}_{b,f,0}(d;q)$, $i=1,2$, we have that $\psi_{sl}(y_i,d)\geq\psi_{cc}(y_i,d)$ and $\psi_{sl}(y_i,d)\geq\psi_{both}(y_i,d)$ for $i=1,2$. Moreover we have that $\psi_{sl}(y_2,d)\geq\psi_{nodef}(c_2^*,b_2^*,f_2^*,y_2,d)$, where $(c_2^*,b_2^*,f_2^*)\in B_{b,f,0,0,0}(d,y_2;q)$ is the optimal choice for non default. We can easily modify the previous argument to show that $\psi_{sl}(y,d)\geq\psi_{cc}(y,d)$, and using another minor modification, that $\psi_{sl}(y,d)\geq\psi_{both}(y,d)$. If $B_{b,f,0,0,0}(d,y;q)=\emptyset$ this suffices to show that $y\in D^{SL}_{b,f,0}(d;q)$. If $B_{b,f,0,0,0}(d,y;q)\neq\emptyset$ then we need also to prove that $\psi_{sl}(y,d)\geq\psi_{nodef}(c^*,b^*,f^*,y,d)$, where $(c^*,b^*,f^*)\in B_{b,f,0,0,0}(d,y;q)$ is the optimal choice for non default. By the third part of Theorem 1 there must be some $\overline{y}\in(y_1,y_2)$ such that $\overline{y}<y$ and $\psi_{nodef}(\overline{c}^*,\overline{b}^*,\overline{f}^*,\overline{y},d)<\psi_{sl}(y,d)$. Thus $y\in(\overline{y},y_2)$ and $\psi_{sl}(y_2,d)\geq\psi_{nodef}(c_2^*,b_2^*,f_2^*,y_2,d)$ and $\psi_{sl}(y,d)=\psi_{nodef}(\overline{c}^*,\overline{b}^*,\overline{f}^*,\overline{y},d)$. Then the same argument as before shows that $\psi_{sl}(y,d)\geq\psi_{nodef}(c^*,b^*,f^*,y,d)$, and, thus $y\in D^{SL}_{b,f,0}(d;q)$. Therefore $D^{SL}_{b,f,0}(d;q)$ is an interval. □

Theorem 4. *Let $q \in Q$, $(b, f, 0) \in \mathcal{S}$.* If $D^{CC}_{b,f,0}(d; q)$ is nonempty then it is a closed interval for all d.

Proof. If $b \geq 0$ then $D^{CC}_{b,f,0}(d; q)$ is empty. If $b < 0$ the proof of the theorem is very similar with the proof of Theorem 3 and we will omit it. □

Theorem 5. *Let $q \in Q$, $(b, f, 0) \in \mathcal{S}$. If the set $D^{Both}_{b,f,0}(d; q)$* is nonempty then it i*s a closed interval* for all d.

Proof. Once again, for $b \geq 0$ the set $D^{Both}_{b,f,0}(d; q)$ is empty. For $b < 0$ the proof is similar with the proof of Theorem 3 □

Theorem 6. *For any price $q \in Q$, $b \in B$ and $h \in H$, the sets $D^{CC}_{b,f,h}(d; q)$ and $D^{Both}_{b,f,h}(d; q)$ shrink when f increases, for all $d \in D$.*

Proof. The theorem follows from the observation that ψ_{nodef} and ψ_{sl} decrease when b decreases while ψ_{cc} and ψ_{both} remain the same. □

Theorem 7. *For any price $q \in Q$, $b \in B$ and $h \in H$, the sets $D^{b}_{b,f,h}(d; q)$ and $D^{bd}_{b,f,h}(d; q)$ shrink when f increases, for all $d \in D$.*

Proof. Notice that for any y we have that ψ_{nodef} and ψ_{sl} increase in f, while $\psi_{cc}(y, d)$ and $\psi_{both}(y, d)$ remain the same. This implies that the set $D^{b}_{b,f,h}(d, q)$ shrinks when f increases. The second part is an immediate consequence of the first part. □

Theorem 8. *For any price q, there is $\varepsilon_1 > 0$ such that if $y - d < \varepsilon_1$ then a defaulter will choose to default on student loans rather then on credit card debt. Furthermore, we can find $\varepsilon_2 > 0$ such that if $b - q_{f'} b_{min}(f) < \varepsilon_2$, where $f' = g(f, b, b_{min}(f))$, then the agent defaults.*

Proof. The third part of Theorem 1 implies that there is $\varepsilon_1 > 0$ such that if $y - d < \varepsilon_1$, then $\psi_{cc}(y, d) < \min\{\psi_{nodef,}\psi_{sl}, \psi_{both}\}$. Thus, if the agent chooses to default, she necessarily chooses to default on student loans or on both student loans and credit card debt.

Using one more time Theorem 1, we can find $\varepsilon > 0$ such that if $y + b - d - q_{f'} b_{min}(f) < \varepsilon$, then $\psi_{nodef}(y, d) < \min\{\psi_{sl}, \psi_{both}\}$. Thus, if we pick $\varepsilon_1 < \varepsilon$ and $\varepsilon_2 < \varepsilon - \varepsilon_1$, the conclusion follows. □

A.3 Proofs of Theorems 9 and 10

Theorem 9. *The incentive to default on any type of loan or on both increases when the market tightens.*

Proof. Suppose that that the economy E_1 features a relatively tighter credit card market compared to the economy E_2. Then $B^{E_1}(f) \subset B^{E_2}(f)$ for all $f \in F$. Therefore

$$B^{E_1}_{b,f,0,0,0}(d, y; q) \subset B^{E_2}_{b,f,0,0,0}(d, y; q)$$

for all $b \in B$, $f \in F$, $d \in D$, $y \in Y$, and $q \in Q$. Thus

$$\max \psi^{E_1}_{nodef}(c, b', f', y, d) \leq \max \psi^{E_2}_{nodef}(c, b', f', y, d),$$

while ψ_{sl}, ψ_{cc}, and ψ_{both} are unaffected by the tightness of the market. Thus the incentive to default increases for the economy E_1 relative to the economy E_2. □

Theorem 10. *The incentive to default on student loans (credit card debt) decreases (increases) in* γ.

Proof. Let $\gamma_1 < \gamma_2$. Then $B^{\gamma_2}_{0,f,1,0,0}(d, y; q) \subset B^{\gamma_1}_{0,f,1,0,0}(d, y; q)$. Therefore $\psi^{\gamma_2}_{sl}(y, d) < \psi^{\gamma_1}_{sl}(y, d)$ and a similar inequality holds for ψ_{cc}. It follows that the incentive to default on student loans decreases with γ. The fourth part of Definition 1 implies that the incentive to default on credit card debt increases in γ. □