Abstract

This paper studies informal default in consumer credit as the start of a process of negotiation with the lender. We consider an economy with uninsurable individual risk where households in debt have also the option of declaring formal bankruptcy. In a calibrated version of the model, informal defaulters are notably wealthier, have lower income, and hold more debt than formal defaulters, an implication consistent with the evidence. Quick settlements are achieved often, with limited discounts. Protracted negotiations feature individuals disaving before they reach agreement or declare bankruptcy. Allowing for negotiations raises default rates but substantially improves welfare as it provides greater insurance opportunities. Thus lowering the cost of informal default, as opposed to that of formal default, raises welfare and dampens consumption volatility. A tighter exemption improves welfare as the bargaining option mitigates the adverse effect on insurance via formal bankruptcy. Attempts at limiting collection outside bankruptcy reduce the incidence of bankruptcy but lower overall welfare.

Keywords: consumer unsecured credit, bankruptcy, bargaining, default, debt negotiations
1 Introduction

This paper investigates consumer defaults which do not result immediately in bankruptcy. In the U.S., default outside formal bankruptcy procedures is common. This is the theme of recent empirical and theoretical studies in Dawsey and Ausubel (2004), Dawsey, Hynes, and Ausubel (2008) and Chatterjee (2010). In the Survey of Consumer Finances 2007, about 1% of the U.S. population had filed under Chapter 7, whereas over 5% held delinquent loans. Informal default might account for the bulk of loan write-offs, a point made in Dawsey and Ausubel (2004). This evidence notwithstanding, only formal bankruptcy filings have attracted analysis and discussion in the recent macroeconomic literature.\footnote{For example, Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), Athreya, Tam, and Young (2009), Livshits, MacGee, and Tertilt (2007), Mateos-Planas (2009) and Mateos-Planas and Rios-Rull (2010).}

In this paper, we view informal default as a process of negotiation between the debtor and the creditor that may ensue after debt repayments fail to be met. It encompasses a wide variety of seemingly desperate defaults. It also intends to account for negotiations through lawyers and bank administrators as well as unserious negotiations that are simply attempts to avoid payment.

The first objective of this paper is to put forward an equilibrium model where bargaining is an alternative to formal Chapter 7 bankruptcy procedures. Our second objective is to study the implications for the characteristics of households with different repayment histories and choices of default. In the data informal defaulters appear different from individuals in formal bankruptcy proceedings. In particular, individuals in informal bankruptcy are wealthier and have more assets and more debt, but lower income, than individuals in formal bankruptcy. We address this type of evidence about the portfolio mix for the first time. Finally, we will analyze positive and welfare responses over a variety of policies that are frequently studied in consumer default, including policies that pertain primarily to delinquent rather than bankrupt households.

The theoretical framework is based on a version of the equilibrium model of heterogeneous households with uninsurable risk typical of much bankruptcy literature – like Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) – extended to account for bargaining and adjustment costs for assets. The latter feature allows us to consider in a simple way households who maintain both debt and assets, a missed feature of many such models in the literature.\footnote{Li and Sarte (2006) is an exception. In Hintermaier and Koeniger (2009) unsecured debts also coexist with positive net worth at the individual level.} We include this feature because the portfolio mix matters for both the value of the default options and the outcome of negotiations. The former feature is more novel and the core contribution of the paper. Default can take the form of a formal procedure modeled on the U.S. bankruptcy code – including an asset exemption – or can lead to a process of bargaining. The bargaining, which involves complete information, takes place through randomly alternating offers between the debtor and the creditors. The two bargain over a plan by which the debtor can avoid Chapter 7 and rejoin the market. A plan consists of
proposed transfers of current resources and debt swaps which are priced at market rates. It also involves a recommendation for allowable consumer assets next period. Negotiations may fail to resolve quickly. Failed negotiations can continue indefinitely or revert to Chapter 7.

We solve the stationary equilibrium of the model for a numerical benchmark. Parameters of the model, including the costs of defaulting, the collection technology, income persistence and adjustment costs, are chosen to broadly match selected empirical targets from the Survey of Consumer Finances. These targets include the formal and informal default rates, the debt to income ratio, and the proportion of population in debt. In this setting, many households hold mixed portfolios, and the possibility of bargaining accounts for a large fraction of total debt in the economy. It is to be noted that formal bankruptcy also happens in equilibrium due to lack of commitment in negotiations.

We find the following main properties of the benchmark economy. First, defaulters who bargain are notably wealthier, have lower income, and hold more debt than Chapter 7 filers, an implication which is borne out by the data. The informal defaulters exploit in their favor the opportunity to borrow heavily and then default when their assets exceed the bankruptcy exemption level. Although higher interest rates price in the risk for lenders, they do not deter these loans given that negotiations will result in partial recovery. This motivation of informal defaulters is in contrast with the motivation for formal defaulters. The latter, holding only low assets, are priced out of large loans as a consequence and fully discharge their debts.

Second, negotiations may resolve quickly or can be protracted. Most informal defaulters settle on a agreed plan within one period but a substantial proportion fail to agree and carry on negotiating. There are marked differences among these two types of households. Failed settlements involve households with relatively high debts and assets. Along the negotiation process the household reduces her assets. Delay precisely reflects the fact that the household holds a larger amount of assets and running them down to achieve a tolerable settlement would be too costly. The process of bargaining allows households to disave gradually rather than hastily, thus smoothing the costs while, at the same time, weakening the bargaining position of the creditors. These individuals can eventually end up in Chapter 7. Negotiations can also be ineffective as well as some consumers report multiple delinquencies in very short periods. So default can be messy.

We conduct some basic policy experiments. Ruling out informal bankruptcy has substantial consequences. Indebtedness declines by about 30 per cent and, in terms of welfare, removing the option to bargain causes a generalized welfare loss, especially for highly indebted households. The variance of consumption increases by about 10 per cent. Therefore, negotiation provides substantial additional insurance and studies that ignore it might underestimate the degree of consumption smoothing achievable through non-contingent debt. On the other hand, reducing the cost of informal default for the household improves welfare; this is in contrast with the welfare loss of a reduction in the cost of formal bankruptcy. Finally, the presence of bargaining mitigates the impact of income volatility.
on individual consumption and might thus help to account for observed recent trends in the U.S.

We also analyze the effects of reducing the exemption level for assets in formal default. A tighter exemption increases the levels of debt in the economy as it makes it easier to borrow, lowers the variability of consumption, and raises the rate of informal as well as formal default. Tightening the exemption level brings about substantial welfare gains at all debt levels. This result contrasts with previous findings as the option of informal default allows households to avoid the cost of fire-selling assets associated with formal bankruptcy. Finally we show the effects of softening collection actions outside of bankruptcy. We show that protecting consumers from outside collection efforts reduces the amount of households in Chapter 7 but does this at the expense of consumers outside default, with a negative net welfare effect.

This paper is a first attempt to modeling default both outside and inside of formal bankruptcy. It is related to the consumer bankruptcy literature. Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007), Athreya, Tam, and Young (2009) and Livshits, MacGee, and Tertilt (2007) consider formal default only without renegotiation. Mateos-Planas and Ríos-Rull (2010) is a model of long-term credit contracts with switching costs which could also be interpreted as a form of renegotiation. Although Li and Sarte (2006) add Chapter 13 bankruptcy to Chapter 7, the arrangement is not subject to negotiation. In the sovereign literature, Eaton and Gersovitz (1981) discuss the behavior of short term debt and, more recently, Benjamin and Wright (2009) have first noted that including bargaining can sharply improve the ability of these models to reproduce data outcomes. However the inclusion of Chapter 7 produces a richer set of outcomes than the results in that model.

2 Model

Our theory concerns the borrowing, lending, and negotiations between private lenders and their creditors. The formal model has two types of agents. The first is infinitely lived risk-averse households who face idiosyncratic income risk. They can hold debt and assets to smooth consumption. They choose whether to default or not, whether to default via Chapter 7 or via bargaining, and, if given the chance, which terms of settlement to propose and, otherwise, whether to accept the bank’s offered terms. The other type of agents is risk neutral creditors who lend competitively to the debtor. When bargaining banks choose, if given the chance, which terms of settlement to propose and, otherwise, whether to accept the household’s offered terms.

**Individual states.** At the beginning of each period, a household is described by an idiosyn-

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3 Other recent examples in the sovereign debt literature include Yue (2010).
cratic productivity state $s$, asset holdings $a$, debt holdings $b$, and credit status $z$. There are four possible situations in terms of credit status for the household $z \in \{0, 1, 2, 3\}$. If $z = 0$, the household has a clean record and can freely borrow and lend or default; if $z = 1$, the household had filed for or been under Chapter 7 bankruptcy in the previous period and not cleared its bankruptcy flag; if $z = 2$ the household had entered bargaining in the previous period and not reached a settlement and the household is the current proposer; if $z = 3$ the household had entered bargaining in the previous period and not reached a settlement and the lender is the current proposer.

Preferences. Preferences are defined over sequences of consumption $c$ over the household’s lifetime. At any point, it is represented as the expected sum of period utility $u(c)$ discounted at the rate $\beta$. The utility function belongs in the CRRA class and is specified as

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where $\sigma$ is the inverse of the rate of risk aversion. Utility is also affected by a non-pecuniary cost to defaulting on debt via Chapter 7 denoted $\chi^7$, and via bargaining denoted $\chi^B$. These costs enter separably and additively in the utility of the household. For the most part we will assume they have a common value $\chi^d$.

There is ex-post individual random noise in the preferences over some choices in some states in the form of an additive and separable shocks to utility. The purpose is primarily technical as this achieves some needed smoothness, although these shocks lend themselves to intuitive interpretation. When $z = 0$, there is a random component to the utility of defaulting which is normally distributed with zero mean and standard deviation $\sigma_{DvsR}$. This is picking up unobservable heterogeneity in factors that affect the default decision. When having to decide between Chapter 7 default or bargaining – that is, when $z = 0$ and having decided to default or when $z = 2, 3$ – there is a random component to the utility of opting for Chapter 7 which is normally distributed with zero mean and standard deviation $\sigma_{7vsB}$.

Technology. Household income $e$ is given by the idiosyncratic productivity type $s$. The income type is a Markov process with transition matrix $\Gamma_{e,e'}$ over a set $E_\kappa$. Changing the level of assets from $a$ to $a'$ incurs an adjustment quadratic cost of $\phi_e \times (a - a')^2$. Banks operate an intermediation technology of deposits into one-period loans. They face a transaction cost at maturity which is a proportion $\lambda$ of debt outstanding.

Markets. The households buy perishable consumption $c$ in competitive markets at a price normalized to 1. They can hold non-contingent debt $b$ and assets $a$. Households and banks interact in two environments, in the market and, when in default, outside of the market. Consider the market here. Households buy assets and banks sell deposits at the risk-free discount price $q_0$. Households sell and banks buy debt $b'$ at a discount price that depends
on all the relevant information: assets and debts carried forward, \( a' \) and \( b' \), and the current income state \( s \). Thus the menu of debt prices is written as \( q(a', b', s) \).

**Bankruptcy and bargaining.** There is imperfect enforcement of contracts. Failure of debts to be repaid may place households and banks outside of the market, being either in Chapter 7 bankruptcy or in a process of bargaining. There affairs will be conducted under specific institutional conditions. Let us consider the timing first. If the household has a clear record \( z = 0 \), it is when it defaults that the exact identity of the would-be proposer, whether the household or the bank, becomes known. Then there is the choice about how to default, whether by Chapter 7 or bargaining. If Chapter 7 follows, then the Chapter 7 flag carries over into the next period \( z' = 1 \) with probability \( \pi^7 \), otherwise \( z' = 0 \). If bargaining follows, then the household will start clean next period \( z' = 0 \) if they reach a settlement, or will carry a bargaining flag \( z' = 2 \) or \( 3 \) if there is no settlement, depending on which party will be the next the proposer. If the household has currently a past-bargaining flag \( z = 2, 3 \), the identity of the would-be proposer is known by definition, and events unfold just like in the previous case, starting with the choice whether to file for Chapter 7 or continue the bargaining. Finally, if the household has a Chapter 7 flag \( z = 1 \), it is again brought over into the next period \( z' = 1 \) with probability \( \pi^7 \), otherwise \( z' = 0 \).

From the above description, the decision whether to repay or default in the first place is made before the bargaining plan's proposer is known, but any later decision on the modality of default is made in that knowledge. The identity of the proposer is stochastic. Without default history, it is the result of an independent draw and \( \gamma_{ho} \) denotes the ex-ante probability that the household is the party who would propose. Once with a bargaining flag, there may be persistence in the proposer's identity, and \( \gamma_{ho,ho} \) and \( \gamma_{lo,lo} \) denote the probability of repeating proposer in this case.

We now turn to the institutional rules governing the different default situations. Consider first the case where the household is under Chapter 7 bankruptcy. The household’s assets above a certain exemption level, \( \bar{a}^7 \), will be seized by banks as repayment, up to a value equivalent to debt outstanding. Banks get to obtain only a fraction \( \gamma^7 \) of those assets, a description of the efficiency of the recovery technology. Households cannot borrow but can save. The probability of clearing the bankruptcy flag, \( 1 - \pi^7 \), describes the punishment to the household in terms of exclusion from the credit market. Consider next the case where the household is in a process of bargaining. The designated party proposes a plan involving new debt \( b' \) and current transfer of goods \( \tau \) and, if the bank proposes, new assets \( a' \). If there is no settlement, the bank uses a exogenous collection process whereby it recovers a fraction given by the collection tax on debt \( \tau_b \). In this situation, the household is free to manage its assets. If there is settlement, the household borrows \( b' \) in the credit market and arranges the transfer \( \tau \) to repay the bank in accordance with the plan agreed.
Equilibrium. We consider competitive equilibria with free entry in the credit market, and SPE in bargaining.\textsuperscript{4} Generically, an equilibrium is a situation where households’ choices maximize the expected value of utility given prices and bank’s (bargaining) responses; banks’ choices maximize the expected recovered value given prices and households’ responses; and prices make banks’ (expected) profits equal to zero given households’ and bank’s decision rules. More specifically, it will be instructive to break it down into two blocks, a no-bargaining equilibrium and a bargaining equilibrium. A no-bargaining equilibrium takes as given the bargaining values for the households and the banks, and determines the price schedule, and the values and decision rules for repayment, default and Chapter-7 bankruptcy. This is akin to the standard equilibrium in recent consumer bankruptcy models (that is, if with arbitrarily bad outcomes of bargaining). A bargaining equilibrium takes as given the values of being clean and under Chapter 7 and prices, and determines the bargaining policy rules and reservation values. An equilibrium is a combination of no-bargaining and bargaining equilibria that are consistent with each other. To make this definition operative, we move on to state formally the decision problems and conditions implied in an equilibrium of the model.

3 Equilibrium conditions and characterization

To help notation, denote \( o \) the party that gets to propose a bargaining plan, and use \( ho \) to index situations when the household offers and \( lo \) for situations when the lender offers. Also, use \( ac \) to index situations where a offer is accepted and \( noac \) for when it is not. This can be expressed recursively.

3.1 Households

Households take as given the market price schedule for debt \( q \) and the bank’s reservation value in the failed bargaining states \( W_o^{D,B} \) conditional on the proposer’s type \( o \in \{lo, ho\} \).\textsuperscript{5} We first consider in the next subsection the key decisions about whether and how to default under each of the states, for given values associated with these choices. Then in the following subsection we will characterize these underlying values of the default options.

\textsuperscript{4}This is not unlike the stationary SPE of Merlo and Wilson (**).

\textsuperscript{5}The superindex \( D,B \) indicates a state of default following a period of failed bargaining, that is one of \( z = 2, 3 \). With the subindex \( lo \) it corresponds to \( z = 2 \); with the subindex \( lo \) it corresponds to \( z = 3 \). Admittedly, this notation may not be the most fortunate one.
3.1.1 Default choices

More specifically, the default decisions yield a number of functions of the state \((a, b, s)\) for the household. Starting from a clean record \(z = 0\), we have the probability of choosing Chapter 7 default conditional on the proposer \(p_{7,o}^E\); the values to defaulting conditional on the proposer \(V^D_o\); the ex-ante value of defaulting \(V^D\) and the corresponding default rate \(d\); the value of a clean record \(V\). On the other hand, starting from a situation of previous failed bargaining \(z = 2, 3\), we have the probability of choosing Chapter 7 default conditional on the proposer \(p_{7,o}^E\); the values conditional on the proposer \(V^D,B_o\).

Consider first the households with a good history \(z = 0\). Having chosen to default, they decide whether to choose Chapter 7 or bargaining. The probability of Chapter 7 and values are, for \(o = lo, ho\),

\[
p_{7,o}^E(a, b, s) = CDF\left(\frac{(V^7(a, s) - \chi^7) - (V^B_o(a, b, s) - \chi^B)}{\sigma_{7vsB}}\right) \tag{1}
\]

\[
V^D_o(a, b, s) = (1 - p_{7,o}^E(a, b, s))(V^B_o(a, b, s) - \chi^B) + p_{7,o}^E(a, b, s)(V^7(a, s) - \chi^7) \tag{2}
\]

where \(V^7\) and \(V^B_o\) denote the bankruptcy and bargaining option values, respectively, and \(CDF\) denotes the cdf of a standard Normal. Note that the \(V^B_o\)’s are ex-post values known after uncertainty about the proposer is revealed. The ex-ante value is

\[
V^D(a, b, s) = \gamma_{ho}V^D_{ho}(a, b, s) + (1 - \gamma_{ho})V^D_{lo}(a, b, s) \tag{3}
\]

Then the choice whether to default of repay yields the default probability

\[
d(a, b, s) = CDF\left(\frac{V^D(a, b, s) - V^R(a, b, s)}{\sigma_{DvsR}}\right) \tag{4}
\]

and ex-ante value

\[
V(a, b, s) = (1 - d(a, b, s))V^R(a, b, s) + d(a, b, s)V^D(a, b, s) \tag{5}
\]

where \(V^R\) stands for the value of repaying.

There is no decision about status for the household while bearing the bankruptcy flag \(z = 1\). In the failed-bargaining states \(z = 2, 3\), the household must decide whether to continue bargaining or file for Chapter 7.\(^6\) The probability of bankruptcy and the resulting value are, for \(o = lo, ho\),

\[
p_{7,o}^B(a, b, s) = CDF\left(\frac{(V^7(a, b, s) - \chi^7) - V^B_o(a, b, s)}{\sigma_{7vsB}}\right) \tag{6}
\]

\[
V^D,o(a, b, s) = (1 - p_{7,o}^B(a, b, s))(V^B_o(a, b, s) - \chi^B) + p_{7,o}^B(a, b, s)(V^7(a, b, s) - \chi^7) \tag{7}
\]

\(^6\)There is no extra non-pecuniary cost to continuing bargaining but there is still the cost to fling bankruptcy. We may or may not want to change this assumption.
3.1.2 The option values

The objects that all the above default choices take as given are the value of repaying $V^R$; the value of filing for bankruptcy $V^7$; the value of failed bargaining $V_o^B$. The value of repaying in the clean state:

$$V^R(a,b,s) = \max_{b',a'} \{u(c) + \beta E[V(a',b',s')|s]\}$$  

subject to

$$c - q(a',b',s)b' + q^0a' \leq e(s) - b + a - \phi(a'/a)$$

The value for the household to filing for Chapter 7 when $z = 0$ or being in the bankrupt state $z = 1$ is determined as follows. (Note that when $z = 1$ the agent holds no debt so it must be that $b = 0$.) The household’s value:

$$V^7(a,b,s) = \max_{a'} \{u(c) + \beta E[\pi^7 V^7(a',0,s') + (1 - \pi^7) V(a',0,s')]\}$$

s.t. $c + q^0a' = e(s)(1 - \theta^7) + a - \max(0,\min(a - a^7, b)) - \phi^7(a'/a)$

Consider now the values of bargaining. Bargaining is over proposed transfers of current resources $\tau$ and debt swaps $b'$. It also involves a choice for assets next period $a'$.

**Household proposes** Suppose the proposer is the household. The value if the offer is not accepted (i.e., of delaying agreement by making an unacceptable offer) is

$$V_{ho,noac}^B(a,b,s) = \max_{a'} \{u(c) + \delta E[I_{ho,ho} V_{ho,ho}^B(a',b,s') + (1 - I_{ho,ho}) V_{lo,ho}^D(a',b(1 - \tau_b),s')]\}$$

subject to the budget $c + q^0a' = e(s)(1 - \theta^B) + a - \phi^B(a'/a) - \min(\tau_b e(s)(1 - \theta^B) + a)$ This gives decision rule for assets $a'_{ho,noac}(a,b,s)$. The value if the offer is accepted is

$$V_{ho,ac}^B(a,b,s) = \max_{a',b'} \{u(c) + \delta E V(a',b',s')\}$$

subject to the budget $c + q^0a' = e(s)(1 - \theta^B) - T(a,b,s,a',b') + a - \phi^B(a'/a)$ where the transfer $T(.)$ is such that

$$T(a,b,s,a',b') + b' q(a',b',s) = \min \{q^0 E[\gamma_{ho,ho} W_{ho}^{D,B}(a'_{ho,ho},a',b,s) + (1 - \gamma_{ho,ho}) W_{lo}^{D,B}(a'_{ho,ho},a',b,s)|s], b]\}$$

Expression (12) says that the household must get the bank to accept by offering either full-repayment or the bank’s option value of postponing agreement. This yields the settlement plan $a'_{ho,ac}(a,b,s)$, $b'_{ho,ac}(a,b,s)$ and $\tau_{ho,ac}(a,b,s) = T(a,b,s,a'_{ho,ac}(a,b,s),b'_{ho,ac}(a,b,s))$. The household’s value of bargaining is thus

$$V_{ho}^B(a,b,s) = \max \{V_{ho,ac}^B(a,b,s), V_{ho,ac}^B(a,b,s)\}$$
Lender proposes Suppose now the proposer is the lender. If the offered plan is not accepted

$$V_{lo,noac}^B(a,b,s) = \max_{a'} \{ u(c) + \delta E[\gamma_{lo,lo} V_{lo}^{D,B}(a',b(1-\tau_b),s') + (1-\gamma_{lo,lo}) V_{ho}^{D,B}(a',b(1-\tau_b),s')] \}$$

(14)

with $c + q' a' = e(s)(1-\Theta^B) + a - \phi^B(a'/a) - \min(\tau_b e(s)(1-\Theta^B) + a)$, which yields the policy rule $a'_{lo,noac}(a,b,s)$. Clearly, the household's acceptance condition must hold as an equality at the plan proposed by the bank. Therefore, the household gets its reservation value $V_{lo,ac}^B(a,b,s) = V_{lo,noac}^B(a,b,s)$. The household's value of bargaining is thus

$$V_{lo}^B(a,b,s) = V_{lo,noac}^B(a,b,s).$$

(15)

3.2 Banks

Banks make decisions when in a failed-bargaining default state $z = 2, 3$. In such a situation, the bank takes as given the market prices $q$, the household's option values of remaining in that state $V_{o,noac}^B$ for $o = lo$ or clearing its record $V$, and the household's decision rules about bankruptcy $p_{7,o}^B$ and no-acceptance savings $a'_{o,noac}$ for $o = lo, ho$.

We first determine the value to the bank in such a state as a function of the bank's option values of bargaining $W_{o}^D$ for $o = lo, ho$ and of bankruptcy $W^7$. This value is, for $l = lo, ho$,

$$W_{o}^{D,B}(a,b,s) = (1 - p_{7,o}^B(a,b,s)) W_{o}^B(a,b,s) + p_{7,o}^B(a,b,s) W^7(a,s)$$

(16)

Second we determine the underlying option values. The recovered value in bankruptcy is

$$W^7(a,b,s) = \begin{cases} 0 & a \leq \bar{a}^7 \\ \gamma^7 \min(a - \bar{a}^7, b) & a > \bar{a}^7 \end{cases}$$

(17)

Household proposes Consider now the bargaining option when the household proposes. The bank's value to accepting is

$$W_{ho,ac}^B(a,b,s) = \tau_{ho,ac}(a,b,s) + b'q(a'_{ho,ac}(a,b,s), b'_{ho,ac}(a,b,s), s),$$

(18)

and the value to rejecting is

$$W_{ho,noac}^B(a,b,s) = q^0 E[\gamma_{ho,ho} W_{ho}^{D,B}(a'_{ho,noac}(a,b,s), b(1-\tau_b),s')] + (1-\gamma_{ho,ho}) W_{lo}^{D,B}(a'_{ho,noac}(a,b,s), b(1-\tau_b),s')] + \min(\tau_b e(s)(1-\Theta^B) + a)$$

(19)

The bank would get its reservation value unless it exceeds full repayment so that $W_{ho,ac}^B(a,b,s) = \min(W_{ho,noac}^B(a,b,s), b)$. Therefore the value of bargaining to the bank is then:

$$W_{ho}^B(a,b,s) = \min(W_{ho,noac}^B(a,b,s), b) - \lambda^B b$$

(20)
**Lender proposes** Consider finally bargaining when the bank proposes. The values to the bank to having the household accepting is

\[ W^B_{lo,ac}(a, b, s) = \max_{a',b'} T(a, b, s, a', b') + b'q(a', b', s), \]

subject to full repayment being the best the bank can propose

\[ W^B_{lo,ac}(a, b, s) \leq b \]

with the transfer satisfying

\[
T(a, b, s, a', b') = -q^0 a' + e(s)(1 - \theta^B) + a - \phi^B(a'/a)
- \left((1 - \sigma) \left[ V^B_{lo,noac}(a, b, s) - \beta EV(a', b', s') \right] \right)^{1/(1-\sigma)}
\]

This last condition (23) says that the bank will make an offer that leaves the household indifferent between accepting and delaying the outcome. This yields the settlement decision rules \( a'_{lo,ac}(a, b, s) \), \( b'_{lo,ac}(a, b, s) \) and \( \tau_{lo,ac}(a, b, s) \). The value if the offer is not accepted:

\[
W^B_{lo,noac}(a, b, s) = q^0 E[\gamma_{lo,lo} W^D_{lo}(a'_{lo,noac}(a, b, s), b(1 - \tau_b), s')] + 
(1 - \gamma_{lo,lo}) W^D_{ho}(a'_{lo,noac}(a, b, s), b(1 - \tau_b), s') | s] + \min(\tau_b b, e(s)(1 - \theta^B) + a)
\]

The bank's value of bargaining here is therefore

\[ W^B_{lo}(a, b, s) = \max[W^B_{lo,noac}(a, b, s), W^B_{lo,ac}(a, b, s)] - \lambda^B b \]

### 3.3 Zero profits

With fee-entry competition in the credit market, the prices of debt imply zero ex-ante profits for banks extending loans. The evaluation of cash flows in the coming periods depends on the households default rule \( d \) and the banks ex-ante value in the event of default \( W^D \). Given these, the zero-profit condition for the lender yields the price schedule

\[ q(a', b', s) = (1 - \pi(a', b', s) - \lambda) q^0 + q^0 \sum_{s'} \Gamma_{s,s'} d(a', b', s') W^D(a', b', s')/b' \]

where the probability of default is

\[ \pi(a', b', s) = \sum_{s'} \Gamma_{s,s'} d(a', b', s'). \]

It remain to determine the ex-ante value of the bank under default with a clear record \( D^D \). It depends on the clean households' decision rules about bankruptcy \( p_{7,o} \) and the
bank’s option values of bargaining $W^B_o$, for $o = lo, ho$, and of bankruptcy $W^7$. The ex-ante recovered value when there is default conditional on the proposer $o = lo, ho$ is

$$W^D_o(a, b, s) = p_{7,o}(a, b, s)W^7(a, b, s) + (1 - p_{7,o}(a, b, s))W^B_o(a, b, s)$$  \hspace{1cm} (27)$$

The ex-ante value is therefore

$$W^D(a, b, s) = \gamma_{ho}W^D_{ho}(a, b, s) + (1 - \gamma_{ho})W^D_{lo}(a, b, s)$$  \hspace{1cm} (28)$$

### 3.4 Formal definition

Define first an equilibrium for given bargaining outcomes. Given the bargaining values for the household $V^B_{ho}$ and $V^B_{lo}$, and for the bank $W^B_{ho}$ and $W^B_{lo}$, a bargaining-conditional equilibrium is a price schedule, value functions and policy rules such that:

(i) Given the value of bankruptcy for the household $V^7$, the value of defaulting $V^D$ and the probabilities of bankruptcy, $p_{7,lo}$ and $p_{7,ho}$, solve (1) to (3).

(ii) Given the ex-ante value $V$ and the price schedule $q$, the value of repaying $V^R$ solves (8).

(iii) Given the ex-ante value $V$, the value of bankruptcy $V^7$ solves (9).

(iv) Given the values to repaying $V^R$ and defaulting $V^D$, the ex-ante value $V$ and default probabilities $d$ solve (5) and (4).

(v) The bank’s value under bankruptcy $W^7$ solves (17).

(vi) Given the probabilities of bankruptcy, $p_{7,lo}$ and $p_{7,ho}$, and $W^7$, the bank’s value to defaulting $W^D_{lo}$, $W^D_{ho}$ and $W^D$ solve (27) and (28).

(vii) Given default probabilities $d$ and bank’s default value $W^D$, the price schedule $q$ solves (26).

Given $q$, (i)-(vi) defines a fixed pint problem in $V$, $V^7$, $V^D$, $V^R$, $W^7$ and $W^D$ and the associated decision rules. Adding (vii) results in another fixed point problem in $q$. These are for given bargaining values $V^B_o$ and $W^B_o$.

A bargaining equilibrium. Given non-bargaining functions $V$, $V^7$, $W^7$, and prices $q$ a bargaining equilibrium are values and decisions functions such that:
(i) Given the values of continuing bargaining for the household $V_{lo}^B$ and $V_{ho}^B$, the probabilities of filing for bankruptcy or bargain and the values, $p^B_{B,lo}$, $p^B_{1,ho}$, $V_{lo}^{D,B}$ and $V_{ho}^{D,B}$, solve (6), (7).

(ii) Given the values to continue bargaining for the household, $V_{lo}^{D,B}$ and $V_{ho}^{D,B}$, and for the bank, $W_{lo}^{D,B}$ and $W_{ho}^{D,B}$, the bargaining value for the household when it proposes, $V_{ho}^B$, solves (10) to (13).

(iii) Given the values to continue bargaining for the household, $V_{lo}^{D,B}$ and $V_{ho}^{D,B}$, the bargaining value for the household when the bank proposes, $V_{lo}^B$ and $V_{lo, noac}^B$, solves (14) to (15).

(iv) Given the probabilities of filing for bankruptcy or bargain and bank values, $p^B_{B,lo}$, $p^B_{1,ho}$, and $W_{lo}^B$, $W_{ho}^B$, the values for the bank of being in default and bargaining $W_{lo}^{D,B}$ and $W_{ho}^{D,B}$ solve (16).

(v) Given the values for the bank of being in default and bargaining $W_{lo}^{D,B}$ and $W_{ho}^{D,B}$ and the household’s behaviour if its proposal is not accepted $V_{ho, noac}^B$, the banks value of entering bargaining when the household proposes $W_{ho}^B$ solves (19) and (20).

(vi) Given the values for the bank of being in default and bargaining $W_{lo}^{D,B}$ and $W_{ho}^{D,B}$ and the household’s behaviour under $V_{lo, noac}^B$, the banks value of entering bargaining when the banks proposes $W_{lo}^B$ solves (21), (22), (23), (24), and (25).

A bargaining equilibrium if a fixed point in $V_o^B$ and $W_o^B$. It takes as given non-bargaining values $V$, $V^7$, and $W^7$, and $q$.

An equilibrium is functions $V_o^B$, $W_o^B$, $V$, $V^7$, $W^7$ and $q$ that satisfy both a bargaining equilibrium and a no-bargaining equilibrium. Finding an equilibrium involves solving a number of nested fixed point problems.

We can show two theoretical results. First taking debt prices as exogenous we can show that a unique bargaining equilibrium exists. Second under limited conditions we can show that an equilibrium to the entire model exists.

4 Calibration

The model has a large number of parameters. In order to facilitate the task of assigning values, we will first select a smaller set of parameters that are either largely responsible for one observation or part of a broad consensus in the literature. These are shown in table 1.
A model’s period corresponds to one year.\(^7\) The discount rate \(\beta\) and risk aversion \(\sigma\) are set to standard values. The price of assets \(q_0\) is set to accord with a 3 per cent risk free interest rate. The intermediation costs \(\lambda\) matches existing estimates of 4 per cent. We assume away asset losses under Chapter 7 which dictates the choice for \(\gamma_7\). The exemption level for assets under bankruptcy varies widely across U.S. states. The choice for \(\bar{a}_7\) is equivalent to exempting assets below average income. We pick the persistence of the formal default state \(\pi_7\) to match the average exclusion from credit after a bout in Chapter 7, which is frequently cited in the literature as under 2 years. Finally we begin with a benchmark calibration which allows offers to be solely suggested by creditors.\(^8\)

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk aversion</td>
<td>(\sigma)</td>
<td>2.00</td>
</tr>
<tr>
<td>Discount</td>
<td>(\beta)</td>
<td>0.90</td>
</tr>
<tr>
<td>Bank’s cost</td>
<td>(\lambda)</td>
<td>0.04</td>
</tr>
<tr>
<td>Capital loss Ch. 7</td>
<td>(\gamma_7)</td>
<td>1.00</td>
</tr>
<tr>
<td>Asset exemption</td>
<td>(\bar{a}_7)</td>
<td>0.50</td>
</tr>
<tr>
<td>Risk-free interest</td>
<td>(1/q_0 - 1)</td>
<td>0.03</td>
</tr>
<tr>
<td>Persistence formal default</td>
<td>(\pi_7)</td>
<td>0.68</td>
</tr>
<tr>
<td>Proposer</td>
<td>(\gamma_{ho})</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(\gamma_{ho, ho}, \gamma_{lo, lo})</td>
<td>1.00  1.00</td>
</tr>
</tbody>
</table>

The remaining few parameters will be chosen in order to produce outcomes consistent with a limited number of reasonable targets related to observed measures of default and debt. We restrict ourselves to situations where the adjustment cost and the default costs are the same across types of default decisions and individual states. The parameters are the collection tax \(\tau_b\), the adjustment cost \(\phi\), the utility costs to defaulting \(\chi_d\), and the income process. The income process is described by two possible states that follow a Markov chain, so one needs to specify two values for income realizations and their degree of persistence.\(^9\) Note also that the variance of the noises, \(\sigma_{7vsB}\) and \(\sigma_{DvsR}\), will need to be set to perform their designed technical function; at 0.05, they do not affect results in any way. The parameters will be aiming to produce outcomes consistent with the following observations. According to the SCF for waves 2004 and 2007, the unsecured debt to income ratio falls between 5 and 7, and the proportion of household in debt is between 20 and 30 per cent. As for debt defaults, one could tentatively take the number of households not

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\(^7\)This is slightly inconsistent with our measure of informal default. We assume an individual behind repayments for two months is engaged in informal default. Thus we include in our theoretical measure of default households who default immediately into bargaining and settle in the very same period as these households are less than one year in default.

\(^8\)We consider policy changes that strengthen the hand of debtors to be important to the analysis and they are discussed in the policy section of the paper.

\(^9\)These parameters are traditionally set outside the model but, given the sparse state space, we select them to match endogenous moments.
current on their debt repayments. In the Survey of Consumer Finances 2007 the figure stands at above 5 per cent.\(^{10}\) Regarding the portion of defaults that take place under the formal bankruptcy procedures (i.e., Chapter 7), this is between 0.6 and 0.9 per cent of the population. Taking the low-income state to represent unemployment, a realization of about a quarter of employment income is a standard choice. Table 2 displays the chosen parameters and the implied values for the targeted variables.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>value</th>
<th>Observable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collection tax</td>
<td>(\tau_b)</td>
<td>0.08</td>
<td>formal default rate 0.0064</td>
</tr>
<tr>
<td>Default costs</td>
<td>(\chi_d)</td>
<td>0.20</td>
<td>informal default rate 0.052</td>
</tr>
<tr>
<td>Income realizations</td>
<td>((e_1,e_2))</td>
<td>(0.20,0.90)</td>
<td>income gap 0.22</td>
</tr>
<tr>
<td>Persistence income</td>
<td>((\Gamma_{1,1},\Gamma_{2,2}))</td>
<td>(0.40,0.80)</td>
<td>prop. in debt 0.26</td>
</tr>
<tr>
<td>Adjustment cost</td>
<td>(\phi)</td>
<td>0.60</td>
<td>debt/inc 0.056</td>
</tr>
</tbody>
</table>

The choices made yield implications fairly close to the empirical targets. The average interest rate implied by the model is 26 per cent, larger than the measured 12 or 14 per cent, but in line with other similar studies. The assumed 8\% of debt lost to collection is roughly consistent with the data on collection. The variance of log income 0.424 is close to the data reported in Krueger and Perri (2006) Fig. 1.

5 Properties

This section reports properties of the benchmark economy, with a focus on the patterns for formal and informal default across different households. One main goal in this section will be to show how the model reproduces qualitative patterns observed in the data.

A preliminary observation is that, in this model, adjustment costs as well as bankruptcy exemptions bring about mixed portfolios for individual households, with positive amounts of both debt and assets. In the stationary distribution, most individuals in debt hold some positive amount of assets. This is a distinctive property of our model, and one obvious precondition for addressing the role of portfolio composition.

5.1 Default outcomes and credit terms

Table 3 displays figures regarding the mass and characteristics of households in different states as well as their decisions.

\(^{10}\)See Díaz-Giménez, Glover, and Ríos-Rull (2011).
Table 3: **Summary distribution and choices**

<table>
<thead>
<tr>
<th>Clean credit:</th>
<th>mass</th>
<th>debt</th>
<th>assets</th>
<th>borrowing</th>
<th>saving</th>
<th>inc</th>
<th>recov</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>0.9740</td>
<td>0.041</td>
<td>0.823</td>
<td>0.038</td>
<td>0.844</td>
<td>0.727</td>
<td></td>
</tr>
<tr>
<td>Formal defaulters</td>
<td>0.0064</td>
<td>0.089</td>
<td>0.204</td>
<td>0.000</td>
<td>0.177</td>
<td>0.401</td>
<td></td>
</tr>
<tr>
<td>Informal defaulters</td>
<td>0.0526</td>
<td>0.284</td>
<td>0.839</td>
<td>0.159</td>
<td>0.543</td>
<td>0.246</td>
<td></td>
</tr>
<tr>
<td>Inf. def. who settle</td>
<td>0.0411</td>
<td>0.269</td>
<td>0.770</td>
<td>0.109</td>
<td>0.500</td>
<td>0.259</td>
<td>0.095</td>
</tr>
<tr>
<td>Inf. def. who delay</td>
<td>0.0115</td>
<td>0.335</td>
<td>1.088</td>
<td>0.335</td>
<td>0.692</td>
<td>0.200</td>
<td></td>
</tr>
<tr>
<td>Repeat defaulters</td>
<td>0.0078</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Failed to agree:      |      |      |        |           |        |      |
|-----------------------|------|------|--------|-----------|--------|------|-------|
| All                   | 0.0116 | 0.313 | 0.685  |           | 0.620  |
| Formal defaulters     | 0.0002 | 0.135 | 0.248  | 0.000     | 0.217  | 0.492|
| Informal defaulters   | 0.0114 | 0.316 | 0.692  | 0.031     | 0.571  | 0.622|
| Inf. def. who settle  | 0.0113 | 0.321 | 0.698  | 0.028     | 0.576  | 0.627| 0.155 |
| Inf. def. who delay   | 0.0001 | 0.119 | 0.244  | 0.120     | 0.069  | 0.200|

Table 4: **Portfolio mix. SCF 2007**

<table>
<thead>
<tr>
<th></th>
<th>bankrupt</th>
<th>late payers</th>
</tr>
</thead>
<tbody>
<tr>
<td>assets in USD</td>
<td>169,085</td>
<td>197,878</td>
</tr>
<tr>
<td>debt in USD</td>
<td>79,200</td>
<td>98,063</td>
</tr>
<tr>
<td>income in USD</td>
<td>40,792</td>
<td>38,471</td>
</tr>
</tbody>
</table>
First we want to compare those who choose default in any form with those who repay. We begin by comparing those who begin with clean credit records. Starting with a clean credit record, the households who default have higher debts, lower assets (on average) and lower incomes. We can also make comparisons between different types of defaulters. Informal defaulters have markedly more debt and assets, and less income, than informal defaulters. We want to see if this central model’s implications find support in the data. We focus on the portfolio mix of households making different decision about debt repayments. Diaz-Giménez, Glover, and Rios-Rull (2011) discuss the financial characteristics of late and timely payers based on data from the SCF 2007. As discussed in the calibration section, the definition of late payers is close to the notion of informal defaulters in the model. Table 4 reproduces figures for measures of assets, debt and income for the two groups of households. By comparison with the figures implied by the model in Table 3, we can conclude that the model captures well observed differences across individuals. In the data, like in the model, late payers have more debts and assets than formal defaulters, the difference in assets being more marked than the difference in debts. Also late payers have lower income. Quantitatively, however, the model appears to overstate the differences between these groups.

There are also differences within the group who choose informal default. About 80 per cent of the households who default to engage in bargaining do accept the lender’s first proposed plan and settle immediately; the remaining 20 per cent of bargainers do not settle and carry on bargaining. Those who settle quickly have lower debts and, especially, fewer assets than those who delay settlement; they also have a slightly higher average income. Settlements involve a recovery of about 30 per cent of debt outstanding. About 19 per cent of those who settle will repeat defaulting next period if income does not improve. These debtors buy time in informal default to run down their assets and borrow again before filing formally. These settlements are in this sense fictional.

Looking at the borrowing and saving decisions, it is clear that as individuals move through informal default their assets fall, an indication that the process of bargaining is being used to run assets down in an orderly less-costly fashion while, at the same time, weakening the creditor’s future negotiating position. In effect, if an exogenous condition banning the sale of assets in bargaining is imposed, then bargaining will never occur in equilibrium. Notice that settlements involve a sharp reduction in debt.

Consider now the group of households who had defaulted informally and never settled. The vast majority will settle in the current period so the average length of negotiations is of two periods. They mainly settle as income improves. Settlers, compared with stayers, have notably higher income, as well as larger debt and assets. Settlements involve a recovery of about half of outstanding debt. Stayers are those hit again with low income in spite of holding relatively lower debts and assets. A few remaining households, about two per

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11 Furthermore, solvent households have higher net worth and lower debt than the two types of defaulters both in the data and in the model. Incidentally, note that, consistently with the data shown, on average defaulters have positive net worth. This is not due to bargaining but to mixed individual portfolios.
cent of non-settled bargainers, file for formal Chapter 7 bankruptcy. Like the stayers, they have successfully ridden their assets below the exemption level and have relatively small debts, but have a higher income on average.

In sum, quick settlements are reached with households who hold comparatively few assets and debts; for most other households, settlements occur later when income recovers.

5.2 The distribution of default

Having discussed average patterns we now present details of the distribution of outcomes. We discuss total default first, without distinguishing its formal and informal modalities. The top section of Figure 1 displays the policy rule for default, \( d \), or default set, over asset and debt levels. As expected, default is monotonically increasing in debt and decreasing in assets. It is also decreasing in income. Interestingly default can occur for individuals with positive net worth. This is a feature of the data which previous work fails to reproduce.\(^\text{12}\) It shows the role of the financial portfolio in the presence of limited commitment. Drawing on assets to smooth consumption while simultaneously repaying debt can lead to a significant expenditure in adjustment costs. The costs of this fire-sale of assets will be a prime driver of repayment troubles in our model.

The bottom section of Figure 1 shows the distribution of the mass of households who default. This is drawn only for individuals who have a clean credit record. Consistently with the averages discussed from Table 3, two differentiated groups of households fail to repay, one with low debts and low assets, and another type with higher debts and high assets above the bankruptcy exemption level. Most default occurs within the high-asset type of defaulting households as this group has larger mass. Interestingly, the asset position of defaulters is increasing in the level of debt outstanding, an indication of the role of assets as (informal) collateral.

Turning to the default mix between formal Chapter 7 bankruptcy and informal bargaining, the top section of Figure 2 shows the decision rule for the fraction of defaulting households that choose bargaining instead of Chapter 7. The preference for informal default shows a sharp rise near the asset exemption level, and is inverted-U-shaped in the level of assets for high income households. These patterns for the preference over the default mix, combined with the characterization of the distribution of overall default, result in the picture for the distribution of the default mix. The low-asset defaulters choose predominantly Chapter 7, whereas the high-asset defaulters opt in favour of bargaining. The mass distribution of bargainers is explicitly shown in the bottom panel of Figure 2, to be compared with the analogous representation for all defaulters earlier in Figure 1. Bargainers make up for the immense majority of the defaulters with high asset positions, but they are only a small proportion of the defaulters with low asset positions. Some formal default happens for assets above the exemption but only for high-income households.

\(^{12}\)This feature is mentioned in Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007).
5.3 Bargaining or Chapter 7?

The households who default via bargaining have a distinctive type of portfolio, holding debt and assets markedly above the rest of defaulters, with assets above the bankruptcy exemption level. Why do these households prefer bargaining? With assets that can be seized under Chapter 7 to pay off a considerable outstanding debt, the option to bargain is more attractive. The household can smooth consumption by either settling and issuing debt or delaying settlement instead of disinvesting hurriedly. By choosing to bargain, the household keeps more assets than under Chapter 7 bankruptcy. Bargainers have lower earnings on average since, as an outcome of the negotiations, lenders will recover more from high earners.

Why is there any Chapter 7 at all? Chapter 7 occurs in the situations shown in Fig. 1 and 2 where households have little in assets so the discharge of debt precludes the bank from grabbing anything from the household. Bargaining means the bank can always seize something until a settlement is reached and becomes a suboptimal choice for the household in these situations. In effect, it can be seen in Figure 5 that recovery rates rise noticeably with the household’s level of assets. On first blush, it may seem puzzling that any debtors would enter formal default at all as some total surplus is lost inside formal default and the participants in the bargaining should be able to work out a deal that strictly improves upon the formal default payoffs. The reason for formal default occurring is a failure of commitment: the timing in negotiations implies that the household cannot reconsider their decision within the current period not to file for bankruptcy when the bank proposes a plan to bargain over. With bargaining power on the side of the creditor, debtors cannot be guaranteed their Chapter 7 value if they choose to negotiate.\textsuperscript{13}

How have bargaining households ended up with their characteristic portfolios? We need to examine the decision rules for clean households. The policy function for savings has the usual properties except for a particularly sharp change when the household savings approach the level of bankruptcy exemption. The policy function for borrowing is in Figure 3 for the low income household.\textsuperscript{14} For portfolios where new savings is below the exemption level the household borrows only small amounts. For some higher level of wealth leading to savings narrowly above the exemption level, the household comes to borrow more. Quite a lot of debt in the economy is accounted for by this group of households. In sum, bargainers are low-income agents who in the previous period made some savings above the bankruptcy exemption level.

Why the large debts the bargaining households carry? These households anticipate they will default and bargain if a low income realization strikes again. Since the level of debt does not much affect negatively the outcome of the negotiations (see (14),(15), (21) (22) (23)\textsuperscript{13}\textsuperscript{14} However, the ability to run down assets in informal default allows consumers to capture some of the negotiation rents.\textsuperscript{14} For the high income realization borrowing is pretty thin. The bargainers state is necessarily reached after one period of low income so we focus on this situation.)
this household issues whatever debt is necessary to maximize the amount of current consumption accruing from borrowing. This would also be true of low-assets Chapter 7 defaulters, but the difference is that under bargaining banks anticipate they will recover something so this debt commands a less steep price schedule compared with that faced by households bound to declare Chapter 7 bankruptcy. These consumers can therefore issue a larger level of debt in order to increase consumption. Figure 4 displays the schedule of debt prices for low income households to demonstrate the point. The schedule is much flatter at high asset positions associated with bargaining default than at the low asset positions associated with Chapter 7 default.

5.4 Delay in settlements

There are situations where a settlement is not achieved immediately upon entering bargaining. For low income households, settlement is not achieved on a region of high debts and high assets. This corresponds to situations where the creditor cannot get much out of an acceptable plan and thus prefers to wait to recover more in future on the expectation that the household may have a good income realization later on, while keeping current collection efforts. This point is supported by Figure 5 showing the recovery rates conditional on settlement. It conveys a trade-off facing the bank when delaying between recovering more in the event that income rises and recovering less as the household runs assets down in the meantime. Bargaining with high income households always leads to settlement.

5.5 Time paths

From the value functions we can simulate the time paths of an economy to show how consumers move into and out of various default chapters. We plot three examples. These are attached as Figures(x-z). These examples show how the inclusion of informal default allows for qualitatively different default experiences than in the traditional model. The first two examples are of individuals who have accumulated assets before having income issues. In these figures, we show how informal default can allow such consumers to insure against shocks. In the first figure we demonstrate how, as part of that insurance, a prolonged stay in informal default can arise. For this simulation we attach four plots. The first plot shows the behavior of income which is the driving factor in creating economic distress and default. In this example and in every example we attach, the adverse income shock occurs prior to default by at least one period. This is a general property of the model. Generally household spends one or more periods after the initial shock accumulating more debt and running down assets and only then default. This can be seen from the second and third

\[ (24) \] in section 3.1.2. and 3.2),\(^{15}\) this household issues whatever debt is necessary to maximize the amount of current consumption accruing from borrowing. This would also be true of low-assets Chapter 7 defaulters, but the difference is that under bargaining banks anticipate they will recover something so this debt commands a less steep price schedule compared with that faced by households bound to declare Chapter 7 bankruptcy. These consumers can therefore issue a larger level of debt in order to increase consumption. Figure 4 displays the schedule of debt prices for low income households to demonstrate the point.\(^{16}\) The schedule is much flatter at high asset positions associated with bargaining default than at the low asset positions associated with Chapter 7 default.

\(^{15}\)Outcomes are always far from the full-repayment constraint in Eq.(22).

\(^{16}\)High income households face an only slightly different set of prices because of the assumed asymmetry in the income process.
subpanel where nominal debt and assets are attached. We also attach in the forth panel debt prices on new debt issued in a given period. We point on one of the most salient features. After the poor income shock, the consumer is able to borrow at a severe discount. This means they are able to borrow then they can guarantee repayment on. Without informal default, consumers in equilibrium are only able to borrow an amount they can likely payback for sure regardless of how income changes the next period. This is what is generally referred to in these models as a tight debt constraint, since any short term insurance through debt will come at the expense of lower consumption once that debt is repaid. But in this model full immediate repayment only occurs if income recovers. Thus the lower debt prices represent more flexibility in the model. It is this state contingent repayment that allows the model to produce insurance.

We can describe some more properties of the extended default which is attached as Figure 6. The default experience is long and lasts for multiple periods. The delinquency fails to resolve in the initial period as the the debtors are effectively debt constrained so there is no benefit in reentering the market. The default itself is quite messy as there is at least one failed workout prior to settlement. Only when income recovers is a lasting settlement reached. Only when income recovers do the in the interest rates consumers borrow at also recover.

In a second panel we have attached a default that begins as a simple delinquency but as assets decline the default is eventually transformed into a formal bankruptcy. This is shown in Fig.7. As in the previous example the initial default leads the household to respond by borrowing at a discount to attempt to cushion the initial shock even though the households borrowing is ultimately likely to be resolved through default. Once borrowing constraints are reached, the household defaults formally. We note in passing that this extended default contains a failed settlement. Generally while in informal default, the household runs down her assets. In this simulation the household’s assets eventually reach the exemption level while in informal default. Here the household enters bankruptcy has her debt forgiven without any further repayment and prepares to reenter the market. Note that as the household’s financial distress worsens the debt prices the household faces steadily worsen as well, culminating in a trough in the period before entering formal bankruptcy and their debt is worthless. It is very common in this model for households in bankruptcy to have no recovery at all, which matches a stylized fact about bankruptcy. It is also common for bankruptcy to be unavoidable once assets decline below the exemption level, regardless of the behavior of income.

We have attached a third panel to show the experience of a household with limited assets. This is attached as 8. Note that the household who possessed no debt before the first adverse shock has an initial response of borrowing to attempt consumption smoothing while living off her limited assets. This is very similar to the previous examples. But there is an immediate caveat which makes this default very different than the previous two. Her ability to borrow is very limited and she can only accumulate a very small amount of debt at a much smaller discount than in the previous simulations. Encouraged by a relatively
favorable shock to stigma, in the next period she immediately defaults into Chapter 7 as it is preferable to repaying her limited debt stock. We emphasize that there are very few insurance opportunities available for this household as there is very little borrowing in this example once the bad shock occurs. Any attempt to add additional debt would be effectively debt constrained by the possibility of Chapter 7.

We can compare these time paths to the time paths generated by a model without Chapter 7. We construct two. One is for a household who possesses assets below the exemption when the initial negative shock occurs. This is attached as Figure 9. Note that the behavior of this default is very similar to the time path for the model with informal bankruptcy. This leads us to the conclusion that for low levels of assets the threat of bankruptcy eliminates the insurance opportunities the model possesses. The more interesting example is for a household who manages to accumulate a significant amount of assets before the initial shock occurs. This is attached as Figure 10. Note that consumers are able to respond to bad shocks by borrowing but that debt prices never substantially decline until the household reaches formal default. This means that consumers are limited to only borrowing an amount that they are likely to repay regardless of the performance of income. Since such repayment is likely to occur before income recovers it is of limited use in terms of providing insurance.

6 The significance of informal default

We discuss why accounting for bargaining is significant for empirical and policy questions. Table 5 illustrates the main findings.

6.1 Welfare, consumption smoothing, and debt

The first experiment compares the model to one in which bargaining is not allowed. This results in the type of environment characteristic of all the existing literature on consumer bankruptcy where default only takes place via the formal procedures. In such a model consumers have a more limited ability to use debt to smooth consumption. Table 5 compares aggregate variables with and without bargaining. The variance of consumption declines with the presence of bargaining by about 10 per cent. In terms of welfare, a policy of penalties that removes the possibility of bargaining has an adverse effect. On average, households are a large 0.60 per cent worse off. The high-debt individuals, who would default and bargain with high probability, loose above 2 per cent. They now opt for repaying. Figure ** displays the distribution of welfare changes over debt for agents with a clean record.

Hence the potential for negotiations provides additional insurance against bad shocks. It allows more households to borrow when they hold asset positions. In this way, they can wait until things get better instead of running down their assets in a costly manner. In ef-
<table>
<thead>
<tr>
<th>Description</th>
<th>$\chi^7$</th>
<th>$\chi^7B$</th>
<th>$\chi^B$</th>
<th>$d^7$</th>
<th>$d^B$</th>
<th>$d_{ac}^B$</th>
<th>debt</th>
<th>indebt</th>
<th>$q$</th>
<th>$c$</th>
<th>varc</th>
<th>% welf</th>
</tr>
</thead>
<tbody>
<tr>
<td>benchmark</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.0064</td>
<td>0.0526</td>
<td>0.0412</td>
<td>0.041</td>
<td>0.27</td>
<td>0.792</td>
<td>0.735</td>
<td>0.0276</td>
<td>–</td>
</tr>
<tr>
<td>no barg.</td>
<td>0.20</td>
<td>0.20</td>
<td>+∞</td>
<td>0.0103</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.029</td>
<td>0.25</td>
<td>0.892</td>
<td>0.735</td>
<td>0.0296</td>
<td>-0.61</td>
</tr>
<tr>
<td>$\chi^7$ down</td>
<td>0.10</td>
<td>0.10</td>
<td>0.20</td>
<td>0.0120</td>
<td>0.0515</td>
<td>0.040</td>
<td>0.030</td>
<td>0.19</td>
<td>0.687</td>
<td>0.734</td>
<td>0.0282</td>
<td>-0.49</td>
</tr>
<tr>
<td>$\chi^B$ down</td>
<td>0.20</td>
<td>0.20</td>
<td>0.05</td>
<td>0.0200</td>
<td>0.0815</td>
<td>0.068</td>
<td>0.038</td>
<td>0.24</td>
<td>0.740</td>
<td>0.735</td>
<td>0.0274</td>
<td>+0.36</td>
</tr>
<tr>
<td>[Low risk economy $e = (0.23, 0.89)$]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>barg.</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.0072</td>
<td>0.0381</td>
<td>0.0278</td>
<td>0.026</td>
<td>0.24</td>
<td>0.809</td>
<td>0.732</td>
<td>0.0266</td>
<td>+1.74</td>
</tr>
<tr>
<td>no barg.</td>
<td>0.20</td>
<td>0.20</td>
<td>+∞</td>
<td>0.0077</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.019</td>
<td>0.24</td>
<td>0.901</td>
<td>0.732</td>
<td>0.0280</td>
<td>+1.43</td>
</tr>
</tbody>
</table>
fect, without bargaining, there would be no debt for households with substantial assets.\textsuperscript{17} Furthermore, households in debt can time their repayment and, if income is slow in recovering, the repayment will decline as assets decline. This adds a de facto state contingency to the debt. Krueger and Perri (2006) note that the standard incomplete markets model overstates consumption inequality. The existence of formal default reduces risk sharing further, a point already made in Athreya, Tam, and Young (2009).\textsuperscript{18} So models which exclude informal default may exaggerate the limitations of one term debt in smoothing consumption, an important consideration for quantitative and applied work. To illustrate this point, we compare the variances of log consumption. The model with bargaining delivers a figure of 0.0786, lower than the 0.0875 in the model without bargaining. This is relevant for recent quantitative studies like Krueger and Perri (2006) or Kaplan and Violante (2010).

Regarding the mass distribution of households, the most visible feature is that without bargaining the overall default is much lower but formal default is higher. Removing bargaining leads to a mild shift of defaulters into formal bankruptcy. Concerning portfolios, the main consequence of removing bargaining is a reduction in indebtedness. Defaulters enter bankruptcy with a lower level of assets. The policy rule for borrowing no longer shows the sharp bump seen above for the bargaining case and, as a consequence, debt is concentrated in the lower range of values. Therefore, the presence of bargaining accounts for a substantial 30 per cent of debt in the economy. This is relevant since generating substantial levels of debt is a challenge for many modes of default (see the seminal Eaton and Gersovitz (1981)).

6.2 Formal and informal default are very different

We consider now a reduction in the non-pecuniary costs of informal and formal default separately. This makes sense since there are costs that affect only one form of default. Concerning informal default, the Fair Debt Collection Practices Act (FDCPA) 1978 (as Title VIII of the Consumer Credit Protection Act) seeks to eliminate abusive practices in the collection of consumer debts.\textsuperscript{19} These are practices that do not occur under formal bankruptcy procedures. Another case in point is Obama’s HAMP programme which intends to facilitate negotiations and prevent foreclosures.

\textsuperscript{17}If they borrowed and were to have persistent bad luck, either formal bankruptcy or repayment imposes considerable costs on these households in terms of asset liquidation.

\textsuperscript{18}Removing bankruptcy also eliminate bargaining since the household has no threat value. We have also considered removing bankruptcy as the first default choice but keeping it as an option after attempting negotiations in this case, bargaining remains.

\textsuperscript{19}These include contacting consumers by telephone outside of the prescribed hours, failure to cease communication upon request, communicating with consumers in any way (other than litigation) after receiving written notice that said consumer wishes no further communication or refuses to pay the alleged debt, causing a telephone to ring or engaging any person in telephone conversation repeatedly or continuously with intent to annoy, abuse, or harass any person at the called number; communicating with consumers at their place of employment after having been advised that this is unacceptable or prohibited by the employer.
Reducing the cost of informal bankruptcy for the household improves welfare. This is accompanied by a reduction in the variance of consumption – although this does not seem to be critical – and an increase in defaults, both informal and formal. By comparison, a reduction in the cost of formal default causes a welfare loss and a rise in consumption volatility, along with a rise in formal default, and a sharp increase in interest rates. Therefore reducing the cost of default and raising default levels does not have the same effect depending on how this is achieved. It is important to distinguish between the two types of default for policy purposes. Facilitating negotiation is clearly different in its effects from facilitating bankruptcy filings.20

6.3 The consequences of income volatility with bargaining

The evidence over the last few decades for the U.S. is that of an increase in the volatility of individual earnings jointly with a much more stable volatility of consumption. We now consider in Table 5 a change in the volatility of income within the model comparable with the change studied in Krueger and Perri (2006). Without bargaining, a rise in income risk implies a sharp rise in the variance of consumption since, as discussed earlier, bankruptcy provides limited opportunities for default. In the economy with bargaining, by contrast, increasing risk leads to a more moderate rise in the variance of consumption and, therefore, appears more congruent with the data. With bargaining, the wider dispersion of income drives formal bankruptcy downwards while increasing informal default, an indication of households seeking insurance. On these two counts, our bargaining economy with one bond resembles more the Krueger and Perri (2006) model of complete markets with borrowing constraints.21

Our results demonstrate that a decrease in the cost of bargaining might help explain the apparent lack of impact of increased income risk on consumption volatility. Specifically, we see that a lower cost of bargaining can cause the variance of consumption to decrease even under an increasing income volatility. It suggests that a fall in the cost negotiating might help understand the changes in income and consumption volatility over the recent decades. This is different from the emphasis on the stigma cost of bankruptcy in recent papers;22 while explaining rising bankruptcy, it goes against the moderate response of consumption volatility.

20 But note a reduction in the collection tax, perhaps to be discussed later, reduces welfare. This is no contradiction since this is a change in a transfer against creditors who do therefore restrict credit. Another distinction worth making is between the efficiency in the collection process (welfare improving) and the abusive practices in pursuing it. One could plausibly argue they are opposite sides of the same coin.

21 Their story suggests also a decline in the incentives to go bankrupt as the penalty is a long exclusion from credit there.

22 See, for example, Livshits, MacGee, and Tertilt (2010).
6.4 Exemption policy

Exemption levels differ across states in the U.S. (see Pavan (2008)). There has also been Federal legislation - in the form of the bankruptcy reform act 2005 - to limit the number of assets and labor income which can be protected from collection or seizure in bankruptcy. We consider the affects of adjusting the exemption level. We illustrate the consequences with an experiment the reduces by 20 percent the amount of assets that the household can protect from seizure under bankruptcy. As we saw, much of the action related to bargaining takes place in connection with this exemption.

We report the results from this experiment in Table 6. Reducing assets exempt causes a notable increase in default, especially default leading to bargaining, and a rise in the proportion of household in debt. Average levels of debt also increase and the average interest rate increases too. In the population distribution, there are more households in high debt positions. The ultimate reason is that there are more people with an asset position near or above the bankruptcy exemption who will try to borrow heavily with a view to defaulting and bargaining in the subsequent period. The profiles of debt prices resembles the one for benchmark economy, so the rise in interest rates is a composition effect.

As for welfare, tightening the asset exemption leads to an overall welfare gain of about 0.38 per cent in equivalent consumption units. Gains are generalized across net worth positions. The main reasons for this is that lowering the exemption level increases creditor recovery which makes it easier to borrow. But a tighter exemption has also a negative effect in that formal bankruptcy procedures provide less insurance to households with assets above the exemption limit. In this model, the existence of informal default weakens the impact of this negative insurance effect. This is in contrast with recent work in Pavan (2008) and Athreya (2006) which, having only formal default, find that the insurance effect dominates and causes a negative welfare effect of tightening the level of exemptions. Hintermaier and Koeniger (2009) find that, quantitatively, that type of impact must be small.

6.5 Collection policy

Many individuals face collection efforts outside default. We look now at the role of collection policy, an issue that has not received attention in literature. Proposals to strengthen consumer’s protection against collection are common among consumer advocates. To test the efficacy of these policies, we consider the effects of reducing collection. Table 7 reports outcomes following a reduction in $\tau_b$.

On the plus side, an expected effect of this policy is an increase in the ability of consumers
Table 6: **Asset exemptions**

<table>
<thead>
<tr>
<th>Description</th>
<th>Benchmark 0.50</th>
<th>reduced 0.40</th>
</tr>
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<tbody>
<tr>
<td>Number of $z = 0$</td>
<td>0.974</td>
<td>0.962</td>
</tr>
<tr>
<td>With positive debts</td>
<td>0.270</td>
<td>0.276</td>
</tr>
<tr>
<td>With positive debts and assets</td>
<td>0.259</td>
<td>0.270</td>
</tr>
<tr>
<td>Defaulters</td>
<td>0.059</td>
<td>0.067</td>
</tr>
<tr>
<td>Defaulters to ch7</td>
<td>0.0064</td>
<td>0.0103</td>
</tr>
<tr>
<td>Defaulters to barg. who settle</td>
<td>0.041</td>
<td>0.041</td>
</tr>
<tr>
<td>Defaulters to barg who reject</td>
<td>0.011</td>
<td>0.016</td>
</tr>
<tr>
<td>Repeat defaulters</td>
<td>0.0078</td>
<td>0.0089</td>
</tr>
<tr>
<td>Debt Holdings</td>
<td>0.041</td>
<td>0.051</td>
</tr>
<tr>
<td>Asset Holdings</td>
<td>0.824</td>
<td>0.824</td>
</tr>
<tr>
<td>Price of debt</td>
<td>0.792</td>
<td>0.774</td>
</tr>
<tr>
<td>Variance log consumption</td>
<td>0.0786</td>
<td>0.0757</td>
</tr>
<tr>
<td>Change in welfare</td>
<td>–</td>
<td>+ 0.38%</td>
</tr>
</tbody>
</table>

Table 7: **Different levels of collection**

<table>
<thead>
<tr>
<th>Description</th>
<th>Benchmark 0.08</th>
<th>reduced 0.04</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean households:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of $z = 0$</td>
<td>0.975</td>
<td>0.913</td>
</tr>
<tr>
<td>With positive debts</td>
<td>0.270</td>
<td>0.174</td>
</tr>
<tr>
<td>With positive debts and assets</td>
<td>0.259</td>
<td>0.165</td>
</tr>
<tr>
<td>Defaulters</td>
<td>0.059</td>
<td>0.0485</td>
</tr>
<tr>
<td>Defaulters to ch7</td>
<td>0.0064</td>
<td>0.0049</td>
</tr>
<tr>
<td>Defaulters to barg. who settle</td>
<td>0.041</td>
<td>0.0030</td>
</tr>
<tr>
<td>Defaulters to barg who reject</td>
<td>0.011</td>
<td>0.0406</td>
</tr>
<tr>
<td>Repeat defaulters</td>
<td>0.0078</td>
<td>0.00</td>
</tr>
<tr>
<td>Debt Holdings</td>
<td>0.041</td>
<td>0.022</td>
</tr>
<tr>
<td>Asset Holdings</td>
<td>0.824</td>
<td>0.791</td>
</tr>
<tr>
<td>Price of debt</td>
<td>0.792</td>
<td>0.751</td>
</tr>
<tr>
<td>Variance log consumption</td>
<td>0.0786</td>
<td>0.0847</td>
</tr>
<tr>
<td>Change in welfare</td>
<td>–</td>
<td>- 0.21%</td>
</tr>
<tr>
<td>Change in welfare informal defaulters</td>
<td>–</td>
<td>+ 0.32%</td>
</tr>
</tbody>
</table>
to use informal default as insurance as their assets are not depleted to repay creditors while in informal default. This effect can be seen by examining the outcomes of the model. To begin with, households who choose to enter informal default spend more time as delinquent. Also conditional on default more households are likely to choose informal over formal default. (X: David: Your reported numbers do not match the table, I think) We note an immediate conclusion of this experiment. Since limiting collection limits overall default, limiting collection reduces the percentage of households who take advantage of Chapter 7. However it does so at the cost of reducing access to debt. Debt and assets are lower without collection. The intuition for these changes follows arguments presented previously. The credit constraint effect is sharp in this case because default into informal default leads to lower repayments. This effect is visible through examining the debt prices which are reduced without collection. The insurance effect is increased by limiting collection for the reasons discussed above.

The aggregate welfare depends on the relationship between these two effects. Aggregate welfare is lowered in this experiment which is especially true of consumers whose financial health keeps them far away from default. However the welfare response is not uniform as delinquent consumers would benefit from this policy change. These are the consumers who benefit mostly from the improvement in insurance. We also report that one worry of the banking industry—that reduced collection would lead to greater delinquency, is not borne out by the experiment. The tighter credit in the long term reduces debt which leads to less default.

7 Conclusions

We have proposed an equilibrium model to study consumer debt default where there is an option to negotiate prior to or alongside declaring formal bankruptcy. The existence of the option to negotiate provides substantial consumption insurance, with implications for understanding consumption inequality in the U.S. Policy questions can we analyzed with this novel perspective. Facilitating negotiation raise welfare for consumers; policies which lower the bankruptcy asset exemption—in contrast with previous findings—also raise welfare across the board; making collection outside bankruptcy more difficult is detrimental to overall welfare.

This paper therefore suggests that taking into account negotiations may be relevant for a number of important empirical and policy questions. Further quantitative analysis will help establish more firmly these implications.

Our approach can also speak to a wide variety of policies. An application beyond the policies studied here is the recent home mortgage modification program. In subsequent

\[^{24}\text{We note that this result differs from the welfare effect of limiting the deadweight losses associated with collection. In that policy all consumers benefited. The reason is that collection is a source of repayment whose presence loosens credit constraints whereas deadweight losses are not a form of repayment.}\]
research we will consider the effect of subsidized settlements on rigidly prescribed terms. There is also room to further explore collection policy in an environment where there is excess incentive for consumers to run down assets.
References


A Figures

Figure 1: Default. Policy function (top) and mass distribution (bottom). Low/high income in left/right panel.
Figure 2: Default mix. Policy function for probability of Chapter 7 vs bargaining and mass distribution of bargainers (bottom). Low/high income in left/right panel.

Figure 3: Decision rule for borrowing for non defaulters.
Figure 4: Debt prices. Low income.

Figure 5: Recovery rates on settlement. Low/high income in left/right panel.
Figure 6: Long default.

Figure 7: From default to bankruptcy.
Figure 8: Simple bankruptcy

Figure 9: No bargaining
Figure 10: From default to bankruptcy. No bargaining