Foreign Debt and the Ricardian Equivalence

Eric Mengus*

February 12, 2012

VERY PRELIMINARY - PLEASE DO NOT QUOTE

Abstract

This paper shows that Bulow and Rogoff’s “no sovereign lending” result does not apply in non-Ricardian economies. When a government strictly prefers debt-based funding to tax-based funding, an endogenous cost arises, prompting the government to repay. More accurately, a government which does not have enough tools to reach the first best (in which the Ricardian equivalence holds), cannot afford to redistribute the gains from default, and therefore net losses to agents will emerge in the economy. Finally, when foreign debt levels are small, the gains of default will not balance these losses.

Keywords: Sovereign debt, Ricardian equivalence, bubbles.

JEL codes: E62, F34, H63.

---

*Toulouse School of Economics. TSE, Université de Toulouse, Manufacture des Tabacs, 21 allées de Brienne, Fr - 31000 Toulouse. Email: eric.mengus@tse-fr.eu. I am deeply indebted to Christian Hellwig for his invaluable guidance. I thank Jean Tirole, Patrick Fève and participants to the internal TSE macro-workshop. All remaining errors are mine.
1 Introduction

What are the incentive to repay when debt contract are not enforceable? In a famous paper, Bulow and Rogoff (1989) show that reputation is not sufficient to provide with these incentives when the potentially defaulting country can save abroad. In response, it has been argued that, when sovereign debt is in form of bonds traded by both domestic and foreign agents, a default would affect also domestic residents and, hence, represents an internal cost of default\textsuperscript{1}. For sure, this cost prompts the government to repay, at least for small values of debt due to foreigners.

This collateral-damage theory of default relies on two strong assumptions for the domestic private sector: 1) the government cannot discriminate between domestic and foreign holders of sovereign debt and 2) the losses caused by the default on bonds held domestically cannot be offset by any transfers or even, by any corresponding tax cuts.

In this paper, I provide a foundation for this collateral-damage theory: I argue that Bulow and Rogoff’s no-lending result holds as long as the domestic economy is Ricardian, but no longer holds as soon as the government has a preference for debt. I also show that this latter preference constitutes a generalization of the standard assumptions in the literature on the collateral-damage theory of default.

The frictions, which make an economy non-Ricardian, prevents at the same time the government to balance the reallocation of resources among domestic agents, leaving some of them strictly worse off. Depending on their political weight or on their impact on the aggregate resources (through production for instance), the government prefers not to default, at least for small values of foreignly held debt.

To be able to establish this result in all its generality, I use a preference relation for government’s choices. This preference relation compares the outcome of the underlying economy produced by each fiscal situation, i.e. the distribution of public debt holdings and the plans of taxes domestic agents have to pay. Then, this relation allows me to state clearly a simple arbitrage argument for the government: when debt is strictly preferred by domestic agents compared with taxes, the government does not default. This corresponds to what I will define as debt-oriented non-Ricardian economies. Conversely, when domestic agents are indifferent between taxes and debt and \textit{a fortiori} when they prefer taxes), the government defaults as in Bulow and Rogoff (1989).

\textsuperscript{1}See Kremer and Mehta (2000), Guembel and Sussman (2009), Brutter (2011), Gennaioli et al. (2011) or Mengus (2012) for recent theoretical contributions and Borensztein and Panizza (2009) or Panizza et al. (2009) for empirical evidence.
I give, then, examples of debt-oriented non-Ricardian economies as the overlapping generations model with public debt as in the seminal contribution of Diamond (1965) or as an Aiyagari’s model where domestic households can save in public bonds\(^2\). More generally, I show that a domestic economy which sustains unbacked public debt is debt-oriented non-Ricardian. This constitutes an additional link between international borrowing and bubbles along the one suggested by Hellwig and Lorenzoni (2009), with the following difference. However, the bubble in Hellwig and Lorenzoni’s approach are between a country and its creditors while, in this paper, the bubble is within the country; and the connection made in this paper relies not only on the possibility of bubbles but also on the desiderability of them in the sense of Diamond (1965) or Tirole (1982, 1985).

**Related literature** Several papers challenged Bulow and Rogoff’s result by introducing new features which temper saving incentives. These features deal with the basic assumptions of Bulow and Rogoff’s result: inability to commit to save (Gul and Pesendorfer, 2004, Amador, 2008), foreign lenders (Cole and Kehoe, 1995, Hellwig and Lorenzoni, 2009) or reputation spillovers (Cole and Kehoe, 1998, among others).

I assume also that domestic agents use public debt as stores of value\(^3\).

My result shares similarities with the literature on bubbles. Sovereign debt and bubbles have been connected by Hellwig and Lorenzoni (2009) through interest rates. Here I emphasize another channel which is through welfare. In some non-Ricardian models, public debt is used as private liquidity: issuance of public debt allows to improve domestic welfare and reciprocally default would decrease it. The positive impact of bubbles on welfare has been studied by Scheinkman and Weiss (1986) or Santos and Woodford (1997) in the incomplete market model, by Farhi and Tirole (2009) in Woodford-style models, by Tirole (1985) in OLG models.

---

\(^2\)As in Aiyagari and McGrattan (1998) for example.

\(^3\)This has been documented by Kumhof and Tanner (2005) or by Krishnamurthy and Vissing-Jorgensen (2010) among others.
2 The environment

This section presents the environment. I consider a small open economy populated by a government, a domestic private sector and foreign investors. Time is discrete and indexed by $t \in \{0, 1, \ldots, N\}$, where $N \in \{1, \ldots, \infty\}$.

Uncertainty For any date $t$, the economy can be affected by both aggregate shocks, denoted by $z_t$, and idiosyncratic shocks to domestic agents, denoted by $h_t$. $s_t = (z_t, h_t)$ summarizes these two components. Besides, I denote the whole history of shocks at time $t$ by: $s^t = \{s_0, s_1, \ldots, s_t\}$.

The unconditional probability of state $s^t$ is $\pi(s^t) > 0$ and $\pi(s^t|s^{t-1})$ is the conditional probability of state $s^t$ knowing the realization of state $s^{t-1}$.

Domestic Agents I denote the set of domestic private agents by $I$. The government’s promised repayment to agent $i$ in period $s^t$ is denoted by $B_i(s^t)$ and the flows of taxes and transfers by $\{T_i(s^T)\}_{s^T > s^t}$.

Remark. I do not provide any further structure for domestic agents to keep the approach as general as possible. In particular, the type of agents may correspond to ex ante heterogeneity (e.g. differences in endowment processes) or ex post heterogeneity (e.g. because of different histories of idiosyncratic shocks as in Aiyagari (1994) or because of indifference between portfolio choices as in Mengus (2012).

Foreign investors Foreign investors are modelled by one risk-neutral representative agent, which discounts the future at a rate $\beta^*$. At each period $s^t$, they receive an endowment $y^*(s^t)$. They can invest either in domestic bonds or in foreign assets.

Foreign assets consist of a complete set of securities, that pays 1 unit of goods in period $s^t$ for $q^*(s^t)$ in the preceding period $s^{t-1} < s^t$.

Foreign investors form beliefs on the domestic government repayment decisions in state $s^{t+1} > s^t$: $\delta^E(s^{t+1}) \in [0, 1]$ and I denote by $B^*(s^t)$ the promised repayment from the domestic government.

Throughout the paper, I make the following assumption:

Assumption 1 (Deep pocket investors). Foreign investors’ endowments are sufficiently large compared with potential domestic public borrowing. In particular:

$$y^*(s^t) > \sum_{T, s^T} \pi(s^T|s^t) \frac{g(s^T)}{1 + \beta^*(s^T)}$$
**Government** The government faces a stream of exogenous expenditures \( \{g(s^T)\}_{s^T > s^t} \). The government raises taxes from domestic agents or can implement transfers to them. These are denoted by \( T^i(s^t) \) for each domestic agent \( i \).

The government can issue contingent debt: each domestic bonds corresponds to a promised repayment of 1 unit of good next period. The price in period \( s^{t-1} \) of a security yielding one unit of good in period \( s^t \) is \( q(s^t) \).

The government has the possibility to repudiate its debt: \( \delta(s^t) \in \{0, 1\} \) denotes the corresponding decision variable which equals 1 when the government decides not to default and 0 otherwise.

The government budget constraint is then:

\[
g(s^t) = \sum_{i \in I} T^i(s^t) + \sum_{s^{t+1} > s^t} q(s^{t+1})B(s^{t+1}) - \delta(s^t)B(s^t) \tag{2}
\]

In the beginning of period \( s^t \), the government may default on its debt. However, it cannot default selectively on the debt held abroad, but has to default on both debts.

After having default, foreign lenders do not borrow anymore to the domestic country, but they cannot prevent this country to save abroad. This corresponds to a restriction on the government’s net foreign position: for a default in period \( t \):

\[
\forall T \geq t, \forall s^T > s^t, B^*(s^T) \leq 0 \tag{3}
\]

I denote the value not to default by \( V^R(s^t) \) and the value of default by \( V^D(s^t) \). The government defaults as long as \( V^R(s^t) < V^D(s^t) \).

**Summary of the timing** Figure 1 summarizes the timing of the economy:

<table>
<thead>
<tr>
<th>Private agents form beliefs and make portfolio choices.</th>
<th>The state of nature is revealed.</th>
<th>The government chooses whether to default and sets future taxes.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period ( s^t )</td>
<td></td>
<td>Period ( s^{t+1} )</td>
</tr>
</tbody>
</table>

Figure 1: Timing
2.1 The preference relation

From now on, I consider only situations when the government prefers not to default, i.e. when $\delta(s^t) = 1$, or when $V^D$ is always lower (at least weakly) than $V^R$, and I will find sufficient conditions under which these situations are equilibria.

To do so, I rely on a set of assumptions and on an intermediary result on interest rates. This one is the following:

**Proposition 1.** Along the equilibrium path, the domestic interest rate is pegged by the foreign interest rate:

$$q(s^t) = q^*(s^t), \text{ for all } s^t$$

**Proof.** See appendix.

I make also the following assumptions on government’s values and domestic agents’ behaviour:

**Assumption 2** (Indirect objective function). $V^R$ and $V^D$ are functions both of foreign and domestic holdings of debt and of tax plans:

$$\{B^*(s^t), \{B_i(s^t)\}_{i \in I}, \{T_i(s^T)\}_{s^T > s^t, i \in I}\}$$

**Remark.** Assumption 2 is satisfied by a large set of standard economies. When the government’s objective includes domestic agents’ utility, and, hence consumption, the government can affect these variables by manipulating, on the one hand, the current resources through $\{B_i(s^t)\}_{i \in I}, \{T_i(s^t)\}_{i \in I}$ and private agents’ future decisions through $\{T_i(s^T)\}_{s^T > s^t, i \in I}$.

More precisely, the effect on current variables goes through the budget constraint as, for every agent $i$, government choices $(B_i(s^t) - T(s^t))$ enter its disposable resources in period $s^t$. This will become clearer when considering examples of economies as in Section 4.

Finally, the vector of taxes is subject to restrictions:

**Assumption 3.** There exists $\Theta(s^t)$ such that:

$$\{T_i(s^T)\}_{s^T > s^t, i \in I} \in \Theta(s^t)$$

$\Theta(s^t)$ is possibly a function of date-$s^t$ allocation and, in particular, of $\{B_i(s^t)\}_{i \in I}$. 
Definition 1 (Admissibility). Any tax vector in $\Theta(s^t)$ is admissible.

Using these elements, and especially Assumption 2, I can remap government’s choices into a preference relation on fiscal variables:

$$\{B^*(s^t), \{B_i(s^t)\}_{i \in I}, \{T_i(s^T)\}_{s^T > s^t, i \in I}\}$$

Indeed, according to Assumption 2, for each of these fiscal variables $$\{B^*(s^t), \{B_i(s^t)\}_{i \in I}, \{T_i(s^T)\}_{s^T > s^t, i \in I}\}$$, there exists a real number $V^R(\{B^*(s^t), \{B_i(s^t)\}_{i \in I}, \{T_i(s^T)\}_{s^T > s^t, i \in I}\})$.

Using the usual order on $\mathbb{R}$, I can define:

Definition 2. Let $\succeq$ be a preference relation such that: if $$\{B_1^*(s^t), \{B_{1i}(s^t)\}_{i \in I}, \{T_{1i}(s^T)\}_{s^T > s^t, i \in I}\} \succeq \{B_2^*(s^t), \{B_{2i}(s^t)\}_{i \in I}, \{T_{2i}(s^T)\}_{s^T > s^t, i \in I}\}$$, the government prefers $$\{B_1^*(s^t), \{B_{1i}(s^t)\}_{i \in I}, \{T_{1i}(s^T)\}_{s^T > s^t, i \in I}\}$$ rather than $$\{B_2^*(s^t), \{B_{2i}(s^t)\}_{i \in I}, \{T_{2i}(s^T)\}_{s^T > s^t, i \in I}\}$$.

$\succ$ indicates strict preference and $\approx$ indifference.

Using this relation, I can rewrite the problem solved by the government, when assuming no default (i.e. $\delta(s^T) = 1$), as

Problem 1 (Government’s problem).

$$\max_{B(s^t), \{T_i\}_{i \in I}} \{B^*(s^t), \{B_i(s^t)\}, \{T_i\}(s^t)\}$$

s.t. $\{T_i\}(s^t) \in \Theta(s^t)$

$$B^*(s^t) + \sum_i B_i(s^t) = B(s^t)$$

$$B_i(s^t) = \lambda_i(1, \{T_i\}(s^t))$$

This problem has at least one solution: $\{B^*(s^t), \{B_i(s^t)\}, \{T_i\}(s^t)\}$. From now on, I drop some indices whenever there is no ambiguity.
2.1.1 General properties of the relation

As the preference relation is related to $V^R$, it inherits the properties of the usual order on $\mathbb{R}$. These are, among others:

\[
\forall \{B_1^*(s'), \{B_{11}(s')\}, \{T_{11}(s')\}\}, \{B_2^*(s'), \{B_{22}(s')\}, \{T_{22}(s')\}\}, \{B_3^*(s'), \{B_{33}(s')\}, \{T_{33}(s')\}\}
\]

(i) **Totality:** either $\{B_1^*(s'), \{B_{11}(s')\}, \{T_{11}(s')\}\} \succeq \{B_2^*(s'), \{B_{22}(s')\}, \{T_{22}(s')\}\}$

or $\{B_2^*(s'), \{B_{22}(s')\}, \{T_{22}(s')\}\} \succeq \{B_1^*(s'), \{B_{11}(s')\}, \{T_{11}(s')\}\}$.

(ii) **Reflexivity:** $\{B_1^*(s'), \{B_{11}(s')\}, \{T_{11}(s')\}\} \succeq \{B_1^*(s'), \{B_{11}(s')\}, \{T_{11}(s')\}\}$.

(iii) **Antisymmetry:** if $\{B_1^*(s'), \{B_{11}(s')\}, \{T_{11}(s')\}\} \succeq \{B_2^*(s'), \{B_{22}(s')\}, \{T_{22}(s')\}\}$ and $\{B_2^*(s'), \{B_{22}(s')\}, \{T_{22}(s')\}\} \succeq \{B_1^*(s'), \{B_{11}(s')\}, \{T_{11}(s')\}\}$, then:

$$\{B_2^*(s'), \{B_{22}(s')\}, \{T_{22}(s')\}\} \succeq \{B_1^*(s'), \{B_{11}(s')\}, \{T_{11}(s')\}\}.$$

(iv) **Transitivity:** if $\{B_1^*(s'), \{B_{11}(s')\}, \{T_{11}(s')\}\} \succeq \{B_2^*(s'), \{B_{22}(s')\}, \{T_{22}(s')\}\}$

and $\{B_2^*(s'), \{B_{22}(s')\}, \{T_{22}(s')\}\} \succeq \{B_3^*(s'), \{B_{33}(s')\}, \{T_{33}(s')\}\}$, then:

$$\{B_1^*(s'), \{B_{11}(s')\}, \{T_{11}(s')\}\} \succeq \{B_3^*(s'), \{B_{33}(s')\}, \{T_{33}(s')\}\}.$$

Thus $\succeq$ is a total order on fiscal variables. I will use these properties in the next sections.

2.1.2 Rescheduling taxes and debt

To be able to deal with the Ricardian equivalence, I need to clarify the way changes in taxes are expressed using the preference relation.

For any change in portfolio \(dB_i(s^t)\) \(_{i \in I}\), for any allocation \(\{B^*(s^t), \{B_i(s^t)\}, \{T_i(s^T)\}\}\), the problem of the government is to find a vector \(\{dT_i(s^T)\}_{i \in I, T \geq t, s^T > s^t}\) such that

$$\{B^*(s^t), \{B_i(s^t) - dB_i(s^t)\}_{i \in I}, \{T_i(s^T) - dT_i(s^T)\}_{s^T > s^t, i \in I}\}$$

maximizes its objective under the constraint of:

- \(\{B^*(s^t), \{B_i(s^t) - dB_i(s^t)\}_{i \in I}, \{T_i(s^T) - dT_i(s^T)\}_{s^T > s^t, i \in I}\}\) is feasible, i.e satisfies the government’s budget constraint (2).

- \(\{B^*(s^t), \{B_i(s^t) - dB_i(s^t)\}_{i \in I}, \{T_i(s^T) - dT_i(s^T)\}_{s^T > s^t, i \in I}\}\) is admissible, i.e the vector of taxes \(\{T_i(s^T) - dT_i(s^T)\}_{s^T > s^t, i \in I}\) are admissible.

**Proposition 2.** This maximization yields at least one solution.
Proof. See appendix.

Remark. Mengus (2012) presents a simple situation of a reschedule of taxes, when the government defaults. More precisely I show that, when facing a set of domestic agents who are not taxes but hold domestic public debt, the government responses hinge on the distribution of portfolios and on the presence of opportunistic agents.

Proposition 2 does not conclude on the unicity of the solution, in terms of allocation: there exists possibly multiple vectors \( \{dT_i(s^T)\}_{i \in I, T \geq t, s^T > s'} \) which solve the problem. However, the government is indifferent between these vectors, otherwise it would contradict the fact that they are maxima. This gives a unicity result which allows to denote by:

\[ \{B^{*}(s') - dB^{*}, B_i(s'), \{T_i - dB_i\}\} \]

this unique value. Similarly, I can define without ambiguity the reschedule of taxes following a change in foreign debt:

\[ \{B^{*}(s') - dB^{*}, B_i(s'), \{T_i - dB_i\}\} \]

2.1.3 Default decisions

As the government cannot distinguish between the debt held by foreigners and the debt held by domestic agents, a default affects both stock of debts. In terms of the preference relation, this corresponds to:

\[ \{0, 0, \{T_i - B_i(s') - B^{*}(s')\}\} \], without taking into account future punishments.

Using the notation defined above, I can express the strict preference not to default as:

\[ \{B^{*}(s'), B(s'), \{T\}(s')\} \succ \{0, 0, \{T - B(s') - B^{*}(s')\}\} \] (4)

Remark, however, that the value of default and to value not to default are equal when there is no outstanding stock of public debt (including both domestically and foreign held debt):

\[ V^D(0, 0, \{T\}) = V^R(0, 0, \{T\}) \] (5)

More generally, when \( \{0, 0, \{T\}\} \) is weakly (strictly) preferred to any other cases, a default is weakly (strictly) preferred rather than not to default. I will use this property to characterize decisions not to default.
**Bulow and Rogoff’s extension to selective default.** Another useful result on default, is a generalization of Bulow and Rogoff’s result as described by the following proposition:

**Proposition 3** (Selective default). A selective default on external debt is always preferred: For any plans \( \{B^*(s'), B(s')\}, \{T\} \),

\[
\{0, B(s'), \{T - B^*(s')\}\} \succeq \{B^*(s'), B(s'), \{T\}\}
\]

with equality if and only if \( B^*(s') = 0 \).

The proof of this proposition goes back to Bulow and Rogoff (1989): when the government defaults and saves, it faces lower flows of funds in any future periods.

### 2.1.4 Ricardian and non-Ricardian economies

Economies where the Ricardian equivalence holds can be characterized using the preference relation. When changing portfolios and tax schedules, Government’s indifference between taxes and debt held by the domestic sector, can be written formally as:

**Definition 3** (Ex post Ricardian economy). An economy is *Ex post Ricardian* if and only if for any given level of foreign debt \( B^*(s') \), for any tax plan \( \{T_i\}(s') \) and any level of domestic debt \( \{B_i(s')\} \), for any change in debt \( dB_i \):

\[
\{B^*(s'), \{B_i(s')\}, \{T_i\}(s')\} \approx \{B^*(s'), \{B_i(s') - dB_i\}, \{T_i - dB_i\}(s')\}
\]

However, this does not correspond to the traditional definition of a Ricardian economy, where the government is *ex ante* indifferent between issuing debt or raising taxes. Two sources of *ex post Ricardianity* can be found, among others:

- When the economy experiences no frictions.

- When the government has enough tools to implement the frictionless economy.

**Non-Ricardian economies** Using this first definition, I can also define non-Ricardian economies, i.e. economies where the government is not indifferent between issuing debt and raising taxes. Here I focus on a particular subclass of non-Ricardian economies, those where debt is preferred to taxes:

**Definition 4** (Debt-oriented non-Ricardian economy). A debt-oriented non-Ricardian economy is such that:

\[
\forall \{dB_i\}, \{B^*(s')\}, \{dB_i\}, \{T_i\} \supseteq \{B^*(s'), \{0\}, (\{T_i - dB_i\})\}
\]

with strict inequality at least for some value of \( dB \).
In other words, a debt-oriented non-Ricardian economy is an economy where debt is weakly preferred to taxes and strictly preferred for some values. Other non-Ricardian economies could be defined as tax-oriented economies or even any mixture between tax-oriented and debt-oriented non-Ricardian economies (i.e. when taxes and debt are alternatively preferred to each others). However, only debt-oriented non-Ricardian economies are relevant for the remaining of the paper.

3 Sovereign debt and non-Ricardian economies

In this section, using the tools introduced in the previous section, I show the two main results of the paper: the generalisation of Bulow and Rogoff’s result to domestic Ricardian economies and the limitation of this result when the domestic economy is not Ricardian by preferring debt.

To begin with, Theorem 4 restates Bulow and Rogoff (1989)’s result in Ricardian economies:

**Theorem 4 (Bulow and Rogoff).** If an economy is Ricardian, the default is weakly preferred, with strict preference if and only if $B^*(s^t) > 0$.

*Proof. See appendix.*

This result relies on a simple cost-benefit comparison of default: when an economy is Ricardian, no internal frictions prevent the government to reduce domestic debt in exchange for lower taxes. Consequently, the only result of a default is the gains from defaulting on the external debt, which are strictly positive whenever the external debt is strictly positive, and, thus, default is always preferred.

Now I turn to the main deviation from this result. To obtain the result, we have to make only one assumption on the preference relation:

**Assumption 4 (Local non-satiation).** For any $B^*(s^t) ≥ 0$ such that

$$\{B^*(s^t), B_1(s^t), \{T_1\}\} > \{B^*(s^t), B_2(s^t), \{T_2\}\}$$

there exists $dB^* > 0$ such that:

$$\forall B^* \left( B^*(s^t) - dB^*, B^*(s^t) + dB^* \right), \{B^*, B_1(s^t), \{T_1\}\} > \{B^*, B_2(s^t), \{T_2\}\}$$

The following remark gives a simple situation where this assumption holds:
Remark. A sufficient condition for Assumption 4 to hold is that $V^R$ is a continuous function of $\{B^*(s^t), \{B_i(s^t)\}_{i \in I}, \{T_i(s^{s^T})\}_{i \in I} \}$.

The following theorem establishes how Theorem 4’s result evolves when an economy is non-Ricardian:

**Theorem 5** (Non-Ricardian economies). Under property 4, at each period $s^t$, there exists $\overline{B}^*(s^t) > 0$ such that the economy (i.e. the government) prefers not to default for any $B^*(s^t) \leq \overline{B}^*(s^t)$ if and only if it is a debt-oriented non-Ricardian economy.

**Proof.** See appendix.

In other words, there exists strictly positive level of foreign debt for which the government prefers not to default.

The intuition of this result is that: when foreign debt equals 0, and when the domestic sector holds some public debt, the government prefers not to default. Then property (4) guarantees that for some sufficiently small amount this preference not to default holds as well.

Government’s choice to repay derives only from the trade-off between the gains from default and the losses suffered by the domestic sector. These losses arise if and only if the economy displays some non-Ricardian features.

Remark that, unlike Hellwig and Lorenzoni (2009), here the interest rate plays no role.

### 4 A geography of economies

This Section illustrates the two results of Theorems 4 and 5 by providing an elementary geography of the standard economies.

The first subsection shows that a sufficient condition for an economy to be debt-oriented non-Ricardian economies is to sustain unbacked public debt. Then I provide two examples of such economies, e.g. an OLG economy à la Diamond (1965) and an Aiyagari economy. The second subsection looks at standard Ramsey problems, first with lump sum taxes and then with distortive taxes. I show that both economies do not sustain foreign debt as the former is Ricardian and for the latter the government has an incentive to cut taxes. The third subsection provides an equivalence result with dynamic self-control preferences introduced by Gul and Pesendorfer (2004).
4.1 Debt-oriented non-Ricardian economies and bubbles

A major source for an economy to be debt-oriented non-Ricardian is the possibility of bubbles. More specifically, I show in this subsection that as long as unbacked public debt exists in the domestic economy, this latter is debt-oriented non-Ricardian.

I assume that there is no public spending: \( g(s^T) = 0 \) for every \( s^T \) and that the government cannot tax nor bail out.

Its budget constraint becomes:

\[
\sum_{s^{t+1} > s^t} q(s^{t+1}) B(s^{t+1}) = \delta(s^t) B(s^t)
\]

(11)

Suppose that there exists unbacked public debt domestically, i.e. there exists strictly positive portfolios \( \{B_i(s^t)\}_{i \in I} \) such that 11 is satisfied. In terms of portfolio allocation, this corresponds to \( \{0, \{B_i(s^t)\}_{i \in I}, 0\} \).

When defaulting, the government’s resources are: \(-B(s^t) - B^*(s^t)\) which it has to transfer to domestic agents. Indeed, the government has no better option for savings and it has no future spendings to finance. Finally the allocation becomes \( \{0, 0, -B(s^t) - B^*(s^t)\} \).

When there is no outside debt, three situations may arise:

Case (i) : \( \{0, \{B_i(s^t)\}_{i \in I}, 0\} \succ \{0, 0, \{-B(s^t)\}\} \)

Case (ii) : \( \{0, \{B_i(s^t)\}_{i \in I}, 0\} \approx \{0, 0, \{-B(s^t)\}\} \)

Case (iii) : \( \{0, 0, \{-B(s^t)\}\} \succ \{0, \{B(s^t)\}_{i \in I}, 0\} \)

In cases (ii) and (iii), the government is weakly better off by taxing rather than issuing debt. However, if the government has this possibility, it could have ex ante limited the inefficiencies that makes unbacked public debt sustainable.

More precisely, if \( \{-B_i(s^t)\} \in \Theta(s^t) \), the government can exactly offset domestic losses due to the default, leading to the possibility of cases (ii) and (iii). In contrast, when there are sufficient restrictions on the government’s ability to transfer, case (i) may arise.

This leads to the following theorem:

**Theorem 6.** If unbacked public debt is sustainable, an economy is debt-oriented non-Ricardian.

**Proof.** See appendix. \( \square \)

The conditions under which a bubble is sustained in an economy are well-known (see Tirole (1982, 1985) and the general criteria in Santos and Woodford (1997)).
This theorem gives also a very simple mapping with Hellwig and Lorenzoni (2009)’s result: their argument for the existence of sovereign debt relies the possibility of outside bubbles, i.e. on bubbles between the foreign lenders and the domestic government, while this paper focuses on inside bubbles, i.e. bubbles between the domestic lenders and the government.

This result is also related to Cole and Kehoe (1998). They emphasize that when reputation on international borrowing markets spills over on relationships with enduring benefits\(^4\), the government does not repudiate its foreign debt. Here second the relationship is also a debt relationship, which has enduring benefits as it has the properties of a bubble.

Remark. This result implicitly assumes that unbacked public debt is the only bubble sustained in the economy. Allowing for private bubbles may crowd out public debt. This will result in a lower ability to borrow abroad as the domestic cost of default is also lowered. Possibly, when private bubbles are stochastic, and when domestic agents are risk-adverse, public debt can be preferred to private bubbles. This holds obviously as long as public debt is sufficiently sure.

The remaining of the subsection presents special cases of economies with unbacked public debt: overlapping generation models and Aiyagari models.

4.1.1 OLG models

In this first example, the domestic private sector consists of overlapping generations of households who live two periods (young and old). In any period \( t \) the endowment to the youngs \( (y^Y) \) is greater than the endowment to the olds \( (y^O) \): \( y^H > y^G \). This gives incentives to the young to save in order to smooth consumption over their lifetime.

Each household \( i \) chooses consumption so as to maximizes its lifetime utility function:

\[
U^i = u(c^Y_i(s^t)) + \sum_{s^{t+1} > s^t} \pi(s^{t+1}|s^t)\beta u(c^Y_i(s^{t+1}))
\]

I denote taxes and transfers by \( T^i(s^t) \). Remark that they can be contingent to household and, hence, to types. The program of one household is then:

\[
\max u(c^Y(s^t)) + \sum_{s^{t+1} > s^t} \beta \pi(s^{t+1}|s^t)u(c^O(s^{t+1}))
\]

s.t. \( c^Y(s^t) = y^Y(s^t) - B^d(s^t) - T^Y(s^t) \)

\( c^O(s^{t+1}) = y^G(s^{t+1}) + \delta(s^{t+1})(1 + r(s^{t+1}))B^d(s^t) - T^O(s^t) \)

The solution of this problem is a non-zero demand for bonds: \( B^h(s^t) > 0. \)

\(^4\)which they distinguish from transient benefits
In case of default, however, if taxes are contingent to types, the government can replicate the revenues of public bonds by giving 0 to youngs and \((1 + r(s^{t+1}))B^d(s^t)\). In addition to that, the government can also redistribute the gains from the default on the foreign debt, by cutting taxes.

In turn, contingent taxes would imply that, in normal times, the government can also redistribute from the youngs to the old, shrinking down heterogeneity and thus the net demand for public bonds.

This gives rise to the following proposition:

**Proposition 7.** When taxes and transfers are contingent to types, an overlapping generation economy is Ricardian.

Thus, I assume that \(T\) does not depend on types. In that case, when the country defaults, one generation may be strictly worse off: the generation which becomes old at the time of the default. Indeed they receive only \(T(s^t)\) and lose \((1 + r(s^t))B^d(s^{t-1})\). As \(T(s^t) = B^*(s^t)\), there exists a level \(\overline{B}(s^t)\) such that for any \(B^*(s^t) < \overline{B}(s^t)\), the net outcome for the old generation is \(T(s^t) - (1 + r(s^t))B^d(s^{t-1}) < 0\).

Every other generations are not negatively affected:

(i) Generations born before the default are not affected at all.

(ii) Generations after the default are positively affected as they gain the difference between saving and borrowing as in Bulow and Rogoff (1989).

**Government default decision** Turning to government decisions, a key parameter is the sensivity of the objective of the government to the welfare of the old generation at the time of the default: as long as this parameter is large enough, the losses suffered by this generation cannot be to compensated by the gains of every future generation.

TBA.

**Proposition 8** (OLG model). In an OLG model, with limited transfers, there exists a strictly positive level of sustainable debt.

*Proof.* See appendix. □

The argument given here in a simple OLG model could be extended in other settings as in Blanchard (1985): a default hurts one generation making default undesirable, depending on its relative weight in the government objective.
Rather than using the standard OLG model with inequal endowment for young and old households, several other demand for stores of value by generations of agents can be introduced: the political economy model as in Guembel and Sussman (2009), a demand by entrepreneurs, either because of a mistiming of investment as in Woodford (1990) or Farhi and Tirole (2009), or due to the expectation of reinvestment shocks as in Holmstrom and Tirole (1998). Brutti (2011) has already considered this latter demand for stores of value as a source of international borrowing. Gennaioli et al. (2011) or Mengus (2012) consider a Woodford (1990)’s style demand for stores of values. When extended to overlapping generation, all these models’ outcome are special cases of the general results of the previous section.

4.1.2 Aiyagari economy

We now turn to a simple Aiyagari economy where agents face an uninsurable idiosyncratic risk.

The domestic private sector is a continuum of mass one of infinitively-lived households. They choose consumption so as to maximize:

$$\max_{t,s} \sum t, s \beta^t \pi(s^t) u(c^t(s^t))$$

Each of them receives a endowment $y(s^t) + \epsilon^i(s^t)$ where $\epsilon^i(s^t)$ is an idiosyncratic risk which is not serially correlated across periods and which satisfies:

$$\int_0^1 \epsilon^i(s^t) di = 0 \tag{12}$$

For simplicity, we assume that $\epsilon^i(s^t)$ can take only two values: $+\epsilon$ or $-\epsilon$ with $0 < \epsilon < \min(y(s^t))$. The probability which satisfy (12) is $1/2$.

As in the previous subsection on overlapping generations models, I assume that taxes and transfers are uncontingent to types: $T(s^t)$. Otherwise, the government would have enough tools to compensate domestic losses, but it would also be able to compensate frictions and, thus, make the economy Ricardian.

The only asset that households use to smooth consumption is public debt. Their holdings is denoted by $B^d$ as previously. Households cannot short public debt: this imposes as that $B^d(s^t) \geq 0$, as in Aiyagari (1994).

Consequently the problem of household $i$ is:

$$\max_{t,s^t,h^t} \sum_{t,s^t,h^t} 1/2 \pi(s^t) u(c^i(s^t, h^t))$$

s.t.  
$$c^i(s^t, h^t) + T(s^t, h^t) = y^i(s^t, h^t) + B^{d,i}(s^t, h^t) - (1 + r(s^t))\delta(s^t)B^{d,i}(s^{t-1}, h^{t-1})$$

$$B^{d,i}(s^t) \geq 0$$

16
This problem leads as well to a non-zero demand for bonds. More precisely, a fraction of households hold public while the others have liquidated their holdings and are constrained.

Following the results of Aiyagari (1994), there exist $N$ holdings levels: $\{0, B_1, ..., B_N\}$. In case of default, the government makes a transfer $T(s^t)$ to households. There exists $i \in \{0, 1, ..., N\}$ such that $T(s^t) \in [(1 + r(s^t))B_i, (1 + r(s^t))B_{i+1}]$. Consequently, every household who holds $B_j$, with $j \geq i + 1$ faces losses equal to $T(s^t)(1 + r(s^t))B_j$.

**Government objective** Using as objective function, we have:

Proposition 9 (Aiyagari economy). In this economy, the holdings of public bonds by domestic households is strictly positive as long as $\epsilon > 0$. This implies that there exists a strictly positive level of sustainable debt.

*Proof.* See appendix.

4.2 Optimal taxation models

A significant deviation from the Ricardian equivalence is perhaps the limited ability for government to raise taxes as emphasized by Judd (1985), Chamley (1986) or Golosov et al. (2003).

Following the Ramsey taxation literature, I consider one representative household who consumes, provides labor and invest in domestic debt and in capital.

The household’s preferences on consumption and labor are:

$$\sum_{s^{t+1} > s^t} \pi(s^T)\beta^t u(c(s^t), l(s^t))$$

with $\beta \in (0, 1)$ the discount factor and $u$ is a concave function, increasing in consumption but decreasing in labor. I assume that $u$ satisfies the standard Inada conditions. The household’s budget constraint reads:

$$c(s^t) = B(s^t) - \sum_{s^{t+1} > s^t} q(s^{t+1})B(s^{t+1})$$

$$+ (1 - \tau^t(s^t))w(s^t)l(s^t) + (1 - \tau^k(s^t)) \left( F(k(s^{t-1}), l(s^t)) - w(s^t)l(s^t) \right) - k_i(s^t) + T_i(s^t)$$
In equilibrium, the household’s first order conditions are:

\[
q(s^{t+1})u_C(c(s^{t}), l(s^{t})) = \beta \pi(s^{t+1}|s^{t})u_C(c(s^{t+1}), l(s^{t+1}))
\]

\[
u_C(c(s^{t}), l(s^{t})) = \sum_{s^{t+1} > s^{t}} \beta \pi(s^{t+1}|s^{t})u_C(c(s^{t+1}), l(s^{t+1})) \left(1 + (1 - \tau(s^{t+1})F_k(s^{t+1}))\right)
\]

\[
\tau^t(s^{t}) = 1 + \frac{u_C(c(s^{t}), l(s^{t}))}{u_C(c(s^{t}), l(s^{t}))w(s^{t})}
\]

\[
w(s^{t}) = F_1(s^{t})
\]

The government’s budget constraint is:

\[
TBA
\]

**Definition 5 (A Ramsey problem).** TBA

**Lump sum taxes** When the government has this ability to raise lump sum taxes (i.e. when TBA does not bind), it is well-known that the Ricardian equivalence holds. In such case, the domestic private sector decisions and allocation depend only on the net present value of futures taxes. Indeed, summing the government’s budget constraint over all future periods, we have:

\[
B(s^0) \leq \sum_{s^{t}} q(s^{t}) \left(T(s^{t}) - g(s^{t})\right)
\]

Only the net present value of taxes matter as in Barro (1974).

**Distortive taxes** To make the problem simple, without loss of generality, I make two assumptions. First the preferences satisfy Zhu (1992)’s condition: u is separable in consumption and labor and is CRRA with respect to consumption. Under this condition, the government will tax only labor as in Judd (1985) or Chamley (1986). Second the utility is convex with respect to labor and the relative curvature is constant. This makes the tax rate on labor constant across states. Finally, the utility function is of the form:

\[
\begin{aligned}
    u(c, n) &= c^{1-\sigma} - n^{1-\xi} \\
    \sigma, \xi &> 1
\end{aligned}
\]

In that context, the budget constraint of the government writes as:

\[
B(s^t) + B^*(s^t) + \sum_{s^{T} > s^{t}} q(s^{T})g(s^{T}) = \tau w \sum_{s^{T} > s^{t}} q(s^{t})w(s^{t})
\]
By defaulting on the whole stock of debt $B(s^t) + B^*(s^t)$, the tax rate on labor after the default $\tau^D_w$ is such that:

$$\sum_{s^T > s^t} q(s^T) g(s^T) = \tau^D_w \sum_{s^T > s^t} q(s^t) w(s^t)$$

and $\tau^D_w < \tau_w$.

This change in tax rate affects domestic welfare in two dimensions: through the amount disposable resources for households and through the change in the distortions.

For the former effect, using the household’s budget constraint, the decrease in taxes is beneficial for the domestic household as he beneficiates from a net tax cut:

$$\tau^D_w \sum_{s^T > s^t} q(s^t) w(s^t) \leq \tau_w \sum_{s^T > s^t} q(s^t) w(s^t) - B(s^t)$$

with equality if and only if $B^*(s^t) = 0$?

For the distortionary effect, the tax cut correspond also to a net gain in terms of utility.

The following proposition sums up these results:

**Proposition 10** (Distortionary taxes). *When taxes are distortionary, a default is always strictly preferred for $B^*(s^t) > 0$.*

This result sheds some light on debt-oriented non-Ricardian economies. The preference for debt is such economies is not only *ex ante*, when issuing debt, but also *ex post*, when debt has to be repaid. With distortionary taxes, debt is desirable *ex ante* as this reduces the welfare cost of distortionary taxes, but not *ex post* as debt repayment implies distortions in the future.

In more complex settings the argument still holds but hinges on required distortions as in Golosov et al. (2003) for capital taxations.

### 4.3 Some equivalence results

This last subsection provides an equivalence result between time-inconsistency explanations as in Gul and Pesendorfer (2004) or in Amador (2008) for sustaining sovereign debt and my result.

Gul and Pesendorfer (2004) have argued that with dynamic self control preferences, debt is preferred to savings, as debt is a way to commit. Consequently, Bulow and Rogoff’s result does not hold anymore with such preferences as the domestic country with dynamic self control preferences prefers to honor its debt in order to commit rather than to default and to save, as it leads to the risk not being able to commit.
In this subsection, I show that in a debt-oriented non-Ricardian economy, the government behaves as if it has dynamic self control preferences, even though domestic agents have standard preferences. The additional terms which imply dynamic self control correspond to a loss function derived from the domestic cost of default. The presence of these terms depends crucially on the Ricardian properties of the underlying economy.

To do so, I use a model of two risk-neutral entrepreneurs, which discounts the future at a rate \( \beta \in (0, 1) \). Each entrepreneur is endowed with a processus \( \omega_1 = \{W, 0, W, 0, \ldots\} \) or \( \omega_2 = \{0, W, 0, W, \ldots\} \). When they do not receive endowments, entrepreneurs have access to a technology of production which transforms within the period \( \rho \) unit of goods out of one unit for investment lower or equal to \( (1 + r)A \) and with a marginal return of 1 above. I assume that entrepreneurs cannot borrow, and, hence, they have to transfer their endowments into the next period of their life.

Finally, I assume for simplicity that the government cannot make transfer to entrepreneurs. Allowing for transfers with imperfect information would lead to similar results.

The mistiming of investment opportunities and the impossibility to bail out make the economy non-Ricardian.

Maximization of entrepreneurs’ utility implies that they invest \( A \) in public bonds, are reimbursed \( (1 + r)A \), which they invest and produce \( \rho(1 + r)A \). They finally consume these \( \{0, \rho(1 + r)A, \ldots\} \) and \( \{\rho(1 + r)A, 0, \ldots\} \).

The government’s program is to maximize domestic utility:

\[
\max \sum_{i=1,2} \sum_{t} \beta^t c_i^t
\]

s.t. \( B_t^* + B_t = \delta_t(1 + r)(B_{t-1}^* + B_{t-1}) + g_t \)

\( c_i^t = \rho \delta_t(1 + r)B_{t-1}^i \)

\( A = B_t^i \)

This maximization problem is equivalent to maximize some Lagrangian:

\[
L_1 = \sum_{t} \beta^t c_i^t + \lambda_t \left( B_t^* + B_t - \delta_t(1 + r)(B_{t-1}^* - B_{t-1}) - g_t \right) + \psi_t \left( c_i^t - \rho \delta_t(1 + r)B_{t-1}^i \right)
\]

Let \( \{\xi_t\} \) be a sequence of scalars and consider the following program:

\[
\max \sum_{i=1,2} \sum_{t} \beta^t c_i^t + \xi_t (c_i^t - \rho \delta_t(1 + r)B_{t-1}^i)
\]

s.t. \( c_i^t + g_t + (1 + r)B_{t-1}^* = B^*(s^t) + \rho(1 + r)A \)
The Lagrangian is:

\[ L_2 = \sum_t \beta^t c_t^i + \xi_t (c_t^i - \rho \delta_t (1 + r) B_{t-1}^i) + \gamma_t \left( c_t^i + g_t + (1 + r) B_{t-1}^* = B^* (s^t) + \rho (1 + r) A \right) \]

\[ L_1 = L_2 \] yields the following two conditions: \( \xi_t + \gamma_t = \psi_t \) and \( \lambda_t = \gamma_t \)

As a consequence, in a deterministic economy, we can write government’s utility as being dynamic self-control preferences.

This is only one example of the way to relate dynamic self controls preference to the internal cost of default. Nevertheless, it has some generality as it is always possible to write the frictions as constraints (cf. Albanesi and Armenter, 2007), and thus it is always possible to rewrite indirect preferences using Lagrange multipliers. This multiplier approach is also connected with Thomas and Worrall (1988) on commitment.

5 Conclusion

In this paper, I identify a link between the Ricardian equivalence and the existence of sovereign debt. As long as an economy is Ricardian, per se or because the government has enough tools to replicate the first best allocation, no sovereign lending is possible. However, when the economy has a preference for debt-financed expenditures, sovereign lending becomes sustainable, at least for some values. I identify a major source for a preference for debt, which is the existence of unbacked public debt. This result is reminiscent of Hellwig and Lorenzoni (2009)’s result on bubbles and debt sustainability, with a different view, as this paper emphasizes the role of internal bubbles.
References


**Appendix**

**Proof of Proposition 1.**

*After the default*, the government lends to a rate $1/q(s^{t+1})$. The domestic sector sector can lend to the government at that rate.

*Before the default*, as $\delta = 1$, foreign investors make the arbitrage between domestic bonds and foreign assets.

**Proof of Proposition 2.**

TBA.

**Proof of Theorem 4.**

Suppose that the economy is Ricardian and suppose that $B^*(s^t) > 0$:

$$\{B^*(s^t), B^d(s^t), \{T\}(s^t)\} \approx \{B^*(s^t), 0, \{T\}(s^t) - B^d(s^t)\}$$

Besides:

$$\{0, 0, \{T\}(s^t) - B^d(s^t)\} \succeq \{B^*(s^t), 0, \{T\}(s^t) - B^d(s^t)\}$$

with equality if and only if $B^*(s^t) = 0$. and then:

$$\{0, 0, \{T\}(s^t) - B^d(s^t)\} \succ \{B^*(s^t), B^d(s^t), \{T\}(s^t)\}$$
Proof of Theorem 5.

Suppose that an economy strictly prefers not to default:

\[ \exists B^*(s') > 0, \{B^*(s'), B(s'), \{T\}\} \succ \{0, 0, \{T - B(s') - B^*(s')\}\} \]

and suppose that the economy is Ricardian:

\[ \{B^*(s'), B(s'), \{T\}\} \approx \{B^*(s'), 0, \{T - B(s')\}\} \]

Then:

\[ \{B^*(s'), 0, \{T - B(s')\}\} \succ \{0, 0, \{T - B(s') - B^*(s')\}\} \]

which contradicts BR property.

Reciprocally: Suppose that the economy is non-Ricardian: in particular, there exists some \( B(s') \) such that:

\[ \{0, B(s'), \{T\}\} \succ \{0, 0, \{T - B(s')\}\} \]

Using the continuity property, there exists \( dB^*(s') > 0, \) such that:

\[ \{dB^*(s'), B(s'), \{T\}\} \succ \{dB^*(s'), 0, \{T - B(s')\}\} \]

and then, using the BR property:

\[ \{dB^*(s'), B(s'), \{T\}\} \succ \{0, 0, \{T - B(s')\}\} \]

Proof of Theorem 6.

Suppose that there exists \( B^d \) such that \( B^d(s^0) > 0 \) and \( B^d(s^t) \leq \sum_{s^{t+1}>s^t} (1+r^*(s^{t+1}))B^d(s^{t+1}) \). We have to compare: \( \{B^*(s'), \{B_i(s')\}, 0\} \) with \( \{B^*(s'), 0, \{-B_i(s')\}\} \).

Proof of proposition 8

The program of each generation is:

\[
\max u(c^Y(s^t)) + \sum_{s^{t+1}>s^t} \beta \pi(s^{t+1}) / \pi(s^t) u(c^O(s^{t+1}))
\]

\[ c^Y(s^t) = y(s^t) - B^h(s^t) - T(s^t) \]

\[ c^O(s^{t+1}) = \delta(s^{t+1})(1 + r(s^{t+1}))B^h(s^t) - T(s^t) \]

As long as \( u \) is concave, we have \( B^h(s^t) > 0 \). In case of default in \( s' \), \( c^O(s^t) = 0 \) which gives the result.
Proof of proposition 9

The program of each household is:

\[
\max_{t,s_t,h_t} \frac{1}{2} \pi(s_t) u(c_t(s_t, h_t))
\]

\[
c_t(s_t) + T(s_t) = y_t(s_t, h_t) + B^{h_t}(s_t) - (1 + r(s_t))B^{h_t}(s_{t-1})
\]

\[
B^{h_t}(s_t) \geq 0
\]

First order conditions are:

\[
\frac{1}{2} \pi(s_t)u'(c_t(s_t, h_t)) = \lambda(s_t, h_t)
\]

\[
\lambda(s_t, h_t) = \sum_{s_{t+1} > s_t} \sum_{h_{t+1} > h_t} \lambda(s_{t+1}, h_{t+1})(1 + r(s_{t+1})) + \phi(s_t, h_t)
\]

We can show then that \( B^{h_t}(h_t, s_t) \geq 0 \) binds for households in the bad state of the idiosyncratic shock and not in the other case as in Aiyagari (1994).

A default leads then to a loss of wealth by half of the households equal to \( B(s_t, +\epsilon) \). However, government is not able to transfer to them wealth with taxes without giving also to households who do not hold public bonds. As a consequence, a default is not Pareto-improving for sure. The government should at least transfer \((1 + r(s_{t+1}))B(s_t, +\epsilon)\) to every households. This implies that for external debt such as \( B^*(s_t) \leq B(s_t, +\epsilon) \), the government does not default.