Is Debt Overhang a Problem for Monetary Policy? *

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Abstract

The U.S economy has accumulated in recent years seemingly excessive levels of household debt. U.S monetary policy has responded to the situation by keeping real interest rates low. Critics of the low real interest rate policy contend that such a policy helps borrowers and punishes savers, thus delaying the necessary adjustment needed to return to balanced growth. We consider a macroeconomic setting which gives voice to these concerns. In the model, lifecycle considerations lead to a natural interaction between young borrowers and middle-aged lenders. Optimism concerning future income prospects will raise borrowing, but if the optimism later turns out to be unwarranted, households will have borrowed too much. We show that, in this setting, overhang drives real interest rates up, not down, and that policy attempts to keep rates lower may drag out the necessary adjustment to the balanced growth path, and lower lifecycle welfare. We compare these findings to recent contributions on debt overhang emphasizing exogenous debt constraints due to Eggertsson and Krugman(2010) and Guerrieri and Lorenzoni(2010)

1 Introduction

1.1 The Policy Issues

The financial crisis of 2008 and the large worldwide recession that ensued ushered in a period of low nominal interest rates that pushed the Federal funds yield near zero since December 2008. Large liquidity injections in 2008-2010 and several rounds of quantitative easing have driven real rates of return on relatively safe debt way down [Figure 1]. The net effect of monetary policy has been to sustain expectations that rates will remain low far into the future.

Is quantitative easing socially optimal in a recession economy with unusually large private debt[Figure 2]? Does it help borrowers adjust to the new realities of slower

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growth and lower incomes? Does it encourage lenders to supply the loans still demanded by overburdened borrowers while they build up their asset positions?

If the answer is in the negative, current monetary policy prolongs the problem caused by private debt overhang and reduces consumer welfare. Could low interest rates ever be the wrong reaction to a large recession? In economies with exogenous debt constraints like those of Bewley (1980) and Aiyagari (1994), low yields relax debt constraints and improve intertemporal trading opportunities; see Eggertsson and Krugman (2010) and Guerrieri and Lorenzoni (2011) for two attempts in that direction. In lifecycle economies without debt constraints, low interest rates favor young households who are natural borrowers, at the expense of middle-aged and older households who are natural lenders.

Recent work by Ford and Vlasenko (2011) and Glover, Heathcote, Krueger and Rios-Rull (2011) shows that this intergenerational redistribution can be quite substantial. Does quantitative easing punish savers too much? The answer would be yes in a lifecycle economy if monetary policy employs a Taylor rule that clashes with the golden rule of accumulation by keeping real yields on relatively safe private debt below the growth rate for extended periods of time. This may occur if the central bank overestimates "full-employment" national income, or shows unjustified optimism about future growth in GDP or expected inflation rates.

1.2 Pieces of the Puzzle
To examine these issues convincingly we need a theoretical model in which borrowing and lending respond to changes in monetary policy. The simplest model available is an overlapping generations one with a three period lifecycle and segmented asset markets in which the nominal interest rate has the power to influence real yields on private debt and, by that credit channel, to alter the allocation of consumption over the lifecycle.

We choose to study an endowment economy with a constant population of households divided into two groups:"participants" and "non-participants". Participants have an income profile that peaks in middle age; they borrow when young, repay when in middle age, and save to provide for their retirement. Non-participants are arbitrarily excluded from the loan market and permitted to hold only currency even though it is dominated by debt in rate of return.

Working with a three-period lifecycle model with segmented markets in a choice that values tractability over and above the twin disadvantages of low frequency and poorly justified demand for money. In our framework, one time period is twenty years and some
people hold money because they have no other options. The first of these drawbacks is
less serious than it looks because lifecycle behavior arises at much higher frequencies
in economies with infinitely-lived households without commitment to repay loans, as
shown in Hellwig and Lorenzoni(2009).

Monetary policy is in the hands of a central bank that sets interest rates through pur-
chases or sales of safe private debt. The bank prints or destroys money to raise or lower
its debt holdings, changing security prices and real yields as it goes along. Initial holders
of currency pass it on to non-participants who hold the entire money stock as an asset
that helps them provide for retirement.

2 The Environment

Our exchange economy starts at $t = 0$ and goes on forever. It is populated by a constant
mass, equal to $1 + \lambda$, of three-period lived cohorts indexed $h = 0, 1, ...$. One cohort is born
every period. At the beginning of time there are also two transitional cohorts, indexed $h = -1, -2$ whose wealth and choices will determine initial conditions for our economy.
Transitional cohorts will carry no weight in our social welfare function, and will be ig-
nored in deriving the socially optimal intergenerational allocation of consumption.

2.1 Households

Households are divided into two types, ”participants” and ”non-participants”, with mass
1 and $\lambda > 0$ respectively. We denote by $c_h(t)$ the consumption of the participant cohort
$h$ at time $t = h, h + 1, h + 2$. Non-participants consume only in old age. Lifecycle utility
for participant cohort $t = 0, 1$ is

$$v_t = u[c_0(t)] + \beta u[c_1(t + 1)] + \beta^2 u[c_2(t + 2)]$$

with $\beta \in [0, 1]$ and $u : R_+ \to R$ a standard iso-elastic utility function of the form
$u(c) = c^{1-\theta}/(1-\theta), \theta \geq 0$.

Endowments are $\omega^t = (\omega_0 \gamma^t, (\omega_1 + \epsilon_{t+1})\gamma^{t+1}, \omega_2 \gamma^{t+2})$ for participants, and $e^t =
(0, (1 + \epsilon_{t+1})\gamma^{t+1}, 0)$ for non-participants where $(\omega_0, \omega_1, \omega_2) \in R_+^3$ and $\gamma > 0$ is an econ-
omywide income growth factor. We normalize incomes so that $\omega_0 + \omega_1 + \omega_2 = 1$ and interpret $\epsilon_t \in [0, \hat{\epsilon}]$ to be a positive income shock that describes skill-biased technological change in favor of middle-aged individuals.

Total income in this economy $y_t = (1 + \lambda + \epsilon_t)\gamma^t$ is an exogenous parameter which
we can easily ”endogenize” by adding a production sector with an exogenous shock to
the productivity of middle-aged labor, and relatively inelastic labor supply.

To guarantee that asset demands are monotone increasing in yields, and also that middle-age participants lend and young participants borrow at all values of aggregate income, we assume:

**A1** All dated consumption goods are normal and gross substitutes.

**A2** \( u'(\omega_0) > \beta u'(\gamma \omega_1) \)

**A3** \( u'(\omega_1) < \beta u'(\gamma \omega_2) \)

### 2.2 Asset Markets

Debt and currency are traded in two distinct competitive markets that clear at each \( t = 0, 1, \ldots \) with real gross yields denoted by \((R_{t+1}, R_{t+1}^M)\) respectively. One unit of consumption saved at time \( t \) pays off \( R_{t+1} \) units at time \( t + 1 \) if the saver buys private debt, \( R_{t+1}^M \) units if the saver holds currency. Participants can operate in both asset markets but will typically choose to hold private debt if currency is dominated in rate of return. Non-participants may only hold non-negative amounts of money.

The central bank prints currency to increase its stock of private securities subject to the accounting constraint:

\[
B_{t+1} = R_t B_t + (M_t - M_{t-1})/p_t
\]

(1)

where \( B_t \) denotes holdings of one-period debt purchased at \( t - 1 \) and maturing at \( t \). As usual, \( M_t \) is the aggregate stock of currency and \( p_t \) is the price level. Letting \( m_t = M_t/p_t \) denote money balances, and \( N_t \) describe the nominal yield on private debt, we rewrite (1) as follows:

\[
B_{t+1} = R_t B_t + m_t - m_{t-1}(R_t/N_t)
\]

(1')

In equilibrium, \( B_{t+1} \) will equal the supply of private securities by participants at time \( t \), and \( m_t \) the demand for currency by non-participants also at time \( t \).

Asset markets clear when the auctioneer chooses at time \( t \) the ”correct” value of the real yield \( R_{t+1} \). That value balances the securities supplied by households with those demanded by the government at a given lifecycle income profile, given the previous real yield \( R_t \), and given the specifics of monetary policy. Policy itself may be expressed at time \( t \) by current and expected future values of the nominal yield sequence \((N_s^\infty)_{s=1}^\infty\) or by the corresponding rates of money creation. One useful way to think about policy at \( t \) is that it fixes either the nominal yield at \( t \) or the inflation rate. These two rates must by consistent with the inherited real yield \( R_t \).
Non-participants save their active middle-age income; the money market clears if the following quantity theory relationship holds:

\[ m_t = \frac{\lambda}{1 + \lambda y_t} \quad \forall t \]  

(2)

Participant members of cohort \( t \geq 0 \) choose asset demands \( A_t^0(R_{t+1}, R_{t+2}|\epsilon_{t+1}) \) in youth, and \( A_t^1(R_{t+1}, R_{t+2}|\epsilon_{t+1}) \) in middle age which are conditioned on the expected income shock \( \epsilon_{t+1} \). These demands fall out of maximizing lifetime utility subject to the budget constraints :

\[
  c_t(t) + A_t^0 = \omega_0 \gamma^t \\
  c_t(t + 1) + A_t^1 = (\omega_1 + \epsilon_{t+1}) \gamma^{t+1} + R_{t+1} A_t^0 \\
  c_t(t + 2) = \omega_2 \gamma^{t+1} + R_{t+2} A_t^1
\]

(3a), (3b), (3c)

Household demand for debt at time \( t \) is the sum of \( A_t^0 \) from the current young and \( A_t^{t-1} \) from the current middle-aged. Because of homotheticity, household demand is proportional to the growth parameter \( \gamma \), that is, 

\[
  A_t^0(R_{t+1}, R_{t+2}|\epsilon_{t+1}) + A_t^{t-1}(R_{t}, R_{t+1}|\epsilon_{t}) = \gamma^{t+1} A(R_{t+1}|\epsilon_{t+1})
\]

where

\[
  R_{t+1} = (R_{t}, R_{t+1}, R_{t+2}), \epsilon_{t+1} = (\epsilon_t, \epsilon_{t+1})
\]

The schedule \( A \) is an increasing function of the real-yield history \( R_{t+1} \) because dated consumption goods are gross substitutes by assumption A1. We make the additional assumption that our economy is "sufficiently impatient", that is,

\[
  A(\gamma|1) = 0
\]

A4  \( A(\gamma|1) = 0 \)

This postulate guarantees that a debt market with no policy intervention and no income shocks will clear at a real yield that is equal to the income growth rate. In other words, we assume that a stationary laissez-faire equilibrium achieves the golden rule.

The credit market clears if excess demand for private debt is zero at each \( t \geq 1 \), that is,

\[
  B_{t+1} + \gamma^{t+1} A(R_{t+1}|\epsilon_{t+1}) = 0
\]

(4)

Combining equations (1)',(2)and (3) we obtain a non-autonomous third order difference equation which is a typical description of equilibria in lifecycle economies with three-period lifespans [see Azariadis, Bullard and Ohanian(2004) for more details]

\[
  -\gamma A(R_{t+1}|\epsilon_{t+1}) + R_t A(R_{t}|\epsilon_t) - \frac{\lambda}{1 + \lambda} [1 + \lambda + \epsilon_t - (1 + \lambda + \epsilon_{t-1}) \frac{R_t}{\gamma N_t}] = 0
\]

(5a)
A different market-clearing condition applies to the initial period $t = 0$ in which the transitional cohort $h = -1$ is still active in the credit market and has a predetermined stock of initial assets $A_1^*$. Suppose also that the monetary authority has no inherited claims on the private sector at $t = 0$ which leads to an initial-period budget constraint:

$$B_1 = m_0 = \frac{\lambda}{1 + \lambda}$$

Proceeding as before, we obtain an initial equilibrium condition for $t = 0$, namely

$$-A(R_1, R_2|\epsilon_1, \epsilon_2) + A_1^* - \frac{\lambda}{1 + \lambda} = 0$$

(5b)

This one connects yields in the initial period with those in the period immediately following.

### 3 The Credit Channel

#### 3.1 Passive and Neutral Policies

Monetary policy is neutral if the central bank either holds no private debt or always holds a constant amount of it. Allocations in that case depend on endowments and preferences alone. For example, if the policy is not to hold any private debt, then equation (5a) reduces to

$$A(R^t|\epsilon^t) = 0$$

(6a)

and real yields depend on income shocks alone. If instead, the policy is to hold a constant amount of private debt and never to issue any currency, equation (6a) becomes

$$R_tA(R^t|\epsilon^t) = \gamma A(R^{t+1}|\epsilon^{t+1})$$

(6b)

which leads to the same conclusion.

The credit channel comes alive if the central bank maintains a constant nominal yield $N_t = N > 1$ for all time. For example, if we have no income shocks, equation (6a) admits stationary equilibria that satisfy

$$(R - \gamma)A(R|0) + \lambda\left(\frac{R}{\gamma N} - 1\right) = 0$$

(7)
Since the LHS of this equation is increasing in the real yield \( R \) for any \( R \geq \gamma \) and decreasing in the nominal yield \( N \), the two yields must move together in any high-interest-rate equilibrium. Thus the real yield is an increasing function of the nominal one in any dynamically efficient equilibrium.

### 3.2 Perfect-Foresight Equilibrium

More generally, sequences of real yields \((R_t)_{t=1}^{\infty}\) are perfect-foresight equilibria if they obey the credit market clearing equation (5a) and the initial condition (5b) for a given choice of monetary policy \((N_t)_{t=1}^{\infty}\). Knowing real yields in this environment is equivalent to knowing everyone’s consumption for each period, as one can see easily from the asset demands \((A_0, A_1)\) and the budget constraints (3a) through (3c).

Policies can be arbitrary sequences of nominal yields or Taylor-like rules that connect nominal yields with lagged, contemporaneous or expected values of output and inflation. Policies of this type attempt to manipulate real debt yields in the manner desired by the central bank. For our purposes, it is more natural to suppose that the central bank seeks to achieve the golden rule by a policy of the form

\[
N_t - N_t^* = \phi_i (\pi_t - \pi^*) + \phi_y \frac{y_t - y_t^*}{y_t^*}
\]

where \( y_t^* \) is trend output, \( y_t - y_t^* \) is the output gap, \((\pi_t^*, N_t^*)\) are targets for the gross inflation rate and gross nominal yield, and \((\phi_i, \phi_y) > 0\) are policy parameters that measure the importance of the inflation and output gap targets. A monetary policy is a specification \((\phi_i, \phi_y, \pi_t^*, N_t^*, y_t^*)\).

To sum up, equilibria are described by the initial condition (5b) and a difference equation like (5a) for zero excess demand in the credit market.

### 3.3 An Example

A first step toward understanding the intergenerational impact of monetary policy rules is to study a simple economy with discount rate \( \beta = 1 \), baseline participant endowment \((\omega_0, \omega_1, \omega_2) = (0, \gamma^{t+1}, 0)\) and utility \( u(c) = \log(c) \). In that case we obtain asset demands:

\[
A_0(R_{t+1}, R_{t+2}|\epsilon_{t+1}) = -\frac{\alpha(1 + \epsilon_{t+1})}{R_{t+1}} \gamma^{t+1}, \quad \alpha = 1/3
\]

\[
A_1(R_{t+1}, R_{t+2}|\epsilon_{t+1}) = \alpha \beta^2 (1 + \epsilon_{t+1}) \gamma^{t+1}
\]

These add up to a community asset demand schedule

\[
\gamma^{t+1} A(R^{t+1}|\epsilon^{t+1}) = \left[ \frac{\alpha}{\gamma} \beta^2 (1 + \epsilon_t) - \alpha(1 + \epsilon_{t+1})/R_{t+1} \right] \gamma^{t+1}
\]
In this case the initial condition (5b) fixes the initial real yield $R_1$. The credit market clears for an arbitrary policy sequence if equation (5a) holds, that is, if

$$
\frac{R_{t+1}}{\gamma} = \alpha R_t + \alpha z_t + \frac{\lambda}{1+\lambda} (\lambda + z_t) - \left[ \alpha \beta^2 z_{t-1} + \frac{\lambda}{1+\lambda} \frac{z_{t-1}}{N_t} \right] \frac{R_t}{\gamma}
$$

(11)

where $z_t = 1 + \epsilon_t$, $\beta = 1$, $\alpha = 1/3$.

In a stationary economic environment with $\epsilon_t = 0$ for all $t$, this expression simplifies to

$$
\frac{R_{t+1}}{\gamma} = \frac{1}{2 + 3\lambda - (1 + \frac{3\lambda}{N_t}) \frac{R_t}{\gamma}}
$$

(12’)

For each constant nominal-yield policy $N_t = N$ for all $t$, The top equation (11) has two steady states, a dynamically inefficient state $\bar{R}_1(N)$ and an efficient one $\bar{R}_2(N)$. Both states depend on the nominal yield $N$. It is easy to check that:

i $\bar{R}_1(N) \leq \gamma \leq \bar{R}_2(N) \leq N$ with strict inequality if $N > 1$

ii $\bar{R}_1$ is asymptotically stable and $\bar{R}_2$ is not.

iii $\bar{R}_1$ is decreasing in $N$; $\bar{R}_2$ is increasing in $N$.

iv $\bar{R}_1(\gamma) = \bar{R}_2(\gamma) = \gamma$

v Equilibrium is unique relative to initial conditions. It is $R_t = \bar{R}_2(N)$ if initial yield is high enough or, equivalently, if the transitional middle-age cohort starts with small enough assets, otherwise equilibrium converges to the inefficient state $\bar{R}_1(N)$.

To rule out the inefficient state $\bar{R}_1(N)$, good activist policy sets the nominal yield $N_t$ to insure that the economy at $t + 1$ achieves the golden rule for any current yield $R_t$. Specifically, we replace $R_{t+1}$ on the LHS of equation (11) with $\frac{\mu_{t+1}}{\mu_t} = \frac{\gamma (\gamma + \lambda)}{(\gamma + \lambda)}$, solve for $N_t$, and obtain a Taylor-like rule that is greatly simplified in a stationary economy with $\epsilon_t = 0 \forall t$. In that case, the optimal interest-rate rule is

$$
N_t = \frac{[\alpha (1 + \beta^2) + \lambda - \alpha] \gamma \pi_t - \lambda}{\alpha \beta^2}
$$

In this economy, target rates $(\pi^*_t, N^*_t)$ are in accord with the golden rule because they satisfy $N^*_t = \gamma \pi^*_t$ for any inflation target $\pi^*_t$. Notice also that the nominal yield overacts to changes in the inflation rate with a coefficient

$$
\frac{dN_t}{d\pi_t} = 1 + \frac{\lambda}{\alpha \beta^2} = 1 + \frac{\lambda (1 + \beta + \beta^2)}{\beta^2} > 1
$$

For $\lambda = 0.1$ and $\beta = 0.6$, this coefficient equals 1.54.
3.4 The Origins of Debt Overhang

We associate the rapid increase in private debt that occurred in the 1990s with unjustifiably optimistic forecasts of future income levels or income growth rates arising from a widespread belief in the "new economy", that is, with widespread expectations of a permanent acceleration in the growth rate of total factor productivity. Those expectations were eventually abandoned but left behind them a mountain of private debt whose burden weighs on the private sector to this day. Central banks have kept yields very low in an attempt to alleviate that burden on borrowers seemingly without thinking about the impact of low-interest-rate policies on lenders.

To analyze this issue within our lifecycle framework, we will need to depart from perfect foresight in a way that permits households to overestimate future incomes for a while before reality sets in and forces them to come to their senses. Specifically, we study a temporary one-time increase in the middle-age income of participants in cohort $t = T$ from $\omega_1$ to $\omega_1 + \epsilon$ where $\epsilon > 0$ is some fixed number. Aggregate income is $1 + \lambda$ for all periods except $T + 1$ at which point $y_{T+1} = 1 + \lambda + \epsilon$.

To simplify matters, we assume that the income change is unexpected before it happens, regarded by all as permanent when does, and understood by all to have been temporarily the period after. The monetary authority and all households share point expectations at time $t$ on the lifecycle income profile $(\omega^t = (\omega_0, \omega^t_1(t + 1), \omega_2)$ of participant cohort $t$ with the property that

$$\omega^t_1(t + 1) = \begin{cases} \omega_1 + \epsilon & t = T + 1 \\ \omega_1 & t \neq T + 1 \end{cases}$$

This formulation says that the cohort that receives the extra income does not anticipate the windfall ahead of time, and neither does the central bank. The next cohort expects at $T + 1$ that the good luck will persist into $T + 2$, at which point it finds out they were wrong. After that, all