Wage Determination and Labor Market Volatility under Mismatch

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Abstract

Shimer (2007) introduced a model of mismatch, in which limited mobility of vacant jobs and unemployed workers provides a microfoundation for their coexistence in equilibrium. Shimer assumed that the short side of a local labor market receives all the gains from trade, and argues that the model helps to explain the volatility of unemployment and the vacancy-unemployment ratio in response to productivity shocks. I show that the assumption on wages is essential for this conclusion by considering alternative assumptions. When wages are determined according to the Shapley (1953b) value, they depend more smoothly on local labor market conditions, but unemployment and the vacancy-unemployment ratio are even more volatile. However, in both cases amplification relative to the Mortensen-Pissarides benchmark arises only because the implied process for wages is more volatile.

JEL Codes: E24, J41, J63, J64.

Key words: mismatch, unemployment, vacancies, volatility, wage determination, Shapley value

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1 Introduction

It is by now well-known that the most familiar model of equilibrium in frictional labor markets, due to Mortensen and Pissarides (Pissarides, 1985; Mortensen and Pissarides, 1994; Pissarides, 2000), has difficulty accounting for the empirical volatility of key labor market variables such as unemployment and the vacancy-unemployment ratio. A large literature, following Shimer (2005) and Costain and Reiter (2008), has attempted to understand what modifications to the model can render it consistent with the observation that unemployment and vacancies are much more volatile than productivity even when the surplus from the average employment relationship is large. A further drawback of the Mortensen-Pissarides framework is that the process of matching between vacancies and unemployed workers is black-box. Although a reduced-form Cobb-Douglas matching function can seldom be rejected empirically (Petrongolo and Pissarides, 2001), the abstractness of that model of matching is an important limitation, since it is difficult to know whether the matching process will remain invariant to policy interventions.

In this context, the model of mismatch due to Shimer (2007) makes an important contribution. In this model, there are many local labor markets which differ in their number of jobs and workers. There are no frictions to matching within a labor market: the number of matches formed is identically equal to the minimum of the number of jobs and the number of workers. However, the process by which workers and jobs are allocated to labor markets exhibits randomness: jobs are randomly allocated to labor markets when they enter, and cannot move, while the process by which workers move from labor market to labor market is independent of the characteristics of origin and destination markets. This ensures that there is random variation in the number of workers and jobs across labor markets, so that some labor markets have excess unemployed workers and some have excess vacancies. This gives a microfoundation for the coexistence of unemployment and vacancies in equilibrium. Shimer shows that the resulting Beveridge curve traced out as the number of jobs varies is indistinguishable from that arising from a Cobb-Douglas matching function.

To generate implications for the volatility of unemployment and vacancies, an assumption on wage determination is needed to determine how much entry will occur and how this will respond to shocks. Shimer’s assumption is that wages are determined by Bertrand competition within a local labor market. Thus, in his paper, in markets with more jobs than workers, workers receive the entire output of the match, while in markets with more workers than jobs, the wage is driven down to a worker’s outside option, namely, the value of home production. Under this assumption, unemployment and vacancies are twice as volatile as in the Shimer (2005) calibration of the the Mortensen-Pissarides model (modified to allow for irreversible entry). It is well-known that in general the process of wage determination is key for understanding the cyclical behavior.

\footnote{Hagedorn and Manovskii (2008) observe that if the employment surplus is small, unemployment and vacancies are more volatile than in Shimer (2005). Different authors have studied the effect of ad hoc wage rigidity (Hall, 2005; Gertler and Trigari, 2009; Blanchard and Gali, 2010; Shimer, 2010), variant bargaining models (Hall and Milgrom, 2008), and asymmetric information (Kennan, 2010).}
of models of frictional labor markets.\(^2\) Therefore, the goal of the current paper is to investigate the implications of alternative wage determination protocols on the volatility of cyclical fluctuations in the mismatch model. This is all the more important since wage determination in Shimer (2007) is perhaps overly simplified in that wages take only two values and can exhibit sudden very large changes in response to small changes in local labor market conditions. It is reasonable to ask whether the aggregate response of the model to productivity shocks depends crucially on this unrealistic feature.

The investigation in the paper proceeds in two ways. I first show how, using the bargaining solution introduced by Shapley (1953b), I can modify the model so that wages in individual labor markets depend in a more continuous way on the number of workers and jobs in the market. When I assume that the bargaining powers of firms and workers are equal, unemployment and vacancies are substantially more volatile than in Shimer (2007), and therefore in turn again substantially more volatile than in the Mortensen-Pissarides benchmark. However, when I recalibrate the bargaining model so that the labor share and the elasticity of average wages with respect to productivity are comparable to those which arise in the benchmark mismatch model, the results are then very similar to those in the mismatch model.

These results suggest that the implications of the mismatch model for the volatility of unemployment and vacancies depend only on the labor share and on the elasticity of the response of wages to productivity. To confirm this hypothesis, I then consider an alternative, more reduced-form, wage determination method in which I simply choose wages to be independent of conditions in the local labor market, but to depend on aggregate productivity in a simple way. With this setup, when I choose the level and productivity elasticity of wages to mimic those arising in various benchmarks (for example, Shimer’s model, the Shapley value model I introduce here, or the Mortensen-Pissarides model), the aggregate behavior of the model is essentially indistinguishable from those benchmarks.

In summary, this paper emphasizes that what matters most for volatility in models of frictional labor markets is the behavior of wages. Shimer (2007) advanced our understanding by providing a microfoundation for the Beveridge curve that did not rely on an ad-hoc imposition of a Cobb-Douglas matching function, but instead derived it as an aggregation result. In addition, as I show here, his framework is tractable enough to allow for natural alternative methods of wage determination under which wages depend less discontinuously on aggregate labor market conditions. However, the ability of his model to account for volatile fluctuations in unemployment and vacancies arose only because his method of wage determination implicitly generated wages that depended more strongly on aggregate productivity than in the Mortensen-Pissarides benchmark. This underlines the necessity of further research into the dependence of wages on aggregate conditions for understanding the volatility of employment and unemployment.

The structure of the remainder of the paper is as follows. Section 2 describes the basic model, which is essentially that of Shimer (2007). Section 3 describes how wages can be determined

\(^2\)For a discussion, see, for example, Section 1.3 of Rogerson and Shimer (2010).
according to the Shapley value, as well as the case when wages depend simply on aggregate productivity. Section 4 describes the implications of the model for aggregate fluctuations, and Section 5 concludes briefly.

2 Model

The technological environment of the model and the preferences of agents are identical to those introduced in Shimer (2007). I describe them here to make the description of the model self-contained, but refer the reader to Shimer (2007) for proofs of the basic aggregative properties. I discuss how wages are determined in Section 3 below.

Time is continuous. There are a fixed number \( \hat{M} \) of workers and a large number of firms. All agents are risk neutral and discount the future at rate \( r \). Firms create jobs in a process to be described further below; denote the number of jobs in the economy at time \( t \) by \( \hat{N}(t) \).

There are \( \hat{L} \) labor markets; at any time \( t \), each worker is attached to a particular labor market, as is each job. The particular labor market to which each agent is attached at time \( t \) is random, with equal probability across all labor markets. Write \( M = \hat{M}/\hat{L} \) and \( N(t) = \hat{N}(t)/\hat{L} \). I consider the limiting case of a large economy in which \( \hat{L} \to \infty \) but regard \( M > 0 \) as a parameter and \( N(t) \geq 0 \) as an endogenous variable. The interpretation of \( M \) is the average number of workers per labor market; similarly, \( N(t) \) gives the average number of jobs per labor market. By the usual abuse of the law of large numbers, I can assume that the fraction of labor markets with precisely \( i \in \mathbb{N} \equiv \{0, 1, 2, \ldots \} \) workers is given by the Poisson formula \( \tilde{\pi}(i; M) = e^{-M}M^i/i! \); analogously, the fraction of labor markets with precisely \( j \in \mathbb{N} \) jobs is \( \tilde{\pi}(j; N(t)) = e^{-N(t)}N(t)^j/j! \). Because workers and jobs are allocated independently across labor markets, the fraction of labor markets with precisely \( i \) workers and \( j \) jobs, denoted \( \pi(i, j; M, N(t)) \), is simply the product of \( \tilde{\pi}(i; M) \) and \( \tilde{\pi}(j; N(t)) \), that is,

\[
\pi(i, j; M, N(t)) = \frac{e^{-(M+N(t))}M^iN(t)^j}{i!j!}.
\]

Workers and firms match in pairs in order to create output of the single good in the economy. At any time \( t \), a matched firm-worker pair produce output \( p(t) \), which is identical across all labor markets and across all matched firm-worker pairs within a labor market, but which follows an aggregate stochastic process to be described further below. An unmatched worker produces \( z < p(t) \) units of the same good in home production; an unmatched firm produces nothing.

There are no frictions within a labor market, and under all of the wage determination protocols to be considered in this paper, it will always be individually rational for as many matches as possible to form. Thus, in a labor market with \( i \) workers and \( j \) jobs, \( \min\{i, j\} \) matches form. If \( i > j \), then \( i - j \) workers are unemployed, while if \( j > i \), then \( j - i \) jobs are vacant. Aggregate employment \( E(N(t)) \), unemployment \( U(N(t)) \) and vacancies \( V(N(t)) \) can be calculated by
summing unemployment and vacancies across labor markets:

\[
E(N(t)) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \min\{i, j\} \pi(i, j; M, N(t));
\]

\[
U(N(t)) = \sum_{i=0}^{\infty} \sum_{j=0}^{i} (i - j) \pi(i, j; M, N(t));
\]

\[
V(N(t)) = \sum_{i=0}^{\infty} \sum_{j=i}^{\infty} (j - i) \pi(i, j; M, N(t)).
\]

Note that \(E(t) + U(t) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} i \pi(i, j; M, N(t)) = \sum_{i=0}^{\infty} i \tilde{\pi}(i; M) = M\), consistent with the notion that the average number of workers per labor market is \(M\). Similarly \(E(t) + V(t) = N(t)\).

There are three ways in which the state of a labor market, that is, the numbers of workers and jobs, can change. First, at Poisson rate \(q\), each worker receives a ‘quit shock,’ that is, a shock to her human capital which causes her to leave her current labor market and move to a new one, randomly selected with equal probability from all markets. This shock is independent of the identity of the worker and of the states of the origin and destination labor markets.

In addition, at any instant, any firm can create as many jobs as desired at constant marginal cost of \(k\). A newly-created job is allocated at random to a particular labor market, and does not move thereafter. At Poisson rate \(l\) there is a ‘layoff shock’ and the job is destroyed; there is no scrapping value. If there are other vacant jobs in this labor market, a worker who was matched with this job moves immediately to another match; if there are no such vacant jobs, the worker becomes unemployed. The job destruction shock is independent of the identity of the job and of the state of the labor market in which it is located.

Shimer (2007) shows that the two shock processes and the entry process preserve the independent Poisson distribution of workers and jobs across labor markets. This is intuitive because the Poisson distribution arises from the cumulation of many independent random events, and the shocks and the identity of the labor market to which an entrant job is allocated are independent of the numbers of firms and workers in this market.

There is a single aggregate shock, which affects productivity \(p(t)\). Productivity can take finitely many possible values. For the sake of comparability, I follow Shimer (2007) and assume that \(p(t) = p_y(t) = e^{y(t)} + (1 - e^{y(t)}) p\), where \(y(t)\) is a jump variable lying in the finite set \(-v \Delta, -(v - 1) \Delta, \ldots, 0, \ldots, v \Delta\). \(p\) is a lower bound on the lowest possible productivity \(p_{-v\Delta}\). \(y(t)\) remains constant at all times except when a shock arrives, which happens according to a Poisson process with arrival rate \(\lambda\). When a shock arrives, \(y(t)\) moves either up or down by \(\Delta\), with the new value \(y'\) satisfying

\[
y' = \begin{cases} 
y + \Delta & \text{with probability } \frac{1}{2} \left(1 - \frac{v}{r\Delta}\right) \\
y - \Delta & \text{with probability } \frac{1}{2} \left(1 + \frac{v}{r\Delta}\right)
\end{cases}
\]
The process for \( y(t) \) approximates an Ornstein-Uhlenbeck process \( dy = -\frac{1}{\nu} dt + \sqrt{\lambda} \Delta dx \), with mean zero, mean reversion parameter \( \lambda/\nu \), and instantaneous variance \( \lambda \Delta^2 \); the approximation lies in the fact that the innovations \( dx \) are not normal, although the distribution of \( x(t) \) conditional on \( x(0) \) does have expectation \( x(0) \) and variance \( t \), like a standard Brownian motion. \( p(t) \) is then a rescaled version of \( y(t) \). I refer the reader to Shimer (2005) for further details.

Assume that in a labor market with \( i \) workers and \( j \) jobs, each job earns expected flow profit \( v_p(i, j) \) when aggregate productivity is \( p \). Section 3 discusses in more detail how wages, and therefore profits, are determined. (In all cases I consider, flow profit \( v \) will be weakly increasing in \( i \), weakly decreasing in \( j \), and continuous and weakly decreasing in \( p(t) \), and strictly decreasing in \( p(t) \) if \( p(t) > 0 \).) Assume also that \( \vartheta(N) \), the expected flow profit of a job, defined by

\[
(2) \vartheta_p(N) = \frac{1}{N} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} j v_p(i, j) \pi(i, j; M, N),
\]

is strictly increasing in \( p \), strictly decreasing in \( N \), and satisfies the Inada condition \( \vartheta_p(N) \to 0 \) as \( N \to \infty \).

Denote by \( J_p(N) \) the expected present discounted value of profits that can be earned by an entering firm. If \( J_p(N) \) were strictly greater than \( k \), then firms would immediately create jobs at constant marginal cost of \( k \), making positive profits. (2) guarantees that this process stops for some finite \( N \). It follows that \( J_p(N) \leq k \) at all times. If \( J_p(N) < k \), then there is no entry; the number of jobs falls as existing jobs are destroyed at Poisson rate \( \lambda \), so that the law of motion for \( N \) is simply \( \dot{N} = -\lambda N \). Moreover, because \( \vartheta_p(N) \) depends on aggregate conditions only through \( p \) and \( N \), it is immediate that the equilibrium is characterized by a sequence of target job numbers \( N_p^* \). If \( N < N_p^* \), then \( N_p^* - N \) jobs are immediately created. If \( N = N_p^* \), job creation exactly offsets job destruction in order to keep \( N \) constant, and so is given by \( 1N \). If \( N > N_p^* \), then no jobs are created, and \( \dot{N} = -\lambda N \). It follows that \( J_p(N) \) satisfies

\[
r J_p(N) = \begin{cases} 
\vartheta_p(N) - \lambda J_p(N) - I N J_p(N) + \lambda E_p [J_p(N) - J_p(N)] & N > N_p^*; \\
rk & N \leq N_p^*.
\end{cases}
\]

Value matching and smooth pasting conditions imply that \( J_p(N_p^*) = k \) and \( J_p'(N_p^*) = 0 \). The same proof as for Proposition 1 of Shimer (2007) applies to establish existence of equilibrium, and the same constructive algorithm as applied there works to construct the sequence of targets \( N_p^* \).

3 Wage Determination

In this section I discuss how wages and profits are determined. What matters for equilibrium and for dynamics are profits, but it is more conventional to frame the discussion in terms of wages. The two are evidently closely related.

The assumption on wage determination made by Shimer (2007) is that there is Bertrand com-
petition. In labor markets where there are more workers than jobs, \( i > j \), then the wage is equal to \( z \), the value of home production. In markets with excess jobs, \( j > i \), workers earn the full value of the match output and the wage is equal to \( p \). A tie-breaking assumption is needed to deal with the case \( i = j \), and Shimer assumes that in this case \( w = z \); this matters for proving constrained efficiency but is unimportant in the calibrated model in which only a small fraction of markets fall into this category. In the notation introduced in the previous section, this implies that \( v_{p}(i,j) = (p - z)1(i \geq j) \), so that \( \delta_{p}(N) = (p - z)S(N) \), where \( S(N) = \sum_{i=1}^{\infty} \sum_{j=1}^{i} j \pi(i,j;M,N) \) is the fraction of jobs located in markets with (weakly) more workers than jobs. This assumption is an attractive benchmark; in labor markets without frictions, perfect competition is a natural assumption. However, the goal of this paper is to investigate alternative possibilities for wage determination. Accordingly, in the remainder of this Section I discuss two other ways of modeling wages and profits.

### 3.1 Wages Defined According to Shapley Value

One alternative to perfect competition between non-cooperating workers and jobs is cooperation. Assume that the coalition consisting of all \( i \) workers and all \( j \) jobs in a particular labor market negotiates between themselves how to split the flow surplus, relative to the workers’ alternative option of home production, that can be produced in that market. (This surplus is simply given by \( (p - z) \min\{i,j\} \).) I assume that agents bargain over the flow surplus only; because there are no matching frictions within a labor market, mobility and job entry and exit are independent of market characteristics and of wages, and all agents are risk neutral, there is no natural dynamic component over which agents might bargain.

The canonical model of splitting a surplus was introduced by Shapley (1953b). The Shapley values for players of an arbitrary cooperative game can be calculated as follows.

1. Starting with an empty coalition, one player at a time (either firm or worker) is added, with all remaining players being equally likely at each step so that all possible orders of inclusion are equally likely to arise.

2. Conditional on the ordering, each player is paid the marginal additional surplus created by his addition to the previous coalition.

The Shapley value of the player is then given by taking the expectation over all possible orderings of the players. Multiple authors have described noncooperative foundations for the Shapley value; see Pérez-Castrillo and Wettstein (2001) and the references therein.\(^3\)

In the environment studied here, the marginal surplus created by the addition of a worker to a coalition with \( \hat{i} \) workers and \( \hat{j} \) jobs is equal to \( p - z \) if \( \hat{j} > \hat{i} \) and 0 otherwise; the extra worker increases the surplus precisely when the coalition included a vacant job before he was added. Similarly, the marginal surplus created by adding a job to such a coalition is \( p - z \) if \( \hat{i} > \hat{j} \) and

\(^3\)Stole and Zwiebel (1996) also provide a noncooperative microfoundation for the Shapley value, but their extensive form game only applies in an environment with decreasing returns and a single firm.
The surplus a worker receives relative to his outside option \( z \) under the Shapley value, therefore, must be proportional to \( p - z \) and otherwise depend only on \( i \) and \( j \), the numbers respectively of workers and jobs in the labor market in which the worker is located. That is, the payoffs of a worker and a firm located in a labor market populated by \( i \) workers and \( j \) jobs can be written respectively as

\[
u_p(i, j) = z + (p - z)\psi(i, j) \text{ and } v_p(i, j) = (p - z)\phi(i, j),
\]

where \( \psi(i, j), \phi(i, j) \in [0, 1] \) denote the shares of the per-match surplus \( p - z \) received by a worker and a firm. Because the total surplus to be divided is \( (p - z)\min\{i, j\} \), it follows that

\[
i\psi(i, j) + j\phi(i, j) = \min\{i, j\}.
\]

I now show how to calculate the functions \( \psi(\cdot) \) and \( \phi(\cdot) \). Because it matters more directly for aggregate dynamics via the free entry condition, I concentrate first on \( \phi(\cdot) \). No surplus is generated when there are no workers, so that \( \phi(0, j) = 0 \) for all \( j \geq 1 \). The value of \( \phi(i, 0) \) is not defined (and is irrelevant for calculating expected profits since no job can earn this value); however, it is convenient to write \( \phi(i, 0) = 0 \) for all \( i \geq 1 \). With this convention, I can define \( \phi(i, j) \) for \( i, j \geq 1 \) according to the following recursion:

\[
\phi(i, j) = \begin{cases} 
\frac{i}{i+j}\phi(i-1, j) + \frac{i-1}{i+j}\phi(i, j-1) & 1 \leq i < j; \\
\frac{i}{i+j}\phi(i-1, j) + \frac{i-1}{i+j}\phi(i, j-1) + \frac{1}{i+j} & i \geq j \geq 1.
\end{cases}
\]

To understand why (4) characterizes the Shapley value of an individual job in a market with \( i \) workers and \( j \) jobs, consider calculating the surplus share accruing to a particular job \( j_0 \) by taking an expectation of the marginal contribution of \( j \) over all orderings of the \( i + j \) agents in the grand coalition of \( i + j \) agents. Divide the set of orderings of agents in three groups, according to the identity of the last agent added to the grand coalition.

Because all orderings are equally likely, in fraction \( i/(i+j) \) of all orderings, the last agent added is a worker. The expectation of the marginal surplus associated with a particular job, taken over all orderings of a coalition of \( i \) workers and \( j \) jobs in which a particular worker comes in the last position, coincides precisely with the expectation of the marginal surplus associated with this job, taken over all orderings of a coalition of \( i - 1 \) workers and \( j \) jobs. This is because the addition of this last worker does not affect the marginal surplus of an agent who is always previous in the orderings under consideration. Thus, in this case the expected value of the surplus share accruing to job \( j_0 \) is \( \phi(i-1, j) \).

In fraction \( (j - 1)/(i+j) \) of all orderings, the last agent added is a job other than the particular job under consideration; this analogously gives an expected contribution of \( \phi(i, j - 1) \) to the value of this job. Finally, in fraction \( 1/(i+j) \) of all orderings, the last agent added is the job of interest; this generates value \( p - z \) if and only if there were excess workers before the addition of this job,
that is, if $i > j - 1$, or equivalently, if $i \geq j$. (4) then follows using the linearity of the expectation operator.

There is no simple closed form for $\phi(i, j)$, but the recursion (4) allows values to be calculated numerically. Figure 1 shows the function $\phi(\cdot)$. Some properties can be established analytically. First, $\phi(i, j) \in [0, 1)$ for all $(i, j)$. By symmetry, $\phi(j, j) = \frac{1}{2}$ for all $j$. Moreover, standard inductive arguments establish that $\phi(i, j)$ is strictly increasing in $i$ and strictly decreasing in $j$ (provided $i, j \geq 1$). That is, wages are higher and profits lower in labor markets in which there are more jobs; conversely, wages are lower and profits higher in labor markets in which there are more workers. As under Bertrand competition, the short side of the market receives a greater share of match surplus, but under the Shapley value the dependence of wages and profits on $(i, j)$ does not exhibit an abrupt change at $i = j$. An immediate corollary of the two previous properties is that $\phi(i, j) > \frac{1}{2}$ for $i > j$ and $\phi(i, j) < \frac{1}{2}$ for $i < j$. Finally, because $j\phi(i, j) \leq \min\{i, j\} \leq i$, it is immediate that for each fixed $i$, $\phi(i, j) \to 0$ as $j \to \infty$.

Using the properties mentioned in the previous paragraph, it is straightforward to verify that $\bar{v}_p(N)$, the expected flow profit of a job located in a randomly chosen labor market as defined by (2), satisfies the properties assumed in Section 2. Specifically, $\bar{v}_p(N)$ is strictly increasing in $p$ and strictly decreasing in $N$, and satisfies $\bar{v}_p(N) \to 0$ as $N \to \infty$.

The surplus share received by a worker, denoted $\psi(i, j)$, can be calculated as a residual using the fact that all surplus is received by either workers or firms, according to (3). Alternatively, $\psi(i, j)$ also satisfies an analogous recursion to (4). First, $\psi(i, 0) = 0$ for $i \geq 1$, and by convention $\psi(0, j) = 0$ for $j > 0$. For $i, j \geq 1$,

$$
\psi(i, j) = \begin{cases} 
\frac{i}{i+j} \psi(i-1, j) + \frac{1}{i+j} \psi(i, j-1) & 1 \leq j < i; \\
\frac{i}{i+j} \psi(i-1, j) + \frac{1}{i+j} \psi(i, j-1) + \frac{1}{i+j} & j \geq i \geq 1.
\end{cases}
$$
Mapping the values \( u_p(i, j) = z + (p - z)\phi(i, j) \) and \( v_p(i, j) = (p - z)\phi(i, j) \) received by workers and jobs to wages and profits is not quite straightforward. There is a precise prediction for the total output of all jobs, namely \( p \min\{i, j\} \), as well as for the total earnings of workers matched with jobs, \( z \min\{i, j\} + (p - z)i\phi(i, j) \) and therefore for the labor share. However, each worker and each job is treated symmetrically, so that each worker is paid the same, independently of whether he works in a job or in home production, and likewise each job is paid the same, again independently of whether it is filled or not. Thus, \( u_p(i, j) \) and \( v_p(i, j) \) are best thought of as expected payoffs before the realization of the lottery that determines employment status. If unemployed workers in fact receive only their home production \( z \), then the expected value of a worker is equal to \( u_p(i, j) \) if the wage \( w_p(i, j) \) received by each employed worker satisfies \( z(i - \min\{i, j\}) + \min\{i, j\}w_p(i, j) = iu_p(i, j) \), or equivalently

\[
w_p(i, j) = z + \frac{i\phi(i, j)}{\min\{i, j\}}(p - z).
\]

I will use (6) for calibration purposes. It is worth emphasizing that the distribution of wage payments between unemployed and employed agents is irrelevant for the aggregate economy, since all agents are risk neutral and all that matters for the only economic decision in the model, firm entry, is the expected profit from creating a job.

It is natural to ask how to generalize the wage determination protocol discussed above by removing the assumption of symmetry between firms and workers. Shapley (1953a) generalized the Shapley value to allow for unequal weightings of the players, which arise when not all orderings of the agents in the grand coalition are equally likely. To generalize the idea of bargaining power familiar from the generalized Nash bargaining model, denote the ‘weight’ of a worker by \( \beta \) and that of a job by \( 1 - \beta \). Assume that the probability that an ordering of the grand coalition of \( i \) workers and \( j \) firms features a worker last is \( \beta i / (\beta i + (1 - \beta)j) \); the probability that it features a firm last is \( (1 - \beta)j / (\beta i + (1 - \beta)j) \). Assume that analogous expressions apply to orderings of smaller subcoalitions of the grand coalition: for example, in a subcoalition with \( \hat{i} \) workers and \( \hat{j} \) firms, the probability some worker is last is \( \beta\hat{i} / (\beta\hat{i} + (1 - \beta)\hat{j}) \). This generates a probability distribution over orderings of the \( i \) workers and \( j \) jobs in a labor market. To define the weighted Shapley value, assume that the value of a player is their expected marginal contribution to the surplus generated by the grand coalition when players are ordered for addition to the grand coalition according to this probability distribution. The profit earned by a job in a labor market with \( i \) workers and \( j \) jobs is then given by \( (p - z)\phi^\beta(i, j) \), where \( \phi^\beta(i, j) \) satisfies \( \phi^\beta(i, 0) = \phi^\beta(0, j) = 0 \) for all \( i \) and \( j \), and is defined for \( i, j \geq 1 \) by the appropriate generalization of (4),

\[
\phi^\beta(i, j) = \begin{cases} 
\frac{\beta i}{\beta i + (1 - \beta)j} \phi(i - 1, j) + \frac{(1 - \beta)(j - 1)}{\beta i + (1 - \beta)j} \phi(i, j - 1) & 1 \leq i < j; \\
\frac{\beta i}{\beta i + (1 - \beta)j} \phi(i - 1, j) + \frac{(1 - \beta)(j - 1)}{\beta i + (1 - \beta)j} \phi(i, j - 1) + \frac{1 - \beta}{\beta i + (1 - \beta)j} & i \geq j \geq 1.
\end{cases}
\]

Note that \( \phi^\beta(1, 1) = 1 - \beta \): when there is only a single worker and a single job in the labor market,
the surplus is shared between the two in ratio $\beta : (1 - \beta)$, as in the generalized Nash bargaining model. Figure 2 shows $\phi^\beta(\cdot)$ for $\beta = 2/3$; the scales and orientations of the two panels are the same as in Figure 1 for ease of comparison.

3.2 Wages Dependent on Aggregate Productivity

In this subsection I briefly present a much simpler alternative model of wage and profit determination, under which the wage paid in a match depends only on aggregate productivity. In particular that the wage paid by a particular job to the worker matched with it is independent both of $N$, the average number of jobs per labor market, $N$, and of $i$ and $j$, the numbers of workers and jobs in the labor market in which the job is located. This wage determination method is interesting mostly as a benchmark in the calibration in Section 4.

More concretely, if wages depend only on aggregate productivity, then I can write the wage as $w_p$. Since a total of $\min\{i, j\}$ jobs are matched in a labor market with $i$ workers and $j$ jobs, it follows that the expected flow profit of a job in such a market is given by

$$v_p(i, j) = (p - w_p) \frac{1}{j} \min\{i, j\}. \tag{8}$$

Write $x_p = p - w_p$. Assuming that $x_p$ is a strictly increasing function of $p$, it is immediate that the average flow profit across all jobs, $\bar{v}_p(N)$, is strictly increasing in $p$. It is also straightforward to verify that $\bar{v}_p(N)$ is strictly decreasing in $N$ and falls to zero as $N$ becomes large; the easiest proof of these two facts comes from observing that $\bar{v}_p(N) = x_p E(N)$: each match produces flow profits of $x_p$, so expected flow profits are simply the product of this with the fraction of jobs that are matched with a worker.

\footnote{Note, however, that $\phi^\beta(j, j) \neq 1 - \beta$ for $j \geq 2$. In fact, if $\beta > \frac{1}{2}$, then $\phi(j, j) < 1 - \beta$ for $j \geq 2$.}
An informal justification of how this simple wage determination method could arise in a de-centralized economy can be given. Consider a discrete-time version of the model, and suppose that matching of workers and jobs is redone completely at random every period. Suppose that once production in a match has begun during a period, then it cannot be interrupted without destroying all the output of the match. (Suppose, however, that the worker retains the ability to engage in home production during the current period should his match be interrupted.) In this case market conditions, in particular, the numbers of workers and jobs in the current labor market, are irrelevant to wage determination, and any wage in the interval \([z, p]\) is consistent with completing production assuming both the worker and the job exhibit individually rational behavior. Thus, any wage in this interval can be justified.

In the calibration in Section 4, I will consider wages defined in such a way that the flow profit of a job \(x_p\) exhibits a constant elasticity with respect to \(p\), that is, \(x_p = a_0 p^{a_1}\) for parameters \(a_0\) and \(a_1\).

4 Implications for Aggregate Fluctuations

I now turn to studying the implications of the alternative models of wage determination introduced above for the volatility of the aggregate labor market.

4.1 Calibration Strategy

Since the only difference between the current paper and Shimer (2007) is how wages are determined, I calibrate the remainder of the model using the same parameterization as used in that paper. As Shimer’s Proposition 4 shows, corresponding to any unemployment and vacancy rates there is a unique pair \((M, N)\) of workers and jobs per labor market. If \(M = 244.2\) and \(N = 236.3\), then the unemployment rate is 5.4 percent and the vacancy rate is 2.3 percent, consistent respectively with data from the Current Population and Job Openings and Labor Turnover Surveys conducted by the Bureau of Labor Statistics for the period December 2000 through April 2006. Thus, \(M = 244.2\) and the entry cost \(k\) is chosen so that \(N = 236.3\) when productivity is at its median value of 1. (The entry cost \(k\) varies according to the model of wage determination; it is 4.0785 as in Shimer’s calibration when his assumption on wages is used.)

Time is measured in quarters. The quit rate \(q\) and the job destruction rate \(l\) are both set to 0.081; the sum \(q + l\) is chosen to match the separation rate into unemployment in the steady state, while the ratio of \(q\) to \(l\) is unimportant for the model’s behavior provided \(l\) is not too small so that the irreversibility of entry is not too severe. The interest rate \(r\) is set to 0.012. The parameters of the productivity process are \(\nu = 1000\), \(\lambda = 86.6\), and \(\Delta = 0.00580276\), corresponding to a process for \(y(t)\) of the form \(dy = -0.0866 dt + 0.054 dx\), where \(dx\) has unit instantaneous standard deviation. Shimer targets these parameters to match the standard deviation and autocorrelation of detrended labor productivity, as discussed further below. Finally, I choose \(p\) to be equal to \(z + (r + l) \cdot 4.0785 = 0.7793\). This is the same lower bound for productivity chosen by Shimer and equal to
$z + (r + l)k$ for his parameterization, so it is the largest lower bound for productivity in his model ensures that entry is always profitable provided few enough jobs exist. Rather than identifying different lower bounds for productivity as I change how wages are determined, instead I keep $p$ constant across the various wage determination models, since to do so without also changing the productivity parameters $\lambda$ and $\Delta$ would change the volatility of labor productivity. (In practice the value of $p$ does not matter very much since $y(t)$ is vanishingly unlikely to reach the minimum possible value $-\nu \Delta$.)

The description and characterization of the equilibrium in Section 2 makes it easy to provide a computational procedure to calculate an equilibrium numerically, given a functional form for profits $v_p(i, j)$. This differs from the procedure in Shimer’s paper only in that the function form for $v_p(i, j)$ is different, and in that I use a slightly different method to calculate labor productivity. More specifically, I use the same computational algorithm suggested by Shimer’s Proposition 1 to calculate the target numbers $N^*_p$ for jobs per market associated with each of the $2\nu + 1$ values for productivity. I choose initial values $p(0)$ and $N(0)$, and draw the time of the first shock that changes the value of labor productivity (this is distributed exponentially with mean $1/\lambda$). I calculate the number of unemployment-to-employment (UE) and employment-to-unemployment (EU) transitions that occur in this time interval, using the appropriate out-of-steady state generalizations of Shimer’s equations (16) and (17). This allows me to calculate the value of $N(t)$ at the time of the first shock. I also aggregate output during the time until the shock arrives. I then draw a new value of labor productivity according to (1), and repeat the process.

At the end of every month (one third of a period), I record the cumulative total numbers of UE and EU transitions that occurred during the month. I record the values of the unemployment and vacancy rates at the end of the month. Finally, I record cumulative output during the month. I calculate a measure of labor productivity by dividing total output during the month by average employment. I also measure the separation rate by dividing the number of EU transitions during the month by the employment rate at the beginning of the month; similarly I measure the job-finding rate by dividing the number of UE transitions by the unemployment rate. I discard the first 25,000 years of data, then generate 20,000 samples each consisting of 53 years (212 quarters) of model-generated data. I take logarithms and express all results as deviations from an HP trend with smoothing parameter $10^5$. I calculate each moment in each of the 20,000 samples, and report the cross-sample means and standard deviations of these values.

4.2 Results

Table 1 replicates the results obtained by Shimer (2007). The volatilities of key labor market vari-

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5I calculate employment and unemployment precisely. Because the instantaneous separation flow is a very slightly non-linear function of $N(t)$, which itself decays slightly non-linearly when it is above the appropriate employment target $N^*_p$, I approximate the measure of total EU transitions within a time interval within which there is no productivity shock by multiplying the duration of this time interval by the average of the instantaneous EU flow corresponding to the initial and final values of $N(t)$. The numerical error associated with this approximation is negligible. A similar approximation is used to calculate UE transitions.
Table 1: Wages determined by Bertrand competition as in Shimer (2007)

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$v$</th>
<th>$v/u$</th>
<th>$f$</th>
<th>$s$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation</td>
<td>0.058</td>
<td>0.079</td>
<td>0.137</td>
<td>0.032</td>
<td>0.032</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.011)</td>
<td>(0.018)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Quarterly autocorrelation</td>
<td>0.874</td>
<td>0.874</td>
<td>0.874</td>
<td>0.728</td>
<td>0.880</td>
<td>0.880</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.031)</td>
<td>(0.031)</td>
<td>(0.061)</td>
<td>(0.030)</td>
<td>(0.030)</td>
</tr>
</tbody>
</table>

### Correlation matrix

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$v$</th>
<th>$v/u$</th>
<th>$f$</th>
<th>$s$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>1</td>
<td>-0.999</td>
<td>-1.000</td>
<td>-0.949</td>
<td>0.992</td>
<td>-0.993</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.000)</td>
<td>(0.011)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>$v$</td>
<td>1</td>
<td>1.000</td>
<td>0.948</td>
<td>-0.990</td>
<td>0.989</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.011)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v/u$</td>
<td>1</td>
<td>0.949</td>
<td>-0.991</td>
<td>0.991</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f$</td>
<td>1</td>
<td>-0.903</td>
<td>0.906</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
<td>(0.021)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>1</td>
<td>-1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tbody>
</table>

The variables are essentially indistinguishable from those reported in his Table 2. The variables shown in the Table are (the deviations from trend of) the unemployment rate $u$, the vacancy rate $v$, the ratio of these two quantities, the UE and EU transition rates $f$ and $s$, and labor productivity $p$. The volatility and autocorrelation of labor productivity are calibration targets. (Very slight differences arise between the results reported here and those reported in Shimer (2007) in the autocorrelation of measured labor productivity, which Shimer measures as the value at the end of each period and I measure by dividing cumulative output during a month by average employment during that month.)

When wages and profits are determined according to the Shapley value as in Section 3.1, the labor market becomes more volatile. This is shown in Table 2. The standard deviation of unemployment increases from 0.060 to 0.080, that is, from around three times the volatility of labor productivity to around four times that volatility. Corresponding increases are seen in the volatilities of vacancies, the vacancy-unemployment ratio (whose standard deviation is by construction approximately given by the sum of the standard deviations of unemployment and vacancies, since these variables are nearly perfectly negatively correlated in the model), and measured UE and EU transition rates.

When wages are determined using the weighted Shapley value, with the bargaining power of workers set to $\beta = 2/3$, on the other hand, unemployment and vacancies are significantly less volatile, as are job-finding and separation rates. The standard deviation of unemployment falls to 0.035, less than twice the standard deviation of labor productivity (which is nearly invariant to the changes in wages). The volatilities of the other labor market variables shown also fall roughly proportionally. Note that the autocorrelation of variables, along with the correlation matrix, are essentially invariant to the change in volatility. These appear to be a feature of the structure of the mismatch environment, along with the chosen detrending procedure (these moments are sensitive
to the smoothing parameter of the HP trend removed).

Why is it that unemployment and vacancies are more volatile under the Shapley value than under Bertrand competition, where they are in turn more volatile than under the weighted Shapley value? To answer this question, I turn to the simpler wage determination protocol introduced in Section 3.2. Assume that profits exhibited a constant productivity elasticity, \( x_p = a_0 p^{a_1} \). I use the following procedure to choose the parameters \( a_0 \) and \( a_1 \). I choose \( a_0 \) so that the expected profit of a job, \( \bar{v}_p(N) \) as defined by (2), is the same as in the nonstochastic steady state under Bertrand wage determination; this requires setting \( a_0 = 0.388 \). This guarantees that in the steady state with \( p = 1 \) and with the same value of the entry cost \( k = 4.0785 \), both models have the same average number of firms per labor market, \( N \), and therefore the same employment and unemployment rates. I then choose \( a_1 \), the productivity elasticity of profits, as follows. In the model with Bertrand wages, I increase \( p \) slightly from \( p = 1 \) to \( p' = 1 + \delta \), and compute the number of firms \( N' \) in the new nonstochastic steady state. I then compute the associated value of \( \bar{v}_p(N') \). I then choose \( a_1 \) so that the value of \( v_{p'}(N') \) is the same in the model with constant productivity elasticity of profits. This requires setting \( a_1 = 0.0909 \). This functional form for profits implies a labor share of \( 1 - a_1 = 0.612 \) and a productivity elasticity of wages equal to \( (1 - a_0 a_1) / (1 - a_0) = 1.577 \) in a neighborhood of the nonstochastic steady state.

The results from simulating the model described in the previous paragraph are shown in Table 4. It is apparent that Table 4 is almost exactly the same as Table 1. That is, knowing the labor share (equivalently, the average wage) and its productivity elasticity is sufficient in the mismatch model to determine the volatility of unemployment and vacancies. Not only are the reported sample moments nearly identical, but in fact conditional on a sequence of productivity draws, the

<table>
<thead>
<tr>
<th></th>
<th>( u )</th>
<th>( v )</th>
<th>( v/u )</th>
<th>( f )</th>
<th>( s )</th>
<th>( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td></td>
<td>-0.997</td>
<td></td>
<td></td>
<td></td>
<td>-0.993</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.011)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>v</td>
<td>1</td>
<td>1.000</td>
<td>0.948</td>
<td>-0.987</td>
<td>0.989</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.011)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( v/u )</td>
<td>1</td>
<td>0.948</td>
<td>-0.990</td>
<td>0.991</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f )</td>
<td>1</td>
<td>-0.902</td>
<td>0.907</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
<td>(0.021)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( s )</td>
<td>1</td>
<td>-1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p )</td>
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<td></td>
<td></td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Standard deviation</th>
<th>Quarterly autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.078 (0.011)</td>
<td>0.873 (0.031)</td>
</tr>
<tr>
<td></td>
<td>0.105 (0.014)</td>
<td>0.874 (0.031)</td>
</tr>
<tr>
<td></td>
<td>0.183 (0.024)</td>
<td>0.874 (0.031)</td>
</tr>
<tr>
<td></td>
<td>0.043 (0.005)</td>
<td>0.728 (0.061)</td>
</tr>
<tr>
<td></td>
<td>0.043 (0.006)</td>
<td>0.880 (0.030)</td>
</tr>
<tr>
<td></td>
<td>0.020 (0.003)</td>
<td>0.879 (0.030)</td>
</tr>
</tbody>
</table>

Table 2: Wages determined by Shapley value
time series of unemployment and vacancies are virtually indistinguishable.

I repeat the exercise described in the previous two paragraphs to calibrate \( x_p \) differently to mimic the Shapley value and weighted Shapley value cases shown in Table 2 and Table 3. The case of Shapley value with equal bargaining powers corresponds to \( a_0 = 0.354 \) and \( a_1 = 0.121 \), or equivalently, a labor share of 0.646 and a productivity elasticity of wages equal to 1.482. The case of weighted Shapley value with worker’s bargaining power \( \beta = \frac{2}{3} \) corresponds to \( a_0 = 0.0334 \) and \( a_1 = 0.0524 \), or equivalently, a labor share of 0.967 and a productivity elasticity of wages equal to 1.033. The results from simulating these two models coincide almost precisely with those reported in Table 2 and Table 3, and are therefore omitted.

As a final exercise to demonstrate that the labor share and productivity elasticity of wages are nearly all that matter for the volatility of unemployment and vacancies in this class of models, I recalibrate these two values to match the behavior of the Mortensen-Pissarides model with a sunk cost of entry. In the environment of Shimer (2005), set the flow vacancy-posting \( c \) to zero and instead institute a sunk cost of entry \( k \). Wages are determined by generalized Nash bargaining, with the outside options of an employed worker and a filled job being to continue as unemployed or as an unfilled job (in particular, the firm does not lose \( k \) should bargaining fail). The worker’s bargaining power is \( \beta \). Some straightforward algebra shows that in the steady-state, market tightness satisfies

\[
\frac{r + s}{q(\theta)} + \beta \theta = (1 - \beta) \left[ \frac{p - z}{(r + s)k} - 1 \right].
\]

If the solution is implemented by continuously rebargained wages, then this wage is given by

\[
w = (1 - \beta)z + \beta p - \beta(1 - \theta)(r + s)k.
\]

Using the calibration of Shimer (2005) and setting \( k \) such that the steady-state vacancy-unemployment ratio is as in the mismatch model (that is, \( v/u = 15 \)).

<table>
<thead>
<tr>
<th></th>
<th>( u )</th>
<th>( v )</th>
<th>( v/u )</th>
<th>( f )</th>
<th>( s )</th>
<th>( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation</td>
<td>0.034 (0.005)</td>
<td>0.045 (0.006)</td>
<td>0.079 (0.011)</td>
<td>0.019 (0.002)</td>
<td>0.018 (0.003)</td>
<td>0.020 (0.003)</td>
</tr>
<tr>
<td>Quarterly autocorrelation</td>
<td>0.873 (0.031)</td>
<td>0.874 (0.031)</td>
<td>0.873 (0.031)</td>
<td>0.728 (0.061)</td>
<td>0.880 (0.030)</td>
<td>0.879 (0.030)</td>
</tr>
</tbody>
</table>

Table 3: Wages determined by weighted Shapley value, \( \beta = 0.667 \)
0.023/0.054 = 0.412, the flow profit of a filled job is $p - w = 0.286$, and the productivity elasticity of profits are 0.0587. I then calibrate the mismatch model with a constant productivity elasticity of profits by setting $a_0 = 0.286$ and $a_1 = 0.0587$ (or equivalently, a labor share of 0.714 and a productivity elasticity of wages of 1.377).

The results are as shown in Table 5. Again, autocorrelations and cross correlations are essentially unaffected by the change in the functional form for wages, while the volatilities of all labor market variables falls relative to the case of Bertrand competition. Unemployment is now just under twice as volatile as labor productivity, and market tightness around 4.5 times as volatile. This is slightly more volatility than would be suggested simply by the comparative statics of steady states in this model. As noted in footnote 10 of Shimer (2007), (9) implies that the productivity elasticity of the $v/u$ ratio in steady state for this calibration is $1.90p/(p - z)$, around 1.84 times as much as in the Shimer (2005) calibration of the model with flow vacancy posting costs. In fact, in the simulation, the $v/u$ ratio is around 2.48 times more volatile than in Shimer (2005). The slight difference between these two numbers must be ascribed to the differences in the microeconomic pattern of matching in the model (in particular, to the fact that in the Mortensen-Pissarides setting, a newly-created job begins its life unmatched for sure, while under mismatch it might be matched as soon as it enters; this makes entry more responsive to current productivity, because a new job is more likely to produce while productivity is temporarily high).

5 Conclusion

The mismatch model of Shimer (2007) makes an important contribution to our understanding of the microfoundations of the aggregate relationship between unemployment and vacancies,
Table 5: Wages determined so profits exhibit constant productivity elasticity; Shimer (2005) calibration

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$v$</th>
<th>$v/u$</th>
<th>$f$</th>
<th>$s$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation</td>
<td>0.038</td>
<td>0.051</td>
<td>0.088</td>
<td>0.021</td>
<td>0.021</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.007)</td>
<td>(0.012)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Quarterly autocorrelation</td>
<td>0.873</td>
<td>0.874</td>
<td>0.874</td>
<td>0.728</td>
<td>0.880</td>
<td>0.879</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.031)</td>
<td>(0.031)</td>
<td>(0.061)</td>
<td>(0.030)</td>
<td>(0.030)</td>
</tr>
</tbody>
</table>

$$
\begin{array}{ccccccc}
\text{Correlation} & u & v & v/u & f & s & p \\
\hline
\text{matrix} & 1 & -0.999 & -1.000 & -0.948 & 0.993 & -0.993 \\
& (0.000) & (0.000) & (0.011) & (0.002) & (0.002) & (0.002) \\
\text{Correlation} & v & 1 & 0.949 & 0.949 & 0.992 \\
& (0.011) & (0.011) & (0.011) & (0.002) & (0.002) & (0.002) \\
\text{Correlation} & f & 1 & -0.904 & 0.908 \\
& (0.022) & (0.022) & (0.022) & (0.021) & (0.021) & (0.021) \\
\text{Correlation} & s & 1 & -1.000 \\
& (0.000) & (0.000) & (0.000) & (0.000) & (0.000) & (0.000) \\
\text{Correlation} & p & 1 \\
\end{array}
$$

by showing how the empirical relationships between unemployment and vacancy rates, as well as UE and EU transition rates, can be consistent with a simple microfoundation. However, the predictions of the model for the volatilities of unemployment and vacancies depend critically on the assumption on how wages are determined. The stylized method of wage determination used in Shimer’s paper generates profits that vary more with productivity than in the Mortensen-Pissarides model, and the results of this paper show that this is the only reason why the model generates more volatility than the MP benchmark. In this paper, I investigated various alternative methods of wage determination in the mismatch model—the simplicity of the mismatch framework in allowing for these alternatives is a significant advantage of the model—and I showed that the resulting volatilities of key labor market variables were determined almost entirely by the labor share and the productivity elasticity of wages. Finally, the model also produces very similar behavior of unemployment and vacancies to the Mortensen-Pissarides model when wages are calibrated so as to match their behavior in that model.

That the behavior of wages is key for understanding business cycle fluctuations in models with labor market frictions is not, of course, a new observation (Caballero and Hammour, 1996; Hall, 2005). The contribution of this paper is to reinforce this consensus, and re-sound a call for more empirical research on this topic, made most recently by Rogerson and Shimer (2010), who underline why existing evidence on this topic is at best inconclusive. Another, more theoretical, direction in which progress could be made would be to move away from the focus on neutral productivity shocks as the driving force in models of frictional labor markets, particularly given that the procyclicality of labor productivity may have changed after the mid-1980s (Galí and Gambetti, 2009; Stiroh, 2009; Galí and van Rens, 2010; Hagedorn and Manovskii, 2011). The mismatch model seems a natural environment for this second line of research, since just as its dynamics are
tractable enough to allow easily for alternative models of wage determination, they are equally tractable enough to allow for alternative types of shocks.
References


