Labor Share and Technology Dynamics

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February 2012

[PRELIMINARY AND INCOMPLETE]

Abstract

In this paper we develop a theory on the joint dynamics of labor share and technology (measured as a Solow residual) at the business cycle frequency. Our main motivating fact is the overshooting property of the labor share: After a positive technology shock, the share of output that corresponds to labor falls temporarily but it quickly rises, at a higher level (in absolute terms) than the initial drop. We propose a framework where firms own heterogeneous production units (putty clay technology) and the labor market functions as in the search and matching literature. The model is capable of reproducing the business cycle facts of labor share and its components and also provides a framework in which sticky wages do not solve the unemployment-productivity correlation or Shimer puzzle.

Keywords: Labor Share, Technology shocks, Putty-Clay, Business Cycles

JEL Codes: E01, E13, E25, E32

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1 Introduction

In this paper we develop a real business cycle model to understand the joint dynamics of labor share and technology. Our main motivating fact is the *overshooting* property of the labor share, as described in Ríos-Rull and Santaéulalia-Llopis (2010) for the U.S. economy.\footnote{After a positive technology shock, the share of output that corresponds to labor falls temporarily but it quickly rises, at a higher level (in absolute terms) than the initial drop. This positive response is persistent and long lasting.} As shown by Choi and Ríos-Rull (2009), these dynamics are quite a challenge for standard models of the business cycle with labor search frictions. This challenge is related to the poor performance of these models with respect to amplification mechanisms from technology innovations to total hours.\footnote{In recent literature, this has been labeled as the *Shimer puzzle*. See Shimer (2005) and Hagedorn and Manovskii (2008).} In this paper we achieve both the replication of the cyclical behavior of the labor share and an improvement of the response of total hours to technology shocks, hence, our contribution is twofold: we suggest a new way of thinking of aggregate technology in business cycle models and relate this to the ongoing debate about how technology affects the labor market in such models.

Our model uses putty-clay technology,\footnote{See Atkeson and Kehoe (1999), Gilchrist and Williams (2000), Wei (2003) and Gourio (2007).} where production takes place through individual productive units. Firms own these units and before installing, they must decide on the "quality" (or size) of the unit and attach one worker to it. Production takes place only if the unit is operated by a worker and unit-worker pairs are subject to aggregate technology shocks. The pre-installation size menu available to firms is Cobb-Douglas (i.e., 'putty'), but once installed, plants remain in place without the possibility of changing its quality (i.e., the capital on a standing plant is like 'clay'). This assumption creates a Leontief productive structure in the short-run, with zero substitutability between capital and labor. In the long-run, the aggregate production function is a Cobb-Douglas.

We embed this technology in an environment with matching frictions, in the sense that vacant machines have to wait one period in order to be matched with a non employed worker. We also use staggered wage contracts, similar to Gertler and Trigari (2009) to analyze the

We then compare the model to standard labor search and matching models, in the spirit of Andolfatto (1996) and Merz (1995). By doing this, we can assess the importance of the putty clay technology plus staggered wages in delivering our results and we also have a comparison standard for the response of total hours to technology shocks in our model.

This paper is related to an ongoing strand of labor-macro literature arguing for the introduction of
some form of wage rigidity in order to increase the amplification mechanism from technology shocks to labor market variables in models with search frictions. In this vein we find Shimer (2005), Hall (2005), Blanchard and Galí (2007), Costain and Reiter (2008) and Hall (2009), among others. The main insight from this literature is clear: matching and the terms of bargaining in real wages matter for labor market outcomes during the cycle. The incentives for firms to post wages during favorable aggregate conditions depend on how much profits they can get from matched employees, or in model terms, in the wedge between labor productivity and real wages. Hence, (more) rigid wages give more incentives for employment creation by increasing the dynamic difference between labor productivity and real wages.

However, the logic above hinges critically on the type of technology used and ceases to be true in the model with putty clay technology, as we show in our simulations. Our results also provide support for a wider use of more flexible types of technology arrangements, away from the standard Cobb-Douglas construct.

Understanding the properties of the labor share and its dynamics can shed light on additional economic issues. For example, Blanchard (1997) and Caballero and Hammour (1998) study the evolution of recent European labor markets: Caballero and Hammour analyze the relationship between the appropriability of specific quasi-rents and factor substitution to explain slow job growth, while Blanchard studies shifts in labor demand and the implementation of labor saving technologies to account for declining labor share; Gali and Gertler (1999) and Sbordone (2002) study Phillips curves empirically using the labor share as a measure of economic activity, instead of the output gap; Ríos-Rull and Santaeulalia-Llopis (2010) identify aggregate properties of the standard business cycle model and their relation with the dynamics of factor shares.

The structure of the rest of the paper is as follows. In the next section we lay out the facts with respect to the labor share and its components in the postwar U.S. experience. In section 3.1 we present our model and its variants. Section 4 discusses our calibration strategy while section 5 presents the numerical results of our simulations. Section 6 concludes.

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4 An exception is the paper by Hagedorn and Manovskii (2008) who argue that a different calibration of the standard search and matching model is able to produce big responses in vacancies and unemployment after a technology shock. However, Rogerson and Shimer (2010) claim that the mechanism in that paper can be reinterpreted also as wage rigidity, adding to the original argument.

5 See for example Mortensen (1992), Caballero and Hammour (1996) and Rogerson and Shimer (2010).
2 Labor Share During the Business Cycle

To analyze the behavior of labor share and its components, we use aggregate quarterly US data from 1954:I to 2004:IV. To construct factor shares (percentage of aggregate payments that pertain to capital and labor) we follow Cooley and Prescott (1995) and Ríos-Rull and Santaeulalia-Llopis (2010) and assume that the ratio of ambiguous labor income to ambiguous total income is the same as the one for unambiguous labor income to unambiguous total income. Detailed information on the construction of this variable, as well as information on sources of data can be found in appendix A.

Next, using the series for labor share ($\zeta_t$), our measure of technology is defined as the residual of a growth accounting exercise. Equation (1) shows our exact definition:

$$z_t = \tilde{y}_t - (1 - \zeta_t)\tilde{k}_t - \zeta_t\tilde{n}_t$$

where $Y$ is output, $K$ is capital and $N$ is total hours. This definition differs from a standard Solow residual since we are using the whole history of factor shares to construct $z_t$, as opposed to its long-run average.

Figure 1 shows the labor share and our constructed measure of technology. Visual inspection of both panels shows that the labor share is volatile and has a slight downwards trend, properties quite distant from the constant assumption used in much of the literature since Kaldor (1957). On the other hand, our measure of technology exhibits the well known ‘moderation’ of business cycles after the mid 1980s, in terms of volatility of the series around its long run trend (zero).

To assess more rigourously the cyclical co-movement of both variables, we estimate a vector autoregression of order 1 for the level of $z_t$ and the demeaned labor share. Using a Cholesky decomposition with the residual $z_t$ ordered first in the system (in order to identify technology shocks), we can produce an impulse response function for the labor share after an innovation in technology. We present this estimation as well as 95% confidence bands in figure 2.

The figure summarizes various facts with respect to the labor share which have been previously discussed in Choi and Ríos-Rull (2009) and Ríos-Rull and Santaeulalia-Llopis (2010): Upon impact, labor share falls giving rise to the well known counter cyclical behavior of the series. Consequently, labor

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6 For extensive robustness exercises and different definitions of the labor share, see Ríos-Rull and Santaeulalia-Llopis (2010).

7 In this definition, we have used a linear detrending procedure, i.e., $\tilde{x}_t \equiv \log(X_t) - \alpha_0 - \alpha_1 t$ where $\alpha_0$ and $\alpha_1$ are ordinary least squares (OLS) estimates and $t$ is a time trend.
Figure 1: U.S. data, 1954:I to 2004:IV

Figure 2: IRF of the labor share to a shock in technology and 95% C.I., from VAR(1)
share starts to rise, and peaks around 15 to 20 quarters after the initial shock. This positive response is persistent and long lasting, making the aggregate gains of labor due to an expansion positive in the medium to long run, more than compensating the initial drop in this share (hence the overshooting label). As seen in the figures, this response is statistically significant at the 95% level.

In figure 3 we show a decomposition of this estimated labor share response. In each panel, we plot the impulse response function of each of its components (output, total hours and real wages), which were computed through bivariate vector autoregressions or order one, between $z_t$ and each component. These responses uncover positive responses in output, total hours and real wages. Total hours (northeastern panel) react sluggishly at first to the technology shock and peak 10 to 20 quarters after its onset, evidence of some form of friction in the adjustment of aggregate hours in the economy. Real wages (southwestern panel) react slightly less than output (first panel) and their response declines almost linearly in time, at a slower rate than that of output.

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The estimates of the bivariate VAR show no effects whatsoever of lagged labor share on productivity. See Ríos-Rull and Santaulalia-Llopis (2010) for details.
3 The Model

The economy is populated by a measure of households, each of them consisting of a mass of consumers/workers (we normalize their size to one). Each household has preferences over consumption and leisure streams of their members. Every household member is part of the labor force, but employment is limited by the number of jobs posted by firms. We assume perfect substitutability across household members and perfect capital markets, so there is no distinction in terms of consumption between employed or unemployed household members.

In this fashion we can write the per period utility function of the family as $u(c) + v(n) = \log(c) + (1 - n)b$, where $c$ is the common consumption level for everyone inside the family and $n$ is the fraction of household members that work this period and $b$ is the additional value of leisure to the non-working members of the family (we can normalize the leisure value to those working at zero). In addition, households discount the future at rate $\beta$ and take the appropriate expectations whenever necessary.

Production of the unique good of this economy takes place in plants that have one worker each and that may differ in the amount of capital installed in each of them. We denote this amount as $k$ and we think of this value as "size" or "quality". Plants take one period to become operational and once built, a plant’s capital cannot be changed. Capital in these plants cannot be de-installed and its scrap value is zero. Thus, to calculate aggregate production in the economy, we need (potentially) information on the whole size-distribution of operating plants.

We assume that firms can find workers in a frictionless way, i.e., every new plant can find an unemployed worker for sure at no cost. However, given the required "incubation" period for plants, aggregate hours in the economy will move with some lag. Once a worker is hired her wage is set by bilateral Nash bargaining between the firm (owner of the plant) and the family of the worker. At the end of every period, workers are discarded by firms (go to temporary unemployment) and firms hire again workers to continue production. These assumptions matter for the wage setting mechanism and "kill" the dependence of individual bargained wages on the entire distribution of current and future plant sizes. In turn, we get both a simplified wage function per plant and an aggregate wage bill that can be handled analytically, despite the existence of a distribution of installed plant sizes.

\footnote{This setup is similar to Andolfatto (1996), where workers are "shuffled" across jobs at the end of every period, as in a game of \textit{musical chairs}.}

\footnote{There is an alternative configuration of assumptions that yield identical results: workers are attached to plants as long as they are operational: in case of breakdown, the firm cannot hire another worker and the plant gets destroyed. Additionally, the job finding rate for households needs to be inelastic to the own number of unemployed individuals.}
Each productive plant has an exogenous break-down probability of $\delta$. For now we will assume that plants are only destructed exogenously and that the firm does not want to stop operations, but later we will argue that this is equilibrium behavior under all of our simulated economies.

A plant of size $k$ produces an amount $y_k = e^z k^\alpha$ of goods, where $z$ is an aggregate productivity shock. The aggregate menu of technologies available to any firm is of the Cobb-Douglas form ex-ante (putty analogy), but once installed, a plant operates according to a fixed Leontief structure (installed capital is like "clay"). The per-period profit of a plant is given by $\pi_k = y_k - w$, where $w$ is the wage paid to the worker, which potentially depends on the size of the particular plant $k$ and the whole distribution of installed plants.

The aggregate state variables for this economy, which we denote as $S$, include the whole distribution of plant sizes $x$, in addition to the technological shock. However, the assumptions we described above (that there are no search frictions, that workers are attached to plants for only one period and that in case of breakdown of negotiations, firms can hire an alternative worker immediately for the following period) simplify the problem, since the outcome of Nash bargained wages under this setting is static; when this is the case, the investment/plant size installation decision problem for the firms is also greatly simplified, which in turn reduces the aggregate state space for the economy to to the technology shock and a notion of installed production capacity.

To establish this, we proceed by posing the model without taking advantage of the existence of a small set of state variables and then we show how, given our assumptions about the bargaining procedure, agents only need to know the reduced set of state variables.

3.1 Setting up the agents’ problems

To describe the problems that the households face, we denote the aggregate state variables by $S$, and by $S' = G(S)$ their law of motion. An individual household is characterized by its assets $a$ and the fraction of its members that are employed $n$. In the context of the pairwise bargaining problem, the plant-specific wage function that each working household member can command will be denoted by $w(S, k, a, n)$, where $k$ is the plant size, $a$ denotes the assets of the household, and $n$ the number of workers in that household. Given state $S$ and household employment $n$ we denote with $W(S, a, n)$ the total labor income of the household. In doing this we are implicitly assuming that workers in all households are equally distributed across plants. Denote the measure of plants $x(S)$, to make explicit its dependence on the state.
Total labor income is just the sum of the earnings of all workers in the household:

\[ W(S, a, n) = \int_{0}^{\infty} w(S, k, a, n) x(S) (dk). \]  

(2)

Denote by \( Q(S) \) the number of new plants installed this period (because there is nothing idiosyncratic about plant creation, all new plants will be of the same size). Then \( Q(S) \) is also the number of newly hired workers for next period. From the point of view of a household, the evolution of its employment \( n \) is given by

\[ n'(S, a, n) = (1 - \delta) n + \left( \frac{1 - n}{1 - N(S)} \right) Q(S) \]  

(3)

where \( N(S) \) is aggregate employment today. This specification implies that the chances of getting jobs for the household members depend on the size of its unemployed members relative to the economy’s unemployment pool.\(^{11}\)

### 3.1.1 Problem of the Household

A household with \( a \) assets and \( n \) members employed takes as given prices and the wages that its members may command in each plant. We write its problem as

\[
\begin{align*}
\Omega(S, a, n) = \max_{c, a'} & \quad u(c) + (1 - n) b + \beta E \left\{ \Omega \left[ G(S), a', n' \right] | S \right\} \\
\text{s.t.} & \quad c + a' = [1 + r(S)] a + W(S, a, n) \\
\end{align*}
\]

(4)

and equation (3)

where \( r(S) \) is the equilibrium return on savings. The solution to this problem is a couple of functions for consumption and savings \( \{c(S, a, n), a'(S, a, n)\} \). Note that the household will take any jobs available, whenever \( w > b/u_c \). The FOC condition for the household requires that \( u_c = \beta E\{[1 + r(S')] u'_c\} \), where \( u_c \) is the marginal utility of consumption.

The value of an additional worker for this household depends on the particular plant that the marginal worker is attached to, and this is given by the derivative of (4) with respect to \( n \), evaluated at whichever plant \( k \) the worker is at. We denote this value, normalized by the marginal utility of consumption to measure it in terms of current goods, by \( V(S, k, n, a) \). Given that having a job today has no bearing

\(^{11}\)This specification underscores the fact that there are no search frictions, since the fraction \( \frac{1 - n}{1 - N(S)} \) is an extreme case of a matching function with no curvature in the number of unemployed individuals.
on whether the worker is employed tomorrow, we have that

$$V(S, k, n, a) = w(S, k, a, n) - (b/u)$$  \quad (6)

### 3.1.2 Firms and Investment Decisions

The value of a plant of capacity $k$ attached to a type $\{a, n\}$ worker is

$$\Pi(S, k, a, n) = e^z k^\alpha - w(S, k, a, n) + (1 - \delta) E \{ R[G(S)] \Pi(G(S), k, G^a(S), N') \}$$  \quad (7)

where we define $R(S) = [1 + r(S)]^{-1}$. Note that the assumption that the plant can costlessly change workers implies that the assets and employment level of tomorrow’s worker are those that will prevail in the economy. Let $G^a(S)$ be the assets and $N' = (1 - \delta) N + Q(S)$ the employment of the representative household tomorrow when the economy is in state $S$. Note that firms may in principle have to assess household’s savings to make decisions, but once a plant is installed, the firm makes no further decisions.

The value of marginally increasing the plant size $k$ is given by

$$\Pi_k(S, k, a, n) = e^z \alpha k^{\alpha - 1} - w_k(S, k, a, n) + (1 - \delta) E \{ R[G(S)] \Pi_k(G(S), k, G^a(S), N') \}$$  \quad (8)

This expression is useful to calculate the optimal installation size, which solves the following problem

$$\max_k -k + E\{ R[G(S)] \Pi(G(S), k, G^a(S), N') \}. \quad (9)$$

where the solution $k^*(S)$ satisfies the first order condition

$$1 = E\{ R[G(S)] \Pi_k(G(S), k^*(S), G^a(S), N') \}$$  \quad (10)

Note that in this economy the fact that the size of the plant cannot be changed forever implies that there isn’t an envelope type condition that eliminates the last term of (10). This is underscored by the presences of $k^*(S)$ in the expected future derivative of $\Pi_k$, which is forward looking and depends on the entire future realizations of the aggregate shock $z$.

On the other hand, the number of plants $Q^*(S)$ that are installed is determined by a zero profit
condition:
\[ E \{ R(S') \Pi(G(S), k^*(S), G^\alpha(S), (1 - \delta)N + Q^*(S)) \} = k^*(S). \]  

Equations (10) and (11) are two equilibrium equations that determine the number and size of entering plants, \( Q^*(S) \) and \( k^*(S) \) respectively (remember that \( Q^*(S) \) also determines aggregate employment). Furthermore, these two variables determine overall capital investment in the current period, \( Q(S)k^*(S) \).

3.2 Wage Determination

We use Nash bargaining between the firm and the household to determine real wages. There is only one worker per plant so the value of the particular worker for the firm is only output minus the wage since the worker can be substituted with no cost. By the same token, the value of the job for the worker’s household is just the wage net of the leisure value of not working in terms of goods \( b/u_c \). This again is due to the fact that the worker loses her job at the end of the period which cancels out the option value of being unemployed in the bargaining problem below.

The plant-specific wage solves the following problem
\[ w(S, k, a, n) = \arg\max_\omega (e^z k^\alpha - \omega)^{1-\mu} (\omega - b/u_c)^\mu \]  
where the bargaining weight of the worker is given by \( \mu \). Taking the FOC and rearranging terms, we get
\[ w(S, k, a, n) = \mu e^z k^\alpha + (1 - \mu)b/u_c \]  

Given this wage function, plant specific profits each period are given by
\[ \pi(S, k, a, n) = (1 - \mu)(e^z k^\alpha - b/u_c) \]  
hence, if \( e^z k^\alpha - b/u_c > 0 \) for all points in \((S, k, a, n)\), the firm would always want to maintain the plant in operation. For the moment, we will assume that this is the case and then show that this is true under our benchmark parameterization of the model. This is not hard to achieve, due to the assumption of no scrap value of installed capital.
3.3 The simplified problems

Instead of dealing with the infinite dimensional state space of this problem (the size distribution of installed plants), here we will guess and verify that under the assumptions laid down until here, the model can be solved using only three state variables: the aggregate technology shock, aggregate employment and average installed capacity. We define this latter variable as total output when productivity is at a long run average ($\bar{z} = 1$) and denote it by

$$\bar{Y} = \int_{0}^{\infty} i^{\alpha} \mathcal{X}(i) (di)$$

(15)

where $\mathcal{X}(k)$ is the aggregate measure of plants in the economy that are smaller than $k$.\(^\text{12}\) If every standing plant is operated until exogenous breakdown, $\bar{Y}$ can be defined recursively

$$\bar{Y}' = (1 - \delta)\bar{Y} + Q^*(S) k^*(S)^{\alpha}$$

(16)

as well as aggregate employment

$$N' = (1 - \delta)N + Q^*(S)$$

(17)

This comes from the fact that the only thing that modifies the distribution of installed plants (hence employment) from period to period is the aggregate breakdown of plants (jobs) at the exogenous rate $\delta$. Notice that total output is defined simply as $Y = e^{\bar{z}} \bar{Y}$.

Since $z$, $\bar{Y}$ and $N$ can be defined recursively, a model containing only these variables as states can be solved at minimal computational cost by means of approximation methods. Using this simplified state space, we have that $Q^*(S) = Q^*(z, \bar{Y}, N)$ and $k^*(S) = k^*(z, \bar{Y}, N)$, so $\bar{Y}'$, $Y$ and $Y'$ are also functions of $z$, $\bar{Y}$ and $N$ only. Given the aggregate feasibility constraint, total consumption (consumption of the representative household) is

$$C = Y(z, \bar{Y}, N) - Q(z, \bar{Y}, N) k^*(z, \bar{Y}, N)$$

(18)

so $C = C(z, \bar{Y}, N)$. It follows the rate of return of the economy depends also on the reduced state

\(^{12}\)This is a specialized version of the measure $x(S)$ presented before.
space because of the first order condition for the representative household\footnote{The representative household has the average wealth of the economy and the average employment rate among its members. We are abusing notation in order to simplify the state space from $(z, Y, N, Y, N)$ to $(z, Y, N)$ for this household.}

\begin{equation}
    u_c[C] = \beta E \{ u_c[C'] (1 + r') \mid z \}
\end{equation}

which implicitly defines $r = r(z, Y, N)$.

Given the above, we can compute total labor income for the household using the wage function in equation (13)

\begin{equation}
    W = \int_0^\infty \left[ \mu e^z k \alpha + (1 - \mu) \left( b/u_c \right) \right] \mathcal{X}(i)(di) = \mu Y + N \left( 1 - \mu \right) \left( b/u_c \right)
\end{equation}

The second term of this last expression comes from the fact that all individuals inside the household are restricted to enjoy the same value of leisure and that workers provide a fixed amount of labor to the plant where they work. Hence, given the same outside option for all workers, the second element in the aggregate wage bill is simply the sum of a constant value $(b/u_c)$ across $N$ workers. From this discussion, it is clear that the wage bill depends on the reduced state space, i.e., $W = W(z, Y, N)$ and that the individual wage for a plant of size $k$, matched with an individual of the representative household is $w = w(z, Y, N)$.

At this point, we have showed that the aggregate state space composed by $(z, Y, N)$ is sufficient to determine the prices ($r$ and $W$) that the household needs to know in order to solve its problem. It follows that the general solution to the problem of the household is given by functions $C(z, Y, N)$ and $a'(z, Y, N)$ (again, for the representative household) and that the optimal plant size and the optimal number of plants to install are given by $k^*(z, Y, N)$ and $Q^*(z, Y, N)$.

### 3.4 Equilibrium

Below we define equilibrium for the economy.

**Definition 1** A recursive equilibrium consists of functional equations for a plant-specific wage $w(z, Y, N, k)$ the rate of return, $R(z, Y, N)$, consumption of the household $C(z, Y, N)$, productive capacity $Y'(z, Y, N)$,
new plant creation $Q^* (z, Y, N)$, new plant size $k^* (z, Y, N)$ and a plant-specific profit function $\Pi (z, Y, N, k)$, such that\footnote{We drop function dependencies on $(z, Y, N)$ to ease exposition.}

i) The current wage equation is defined by:

$$w(z, Y, N, k) = \mu e^z k^\alpha + (1 - \mu) \left( \frac{b}{u_c} \right)$$  \hspace{1cm} (21)$$

ii) Households optimize:

$$u_c [C] = \beta E \{ R' u_c [C'] \} $$  \hspace{1cm} (22)$$

ii) Firms optimize, i.e., profits, optimal current plant size and optimal current number of plants to install are respectively given by:

$$\Pi (z, Y, N, k) = e^z k^\alpha - w(z, Y, N, k) + (1 - \delta) E \left\{ R' \Pi (z', Y', N', k) \right\}$$  \hspace{1cm} (23)$$

$$1 = E \left\{ R' \Pi_k \left( z', Y', N', k^* (z, Y, N) \right) \right\} $$  \hspace{1cm} (24)$$

$$k^* (z, Y, N) = E \left\{ R' \Pi \left( z', Y', N', k^* (z, Y, N) \right) \right\} $$  \hspace{1cm} (25)$$

iii) There is market clearing and consistency in the evolution of aggregates:

$$C = e^z Y - k^* Q^* $$  \hspace{1cm} (26)$$

$$Y' = (1 - \delta) Y + (k^*)^\alpha Q^* $$  \hspace{1cm} (27)$$

$$N' = (1 - \delta) N + Q^* $$  \hspace{1cm} (28)$$

Note that using this definition, the wage bill can be calculated simply using equation (20) and labor share by calculating the ratio between the wage bill and total output.

4 Calibration

The model period is a quarter. We use log utility for consumption and parameterize $b$ as a constant. The rest of the model relies on a fairly small number of parameters: the discount factor $\beta$, the rate of plant destruction $\delta$, the coefficient of capital intensity in the production menu $\alpha$ and the bargaining power of workers $\mu$. On the other hand, we use the standard AR(1) specification for the aggregate
shock,
\[ z' = \rho z + \epsilon' \]  
(29)
with \( \rho \in (0, 1) \) and \( \epsilon \sim iid(0, \sigma) \). We pick \( \{\beta, \delta, \alpha, b\} \) to match long run averages of consumption-output ratio (75%), capital-output ratio (9.24\(^{15}\)), labor share (68%) and unemployment (5%). Also, we add as a target the contemporaneous effect of the technology shock on real wages in order to parameterize \( \mu \). We measure this contemporaneous effect as the first element in the calculated impulse response function of the real wage with respect to a technology shock after estimating a VAR(1) between model-simulated data for wages and technology. This amounts to a 0.34% deviation from the steady state of real wages.\(^{16}\).

To calibrate the aggregate shock, we pick values of \( \rho \) and \( \sigma \) to match the autocorrelation coefficient of the measured technological shock (0.95) and the first element in the impulse response function of output to technology shocks.

Table 1 presents the parameter values of the baseline calibration. As in Hagedorn and Manovskii (2008), the value of the worker’s bargaining power is low (\( \mu = 0.12 \)) significantly different from the usual symmetric calibration used in much of the literature. The rest of the parameters fall in line with previous studies.

<table>
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<th>( \mu )</th>
<th>( \beta )</th>
<th>( \delta )</th>
<th>( \alpha )</th>
<th>( b )</th>
<th>( \rho )</th>
<th>( \sigma )</th>
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<td>0.3718</td>
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Table 1: Parameter values, baseline model

5 Results

Given the parameterization of the model, we solve its simplified version using a first order Taylor approximation of the forward looking equilibrium conditions and compute statistics from simulated data.\(^{17}\) After retrieving the optimal policy functions, we simulate the model for 10100 periods and discard the first hundred observations to avoid influence from initial conditions. Then, we calculate

\(^{15}\)This represents an annual capital-output ratio of 2.81, value computed using the same data as in Ríos-Rull and Santeulalia-Llopis (2010)
\(^{16}\)A similar calibration strategy is used in Hagedorn and Manovskii (2008)
\(^{17}\)We solved the model using Dynare v. 4.1.1 alongside Matlab. We checked robustness of our simulations both to higher order approximations and to changes in the size of the simulated sample and found negligible changes in results.
the implied technology shock residual as in section 2 and the impulse response functions between this variable and the rest of the components of labor share. Results from the baseline model are in figure 4. As seen from the impulse response functions in the figure, the model is able to replicate the over-shooting of the labor share, as well as the positive and persistent responses in the rest of its components, namely output, total hours and real wages. Quantitatively, the responses in our baseline are overstated with respect to US data in the case of output and understated for total hours. The response of real wages falls in line with the data, since the simulated response lays entirely inside the empirical confidence bands (the same is true for the labor share).

### 5.1 Labor Search and Matching

As means of comparison, we also simulate and compute the responses of a standard search and matching framework in general equilibrium. In particular, we follow the model in Andolfatto (1996) and Cheron and Langot (2004). See appendix B for a description of that model and its calibration. We further consider two versions of the model, differing only in the assumed aggregate technology used: Cobb-Douglas versus CES.
We present the results of this exercise in figures 5 and ??, where we can see that the cyclical response of the labor share is non existent, while the response in total hours is significantly lower than in our baseline. As discussed in Choi and Rios-Rull (2009), different calibrations of the standard model are able to produce higher responses in total hours but the non-responsiveness of the labor share is pervasive.

5.2 Sticky Wages?

[ TO BE COMPLETED ]

6 Conclusions

In this paper we develop a theory on the joint dynamics of labor share and technology innovations at the business cycle frequency. The target of the theory is the response of Labor Share to total factor productivity (or output) innovations: negative contemporaneous correlation followed by a strong hump shaped response. This overshooting of the labor share uncovers positive responses of both aggregate real wages and labor input during an upturn, thus providing an alternative way of disciplining how we
model the mechanisms through which technology innovations affect labor market volatility.

In the backdrop of the standard Real Business Cycle framework, our proposed theory incorporates putty-clay technology (production takes place through individual plants which, once installed, cannot change size), decentralized non-competitive wage setting for worker-firm pairs (akin to bilateral Nash bargaining) and an aggregate technological shock that is biased toward newer hires. These modeling choices make new workers more productive than the average labor force member but also hard to substitute with capital in the short run, producing the desired joint dynamics between technology innovations, real wages and aggregate hours.

Further, we have also showed that in our model, rigid wages are not needed to produce high job creation during cycles, a common prescription for similar models with frictions in the labor market.
References


Appendix

A Data

We compute the labor share as in Rios-Rull and Santaeulalia-Llopis (2010), using the assumption that the proportion of ambiguous labor income to total ambiguous income is equal to the proportion of unambiguous labor income to total unambiguous income.\(^{18}\) Our calculations span the period between the first quarter of 1954 and the fourth quarter of 2004. For the rest of our analysis we use the following series:

1. total hours is comprised by employment \((L)\) and hours per worker \((H)\). Both series are taken from the Bureau of Labor Statistics: series ID CES0000000081 for employment and series CES0500000082 for average weekly hours. We normalize employment by the size of the population \((POP)\)^{19} while weekly hours are divided by 120, i.e., the number of total hours in a 5 day working cycle (week). Finally, we define total hours simply as \(N_t = \left( \frac{L_t}{POP_t} \right) \left( \frac{h_t}{120} \right)\) where \(t\) is the corresponding quarter.

2. For Gross Domestic Product \((Y_t)\), we use the data in table 1.1.6 from the Bureau of Economic Analysis (BEA), National Economic Accounts and normalize this output series by the size of the population.

3. Our measure of capital \((K_t)\) is taken directly from Rios-Rull and Santaeulalia-Llopis (2010), who use data from BEA (chain-type quantity index from table 1.2 and the current cost net stock in 2000 from table 1.1) to construct an annual capital series, which is transformed to quarterly frequency by means of interpolation \((K)\). We normalize this series also by the size of the population.

4. Our real wage index is constructed using our constructed Labor Share \(\zeta_t\) and the variables described above: \(W_t = \zeta_t \frac{Y_t}{N_t}\)

\(^{18}\)One example is proprietor’s income in national accounts, which is not clear whether it belongs to labor or capital

\(^{19}\)Series CNP16OV of FRED II (http://research.stlouisfed.org/fred2). The frequency of the original series is monthly, so we take end of quarter numbers as our measure of population.
B Standard Search and Matching Model

We use the model put forward by Andolfatto (1996), which is a general equilibrium version of the standard search and matching framework. The model is described by perfect insurance inside the household (so employed and unemployed workers consume the same) and search and matching frictions in the labor market. Households derive utility from consumption and the leisure enjoyed by its members, with per period utility given by \( \log(C) + (1 - N)b \), where \( C \) is consumption, \( N \) is the measure of household members who are unemployed while \( b \) is a parameter related to the extra amount of leisure enjoyed by the unemployed. Households decide how much to consume and save every period. Firms rent capital \( K \) and search for workers by posting vacancies \( V \), at flow cost \( c_v \). Matchings in the economy are given by the matching function \( m(V, 1 - N) = \omega V^\psi (1 - N)^{1-\psi} \). Real wages are the solution to bilateral Nash bargaining between the firm and the household, with the bargaining weight of the worker being \( \mu \). We consider two alternative versions of the model: one with Cobb-Douglas technology and one with CES technology. Calibration of the parameters in the Cobb-Douglas technology is standard, while for the CES, we pick its parameters in order to match steady state labor share (0.68) and the response of labor share to a technology shock.

The following equations characterize the equilibrium of this model, while its parameters are in table 2. The calibration follows Cheron and Langot (2004) very closely, except for the employment target and the parameterization of \( \mu \) (the bargaining parameter for workers): for the earlier, we target steady state employment of 0.95 while their target is 0.56. We do this to compare model predictions to the literature started by Shimer (2005), Hall (2005), Hagedorn and Manovskii (2008), among others who use this figure and because of issues raised by Costain and Reiter (2008), who argue that a low target for employment increases artificially model responses in job creation; to calibrate \( \mu \), we follow the same approach as in the putty clay model (target the contemporary response of real wages to a technology shock).
Aggregate consistency and evolution of states:

\[ Y = e^z F(K, N) \]
\[ Y = I + C + c_v V \]
\[ N' = (1 - \chi)N + \omega V^\psi (1 - N)^{1-\psi} \]
\[ K' = (1 - \delta)K + I \]
\[ z' = rz + \epsilon' \]

Job Filling Rate, \( m/V \):

\[ \Phi = \omega V^{\psi-1} (1 - N)^{1-\psi} \]

Euler equation:

\[ 1 = \beta E \left[ \frac{C}{C'} \left( 1 - \delta + \frac{\partial Y'}{\partial K'} \right) \right] \]

Optimal vacancy posting decision:

\[ \frac{c_v}{\Phi} = \beta E \left[ \frac{C}{C'} \left( \frac{\partial Y'}{\partial N'} - w' + (1 - \chi) \frac{c_v}{\Phi'} \right) \right] \]

Nash bargaining for the wage bill:

\[ w = \mu \left[ \frac{\partial Y'}{\partial N} + c_v \frac{V}{1 - N} \right] + (1 - \mu)Cb \]

Technology:

\[ F = K^\alpha N^{1-\alpha} \quad \text{(Cobb-Douglas case)} \]
\[ F = \left[ \alpha K^{-\nu} + (1 - \alpha)N^{-\nu} \right]^{-\frac{1}{\nu}} \quad \text{(CES case)} \]
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Table 2: Parameter values for the standard search and matching model.