Housing and Liquidity*

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Abstract

We study economies where houses, in addition to providing utility as shelter, may also facilitate credit transactions, since home equity can be used as collateral. We document there were big increases in home-equity-backed consumption loans coinciding with the start of the house price boom, and suggest an explanation. When it can be used as collateral, housing can bear a liquidity premium. Since liquidity is endogenous, even when fundamentals are constant and agents fully rational house prices can display complicated equilibrium paths resembling bubbles. Our framework is tractable, with exogenous or with endogenous supply, yet captures several salient features of housing markets. The effects of monetary policy are also discussed.

JEL Classification: E44, G21, R21, R31

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1 Introduction

We study economies in which housing plays two roles. First, houses provide utility, either directly as durable consumption goods or indirectly as inputs into household production. Second, houses are assets that can facilitate transactions when credit markets are imperfect. In the presence of limited commitment/enforcement, it can be difficult to get unsecured consumer loans, and this generates a role for home equity as collateral. This implies equilibrium house prices can bear a liquidity premium, since people are willing to pay more than the fundamental price, as defined below, because home ownership provides security in the event that one needs a loan. Given this, we show how equilibrium house prices can display a variety of interesting dynamic paths, some of which look like bubbles. Intuitively, liquidity is at least to some extent a self-fulfilling prophecy, which means that the price of a liquid asset is to some extent a matter of beliefs. In this sense houses are similar, although not the same as, money – e.g., both ameliorate trading frictions, but houses generate direct utility and can be produced by the private sector.

Our goal is to make these ideas precise and study their implications. This seems interesting for several reasons, primarily because it is consistent with experience since the turn of the millennium. It is commonly heard that there was a bubble in house prices during this period, which eventually burst, leading to all kinds of economic problems. And it has been noted that there was over the period a huge increase in home equity loans. Reinhart and Rogoff (2009) contend developments in financial markets allowed consumers "to turn their previously illiquid housing assets into ATM machines."[1] Ferguson (2008) also says this "allowed borrowers to treat their homes as cash machines," and reports that between 1997 and 2006 "US consumers withdrew an estimated $9 trillion in cash from the equity in their homes." Mian and Sufi (2011), estimate that homeowners extracted 25 cents for every dollar increase in home equity, and these loans added $1.25 trillion to household debt between

[1] The financial development they have in mind is *securitization.* As Holmström and Tirole (2011) put it, "In the runup to the subprime crisis, securitization of mortgages played a major role. ... Securitization, by making [previously] nontradable mortgages tradable, led to a dramatic growth in the US volume of mortgages, home equity loans, and mortgage-backed securities in 2000 to 2008, partly in response to increased globabl demand for savings instruments."

2002 and 2008. These authors also emphasize that the borrowed funds were used by households for consumption, rather than, e.g., paying off credit card debt or purchasing financial assets.[2]

Figure 1 shows some data for the US over the relevant period (exact data definitions and sources are given below; all figures are at the end of the paper). Housing prices are deflated in two ways. One divides by the CPI to correct for the purely nominal impact of inflation. The other divides by an index of rental rates, to correct for inflation plus changes in the demand for shelter relative to other goods and services, yielding the inverse of the rent-price ratio. Even before we define terms precisely, this illustrates what people have in mind when they talk about a bubble: a dramatic run up followed by collapse in housing prices. Also shown are measures of home equity loans, this time normalized in three ways. The first again uses the CPI, showing a huge increase in real home equity loans over the period. The second normalizes by all bank loans, and still shows a big increase, to establish that the increase in home equity loans is not merely part of an overall increase in lending. The third normalizes by all real estate loans, to make it clear that the increase in home equity loans is not an artifact of an increase in the value of real estate generally – e.g., it is not merely the case that housing prices, mortgage loans and home equity loans all go up by the same factor. Finally, we show a measure of housing investment, normalized by GDP, to get an idea about what was happening to supply.[3]

The message we take away from all this is: coinciding with the start of the boom in house prices, there is a large increase in the real value of home equity loans and a moderate increase in

[2] Mian and Sufi (2011) also find that home equity loans were used more by younger households and those with lower credit scores, consistent with our approach. In related work, Greenspan and Kennedy (2007) find that home equity withdrawal financed about 3% of personal consumption from 2001 to 2005, and Disney and Gathergood (2011) find one-fifth of the recent growth in household indebtedness can be explained by rising house prices. Also related is the evidence reported in Ferraris and Watanabe (2008) that in 2004, 47.9 percent of the US households had home-secured debt (and they point out that real estate is also used to collateralize commercial and industrial bank loans accounting for 46.9 percent of the total value in the US, but the emphasis here is on consumption loans).

[3] Definitions of the series used in to construct Figure 1 are as follows: Home Prices uses the FHFA Purchase Only price index. To turn it into a real variable, it is divided by the CPI, or by the BLS Rent index, with these real series then normalized to 1 in 1991. Loan data are from the Federal Reserve and are for all commercial banks in the US. Home Equity Loans are similarly divided by nominal GDP, by Bank Loans and Leases, and by Real Estate Loans, with the resulting three series normalized to 0.3 in 1991. Bank Loans and Leases includes Commercial and Industrial Loans, Real Estate Loans, Consumer Loans and Other Loans and Leases. Real Estate Loans contains all loans secured by real estate, including Revolving Home Equity Loans, Closed-end Residential Loans and Commercial Real Estate Loans. Finally, the Residential Fixed Investment Index is from BEA; it is divided by real GDP and then normalized to 0.7 in 1991.

housing investment; later, prices fall, while home equity borrowing stays high and investment drops. This suggests to us that the role of home equity as collateral is potentially important for trying to understand the episode. If one considers a house only as a durable consumption good, with its value determined by the utility it provides, the ratio of rent (which should measure the utility flow) to the house price (the value of owning) should be roughly the sum of the discount and depreciation rates. There can be other costs and benefits of owning, including tax implications, but while these may affect the level of the rent-price ratio, as long as they are approximately constant, this should not generate the time series in Figure 1.[4] Our position, following the authors quoted above, is that financial development led to a bigger role for home equity in the credit market, this fueled an increase in the housing demand, and that led to an increase in price in the short run and quantity in the long run. We show that once one takes account of the liquidity role for home equity, it is not hard to generate equilibria consistent with this position.

As we said, many people seem to think that the data indicate bubbles, although it is not always obvious what they mean. As Case and Shiller (2003) put it, "The term 'bubble' is widely used but rarely clearly defined. We believe that in its widespread use the term refers to a situation in which excessive public expectations of future price increases cause prices to be temporarily elevated." Shiller (2011) more recently says "In my view, bubbles are social epidemics, fostered by a sort of interpersonal contagion. A bubble forms when the contagion rate goes up for ideas that support a bubble. But contagion rates depend on patterns of thinking, which are difficult to judge." Phenomena like "excessive public expectations, social epidemics, and interpersonal contagion" are nothing if not bewitching, but here we emphasize a more pedestrian idea of liquidity, and a bubble is simply an asset price different from its fundamental value, given by the present value of holding the asset (this is made precise below). This is consistent with, e.g., Stiglitz (1990), who says that "if the reason that the price is high today is only because investors believe that the selling price is high tomorrow

[4] Others have considered the data discussed above. Harding, Rosenthal, and Sirmans (2007), e.g., estimate the depreciation rate on houses to be around 2.5 percent, so if the discount rate is around 3 percent, the rent-price ratio should be around 5. In Campbell, Davis, Gallin, and Martin (2009), from 1975 to 1995, this ratio is indeed around 5, but then declines to around 3.7 percent in 2007. This is very much consistent with our general position.

– when ‘fundamental’ factors do not seem to justify such a price – then a bubble exists.” We show that house prices can differ from fundamental values due to a liquidity role for home equity.

In emphasizing credit frictions, we follow a large literature summarized in Gertler and Kiyotaki (2010) and Holmström and Tirole (2011), as well as a related literature surveyed in Nosal and Rocheteau (2011) and Williamson and Wright (2010a,b) that endeavors to be relatively explicit about the process of exchange by going into detail concerning how agents trade (bilateral, multilateral, intermediated etc.), using which instruments (barter, money, credit etc.) and at what terms (price taking, posting, bargaining etc.). A direct antecedent to our approach is the body of work emanating from Kiyotaki and Moore (1997, 2005). In terms of technical details, the model is similar to Rocheteau and Wright (2005, 2010), although differences arise because we study housing as opposed to some generic asset. To illustrate, a typical asset generates a dividend stream that enters your budget equation; housing does this, too, but additionally enters your utility function, and this matters for results. Thus, it can change the circumstances under which bubbles exist from one of low supply to one of either low or high supply depending on preferences. Also, it means welfare might decrease with an exogenous increase in the stock of houses, which does not happen with financial assets.

Our approach is related to a body of work on bubbles and liquidity, in general, which is too large to survey here, so we refer readers to Farhi and Tirole (2011) for references. Again, there are several interesting differences between houses and generic assets that come up in our analysis. In terms of research on housing markets, there are several other papers that also try to take seriously the precautionary or collateral function of home equity. A technical difference from some of this work is that we focus on fully rational agents, with homogenous beliefs, and indeed we can generate bubble-like equilibria under perfect foresight. We can also do this with an endogenous supply of housing, which seems relevant since it has been suggested by, e.g., Shiller (2011), that “The housing-price boom of the 2000’s was little more than a construction-supply bottleneck, an inability to satisfy investment demand fast enough, and was (or in some places will be) eliminated with massive

increases in supply" (see also Glaeser, Gyourko, and Saiz (2008)). The housing literature more generally is sufficiently voluminous that, at the risk of neglecting some relevant contributions, we can only cite a few example that influenced our thinking on the issues.[5]

We do highlight a recent paper by Liu, Wang, and Zha (2011) that independently develops a complementary model. They also assume real estate can be used as collateral, but by producers, as in the usual application of Kiyotaki and Moore (1997, 2005), not by consumers, They present calibration results, while we emphasize more theoretical results, in part because for their parameter values equilibrium exhibits saddle-path stability. If one knew for sure that their exact specification and parameters were correct, one might argue that in the empirically relevant case exotic dynamics of the type we highlight cannot happen. We think this would be hasty, and in any case one should want to know how models behave, more generally, and to see just what it takes to generate bubbles in housing markets, in particular. We also mention work by Miao and Wang (2011) and Liu and Wang (2011) that discuss bubbles or cycles in economies with credit constraints, but again, on firms and not on households. We are interested in applying theories of credit frictions not to producers using generic capital goods to finance investment, but to households using home equity to finance consumption.

The rest of the paper is organized as follows. Section 2 lays out the basic environment. Section 3 discusses steady state equilibrium. Section 4 discusses dynamics, presenting explicit examples to show how bubble-like outcomes may arise. Section 5 endogenizes the supply of housing, discusses the multiplicity of equilibria, and presents an example that looks somewhat like recent experience, in terms of price, quantity and the use of home equity loans. Section 6 presents a monetary version

[5] Carroll, Dynan, and Krane (2003) study the impact of a precautionary (against job loss) motive on the demand for housing, and find evidence for it at moderate and higher income. Hurst and Stafford (2004) find that unemployment shocks and low asset positions increase households' likelihood of using home-equity loans. Campbell and Hercowitz (2005) study a growth model where demand for housing increases over time, and existing houses alleviate borrowing constraints. Arce and Lopez-Salido (2011) study a life-cycle model where some agents hold houses purely as a saving vehicle. Other work emphasizing the role of asset shortages includes Caballero and Krishnamurthy (2006) and Fostel and Geanakoplos (2008). Brady and Stimel (2011) find that household's response to house price shocks has shifted since 1998, consistent with our timing assumptions. Aruoba, Davis, and Wright (2011) study the interaction between housing markets and inflation, as we do below, and give citations to previous work along these lines. Other recent work on housing market dynamics includes Burnside, Eichenbaum, and Rebelo (2011), Coulson and Fisher (2009), Ngai and Tenreyro (2009) , Novy-Marx (2009), Piazzesi and Schneider (2009b,a) and Jaccard (2011).

of the model to study interplay between housing and inflation. Section 7 concludes. To close these introductory remarks, we emphasize that this paper is *not* about imperfect housing markets: houses are traded in frictionless markets, the way other forms of capital are traded in standard growth theory. This is not because we think imperfect housing markets are uninteresting, but because we want to focus on other issues, especially the role of home equity in imperfect credit markets.[6]

2 The Basic Environment

Each period in discrete time agents interact in two distinct markets. First, they participate a decentralized market, labeled DM, with explicit frictions detailed below; then they trade in a frictionless centralized market, labeled CM. At each date t, in addition to labor ℓ_t, there are two nonstorable consumption goods x_t and y_t, plus housing h_t. We assume ℓ_t, x_t and h_t are traded in the CM, while y_t is traded in the DM. The utility of a household is given by

$$\lim_{T\to\infty} \mathbb{E} \sum_{t=0}^{T} \beta^t \left[\mathcal{U}(x_t, y_t, h_t) - A\ell_t\right], \tag{1}$$

where $\beta \in (0, 1$.[7] Much tractability comes from quasi-linear utility, although this can be replaced with indivisible labor à la Rogerson (1987), which also generates unemployment (Rocheteau, Rupert, Shell, and Wright (2008)). To ease the presentation, we assume $\mathcal{U}(x_t, y_t, h_t) = U(x_t, h_t) + u(y_t)$, where $U(\cdot)$ and $u(\cdot)$ satisfy the usual assumptions, including $u(0) = 0$.

For now there is a fixed stock of housing H. In terms of CM goods, ℓ_t can be converted one-for-one into x_t (the framework is easily extended to more general production functions). In terms of DM goods, some agents can produce y_t using a technology summarized by the cost function $v(y_t)$. In many related models, households produce for each other in the DM, and $v(y_t)$ is interpreted as a direct disutility; in other models, DM producers are retail firms. Although it does not matter much for the results, in this paper, we follow the latter approach, with households buying y_t from DM

[6] Housing market models with search go back to Wheaton (1990). See Albrecht, Anderson, Smith, and Vroman (2007) or Head, Lloyd-Ellis, and Sun (2010) for more recent examples and citations to the literature.

[7] We assume here the limit in (1) exists; if not, one can use more advanced optimization techniques (see the discussion and citations in Rocheteau and Wright (2010)).

retailers. Our retail technology works as follows: by investing at $t-1$ a fixed amount, normalized to 1, of the CM numeraire x_{t-1}, a retailer can at t convert it into any amount $y_t \leq 1$ of the DM good and some amount $x_t = F(1-y_t)$ of the CM good. The profit from this activity, conditional on selling y_t in the DM at t for revenue R_t, measured in period t numeraire, is $R_t + F(1-y_t) - (1+r)$, given the initial investment at $t-1$ is repaid in the CM at t at an interest rate $1+r = 1/\beta$.

Not all retailers earn the same payoff, since not all trade, in the DM. Let α_f be the probability a retailer trades, often interpreted as the probability of meeting a household, in the DM, and symmetrically let α_h be the probability a household trades. Also, assume $y \leq 1$ is not binding, as must be the case, e.g., if $F'(0) = \infty$. Then expected profit is

$$\begin{aligned} \Pi_t &= \alpha_f \left[R_t + F(1-y_t)\right] + (1-\alpha_f)F(1) - (1+r) \\ &= \alpha_f \left[R_t - v(y_t)\right] + F(1) - (1+r), \end{aligned} \tag{2}$$

where $v(y_t) \equiv F(1) - F(1-y_t)$ as the opportunity cost of selling y_t in the DM. In general, if there is a $[0,1]$ continuum of households and a $[0,N]$ continuum of retail firms, the trading probabilities can be endogenized by $\alpha_f = \alpha(n)/n$ and $\alpha_h = \alpha(n)$, where $\alpha(\cdot)$ comes from a standard matching technology and $n \leq N$ is the measure of firms in the DM. Firms may have to pay a DM participation cost, in addition to their initial investment in goods, and n can be determined by the usual free-entry condition. To make our main points, however, we can assume the cost is small and $1+r < F(1)$, so that $n = N$, and α_h and α_f are fixed constants.

It is not necessary to invoke search or matching, although it is common in these types of models.[8] An alternative story that is equivalent for our purposes is that households sometimes realize a demand for y_t due to preference or opportunity shocks. Nice examples include the possibility that one has an occasion to throw a party, or an opportunity to buy a boat at a good price; not-so-nice examples include the possibility that one has an emergency medical breakdown, or one's boat does. In any

[8] In terms of the literatures discussed in the Introduction, it is common to have search, and with it bargaining, in the papers surveyed by Nosal and Rocheteau (2011) or Williamson and Wright (2010a,b), but not thoses surveyed by Gertler and Kiyotaki (2010) or Holmström and Tirole (2011). A side product of this project is to help integrate the two approaches – i.e., we provide a search-and-bargaining version of Kiyotaki and Moore (1997, 2005).

case, the probability of such an event is α_h. Then we can assume that the DM clears in various ways, and could involve bilateral or multilateral matching. We consider various options, but to simplify notation we always assume the same number of agents trade on each side of the market, so that $\alpha_h = N\alpha_f$. More significantly, in the DM, households use *credit*, since they have nothing to offer by way of quid pro quo: y_t is acquired in exchange for a debt obligation d_t to be retired in the next CM, where one-period debt is imposed without loss of generality.

Credit is limited, however, by lack of commitment/enforcement: households are free to default, albeit possibly at the risk of some punishment. At one extreme, punishment can be so severe that credit is effectively perfect. At the other extreme is no punishment, not even exclusion from future credit as in Kehoe and Levine (2001) or Alvarez and Jermann (2000), say, because borrowers are anonymous. This would completely rule out unsecured credit, and generate a role for collateral. In general, we use a debt limit $d \leq D = D(e_t)$ with $e_t = \psi_t h_t$, where ψ_t is the price of h_t in terms of x_t. We focus on the linear case $D(e_t) = D_0 + D_1 e_t$ with $D_0, D_1 \geq 0$. If D_0 is big, the limit never binds, and unsecured lending works well; if D_0 is small, however, the limit binds with insufficient home equity. A natural specification is $D(e_t) = e_t$, but it also makes sense to consider $D(e_t) < e_t$ since, when if debtors renege we can seize only part of their assets (e.g., we get the house but they run off with the appliances), or if some value is lost due to litigation and other costs. And $D(e_t) > e_t$ is interesting if we have punishments beyond confiscating collateral. In any case, we take $D(e_t)$ to be exogenous, but it would be interesting to endogenize it, along the lines of Kehoe and Levine (2001) or Alvarez and Jermann (2000).

Let $W_t(d_t, h_t)$ be a household's value function entering the CM at t, with debt d_t and house h_t brought in from $t-1$. Since d_t is paid off each period in the CM, households start debt free in next period's DM, where $V_{t+1}(h_{t+1})$ is the value function. The CM problem is

$$W_t(d_t, h_t) = \max_{x_t, \ell_t, h_{t+1}} \{U(x_t, h_t) - A\ell_t + \beta V_{t+1}(h_{t+1})\} \tag{3}$$

$$\text{st } x_t + \psi_t h_{t+1} = \ell_t + \psi_t h_t + T_t - d_t \text{ and } \ell_t \in [0, \bar{\ell}]$$

where T_t is other wealth, including government transfers, returns from investments etc., but since wealth does not affect anything except leisure, with our quasi-linear utility function, this is all left implicit.[9]

Before proceeding, we take a brief digression into the theory of household production. According to this theory, households value two types of goods: those acquired on the market x_t^1, and those produced in the home x_t^2. They also engage in two types of labor: work in the market ℓ_t^1, and work in the home ℓ_t^1. Let $x_t^2 = G_t\left(\ell_t^2, h_t\right)$ be the household production function, mapping home time and home capital – i.e., housing, broadly defined to include home furnishings, appliances etc. – into home goods. Then the generalized version of (3) is

$$W_t(d_t, h_t) = \max_{x_t^1, x_t^2, \ell_t^1, \ell_t^2, h_{t+1}} \left\{ U(x_t^1, x_t^2) - A_1 \ell_t^1 - A_2 \ell_t^2 + \beta V_{t+1}\left(h_{t+1}\right) \right\} \tag{4}$$
$$\text{st } x_t^1 + \psi_t h_{t+1} = \ell_t^1 + \psi_t h_t + T_t - d_t,\ x_t^2 = G_t\left(\ell_t^2, h_t\right) \text{ and } \ell_t^1 + \ell_t^2 \in [0, \bar{\ell}].$$

In this specification h_t does not enter $U(\cdot)$ directly, but indirectly as an input to $G_t(\cdot)$, although one could generalize this, by disaggregating home capital into a part that produces goods or services (say, stoves and vacuums) and a part that generates utility directly (say, televisions and beds).

It is well known that one can substitute out $x_t^2 = G_t\left(\ell_t^2, h_t\right)$ and maximize out ℓ_t^2, for any given values of x_t^1, ℓ_t^1 and h_{t+1}, to derive a reduced-form model that only depends on market variables, as in our baseline specification. Still, in general, there are several reasons for being explicit about household production, especially for quantitative work. In the interests of space, however, for the purposes of this paper we proceed using as a benchmark model the version where home production is not made explicit. We merely wanted to make the point that our general approach is consistent with the more detailed work on home production in both the micro and macro literatures (see Gronau (1997) and Greenwood, Rogerson, and Wright (1995) for overviews of these literatures; see Aruoba, Davis, and Wright (2011) for a recent application with references to other more recent papers). We now return to the baseline specification, without an explicit home production sector.

[9] This requires that wealth other than home equity cannot be used to collaterize loans, perhaps because it is hard to seize; in the language of Holmström and Tirole (2011), h_t is and $T_t - d_t$ is not *pledgeable*.

Assuming $\ell_t \in [0, \bar{\ell}]$ does not bind, we eliminate ℓ_t using the budget equation to write

$$W_t(d_t, h_t) = \psi_t h_t + T_t - d_t + \max_{x_t} \{U(x_t, h_t) - x_t\} + \max_{h_{t+1}} \{\beta V_{t+1}(h_{t+1}) - \psi_t h_{t+1}\}, \tag{5}$$

where we have normalized the utility parameter $A = 1$, without loss of generality. Immediately (5) implies that choices at t, and in particular h_{t+1}, are independent of (d_t, h_t), which simplifies the analysis because we do not have to keep track of distributions across agents.[10] The FOC for the maximization in (5) are

$$U_1(x_t, h_t) = 1 \text{ and } \psi_t = \beta \frac{\partial V_{t+1}}{\partial h_{t+1}}. \tag{6}$$

Also, we have

$$\frac{\partial W_t}{\partial d_t} = -1 \text{ and } \frac{\partial W_t}{\partial h_t} = U_2(x_t, h_t) + \psi_t, \tag{7}$$

so that, in particular, W is linear in debt (more generally, in net worth).

We now describe what happens in the DM in a trading opportunity, which we recall arises with probability α_h for a household and α_f for a retail firm each period. In such an opportunity, firms (or sellers) produce, while households (or buyers) consume y_t, in return for which the latter issue a promise of payment $d_t \leq D(e_t)$ in the CM. The terms of trade (y_t, d_t) can be determined in many ways in this type of model, as discussed in the surveys mentioned in the Introduction. We begin by describing competitive Walrasian pricing, where, to motivate this, we can assume agents with opportunities to trade meet in large groups, rather than bilaterally, in the DM. First, note that buyers' trade surplus from (y_t, d_t) is $S_{bt} = u(y_t) + W(d_t, h_t) - W(h_t) = u(y_t) - d_t$, since $W(\cdot)$ is linear in d_t by (7). Similarly, for sellers, $S_{st} = d_t - v(y_t)$. Then sellers and buyers solve

$$\begin{aligned} (y_t, d_t) &= \arg\max S_{st} \text{ st } d_t = p_t y_t \\ (y_t, d_t) &= \arg\max S_{bt} \text{ st } d_t = p_t y_t \leq D_t, \end{aligned}$$

taking as given the price p_t and the the debt limit D_t.

[10] These results follow obviously if the constraint $\ell_t \in [0, \bar{\ell}]$ is slack. More generally, people with very low or high net worth may be unable to set ℓ_t high or low enough to settle all debt or get to their preferred h_{t+1} in a given period, and, in this case, they would need to borrow or save between CM meetings using frictionless (mortgage) credit. But if we start with a distribution of h_t and d_t that is not too disperse, relative to $[0, \bar{\ell}]$, households can settle all their debt and choose the same housing in each CM without borrowing or lending.

To solve this model, as a preliminary step, first note that equilibrium without the debt constraint is given by $y_t = y^*$, where $u'(y^*) = v'(y^*)$, and $p_t = p^* = v'(y^*)$. Define $d^* = p^* y^* = v'(y^*)\, y^*$. If $d^* \leq D_t$ then the DM equilibrium at t is what we found ignoring the constraint. But if $d^* > D_t$ then the DM equilibrium at t is instead given by $p_t = v'(y_t)$, from the sellers' FOC, and $y_t = D_t/p_t$, from the buyers' constraint. In other words, when $d^* > D_t$ we get a constrained equilibrium with $d_t = D_t$, and y_t is the solution to $v'(y_t)\, y_t = D_t$. For future reference, define $g(y_t) = v'(y_t)\, y_t$, so that when $d^* > D_t$ the equilibrium output can be written $y_t = g^{-1}(D_t) < y^*$. Summarizing:

Proposition 1 *Let y^* and $p^* = v'(y^*)$ be the equilibrium ignoring the constraint $d_t \leq D_t$, and let $d^* = g(y^*)$. Then, as shown in Figure 2, equilibrium in the DM at t is given by:*

$$y_t = \begin{cases} g^{-1}(D_t) & \text{if } D_t < d^* \\ y^* & \text{if } D_t \geq d^* \end{cases} \quad \text{and } d_t = \begin{cases} D_t & \text{if } D_t < d^* \\ d^* & \text{if } D_t \geq d^* \end{cases} \tag{8}$$

As an alternative trading mechanism, suppose we pair off buyers and sellers and have them bargain bilaterally. One approach is to use the generalized Nash solution

$$(y_t, d_t) = \arg\max S_{bt}^{\theta} S_{st}^{1-\theta} \text{ st } d_t \leq D_t.$$

One show that the outcome is the same as (8), except that instead of $g(y) = v'(y)\, y$, we redefine

$$g(y) = \frac{\theta v(y)\, u'(y) + (1-\theta)\, u(y)\, v'(y)}{\theta u'(y) + (1-\theta)\, v'(y)} \tag{9}$$

(see Lagos and Wright (2005)). In particular, with Nash we have $d^* = g(y^*) = \theta v(y^*) + (1-\theta)\, u(y^*)$, while with Walras we have $d^* = v'(y^*)\, y^*$, but in either case Proposition 1 holds exactly as stated. Alternatively, one can use the proportional bargaining solution in Kalai (1977),

$$(y_t, d_t) = \arg\max S_{bt} \text{ st } S_{bt} = \theta\, [u(y_t) - v(y_t)] \text{ and } d_t \leq D_t,$$

which has some advantages in this class of models (see Aruoba, Rocheteau, and Waller (2007)). Kalai gives the same outcome as Nash iff the constraint is slack, but in any case, the outcome still satisfies (8), except now we redefine

$$g(y) = \theta v(y) + (1-\theta)\, u(y). \tag{10}$$

As a final example, consider the following extensive-form bargaining game:[11]

Stage 1: The seller offers (y_t, d_t).

Stage 2: The buyer responds by accepting or rejecting, where:

- accept implies trade at these terms;
- reject implies they go to stage 3.

Stage 3: Nature moves (a coin toss) with the property that:

- with probability θ, the buyer makes a take-it-or-leave-it offer;
- with probability $1-\theta$, the seller makes a take-it-or-leave-it offer.

Any offer must satisfy $d_t \leq D_t$. In Appendix A we show that there is a unique SPE, characterized by acceptance of the initial Stage 1 offer, given by

$$(y_t, d_t) = \arg\max S_{st} \text{ st } S_{bt} = \theta \left[u(\bar{y}_t) - \bar{d}_t \right] \text{ and } d \leq D_t, \tag{11}$$

where $\left(\bar{y}_t, \bar{d}_t\right)$ is the offer a buyer would make if (off the equilibrium path) we were to reach Stage 3.

For each of the above trading mechanisms, an in principle many others, the equilibrium outcome is given by (8) for a particular choice of $g(\cdot)$. Then the DM value function can be written

$$V_t(h_t) = W(0, h_t) + \alpha_h \left\{ u\left[y(\psi_t h_t) \right] - d(\psi_t h_t) \right\}, \tag{12}$$

where it is understood that $y(\psi_t h_t)$ and $d(\psi_t h_t)$ are given by (8) with $D_t = D\left(\psi_t h_t\right)$. Using (7), we have

$$\frac{\partial V_t}{\partial h_t} = U_2(x_t, h_t) + \psi_t + \alpha_h \psi_t \left[u'\left(y\right) y'(\psi_t h_t) - d'(\psi_t h_t) \right].$$

Differentiating y and d using (8), we reduce this to

$$\frac{\partial V_t}{\partial h_t} = U_2(x_t, h_t) + \psi_t + \alpha_h \psi_t D_1 \mathcal{L}\left[y\left(\psi_t h_t\right) \right], \tag{13}$$

[11] Wong and Wright (2011) proved a recent discussion of why one might want to use an explicit noncooperative game, rather than some axiomatic solution, in dynamic search-and-bargaining models, especially with nonlinear utility, along with a discussion of the particular game used here, and references to the literature.

where to reduce notation we use

$$\mathcal{L}(y) \equiv \begin{cases} u'(y)/g'(y) - 1 & \text{if } y < y^* \\ 0 & \text{otherwise} \end{cases}. \tag{14}$$

The function $\mathcal{L}(y)$, which represents a *liquidity premium*, is common in the papers in the surveys mentioned in the Introduction, and can be shown to equal the Lagrangian multiplier on the credit constraint. Inserting these results into the FOC for h_{t+1} in (6), and using $1 + r = 1/\beta$, we get the Euler equation for housing

$$(1+r)\psi_t = U_2(x_{t+1}, h_{t+1}) + \psi_{t+1} + \alpha_h \psi_{t+1} D_1 \mathcal{L}\left[y\left(\psi_t h_t\right)\right]. \tag{15}$$

The three terms on the RHS describe the three benefits of owing a bigger house: 1) it yields more utility; 2) it makes you wealthier; and 3) it allows you to borrow more if necessary. Setting $h_t = H$, and using the FOC $U_1(x_t, H) = 1$ to define $x = X(H)$, (15) defines a difference equation in the price of housing, $\psi_t = \Psi\left(\psi_{t+1}\right)$. An *equilibrium* is any sequence $\{\psi_t\}$ solving $\psi_t = \Psi\left(\psi_{t+1}\right)$ that is nonnegative and bounded, boundedness being required to satisfy a standard transversality condition (see, e.g., the discussion in Rocheteau and Wright (2010)). Given $\{\psi_t\}$, we can recover the other variables, $e_t = \psi_t H$, $D_t = D\left(e_t\right)$, $y_t = y\left(e_t\right)$ etc.

3 Steady State Equilibrium

A stationary equilibrium, which here is equivalent to a steady state, is a solution to $\psi = \Psi\left(\psi\right)$. In steady state there are no capital gains, since $\psi_{t+1} = \psi_t$, and (15) becomes

$$r\psi = U_2\left[X(h), h\right] + \alpha_h \psi D_1 \mathcal{L}\left[y\left(e\right)\right]. \tag{16}$$

One can interpret this as the long-run demand for housing. The slope is

$$\frac{\partial h}{\partial \psi} = \frac{U_{11}\left(U_2 - \alpha_h \psi^2 h \mathcal{L}' y'\right)}{\left(U_{11}U_{22} - U_{12}^2 + U_{11}\alpha_h \psi^2 \mathcal{L}' y'\right)\psi} < 0, \tag{17}$$

after inserting $X'(h) = -U_{12}/U_{11}$ and using (15) to eliminate r. In this paper we assume $\mathcal{L}'\left(y\right) < 0$, which immediately implies that demand slopes downward, but it is important to emphasize that we

do *not* actually need this assumption, as one can show all of the same results without $\mathcal{L}'(y) < 0$ using the method in Wright (2010). Here we prefer to avoid these technicalities, especially since $\mathcal{L}'(y) < 0$ follows automatically for many of the mechanisms used here anyway, including Walras and Kalai, if not Nash, pricing.

The fact that demand is downward sloping means, given a fixed supply, steady state is unique. This highlights another big difference between housing and fiat currency: in any decent monetary model, whenever there exists a steady state where currency is valued there exists another where it is not. This is not true when money is replaced by home equity as a way to facilitate transactions. For the record we summarize this result as:

Proposition 2 *Given fixed supply H, steady state equilibrium is unique.*

If in steady state $e = \psi H > e^*$, then $\mathcal{L}(e) = 0$ and (16) implies $\psi = \psi^* \equiv U_2 [X(H), H] / r$, where ψ^* is *the fundamental price*, defined as the present value of the marginal utility of living in house H forever. In this case, households have enough home equity that they are never liquidity constrained, and houses bear no premium. But if $e < e^*$ then $\mathcal{L}(y) > 0$ and (16) implies $\psi > \psi^*$. In this case, home equity is scarce and houses bear a liquidity premium, which means price is above the fundamental value, which constitutes a *bubble*. It happens to be a stationary bubble, since for now we focus on steady state, but it is a bubble nonetheless. The idea behind the liquidity premium is this: if at the fundamental price $e = H\psi^* < e^*$, there will be excess demand, because agents not only get a utility flow from shelter, they also use it to collateralize loans. If the credit constraint binds, agents would pay a premium for assets that relax it. The exact outcome depends on the mechanism – e.g., Walras versus Kalai pricing – but a bubble emerges whenever $e < e^*$.

There are related results in similar models with different assets, including not only fiat currency, but Lucas trees and neoclassical capital, all of which can bear liquidity premia in some circumstances (Geromichalos, Licari, and Suárez-Lledó (2007); Lagos and Rocheteau (2008); Lester, Postlewaite, and Wright (2011)). The economics is different with houses. With trees, there is a liquidity premium

iff the exogenous supply is low; and with capital there is a liquidity premium iff the endogenous supply is low, but the endogenous supply without liquidity considerations is itself pinned down by productivity. With housing, there can be a liquidity premium iff e is low, but e can be low either when H is low or when H is high, depending on the elasticity of demand. In particular,

$$\frac{de}{dH} = \psi + H\frac{d\psi}{dH} = \frac{\psi H\left(U_{22}U_{11} - U_{21}^2\right) + \psi U_2 U_{11}}{U_{11}\left(U_2 - \alpha\psi^2 H\mathcal{L}'\right)} \simeq -H\left(U_{22}U_{11} - U_{21}^2\right) - U_2 U_{11},$$

where the notation $A \simeq B$ means that A and B have the same sign.

Figure 3 shows housing demand for the example

$$U(x,h) = \tilde{U}(x) + \frac{h^{1-\sigma}}{1-\sigma}.$$

When e can be used as collateral, demand is given by the solid curve; when e cannot be used in this way, say because α_h or D_1 is 0, it is given by the dotted curve. When the solid is above the dotted curve, e is scarce and housing bears a premium. For $\sigma < 1$ this happens when H is low, while for $\sigma > 1$ this happens when H is high. Perhaps less obviously, this means that welfare W might fall as H increases, which is typically not the case with financial assets in these models. If H increases here, even though CM utility must rise, if ψH falls DM utility might fall by enough to reduce total utility. We give an example in the next Section. For now, we summarize the results by:

Proposition 3 *Suppose $h = H$ is fixed. If $e > e^*$ we get the fundamental price $\psi^* = U_2[X(H), H]/r$; if $e < e^*$ we get a premium, or bubble, $\psi > \psi^*$. We can have $e < e^*$ and hence $\psi > \psi^*$ either when H is low or when H is high, depending on utility. It is possible to have $\partial W/\partial H < 0$.*

4 Dynamics: Cyclic, Chaotic and Stochastic Equilibria

Although we also discuss stochastic (sunspot) outcomes, we are mainly interested here in deterministic equilibria, given by nonnegative and bounded solutions to

$$(1+r)\psi_t = \Psi\left(\psi_{t+1}\right) = U_2[X(H), H] + \psi_{t+1} + \alpha_h \psi_{t+1} D_1 \mathcal{L}\left[y(\psi_{t+1}H)\right]. \tag{18}$$

The first observation is that any interesting dynamics must emerge from liquidity considerations, which show up in the nonlinear term $\mathcal{L}\left[y(\psi_{t+1}H)\right]$. To see this, set α_h or D_1 to 0. Then (18) is a linear difference equation, which can be rearranged as a mapping from the price today to the price tomorrow,

$$\psi_{t+1} = -U_2\left[X\left(H\right), H\right] + \left(1+r\right)\psi_t.$$

This has a unique steady state at the fundamental price $\psi = \psi^*$, which is also the unique equilibrium, since any solution to (18) other than $\psi_t = \psi^*\ \forall t$ is unbounded or becomes negative. There are no dynamics unless the liquidity motive is operative.[12]

When $\alpha_h > 0$, as long as $H\psi_{t+1} < e^*$ we have $\mathcal{L}\left[y(\psi_{t+1}H)\right] > 0$, and the nonlinear part of (18) kicks in. We analyze this in $\left(\psi_{t+1}, \psi_t\right)$ space, where it is natural to think of ψ_t as a function of ψ_{t+1}, because for any ψ_{t+1} there is a unique individual demand h_t given ψ_t, so market clearing pins down ψ_t. However, as usual, there can be multiple values of ψ_{t+1} for which this mapping yields the same ψ_t, so that the inverse $\psi_{t+1} = \Psi^{-1}\left(\psi_t\right)$ is a correspondence. Of course, Ψ and Ψ^{-1} cross on the 45^o line in $\left(\psi_{t+1}, \psi_t\right)$ space at the unique steady state. Textbook methods (e.g., Azariadis (1993)) tell us that whenever Ψ^{-1} and Ψ cross off the 45^o line there exists a cycle of period 2 – i.e., a solution $\left(\psi^1, \psi^2\right)$ to $\psi^2 = \Psi\left(\psi^1\right)$ and $\psi^1 = \Psi\left(\psi^2\right)$ that is nondegenerate in the sense that $\psi^1 \neq \psi^2$ – and that this happens whenever Ψ has a slope less than -1 on the 45^o line. In this 2-cycle equilibrium, even though fundamentals are constant, ψ oscillates between ψ^1 and ψ^2 as a self-fulfilling prophecy. This is a nonstationary bubble.

Before discussing the economic intuition, consider higher-order n-cycles – i.e., nondegenerate solutions to $\psi = \Psi^n\left(\psi\right)$. To reduce notation, normalize $H = 1$, and consider by way of example $v\left(y\right) = y$, $D\left(e\right) = e$, and[13]

$$U\left(x, h\right) = \tilde{U}\left(x\right) + \kappa\frac{h^{1-\sigma}}{1-\sigma} \text{ and } u\left(y\right) = \eta\frac{\left(y+\varepsilon\right)^{1-\gamma} - \varepsilon^{1-\gamma}}{1-\gamma}.$$

[12] This is a standard no-bubble result, versions of which can be found in many places. Heuristically, the only way to have $\psi_t > \psi^*$ is for agents to believe ψ_{t+1} will be even higher, by at least enough to make up for discounting, and this means $\psi_t \to \infty$. Such a belief is inconsistent with rationality because $\psi_t \to \infty$ is inconsistent with equilibrium.

[13] The form of $\tilde{U}$ is irrelevant for all the restults. The role of ε in $u\left(y\right)$ is simply to force $u\left(0\right) = 0$. Also, the value of σ is irrelevant, since it vanishes from $\partial U/\partial h$, given $h = H = 1$.

To show our results are robust to the choice of mechanism used to determine the terms of trade in the DM, we consider Walrasian pricing, Kalai bargaining and the extensive-form game in Section 2. These choices and the parameter values across examples are shown in Table 1.

	Example 1	Example 2	Example 3	Example 4	Example 5
Mechanism	Walras	Walras	Kalai	Game	Walras
α_h	0.5	0.5	0.9	0.5	0.5
β	0.8	0.6	0.6	0.8	0.9
θ	n/a	n/a	0.9	0.6	n/a
κ	0.125	0.3333	0.1	0.125	1
σ	n/a	n/a	n/a	n/a	7
η	1.5125	3.2479	0.5882	3.0368	1
γ	2	7	9	8	0.1
ε	0.1	0.1	0.5	0.1	0.01

Table 1: Parameter Values for the Examples

Example 1, with Walrasian pricing, is shown in Figure 4a, where the unique equilibrium is the steady state. Example 2, also with Walrasian pricing, but different parameters, is shown in Figure 4b, which exhibits a 2-cycle. In this case, the unconstrained value of y is $y^* = 1.0833$, the constraint binds iff $\psi < y^*$, and this happens in alternate periods. Figures 4c and 4d show similar results for Examples 3 and 4, using Kalai and strategic bargaining, to show the effects come not a particular pricing mechanism but from general liquidity considerations. It is not hard to generate higher order cycles. Example 2 also has a 3-cycle, with $\psi^1 = 0.8680 < y^*$, $\psi^2 = 1.5223 > y^*$, $\psi^3 = 1.1134 > y^*$. When a 3-cycle exists, by the Sarkovskii theorem and the Li-Yorke theorem (again see Azariadis (1993)), there exist cycles of all orders and chaotic dynamics. Chaos is defined in the context of our model as a nonnegative and bounded solution $\{\psi_t\}$ to (18) with the property that $\psi_s \neq \psi_t \ \forall s \neq t$. Figure 5 shows an equilibrium price path with chaotic behavior. Figure 6 is not about dynamics, but depicts steady state welfare W as a function of H, for Example 5, to illustrate the possibility that W may decrease over some range of H, as mentioned above.

A model that takes into account the collateral role of home equity can generate equilibria where house prices display cycles of arbitrary periodicity, even though fundamentals are stationary, and agents have perfect foresight. House prices can fluctuate simply as a self-fulfilling prophecy. This is

because houses have liquidity value in imperfect credit markets, and, as we said above, liquidity is at least partially a matter of beliefs. Heuristically, in a 2-cycle, if at t one believes ψ_{t+1} and hence e_{t+1} will be high, then liquidity will be plentiful at $t+1$, which lowers the price people are willing to pay for it today. Therefore low ψ_t is consistent with high ψ_{t+1}, and by the same logic high ψ_{t+1} is consistent with low ψ_{t+2}, and so on. One might wonder why rational agents are willing to pay a lot for an asset at $t+1$ when they know its price is about to drop at $t+2$? It is precisely *because* the price is about to drop that liquidity will soon become scarce, and thus liquidity is currently in high demand. A cycle of order $n > 2$, or a chaotic path, is a more complicated self-fulfilling prophecy, but in all these cases the model displays housing prices bubbles.

Additionally, following standard methods, we can construct sunspot equilibria, or stochastic cycles. In such an equilibrium, e.g., the price can be ψ^1 and jump to $\psi^2 > \psi^1$ with some probability λ^1 each period; then when it is ψ^2 it can fall back to ψ^1 with some probability λ^2 each period.[14] Again, agents are rational and all know the stochastic structure of the equilibrium – in this example, they know λ^1 and λ^2 – yet we can still have random price fluctuations when fundamentals are deterministic. In rational expectations equilibrium, demand for liquid assets and hence ψ^2 will be high not because the price is about to drop to ψ^1 for sure, as in a 2-cycle, but because it drops with some probability; still, the basic economic intuition is similar.

Of course this is all related to monetary theory, where it is well know how to generate similar exotic dynamics (again Azariadis (1993) is a textbook reference). But H is different from M. Any reasonable model of fiat currency has at least two steady states, one where M is valued and one where it is not, while our housing model has a unique steady state. Obviously, $\psi = 0$ cannot be an equilibrium when H has fundamental value as shelter. To give credit where credit is due, part of our motivation for this project came from an example of Kocherlakota, where something he called housing could exhibit somewhat interesting dynamics – not as interesting as described here, but at

[14] There are different methods for constructing sunspot equilibria. Azariadis and Guesnerie (1986) note that in the limit when $\lambda^1 = \lambda^2 = 1$ the sunspot equilibrium described in the text reduces to a 2-cycle, which exists under conditions described above, and appeal to continuity. This method applies in our model.

least an equilibrium where price jumps stochastically from $\psi > 0$ to an absorbing state with $\psi = 0$ (a special case of the sunspot equilibria discussed above, special because once ψ goes down it never comes back). But what he calls housing is in fact a fiat object, since he endows it with fundamental utility value 0, different from our model. Giving H intrinsic value not only rules out equilibria where $\psi = 0$, it also rules out equilibria where $\psi_t \to 0$, either stochastically or deterministically, leading us to claim that the dynamics here are not merely reinterpretations of standard results in monetary theory, as one might say of Kocherlakota's example.

Having said that, at least simple examples of equilibria with cycles, chaos or sunspots do not resemble recent housing market experience that well – prices tend to go up and down rather too regularly, and do not display the stereotypical bubble pattern of a prolonged boom followed by collapse. Perhaps in conjunction with slowly moving real changes to preferences or technology these effects might generate more realistic time series, and this may be worth exploring. In the next Section, instead, we generate equilibria that resemble better recent data by making H look less like M, not more, when we allow houses to be produced by the private sector.

5 Endogenous Housing Supply

Assume now that in the CM, in addition to the technology for converting ℓ_t into x_t, we introduce a technology for the construction of H, with cost function $x_t = c(\Delta h_t)$. Thus, Δh_t units of new housing require an input of $c(\Delta h_t)$ units of numeraire. Construction, like other CM activity, is perfectly competitive. Hence, profit maximization implies

$$\psi_t = c' \left[h_{t+1} - (1-\delta) h_t \right], \tag{19}$$

equating price to the marginal cost of augmenting the supply from $(1-\delta)h_t$ to h_{t+1}, where δ is the depreciation rate. The households' CM problem is unchanged, except now $e_t = \psi_t (1-\delta) h_t$, and so the Euler equation becomes

$$(1+r)\psi_t = U_2 \left[X(h_{t+1}), h_{t+1} \right] + \psi_{t+1}(1-\delta) + \alpha_h (1-\delta) \psi_{t+1} D_1 \mathcal{L} \left[y(e_{t+1}) \right]. \tag{20}$$

In steady state, (19)-(20) can be written

$$(r+\delta)\,\psi = U_2\left[X\left(h\right),h\right] + \alpha_h\left(1-\delta\right)\psi D_1 \mathcal{L}\left[y\left(e\right)\right] \tag{21}$$

$$\psi = c'\left(\delta h\right), \tag{22}$$

where (21) is the straightforward generalization of long-run demand (16), while (22) is a long-run supply relation. It is easy to show they intersect uniquely.[15] As in Section 4, we can get a liquidity premium when the (now endogenous) supply of housing is high or when it is low, depending on elasticities. Therefore, we have:

Proposition 4 *The results in Propositions 2-3 all hold with H endogenous.*

Moving beyond steady states, equilibrium generally is defined by a path for $\{h_t, \psi_t\}$ satisfying the dynamical system (19)-(20). One should anticipate the existence of interesting dynamics in this system, given the results in Section 4, even if the methods are somewhat different for the bivariate discrete-time system. We can imagine taking limits as the period length gets small to obtain a continuous-time version, or building a model from the ground up in continuous time, and analyzing the system with standard differential equation methods. We leave this as an exercise, and instead use the discrete-time version to help organize a particular narrative concerning recent events. As the story goes, at the start of the episode in question, financial innovation gave households easier access to home equity loans: this is what it means to say financial developments allowed households "to turn their previously illiquid housing assets into ATM machines." We now show that this can lead to dramatic increase in house prices followed by a crash.

Suppose at $t = 1$ the economy is at the unique steady state, where housing is hard to use as collateral, say $D\left(e\right) = D_1 e$ with D_1 small. To make the point stark, in the CM at $t = 2$, suppose an unexpected once-and-for-all financial innovation occurs that we formalize by increasing D_1. The

[15] Combine (21) and (22) to get

$$r+\delta = \frac{U_2\left[x\left(h\right),h\right]}{c'\left(\delta h\right)} + \alpha_h\left(1-\delta\right) D_1 \mathcal{L}\left[\left(1-\delta\right) c'\left(\delta h\right) h\right].$$

The RHS goes to ∞ as h goes to 0, and vice-versa, and it is stirctly decreasing

resulting transition path depends on parameters. For some parameter values, we get the analog of saddle-path stability: there is a unique equilibrium where the price ψ jumps with the change in D_1, and monotonically declines to its new steady state value as constructing raised the housing stock to its new steady state value. For other parameters, the system displays a classic indeterminacy associated with a stable steady state: in this case, there are many transition paths from the initial to the new steady state. This means we can let ψ jump after the change in D_1 to any value in some range before beginning the transition, giving us some freedom to pick a path that we find reminiscent of recent experience.

A particular transition is shown in Figure 7, constructed under Walrasian pricing, using parameters such that the constraint $y \leq D(e)$ is binding, and verifying numerically that both eigenvalues are real and less than 1 at the new steady state. The equilibrium path in Figure 7 looks like the data in Figure 1, starting around 2000, in the following sense. We do not mean they look *exactly* the same, which would be more than one should expect (primarily because the actual data were presumably generated by something other than a one-time surprise increase in D_1, given that it may take time for financial developments to evolve, for agents to understand and take advantage of them, etc.). All we mean is that the paths look qualitatively similar, in that housing prices first soar then tumble, whether we measure them by the relative price ψ_t or the price-rent ratio. Also, home equity loans go up, and stay up, as households take advantage of financial innovation. Construction also rises, then drops as we approach the new steady state. Note home equity loans rise quickly even though the housing stock increase takes time because $e_t = p_t h_t$ rises before h_t.

We conclude that the model can generate price dynamics with the characteristic boom and bust of a bubble. What is perhaps more subtle is that welfare increases over the period, as also shown in the Figure 7. Financial development, formalized as an increase in D_1, is good because it relaxes credit constraints, even though it can set us off on a path that resembles a bubble, complete with the ultimate collapse.[16]

[16] More work is warranted on the welfare implications of bubbles, but we think it is fair to say that both economists

6 Intermediated Collateralized Lending: Banks

In the model, so far, households put up home equity as collateral for consumption loans. This is consistent in a stylized way with experience, but in reality, more typically, households use home equity to borrow cash from a bank or related institution, then use the cash to buy consumption goods. Here we model this explicitly, not only for the sake of realism, but to investigate the interaction between monetary policy and housing.[17] We assume for the sake of this discussion that money is the only means of payment accepted in the DM due to the fact that buyers and sellers trade anonymously (as is standard in modern monetary economics; see the references in the Introduction). Intuitively, if a seller in the DM does not know the identity of individual buyers, and if this seller were to offer a buyer a consumption loan collateralized by equity in a house, the latter could put up a claim on a nonexistent house, or the house of a stranger, or one that is under water, etc. Nevertheless, buyers here have relationships with their bankers, who keep records and know their identities. Hence, bank loans are possible, while pure consumption loans from retailers are not.

To ease the presentation we fix the housing stock at H and set $\delta = 0$. Also, we are more explicit about preference/opportunity shocks. At the beginning of each period, with probability α_h a household wants to consume the DM good, and with probability $1 - \alpha_h$ does not. Conditional on wanting to consume, for simplicity, the household trades in the DM with probability 1. At the start of each period, before the DM opens, households not only have access to any cash brought in from the previous period, they can also access a financial market, FM. We describe FM in terms of intermediaries that we call banks and that work as follows. Households that want to consume in

and the general public seem to feel they must be bad. According to *Wikipedia*, e.g., "The impact of bubbles is debated within and between schools of economic thought; they are not generally considered beneficial, but it's debated how harmful their formation and bursting is." Yet rudimentary monetary economics shows that valued fiat money – a bubble – is beneficial. *Wikipedia* goes on to say, by the way, that "Another important aspect of economic bubbles is their impact on spending habits. Market participants with overvalued assets tend to spend more because they 'feel' richer (the wealth effect)." According to our theory, this is not quite right: here agents do indeed spend more, but because their liquidity constraints are relaxed, not because they 'feel' wealthier (although this may be merely semantic).

[17] Here we are motivated by Ferraris and Watanabe (2008) and Li and Li (2010), who also provide detailed models where real assets are used to secure money loans. Intuitively, in such models, money and other assets can be complements, while usually they are substitutes, in a Tobinesque kind of way, because they provide different ways to store wealth and facilitate trade (see, e.g., Lester, Postlewaite, and Wright (2011) for a recent discussion and more references).

the DM (borrowers) may withdraw cash from banks to increase their purchasing power, while those that do not (depositors) keep their money in the banks. The FM operates as a competitive market, with settlement in the CM, but we maintain the assumption of limited commitment: households can renege on bank loans, if they like, but then banks can seize their houses. It is not important, and cannot be determined, who carries money out of the CM into the next period (see below). Therefore, we assume all currency is put into banks at the end of each CM, and those that want to consume in the DM withdraw – generally, more than their deposits – while the rest leave it in the bank until the next CM.[18]

The generalized CM value function is $W_t(d_t, h_t, m_t)$, where a household's portfolio now consists of real debt d_t, housing h_t and money in the bank m_t. The FM value function next period is $J_{t+1}(h_{t+1}, m_{t+1})$, given that all debt is paid off in the CM, without loss of generality, as in the baseline model. The CM problem is

$$W_t(d_t, h_t, m_t) = \max_{x_t, \ell_t, h_{t+1}, m_{t+1}} \{U(x_t, h_t) - \ell_t + \beta J_{t+1}(h_{t+1}, m_{t+1})\} \tag{23}$$
$$\text{st } x_t + \psi_t h_{t+1} + \phi_t m_{t+1} = \ell_t + \psi_t h_t + T_t + \phi_t m_t - d_t$$

where ϕ_t is the value of a dollar in terms of the CM numeraire (the inverse of the nominal price level). As in the baseline model we can eliminate ℓ_t and derive the FOC

$$U_1(x_t, h_t) = 1,\ \psi_t = \beta \frac{\partial J_{t+1}}{\partial h_{t+1}} \text{ and } \phi_t = \beta \frac{\partial J_{t+1}}{\partial m_{t+1}}, \tag{24}$$

showing (x_t, h_{t+1}, m_{t+1}) is independent of (d_t, h_t, m_t), and as usual W_t is linear in wealth.

The FM value function satisfies

$$J_t(h_t, m_t) = \alpha_h \max_{\hat{m}_t} V_t[(1+\rho_t)(\hat{m}_t - m_t)\phi_t, h_t, \hat{m}_t] + (1-\alpha_h) W_t[-(1+\rho_t) m_t \phi_t, h_t, 0] \tag{25}$$
$$\text{st } (1+\rho_t)(\hat{m}_t - m_t)\phi_t \le D(\psi_t h_t),$$

[18] This setup is basically the same as the model of banking in Berentsen, Camera, and Waller (2007), except they assume households carry cash between periods, and all intermediaries do is reallocate it after the shocks are realized. We assume banks hold the cash between periods to highlight the similarity at a general level to the literature following Diamond and Dybvig (1983), where again the role of intermediation is to allocate liquidity. The key feature in our model is that home equity is required to secure bank loans. The reason it does not matter if one's cash is kept in one's pocket or one's bank account between periods is that there is no interest paid on idle balances across periods, only those kept in the bank between the FM and CM, while thee CM is open, since these can be used to make loans.

where ρ_t is the interest rate and $D(\psi_t h_t)$ the borrowing limit on bank loans. Thus, with probability α_h the household increases money balances from m_t to $\hat{m}_t$, takes it all to the DM, and has a real debt obligation in the next CM of $(1+\rho_t)(\hat{m}_t - m_t)\phi_t$. And with probability $1-\alpha_h$ the household leaves all money in the bank, skips the DM, and goes straight to the next CM with a negative real liability (i.e., a credit) $-(1+\rho_t)m_t\phi_t$. Notice that V_t is now the DM value function conditional on wanting to being able to consume, and it depends on debt and cash, as well as housing (in the baseline model there was no debt entering the DM and there was no cash). It satisfies

$$V_t(d_t, h_t, m_t) = u(y_t) - \phi_t \hat{m}_t + W_t(d_t, h_t, m_t) \tag{26}$$

where, following Proposition 1 in the baseline model, y_t is determined by some mechanism according to $g(y_t) = \phi_t \hat{m}_t$.

Equilibrium can be divided into two cases according to whether the FM borrowing constraint, $(1+\rho_t)(\hat{m}_t - m_t)\phi_t \le D(\psi_t h_t)$, is binding; and if it is binding, there are two subcases, as discussed below. In case 1, where it is slack, from (25) we get the FOC for $\hat{m}$

$$-(1+\rho_t)\phi_t + \frac{\partial V_t}{\partial \hat{m}_t} = 0, \tag{27}$$

using the fact that the derivative of V_t wrt debt is simply -1, since debt in the DM is simply carried over into the CM. Using (26), it is easy to show this reduces to $\mathcal{L}(y_t) = \rho_t$. This says the amount of cash one brings to the DM, inclusive of loans, sets the marginal benefit of liquidity $\mathcal{L}(y_t)$ to the marginal cost, which here is the loan rate ρ_t.

In terms of the choice of cash deposits coming out of the CM, as opposed to the choice coming out of the FM $\hat{m}$, the FOC for m_{t+1} in (24) reduces to the Euler equation

$$\phi_t = \beta\left(1+\rho_{t+1}\right)\phi_{t+1}. \tag{28}$$

To understand this, let i_t denote the nominal interest rate agents would require to do the following deal: give up a dollar in the CM at t and get back $1+i_t$ dollars in the CM at $t+1$.[19] The standard

[19] Note that we are *not* thinking of a tangible nominal bond here – the loan of a dollar is simply a book entry on some balance sheet, and cannot be traded in the DM. We refer to such a asset (this claim on a dollar) as illiquid.

Fisher equation says $1 + i_t = \phi_t/\beta\phi_{t+1}$, since $\phi_t/\phi_{t+1} = 1 + \pi_t$ is inflation and $1/\beta = 1 + r_t$ is the real interest rate. Hence (28) simply says $\rho_{t+1} = i$. This is equivalent to our earlier assertion that agents are indifferent about carrying cash deposits out of the CM, since they can always adjust $\hat{m}$ in the FM, depending on whether or not they want to consume in the DM. Similarly, the Euler equation for housing in case 1 is

$$(1 + r)\,\psi_t = \psi_{t+1} + U_2\left(x_{t+1}, h_{t+1}\right).$$

As in the baseline model, this says that housing must be priced fundamentally when the borrowing constraint is slack, the only difference being that now the borrowing constraint applies to cash loans in the FM rather than consumption loans in the DM.

We now turn to case 2, where the borrowing constraint is binding. Then households who want to consume in the DM borrow to their limit in the FM, and

$$J_t\left(h_t, m_t\right) = \alpha_h V_t\left[D\left(\psi_t h_t\right), h_t, m_t + \frac{D\left(\psi_t h_t\right)}{\left(1 + \rho_t\right)\phi_t}\right] + \left(1 - \alpha_h\right) W_t\left[-\left(1 + \rho_t\right)\phi_t m_t, h_t, 0\right]$$

In this case, the Euler equation for money is

$$\phi_t = \alpha_h\left[\mathcal{L}\left(y_{t+1}\right) + 1\right]\phi_{t+1} + \left(1 - \alpha_h\right)\rho_{t+1}\phi_{t+1}, \tag{29}$$

which is very similar to what one sees in related models along the lines of Berentsen, Camera, and Waller (2007). Intuitively, with probability α_h you desire DM goods, and an extra dollar in the FM relaxes your constraint, while with probability $1 - \alpha_h$ you do not desire DM goods, so you keep your money in the bank. The Euler equation for housing in this case is

$$(1 + r)\,\psi_t = U_2\left(x_{t+1}, h_{t+1}\right) + \psi_{t+1} + \frac{\alpha_h D_1 \psi_{t+1}}{1 + \rho_{t+1}}\left[\mathcal{L}\left(y_{t+1}\right) - \rho_{t+1}\right]. \tag{30}$$

This is similar to (15), except in the final term, the marginal value of liquidity depends upon $\mathcal{L}\left(y_{t+1}\right)$ (because more housing relaxes your borrowing constraint) minus ρ_{t+1} (because you have to pay it back).

We now distinguish two subcases. In case 2a there is no idle cash in the FM at $t+1$, in the sense that borrowing exhausts the deposits, so we need $\rho_{t+1} > 0$ to clear the market. This means

$$\alpha_h D\left(\psi_{t+1} h_{t+1}\right) = \left(1-\alpha_h\right) \phi_{t+1} m_{t+1},$$

which together with $\phi_{t+1} m_{t+1} = g\left(y_{t+1}\right)$ yields

$$g\left(y_{t+1}\right) = \frac{1}{1-\alpha_h} D\left(\psi_{t+1} h_{t+1}\right).$$

Second, in case 2b, there may be idle cash in the FM at $t+1$, in the sense that when all the borrowers borrow up to the constraint, total demand for loans does not exhaust the cash on deposit, which means the $\rho_{t+1} = 0$. In case 2b, the Euler equation for housing is

$$(1+r)\psi_t = \psi_{t+1} + U_2\left(x_{t+1}, h_{t+1}\right) + D_1 \psi_{t+1} i.$$

At this point we focus on steady state. To determine which case obtains, two conditions are relevant. One is the individual debt limit: can individuals in FM borrow as much as they want? If $D(e)$ is low, borrowers are constrained in the FM and housing bears a liquidity premium. The other relevant condition is the aggregate debt limit: are total deposits more than constrained borrowers can borrow? If so, there is idle cash, and $\rho = 0$. There are three possibilities: 1) The aggregate and individual debt limits both slack; 2a) the individual debt limit binds but the aggregate is slack; and 2b) both are binding (the other combination, where the individual constraint is slack and the aggregate binds, cannot happen). Since the outcome depends on $D\left(\psi_t h_t\right) = D_0 + D_1 \psi_t h_t$, naturally, we partition $\left(D_0, D_1\right)$ space into regions according to which case obtains.

To this end, let $\tilde{y}$ and $\bar{y}$ satisfy $\mathcal{L}(\tilde{y}) = i/\alpha_h$ and $\mathcal{L}(\bar{y}) = i$, and define

$$\begin{aligned}
B_1\left(D_0\right) &= \begin{cases} \dfrac{r\left[g(\tilde{y})\left(1-\alpha_h\right) - D_0\right]}{ig(\tilde{y})\left(1-\alpha_h\right) - iD_0 + HU_2} & \text{if } D_0 < g(\tilde{y})\left(1-\alpha_h\right) \\ 0 & \text{if } D_0 > g(\tilde{y})\left(1-\alpha_h\right) \end{cases} \\
B_2\left(D_0\right) &= \max\left\{\frac{r\left[g(\bar{y})\left(1-\alpha_h\right)(1+i) - D_0\right]}{HU_2}, 0\right\}.
\end{aligned}$$

As shown in Figure 8, $B_2 \geq B_1$, both are decreasing in D_0, and they partition $\left(D_0, D_1\right)$ space into 3 regions, each of which corresponds to one of the cases described above. For large D_0 and D_1, we get

case 1, where deposits are plentiful and borrowers are unconstrained. As D_0 and D_1 decrease, we move to case 2a, where all deposits are borrowed, and borrowers are constrained in the sense that they do not bring enough cash to the DM to buy y^*. In this case, $\rho \in (0, i)$, so deposits pay, and loans charge, a positive interest rate, but less than the rate on illiquid nominal assets, i. As D_0 and D_1 decrease further, we move to case 2b, where there are idle deposits and $\rho = 0$.

Suppose the central bank increases the nominal interest rate i.[20] What happens to house prices? In case 1, housing is priced fundamentally, households can borrow as they like given ρ in FM, and $\rho = i$ makes them indifferent between taking holding cash or deposits coming out of the CM. In this case, monetary policy does not affect real home values, since $\psi = \psi^*$ is the fundamental price. In case 2a, where $\rho \in (0, i)$, the results are ambiguous: real house prices may go up or down, depending on parameters, although they must fall under the condition given in the Proposition below (which is analogous to a condition in Li and Li (2010), although they only have financial assets). In case 2b, where $\rho = 0$, one can show real house prices rise. Intuitively, in this case a higher nominal interest-cum-inflation rate makes money less valuable, but because there are idle deposits ρ does not adjust. With the same amount of home equity, borrowers can borrow the same amount of real balances, so they want to hold in their portfolios more housing and less cash, leading to higher real house prices.

Details of the calculations are provided in Appendix B. Here, we summarize some key results as follows.

Proposition 5 ***Proposition 6*** *There is a unique steady state equilibrium, and it satisfies:*

1) if $B_2(D_0) < D_1$ *then* $\rho = i$, $\psi = U_2/r$ *and* $\partial\psi/\partial i = 0$.

2a) if $B_1(D_0) < D_1 < B_2(D_0)$ *then* $\rho \in (0, i)$ *and* $\partial\psi/\partial i$ *is ambiguous, although* $\partial\psi/\partial i < 0$ *if* $g(y)[\mathcal{L}(y) + 1]$ *is increasing in* y.

2b) if $B_1(D_0) > D_1$ *then* $\rho = 0$, $\psi = U_2/(r - iD_1)$ *and* $\partial\psi/\partial i > 0$.

[20] For this exercise it does not matter *how* they increase i – by Fisher Equation they can do so by increasing the inflation rate, and by the Quantity Equation they can do that by increasing the rate of monetary expansion M_{t+1}/M_t (either via lump sum transfers or purchases of x_t in the CM).

The main economic finding in terms of monetary policy is that higher nominal interest-cum-inflation rates have ambiguous effects on the housing market, but we provide precise conditions concerning how these effects depend on the tightness of home-equity borrowing constraints and other parameters. Other results can be derived – e.g., one can show that y decreases with inflation in all cases, one can study the model with an endogenous housing supply, one can consider optimal policy, and so on. We leave further exploration of this to future work, since the current project was more about nonlinear dynamics and housing bubbles. The purpose of this Section is mainly to show that once we have a liquidity-based theory of the housing market we can naturally analyze the effects of monetary policy in an interesting new light.

7 Conclusion

This goal of this project was to study economies where housing, in addition to providing utility as shelter, can be used as collateral to facilitate credit transactions. We showed that in equilibrium a house can bear a liquidity premium, and can hence be priced above its fundamental value (the discounted stream of marginal utility from living in it). Intuitively, this follows from the idea that liquidity is to some extent a self-fulfilling prophecy, as is evidently the case with fiat money. However, houses are also very different from fiat currency, because they can be produced by the private sector, and because they either generate direct utility or are inputs into household production, which means they can never have a price of 0, the way money can. In a sense, all we did is formalize some ideas discussed in the Introduction about financial developments allowing consumers to "treat their homes as cash machines," but we think such formalization is useful. We found that equilibrium house prices can display a variety of dynamic equilibrium paths, some of which look like bubbles, and some of which are qualitatively consistent with the data on prices, home equity loans and housing investment. Perhaps future work can investigate whether this kind of theory can do well quantitatively.

Appendix A

Here we solve the bargaining game in Section 2. The first observation is that if (off the equilibrium path) bargaining were to go to Stage 3 and the buyer gets to make the final take-it-or-leave-it offer, he would offer $(\bar{y}, \bar{d})$ where:

$$\bar{y} = \begin{cases} v^{-1}(D_t) & \text{if } D_t < v(y^*) \\ y^* & \text{if } D_t \geq v(y^*) \end{cases} \quad \text{and } \bar{d} = \begin{cases} D_t & \text{if } D_t < v(y^*) \\ v(y^*) & \text{if } D_t \geq v(y^*) \end{cases}$$

Now there are four possible cases: 1) the constraint $d \leq D$ is slack at the initial and the final offer stage; 2) it binds in the initial but not the final offer stage; 3) it binds in both; and 4) it binds in the final but not the initial offer stage. It is easy to check that case 4 cannot arise, so we are left with three.

Case 1: In the final offer stage, if the buyer proposes, his problem is

$$\max_{y,d} \{u(y) - d\} \text{ st } d = v(y),$$

with solution $y = y^*$ and $d = v(y^*)$. If the seller proposes the buyer gets no surplus, so the buyer's expected surplus before the coin flip is $\theta [u(y^*) - v(y^*)]$. Therefore, in the initial offer stage, the seller's problem is

$$\max_{y,d} \{d - v(y)\} \text{ st } u(y) - d = \theta [u(y^*) - v(y^*)],$$

with solution $y = y^*$ and $d = d^* = (1 - \theta) u(y^*) + \theta v(y^*)$. Since $d^* > v(y^*)$, this case occurs iff $D > d^*$.

Case 2: The buyer's expected payoff before the coin flip is again $\theta [u(y^*) - v(y^*)]$, but at the initial offer stage the constraint binds, so the seller solves

$$\max_{y} \{D - v(y)\} \text{ st } u(y) - D = \theta [u(y^*) - v(y^*)].$$

The solution satisfies $u(y) = D + \theta [u(y^*) - v(y^*)]$ and $d = D$. This case occurs iff $v(y^*) < D < d^*$.

Case 3: In the final offer stage, if the buyer proposes, his problem is

$$\max_{y} \{u(y) - D\} \text{ st } D = v(y).$$

This implies $y = v^{-1}(D)$, and his expected surplus before the coin flip is $\theta \left[u \circ v^{-1}(D) - D\right]$. At the initial offer stage, the seller's problem is

$$\max_{y} \{D - v(y)\} \text{ st } u(y) - D = \theta \left[u \circ v^{-1}(D) - D\right].$$

The solution satisfies $u(y) = \theta u \circ v^{-1}(D) + (1 - \theta) D$ and $d = D$. This case occurs iff $D < v(y^*)$ and $D < u(y^*) - \theta u \circ v^{-1}(D) + \theta D$, the last inequality coming from the observation that, at the

first stage, if the constraint is slack, the buyer pays $u(y^*) - \theta u \circ v^{-1}(D) + \theta D$ to get y^*. This last inequality is equivalent to $(1-\theta) D < u(y^*) - \theta u \circ v^{-1}(D)$, which always holds if $D < v(y^*)$.

To sum up, $d = D$ if $D < d^*$, and otherwise $d = d^*$; and y is given by

$$y = \begin{cases} u^{-1}\left[\theta u \circ v^{-1}(D) + (1-\theta) D\right] & \text{if } D < v(y^*) \\ u^{-1}\left[D + \theta\left[u(y^*) - v(y^*)\right]\right] & \text{if } v(y^*) < D < d^* \\ y^* & \text{if } D > d^* \end{cases}.$$

If we look at the derivative dy/dD, we have

$$\frac{dy}{dD} = \begin{cases} \frac{\theta u'\left[v^{-1}(D)\right] + (1-\theta) v'\left[v^{-1}(D)\right]}{u'(y) v'(v^{-1}(D))} & \text{if } D < v(y^*) \\ \frac{1}{u'(y)} & \text{if } v(y^*) < D < d^* \\ 0 & \text{if } D > d^* \end{cases}.$$

Thus we get $y = g^{-1}(D)$ as a differentiable and strictly increasing function of D for $D < d^*$. ■

Appendix B

Here we verify Proposition 6 by considering each case in turn.

Case 1: The borrowing constraint is not binding. In steady state, we have

$$\begin{aligned} i &= \mathcal{L}(y), \ \rho = i \\ r\psi &= U_2\left[X(H), H\right] \\ g(y) &< \frac{D_0 + D_1 \psi H}{(1+\rho)(1-\alpha_h)}. \end{aligned}$$

The last condition comes from two observations: when $\rho > 0$, to clear the market we must have $g(y) = \phi_t M_t / \alpha_h$, as borrowing exhausts deposits; and when the borrowing constraint is not binding, $(1-\alpha_h) \phi_t M_t < \alpha_h (D_0 + D_1 \psi H) / (1+\rho)$ .We can easily see that this equilibrium exists iff

$$g(\bar{y}) < \frac{D_0 + D_1 \psi H}{(1-\alpha_h)(1+\rho)}$$

with $\psi = U_2/r$, or $D_1 > g(\bar{y})(1+i)(1-\alpha_h) - D_0$. Uniqueness follows immediately. Furthermore, $\partial\psi/\partial i = 0$ and $\partial y/\partial i < 0$.

Case 2: The borrowing constraint is binding. In steady state,

$$i = \alpha_h \mathcal{L}(y) + (1-\alpha_h) \rho \tag{31}$$

$$r\psi = \alpha_h \left[\mathcal{L}(y) - \rho\right] \frac{\psi D_1}{1+\rho} + U_2\left[X(H), H\right] \tag{32}$$

$$g(y) = \phi_t M_t + \frac{D_0 + D_1 \psi H}{1+\rho}. \tag{33}$$

We now consider the subcases separately.

Case 2a: If $\rho > 0$, market clearing and a binding borrowing constraint imply

$$\phi_t M_t = \frac{\alpha_h \left(D_0 + D_1 \psi h\right)}{\left(1-\alpha_h\right)\left(1+\rho\right)}. \tag{34}$$

Using (31), we get $\rho = \left(i - \alpha_h \mathcal{L}\right) / \left(1-\alpha_h\right)$. This, (34) and (33) yield

$$\psi = \frac{g\left(y\right)\left[1+i-1-\alpha_h \mathcal{L}\left(y\right)\right] - D_0}{D_1 H}.$$

Substituting these into (32), we get

$$\frac{r}{D_1} = \frac{\alpha_h \mathcal{L}\left(y\right) - i}{1+i-\pi\left[1+\alpha_h \mathcal{L}\left(y\right)\right]} + \frac{HU_2\left[X\left(H\right),H\right]}{g\left(y\right)\left[1+i-\pi-\pi\alpha_h \mathcal{L}\left(y\right)\right] - D_0} = \Phi\left(y\right). \tag{35}$$

The RHS is decreasing in y, so there is at most one solution. Note in this subcase $\rho < \mathcal{L}\left(y\right)$, implying $0 < \rho < i$. This and (31) imply $\alpha_h i < \mathcal{L}\left(y\right) < i$. Consequently, this equilibrium exists iff (35) has a solution in $\left(\tilde{y}, \bar{y}\right)$, where $\mathcal{L}\left(\tilde{y}\right) = i/\alpha_h$ and $\mathcal{L}\left(\bar{y}\right) = i$. This requires $\Phi\left(\tilde{y}\right) > r/D_1$ and $\Phi\left(\bar{y}\right) < r/D_1$, or $B_1\left(D_0\right) < D_1 < B_2\left(D_0\right)$. One can derive

$$\begin{aligned}
\frac{\partial y}{\partial i} &\sim -\frac{gD_1\left(\mathcal{L}\left(y\right)+1\right)\alpha_h}{\psi\left(1+\rho\right)^2} - \frac{gU_h}{\left(1+\rho\right)\psi^2} < 0, \\
\frac{\partial \rho}{\partial i} &\sim -\frac{\alpha_h \mathcal{L}' D_1 g}{\left(1+\rho\right)\psi} + \frac{U_2\left(X\left(H\right),H\right)g'}{\psi^2} > 0, \\
\frac{\partial \psi}{\partial i} &\sim -D_1 \alpha_h \left[g\mathcal{L}' + g\left(\mathcal{L}\left(y\right)+1\right)\right] \sim -\frac{d}{dy}\left[\mathcal{L}\left(y\right)+1\right]g\left(y\right).
\end{aligned}$$

Therefore, if $\left[\mathcal{L}\left(y\right)+1\right]g\left(y\right)$ is increasing then $\partial\psi/\partial i < 0$.

Case 2b: If $\rho = 0$, steady state is characterized by

$$i/\alpha_h = \mathcal{L}\left(y\right) \tag{36}$$

$$r\psi = i\psi D_1 + U_2\left[X\left(H\right),H\right] \tag{37}$$

$$g\left(y\right) > \frac{D\left(\psi H\right)}{1-\alpha_h}. \tag{38}$$

In this subcase, (36) determines y and (37) determines ψ. This equilibrium exists iff (38) holds given y and ψ, which leads to $B_1\left(D_0\right) > D_1$. It is obvious in this case that $\partial y/\partial i < 0$ and $\partial \psi/\partial i > 0$. ■

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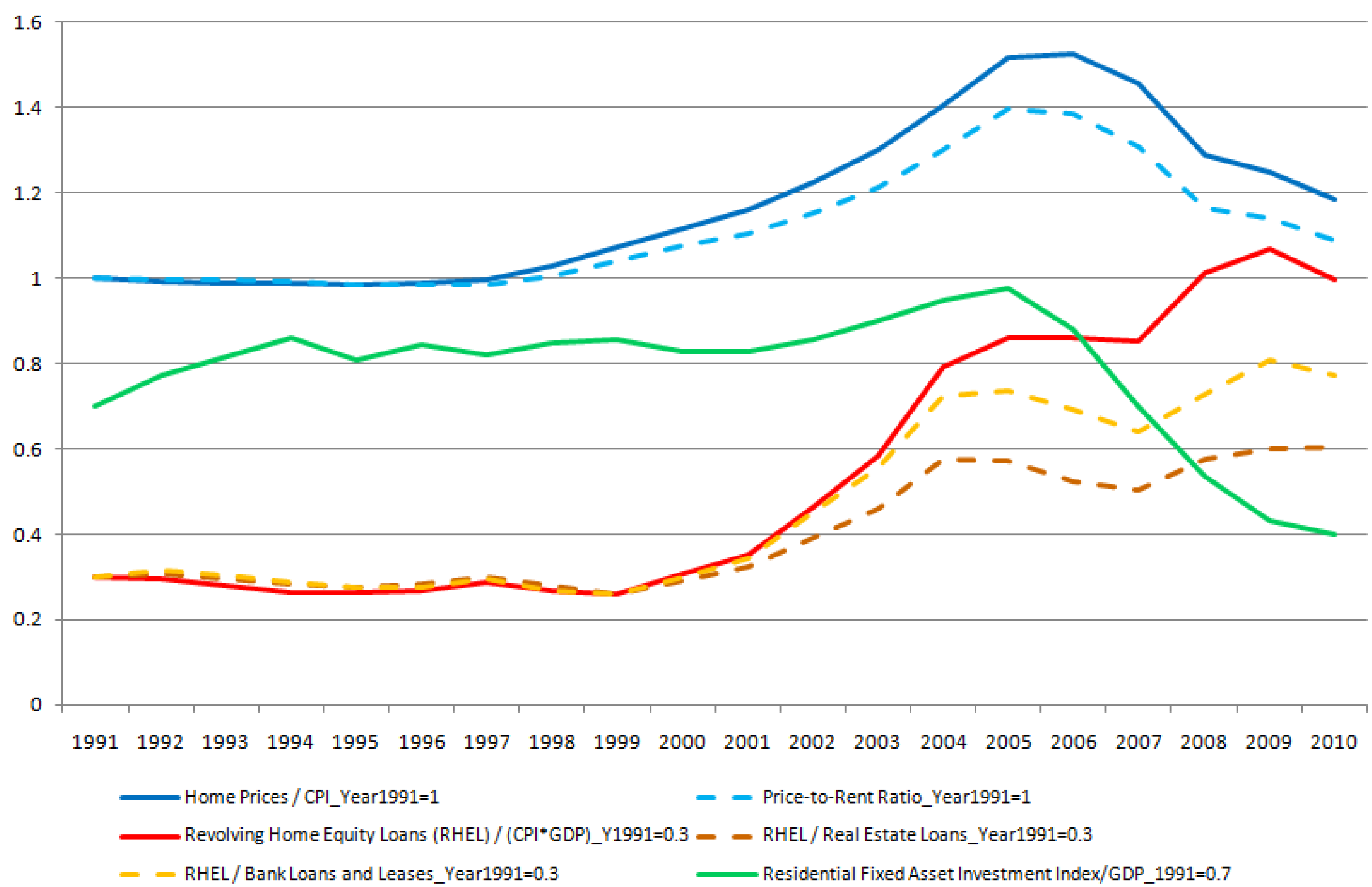


Figure 1: Housing Sector and Home Equity Premium

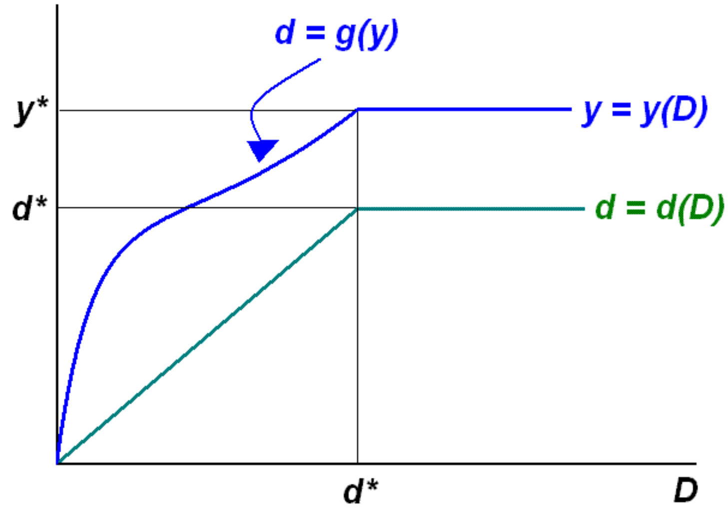


Figure 2: Trading Mechanism

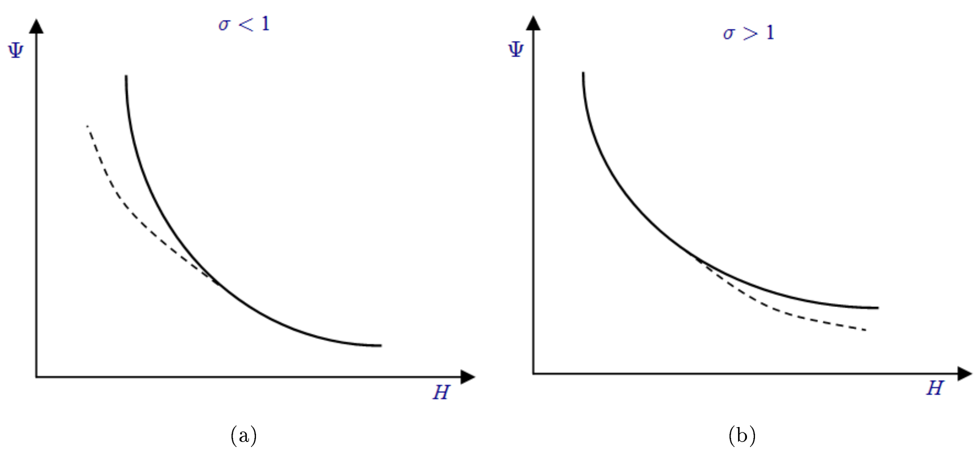
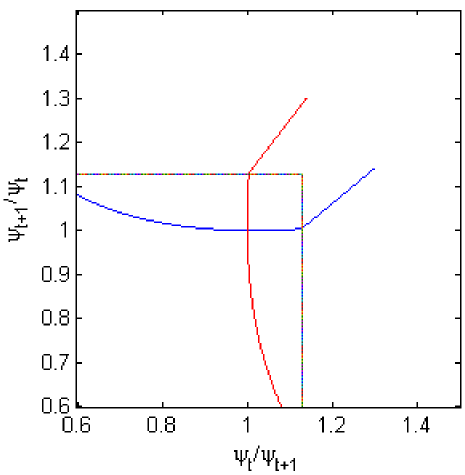
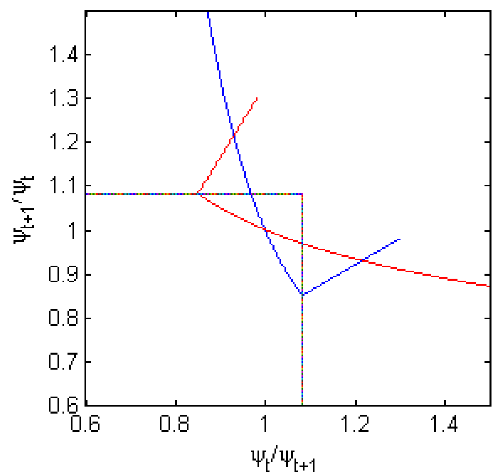


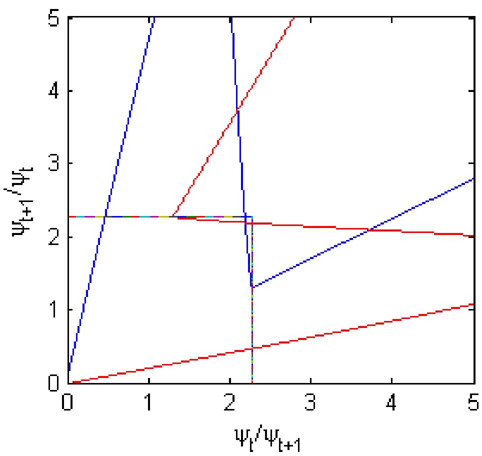
Figure 3



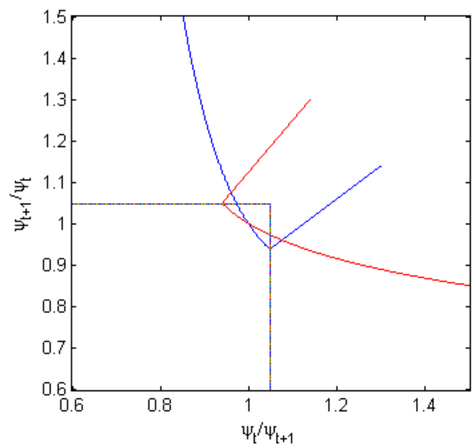
(a) Example 1



(b) Example 2



(c) Example 3



(d) Example 4

Figure 4

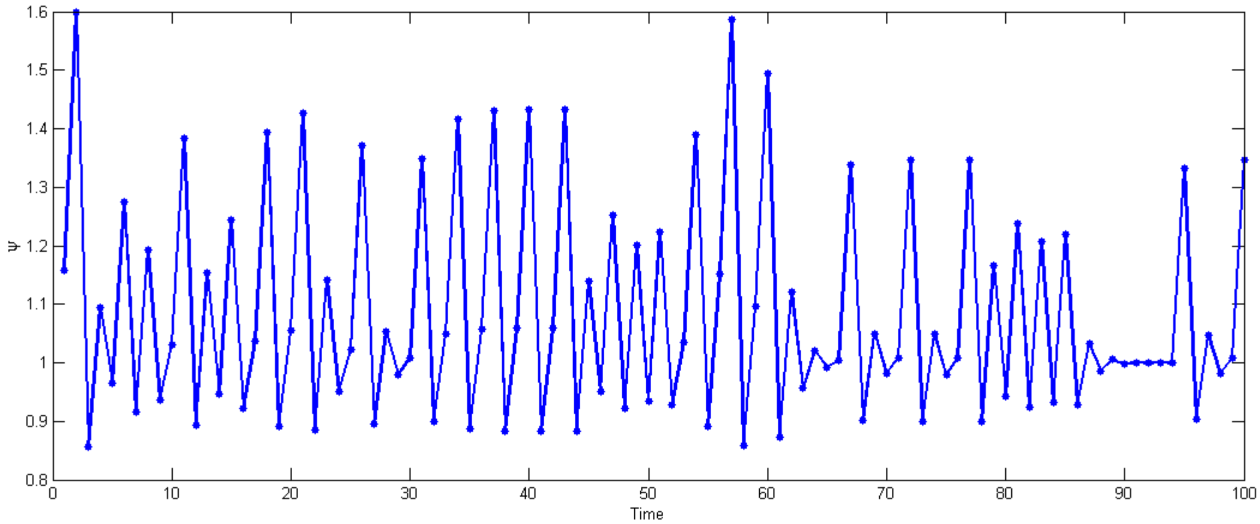


Figure 5: Chaotic Dynamics

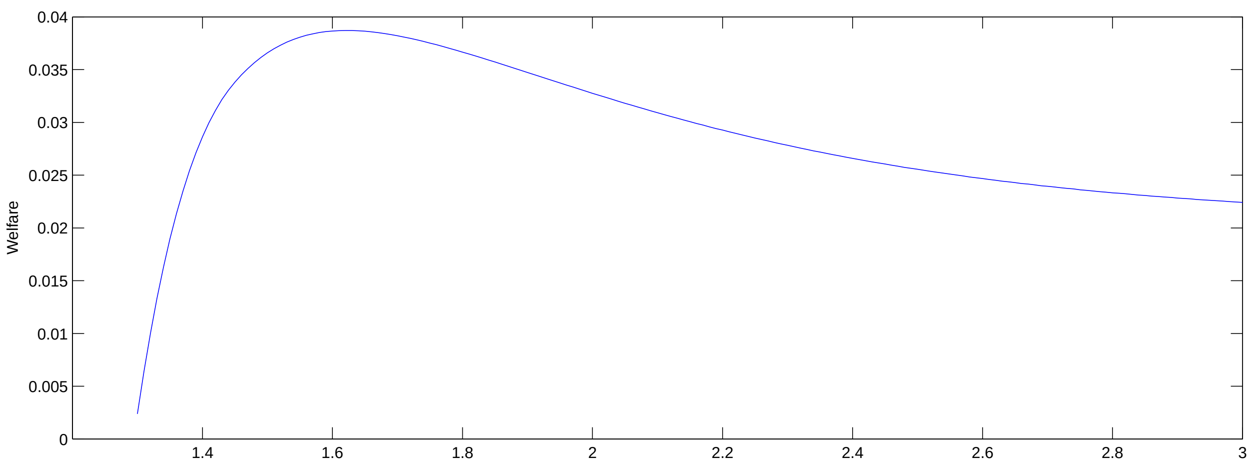


Figure 6: Welfare and Housing Stock, An Example

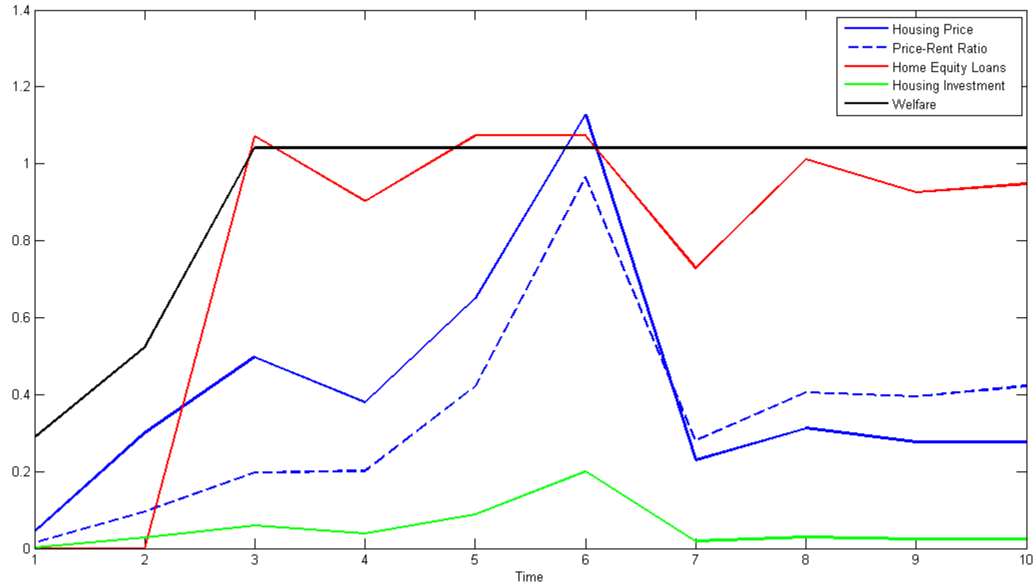


Figure 7: $\alpha_h = 0.5, \beta = 0.6, \gamma = 8, \sigma = 3, \eta = 3.52, \kappa = 1/3$

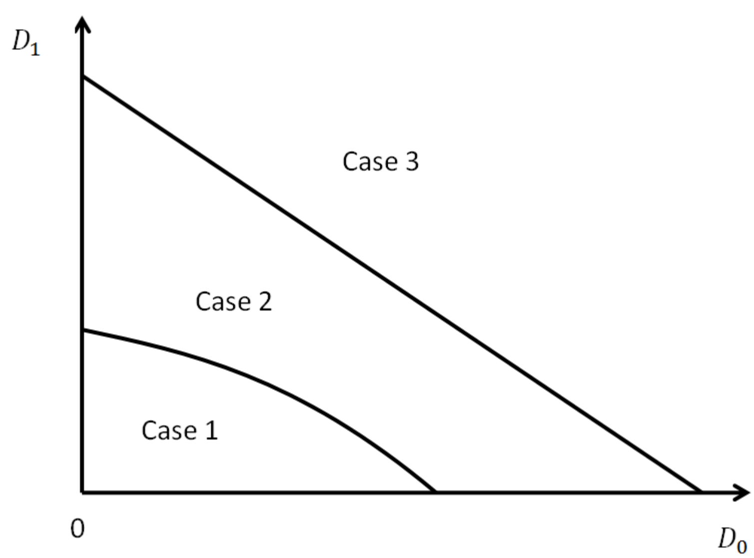


Figure 8