Abstract

This paper presents a parsimonious equilibrium business cycle model with trade frictions in the product and labor markets. The model features unemployment and unsold production and its general equilibrium can be represented very simply: as the intersection of an aggregate supply and an aggregate demand, with product market tightness acting as a price. The aggregate supply represents the expected amount of sales by firms given product market tightness and optimal hiring on the labor market. The aggregate demand represents optimal product consumption given product market tightness—consumers can also spend their income on an unproduced good. We use a search-and-matching structure to realistically represent trade frictions in the product and labor markets. In such a structure, it is not price or wage but market tightness that equalizes supply to demand. In fact, the frictions create situations of bilateral monopoly in price and wage setting that make price and wage indeterminate. To resolve this indeterminacy, we take price and wage as parameters, thus disconnecting price and wage determination from our analysis. Since the equilibrium representation is very transparent and tractable, we are able to obtain a broad range of comparative statics with respect to demand and supply shocks. The model is also suited to think about inventories, labor hoarding, income and wealth inequality. It can be extended to a dynamic environment.
1 Introduction

A vast literature in macroeconomics has developed microfounded models of business cycle fluctuations. In our view, such models should be transparent and tractable and have four key features: (1) involuntarily unemployment happens and firms cannot sell any quantity at the going price; (2) cyclical fluctuations in unemployment and consumption arise and are highly correlated with cyclical fluctuations in tightness in the labor and product markets; (3) some fluctuations are driven by aggregate demand shocks and some are driven by aggregate supply shocks; and (4) the economic equilibrium should be pairwise Pareto efficient in the sense that no mutually advantageous trades between two agents is possible. Although simple models eventually need to be extended into large-scale calibrated dynamic stochastic general equilibrium models, the transparence and tractability afforded by a simple model is valuable to develop our understanding of the macroeconomic mechanisms at play over the business cycle.

Thus, in this paper, we propose a transparent and tractable business cycle model that possesses these four features. The key assumption we make is that there are trade frictions in the product and labor markets. We represent these frictions using a search-and-matching structure. The general equilibrium of this model can be represented as the intersection of an aggregate supply and an aggregate demand that respond to a broad range of macroeconomic shocks. We use the model to study business cycle fluctuations in consumption and unemployment.

It is well known that in search-and-matching models, prices are indeterminate because they are set in situations of bilateral monopoly [Hall, 2005; Howitt and McAfee, 1987]. Indeed by the time a match is realized, search costs are sunk and there is a surplus to share; therefore, there is a band of potential prices acceptable to both seller and buyer. To resolve this indeterminacy, we do not offer a theory of price determination but instead consider reduced-form price schedules that may depend on economic circumstances and can be calibrated to match empirical price behavior. While it is possible to add a microfounded theory of price determination to our model, this is not necessary for our analysis, and is therefore left for future research. Our model disconnects the analysis of fluctuations in quantity and tightness from the price-setting mechanism, which greatly simplifies the analysis relative to other macroeconomic models.

In our model, tightness plays the role of a price: it equilibrates supply and demand. When the economy is slack, quantities respond strongly to tightness. Conversely, when the economy is at capacity, quantities do not respond any more to tightness. This is because, with a typical matching function, the number of trades made by a seller is an isoelastic, increasing, and concave function of market tightness. Thus, in bad times, shocks translate into large quantity responses when prices are rigid.
Although our model proposes a significant conceptual departure from the Real Business Cycle model, it is mathematically similar to a Real Business Cycle model in which prices are replaced by tightnesses. In both models, there are two endogenous variables—a quantity and a price—in each market. These variables are determined in equilibrium by the equality of supply and demand. Our model remains an equilibrium model which avoids the pairwise Pareto inefficiencies that arise in many New Keynesian models. We discuss existing business cycle models and their properties in Section 2.1. We also explain how our framework relates to these models and the value added by our approach.

In Section 2.2, we survey the scattered literature that documents trade frictions on product markets. It is now accepted that the search-and-matching framework provides a realistic description of the labor market, but most researchers are not used to thinking about the product market with this framework. For this reason, we collect systematic evidence from the product market to motivate our modeling choice. The evidence suggests that trade frictions lead firms to build long-term customer relationships.

In Section 3, we present the simplest model of aggregate supply and aggregate demand embodying trade frictions on the product market. We consider an economy of self-employed workers who produce and sell a good. In the model, all workers are both producers and buyers. If the producers cannot find a buyer, their production is lost. Buyers search for producers and divide their income between the produced good and an unproduced good. As mentioned above, the bilateral monopoly structure allows us to consider the product price as a parameter. We define product market tightness as the ratio of the shopping effort of buyers to output for sale. When tightness is high, it is easy for producers to find buyers while it is difficult for buyers to find sellers. Thus, product market tightness is perceived as price because producers like a high tightness while consumers dislike it. The model naturally generates an aggregate demand function and an aggregate supply function. Unlike traditional supply and demand functions that take product price as argument, our supply and demand functions take product market tightness as an argument. For a given product market tightness, the aggregate demand gives the desired purchases of produced good when workers optimally allocate their income between consumption of produced good and consumption of unproduced good. The aggregate demand function decreases with tightness as customers reduce their demand for produced good relative to unproduced good when purchasing products is difficult. For a given product market tightness, the aggregate supply gives the expected sales of produced good. The aggregate supply function increases with tightness as producers are more likely to sell when tightness is high. The general equilibrium can be represented as the intersection of aggregate demand and aggregate supply, with product market tightness acting as a price.
It is easy to compute comparative statics for a variety of shocks. On the one hand, an increase in the product price or an increase in the taste for the unproduced good lead to an adverse shift in aggregate demand, reducing consumption and product market tightness. On the other hand, a reduction in productivity leads to an adverse shift in aggregate supply, reducing consumption but increasing product market tightness. Some degree of price rigidity is needed to obtain response to demand shocks and hence generate realistic business cycle fluctuations.

We can interpret the unproduced good as future consumption to consider consumption/saving decisions. In that case, current production is divided into current consumption and investment used to produce future consumption. Investment can be transformed into future consumption with an increasing and concave production function. The price, which corresponds to the gross return to savings, determines investment and this model behaves exactly as the basic model.

In Section 4, we add a labor market to the basic model of Section 3 to provide a richer mode of aggregate supply. To represent the labor market and its trade frictions, we use the search-and-matching framework of Michaillat [2012a]. Instead of considering an economy of self-employed individuals, we assume that individuals are hired by firms through a matching process. Firms produce output with decreasing marginal returns of labor. The wage is indeterminate and labor market tightness acts as a price equilibrating labor supply and labor demand. As in Section 3, this model can also be summarized by aggregate demand and aggregate supply curves in the (product market tightness, produced good quantity) diagram keeping the labor market in equilibrium in the background.

We obtain richer comparative statics than in the model without labor market. Negative demand shocks now affect the labor market by depressing employment and labor market tightness. Negative supply shocks tend to depress employment with ambiguous effects on labor market tightness. Depending on parameters, positive technology shocks can either boost employment, as in standard models, or depress employment, as in some New Keynesian models with price rigidity [Galí, 1999].

In Section 5, we consider various extensions of the model. First, we show how the model can be embedded into a dynamic stochastic environment. Second, we introduce inequality to provide a richer model of the aggregate demand function. We introduce inequality by assuming that a fraction of workers own a disproportionately large share of firms and have higher taste for the unproduced good; these workers represent the rich. With inequality, transfers from the rich to the poor typically stimulates consumption as the rich have a lower propensity to consume the produced good out of income.

To conclude, Section 6 discusses how price and wage determination could be modeled in future work.
2 Relation to the Literature and Empirical Motivation

The search-and-matching representation provides a conceptual framework to think about product and labor markets. In this section, we argue that this conceptual framework shares the desirable theoretical properties of other representations of product and labor markets (perfect competition, fixprice model, monopolistic competition) that have been used as key building blocks in the macroeconomic literature. While it is well known that matching frictions are empirically important in the labor market, there is less empirical evidence on the product-market side and no systematic survey of available empirical evidence. Hence, we provide a survey of empirical evidence that shows that matching frictions are also a first-order phenomenon in the product market, motivating our modeling choices.

2.1 Existing Business Cycle Models

This section relates our model to other macroeconomic models in the literature. It argues that our model is a natural evolution of previous macroeconomic models, and that it shares the desirable theoretical properties of these models. This section focuses on macroeconomic models of the business cycle that are microfounded (based on optimizing behavior of firms and workers) and that are in general equilibrium (modeling explicitly labor and product markets).

Table 1 summarizes our argument. Panel A presents the four market structures have been commonly used to build business cycle models: fixprice, perfect competition, monopolistic competition, and search and matching. In our view, an adequate market structure to describe product and labor market over the business cycle ought to have three properties, listed in the first three columns of the table. First, it should be able to describe a nonclearing market—that is, a non-Walrasian market in which there may be some slack. Second, the amount of slack should be able to vary over the business cycle. Third, all trades should be pairwise Pareto efficient (i.e., in equilibrium, there should be no feasible pairwise trade desirable to both parties). To illustrate the mathematical structure of the equilibrium within each market, the last two columns of the table summarize the conditions that give equilibrium price and quantity for each market.

Panel B presents the four most widely used classes of business cycle models: the general disequilibrium models, created in the early 1970s, which use a fixprice representation of the product and labor markets; the real business cycle models, created in the early 1980s, which use perfect competition to represent both markets; various New Keynesian models, created in the late 1980s, which use monopolistic competition to represent the product market and sometimes the labor market; and finally the
recent search-search models, which use a search-and-matching representation of both product and labor markets. For completeness, Panel C lists a few of recent business cycle models that mix a search-and-matching representation for one market with a more standard representation (perfect or monopolistic competition) for the other market.

Classical macroeconomists of the beginning of the 20th century assumed that all markets were perfectly competitive and that prices cleared all markets at all time, a set of assumptions dating back to Walras. To explain persistent unemployment during the Great Depression, Keynes [1936] replaced the assumption that the wage clears the labor market by that of a wage floor under which nominal wages would not fall. The idea of Keynes [1936] was formalized by Hicks [1937], who developed a fixprice model in which, as an analytical simplification of the short-run mechanics of the macroeconomy, prices and wages are assumed to be fixed and adjustments take only place through quantity rationing. There is a critical issue with this theory. Since the labor demand linking real wage to employment is fixed over the cycle, it requires countercyclical real wages to generate countercyclical unemployment fluctuations.\(^1\) But it is clear in the data that real wages are mildly procyclical or at best acyclical.\(^2\) It is only once the fixprice method was introduced into a multimarket model, with spillovers across markets, that this critical issue was addressed.\(^3\) The general disequilibrium theory was developed in Barro and Grossman [1971, 1974] to analyse booms and recessions with a macroeconomic model in which price and wage are fixed and do not clear the product and labor market.\(^4\) In this model, there is a fixed amount of money that enters the household’s utility function together with consumption. The utility-maximizing behavior of the household generates an aggregate demand curve, which expresses consumption as a decreasing function of the price. The profit-maximizing behavior of firms generates a labor demand curve, which expresses labor as a decreasing function of the real wage. Once the real wage is determined (with a market clearing condition or otherwise), the labor demand curve combined with the production function gives an inelastic aggregate supply curve. If the price is above the market-clearing level, firms are unable to sell the quantity of output given by the aggregate supply curve. Thus, firms demand a smaller quantity of labor than that given by the labor demand curve, giving rise to unemployment. To conclude, the theory is able to generate unemployment without placing any restrictions on the level or movement of the real

\(^1\)See Okun [1981, Chapter 1] for a clear exposition of Keynes’ motives and the limitations of his theory.


\(^3\)See Patinkin [1965], Clower [1965], Leijonhufvud [1968], and Solow and Stiglitz [1968] for early contributions.

\(^4\)See Bénassy [1993] for a simple exposition of the theory.
Table 1: Properties of existing business cycle models

Panel A: Market structures

<table>
<thead>
<tr>
<th></th>
<th>Nonclearing market</th>
<th>Cyclic fluctuation</th>
<th>Pairwise efficiency</th>
<th>Equilibrium</th>
<th>price</th>
<th>quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixprice (F)</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td></td>
<td>( p ) given</td>
<td>( q = \min { q^d(p), q^*(p) } )</td>
</tr>
<tr>
<td>Competitive (C)</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td></td>
<td>( q^d(p) = q^*(p) )</td>
<td>( q = q^*(p) )</td>
</tr>
<tr>
<td>Monopolistic</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- constant markup (MC)</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td></td>
<td>( mr(p) = q^*(p) )</td>
<td>( q = q^d(p) )</td>
</tr>
<tr>
<td>- varying markup (MV)</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td></td>
<td>( mr(p) = q^*(p) )</td>
<td>( q = q^d(p) )</td>
</tr>
<tr>
<td>- menu cost (MM)</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td></td>
<td>decision rule</td>
<td>( q = q^d(p) )</td>
</tr>
<tr>
<td>Search &amp; matching</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- flexible price (SF)</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td></td>
<td>( q^d(\theta) = q^*(\theta) )</td>
<td>( q = q^*(\theta) )</td>
</tr>
<tr>
<td>- rigid price (SR)</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td></td>
<td>( q^d(\theta) = q^*(\theta) )</td>
<td>( q = q^*(\theta) )</td>
</tr>
</tbody>
</table>

Panel B: Symmetric models

<table>
<thead>
<tr>
<th></th>
<th>Product market</th>
<th>Labor market</th>
<th>Third good</th>
<th>Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>General disequilibrium</td>
<td>F</td>
<td>F</td>
<td>money</td>
<td></td>
</tr>
<tr>
<td>Real Business Cycle</td>
<td>C</td>
<td>C</td>
<td>saving</td>
<td></td>
</tr>
<tr>
<td>New Keynesian</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Blanchard and Kiyotaki [1987, Section I]</td>
<td>MC</td>
<td>MC</td>
<td>money</td>
<td></td>
</tr>
<tr>
<td>- Kimball [1995], Yun [1996]</td>
<td>MC/F</td>
<td>C</td>
<td>saving</td>
<td></td>
</tr>
<tr>
<td>- Gertler and Leahy [2008]</td>
<td>MM</td>
<td>C</td>
<td>saving, money</td>
<td></td>
</tr>
<tr>
<td>- Rotemberg and Woodford [1992]</td>
<td>MV</td>
<td>C</td>
<td>saving</td>
<td></td>
</tr>
<tr>
<td>- Cole and Ohanian [2004]</td>
<td>MV</td>
<td>MV</td>
<td>saving</td>
<td></td>
</tr>
<tr>
<td>- Erceg, Henderson and Levin [2000]</td>
<td>MC/F</td>
<td>MC/F</td>
<td>saving</td>
<td></td>
</tr>
<tr>
<td>Search-search</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Hall [2008]</td>
<td>SR</td>
<td>SR</td>
<td>none</td>
<td>linear</td>
</tr>
<tr>
<td>- Lehmman and Van der Linden [2010]</td>
<td>SF</td>
<td>SF</td>
<td>money</td>
<td>linear</td>
</tr>
<tr>
<td>- this model</td>
<td>SR</td>
<td>SR</td>
<td>unproduced</td>
<td>concave</td>
</tr>
</tbody>
</table>

Panel C: Mixture models

<table>
<thead>
<tr>
<th></th>
<th>Product market</th>
<th>Labor market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Andolfatto [1996], Merz [1995]</td>
<td>C</td>
<td>SF</td>
</tr>
<tr>
<td>Chéron and Langot [2000], Walsh [2003]</td>
<td>MC/F</td>
<td>SF</td>
</tr>
<tr>
<td>Krause and Lubik [2007], Blanchard and Galí [2010]</td>
<td>MC/F</td>
<td>SR</td>
</tr>
<tr>
<td>Gourio and Rudanko [2011], Bai, Rios-Rull and Storesletten [2012]</td>
<td>SF</td>
<td>C</td>
</tr>
<tr>
<td>Arseneau and Chugh [2007]</td>
<td>SR</td>
<td>C</td>
</tr>
</tbody>
</table>
wage because unemployment may arise from insufficient aggregate demand.\footnote{See Malinvaud [1977] and Muellbauer and Portes [1978] for the implications of the theory for unemployment and macroeconomics. See Portes [1981] for an application to centrally planned economies. The general disequilibrium theory was refined, with a focus on mathematical foundations, by Bénassy [1975, 1977], Drèze [1975], Grandmont [1977a], Grandmont and Laroque [1976], and Grandmont, Laroque and Younès [1978], among others. For empirical research that estimates general disequilibrium models, see for instance Quandt [1978], Rosen and Quandt [1978], Ashenfelter [1980], Artus, Laroque and Michel [1984]. See Grandmont [1977b] and Drazen [1980] for surveys of this literature. See Mankiw and Weinzierl [2011] for a recent application of the fixprice framework to the design of optimal fiscal and monetary policy.}

The general disequilibrium theory generates a Keynesian consumption function, business cycle fluctuations, unemployment caused either by excessive real wages—classical unemployment—or deficient aggregate demand—Keynesian unemployment. Nevertheless, the theory suffers from several limitations. First, they are often extremely complex [Drazen, 1980]. Even in their simple form, such as the elegant model of Barro and Grossman [1971], it is always necessary to keep track of which side of the market is rationed, which quantity constraint is binding, and eventually, in which regime the economy is (for instance, Keynesian unemployment with excess supply in labor and product markets, or classical unemployment, with excess supply in the labor market but excess demand in the goods market). Second, the theory does not explain why, if prices are above market-clearing levels, a rationed seller does not reduce his price by a small amount in order to increase sales [Howitt, 1985]. Third, these models run into the critique of Barro [1977]: because most workers are in long-term employment relationships, it is unclear why their employers would choose to determine employment from the static labor demand schedule in each history; there may be significant gains to long-term contracts. Fourth, many policy questions are muted in these models. Consider the labor market with classical unemployment. Employment is given by the intersection of labor demand and wage. Labor supply is irrelevant and policies such as unemployment insurance, which only affect labor supply, have no negative effect on employment.

The Real Business Cycle (RBC) model, developed by Kydland and Prescott [1982], Long and Plosser [1983], Barro and King [1984], and Prescott [1986], addresses the theoretical problems arising in general disequilibrium theory by assuming that all markets are perfectly competitive and that prices clear all markets at all time. The RBC model is much simpler than the general disequilibrium model because its can be described by a couple of optimality and market-clearing conditions. The RBC model suffers, however, from several limitations. An implication of the assumption that all markets clear is the absence of unemployment, which is obviously counterfactual. Another one is that firms are able to sell any amount of output at the going price, which is at odds with the existence of inventory, advertising, and labor hoarding. Indeed, these phenomena reflect the inability of firms to sell its desired output at the
going price. It is also difficult to obtain large monetary nonneutralities with a RBC model.\(^6\) This is an issue given the large amount of empirical evidence indicating that monetary policy has large effect on output.\(^7\) It is also difficult to obtain realistic joint fluctuations in employment and real wages with a standard RBC model. The argument, borrowed from Rotemberg and Woodford [1991], has two parts. If business cycles are driven by demand shocks, the labor demand should remain fixed because it only depends on capital and technology, which do not respond to a demand shock in the short run. Then cyclical fluctuations come from labor supply shifts, which result in countercyclical real wages (this is the same argument as for Keynes’ theory). If business cycles are driven technology shocks, the labor demand does shift over the business cycle and the labor supply remains fixed. But these shifts result procyclical movements in real wages that are too large, and procyclical movements in employment that are too small, because labor supply is not very elastic empirically.

To resolve some of the issues of RBC models, business cycle models with monopolistic competition were developed.\(^8\) ,\(^9\) Blanchard and Kiyotaki [1987, Section I] describes a macroeconomic model with monopolistic competition in product and labor markets, which nests the RBC model as a special case. They introduce real money balances in the utility function: money plays the role of a nonproduced good that provides services. Workers choose between buying goods and holding money, generating an aggregate demand curve. The model is elegantly represented with a price curve and wage curve (equivalent of labor supply and labor demand) in a real wage-output diagram. Furthermore, this simple model generates excess supply of goods and excess supply of labor because price-makers set prices at a markup above their marginal cost, forcing demand to be strictly above supply at the equilibrium quantity. With monopolistic competition on the labor market, there is unemployment in the sense that some workers would like to work at the going wage (the marginal rate of substitution is below the wage).\(^10\) But this employment is voluntary: workers behave as a union that withholds labor to extract the highest surplus from the firm. In markets without labor unions, this seems implausible.

\(^6\)In the basic RBC model, money plays no role. To give a role to money, King and Plosser [1984] build a RBC model with a banking sector, and Cooley and Hansen [1989] build a RBC model with a cash-in-advance constraint. But even in these models, money nonneutralities are small [Kimball, 1995].

\(^7\)The evidence collected by Friedman and Schwartz [1963] convinced many economists that contractionary monetary policy can have large effects on output. For more recent evidence, see Romer and Romer [2004].

\(^8\)Other approaches were proposed to resolve some issues of the RBC model. For instance, Donaldson and Danthine [1990] and Alexopoulos [2004] incorporate the efficiency-wage theories of Akerlof [1982] and Shapiro and Stiglitz [1984] into the RBC framework in order to introduce unemployment.


\(^10\)See Gali [2011] for such an interpretation of a monopolistically competitive labor market.
By itself, the simple model of monopolistic competition does not generate realistic business cycle fluctuations. As explained by Rotemberg and Woodford [1991], one way to obtain fluctuations is to assume countercyclical fluctuations in the markups charged by monopolists on the product market. These fluctuations in the product market markup lead to labor demand shifts that generate procyclical employment and real wages. Several mechanisms generate countercyclical markups on the product market. Rotemberg and Woodford [1992] show that the game-theoretic model of oligopolistic competition of Rotemberg and Saloner [1986] leads to countercyclical markups because oligopolists engage in price wars in expansions. Rotemberg and Woodford [1991] show that the customer market of Phelps and Winter [1970] yields the same results. Gali [1994] proposes a model in which countercyclical markups arise from changes in the composition of demand over the cycle. Finally, Cole and Ohanian [2004] develop a model with monopolistic competition in product and labor markets and show that the substantial increase in cartelization of several industries during the Great Depression can explain the increase in underemployment observed in the US at that time. Cole and Ohanian [2004] argue that cartelization occurred both in the product and labor markets as a result of government policies markups.11 Finally, a wage markup shock accounts for an important share of the variance in output and inflation in modern estimated NK models [Smets and Wouters, 2003, 2007]. This result implies that fluctuations in wage markups are an important source of business cycle shocks.

Another issue with the simple model of monopolistic competition is that money is neutral, affecting all nominal prices and wages proportionately, and leaving output and employment unchanged. The presence of price-makers in the model allows researchers, however, to study microfoundations for price and wage rigidity, which introduce money nonneutrality. One approach is to impose the menu cost of Mankiw [1985] and Akerlof and Yellen [1985] for each price change, which generate endogenous price rigidity [for example, Blanchard and Kiyotaki, 1987; Dotsey, King and Wolman, 1999; Gertler and Leahy, 2008]. Other microfoundations to price rigidity have been introduced into macroeconomic models, such as habit formation or informational frictions.12 These microfoundations are often fairly complex, which may explain why the models have not been used more widely to conduct policy analysis. The introduction by Kimball [1995] and Yun [1996] of the price-setting friction of Calvo [1983] into macroeconomic models with monopolistic competition on the product market was a major simplification.

11 As pointed out by [Shimer, 2009], however, the idea that all recessions are periods of widespread monopolization of the labor market seems empirically implausible.

The resulting model constitutes the baseline New Keynesian (NK) model. The early NK model was further expanded by Erceg, Henderson and Levin [2000] and Christiano, Eichenbaum and Evans [2005] to add nominal wage rigidity caused by the same Calvo friction mechanism. While the Calvo friction provides a more tractable model of price and wage stickiness, the resulting fluctuations in sales and employment are again subject to the critique of Barro [1977]. Indeed, because most workers are in long-term employment relationships and most sellers are in long-term customer relationships, it is unclear why employers and buyers would choose to determine employment and purchase from the static labor demand and product demand schedules in each history. There may be significant gains to explicit or implicit long-term contracts both in labor and product markets.

In this paper, we assume bilateral exchanges in the labor market and the product market following the search-and-matching tradition. That is, markets are not organized as auctions markets in which everybody can sell and buy as much as they desire at the going price. Instead, sellers and buyers can only sell and buy if they are matched with a buyer and seller. The number of matches made is governed by a matching function that takes buying and selling effort as inputs. On both the labor and product market, seller and buyer must decide a price in a bilateral monopoly situation because of the positive surplus that arise from each customer-seller and firm-worker match. It has been known for a long time that the resulting price is fundamentally indeterminate. Hall [2005] exploited this indeterminacy to introduce a rigid wage into a search-and-matching model of the labor market. He showed that the rigidity lead to large unemployment fluctuations. In our model, we introduce a wage schedule and price schedule

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14 A vast literature developed search-and-matching models of the labor market. Classic references include Diamond [1982b], Mortensen [1982], Pissarides [1985], Mortensen and Pissarides [1994], Pissarides [2000]. See Mortensen and Pissarides [1999], Rogerson, Shimer and Wright [2005] and Rogerson and Shimer [2010] for surveys of the literature. Papers have also applied the search-and-matching framework to the product market. Classic references include Diamond [1982a] and Diamond [1984]. More recent works include Gourio and Rudanko [2011] and Shi [2011]. These models have been incorporated into otherwise standard macroeconomic models. RBC models have been augmented with search-and-matching models of the labor market [for example, Andolfatto, 1996; Merz, 1995]; they have also been augmented with search-and-matching models of the product market [for example, Bai, Rios-Rull and Storesletten, 2012; Mathé and Pierrard, 2011]. Arseneau and Chugh [2007] augment and RBC model with a search-and-matching model of the product market and rigid prices. NK model have also been augmented with search-and-matching models of the labor market [for example, Chéron and Langot, 2000; Monacelli, Perotti and Trigari, 2010; Moyen and Sahuc, 2005; Trigari, 2009; Walsh, 2003].

15 Hieser [1970] explains that Edgeworth [1881] was the first to highlight this indeterminacy. He was followed by Pigou [1932]. Howitt and McAfee [1987] and Pissarides [1989] emphasized that the indeterminacy of the solution to the bilateral monopoly problem induced the indeterminacy of the equilibrium wage in a search-and-matching model of the labor market.

16 Other researchers who introduced wage rigidity into search-and-matching models of the labor market include Shimer [2004, 2010, 2011], Hall and Milgrom [2008], Gertler and Trigari [2009], and Michaillat [2012a]. Some recent NK models feature a search-and-matching model of the labor market with a rigid wage [for example, Blanchard and Galí, 2010; Faia, 2008; Gertler, Sala and Trigari, 2008; Krause and Lubik, 2007; Michaillat, 2012b; Thomas, 2008].

17 The common way to resolve this indeterminacy is to set wages using the Nash [1953] bargaining solution [Diamond, 1982b; Mortensen, 1982]. But this solution generates unrealistically small unemployment fluctuations [Shimer, 2005].
that are functions of the parameters of the model. The indeterminacy in price- and wage-setting allows us to choose the types and amounts of price and wage rigidity required to obtain realistic business cycle fluctuations. Thus, we disconnect the theory of price and wage determination from our analysis. Our model can use the standard Nash bargaining pricing as well as alternatives such as partially rigid prices. The key point is that adding an alternative price or wage determination theory does not require to modify the other parts of the model.

Finally, following Hart [1982], we introduce an unproduced good in the model in addition to labor and the produced good. This choice follows from the need to avoid Say’s law, the result that the supply of goods produced automatically generates its own demand. With an unproduced good that enters workers’ utility function, workers allocate their income between consumption of produced and unproduced good as a function of the relative price of these goods. This optimal choice traces an aggregate demand curve. In the standard macroeconomic model, the choice is between consumption and savings, and in fact we show in Section 3.5 that the unproduced good can be reinterpreted as savings. The unproduced good allows us to introduce an aggregate demand in a static model without savings.18

We show that the resulting macroeconomic model shares the desirable properties of previous macroeconomic models. The model features (i) nonclearing market such that involuntary unemployment prevails at the going wage and firms cannot sell their entire production at the going price; (ii) shocks lead to large fluctuations in output and unemployment; (iii) both aggregate demand and aggregate supply shocks lead to fluctuations; (iv) while both price and wage are rigid, the economic equilibrium is pairwise Pareto efficient in the sense that no mutually advantageous trades between two agents are possible; and (v) the equilibrium can be described parsimoniously by a few optimality conditions and the equality of supply and labor in all markets in tightness-quantity plans. (Mathematically, our equilibrium is similar to that of a standard RBC model once price and wage are replaced by product market and labor market tightness.)

Our paper is not the first one to propose a macroeconomic model that merges search-and-matching models of the product and labor market.19 Lehmann and Van der Linden [2010] builds a model with

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18 In a series of papers, Farmer [2011, 2008, 2009, 2010] exploits the indeterminacy arising from the bilateral monopoly problem in wage setting to select an labor market equilibrium determined by the aggregate demand for commodities. In Farmer’s models, the equilibrium wage will adjust to the point where neither firms not workers have an incentive to vary their search intensities and aggregate demand is met. For example in Farmer [2008], aggregate demand is determined by the government tax rate, government spending, and the budget constraint of the government. In Farmer [2009], aggregate demand is determined by the self-fulfilling beliefs of stock market participants. And in Farmer [2010], aggregate demand is determined by a belief function giving expectations of nominal income growth of consumers.

19 Several papers propose macroeconomic models with trade frictions on the product and labor market that do not take the standard form of the search-and-matching frictions. In response to the demise of general disequilibrium models, Howitt [1985] proposes a model with transactions costs on the product and labor market to account for unemployment without being subject to the critique of Barro [1977]. Unlike general disequilibrium models, this model has flexible price, flexible wage,
search-and-matching fractions in the product and labor market. In their model, money is required for transactions, and the presence of money gives rise to an aggregate demand. Since the authors are interested in the long-run interaction between unemployment and inflation, they model prices and wages as completely flexible. While this is a good choice for their purpose, it would be difficult to use their model to study the business cycle. Building on the work of Wasmer and Weil [2004], Wasmer [2011] and Petrosky-Nadeau and Wasmer [2011] propose a model with search-and-matching structure in the labor, financial, and product markets. All prices are determined by Nash bargaining so prices and wages are completely flexible. Finally, Hall [2008] merges search-and-matching models of the product and labor market. Prices and wages are rigid relative to the price of an intermediate good. Hall’s model is therefore the most closely related to ours, but there are two differences. First, Hall [2008] does not have a third good, beyond labor and the produced good, that would break Say’s law and generate an aggregate demand curve; in Hall’s model, supply generates its own demand. Second, Hall assumes linear production and utility functions. These assumptions lead to special equilibria in which labor demand and product supply are perfectly elastic. Instead we assume concave production and utility functions to obtain demand and supply relations that are more general. The assumptions lead to downward-sloping demand curves and upward-sloping supply curves, of which Hall’s curves are a special case. This generalization matters because our model admits nondegenerate equilibria for a much wider range of price and wage, allowing us to interpret tightness as the equilibrating pricing mechanism. As we show in Section XXX, this generalization also matters economically.

2.2 Evidence of Trade Frictions

Our search-and-matching framework incorporates the following four stylized facts: (i) there are costs of selling and buying that are sunk at the time of trading; (ii) buying and selling is uncertain for buyer and seller not engaged in a relationship, (iii) buyers and sellers engage in long-term relationships; and (iv) in long-term relationships, prices are determined by complex norms. In this section, we briefly summarize the evidence regarding these stylized facts for the labor market, and we present in detail evidence from a broad range of sources regarding these stylized facts in the product market. It is now and no aggregate demand effects. More recently, Kaplan and Menzio [2013] propose a macroeconomic model with search-and-matching frictions on the labor market and search fractions as in Burdett and Judd [1983] on the product market, which give rise to equilibrium price dispersion. Shopping externalities appear: a firm employing an additional worker generates positive externalities because employed workers have more income to spend and have less time to shop for low prices than unemployed workers. When the externality is sufficiently strong, it generates multiple rational expectation equilibria and self-fulfilling fluctuations in unemployment. The dynamics between equilibria are complex and are studied numerically.
accepted that the search-and-matching framework provides a realistic description of the labor market, but most researchers are not used to thinking about the product market with this framework. For this reason, we collect systematic evidence from the product market to make our case.\(^{21}\) To our knowledge, there exists no systematic survey of those studies in the literature. For comparison purposes, we first briefly summarize the large literature on labor markets and then turn to product markets.

**Review of Evidence for the Labor Market.** A historical perspective on the US labor market is useful, because it shows how the market structure evolved over time from what was essentially an auction market to a market best described by the search-and-matching framework. The historical work of Jacoby [1984] documents how, in the 20th century, the labor market evolved from impermanent and market-oriented to bureaucratic, rule-bound, and secure. His historical analysis explains that internal labor markets have been adopted by most firms because they were more equitable than the older “drive system”, so they improved workers’ morale and productivity, and eventually increased the profits of firms. Jacoby explains that trade unions, government interventions during the world wars and the Great Depression, and the personnel management movement were instrumental to the adoption of internal labor markets.\(^{22}\)

The fact that employment contracts are long-term contracts appears clearly in US data. For instance, the average monthly job separation rate is below 4%.\(^{23}\) This separation rate would imply that the average employment contract last for \(1/0.04 = 25\) months, or roughly two years. But employment is much more stable than this duration implies. The reason is that most of jobs separation come form a small fraction of the workforce that stays only a short time on jobs. The study of Hall [1982] shows that US labor market provides stable, near-lifetime employment to an important fraction of the labor force. Hall finds that the typical worker today is holding a job which has lasted or will last about eight years. Over a quarter of all workers are holding jobs which will last twenty years or more. Sixty percent hold jobs which will last five years or more. The jobs held by middle-aged workers with more than ten years of tenure are extremely stable. Over the span of a decade, only twenty to thirty percent come to an end. And jobs seem to be even more stable nowadays. In a recent study, Davis et al. [2010] finds that employment is maven more stable now than when Hall [1982] wrote, due to a decline in job destruction intensity from

\(^{21}\)Of course, not all exchanges are conducted through bilateral trade, especially in the product market. As pointed out by Okun [1981], while most goods are exchanged through bilateral trade, some goods are indeed exchanged on auction markets: financial assets, agricultural commodities, mining products, or art work. But we provide evidence that a majority of them are.\(^{22}\)For a detailed analysis of the internal labor market in a large US firm and its career organization, see Baker, Gibbs and Holmstrom [1994a].

\(^{23}\)Using the seasonally-adjusted monthly series for total separations in all nonfarm industries constructed by the Bureau of Labor Statistics (BLS) from the Job Openings and Labor Turnover Survey (JOLTS) for the December 2000–June 2010 period, we find an average separation rate of 3.7%.
the early 1980s to the mid 1990s.

The reason why employment relationships last so long is that a surplus is associated with a firm-worker match. In our model, the surplus arises because worker and firm incur costs before matching. These costs are visible in the data. Using the microevidence provided by Barron, Berger and Black [1997] and Silva and Toledo [2009], Michaillat [2012a] finds that recruiting costs, including labor costs, advertising costs, agency fees, and travel costs, amount to 32% of a worker’s monthly wage. In aggregate, this number implies that 0.5%–1% of GDP is spent on recruiting. The surplus also arises because it takes time for workers to find a job: the average monthly job-finding probability is only around 60%.24

Finally, surveys of business executives are informative because they illustrate the complexity of the wage-setting process, and they provide reasons for nominal wage rigidity that are difficult to model with economic tools. As emphasized by Bewley [1999], pay cuts lower workers’ morale, antagonize workers and ultimately reduce productivity and profitability. The work of Doeringer and Piore [1971] leads to the same conclusion: they highlight that the internal pay structure does not respond to competitive forces in the external labor market, and that it is governed by a set of rules and procedures that are not consistent with pricing which would prevail in a competitive market. Having reviewed the labor market evidence for the search-and-matching framework, we discuss the product market evidence.

**Microeconomic and Macroeconomic Data for the Product Market.** The most obvious cost of buying goods is shopping time. The average shopping time is not negligible. The American Time Use Survey (ATUS), conducted annually by the US Bureau of Labor Statistics (BLS), measures the amount of time people spend doing various activities, such as paid work, childcare, volunteering, and socializing. On average between 2003–2011 in the US civilian population, people spend 47 minutes a day purchasing goods and services (broadly half spent purchasing consumer goods and the other half spent in travels related to purchasing goods and services). As a comparison, people spend on average 3 hours and 40 minutes a day working.

Selling is also costly for firms. Firms spend substantial resources on marketing and selling. Using various data sources, Arkolakis [2010] measures marketing and advertising expenditures in the US. He finds that the advertising expenditure to GDP ratio is stable, around 2%–2.5% of GDP for the 1982–2007 period. This number includes advertising on television, newspapers, radio, magazines, telephone

24Using seasonally-adjusted monthly series for number of hires in all nonfarm industries and for the unemployment level, constructed by the BLS from JOLTS and the Current Population Survey (CPS) over the December 2000–June 2010 period, we calculate the job-finding probability as the ratio of hires to unemployment, and find an average monthly probability of 60.9%.
directories, and the Internet. Including brand sponsorship and public relations, sales promotion, and interactive marketing, he finds that marketing spending is as much as 4%–5% of GDP for 2001–2004. Finally, including marketing events such as trade shows, telephone sales, supporting product materials, hiring of outside marketing personnel, the estimated marketing spending can rise up to 7.7% of GDP.\footnote{For a survey of the economic analysis of advertising, see Bagwell [2007].}

Firm spend quite a lot attracting and retaining customers. We argued above that US firms spend less than 1% of GDP on recruiting. Therefore, US firms seem to spend much more on attracting and retaining customers than on recruiting employees.

Furthermore, buyers face uncertainty on the availability of goods they desire to purchase. Bils [2004] finds that temporary stockouts are quite common. He uses information in the monthly microdata underlying the Consumer Price Index (CPI), the CPI Commodities and Services Survey conducted by the BLS. The survey tracks a large set of prices specific to a particular product at a particular outlet. The survey provides information for constructing occurrences of temporary stockouts: a stockout is an item not available for sale, continuing to be carried by the outlet, and not seasonally unavailable. Bils uses 999,432 observations belonging to 63 categories of consumer durables (a category is a BLS Entry Level Item (ELI)) from January 1988 to June 2004. The expenditure share of these 63 ELIs in the CPI is 6.6%. The stockout rate averages 8.8%. If the stockout rate for each ELI is weighted by its expenditure share in 1997, Bils obtains a weighted stockout rate of 9.2%. The existence of stockouts is evidence that a visit to the store for a buyer that is not in a relationship with a seller may not result in a purchase because the desired product may not be available.

Inventory accumulation also suggests that sellers face significant uncertainty regarding demand for their product, and accumulate inventory to avoid stockout and therefore lost sales.\footnote{The variance of production has been found to exceed the variance of sales—even after controlling for obvious seasonal factors [for example, Blanchard, 1983]. This fact suggests that the common production-smoothing theory of inventory is of secondary importance.} Using data for the US automobile industry and a model developed in Kahn [1987], Kahn [1992] argues that inventory behavior can be attributed to firms’ responses to demand uncertainty with a nonnegativity constraint on inventories. Features of the data such as the levels of inventories, the high variance of production relative to the variance of sales, the synchronization of production and sales, and the procyclicality of inventory investment are all captured by the model with only demand uncertainty and the stockout-avoidance motive. Bils and Kahn [2000] also propose, and find empirical support for, a model in which accumulating inventories is costly but facilitate sales. This productive view of inventories is consistent with the presence of trading frictions on the product market.
Using firm-level data from Israel and Scandinavian countries, Yashiv [2000] and Carlsson, Eriksson and Gottfries [2012] empirically show that firms face a constraint on how much goods they can sell in the short run. They also compare the role of demand shocks to that of hiring costs in the employment policies of firms, and find that local demand shocks play an important role in the recruiting behavior of firms.

In a search-and-matching model, firms with an existing customer base sell more than a firm offering its product for sale for the first time with no clientele. Okun [1981] argued that virtually any firm must face less uncertainty and realize more sales with customers. Recent empirical work by Foster, Haltiwanger and Syverson [2012] support this assumption. Foster, Haltiwanger and Syverson [2012] use microdata from the Census of Manufactures, a dataset constructed by the US Census Bureau that covers the universe of manufacturing plants. The dataset contains information on plants’ output quantities and prices. They analyse data for 1977, 1982, 1987, 1992, and 1997, a total of roughly 17,000 plant-year observations for producers of one of ten products (fiber boxes, white pan bread, carbon black, roasted coffee beans, ready-mixed concrete, oak flooring, block ice, processed ice, hardwood plywood, and raw cane sugar). Their main finding is the importance of a customer base for firm expansion. While young plants’ technical efficiency levels are similar to established plants’ levels, new establishments grow only slowly because they face a demand gap relative to existing ones which closes only slowly over time. In other words, despite of lower prices, new factories grow only slowly because they need time to attract new customers.

Existence of long-term relationships is evidence of a trade surplus. Data suggest that long-term customer relationships are important because customer turnover rates are fairly low. Using various sources from the marketing literature, Gourio and Rudanko [2011] find that monthly turnover rates of 1%–2.5% for cell phone service providers (it is so low probably because of explicit contracts), annual turnover rates in the 10%–20% range for online banking, annual turnover rates of 10%–25% for supermarkets.

**Firm Surveys.** Large-scale surveys of firms have been conducted in most western economies. The idea of these surveys was to ask firms direct questions, expressed in laymen’s terms, on how they reason and act when they set their prices. The samples include firms in a specific sector, or is representative of all sectors. These surveys aim to describe patterns of price fluctuations, determine price-setting rules, and identify sources of price rigidity.

The results of these surveys show that most firms engage in long-term relationships with their cus-
omers. Table 2 gives the fraction of sales that go to regular customers, as opposed to occasional buyers across such surveys. The numbers are high. For instance in the US, 85% of sales of the firms surveyed by Blinder et al. [1998] go to long-term customers. As an illustration, the average share across these eleven studies of sales going to long-term customers is 77%. The table also report the typical customer of the firms. It appears to be another firm and not a final consumer.

The results of these surveys also suggest that prices are determined by complex norms in long-term customer relationships. Table 3 lists the most important reasons given by firms for keeping stable prices. It shows that in many studies, the most important reason for not changing prices is an implicit contract with customers, and a fear to antagonize customers in the event of a price change.\(^{27}\) This factor is not new for firms. Already in the 1930s, the survey of UK manufacturing companies conducted by Hall and Hitch [1939] indicates that changes in price are very costly, “a nuisance to salesmen”. Entrepreneurs explained that customers are attached to conventional prices, and that price changes are therefore disliked by customers.\(^{28}\)

Table 3 suggests that prices are not adjusted upwards immediately after a positive demand shock when firms have an implicit understanding with their customers. How do firms respond to a positive demand shocks? Hall, Walsh and Yates [2000] address this question in their survey. They find that 62% of firms would respond to a positive demand shock with more overtime (higher capacity utilization), 12% of firms would increase their workforce, and only 12% of firms would increase the price of their products. Therefore, firms seem to have more flexibility in overtime and employment than in prices.\(^{29}\)

**Case Studies in Economics and Sociology.** Many case studies in economics and sociology explain the necessity of long-term relationships to conduct trade. Unlike the labor market, these relationships may not be contractual. But they seem to be sustainable nonetheless, which suggests the existence of a large

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\(^{27}\)The implicit contract theory was described as follows across surveys: “the idea is that firms have implicit understandings with their customers—who expect the firms not to take advantage of the situation by raising prices when the market is tight” in Blinder et al. [1998]; “prices could not change more often without disturbing customer relations” in Amirault, Kwan and Wilkinson [2006]; “the price could not be changed more often without a risk of upsetting customer relations” in Apel, Friberg and Hallsten [2005]; “some customers consider price increases resulting from higher demand less fair than those resulting from higher costs; do you keep prices constant despite demand fluctuations because you do not want to jeopardize your customer relationships’ in Kwapi, Baumgartner and Scharler [2005]; “the existence of an implicit contract (regular contact with a customer without any written)” in Loupias and Ricart [2004]; “Our customers prefer a stable price and a change could damage customer relations; even if our competitors also change their price contract” in Lunnemann and Matha [2006]; “Our customers expect us to keep prices as stable as possible” in Hoeberichs and Stokman [2005]; “the preference of our customers for stable prices; changing prices frequently could threaten customer relations” in Martins [2006]; “the possibility of losing customers (even if competitors also raise their prices)” in Alvarez and Hernando [2005].

\(^{28}\)See Table 5 in Hall and Hitch [1939].

\(^{29}\)The results are reported in Table 5 in Hall, Walsh and Yates [2000].
Table 2: Importance of customer relationships in firm surveys

<table>
<thead>
<tr>
<th>Study</th>
<th>Country</th>
<th>Period</th>
<th>Sample</th>
<th>Sales to customers</th>
<th>Type of buyer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blinder et al. [1998]</td>
<td>US</td>
<td>1990–92</td>
<td>200</td>
<td>85%</td>
<td>70% 21%</td>
</tr>
<tr>
<td>Hall, Walsh and Yates [2000]</td>
<td>UK</td>
<td>1995</td>
<td>654</td>
<td>59%</td>
<td>–</td>
</tr>
<tr>
<td>Apel, Friberg and Hallsten [2005]</td>
<td>Sweden</td>
<td>2000</td>
<td>626</td>
<td>86%</td>
<td>77% 12%</td>
</tr>
<tr>
<td>Hoeberichts and Stokman [2005]</td>
<td>Netherlands</td>
<td>2004</td>
<td>1,246</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Martins [2006]</td>
<td>Portugal</td>
<td>2004</td>
<td>1,370</td>
<td>83%</td>
<td>84% 13%</td>
</tr>
<tr>
<td>Aucremanne and Druant [2005]</td>
<td>Belgium</td>
<td>2004</td>
<td>1,979</td>
<td>78%</td>
<td>56% 40%</td>
</tr>
<tr>
<td>Lunnemann and Matha [2006]</td>
<td>Luxembourg</td>
<td>2004</td>
<td>367</td>
<td>85%</td>
<td>–</td>
</tr>
<tr>
<td>Loupias and Ricart [2004]</td>
<td>France</td>
<td>2004</td>
<td>1,662</td>
<td>54%</td>
<td>66% 30%</td>
</tr>
<tr>
<td>Stahl [2005]</td>
<td>Germany</td>
<td>2004</td>
<td>1,200</td>
<td>57%</td>
<td>89% 7%</td>
</tr>
<tr>
<td>Fabiani, Gattulli and Sabbatini [2004]</td>
<td>Italy</td>
<td>2003</td>
<td>333</td>
<td>98%</td>
<td>73% 25%</td>
</tr>
<tr>
<td>Kwapil, Baumgartner and Scharler [2005]</td>
<td>Austria</td>
<td>2004</td>
<td>873</td>
<td>81%</td>
<td>84% 9%</td>
</tr>
<tr>
<td>Alvarez and Hernando [2005]</td>
<td>Spain</td>
<td>2004</td>
<td>2,008</td>
<td>86%</td>
<td>58% 39%</td>
</tr>
</tbody>
</table>

Notes: In Blinder et al. [1998], the results for the distribution of sales among customers are in Table 4.11 and the results for the share of sales to long-term customers are in Table 4.12. For Hall, Walsh and Yates [2000], the share is the fraction of customer relationships that last over 5 years, which is reported in Figure 5. In Apel, Friberg and Hallsten [2005], the results are in Table 1. The eight studies pertaining to euro-area countries (Martins [2006], Aucremanne and Druant [2005], Lunnemann and Matha [2006], Loupias and Ricart [2004], Stahl [2005], Fabiani, Gattulli and Sabbatini [2004], Kwapil, Baumgartner and Scharler [2005], and Alvarez and Hernando [2005]) were conducted within the Inflation Persistence Network, a joint research program on inflation persistence in the euro area coordinated by the European Central Bank. The results for these nine studies are summarized in Table B.3 in Fabiani et al. [2005].

surplus from long-term customer relationships in many industries. Uzzi and Lancaster [2004] show that repeated interactions between corporate lawyers and their clients allow to establish trust between the parties and reduce costs for both sides. Okun [1981] argue that professional buyers prefer long-term, continuing, relationships with sellers, even when they procure physically homogenous products. He gives the examples of the US steel and copper markets: US firms and their customers are in long-term relationships, despite the existence of a spot market for imported metal. Kranton and Minehart [2001] also provides compelling evidence that, in contrast to the assumption that buyers and sellers are anonymous, a buyer and seller must have a relationship, or link, to engage in exchange. They provide evidence from the work of Uzzi [1996] on New York City’s garment market, the historical study of Greif [1993] describing customer-seller networks among Maghribi traders, or the work of McMillan and Woodruff [1999] on trade credit in customer-seller relationships in Vietnam.

The negative effects of price changes on customer-firm relationships seem large. Using time-and-motion methods, Zbaracki et al. [2004] study the pricing practices of a one-billion-dollar industrial firm. They develop a detailed description of the pricing process to account for physical, managerial, and
Table 3: Which factors slow down price adjustment?

<table>
<thead>
<tr>
<th>Study</th>
<th>Most important factors</th>
<th>Rank of “implicit contracts”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blinder et al. [1998]</td>
<td>CF, CB, NC</td>
<td>4</td>
</tr>
<tr>
<td>Hall, Walsh and Yates [2000]</td>
<td>EC, CB, CF</td>
<td>5</td>
</tr>
<tr>
<td>Amirault, Kwan and Wilkinson [2006]</td>
<td>CB, IC, EC</td>
<td>2</td>
</tr>
<tr>
<td>Apel, Friberg and Hallsten [2005]</td>
<td>IC, CB, EC</td>
<td>1</td>
</tr>
<tr>
<td>Kwapil, Baumgartner and Scharler [2005]</td>
<td>IC, EC, CB</td>
<td>1</td>
</tr>
<tr>
<td>Aucremanne and Druant [2005]</td>
<td>IC, EC, CB</td>
<td>1</td>
</tr>
<tr>
<td>Loupias and Ricart [2004]</td>
<td>CB, CF, EC</td>
<td>4</td>
</tr>
<tr>
<td>Lunnemann and Matha [2006]</td>
<td>IC, CB, EC</td>
<td>1</td>
</tr>
<tr>
<td>Hoeberichts and Stokman [2005]</td>
<td>IC, EC, JQ</td>
<td>1</td>
</tr>
<tr>
<td>Martins [2006]</td>
<td>IC, CF, HF</td>
<td>1</td>
</tr>
<tr>
<td>Alvarez and Hernando [2005]</td>
<td>IC, CF, EC</td>
<td>1</td>
</tr>
</tbody>
</table>

Notes: Respondents to the surveys were presented with a set of economic theories of price stickiness, and were asked to rate how important each theory was as a cause of price stickiness in their own firms. Column 2, 3, and 4 presents the first, second, and third most popular theories among respondents. CF is “coordination failure”; CB is “cost-based pricing”; NC is “nonprice competition”; EC is “explicit contracts”; IC is “implicit contracts”; JQ is “judging quality by price”; HF is “high fixed costs”. Column 5 gives the rank of the “implicit contracts” theory with customers. Table 5.1 in Blinder et al. [1998] provides a short summary of the theories commonly proposed to respondents. “Coordination failure” is that firms hold back on price changes, waiting for other firms to go first. “Cost-based pricing” is that price rises are delayed until costs rise, and these delays cumulate through a multi-stage production process. “Nonprice competition” is that firms vary non price elements such as delivery lags, service, or quality. “Explicit contracts” is that prices are fixed by contracts. “Judging quality by price” is that firms fear customers will mistake price cuts for reductions in quality. In Martins [2006], “high fixed costs” refers to the constraint that the presence of high fixed costs puts on a firm’s decision to change its price. “Implicit contracts” is that firms tacitly agree to stabilize prices, perhaps out of fairness to customers. The ranking of the theories are reported in the following tables: in Table 5.2 in Blinder et al. [1998] (with much more insight on implicit contract theory given in Chapter 7); in Table 3 in Hall, Walsh and Yates [2000]; in Table 8 in Amirault, Kwan and Wilkinson [2006]; in Table 3 in Apel, Friberg and Hallsten [2005]; in Table 5 in Kwapil, Baumgartner and Scharler [2005]; in Table 18 in Aucremanne and Druant [2005]; in Table 6.1 in Loupias and Ricart [2004]; in Table 8 in Lunnemann and Matha [2006]; in Table 10 in Hoeberichts and Stokman [2005]; in Table 4 in Martins [2006]; and in Table A17 in Alvarez and Hernando [2005].

Customer costs of changing prices. They find that customer costs, which include communication and negotiation costs with customers, amount for more than 20 times menu costs. They also show that these high costs are the source of price rigidity. In addition, the authors provide qualitative evidence that costs of price adjustment due to customers are higher than their dollar estimates, because these estimates do not take into account the customer antagonization cost. Unjustified price changes will harm the firm’s reputation, and may lead customers to stop doing business with the firm.30

Dore [1983] carried out a comparative survey of the weaving segment of the textile industry in the UK and in Japan. He conducted a series of interviews in 1980 in western Japan to understand relational contracting. He found that repeated interactions between firms brings economic advantages. In these

30 See Zbaracki and Bergen [2010] for more details on the price-adjustment process in the firm.
relationships, prices are the outcome of long-term relational contracts in production markets in Japan. This kind of arrangement allows sellers and buyers to avoid bargaining and price competition, and grants more importance to quality. Dore explains that customer/seller relations involve a “sense of diffuse personal obligation which accrue between individuals engaged in recurring [...] economic exchange”. These obligations are based on traditional values and sentiments. Dore gives a precise example of what these obligations may involve. Even if a finisher re-equips with a new and more efficient dyeing process which gives him a cost advantage and the opportunity of lowering prices, he does not immediately get all the business. He may win business from one or two converters if they had some other reason for being dissatisfied with their own finisher. But the more common consequence is that the other merchant-converters go to their finishers and say: “Look how X has got his price down. We hope you can do the same because we really would have to reconsider our position if the price difference goes on for months. If you need bank finance to get the new type of vat we can probably help by guaranteeing the loan”.

The last case study is that of Kennedy [2005], who studies trade credit through a series interviews conducted in the Bay Area around 2004. It illustrates that customer-seller relationships are invaluable for sellers, and that they also benefit buyers. The following two explanations for trade credit were ranked respectively 1st and 7th (out of 13 possible explanations). The first explanation suggests that customer relationships are critical for the firms, and that they offer trade credit to support them: “Since we have offered credit in the past, if we were to now require immediate cash payment, many of our relationships would deteriorate and some would take their business away”. The second supports again the existence of a reciprocity between customers an sellers: “By giving customers trade credit, we show commitment to them, helping to create customer loyalty”.

Marketing Literature. In the marketing literature, Arndt [1979] notes a change in market structure and develops the concept of “domesticated markets”, which replace competitive markets. He explains that the “competitive market is eroding”, and that ”transactions are occurring in internal markets within the framework of long-term relationships”. This study analyzes changes in market structure and their implications for marketing theory and practice. Arndt describes a process in which many earlier competitive markets are structured as a result of voluntary, long-term, binding commitments among the organizations involved. Examples of interorganizational systems include conglomerates, franchising, vertical and horizontal integration, joint ventures, joint product development, joint physical distribution plans, and labor-management peace agreements. This observation echoes that of Jacoby [1984], who explains
how labor markets transformed from auction market to internal labor market in the 1920s and 1930s in the US. Even for firms who deal with other firms outside of their own organization, it is well known that having customers is crucial. Dwyer, Schurr and Oh [1987] detail cost and benefits of customer relationship, and explain how to manage it. Keaveney [1995] explains the reasons for which customers switch seller, and provides advice to managers to avoid losing customers in the service industry.

**Survey of French Bakers.** The study by Blinder et al. [1998] reports the share of sales going to long-term customers by industry. While 92% of sales go to long-term customers in manufacturing and 94% in wholesale trade, the number is only 71% in retail trade. With the prevalence of supermarkets and other large distributors in retail trade, one wonders whether customer relationship are limited to trade between firms, or whether trade with consumers also gives rise to long-term customer relationships. France provides a good example of a market in which seller and buyer are in long-term customer-seller relationships. Despite increasing competition from supermarkets and other nontraditional bread retailers, local bakeries still capture about 70% of the market for bread. And the clientele of French bakeries are long-term customers: more than 65% of households patronize a bakery at least once a week; those who patronize a bakery weekly average 4 visits a week [Eymard, 1999].

To understand the reasons for the existence of long-term customer relationships with consumers and their influence on bread prices, we interviewed 31 bakers in France in the summer of 2007. Following the approach of Bewley [1999], the interviews were only loosely directed. We sampled bakeries from large cities, small towns, and small rural villages, in three regions: around Grenoble and Aix-en-Provence in the southeast, around Paimpol in the northwest, and around Paris. The number of answers does not permit statistical analysis, but the answers provide insight about the role customer relationships.

Customer relationships allow both seller and buyer to alleviate uncertainty linked to random supply and demand, captured formally in our model by the matching function that applies to buyers and sellers who are not in a relationship. Indeed, a baker defined a “real customer” as somebody who comes every day. However, norms of behavior evolve, and what it means to be a customer changed over the past decades. A baker in Aix-en-Provence explained that nowadays the demand is nearly unpredictable because of the development of tourism and because people do not buy bread everyday anymore, and are not as loyal as in the past. They may mainly patronize a bakery, but they sometimes shop in a supermarket

---

31 See Table 4.12 in Blinder et al. [1998].
32 The market share is from www.boulangerie.org. In France, supermarkets capture more than 77% of the retail food market and the share of artisanal food retailer is only 15% [Fraichard, 2006].
or in another bakery. On the other hand, in the 1960s the baker’s father used to prepare the quantity of bread that he had produced on the same day the previous year. Predicting demand was simple. In spite of these changes, four bakers emphasized the importance of having a large clientele of loyal customers who make it a habit to purchase goods in the shop. Given that customers are loyal to their bakery, it takes time for a new bakery to establish a customer base. The new bakeries face the same problem as the new firms in the study by Foster, Haltiwanger and Syverson [2012]. According to a respondent who opened a bakery recently, it takes about two years to secure a clientele of regular customers.

For buyers, being a customer means that bakeries will put aside some bread when they are running low, such that regular customers are sure to have the bread they like when they come. Five respondents mentioned that a privilege of being a customer is that the baker reserves bread when the bakery is running out of some kind of bread. In fact, bakers know exactly what customers order every day. In particular, it is common for bakers to reserve bread for regular customers on weekends when the bread goes quickly. A baker mentioned that it would be unacceptable to have no bread for a regular customer, and that customers would probably leave the bakery if the baker did not keep them a bread. The baker also explained that these forced bakers to bake too much bread, and that this bread was lost at then end of the day if it is not sold—in fact, the bread is given away to a local farmer for his horses and rabbits.

In addition, customers have privileges similar to those discussed in the case studies of Dore [1983] and Kennedy [2005]. Customer relationship allow the seller to tailor his product to the needs of buyer or to offer advantages such as trade credit. For a customer, a baker can prepare special cakes to accommodate special diets, they can prepare bread in large quantity, etc. In addition, five respondents explained that social interaction with their customers is important to support a customer relationship. This aspect was also present in the study of Dore [1983]. This personal relationship with the baker seems to be valued both by customers and bakers. One aspect of the personal relationship is that it starts with the formal “vous”, then the informal “tu”, and finally interactions on a first-name basis. One baker mentioned that customers would sometimes leave when the cashier was replaced.

The effort to keep customers loyal constrain price variations. Prices adjustments are guided by social norms of fairness to avoid antagonizing customers. As argued by Okun [1981], while the monopoly surplus in customer-seller relationships is large, it can be destroyed if suppliers raise prices in a way that antagonizes customers. In these customer relationships, Okun [1981, Chapter IV] proposes that prices follow the following norm: fixed-time scheduling, prenotification, meeting competition, and cost-oriented pricing. Interestingly, the same four norms appeared in our survey.
Fixed-time scheduling is clearly present. Four bakers explain that they only change their price once a year, on July 1st. Respondents proposed several reasons for this date: the price of bread was controlled until 1978, and was then updated on July 1st to take inflation into account, and bakers stuck to that habit; some bakers explain that their cost is the wage of their employees, which is linked to the minimum wage, which is updated on July 1st by the government; some bakers think that this rule comes from the time when union where giving instructions regarding the price of bread (until 1997). One respondent always changes the price at the same date and always increases the price by the same amount of 2.5%.

Prenotification is also present. When there is a price increase, it is announced long in advance and explained carefully. We have seen several signs posted on the counter of the bakery that explain that in x weeks, the price of bread will increase due to an increase of ingredient y by z percent.

Competition do matters for prices. Part of the reason why bakers are reluctant to increase prices is that they fear losing customers to competitors. New bakers who enter the market also set up their prices at the level of the competitors. Generally, bakers knows the prices of the competitors, the only discrepancies resulting from differences in quality. They were also evidence of collusion, especially in small villages: bakers talk to each other to settle on a price, and coordinate price changes.

Finally, cost-based pricing is widely used. Seven bakers explained that they would rise the price of bread when the price of flour goes up (generally once a year in September at the end of harvest), when utilities go up (especially gas, required to operate the oven), or when the wage of their employees go up. One baker explained that they did not increase the price of bread over the last two years since the price of flour was stable. Interestingly, bakers try to smooth cost increases in various ways. Some distribute the increase over the prices of different kind of breads, some increase the price of luxury items (cakes, pastries) instead of necessity items (baguette), some increase the price only every second cost increases, and some price at an average cost instead of continuously varying the cost (especially for fruit tarts whose cost varies a lot during the year).

3 A Basic Model of Aggregate Supply and Aggregate Demand

This section presents the simplest model of aggregate supply and aggregate demand embodying trade frictions on the product market. The model is static. The economy is populated by self-employed workers who produce and sell a good. Workers receive income from the sale of their production and from the sale of an unproduced good with which they are endowed. Workers allocate their income to the
consumptions of produced and unproduced good. A fraction of workers are in a customer relationship with a seller so they can buy their desired quantity of produced good with certainty. The rest of the workers are not in a customer relationship with a seller. Because of trade frictions, which we model with a matching function, these buyers need to shop for a seller. How much they need to shop to find a seller depends on the product market tightness. Similarly, producers with customers sell their production to their customers with certainty. Because of trade frictions, producers who do not have customers sell their production only if they can find a buyer. The probability to find a buyer depends on the product market tightness. We consider the price as a parameter instead of an endogenous object determined by market-clearing conditions or bargaining assumptions. Although the price is a parameter, the equilibrium is pairwise Pareto efficient because of the bilateral monopoly situation when seller and buyer meet.

Our model is tractable: the equilibrium can be represented as the intersection of an aggregate supply and an aggregate demand, with product market tightness acting as a price. The aggregate supply represents the optimal consumption of produced good given product market tightness. The aggregate demand represents the expected amount of sales of produced good given product market tightness. The notations used in the model, and in its extensions, are summarized in Table 4.

For concreteness, it is useful to think about the produced good as a perishable consumption good such as bread and about the unproduced good as land. All individuals are bakers, but they cannot eat their own bread. Some bakers have customers and sell bread to them for sure. Bakers who do not have customers only sell their bread with some probability. The bread that is not sold is discarded. Similarly, buyers who are regular customers obtain bread for sure. Buyers who are not customers to a bakery shop for bread, which requires effort. The trading frictions captured by the matching function take various forms: a baker may have sold all his bread when the buyer arrives, demand uncertainty at the microlevel may lead buyers toward one bakery more than the other, a buyer may not like the bread prepared by the baker for idiosyncratic reasons, etc. These uncertainties linked to demand were discussed in our survey of French bakers. These uncertainties also explain why firms build inventories, beyond production-smoothing motives.

There are other models of customer markets in the literature: Phelps and Winter [1970], Nishimura [1989], Bagwell [2004], and Vincent [2012] develop search models in which consumers have imperfect information about prices and face searching costs; and Klemperer [1987, 1995] develops models in which consumers face switching costs; Nelson [1970] and Bils [1989] develop models in which consumers have imperfect information products, whose quality is only revealed after purchase; Rotemberg [2005, 2010, 2011] develops a series of models that capture the anger and regret of customers in response to price changes; finally, Nakamura and Steinsson [2011] develop a model of habit-formation for individual goods that leads to customer relationships.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Name</th>
<th>Relation with other variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>Output</td>
<td></td>
</tr>
<tr>
<td>s</td>
<td>Shopping effort</td>
<td>$s = (c - \kappa) / q(x)$</td>
</tr>
<tr>
<td>o</td>
<td>Recruiting effort</td>
<td>$o = (n - (1 - u)) / \hat{q}(\theta)$</td>
</tr>
<tr>
<td>x</td>
<td>Product market tightness</td>
<td>$x \equiv s / (y - \kappa)$</td>
</tr>
<tr>
<td>\theta</td>
<td>Labor market tightness</td>
<td>$\theta \equiv x / s$</td>
</tr>
<tr>
<td>c</td>
<td>Produced-good consumption</td>
<td>$c = \kappa + f(x) \cdot (y - \kappa)$</td>
</tr>
<tr>
<td>m</td>
<td>Unproduced-good consumption</td>
<td></td>
</tr>
<tr>
<td>\sigma</td>
<td>Investment</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>Employment</td>
<td>$n = 1 - u + u \cdot \hat{f}(\theta)$</td>
</tr>
<tr>
<td>\pi</td>
<td>Real profits</td>
<td>$\pi = c - w \cdot n$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Function</th>
<th>Name</th>
<th>Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>\nu(c, m, s)</td>
<td>Utility function</td>
<td>quasilinear or CRRA</td>
</tr>
<tr>
<td>q(x)</td>
<td>Product-finding rate</td>
<td>$q(x) = \omega \cdot x^{-\eta}$</td>
</tr>
<tr>
<td>f(x)</td>
<td>Selling probability</td>
<td>$f(x) = \omega \cdot x^{1-\eta}$</td>
</tr>
<tr>
<td>\hat{q}(\theta)</td>
<td>Recruiting rate</td>
<td>$\hat{q}(\theta) = \hat{\omega} \cdot \theta^{-\nu}$</td>
</tr>
<tr>
<td>\hat{f}(\theta)</td>
<td>Job-finding probability</td>
<td>$\hat{f}(\theta) = \hat{\omega} \cdot \theta^{1-\nu}$</td>
</tr>
<tr>
<td>c^d(x)</td>
<td>Aggregate demand</td>
<td>decreasing</td>
</tr>
<tr>
<td>c^s(x)</td>
<td>Aggregate supply</td>
<td>increasing</td>
</tr>
<tr>
<td>n^d(\theta, x)</td>
<td>Labor demand</td>
<td>decreasing</td>
</tr>
<tr>
<td>n^s(\theta)</td>
<td>Labor supply</td>
<td>increasing</td>
</tr>
<tr>
<td>z(n)</td>
<td>Production function</td>
<td>$z(n) = n^\alpha$</td>
</tr>
<tr>
<td>g(n, \theta)</td>
<td>Production function</td>
<td>$g(n, \theta) = z(n) - [r / \hat{q}(\theta)] \cdot (n + u - 1)$</td>
</tr>
<tr>
<td>Y(\theta)</td>
<td>Equilibrium output</td>
<td>$Y(\theta) \equiv a \cdot g(n^s(\theta), \theta)$</td>
</tr>
<tr>
<td>m(\sigma)</td>
<td>Production function of capital</td>
<td>increasing, concave</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Assumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>\kappa</td>
<td>Customer base</td>
<td>$\kappa &lt; y$</td>
</tr>
<tr>
<td>\beta</td>
<td>Taste for unproduced good</td>
<td></td>
</tr>
<tr>
<td>\epsilon</td>
<td>Decreasing marginal utility</td>
<td>$\epsilon &gt; 1$</td>
</tr>
<tr>
<td>r</td>
<td>Shopping cost</td>
<td>$r &gt; 0$</td>
</tr>
<tr>
<td>\hat{r}</td>
<td>Recruiting cost</td>
<td>$\hat{r} &gt; 0$</td>
</tr>
<tr>
<td>\mu</td>
<td>Endowment of unproduced good</td>
<td></td>
</tr>
<tr>
<td>p</td>
<td>Product price</td>
<td></td>
</tr>
<tr>
<td>w</td>
<td>Real wage</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>Aggregate demand parameter</td>
<td>$D \equiv \mu^{1/\epsilon} / (\beta \cdot p)$</td>
</tr>
<tr>
<td>\eta</td>
<td>Elasticity of product matching</td>
<td>$\eta \in (0, 1)$</td>
</tr>
<tr>
<td>\nu</td>
<td>Elasticity of labor matching</td>
<td>$\nu \in (0, 1)$</td>
</tr>
<tr>
<td>\omega</td>
<td>Effectiveness of product matching</td>
<td></td>
</tr>
<tr>
<td>\hat{\omega}</td>
<td>Effectiveness of labor matching</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>Technology</td>
<td></td>
</tr>
<tr>
<td>u</td>
<td>Initial unemployment</td>
<td></td>
</tr>
<tr>
<td>\alpha</td>
<td>Diminishing marginal returns to labor</td>
<td>$\alpha &lt; 1$</td>
</tr>
<tr>
<td>\chi</td>
<td>Measure of rich workers</td>
<td></td>
</tr>
<tr>
<td>\psi</td>
<td>Per-capita share of endowment per class</td>
<td></td>
</tr>
<tr>
<td>\phi</td>
<td>Per-capita share of profits per class</td>
<td></td>
</tr>
</tbody>
</table>
3.1 The Model

Producers. The economy is composed of a measure one of identical self-employed workers. Each worker produces a quantity \( y \) of good. The assumption that total production is exogenous is relaxed in Section 4. In that section, we model explicitly firms that hire workers on the labor market and use labor to produce output. We explore the consequences of the generalization for the aggregate supply function. We also explore the interaction between product market and labor market.

Product Market. Each producer has a number of customers who buy a quantity \( \kappa < y \) of goods. The remaining \( y - \kappa \) units of goods are available for purchase by buyers who are not customers of the producer, and whom we call shoppers. Since shoppers are not in a customer relationship with a seller, they need to exert a shopping effort \( s \) to purchase the \( y - \kappa \) units of good for sale. The number of shopper-seller matches is given by a Cobb-Douglas matching function: 

\[
h = \omega \cdot (y - \kappa)^\eta \cdot s^{1-\eta},
\]

with the restriction that \( h \leq y - \kappa \). The parameter \( \omega > 0 \) measures the effectiveness of matching, and \( \eta \in (0, 1) \) is the elasticity of matching with respect to goods for sale. The matching function summarizes the complex exchange process on the product market. It captures many frictions that are not made explicit, such as coordination failures that can be represented with an urn-ball model [Petrongolo and Pissarides, 2001].

Each buyer-seller match translates into the sale of one unit of good. We define the product market tightness \( x \) as the ratio of shopping effort to production available for sale to shoppers: 

\[
x \equiv s/(y - \kappa).
\]

Because of the trade frictions, the seller is only able to sell a fraction of these goods. A producer sells one unit of good to a buyer with probability 

\[
f(x) = h/(y - \kappa) = \omega \cdot x^{1-\eta},
\]

with the restriction that \( f(x) \leq 1 \). We refer to \( f(x) \) as the selling probability. A unit of search effort leads to 

\[
q(x) = h/s = \omega \cdot x^{-\eta}
\]

successful purchases. We refer to \( q(x) \) as the product-finding rate.

The function \( f \) is increasing. When buyers do not visit producers, the product market is slack, and only a small fraction of the production that is not purchased by customers is sold. Conversely, when buyers shop intensively, the product market is tight and most of the production that is not purchased by customers is sold. The function \( q \) is decreasing. When a large quantity of production is for sale, the product market is slack, and shopping effort translate into many purchases. Conversely when a small quantity of production is for sale, the product market is tight and shopping effort translates into few purchases.
Given the matching process, the number of goods actually sold is

\[ c = \kappa + f(x) \cdot (y - \kappa), \]

where \( \kappa \) is the number of goods sold to customers, \( y - \kappa \) is the amount of good available for sale to shoppers, and \( f(x) \) is the fraction of goods sold when product market tightness is \( x \).

**Unproduced Good Market.** In addition to the good produced by the self-employed workers, there is another consumption good that cannot be produced. The unproduced good is traded on a perfectly competitive market. Each worker has an endowment of unproduced good, given by \( \mu \). We are allowed to normalize one price in the model; therefore, we assume that the unproduced good is traded at a price normalized to 1.

**Consumers.** The representative worker is also a consumer with quasilinear utility

\[ v(c, m, s) = \frac{\epsilon}{\epsilon - 1} \cdot c^{\frac{\epsilon - 1}{\epsilon}} + \beta \cdot m - r \cdot s \]

over consumption \( c \) of produced goods, consumption \( m \) of unproduced good, and shopping effort \( s \). The parameter \( 1/\epsilon > 0 \) captures diminishing marginal utility of produced-good consumption, \( \beta > 0 \) measures the marginal utility of unproduced-good consumption, and \( r > 0 \) measures the disutility of shopping.

A worker has two sources of expenditures: the purchase of \( m \) units of unproduced good at price 1; and the purchase of \( c \) units of produced good at price \( p \). A worker has two sources of income: the sale of \( \mu \) units of unproduced good at price 1; and and the sale of \( \kappa + f(x) \cdot (y - \kappa) \) units of produced good at price \( p \). Thus, a consumer’s budget constraint is

\[ m + p \cdot c = \mu + p \cdot [\kappa + f(x) \cdot (y - \kappa)]. \]

In addition, the consumer must exert shopping effort \( s \) if he desires to consume more than \( \kappa \) units of produced good. Shopping effort lead to a purchase at rate \( q(x) \) so the number of purchases above \( \kappa \) is

\[ c - \kappa = s \cdot q(x). \]
For tractability, we assume away randomness at the consumer’s level: shopping effort $s$ leads to $s \cdot q(x)$ purchases for sure. Similarly, we assume away randomness at the producer’s level: each producer sells exactly $f(x) \cdot (y - \kappa)$ to shoppers. Since a positive surplus arises from all the $\kappa$ customer relationships, all relationships are sustained in equilibrium. Therefore, $c \geq \kappa$ in equilibrium.

Given product market tightness $x$ and product price $p$, the representative worker chooses consumptions and shopping effort $\{c, m, s\}$ to maximize utility (2) subject to the budget constraint (3) and the matching constraint (4). Let $\lambda$ be the Lagrange multiplier on (3) and $\phi$ the Lagrange multiplier on (4). The first-order conditions with respect to $c$, $m$, and $s$ yield

$$
\frac{\partial v}{\partial c} = p \cdot \lambda + \phi \quad (5)
$$

$$
\frac{\partial v}{\partial m} = \lambda \quad (6)
$$

$$
r = q(x) \cdot \phi. \quad (7)
$$

Since $\partial v/\partial c = c^{-1/\epsilon}$ and $\partial v/\partial m = \beta$, we obtain a simple expression for the optimal consumption of produced good:

$$
c^{-1/\epsilon} = \frac{1}{D} + \frac{r}{q(x)}, \quad (8)
$$

where $D \equiv 1/(\beta \cdot p)$ is a constant that summarizes the level of aggregate demand for the produced good. Optimal consumption $c$ increases with the aggregate demand constant $D$ and it decreases with product market tightness $x$. In fact when the product market is tight, either because few goods are available for sale or because many shoppers are trying to buy, purchasing a good is costly because it requires a lot of shopping effort; therefore, consumers reduce their desired consumption of produced good.

**Product Price.** The product price is set once shopper and producer have matched. If shopper and producer cannot agree on a price, the producer does not sell the good and the good perishes; the shopper does not consume the good even though he has a positive marginal utility for it; accordingly, there always are mutual gains from trade when a shopper and a producer match. As discussed in Section 2.1, there is no compelling theory of price determination in such a situation of bilateral monopoly. Hence we assume that the product price is a parameter $p > 0$.

Since the produced good has no value to a seller if it is not sold, a seller would accept any positive price. On the other hand the price must be low enough such that a consumer prefers to keep all of his $\kappa$ customers relationships, which requires the following assumption:
**ASSUMPTION 1.** The product price is low enough to be Pareto efficient in \(\kappa\) customer-seller matches:

\[
p < \frac{\kappa^{-1/\epsilon}}{\beta}.
\]

Under Assumption 1 the marginal utility from consuming the \(\kappa\)th unit of produced good, \(\kappa^{-1/\epsilon}\), is higher than the utility from consuming \(p\) units of unproduced good, \(p \cdot \beta\). A consequence of this assumption is that the demand for produced good at \(x = 0\) is always greater than the number of customer relationships and all consumers exert some shopping effort.

Before describing the equilibrium, we briefly determine the product price that implements the optimal allocation \(\{x^{opt}, c^{opt}\}\). The optimal allocation is the product market tightness and consumption of produced good that maximize welfare subject to matching frictions on the product market. Formally, \(x^{opt}\) maximizes \(v(\kappa + f(x) \cdot (y - \kappa), \mu, x \cdot (y - \kappa))\). The first argument is the amount of consumption obtained with tightness \(x\) given the matching process (1). The second argument is the endowment of unproduced good. The third argument is that amount of shopping effort required to sustain a tightness of \(x\) if \(y - \kappa\) goods are available for sale to shoppers (since \(s \cdot q(x) = f(x) \cdot (y - \kappa)\), then \(s = x \cdot (y - \kappa)\)). A high \(x\) increases consumption because it increases the fraction of goods sold. But a high \(x\) also requires more shopping effort. At the optimum, the marginal utility from consumption equals the marginal cost from shopping:

\[
\frac{\partial v}{\partial c} = \frac{1}{1 - \eta} \cdot \frac{r}{q(x)},
\]

which is the pendant of the Hosios [1990] condition in the product market. Recall that the household’s optimal consumption condition imposes \(\partial v/\partial c = p \cdot \partial v/\partial m + r/q(x)\). By identification, the optimal allocation can be implemented in the product market if the price satisfies \(p \cdot \partial v/\partial m \bigg|_{m=\mu} = \left[\eta/ (1 - \eta)\right] \cdot [r/q(x)]\). With a quasilinear utility function, this condition imposes that

\[
p = \frac{P}{\beta}.
\]

The constant \(P\) is defined by \(P \equiv \left[\eta/ (1 - \eta)\right] \cdot [r/q(x^{opt})]\), where \(x^{opt}\) satisfies \(\kappa + f(x^{opt}) \cdot (y - \kappa) = [r/ [(1 - \eta) \cdot q(x^{opt})]]^{-\epsilon}\). The constant \(P\) does not depend on \(\beta\) so the optimal price schedule is inversely proportional to the marginal utility of unproduced-good consumption, \(\beta\).
3.2 Equilibrium

The model has two endogenous variables, $c$ and $x$. The two equations that describe the equilibrium are (8) and (1). To study the equilibrium, we define an aggregate supply and an aggregate demand:

**DEFINITION 1.** The aggregate demand is a function of product market tightness defined by

$$c^d(x) = \left[ \frac{1}{D} + \frac{r}{q(x)} \right]^{-\epsilon}$$

for all $x \in [0, x^*]$ and by $c^d(x) = \kappa$ for $x > x^*$. The constant $x^* > 0$ satisfies

$$\kappa = \left[ \frac{1}{D} + \left( \frac{r}{q(x^*)} \right) \right]^{-\epsilon}.$$ 

The aggregate supply is a function of product market tightness defined for all $x > 0$ by

$$c^s(x) = \kappa + f(x) \cdot (y - \kappa).$$

Unlike traditional supply and demand functions that take product price as argument, our supply and demand functions take product market tightness as an argument. For a given product market tightness, the aggregate demand gives the consumption of produced good that satisfies the worker’s optimal consumption condition (8) and the aggregate supply gives the consumption of produced good that satisfies the product market matching condition (1). Before characterizing the equilibrium, we establish a few properties of supply and demand:

**LEMMA 1.** Aggregate demand and aggregate supply satisfy the following properties:

- Under Assumption 1, $c^d(x)$ is strictly decreasing, $\lim_{x \to 0} c^d(x) = D^c$, and $\lim_{x \to +\infty} c^d(x) = \kappa$.

- $c^s(x)$ is strictly increasing, $\lim_{x \to 0} c^s(x) = \kappa$, and $\lim_{x \to +\infty} c^s(x) = y$.

The properties of aggregate demand and supply derive directly from the properties of the product-finding rate $q(x)$ and the selling probability $f(x)$. Proposition 1 establishes that the equilibrium admits a simple representation: it can be described by the intersection of aggregate supply and aggregate demand.

**PROPOSITION 1.** Under Assumption 1, the equilibrium exists and is unique. In equilibrium, product market tightness equalizes aggregate supply and aggregate demand:

$$c^s(x) = c^d(x).$$
The diagram in Figure 1 represents the product market equilibrium in a price $x$-quantity $c$ plan. Product market tightness $x$ acts as a price because it equalizes aggregate supply and aggregate demand in equilibrium. For example, imagine that demand is greater than supply. An increase in $x$ reduces demand by diminishing the product-finding rate and increasing the marginal cost of a purchase; it increases supply mechanically by increasing the selling probability; until supply and demand are equalized. In practice, the equilibrium is reached through adjustment of shopping effort. In our example, demand is greater than supply at the current tightness. Shopping effort by consumers is not sufficient to purchase the desired number of goods. Consumers shop more intensively, increasing tightness. The selling probability rises so producers sell more goods. We observe a movement along the aggregate supply curve. At the same time, the product-finding probability falls, purchasing costs rise, and the amount of produced good desired by consumers fall. We observe a movement along the demand curve. The adjustment of shopping effort continues until consumers close the gap between supply and demand.

Product market tightness acts as a price and is also perceived as a price by producers and consumers: producers like a high product market tightness, but consumers dislike it. The reason is that when product market tightness is high, producers sell a large fraction of their goods but shoppers find it difficult to buy goods.
3.3 Supply and Demand Shocks

We analyse the response of the economy to a variety of macroeconomic shocks using comparative statics. Our analysis relies on the following equilibrium condition:

\[
\left[\frac{1}{D} + \frac{r}{q(x)}\right]^{-\epsilon} = \kappa + f(x) \cdot [y - \kappa].
\]  

(13)

**Demand Shock.** A negative demand shock is a shock that lowers the constant \( D = 1/(\beta \cdot p) \). The two possible sources of a negative demand shock are an increase in consumers’ taste for the unproduced good, \( \beta \), and an increase in the product price, \( p \). On the equilibrium diagram in Figure 1, the aggregate demand shifts in and the aggregate supply does not respond after a negative demand shock. Therefore, a negative demand shock leads to lower consumption and lower product market tightness. Hence after a negative demand shock, the selling probability \( f(x) \) decreases and the production that cannot be sold, \( [1 - f(x)] \cdot (y - \kappa) \), increases.

**Supply Shock.** A negative supply shock is a reduction in the productivity of producers, \( y \). On the equilibrium diagram in Figure 1, the aggregate supply shifts in and the aggregate demand does not respond to the negative supply shock. Therefore, a negative supply shock leads to lower consumption and higher product market tightness. Hence after a negative supply shock, the selling probability \( f(x) \) increases and the production that cannot be sold, \( [1 - f(x)] \cdot (y - \kappa) \), decreases. The decrease in the amount of unsold production is due both to a higher selling probability and a lower amount of production.

**Role of Price Rigidity.** Consider demand shocks arising from a change in consumers’ taste for the unproduced good, \( \beta \). If the product price \( p \) followed the optimal price schedule (9), then \( p \cdot \beta \) and the constant \( D = 1/(\beta \cdot p) \) would remain constant after a shock to \( \beta \). In that case, shocks to \( \beta \) would have no effect on aggregate demand, on equilibrium consumption, and on equilibrium tightness. But as long as \( p \) is less than inversely proportional to \( \beta \), then \( D \) responds to a shock to \( \beta \). To summarize, if prices are rigid in the sense that \( p \propto \beta^{-\gamma} \) with \( \gamma < 1 \), then a negative demand shock leads to a contraction. But if prices are flexible in the sense that \( p \propto \beta^{-1} \), then demand shocks are neutral.

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34 An increase of the disutility of shopping, \( r \), could also be considered as negative demand shock. Such an increase has exactly the same effects as a decrease of \( D \). It is, however, more difficult to interpret a shock to \( r \).

35 Formally, \( \partial c^d/\partial D \big|\big|_x = 0, \partial c^d/\partial D \big|\big|_x > 0, \partial c^d/\partial x \big|\big|_D > 0, \partial c^d/\partial x \big|\big|_D < 0, \) thus, \( dx/dD > 0 \) and \( dc/dD > 0 \).

36 Formally, \( \partial c^s/\partial y \big|\big|_x = 0, \partial c^s/\partial y \big|\big|_x < 0, \partial c^d/\partial y \big|\big|_y > 0, \partial c^d/\partial y \big|\big|_y < 0, \) thus, \( \partial x/\partial y < 0 \) and \( \partial c/\partial y > 0 \).
3.4 Fiscal Multipliers

We analyse the response of the economy to a variety of policy interventions. To summarize the impact of a policy intervention, we compute a multiplier. Before turning to the multiplier, we briefly derive the aggregate demand curve with a CRRA utility

\[ v(c, m, s) = \frac{\epsilon}{\epsilon - 1} \left[ c^{\frac{\epsilon - 1}{\epsilon}} + \beta \cdot m^{\frac{\epsilon - 1}{\epsilon}} \right] - r \cdot s. \tag{14} \]

This specification is useful to understand the effect of government intervention. As with the quasilinear utility above, the aggregate demand remains given by (10). But the constant \( D \) is now given by

\[ D = \frac{\mu^{1/\epsilon}}{\beta \cdot p}, \tag{15} \]

where \( \mu \) is the endowment of unproduced good, \( p \) is the product price, and \( \beta \) measures the preference for the unproduced good. A fall in \( \mu \) generates a negative demand shock exactly as an increase in \( p \) or an increase in \( \beta \). In this section, we assume that consumers have CRRA utility.

The multiplier depends on the fiscal policy implemented. Consider an income tax \( \tau \) that finances the purchase of \( g \) units of produced good. Total purchases are the sum of purchases by consumers, \( c \), and purchases by the government, \( g \), so (1) becomes \( c + g = \kappa + f(x) \cdot (y - \kappa) \). The consumer’s budget constraint becomes

\[ m + p \cdot c = (1 - \tau) \cdot [\mu + p \cdot [\kappa + f(x) \cdot (y - \kappa)]]. \]

The government’s budget balance imposes that

\[ p \cdot g = \tau \cdot [\mu + p \cdot [\kappa + f(x) \cdot (y - \kappa)]]. \]

The consumer and government budget balance, together with equilibrium on the product market, ensure that \( m = \mu \) in equilibrium. Furthermore, we assume that government consumption enters separately into consumers’ utility function such that it does not affect their consumption decisions. Then, the consumer’s demand remains given by (8). Accordingly when the government consumes \( g \), equilibrium product market tightness \( x \) is given by

\[ c^s(x) = c^d(x) + g, \tag{16} \]

where aggregate supply \( c^s(x) \) and private demand \( c^d(x) \) are defined by (11) and (10). Equilibrium private consumption is given by \( c = c^d(x) \). We define the balanced-budget multiplier \( \lambda^{BB} \) as the effect of \( g \) on total purchases \( c + g \). Differentiating this equilibrium condition with respect to \( g \) yields

\[ \frac{\partial c^s}{\partial x} \cdot \frac{\partial x}{\partial g} = \frac{\partial c^d}{\partial x} \cdot \frac{\partial x}{\partial g} + 1. \]
This equation allows us to express $\partial x/\partial g$ as a function of the elasticity of aggregate supply with respect to tightness, $e^s \equiv (x/e^s) \cdot (\partial e^s/\partial x) > 0$, and the elasticity of private demand with respect to tightness, $e^d \equiv -(x/e^d) \cdot (\partial e^d/\partial x) > 0$. (The elasticity $e^d$ is normalized to be positive.) We obtain $\partial x/\partial g = (x/c) \cdot \left[1/(e^d + e^s)\right]$. Using the fact that in equilibrium $\partial c/\partial g = (\partial e^d/\partial x) \cdot (\partial x/\partial g)$, we obtain a formula for the balanced-budget multiplier:

$$\lambda^{BB} \equiv 1 + \frac{\partial c}{\partial g} = \frac{e^s}{e^d + e^s}. \quad (17)$$

It follows that the balanced-budget multiplier is positive, but necessarily less than one. This means that government consumption necessarily crowds out, at least partially, private consumption. After the increase in government purchases, the aggregate demand curve shifts outward. To reach the new equilibrium, product market tightness increases. The product-finding rate falls: it is more difficult for consumers to purchase goods because the government purchase some of the goods available. Thus, consumers reduce consumption, a movement inward along the private demand curve. The size of the multiplier depends on the elasticity of private demand relative to the elasticity of aggregate supply. If private demand is completely inelastic ($e^d = 0$), the multiplier is exactly one. This happens if shopping is costless to consumers ($r = 0$). If the elasticities fluctuate systematically over the business cycle, then the multiplier does fluctuate over the business cycle.

In our model, government consumption financed by deficit spending could have a much larger effect that a balanced-budget intervention. Consider that the government owns some unproduced good, $\tau \cdot \mu$, and that it sells it to purchase some produced good, $g$. In that case the multiplier may be much larger. Total purchases are the sum of purchases by consumers and purchases by the government so $c + g = \kappa + f(x) \cdot (y - \kappa)$. The consumer’s budget constraint remains given by (3). The government’s budget balance imposes that $p \cdot g = \tau \cdot \mu$. The consumer and government budget balance, together with equilibrium on the product market, impose that $m = (1 + \tau) \cdot \mu = \mu + p \cdot g$ in equilibrium. The consumer’s income increases because the government depletes its endowment of unproduced good. Furthermore, we assume that government consumption enters separately into consumers’ utility function such that it does not affect their consumption decisions. Accordingly when the government consumes $g$, equilibrium product market tightness $x$ is given by

$$c^s(x) = c^d(x, g) + g, \quad (18)$$
where aggregate supply \( c^s(x) \) is defined by (11) and private demand \( c^d(x, g) \) is defined by (10) with
\[
D = \frac{(\mu + p \cdot g)^{1/\epsilon}}{\beta \cdot p}.
\]

Equilibrium private consumption is given by \( c = c^d(x, g) \). We define the deficit-financed multiplier \( \lambda^{DF} \) as the effect of \( g \) on total purchases \( c + g \). Differentiating this equilibrium condition with respect to \( g \) yields
\[
\frac{\partial c^s}{\partial x} \cdot \frac{\partial x}{\partial g} = \frac{\partial c^d}{\partial g} + \frac{\partial c^d}{\partial x} \cdot \frac{\partial x}{\partial g} + 1.
\]

We find that \( \frac{\partial c^d}{\partial g} = \beta \cdot p^2 \cdot \left(\frac{c}{\mu}\right)^{(\epsilon+1)/\epsilon} \) when the derivative is evaluated at \( g = 0 \). We obtain \( \frac{\partial x}{\partial g} = \left(\frac{x}{c}\right) \cdot \left[1 + \beta \cdot p^2 \cdot \left(\frac{c}{\mu}\right)^{(\epsilon+1)/\epsilon}\right] / \left(\epsilon^d + \epsilon^s\right) \). Using the fact that in equilibrium, \( \frac{\partial (c+g)}{\partial g} = (\frac{\partial c^s}{\partial x}) \cdot (\frac{\partial x}{\partial g}) \), we obtain a formula for the deficit-financed multiplier:
\[
\lambda^{DF} \equiv 1 + \frac{\frac{\partial c}{\partial g}}{\frac{\partial c^s}{\partial x}} = \left[1 + \beta \cdot p^2 \cdot \left(\frac{c}{\mu}\right)^{(\epsilon+1)/\epsilon}\right] \cdot \frac{\epsilon^s}{\epsilon^d + \epsilon^s}.
\]

It follows that the deficit-financed multiplier is positive, necessarily larger than the balanced-budget multiplier, and possibly greater than one. The deficit-financed multiplier is larger than the budget-balanced multiplier because the government raises consumers’ income by purchasing produced goods. Higher income stimulates private demand, following the same mechanism as the multiplier effect arising with a Keynesian Cross. Once again, the size of the multiplier depends on the elasticity of private demand relative to the elasticity of aggregate supply. If private demand is completely inelastic \( (\epsilon^d = 0) \), the multiplier is strictly above one.

### 3.5 An Interpretation of Unproduced Good as Saving, and A Paradox of Thrift

The unproduced good can be interpreted as future consumption. Suppose that there is a single consumption good. The good is produced in the current period, and it can be consumed in the current period or invested for future consumption. Let \( c \) be the amount consumed in the current period, \( m \) be the amount consumed in the future, and \( \sigma \) be the amount invested. The investment is transformed into future consumption by an increasing and concave production function: \( m = m(\sigma) \). Let \( p \) be the price of current production in terms of future consumption. The price can be interpreted as the gross return to savings:
$p = 1 + r$ where $r$ is the interest rate. For simplicity, we assume that the entrepreneurs transforming investment into future consumption are not subject to matching frictions when buying production. (The entrepreneurs are part of established customer-seller relationships.) Given $p$, the entrepreneur chooses $\sigma$ to maximize profit $\pi = m(\sigma) - \sigma \cdot p$. The first-order condition is $m'(\sigma) = p$. It defines the optimal investment and optimal production as decreasing functions of $p$: $\sigma(p) = (m')^{-1}(p)$ and $\mu(p) = m(\sigma(p))$. The profits of the entrepreneurs are redistributed uniformly to all consumers.

Current and future consumption enter a consumer’s utility function according to the CRRA utility (14). The parameter $\beta$ can be interpreted as a time discount factor. Given $x$ and $p$, consumers choose $c$ and $m$ to maximize (14) subject to the budget constraint $m + p \cdot c = [\mu(p) - \sigma(p) \cdot p] + p \cdot [\kappa + f(x) \cdot (y - \kappa)]$ and to the matching process on the product market: $c + \sigma(p) = \kappa + s \cdot q(x)$. The first-order conditions with respect to $c$, $m$, and $s$ are the same as in the model with unproduced good. In addition, combining the matching constraint with the budget constraint yields $m = \mu(p)$. Thus as in Section 3.4, the demand for current consumption is given by (10) with $D = \mu(p)^{1/\epsilon}/(\beta \cdot p)$. Compared to the model with unproduced good, the endowment $\mu$ is no longer a parameter but a decreasing function of $p$. The constant $D$, which indicates the level of demand for current consumption, is a decreasing function of $p$ and $\beta$. In fact, a high $p$ has two negative effects on current consumption. A high $p$ has a substitution effect: it leads consumers to shift consumption to the future. A high $p$ also has an income effect: by increasing the price of the input used for the production of future consumption, it reduces the amount of future consumption produced, which leads consumers to reduce current consumption further. The supply for current consumption is given by $c^s(x) = [\kappa - \sigma(p)] + f(x) \cdot [y - \kappa]$. Compared to the model with unproduced good, the supply function includes the term $-\sigma(p)$ that measures current production saved and thus not available for current consumption.

In the model with saving, a shock to the time discount factor $\beta$ leads to a paradox of thrift. Suppose that $\beta$ increases. As in the model with unproduced good, the demand for current consumption shifts in whereas the supply is unaffected in a price $x$-quantity $c$ diagram. Therefore in equilibrium, current consumption falls. Since $p$ remains constant, future consumption, given by $m = \mu(p)$, remains constant. To summarize the model exhibits a paradox of thrift: even though consumers value future consumption more, the new equilibrium is such that consumers consume the same in the future but consumer less in the current period. The reason is that the price $p$ of current consumption in terms of future consumption does not adjust to the change in preference.

The effect of a shock to the price $p$ is more complex than in the model with unproduced good because
it affects both demand and supply of current consumption. Suppose that $p$ increases. The constant $D$ and the investment $\sigma(p)$ both decrease. Therefore the demand curve shifts in while the supply curve shifts out in a price $x$-quantity $c$ diagram. It is clear that product market tightness $x$ decreases. What happens to current consumption $c$ is unclear. If the demand curve is inelastic enough, current consumption falls. In that case, a shock to $p$ has the same effect as in the model with unproduced good. Last, a shock to $y$ has exactly the same effect as in the model with unproduced good.

### 3.6 Discussion

We have presented a model of the product market that provides microfoundations for an aggregate supply function and an aggregate demand function. In fact, our product market with quasilinear utility is isomorphic to the labor market in Michaillat [2012a]: the produced good replaces labor, the unproduced good replaces the numéraire consumption good, the utility function replaces the profit function, buyer-seller matches replace firm-worker matches consumers replace firms, producers with customers replace incumbent workers, producers looking for buyers replace jobseekers, product price replaces real wage, and product market tightness replaces labor market tightness. Accordingly, our product market admits the same equilibrium representation as the labor market of Michaillat [2012a]: the equilibrium is given by the intersection of a downward-sloping product demand curve and an upward-sloping product supply curve in a product market tightness-consumption plane instead of being given by the intersection of a downward-sloping labor demand curve and an upward-sloping labor supply curve in a labor market tightness-employment plane.\(^{37}\) Another consequence of this isomorphism is that the properties of our balanced-budget multiplier are quite similar to the properties of the public-employment multiplier studied by Michaillat [2012b] using the labor market of Michaillat [2012a].

### 4 Enriching Aggregate Supply with a Labor Market

This section introduces into the previous model profit-maximizing firms that use labor to produce a consumption good. Firms and labor market are modeled using the framework of Michaillat [2012a]. The production function of firms has diminishing marginal returns to labor. Firms hire workers on a labor market with trade frictions. These frictions are captured by the standard search-and-matching structure.

\(^{37}\)The equilibrium representation of the labor market of Michaillat [2012a] in a labor market tightness-consumption plane is developed in Landais, Michaillat and Saez [2010].
As in the product market, trade friction in the labor market create a situation of bilateral monopoly in wage setting. The indeterminacy is resolved by assuming an exogenous real wage schedule.

Even though the model comprises a labor market and a product market, it remains tractable. The general equilibrium can be represented as the intersection of an aggregate supply function and an aggregate demand function, with product market tightness acting as a price. The aggregate demand function solves the trade-off between consumptions of produced and unproduced good, given product market tightness. The aggregate supply function represents the expected amount of sales by firms given product market tightness and optimal hiring when the labor market is in equilibrium. Furthermore, given equilibrium product market tightness, we represent the labor market equilibrium as the intersection of a labor supply function and a labor demand function, with labor market tightness acting as a price. The labor supply represents the expected number of workers who have a job after matching, given labor market tightness. The labor demand represents the desired employment level in firms given labor market tightness, and given that the probability to sell production is controlled by the equilibrium product market tightness.

4.1 Labor Market and Firms

Labor Market. There is a measure one of workers. Initially $u \in (0,1)$ workers are unemployed and search for a job while $1-u$ workers are employed by firms. Firms post $o$ vacancies to recruit workers. The number of firm-worker matches is given by a Cobb-Douglas matching function $\hat{h} = \hat{\omega} \cdot u^\mu \cdot o^{1-\mu}$, with the restriction that $\hat{h} \leq u$. The parameter $\hat{\omega} > 0$ measures the effectiveness of matching, and $\mu \in (0,1)$ is the elasticity of matching with respect to unemployment. We define the labor market tightness $\theta$ as the ratio of vacancies to unemployment: $\theta \equiv o/u$. Because of the trade frictions, not all unemployed workers can find a job. A jobseeker finds a job with probability $\hat{f}(\theta) = \hat{h}/u = \hat{\omega} \cdot \theta^{1-\mu}$, with the restriction that $\hat{f}(\theta) \leq 1$. We refer to $\hat{f}(\theta)$ as the job-finding probability. A vacancy leads to $\hat{q}(\theta) = \hat{h}/o = \hat{\omega} \cdot \theta^{-\mu}$ hires. We refer to $\hat{q}(\theta)$ as the hiring rate. The function $\hat{f}$ is increasing and the function $\hat{q}$ is decreasing. When the labor market is slack it is difficult for jobseekers to find a job but easy for firms to fill vacancies. The converse is true when the labor market is tight. Given the matching process, employment at the end of the matching process is

$$n = 1 - u + u \cdot \hat{f}(\theta). \quad (20)$$
**Firms.** The representative firm uses labor $n$ to produce output $y$ according to the production function

$$y = a \cdot \left[ z(n) - \frac{\hat{r}}{\hat{q}(\theta)} \cdot (n - (1 - u)) \right] \equiv a \cdot g(n, \theta). \quad (21)$$

The parameter $a$ measures the technology of the firm. The function $z(n) \equiv n^\alpha$ where $\alpha \in (0, 1)$ captures decreasing marginal returns to labor. The output cost of posting a vacancy is $\hat{r} \cdot a$, where $\hat{r} > 0$ measures the resources spent on recruiting. We assume away randomness at the firm level: the firm hires a worker with certainty by opening $1/\hat{q}(\theta)$ vacancies. Therefore, the firm foregoes $\hat{r} \cdot a/\hat{q}(\theta)$ units of output when it hires a worker. The number of workers hired by the firm is $n - (1 - u) > 0$.

Given the trade frictions on the product market the firm sells the first $\kappa$ units of output to customers, but it only sells a fraction $f(x)$ of the remaining $y - \kappa$ units of output to shoppers. The firm also incurs labor costs: it pays a real wage $w$ to its $n$ employees, where real wage denotes wage value expressed in units of produced good. Therefore, the real profits of the firm are

$$\pi = \kappa + f(x) \cdot a \cdot \left[ z(n) - \frac{\hat{r}}{\hat{q}(\theta)} \cdot (n - (1 - u)) - \kappa \right] - w \cdot n. \quad (22)$$

Given labor market tightness $\theta$, product market tightness $x$, and real wage $w$, the representative firm chooses employment $n$ to maximize real profits (22). After dividing by $a$, the first-order condition is

$$z'(n) = \frac{w}{a \cdot f(x)} + \frac{\hat{r}}{\hat{q}(\theta)}. \quad (23)$$

Equation (23) shows two departures from the neoclassical first-order condition, $a \cdot z'(n) = w$, in which the marginal product of labor equals the real wage. First, $w/f(x)$ replaces $w$ because the production of the marginal worker is only sold with probability $f(x)$, thereby increasing the real wage per unit of effective labor by a factor $1/f(x)$. Second, a new term $\hat{r} \cdot a/\hat{q}(\theta)$ is added to the real wage because recruiting the marginal worker requires posting $1/\hat{q}(\theta)$ vacancies at a cost $\hat{r} \cdot a$ per vacancy, thereby increasing the marginal cost of labor.

As in the previous section, we require that $y > \kappa$ such that all existing customer-seller relationships are maintained. Thus, it is necessary to make the following assumption:

**ASSUMPTION 2.** The number of existing customers-seller relationships is small enough:

$$\kappa < a \cdot z(1 - u).$$
Under Assumption 2 the production of the firm, even without new hires, is greater than the customer base. The assumption ensures that the firm sells some goods to shoppers.

**Real Wage.** The real wage is set once worker and firm have matched. If firm and worker cannot agree on a wage, the worker remains unemployed and the firm foregoes some production even though this production yields a positive marginal profit. Accordingly, there are always mutual gains from trade when a worker and a firm match. As discussed in Section 2.1, there is no compelling theory of wage determination in such a situation of bilateral monopoly. Hence we assume that the real wage is a parameter \( w > 0 \).

Since a worker incurs no disutility from working, a jobseeker would accept any positive wage. On the other hand the wage must be low enough such that a firm prefers to keep all of its \( 1 - u \) employees, which requires the following assumption:

**ASSUMPTION 3.** Given product market tightness \( x \), the real wage is low enough to be Pareto efficient in \( 1 - u \) firm-worker matches:

\[
    w < a \cdot f(x) \cdot z'(1 - u).
\]

Under Assumption 3, the marginal revenue from employing the \( 1 - u \)th worker, \( a \cdot f(x) \cdot z'(1 - u) \), is higher than the marginal cost of employing the worker, \( w \). A consequence of this assumption is that the demand for labor at \( \theta = 0 \) is always greater than the number of existing employees and all firms post some vacancies to recruit new workers.

**4.2 Equilibrium**

The model has five endogenous variables, \( y, c, n, \theta, \) and \( x \). The five equations that describe the equilibrium are (8), (1), (23), (20), and (21). To study the equilibrium, we first define a labor supply and a labor demand:

**DEFINITION 2.** The labor demand is a function of labor market tightness and product market tightness defined by

\[
    n^d(\theta, x) = \left[ \frac{w}{a \cdot f(x)} + \frac{\hat{r}}{\hat{q}(\theta)} \right]^{-1/(1-\alpha)}
\]

for all \( \theta \in [0, \theta^*(x)] \) and by \( n^d(\theta, x) = 1 - u \) for \( \theta > \theta^*(x) \). The function \( \theta^*(x) > 0 \) is implicitly defined by \( 1 - u = \left[ w / \left( a \cdot f(x) + \hat{r} / \hat{q}(\theta^*) \right) \right]^{-1/(1-\alpha)} \). The labor supply is a function of labor market tightness
defined for all $\theta > 0$ by

$$n^s(\theta) = 1 - u + u \cdot \hat{f}(\theta).$$

(25)

Unlike traditional labor supply and labor demand functions that take real wage as argument, our labor supply and labor demand functions take labor market tightness as an argument. For a given labor market tightness, the labor demand gives the level of employment that satisfies the firm’s profit-maximization condition (23) and the labor supply gives the employment rate that satisfies the labor market matching condition (20). Lemma 2 establishes a few properties of labor supply and labor demand:

**LEMMA 2.** Labor demand and labor supply satisfy the following properties:

- Under Assumption 3, $n^d(\theta, x)$ is strictly decreasing in $\theta$, strictly increasing in $x$, $\lim_{\theta \to +\infty} n^d(\theta, x) = 1 - u$, $\lim_{\theta \to 0} n^d(\theta, x) = n^*(x)$ where the function $n^*(x)$ is implicitly defined by $z'(n^*) = w/[a \cdot f(x)]$.

- $n^s(\theta)$ is strictly increasing, $\lim_{\theta \to 0} n^s(\theta) = 1 - u$ and $\lim_{\theta \to +\infty} n^s(\theta) = 1$.

The properties of labor demand and labor supply derive directly from the properties of the functions $f$, $\hat{f}$, and $\hat{q}$. When $n^*(x) < 1$, $1 - n^*(x)$ is rationing unemployment in the sense of Michaillat [2012a]: it is the unemployment level that would prevail absent any matching frictions on the labor market ($\hat{r} = 0$).

Proposition 2 establishes that the partial equilibrium on the labor market admits a simple representation: it can be described by the intersection of labor supply and labor demand.

**PROPOSITION 2.** Let $x > 0$ be the product market tightness. Under Assumption 3, there exists a unique pair of labor market tightness $\theta(x) > 0$ and employment $n(x) \in [1 - u, 1]$, such that (23) and (20) hold. Equilibrium labor market tightness $\theta(x)$ equalizes aggregate supply and aggregate demand:

$$n^s(\theta) = n^d(\theta, x).$$

(26)

Equilibrium employment is $n(x) = n^s(\theta(x))$. Tightness $\theta(x)$ and employment $n(x)$ are strictly increasing functions of $x$.

The diagram in Figure 2(a) represents the labor market equilibrium in a wage $\theta$-quantity $n$ plan, taking product market tightness $x$ as given. Labor market tightness $\theta$ acts as a wage because it equalizes labor supply and labor demand in equilibrium. For example, imagine that labor demand is above labor
supply. An increase in $\theta$ reduces labor demand by increasing the marginal recruiting cost; it increases labor supply by increasing the job-finding rate; until labor supply and labor demand are equalized. In practice, the equilibrium is reached through posting of vacancies. For instance if labor demand is above labor supply at the current tightness, the number of vacancies posted by firms is not sufficient to hire the desired number of workers. Firms post more vacancies, increasing tightness. The job-finding rate rises so more jobseekers find a job and jobseekers search more. We observe a movement along the labor supply curve. At the same time, the vacancy-filling probability falls, hiring costs rise, and the employment desired by firms fall. We observe a movement along the labor demand curve. The adjustment continues until firms close the gap between supply and demand by posting vacancies.

Labor market tightness acts as a wage and is also perceived as a wage by workers and firms: workers like a high labor market tightness, but firms dislike it. The reason is that when labor market tightness is high, it is easy for workers to find a job but it is difficult for firms to hire new workers.

Having characterized the partial equilibrium on the labor market given product market tightness, we can now define a new aggregate supply:

**DEFINITION 3.** The aggregate supply is a function of product market tightness defined for $x > 0$ by

$$c^s(x) = \kappa + f(x) \cdot (a \cdot g(n(x), \theta(x)) - \kappa).$$  

(27)

The aggregate supply incorporates the condition that the labor market is in equilibrium for a given product market tightness, which determines $\theta(x)$ and $n(x)$ as specified in Proposition 2. The aggregate demand is described in Definition 1, and satisfies the properties listed in Lemma 1.

At this point, it is useful to define a few function. Let $Y(\theta) \equiv g(n^s(\theta), \theta)$. Combining (20) and (21), we have $Y(\theta) = z(1-u + u \cdot \hat{f}(\theta) - u \cdot \hat{r} \cdot \theta).$ Noting that $\hat{f}(\theta)$ has elasticity $1-\eta$ and that $\hat{q}(\theta) = \hat{f}(\theta)/\theta$, $Y(\theta)$ is maximized for $\theta = \theta^H$ where

$$z'(n^s(\theta^H)) = \frac{1}{1-\eta} \cdot \frac{\hat{r}}{\hat{q}(\theta^H)}.$$  

(28)

The labor market tightness $\theta^H$ maximizes output in the labor market, and it was first described by Hosios [1990]. Since $Y$ is concave, $Y$ is increasing for $\theta < \theta^H$.38 Let $x^H$ be the product market tightness such that $\theta(x^H) = \theta^H$. We can only show that $c^s(x)$ is strictly increasing for $0 \leq x \leq x^H$. For $x > x^H$,

---

38The function $Y$ is concave because $Y''(\theta) = u \hat{f}''(\theta) \cdot z'(n) + u^2 (\hat{f}'(\theta))^2 \cdot z''(n) < 0$ as both $z$ and $\hat{f}$ are concave and increasing.
(a) Labor supply, labor demand, and partial equilibrium on the labor market

(b) Aggregate supply, aggregate demand, and general equilibrium

Figure 2: Representation of the general equilibrium
the selling probability \( f(x) \) increases with \( x \) but the output \( Y(\theta(x)) \) decreases with \( x \) because \( Y(\theta) \) decreases with \( \theta \). In this region of product market tightness, output decreases with labor market tightness and employment because the labor market is so tight that reducing unemployment requires to devote more resources to recruiting than it creates output. To avoid this situation in which the labor market is overheated to the point that aggregate supply may bend backward, we make an assumption such that \( \theta < \theta^H \) for all \( x \):

**ASSUMPTION 4.** The wage is high enough such that for any product market tightness, reducing unemployment increases output:

\[
\frac{w}{a} > \frac{\eta}{1 - \eta} \cdot \frac{\hat{r}}{q(\theta^H)}.
\]

Under this assumption, we are able to establish the following properties of aggregate supply:

**LEMMA 3.** Under Assumption 4, aggregate supply satisfies the following properties:

- \( c^s(x) \) is increasing for all \( x \geq 0 \).
- \( \lim_{x \to 0} c^s(x) = \kappa \).
- \( \lim_{x \to +\infty} c^s(x) = y^M < a \cdot Y(\theta^H) \), where \( y^M \equiv a \cdot Y(\theta^M) \) and \( \theta^M \) satisfies \( z'(n^s(\theta^M)) - \frac{\hat{r}}{\hat{q}(\theta^M)} = \frac{w}{a} \).

Proposition 3 establishes that the equilibrium admits a simple representation: it can be described by the intersection of aggregate supply and aggregate demand.

**PROPOSITION 3.** Under Assumption 1 and 4, the equilibrium exists and is unique. In equilibrium, product market tightness equalizes aggregate supply and aggregate demand:

\[
c^s(x) = c^d(x).
\] (29)

The diagram in Figure 2(b) represents the general equilibrium in a price \( x \)-quantity \( c \) plan.

### 4.3 Some Special Cases

The model nests as special case a fixprice-fixwage model as well as several macroeconomic models with trade frictions.
**Fixprice-Fixwage Model.** Assume CRRA utility. Suppose that there are no existing customer-seller relationships ($\kappa = 0$), no existing worker-firm relationships ($u = 1$), no shopping costs ($r = 0$), and no recruiting cost ($\hat{r} = 0$). The equilibria in the labor and product markets are depicted on Figure 3.

Aggregate demand is given by $c^d = (\beta \cdot p)^{-\epsilon} \cdot \mu$. Aggregate demand is a downward-sloping curve in a traditional price $p$-quantity $c$ plane. Given $p$, it is a vertical curve in our price $x$-quantity $c$ plane. The vertical demand curve is set at $c^d$ defined as the intersection of price $p$ and traditional aggregate demand. In a regime of Keynesian unemployment with excess supply on the product market, the equilibrium is on this vertical curve.

Labor demand is given by $z'(n^d) = w / [a \cdot f(x)]$. The notional labor demand, obtained when the firm sells all of its output ($f(x) = 1$), is obtained by $z'(n^d) = w/a$. Both labor demand and notional labor demand are downward-sloping curve in a traditional wage $w$-quantity $n$ plane. Given $w$, it is a vertical curve in our wage $\theta$-quantity $n$ plane. The vertical curve is set of $n^d$ defined as the intersection of wage $w$ and traditional labor demand. In a regime of Keynesian unemployment with excess supply on the labor market, the equilibrium is on this vertical curve.

To determine the level of employment in equilibrium, we need to determine equilibrium selling probability $f(x)$. To do so, we need to specify the matching process on the product market, which corresponds to the rationing rule in fixprice models. In the typical fixprice model, such as in Barro and Grossman [1971], it is assumed that the first $c^d$ units are sold with probability $f(x) = 1$, and that the rest of the production above $c^d(x)$ is not sold ($f(x) = 0$). This matching rule implies that equilibrium employment $n$ satisfies $z'(n) = w/a$ or $a \cdot z(n) = c^d$, whichever is smaller. If the resulting $n < 1$, then there is unemployment $1 - n > 0$. If there is unemployment and $z'(n) = w/a$, the economy experiences classical unemployment because there is excess demand on the product market (as $a \cdot z(n) < c^d$). If there is unemployment and $a \cdot z(n) = c^d$, the economy experiences Keynesian unemployment because there is excess supply in both labor and product market. In this regime only, the real wage is irrelevant for the prevailing level of unemployment. With this specific matching process, there are several regimes that complicate the equilibrium analysis. In addition, the product market tightness is irrelevant.

The result that the real wage is irrelevant for the prevailing level of unemployment is a special one. It relies on the special matching process assumed in the paragraph above. The result disappears if we assume instead that each unit of output is sold with the same probability $f(x)$, and that $f(x)$ adjusts such that $c^d = f(x) \cdot a \cdot z(n) \equiv c^s(x)$. This assumption corresponds to a standard search-and-matching
framework. With this assumption, the aggregate supply simplifies to

\[ c^s(x) = f(x) \cdot a \cdot z(n(x)) = \alpha^{\alpha/(1-\alpha)} \cdot w^{-\alpha/(1-\alpha)} \cdot [a \cdot f(x)]^{1/(1-\alpha)}. \]  

(30)

The aggregate supply function is increasing. The equilibrium product market tightness \( x \) is uniquely defined by \( c^s(x) = c^d \), which implies \( a \cdot f(x) = 1/[A \cdot w^{-\alpha}] \), where \( A \) is a constant, function of the parameters \( \alpha, \mu, \beta, \epsilon, \) and \( p \). Thus, equilibrium employment \( n \) satisfies \( z'(n) = A \cdot w^{1-\alpha} \) and \( n \) is a decreasing function of the real wage \( w \). Once we replace the special matching process assumed in fixprice models by a standard matching process from the search-and-matching literature, the real wage always influences unemployment, even in the Keynesian unemployment regime.

**Model with Linear Production Function and Linear Utility Function.** Assume that \( \alpha = 1 \) and \( \epsilon \to +\infty \) such that production and utility function be linear. Those are common assumption in the search-and-matching literature [for example, Hall, 2008]. The equilibria in the labor and product markets are depicted on Figure 4.

The aggregate demand relationship becomes

\[ 1 = p \cdot \beta + \frac{q(x)}{q(x)}, \]

which determines equilibrium product market tightness \( x(p) \) as a decreasing function of the product price \( p \). For the equilibrium to exist, it is necessary that \( p < 1/\beta \). The labor demand relationship becomes

\[ 1 = \frac{w}{a \cdot f(x)} + \frac{\hat{r}}{\hat{q}(\theta)}, \]

which determines equilibrium labor market tightness \( \theta \) as a function of the wage \( w \) and the product market tightness \( x \). For the equilibrium to exist, it is necessary that \( w < a \). If \( f(x(p)) < 1 \), \( w \) must be lower such that \( w \leq a \cdot f(x(p)) \). A change in wage only affects labor market tightness whereas a change in price affects both labor and product market tightnesses.

Both labor demand and aggregate demand are perfectly elastic. In the diagrams of Figure 4, which represent product and labor markets under these assumptions, both labor demand and aggregate demand are represented by horizontal curves. The quantities are simply read off the supply curves at the equilibrium tightness: for equilibrium employment, \( n = n^s(\theta) \); for equilibrium consumption, \( c = c^s(x) \).
Labor supply $n^s(\theta)$

Labor demand $n^d(x)$

Aggregate supply $c^s(x)$

Aggregate demand $c^d$

(a) Labor supply, labor demand, and partial equilibrium on the labor market

(b) Aggregate supply, aggregate demand, and general equilibrium

Figure 3: General equilibrium of a fixprice-fixwage model in Keynesian unemployment regime
(a) Labor supply, labor demand, and partial equilibrium on the labor market

(b) Aggregate supply, aggregate demand, and general equilibrium

Figure 4: General equilibrium with linear utility function and linear production function
Table 5: Comparative static effect of macroeconomic shocks

<table>
<thead>
<tr>
<th>Shock</th>
<th>Product market</th>
<th>Labor market</th>
<th>Type of shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c$</td>
<td>$x$</td>
<td>$n$</td>
</tr>
<tr>
<td>Increase of product price $p$</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Increase of taste for unproduced good $\beta$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Decrease of endowment $\mu$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Increase of real wage $w$</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Decrease of technology $a$</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Decrease of labor supply (search effort)</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Increase of labor market mismatch</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: These comparative statics, derived in Section 4.4, show how various shocks affect the key variables. A “+” sign indicates an increase, a “-” sign indicates a decrease. The variable $c$ is consumption of produced good, $x$ is product market tightness (the selling probability $f(x)$ moves like $x$ as $f$ is increasing), $n$ is employment, $\theta$ is labor market tightness, and $y$ is output (including unsold production). In those comparative statics, the product price, $p$, and the real wage, $w$, are kept fixed unless the shock is a change to $p$ or $w$.

4.4 Macroeconomic Shocks

We study the effect of a variety of macroeconomic shocks in the general equilibrium model using comparative statics. We focus on the case with CRRA utility. The responses of the economy to these shocks are summarized in Table 5. The analysis relies on auxiliary functions that we define here. Let $W(x) \equiv w/[a \cdot f(x)];$ let $\Theta(W)$ be the function implicitly defined by $z'(n^s(\Theta)) - \dot{r}/\dot{q}(\Theta) = W;$ and let $N(W) \equiv n^s(\Theta(W)).$ $\Theta(W)$ and $N(W)$ both decrease with $W$. We refer to $W$ as the effective wage. Recall that $Y(\theta) \equiv g(n^s(\theta), \theta).$ We assume that Assumptions 1, 3, and 4 hold such that $c^d(x)$ is strictly decreasing in $x$, $n^d(\theta, x)$ is strictly decreasing in $\theta$, $c^s(x)$ is increasing in $x$, and $Y(\theta)$ is increasing in $\theta$.

We obtain comparative statics from the following condition:

$$
\left[ \frac{1}{D} + \frac{r}{q(x)} \right]^{-\epsilon} = \kappa + f(x) \cdot \left[ \frac{a \cdot Y(\Theta(W(x))) - \kappa}{c^s(x)} \right].
$$

(31)

**Demand Shock.** A negative demand shock is a shock that lowers the constant $D = \mu^{1/\epsilon}/(\beta \cdot p)$. The three possible sources of a negative demand shock are an increase in consumers’ taste for the unproduced good, $\beta$, an increase in the product price, $p$, and a reduction in the endowment of unproduced good, $\mu$.\(^{39}\)

After a negative demand shock, in Figure 2(b), the aggregate demand shifts in and the aggregate supply

\(^{39}\)An increase of the disutility of shopping, $r$, could also be considered as negative demand shock. Such an increase has exactly the same effects as a decrease of $D$. It is, however, more difficult to interpret a shock to $r$.  

49
is unaffected. Therefore, product market tightness and consumption of output decrease after the negative demand shock.

The demand shock affects the product market directly. The response of the product market translates into a response of the labor market. After the negative demand shock the product market tightness decreases, which causes the selling probability to fall. In Figure 2(a), the labor demand shifts in but the labor supply is unaffected. Therefore, labor market tightness and employment decrease after the negative demand shock.\footnote{Formally, $dx/dD > 0$, $W'(x) < 0$, and $N'(W) < 0$, such that $d\theta/dD > 0$ and $dn/dD > 0$.}

Furthermore, output decreases.\footnote{Formally, $d\theta/dD > 0$, $Y'(\theta) > 0$, such that $dy/dD > 0$.} After a negative demand shock, the product market tightness decreases so the selling probability $f(x)$ decreases. Output $y$ also decreases. Therefore, it is unclear whether the amount of output that cannot be sold, $[1 - f(x)] \cdot (y - \kappa)$, increases or decreases.

**Real Wage Shock.** Suppose that the real wage $w$ increases. It is more expensive for firms to hire workers so firms reduce employment. In Figure 2(b), the aggregate supply shifts in and the aggregate demand is unaffected. Therefore, product market tightness increases and consumption of output decreases.\footnote{Formally, $\partial W/\partial w \bigg|_x > 0, \theta'(W) < 0, Y'(\theta) > 0$. Using the definitions in (31), we infer that $\partial c^d/\partial w\bigg|_x = 0$, and $\partial c^s/\partial w\bigg|_\theta < 0$. Since $\partial c^s/\partial x\bigg|_w > 0$ and $\partial c^d/\partial x\bigg|_w < 0$, it is obvious that $dx/dw > 0$ and $dc/dw < 0$.}

The effect of the real wage shock on the labor market depends on the equilibrium response of the effective wage, $W = w/(a \cdot f(x))$. The response is ambiguous because $w$ and $f(x)$ both increase. We show that $W$ necessarily increases: the initial increase of $w$ dominates the equilibrium increase of $f(x)$. Assume that $W$ decreases. Then $\theta = \Theta(W)$ increases, $y = Y(\theta)$ increases, and $c = \kappa + f(x) \cdot (y - \kappa)$ increases: we reach a contradiction. When the real wage increases, the effective wage increases so, in Figure 2(a), labor demand shifts in but labor supply is unaffected. Hence, labor market tightness and employment decrease. Output also decreases.\footnote{Formally, $d\theta/dw < 0$ and $dn/dw < 0$.}

When the real wage increases, the product market tightness increases so the selling probability increases. Output decreases. Therefore, the amount of output that cannot be sold necessarily decreases.

**Technology Shock.** Suppose that technology $a$ decreases. Workers are less productive. Furthermore, the effective wage increases so it is more expensive for firms to hire workers and firms reduce employment. In Figure 2(b), the aggregate supply shifts in and the aggregate demand is unaffected. Therefore, 

\footnote{Formally, $dx/dD > 0$, $W'(x) < 0$, and $N'(W) < 0$, such that $d\theta/dD > 0$ and $dn/dD > 0$.}

\footnote{Formally, $d\theta/dD > 0$, $Y'(\theta) > 0$, such that $dy/dD > 0$.}

\footnote{Formally, $\partial W/\partial w \bigg|_x > 0, \theta'(W) < 0, Y'(\theta) > 0$. Using the definitions in (31), we infer that $\partial c^d/\partial w\bigg|_x = 0$, and $\partial c^s/\partial w\bigg|_\theta < 0$. Since $\partial c^s/\partial x\bigg|_w > 0$ and $\partial c^d/\partial x\bigg|_w < 0$, it is obvious that $dx/dw > 0$ and $dc/dw < 0$.}

\footnote{Formally, $dW/dw > 0$, $\theta'(W) < 0, Y'(\theta) > 0$. Using the definitions in (31), we infer that $\partial c^d/\partial w\bigg|_x = 0$, and $\partial c^s/\partial w\bigg|_\theta < 0$. Since $\partial c^s/\partial x\bigg|_w > 0$ and $\partial c^d/\partial x\bigg|_w < 0$, it is obvious that $dx/dw > 0$ and $dc/dw < 0$.}
product market tightness increases and consumption of output decreases.44

The effect of the technology shock on the labor market is ambiguous. The response of the labor market depends on the equilibrium response of the effective wage, \( W = w/(a \cdot f(x)) \). The response is ambiguous because when \( a \) decreases, \( f(x) \) increases. In fact, we can find scenarios under which \( W \) increases and scenarios under which \( W \) decreases.

A negative technology shock can be contractionary on the labor market. This is the standard case in the literature. Assume that the customer base \( \kappa = 0 \). Then the equilibrium condition is \( c^d(x) = f(x) \cdot a \cdot Y (\Theta(w/ \{f(x) \cdot a\})) \). In equilibrium, \( x \) increases and \( c^d(x) \) decreases. Since \( Y'(\theta) > 0 \) and \( \Theta'(W) < 0 \), it must be that \( f(x) \cdot a \) decreases. Therefore, \( W \) increases and both \( \theta = \Theta(W) \) and \( n = N(W) \) decrease. To conclude, the negative technology shock is contractionary on the labor market.

But a positive technology shock can also be contractionary on the labor market. This is a nonstandard case in the literature. Assume that the aggregate demand function \( c^d(x) \) is totally inelastic because \( r = 0 \). In that case, \( c^d(x) = D^e \) for all \( x \). Then the equilibrium condition becomes \( D^e - \kappa \cdot [1 - f(x)] = f(x) \cdot a \cdot Y(\Theta(W(x))) \). In equilibrium, \( x \) decreases so \( 1 - f(x) \) increases and the left-hand side of the equation decreases. Since \( Y'(\theta) > 0 \) and \( \Theta'(W) < 0 \), \( f(x) \cdot a \) decreases. Therefore, \( W \) increases and both \( \theta = \Theta(W) \) and \( n = N(W) \) decrease. To conclude, the positive technology shock is contractionary on the labor market.

Finally, a technology shock can be neutral on the labor market. This is a knife-edge case obtained by bringing together the two previous cases. Assume that the aggregate demand function \( c^d(x) \) is totally inelastic because \( r = 0 \) and that \( \kappa = 0 \). In that case, \( f(x) \cdot a \) is constant in equilibrium. Therefore, the effective wage \( W \) is constant and the technology shock has no effect on the labor market.

**Labor Supply Shock.** Assume that the search effort of jobseekers decreases from 1 to \( e < 1 \). The reduction in search effort could be caused by an increase of the generosity of unemployment insurance or by a preference shock that makes searching more unpleasant. It becomes more costly to hire workers on the labor market and firms reduce employment. In Figure 2(b), the aggregate supply shifts in and the aggregate demand is unaffected. Therefore, product market tightness increases and consumption of output decreases.45

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44Formally, \( \partial W/\partial a|_x < 0 \), \( \Theta'(W) < 0 \), \( Y'(\theta) > 0 \). Using the definitions in (31), we infer that \( \partial c^d/\partial a|_x = 0 \), \( \partial e^s/\partial a|_{x,W} > 0 \), \( \partial e^s/\partial W|_{x,a} < 0 \), which implies that \( \partial e^s/\partial a|_x > 0 \). Since \( \partial e^s/\partial x|_a > 0 \) and \( \partial c^d/\partial x|_a < 0 \), it is obvious that \( dx/da < 0 \) and \( dc/da > 0 \).

45Formally, \( \partial e^s/\partial e|_\theta > 0 \), \( \partial e^s/\partial e|_x < 0 \), \( \partial n/\partial e|_x > 0 \), \( \partial g(n,\theta)/\partial e|_x > 0 \). It is clear that \( \partial e^d/\partial e|_x = 0 \) and \( \partial e^s/\partial e|_x > 0 \). Since \( \partial e^s/\partial x|_e > 0 \) and \( \partial e^d/\partial x|_e < 0 \), it is obvious that \( dx/de < 0 \) and \( dc/de > 0 \).
Consider now the effect of the decrease in search effort on the labor market. In Figure 2(a), the labor supply shifts in. The labor demand shifts out after the equilibrium increase in product market tightness that follows that decrease in search effort. Therefore, it is obvious that labor market tightness increases. While the equilibrium response of employment is not obvious, we can determine it with some algebra. Combining (1) and (21), we obtain \( c = \kappa + f(x) \cdot [a \cdot z(n) - a \cdot \{\hat{r}/\hat{q}(\theta)\} \cdot \{n - (1 - u)\} - \kappa] \). The optimal hiring decision of the firm, given by (23), implies that \( \hat{r}/\hat{q}(\theta) = \frac{z'(n)}{w} / [a \cdot f(x)] \). Hence,

\[
\frac{\partial c}{\partial n} = w \cdot \{n - (1 - u)\} + f(x) \cdot [a \cdot z(n) - a \cdot z'(n) \cdot \{n - (1 - u)\} - \kappa].
\] (32)

Since \( z(n) \) is increasing and concave, \( a \cdot z(n) - a \cdot z'(n) \cdot \{n - (1 - u)\} \) is increasing in \( n \) and larger than \( a \cdot z(1 - u) \). Under Assumption 2, \( a \cdot z(1 - u) > \kappa \) so \( a \cdot z(n) - a \cdot z'(n) \cdot \{n - (1 - u)\} - \kappa > 0 \). Suppose that \( e \) decreases. We proved that \( c \) decreases and \( f(x) \) increases. Therefore, the right-hand side of (32) increases while the left-hand side decreases. Since the right-hand side is increasing in \( n \), employment \( n \) necessarily decreases in equilibrium.

Output, given by \( y = a \cdot g(n, \theta) \), increases with \( n \) and decreases with \( \theta \). After the negative labor supply shock, it is clear that output decreases. Since the selling probability increases, unsold output decreases unambiguously.

**Labor Market Mismatch Shock.** Assume that mismatch becomes worse on the labor market: the parameter \( \hat{\omega} \) that controls the effectiveness of matching on the labor market decreases. The effects are similar to those of a negative labor supply shock, with the addition of an adverse shift of the labor demand curve. It becomes more costly to hire workers on the labor market and firms reduce employment. In Figure 2(b), the aggregate supply shifts in and the aggregate demand is unaffected. Therefore, product market tightness increases and consumption of output decreases.

Consider now the effect of the increase in mismatch on the labor market. In Figure 2(a), the labor supply shifts in. But the response of the the labor demand is not obvious. On the one hand, the equilibrium increase in product market tightness that follows the increase in mismatch tends to shift labor demand out. On the other hand, the increase in mismatch itself tends to shift labor demand in. As with the negative labor supply shock, we examine (32) to determine the equilibrium response of employment.

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\[\text{Formally, } \frac{\partial s}{\partial e}|_\theta > 0, \frac{\partial n^s}{\partial x}|_\theta = 0, \frac{\partial n^d}{\partial e}|_\theta = 0, \frac{\partial n^d}{\partial x}|_\theta > 0. \] In addition, \( dx/de < 0, \frac{\partial n^s}{\partial \theta}|_e > 0, \frac{\partial n^d}{\partial \theta}|_e < 0. \) Therefore, it is clear that \( dx/de < 0. \)

\[\text{Formally, } \frac{\partial s}{\partial \hat{\omega}}|_\theta > 0, \frac{\partial r}{\partial \hat{\omega}}|_\theta > 0, \frac{\partial r}{\partial \hat{\omega}}|_\theta > 0, \frac{\partial \hat{\omega}}{\partial \hat{\omega}}|_\theta < 0, \frac{\partial n}{\partial \hat{\omega}}|_\theta > 0, \frac{\partial y(n, \theta)}{\partial \hat{\omega}}|_\theta > 0. \] It is clear that \( \frac{\partial c}{\partial \hat{\omega}}|_\theta > 0 \) and \( \frac{\partial e}{\partial \hat{\omega}}|_\theta > 0. \) Since \( \frac{\partial c}{\partial x}|_\theta > 0 \) and \( \frac{\partial e}{\partial x}|_\theta < 0, \) it is obvious that \( dx/d\hat{\omega} < 0 \) and \( dc/d\hat{\omega} > 0. \)
Using exactly the same argument as for the negative labor supply shock, we conclude that employment decreases. However, the equilibrium response of labor market tightness is ambiguous and depends on parameters.

Output, given by \( y = a \cdot [z(n) - \{\hat{r}/\hat{q}(\theta)\} \cdot \{n - (1 - u)\}] \), increases with \( n \) and decreases with \( \hat{r}/\hat{q}(\theta) \). After the increase in mismatch, it is clear that output decreases. Since the selling probability increases, unsold output decreases unambiguously.

### 4.5 Connection with Available Empirical Evidence

The main result arising from our comparative static analysis is that product market tightness and selling probability are procyclical under demand shocks, whereas they are countercyclical under supply shocks. In this section, we connect the comparative static results with available empirical evidence. Available evidence suggests that demand shocks may be the main force behind business cycles.

The first piece of evidence that we examine is the work of Bils and Kahn [2000]. They empirically study the cyclical behavior of the ratio of sales to stock available for sales. Their empirical work focuses on manufacturing for 1959 through 1997. They find that this ratio is strongly procyclical: it decreases dramatically in each recession, typically by 5 to 10 percent. These decreases do not simply reflect transitory sales surprises, but are highly persistent for the duration of each recession. In our static model, stock available for sales is \( y \) and sales are \( c = \kappa + f(x) \cdot (y - \kappa) \). Assume \( \kappa = 0 \). The sale to stock ratio is \( c/y = f(x) \). If \( \kappa > 0 \), \( c/y = f(x) + [1 - f(x)] \cdot \kappa/y \).

If the flows on the product market are balanced in a dynamic version of the model (a typical assumption in search-and-matching models of the labor market), and if \( \delta \) is the rate at which customer relationships are destroyed, then, as we show in Section 5.1,

\[
\frac{c}{y} = \frac{f(x)}{\delta + (1 - \delta) \cdot f(x)}.
\]

Hence, the sale-stock ratio remains positively related to \( f(x) \) this dynamic extension. The evidence in Bils and Kahn [2000] that inventories sell with predictably higher probability at peaks suggests, in light of our framework, that business cycle fluctuations are mostly driven by demand shocks. While the empirical analysis focuses on manufacturing, they find that the sale-stock ratio behaves similarly for finished goods inventories and works-in-process for new housing construction and for finished goods inventories in whole sale and retail trade.

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\( \text{48} \) The results also hold at the industry level: for every industry commonly identified as production-for-stock industries (tobacco, apparel, lumber, chemicals, petroleum, and rubber), the ratio of sales to stock available is highly procyclical.
The second piece of evidence that we examine is the work of Bils, Klenow and Malin [2012]. Using data across US industries from 1990–2011, they test whether sellers deviate from setting the marginal product of labor proportional to the real wage, by setting lower price markups when demand for goods is high. To identify movements in goods demand, they exploit how durability varies across 70 categories of consumption and investment that cover 60% of GDP, and use the result that any macroeconomic shock affects demand for durables more dramatically than demand for nondurables. Their main result is that industries producing goods that are more durable, while displaying much more cyclical employment and output, exhibit countercyclical relative markups. They also find that durables display little fall in relative prices in recessions, even though their demand falls dramatically. This is consistent with the predictions of our model under demand shocks. In our model, prices would not respond to a demand shock. Abstracting from recruiting costs, which are a small share of marginal costs, the markup is the marginal product of labor divided by the real wage: \( a \cdot z'(n)/w = 1/f(x) \). The finding of Bils, Klenow and Malin [2012] that the price markup is countercyclical implies that the selling probability \( f(x) \) is procyclical. This finding is only consistent with demand shocks driving business cycles.

The third piece of evidence that we examine is the work of Basu, Fernald and Kimball [2006]. Using US data, they find that input use decreases sharply after a positive technology shock. (They also find that output changes little.) While the response of employment to a technology shock is ambiguous in our model, the response of product market tightness and the selling probability is not: both decrease when technology increases. If output is a service good, then the selling probability corresponds exactly to labor utilization. With this interpretation, our model implies that labor utilization decreases when technology improves. This prediction is consistent with the findings of Basu, Fernald and Kimball [2006].

5 Extensions

In this section, we develop various extensions of the model of Section 4. In the first extension, we cast the static model into a dynamic environment. We explain how the number of employees and the number of customers of each firm evolves over time. We also show that in steady state, it is not necessary to impose the restrictions on the price and wage level of the static model (Assumption 1 and 3) to enforce pairwise Pareto efficiency. In the second extension, we introduce inequality in wealth and income in a

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49In our model, technology shock can be expansionary but they can also be contractionary if aggregate demand is inelastic enough. Gali [1999] find evidence that technology shocks are contractionary in the short run, suggesting the aggregate demand may be quite inelastic.
simple way. We discuss how the distribution of economic resources influences aggregate demand, and how redistribution, through transfers or adjustment in payroll taxes, shifts aggregate demand.

5.1 A Dynamic Model

We embed the static model of Section 4 into a dynamic environment. For simplicity, we abstract from shocks and focus on a deterministic environment. It is clear, however, that the model can accommodate any stochastic shock process (for instance, a stochastic process for technology, \( \{a_t\} \), for the price, \( \{p_t\} \), for the real wage, \( \{w_t\} \), for labor market mismatch, \( \{\hat{\omega}_t\} \), for the endowment of unproduced good, \( \{\mu_t\} \), or for the demand for unproduced good, \( \{\beta_t\} \)).

Unproduced Good Market. The market for unproduced good is exactly the same as in the static model.

Labor Market. The labor market is similar to that in the static model. Once a match is made in period \( t - 1 \), worker and firm engage in a long-term employment relationship. The long-term relationship is supported by the positive surplus that arises because in a match, firms do not need to incur recruiting cost and workers do not face uncertainty about finding a job. At the end of period \( t - 1 \), a fraction \( \hat{\delta} \) of the \( n_{t-1} \) existing worker-firm matches is exogenously destroyed. Workers who lose their job become unemployed, and start searching for a new job at the beginning of period \( t \). At the beginning of period \( t \), the number of unemployed workers looking for a job is \( u_t = 1 - (1 - \hat{\delta}) \cdot n_{t-1} \). After matching the number of workers with a job is

\[
n_t = (1 - \hat{\delta}) \cdot n_{t-1} + \left[ 1 - (1 - \hat{\delta}) \cdot n_{t-1} \right] \cdot f(\theta_t). \tag{33}
\]

50 There are other models of inequality or heterogeneity in the macroeconomic literature. They are usually quite complex. In some models, idiosyncratic income fluctuations over the lifetime create wealth inequality [for example, Aiyagari, 1994; Bewley, 1987; Huggett, 1993; Huggett, Ventura and Yaron, 2011; Krusell, Mukoyama and Şahin, 2010]. Another way to introduce inequality is to assume that some workers are more impatient than others, such that some workers are always lenders while the others are always borrowers [for example, Eggertsson and Krugman, 2012]. Equivalently, some models introduce workers who differ in the sophistication of their behavior: some workers optimize perfectly their intertemporal consumption problem while others simply consume all they income each period [for example, Gáf, Lopez-Salido and Valles, 2007; Mankiw and Weinzierl, 2011].

51 Related papers that examine the macroeconomic implications of inequality and redistribution include Kumhof and Ranciere [2011] and Kumhof et al. [2011].
In steady state, inflows to unemployment, \( \hat{\delta} \cdot n \), must equal outflows from unemployment, \( \left[ 1 - (1 - \hat{\delta}) \cdot n \right] \cdot \hat{f}(\theta) \). Thus, employment satisfies
\[
\frac{\hat{f}(\theta)}{\delta + (1 - \hat{\delta}) \cdot \hat{f}(\theta)}.
\]
This steady-state relationship between employment and labor market tightness is equivalent to a Beveridge curve.

**Product Market.** The product market is also similar to that in the static model. Once a match is made in period \( t - 1 \), seller and buyer engage in a long-term customer-firm relationship. That is, all consumers who purchase a good from a firm become (or remain if they already were) customers of that firm. The long-term relationship is supported by the positive surplus that arises because in a match, customers do not need to incur shopping cost and firms do not face uncertainty about selling (the customer knows that the desired product is always available from the buyer, and the buyer knows that the customer buys a product every period). The number of customer-firm relationships at the end of period \( t \) is \( c_{t-1} \). At the end of period \( t - 1 \), a fraction \( \delta \) of the \( c_{t-1} \) existing customer-firm matches is exogenously destroyed. Customers who lose their relationship become shoppers, and they need to shop to purchase produced goods in period \( t \). At the beginning of period \( t \), the number of customer-firm matches is \( \kappa_t = (1 - \delta) \cdot c_{t-1} \). At the end of period \( t \) the number of customer-firm matches, which is also the number of sales in period \( t \), is
\[
c_t = (1 - \delta) \cdot c_{t-1} + f(x_t) \cdot \left[ y_t - (1 - \delta) \cdot c_{t-1} \right].
\]
In steady state the number of relationships destroyed each period, \( \delta \cdot c \), must equal the number of new relationships each period, \( f(x) \cdot \left[ y - (1 - \delta) \cdot c \right] \). Thus, sales satisfy
\[
c = \frac{f(x)}{\delta + (1 - \hat{\delta}) \cdot f(x)} \cdot y.
\]

**Firms.** The representative firm hires new workers and finds new customers each period. Both workers and customers are engaged in relationships with the firm for several periods. Therefore, the firm’s problem is inherently dynamic: the firm takes into account the effects on future profits of hiring new workers and finding new customers in the current period. Let \( \beta < 1 \) be the time discount factor of the firm. Given sequences of labor market tightness, product market tightness, and real wage, \( \theta_t, x_t, w_t \)_{t=0}^{+\infty}, the
A representative firm chooses a sequence of sales and employment, \( \{c_t, n_t\}_{t=0}^{+\infty} \), to maximize the present discounted value of profits

\[
\sum \beta^t \cdot [c_t - w_t \cdot n_t]
\]

subject to the production constraint, given by (21), and to the constraint on sales, given by (35). We substitute \( y_t \) out of the sales constraint using the production constraint. We denote by \( \lambda_t \) the Lagrange multiplier on the resulting constraint. We obtain the Lagrangian of the firm’s problem:

\[
\sum \beta^t \left\{ c_t - w_t n_t - \lambda_t \left[ c_t - (1 - \delta) c_{t-1} [1 - f(x_t)] - f(x_t) a \left[ z(n_t) - \frac{\hat{r}}{\hat{q}(\theta_t)} [n_t - (1 - \hat{\delta})n_{t-1}] \right] \right] \right\}.
\]

The first-order condition with respect to \( c_t \) yields

\[
1 = \lambda_t - (1 - \delta) \beta \cdot \lambda_{t+1} \cdot [1 - f(x_{t+1})].
\]

The first-order condition with respect to \( n_t \) yields

\[
z'(n_t) = \frac{w_t}{a \cdot \lambda_t \cdot f(x_t)} + \frac{\hat{r}}{\hat{q}(\theta_t)} - (1 - \delta) \beta \cdot \frac{\lambda_{t+1} \cdot f(x_{t+1})}{\lambda_t \cdot f(x_t)} \cdot \frac{\hat{r}}{\hat{q}(\theta_{t+1})}.
\] (37)

In steady state, the Lagrange multiplier on the sales constraint is \( 1/\lambda = 1 - (1 - \delta) \beta \cdot [1 - f(x)] \).

Hence, the first-order condition becomes

\[
z'(n) = \frac{1 - (1 - \delta) \beta \cdot [1 - f(x)]}{f(x)} \cdot \frac{w}{a} + \left[ 1 - \beta \cdot (1 - \hat{\delta}) \right] \frac{\hat{r}}{\hat{q}(\theta)}.
\]

Compared to the first-order condition in the static model, given by (23), this condition introduces a correction \( 1 - (1 - \delta) \beta \cdot [1 - f(x)] < 1 \) on the real wage and a correction \( 1 - \beta \cdot (1 - \hat{\delta}) < 1 \) on hiring costs. The two corrections lower the marginal cost of labor. They capture the fact that both customers and workers are retained from one period to the other, which effectively reduces production and hiring costs.

**Consumers.** We assume that consumers live for one period before dying and bequesting all their unproduced good to their descendants. We assume that the utility that consumers derive from owning the unproduced good captures both their enjoyment from consuming the unproduced good and their consid-
eration for the well-being of their descendants, as in Piketty and Saez [2012]. In that case the problem of the representative consumer is the same as in the static model, and the optimal consumption of produced good in period $t$ is given by (8).

**Equilibrium.** We can now formally define an equilibrium in the dynamic model. Given initial customer-firm and worker-firm relationships, $n_{-1}$ and $c_{-1}$, and given sequences of price and wage, $\{p_t, w_t\}_{t=0}^{+\infty}$, an equilibrium is a collection of five sequences $\{\theta_t, n_t, x_t, c_t, y_t\}_{t=0}^{+\infty}$ that satisfy the following five relationships: the law of motion (35) for customer-firm relationships; the law of motion (33) for worker-firm relationships; the condition (8) for optimal consumption by workers; the condition (37) for optimal hiring by firms; and the production function (21).

### 5.2 Enriching Aggregate Demand with Inequality

We introduce inequality in the model of Section 4 to analyse how income and wealth inequality affect aggregate demand. The aggregate supply curve is described in Definition 3, and satisfies the properties listed in Lemma 3.

**The Rich and The Poor.** The measure 1 of workers is divided into into two classes. We assume that one class, which we call the rich, likes the unproduced good more than the other class, which we call the poor.

**ASSUMPTION 5.** Utility functions take the CRRA form as in (14). The rich have a greater taste for the unproduced good more than the poor: $\beta^p < \beta^R$.

This assumption captures the empirical fact that wealthier individuals have a much smaller marginal propensity to consume than poorer individuals. Assuming CRRA utility instead of quasilinear utility is critical to obtain income effects, and hence non-neutrality of transfers across classes. In addition, the two classes may differ in their endowment of unproduced good and their ownership of firms.

A measure $\chi \in (0, 1)$ of workers belong to the rich and a measure $1 - \chi$ of workers belong to the poor. Rich and poor have the same productive ability. From firms’ point of view, all workers are identical irrespective of their social class; therefore, all workers receive the same real wage, have the same unemployment rate at the beginning of the day, have the same productivity, and face the same job-finding probability. Accordingly, the labor income of the rich is $\chi \cdot w \cdot n$ and the labor income of the poor is $(1 - \chi) \cdot w \cdot n$. 

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The firms’ profits are distributed to workers. The rich receive a share $\phi_R$ of profits per capita whereas the poor receive a share $\phi_P$ of profits per capita. We assume that the rich own a disproportionally large share of firms:

**ASSUMPTION 6.** The rich receive a larger share of profits per capita than the poor: $\phi_R > \phi_P$.

By definition, $\phi_R \cdot \chi + \phi_P \cdot (1 - \chi) = 1$; therefore, the assumption implies that $\phi_R > 1$ and $\phi_P < 1$.

The rich are endowed with a quantity $\psi_R \cdot \mu$ of unproduced good per capita whereas the poor are endowed with a quantity $\psi_P \cdot \mu$ of unproduced good per capita. By definition, $1 = \chi \cdot \psi_R + (1 - \chi) \cdot \psi_P$.

Finally, the rich and the poor send customers and shoppers to purchase production. The rich and the poor pay the same product price. To simplify the exposition of the model with inequality, we consider the simple case in which consumption decisions are not affected by product market tightness. We assume that it is costless to shop: $r = 0$. Under this assumption, however, we need to assume that $\mu$ is not too large such that the aggregate demand intersect the aggregate supply. (With $r > 0$, we did not need to impose this restriction.)

**Aggregate Demand.** Let $c^R$ and $m^R$ be the per-capita consumption of produced and unproduced good for the representative rich worker. Let $c^P$ and $m^P$ be the per-capita consumption of produced and unproduced good for the representative poor worker. The budget constraints of the representative rich and poor workers are

$$m^R + p \cdot c^R = \psi_R \cdot \mu + \phi_R \cdot \pi + n \cdot w$$
$$m^P + p \cdot c^P = \psi_P \cdot \mu + \phi_P \cdot \pi + n \cdot w.$$  (38)  (39)

Combining the first-order conditions with respect to $c^R$ and $m^R$, and the first-order conditions with respect to $c^P$ and $m^P$, we realize that the consumptions of produced good and unproduced goods are proportional:

$$c^R = (p \cdot \beta^R)^{-\epsilon} \cdot m^R$$
$$c^P = (p \cdot \beta^P)^{-\epsilon} \cdot m^P.$$
Accordingly, workers in each class devote a constant fraction of their income to the consumption of unproduced good:

\[ m^R = \frac{1}{1 + p^{1-\epsilon} \cdot (\beta^R)^{-\epsilon}} \cdot [\psi^R \cdot \mu + \phi^R \cdot p \cdot \pi + n \cdot p \cdot w] \]

\[ m^P = \frac{1}{1 + p^{1-\epsilon} \cdot (\beta^P)^{-\epsilon}} \cdot [\psi^P \cdot \mu + \phi^P \cdot p \cdot \pi + n \cdot p \cdot w] . \]

The fraction \( \frac{1}{1 + p^{1-\epsilon} \cdot \beta^{-\epsilon}} \) of income devoted to the consumption of unproduced good can be interpreted as a marginal propensity to save. Under Assumption 5, the rich have a higher marginal propensity to save than the poor.

We can now determine the aggregate demand for the produced good. The unproduced-good market clears, which imposes \( \mu = \chi \cdot m^R + (1 - \chi) \cdot m^P \). We define the following constants and functions:

\[ B^R \equiv \frac{1}{1 + p^{1-\epsilon} \cdot (\beta^R)^{-\epsilon}} \]

\[ B^P \equiv \frac{1}{1 + p^{1-\epsilon} \cdot (\beta^P)^{-\epsilon}} \]

\[ X(x) \equiv [x \cdot B^R + (1 - x) \cdot B^P] . \] (40)

Under Assumption 5, \( B^R > B^P \) and \( X'(x) > 0 \). We rearrange the market-clearing condition as

\[ \mu = \chi \cdot B^R \cdot [\psi^R \cdot \mu + \phi^R \cdot p \cdot \pi + n \cdot p \cdot w] + (1 - \chi) \cdot B^P \cdot [\psi^P \cdot \mu + \phi^P \cdot p \cdot \pi + n \cdot p \cdot w] \]

\[ [1 - X(\Psi)] \cdot \frac{\mu}{p} = n \cdot w \cdot X(\chi) + \pi \cdot X(\Phi) \]

where

\[ \Psi \equiv \chi \cdot \psi^R = 1 - [(1 - \chi) \cdot \psi^P] \] (41)

\[ \Phi \equiv \chi \cdot \phi^R = 1 - [(1 - \chi) \cdot \phi^P] . \] (42)

Using the fact that \( \pi = c - w \cdot n \), we obtain the link between consumption of produced good, endowment of unproduced good, and labor income when both classes spend their income optimally and the market for unproduced good clears:

\[ c = \frac{1 - X(\Psi)}{X(\Phi)} \cdot \frac{\mu}{p} + \left[ 1 - \frac{X(\chi)}{X(\Phi)} \right] \cdot w \cdot n. \]
As established by Proposition 2, employment is a function \( n(x) \) of product market tightness when the labor market is in partial equilibrium. Using this result, we can define a new aggregate demand:

**DEFINITION 4.** The aggregate demand is a function of product market tightness defined for \( x > 0 \) by

\[
c^d(x) = \frac{1 - X(\Psi)}{X(\Phi)} \cdot \frac{\mu}{p} + \left[ 1 - \frac{X(\chi)}{X(\Phi)} \right] \cdot w \cdot n(x),
\]

(43)

where the function \( X \) is defined by (40) and the constants \( \Psi \) and \( \Phi \) are defined by (41) and (42).

If the poor and the rich have the same preferences, \( \beta^R = \beta^P = \beta \), then \( B^R = B^P \) and \( X(x) = 1/[1 + p^{1-\epsilon} \cdot \beta^{-\epsilon}] \) for any \( x \). As a consequence \( c^d(x) = \mu/(p \cdot \beta)^\epsilon \), which is just the aggregate demand of Definition 1 under the assumption that \( r = 0 \).

Under Assumption 6, \( \phi^R > 1 \) so \( \Phi > \chi \). Under Assumption 5, \( X'(x) > 0 \). Combining both assumptions, we have \( 1 - [X(\chi) < X(\Phi)] > 0 \). Thus, we are able to establish the following properties of aggregate demand:

**LEMMA 4.** Under Assumptions 5 and 6, \( c^d(x) \) is increasing for all \( x \geq 0 \).

Once again, the equilibrium can be described by the intersection of aggregate supply and aggregate demand. But the equilibrium may not exist and may not be unique. The diagram in Figure 5 represents the general equilibrium in a price \( x \)-quantity \( c \) plan.

**Macroeconomic Shocks.** Using comparative statics we study the effects of several macroeconomic shocks, with a focus on shocks that affect inequalities. Assumptions 5 and 6 hold throughout.

Consider a transfer of wealth from the rich to the poor. The transfer amounts to reducing the per-capita endowment of unproduced good received by the rich, \( \psi^R \). The poor have a higher propensity to consume out of their income than the rich so the transfer increases aggregate demand.\(^{52}\) In equilibrium, product market tightness and consumption of produced good increase. The increase in product market tightness in turn stimulates labor demand in the labor market. In equilibrium, employment, labor market tightness, and output increase.

Next, consider a shit in payroll tax from workers to employers. The shift amounts to increasing the real wage paid by firms and received by workers, \( w \). The aggregate supply shifts in because it becomes more expensive for firms to hire workers. Unfortunately, the response of the aggregate demand

\(^{52}\)Formally, a reduction in \( \psi^R \) leads to a reduction in \( \Psi \) and \( X(\Psi) \), and therefore to an increase in \( [1 - X(\Psi)] / X(\Phi) \) and an outward shift in \( c^d(x) \).
is ambiguous. On the one hand, an increase in wage tends to increase labor income and reduce profits, which tends to increase aggregate consumption. The reason why the income shift from profits to labor income stimulates demand is that the poor depend disproportionately on labor income and at the same time, they have a higher marginal propensity to consume. On the other hand, the increase in wage depresses employment and therefore production and income. In (43), $w$ increases but $n(x)$ decreases for a given $x$.

6 Conclusion

This paper has presented a parsimonious macroeconomic model based on a search-and-matching representation of the product and labor markets. The model conforms to empirical observations from microdata, macrodata, survey of firms and workers, ethnographic studies of firms and markets, and other sources. The model provides a microfoundation for the aggregate demand and aggregate supply curves, as well as for the labor demand and labor supply curves. The model rationalizes unemployment as well as labor hoarding or inventories. In future work, we plan on using the model to analyze optimal fiscal and monetary policy over the business cycle. For instance the optimal unemployment insurance analysis
of Landais, Michaillat and Saez [2010], conducted in a model without an operative aggregate demand, could be extended using the model. By redistributing resources across individuals with different wealth and preferences, unemployment insurance could affect aggregate demand and hence affect optimal unemployment insurance analysis.

As discussed, a key feature of our model is that price and wages are parameters and that product and labor market tightnesses equilibrate supply and demand. Therefore, our model disconnects the analysis of fluctuations in quantities and tightnesses from the price-setting mechanism. As we showed, considering rigid prices leads to fairly simple comparative statics and significant fluctuations in quantities following shocks. In principle, our modeling could accommodate many other forms of price schedules, including the traditional Nash bargaining price determination. We view this flexibility as a virtue. Naturally, developing a microfoundation of price and wage determination that matches empirical evidence would be valuable. To conclude this paper, we indicate sources of information that could guide the modeling of price setting and wage setting.

The large volume price microdata would be useful to understand price-setting practices. Following the seminal survey of Kahneman, Knetsch and Thaler [1986], economists have examined the response of consumers to price level and price changes and the impact of pricing on profits. A large marketing literature also provides information about price perception and the effects of pricing on repurchase intentions. Case studies that analyse the pricing procedure in a specific firm are also particularly useful [for example, Zbaracki et al., 2004]. Finally, the survey of firms described in Section 2.2 provide a trove of insights on price response to demand and cost shocks, as well as sources of price rigidity.

Large and high frequency wage and earning microdata are also available to understand wage-setting practices. Economists have also examined the response of workers to wage level, wage changes, lay-

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54 In particular, many laboratory experiments have been conducted to address these questions [for example, Cason and Friedman, 2002; Renner and Tyran, 2004].


56 For analyses of US wage and earning microdata, see Bils [1985], Beaudry and DiNardo [1991], Solon, Barsky and Parker [1994], Devereux [2001], and Haeck, Sonntag and van Rens [2008]. For analyses of European wage and earning microdata, see Devereux and Hart [2006], Carneiro, Guimaraes and Portugal [2009], and Martins, Solon and Thomas [2012]. For a survey of this literature, see Pissarides [2009]. For evidence of downward wage rigidity in the US, see Kahn [1997], Card and Hyslop [1996], Altonji and Devereux [1999], Lebow, Saks and Wilson [2003], and Barattieri, Basu and Gottschalk [2010]. For a survey on downward wage rigidity, see Kramarz [2001]. Last, Dickens et al. [2007] reports the results of a large-scale project that provides microevidence on how wages change for continuing workers. The project uses 31 datasets containing individuals’ earning changes in 16 countries for periods ranging from the early 1970s to the early 2000s. They obtain 360
offs decisions, and other aspects of the employment contract. A large literature in management and human resource science also focuses on the design of wage policies to improve productivity and profitability. Case studies that analyse the wage structure in a specific firm are also very informative [for example, Baker, Gibbs and Holmstrom, 1994b]. The studies of anthropologist, sociologist, and social psychologists—such those described by Akerlof [1982, 1984] and Akerlof and Yellen [1990]—also provide information that would be helpful to design realistic and tractable wage-setting mechanism. Researchers have conducted surveys in different countries to investigate wage-setting practices and the possible reasons for wage rigidity. Another source of information is the historical work of Raff and Summers [1987] and Raff [1988], who study the reasons behind, and the consequences, of the institution in Ford’s automobile factory in January 1914 of a five-dollar-a-day minimum wage, which doubled the pay of most workers.

Obtaining a realistic but tractable model of price setting and wage setting is a major challenge. Both price and wage are the product of a complex set of factors such as institutions, contracts, monopolies and unions, norms of fairness, transfers of risk, or informational frictions. To make progress on this research agenda, bargaining models are available and could be used fruitfully as a starting point to determine prices and wages with bilateral monopoly. Efficiency-wage models, in which firms recognize that higher wages may lead to higher productivity and that lowering wages all the way down to the market-clearing level may reduce profit, could also serve as a useful starting point. The works of Menzio [2005, 2007], who propose theories of wage and price rigidity due to search frictions, of Rudanko [2009], who characterizes implicit contracts between firm and worker in a search-and-matching framework, offer a promising start on this research agenda.

wage change distribution—one for each year in each dataset—capturing 21 million wage changes.

58 For studies using surveys, see Kahneman, Knetsch and Thaler [1986] and Charness and Levine [2000]. For studies using laboratory experiments, see Falk, Fehr and Zehnder [2006] and Charness and Kuhn [2007]. For studies using natural experiments, see Krueger and Mas [2004], Mas [2006]. For a survey of the literature examining the role of wages and fairness on effort, see Fehr, Goette and Zehnder [2009].

59 For example, Bewley [1999] interviewed 300 executives and labor market participants in the US in the early 1990s to understand wage-setting practices. Doeringer and Piore [1971] interviewed managers and union officials in 75 companies in the US in the late 1960 to understand the concept of internal labor market. The results of other field surveys are described in Kaufman [1984], Blinder and Choi [1990], Levine [1993], Agell and Lundborg [1995, 1999], and Campbell and Kamlani [1997]. To understand the wage setting of newly hired workers, a rich source of information is the survey of 13,000 firms in 14 European countries described in Galuscak et al. [2008].

References


