An Equilibrium Model of the African HIV/AIDS Epidemic*

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Abstract

Eleven percent of the Malawian population is HIV infected. Eighteen percent of sexual encounters are casual. A condom is used one quarter of the time. A choice-theoretic general equilibrium search model is constructed to analyze the Malawian epidemic. In the developed framework, people select between different sexual practices while knowing the inherent risk. The analysis suggests that the efficacy of public policy depends upon the induced behavioral changes and general equilibrium effects that are typically absent in epidemiological studies and small-scale field experiments. For some interventions (some forms of promoting condoms or marriage), the quantitative exercise suggests that these effects may increase HIV prevalence, while for others (such as male circumcision or increased incomes) they strengthen the effectiveness of the intervention. The underlying channels giving rise to these effects are discussed in detail.

Keywords: Bayesian learning, circumcision, condoms, disease transmission, HIV/AIDS, homo economicus, Malawi, marriage, policy intervention, sex markets, search, STDs

Comments welcome

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1 Introduction

HIV/AIDS is a major cause of death, currently killing about 2 million people worldwide each year. The number of new infections is even higher than 2 million, suggesting an even more severe problem in the future. The most affected continent is Africa, which hosts about two thirds of all HIV/AIDS infected people. Within Africa most transmissions occur through heterosexual sex. Furthermore, the majority of the HIV-positive population is female, compared to less than one third in most developed countries. The current analysis spotlights one part of Africa: The Republic of Malawi. Malawi has a relatively high HIV/AIDS rate and the prevalence for women is higher than for men.\textsuperscript{1} It also has very good data related to HIV/AIDS and sexual behavior. So, the facts are clear. Traditionally, two approaches have been taken to studying the transmission of HIV/AIDS in Africa, viz epidemiological studies and field experiments.\textsuperscript{2} Epidemiological studies are sophisticated in their treatment of equilibrium but usually lack a feedback loop that the captures behavioral responses. Field-experiments are often on a small scale and do not readily allow for the assessment of general equilibrium effects.

The current paper advocates the use of choice-theoretic general equilibrium modelling as an additional tool for studying the African HIV/AIDS epidemic. First, the main benefit that economics can bring to the field of epidemiology is \textit{homo economicus}. Here the assumption is that humans pick their sexual behavior on the basis of a rational benefit/cost calculation. This assumption allows one to study the potential behavioral responses of individuals with respect to particular policies (e.g., adjustments in the number of casual relationships prior to marriage in response to changes in transmission risk). Second, at the heart of the HIV/AIDS epidemic is an externality, the transmission of a virus. General equilibrium modelling is well suited for the study of externalities. One can analyze how individual shifts in behavior feed back on each other in equilibrium (e.g., if the rate of infection for some individuals rise, then the rate of infections for their partners may increase in turn, etc.). Thus, the great advantage of choice-theoretic equi-

\textsuperscript{1}Section 3 is devoted to providing background on the Malawian HIV/AIDS epidemic.
\textsuperscript{2}A detailed review of the literature is contained in Section 2.
librium modelling is the joint assessment of both behavioral change and equi-
librium adjustment in response to proposed policy interventions. Nothing is for
free, of course. Some abstractions are needed to operationalize choice-theoretic
equilibrium modelling that may render it less rich along some dimensions. As
such, choice-theoretic equilibrium modelling complements the above two exist-
ing approaches to the study of disease transmission.

A choice-theoretic general equilibrium search model is built here to study the
Malawian HIV/AIDS epidemic. Despite the fact that computational general
equilibrium models have made great progress in recent decades, there is essen-
tially no application to the field of disease transmission (except for few purely
theoretical studies that form the basis of departure for this analysis but are too
stylized to be directly mapped to data; see the review in the next section). The
first applied general equilibrium framework to be used for this purpose is devel-
oped here. The constructed model has three main ingredients. First, individuals
select the type of sexual activity that they wish to participate in. They do this
based upon beliefs about the riskiness of each type activity. There are four types
of sexual activity. A person may have a long-term relationship with a partner.
They may also have short-term relationships with people. These short-term rela-
tionships are further subdivided into ones that use a condom and ones that don’t.
Finally, a person can select to be abstinent.

Second, beliefs about the riskiness of various forms of sexual activity are formed
rationally. In the analysis a person’s past sexual history is private information.
Still, the fact that someone desires a short-run sexual encounter involving no con-
dom, say as opposed to seeking a long-term one, may signal something about his
past sexual behavior. In particular, it may indicate a proclivity to engage in risky
behavior. Hence, in the analysis, people form rational forecasts about the like-
lihood of a partner having HIV/AIDS based upon the type of relationship that
they are seeking. A person can then forecast the odds of getting HIV/AIDS if
s/he engages in a particular type of relationship. An individual’s choice about
what type of sexual activity to engage may be influenced by his or her belief
about their own health. People assess their odds of having HIV/AIDS rationally
using Bayes’ rule. They understand of how various sexual activities affect their
future health, which influences their current decisions about which type of sex-

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ual behavior to engage in. People who believe that they have HIV/AIDS may be more likely to participate in risky behavior than those that do not because they think that they have little to lose. This channel will worsen the health of participants seeking short-term unprotected sex and amplify the risk of a relationship with them.

Third, the analysis is general equilibrium in nature. There are “markets” or “meeting places” for the different types of sexual activities. People have differing tastes over various types of relationships and search accordingly on the various markets to fulfill their desires. They can choose how intensively to search for a partner on a particular market (or abstain by not searching). They do this recognizing that some of these meeting places will be riskier than others. For example, a short-term relationship using a condom is safer than one that does not. Yet, condoms may fail and hence are not perfectly safe. The model embeds the well-documented fact of monetary transfers within relationships. Each market is characterized by a transfer that one of the partners makes to the other. These transfers depend on the number of men relative to the number of women that seek a particular type of relationship, and equilibrate the two sides of the market. For example, in the absence of transfers there may be more men desiring to engage in short-term unprotected sex than women. To attract women towards this risky activity, men may have to make some form of transfer payment. The market structure eliminates any joint decision problem between partners about whether or not to use a condom. They have the same desires when choosing the same market. Computing a general equilibrium involves solving a fixed-point problem on a computer.

The constructed model is then tuned to fit aspects of the Malawian data. In particular, it is calibrated to match the HIV/AIDS rates for men and women, separately, the fraction of sexual relationships that are short term, the fraction of short-term sexual encounters that use a condom, and the fraction of deaths that arise from HIV/AIDS. The model’s ability to match some non-targeted lifecycle observations is then examined. These include, the profile by age of a symptom-free person’s belief about being HIV/AIDS infected, the HIV/AIDS infection rate by age, and the likelihood of a casual sexual encounter by age. The model does very well at matching the data along these dimensions.
Last, some policy experiments are conducted. To name some, policies are examined such as male circumcision, which some believe reduces the risk of HIV/AIDS transmission from females to males, the promotion of condom use in casual sex, and income transfers to females. Earlier purely theoretical work (discussed in the following literature section) focused on reductions in transmission risk and emphasized that it can lead to higher overall prevalence because people could get excessively risky. The quantitative part of this study does not confirm that reduced transmission risk (either for both genders or for only one gender) fully negates the positive effects of the treatment. Nevertheless, it is important to account for behavioral adjustments and equilibrium effects as they do strongly affect the predicted effectiveness compared to an epidemiological version without behavioral adjustment or small scale interventions without equilibrium effects. For example, the former would predict a substantial increase in the effectiveness of general transmission reduction while the latter would predict less than half. For other policies the behavioral responses are larger, and indicate the possibility of negating the positive primary effect. For example, the rate of HIV/AIDS displays a \( \cap \)-shaped relationship in the psychic pleasure that people get from sex using condoms.

This study also showcases channels beyond the simple effect that some agents increase their risky sexual activity. For example, some forms of increasing long-term relationships rather than short-term ones seem to raise HIV prevalence not so much because individual agents become riskier but mainly because some risky people now join the previously safe haven of marriage. While their move towards marriage is usually seen as a reduction in their own risky behaviour, it increases the infection risk for their marriage partners who previously had a higher chance of finding a safe match. Such change in the mixing patterns seriously affects the effectiveness of the policy. For other interventions such one-sided reduction in transmission risk (e.g., male circumcision) or increases in income (especially for women) the behavioral adjustment and equilibrium effects magnify the efficacy of the policy: Especially women react strongly to higher incomes, which makes them substantially less willing to engage in risky activities. These changes feed back on the men and lead to a virtuous feedback loop.

Overall, this research program aims to develop tools to aid researchers and prac-
tioners in their attempts to think through the various channels that are present in different interventions, and highlights areas where further and more in-depth research should be conducted to assess with more confidence the magnitude of these channels.

The remainder of this paper is organized as follows. The next section discusses the relation to the literature on HIV/AIDS. Section 3 provides background information on sexual behavior and HIV/AIDS in Malawi. Section 4 sets up the economic environment, while Section 5 defines the equilibrium. Section 6 describes the benchmark parametrization of the model. Section 7 presents the results of the policy experiments, as well as additional relevant literature. Section 8 offers some concluding remarks.

2 Relationship to the Literature

This appears to be the first applied general equilibrium model of disease transmission with purposive decision making. Previous papers are either purely theoretical in nature or do not consider the endogeneity of rational human behavior.

The work-horse epidemiological model of disease transmission is the susceptible-infected (SI) model with random mixing; see, e.g., Anderson and May (1992). In such a model people are in either one of two states: infected or susceptible. If a person is infected he transmits the disease to susceptible (non-infected) people until he leaves the sexually active population. In models of HIV/AIDS there is no stage of recovery. In these models people encounter other individuals in the population randomly. Most epidemiological models take the number of encounters, i.e., the number of sexual partners, as exogenously given. The assumption that individuals do not change their behavior in response to their environment—in particular in response to the overall prevalence of the disease—is problematic for human populations. Survey evidence and several empirical studies suggest that people react to a higher presence of HIV/AIDS by adjusting the number of

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3Recovery and, either resistance against further infection, or the possibility of reinfection, are explicitly modeled for other infectious diseases. See Hethcote (2000) for a comprehensive overview of the mathematical modeling of infectious disease.
partners, the type of sexual relationships, and the protective measures that they use [Wellings et al. (1994)]. Even those contributions in epidemiology that relax simplifying assumptions regarding the transmission process by relying on computer simulations [see e.g. Clark and Eaton (2008)] usually do not incorporate behavioral changes that arise from the risk assessments of individual agents.

Theoretical models of risk assessment and equilibrium analysis have been introduced by economists. For tractability, most of this work has focussed on only one dimension of risky behavior and abstracts from intertemporal considerations of an individual over his lifetime. Philipson and Posner (1993) provide the first economic analysis of HIV/AIDS. Their interesting book gives a great overview of the different aspects involved in treating HIV/AIDS – from testing, to regulatory intervention, to medical research and the political economy of treatment. Most of their analysis is verbal, however. They provide only a very simple formal partial equilibrium model of a one-time choice between protected and unprotected sex, taking the risk of infection into account.

Kremer (1996) augments the basic SI model by determining the number of partners endogenously as a choice by rational agents that depends on the prevalence of the disease. He focuses on the theoretical implications of such endogeneity. He shows that a reduction in the transmission probability can increase the overall prevalence rate because individual agents behave in a more risky fashion. Moreover, in a heterogeneous population an increase in the prevalence rate for the pool of potential partners might make people with low sexual activity more cautious while making high activity people more reckless, since the latter expect such a high probability of contracting the disease that the marginal increase in infection due to an additional sexual act is small. This might in turn lead to a higher overall prevalence rate.

To derive these predictions theoretically, Kremer (1996) abstracts from a number of issues. People only consider the lifetime probability of not contracting HIV/AIDS, but do not care about the exact time they contract the disease. They choose the same behavior in every period of their life and do not update their behavior based on their sexual experience. Condom use is not explicitly modeled, which abstracts from the joint decision problem in using a condom. All relationships have the same length, so a distinction between long-term partners
and short-term sex is not possible. And all agents meet randomly. Finally, the model is a one-gender model that makes the analysis of gender asymmetries in the disease dynamics impossible. The current paper shows how these richer elements can be modeled, and evaluates quantitatively the impact of these various channels for disease transmission, but at the cost of additional complexity.

There are other recent papers who incorporate decision-making into epidemiological models of infectious diseases (not only restricted to HIV/AIDS). One example is Fenichel et al. (2011), which models the trade-offs that drive person-to-person contact decisions. They argue that including adaptive human behavior might change the course of epidemics in such models. Toxvaerd (2012) proposes a model in which agents may engage in costly preventive behavior and studies subsidy policies in such a setting. These papers are purely theoretical, however. Klein et al. (2007) also discuss the benefits of adding economic insights into epidemiological models.

Kremer and Morcom (1998) abstract from homo economicus but introduce selective mixing instead of random mixing: Individuals have a higher probability of meeting people like themselves as opposed to others. This idea also features prominently in this paper, but is applied to the type of sexual behavior that a person seeks. For example, an individual that seeks a long-term relationship can search in a way that makes it particularly likely to meet a partner that also seeks a long-term relationship. The special case where people exclusively meet others who are seeking the same type of relationship allows for selective mixing and avoids modeling any conflict of interest in relationship formation. This case will be the focus in this paper. As in Kremer (1996) the current work assumes that other characteristics such as age, past sexual activity and sexual preferences are private information and abstracts from the issue that these characteristics are (imperfectly) observable in real life. Additionally, as in Kremer (1996), attention is restricted to steady states, with the justification that prevalence rates in Malawi have remained roughly constant in recent years.

The only other model that explicitly models men and women is Magruder (2011) who develops a Jovanovic (1979) style matching model of marital search to analyze the HIV/AIDS epidemic in South Africa. The idea is that partners enter trial marriages and explore whether or not they are good matches. During this
period couples have sex. There is undoubtedly truth to this story. In his setting there is no decision about whether or not to use a condom during these trial marriages. Also, the analysis is not general equilibrium in nature. The decision about whether to accept or reject a partner is not affected by the prevalence rate of HIV/AIDS in society, or by any beliefs that the individual may have about whether or not s/he has the virus based upon her or his past sexual history. This may be important because healthy (young) individuals might self-select into the safety of marriage while those who believe they are infected have less to gain from safety and might opt for more risky alternatives.

The application of economic analysis to sexual behavior has a long tradition in mate selection [e.g. Becker (1973)]. The importance of delay in the search of a partner has been acknowledged both in theoretical work [e.g., Burdett and Coles (1997) and Shimer and Smith (2000)] and in applied work on marriage formation in developing countries [e.g., Boulier and Rosenzweig (1984)]. In this paper, delay arises because of a search cost of finding a partner. The implications for short-term sex that arise while waiting for a long-term partner are taken into account. The simulation connects to the simulation-based analysis of sexual behavior in Greenwood and Guner (2010). This paper also features search for partners when different people have different attitudes towards sex, but focuses on premarital sex in the absence of disease transmission. Similar to the approach taken here, depending on what type of relationship people are interested in, people will direct their search effort to the appropriate market rather than randomly meeting people they do not want to match with. The idea that people can rationally target their search behavior to particular markets is also present in many recent theoretical models [e.g. Jacquet and Tan (2007), Eeckhout and Kircher (2010), Gautier, Svarer, and Teulings (2010)] but has not yet been applied to disease transmission.

There is also an empirical literature that studies HIV/AIDS prevention policies, either using randomized field experiments or epidemiological data. The policies that are most closely related to the current paper are those that decrease the one-time transmission risk. Oster (2005) argues that the treatment of other STDs reduces the infection risk and thereby lowers the incidence of HIV/AIDS. Others find that male circumcision lowers the infection risk for males [see, e.g., Auvert et al. (2005) and Gray et al. (2007)]. The effect of income subsidies is also analyzed.
here. Most closely related to this is Duflo et al. (2006) who find that subsidizing school uniforms for girls reduces sexual behavior in girls. In their case, it increases the opportunity cost of dropping out of school due to pregnancy. For a survey of a number of other studies based on randomized controlled trials, see Padian et al. (2010).

One example of an applied epidemiological study is Low-Beer and Stoneburner (1997), in which the authors propose a life-cycle and sex-dependent model of HIV. They use their model in several applications such as forecasting HIV incidence in East Africa. They do not model individual’s rational behavior nor analyze prevention policies, however. A more recent example is Johnson (2008) that provides a very detailed description of several facets of HIV/AIDS epidemics (short- and long-term relationships, condom usage, etc.). Again, the model doesn’t incorporate a choice-theoretic framework.

These studies complement the approach taken here. First, theory alone cannot infer the elasticity of sexual behavior in response to policy, for example. Additionally, pure epidemiological models without explicit rational behavior fail in taking into account potential behavioral disinhibition that may rise in response to a policy. Moreover, a purely empirical approach has other shortcomings. Typically a randomized field experiment is conducted on a small subgroup of the population which does not allow for general equilibrium effects due to adjustments in prevalence, prices, and sorting patterns. This might be particularly important in the context of HIV/AIDS which may affect men and women differently, but often data is collected on one gender only, as in many studies of male circumcision.

Other empirical studies analyze testing, treatment, or information campaigns which are outside the scope of this paper. Lakdawalla, Sood, and Goldman (2006) study the effects of anti-retroviral drugs and find that these led to an increase in sexual behavior in the U.S. Thornton (2008) uses evidence from a randomized field experiment in rural Malawi to conclude that testing is not a very cost-effective prevention policy. Dupas (2011) shows in a randomized field experiment in Kenya that teenage girls who are given information about the HIV status of different groups of men respond by shifting sexual behavior to the lower risk groups. De Walque (2007) finds, based on evidence from Uganda, that HIV/AIDS information campaigns have a larger impact on more educated
3 Families, Sexual Behavior, and HIV/AIDS in Malawi

The Republic of Malawi serves as a focal country to which the analysis is applied. Therefore, this section briefly describes some information on the HIV/AIDS epidemic in Malawi, together with details about sexual behavior and family life. This background will be useful in guiding the modeling choices. For example, it is argued that it is reasonable to ignore in the analysis mother-to-child transmissions and homosexual sex.

The Republic of Malawi is a country in southeast Africa. It has a population of 14 million people and a land mass of 118,000 square km, making it one of the most densely populated nations in the world. Malawi suffers greatly from the HIV/AIDS epidemic.\(^4\) Twelve percent of the adult population is currently infected. This is well above the average within Sub-Saharan Africa (SSA), which has an adult prevalence rate of about 7.2%—see Canning (2006). It is also well below the HIV rate of the most affected countries, such as Botswana with an adult prevalence rate of 37%, or South Africa where 22% of all adults are infected. The Malawian HIV rate has been roughly constant (ranging between 12-14%) since the mid 1990s, yielding some indication that the disease dynamics have settled into a steady-state.

The principal mode of HIV transmission in Malawi is through heterosexual sex. Mother-to-child transmissions are also important, accounting for about 10% of all new HIV infections. This fact is ignored here. Most people born with HIV die before they reach sexual maturity (about half of all babies infected during the perinatal period die before their fifth birthday), and therefore do not add to the propagation of HIV. Like in the rest of SSA, more than half of the HIV-infected population in Malawi is female. By contrast two thirds of the infected population is male in the Western world—see World Development Indicators (2009). In the West, HIV is largely a disease of drug users and homosexuals. In SSA, by

\(^4\)Unless noted otherwise, information on HIV prevalence and patterns of sexual behavior are from the 2004 Demographic and Health Survey’s (DHS) Final Report for Malawi.
comparison, HIV is a disease that disproportionately affects married (and divorced/widowed) women. The HIV rate among adult women is currently about 13%, compared to 10% among men, suggesting important gender differences. Women are also affected by HIV much earlier in life. For example, 3.7% of women aged 17-19 are HIV-positive, compared to just 0.4% for men in the same age category.

A rational model of HIV only makes sense if people understand what HIV is, are aware of how it gets transmitted, and know how to avoid it. This seems largely to be the case in Malawi. Almost 100% of the surveyed Malawians had heard of HIV or AIDS. About 57% of women and 75% of men correctly identified the use of condoms as a means to protect against HIV infection. Finally, an overwhelming majority of adults in Malawi—74% for women and 86% for men—know of a source to get condoms. Finally, Delavande and Kohler (2009) document that people in Malawi are relatively good in assessing their own probability of being infected with HIV. Thus, a rational model of risky sexual behavior is a reasonable approximation for the Malawian epidemic.

Sexual behavior conducive to the spread of the disease is relatively common in Malawi. Condoms are used by less than half of all respondents in their last sexual act—30.1% of women reported using a condom during their last sexual activity, compared to 47.1% of the men. Interestingly, Malawian women have sex at earlier ages than Malawian men. Large age gaps in sexual relationships are quite common. It is also considered normal for unmarried people to change partners often. Undie, Crichton, and Zulu (2007) single out the following quote from a female teenager in order to capture the overall attitude that they found towards varying partners in their interviews: “[Boys say] ‘Do you just eat vegetables daily? Sometimes, you change [your diet]’... Girls say, ‘You don’t need to have one cloth [outfit] only.’”. Furthermore, divorce is relatively common. Reniers (2003) reports that 45% of marriages end in divorce within 20 years. The quantitative evidence from the DHS suggests that men engage in more risky sexual behavior than women. This difference might be partially due to a social bias as to what is acceptable behavior. Miller, Watkins, and Zulu (2001) analyze gender differences in survey responses in Malawi and find that when husband and wife answers to a variety of questions contradict each other, the wife is more likely to
have said ‘no’ while the husband is more likely to have said ‘yes.’ In other words, gender differences in reported sexual behavior have to be interpreted with caution. Several other forms of risky behavior will be abstracted from in the paper. For example, the model does not have concurrent relationships, such as extramarital affairs or polygyny, both of which are relatively common in Malawi. In 2004, 8.3% of all married men admitted to having had an affair in the last year. Women admit to much fewer affairs. Polygyny is also fairly common in Malawi. As recently as 2004, 10% of all men had more than one wife. The model abstracts from concurrent relationships to keep it tractable. Future work should include these phenomena.

The high prevalence of risky behavior does not necessarily imply that people are uninformed or irrational: it is more likely due to the trade-off between increased safety versus less pleasure. Undie, Crichton, and Zulu (2007) highlight this with the following quote from an interview with a Malawian female about protected sex: “You can’t eat [candy] while it’s in the wrapper. It doesn’t taste [good].” In Malawi, condom use within marriage is essentially non-existent [Chimbiri (2007)]. One reason is that marital sex is often aimed at reproduction. Furthermore, using a condom in marriage may be interpreted as a signal of infidelity. Bracher, Santow, and Watkins (2004) write that “in essence, using condoms within marriage is a sign that it is ‘not a real marriage’” and quote a Malawian saying “she does not protect herself with her husband, for it is marriage.” Note also that while using a condom lowers the transmission risk substantially, it does not decrease the risk to zero. Bracher, Santow, and Watkins (2004) cite a study that finds that for new condoms, the average breakage rate is 4%; this rate jumps to 19% for condoms that are 7 years old. The higher breakage rate may, in fact, be the more relevant figure in the context of Malawi since condom quality degrades faster in a tropical climate where the temperature often exceeds the recommended storage temperature of 25 degrees Celsius.

Poulin (2007) documents that money and gift transfers in sexual partnerships are part of the courting practices of young Malawian women and men. In addition to an expression of love and commitment, she argues that these transfers are a way of acquiring sex for men and about meeting their financial needs for women. A gift might be in the form of sugar or soap, but also in cash. Men who give gifts
expect to receive sex, and they expect it sooner rather than later. Transfers are not made directly before or after sex (as with prostitution), however; rather gift giving is an integral part of a relationship that may depend on the need (e.g. for soap) of the recipient as well as the availability of cash for the giver. Similar evidence is also given in Swidler and Watkins (2007). The model developed here will allow for such transfers between men and women in sexual relationships.

Finally, note that both testing and treatment (such as anti-retroviral drugs) have been fairly uncommon in Malawi until very recently. Testing was introduced in 2004 within the context of the Malawi Diffusion and Ideational Change Project (MDICP). This has led to 2,686 women and 2,581 men being tested (as of the publication of the 2004 DHS Final Report on Malawi), providing the first national population-based HIV prevalence estimate for Malawi.

4 Economic Environment

Imagine a world populated by males and females. Males and females desire relationships with the opposite sex. There are two types of relationships, viz short-term and long-term ones. Within a relationship individuals engage in sex. Sex is risky because of the presence of the HIV/AIDS virus in society. There are two types of sex, protected and unprotected. Protected sex offers a better defense against the transmission of HIV/AIDS across partners. It provides less enjoyment, though. Individuals interested in a short-term relationship must decide what kind of sex they desire. Put simply, they must weigh the extra momentary utility associated with unprotected sex against the increased odds of being afflicted with the HIV/AIDS virus in the future. As motivated in Section 3, sex is always unprotected in long-term relationships. Further, suppose that a person can only engage in one relationship at a time.

Denote the utility from unprotected sex by $u$ and the utility from protected sex by $p$, with $u \geq p > 0$. The utility flow in a long-term relationship is $u + l$, where $l$ may be negative. A positive $l$ can be interpreted as a taste for long-term attachment, while a negative $l$ signifies taste for variety in partners. Individuals also realize utility from the consumption of goods. Let this utility be given by $\ln(c)$, where
c is consumption. Each period a person receives income in the amount \( y \). There is no borrowing or lending in the economy. An individual discounts the future at a stochastic rate that takes two values, viz \( \iota \) and \( \beta \) with \( \iota \leq \beta \). Individuals start off life with the low rate \( \iota \). This low rate reflects the impatience of youth, which may lead to a predilection to engage in risky behavior. Then, at every period, a person may switch permanently to the high rate with probability \( \eta \).

These two discount factors reflect the probability of dying from a natural death each period given by \( \delta \); that is, \( \iota = \tilde{i}(1 - \delta) \) and \( \beta = \tilde{\beta}(1 - \delta) \) where \( \tilde{i} \) and \( \tilde{\beta} \) are the underlying subjective discount factors. The values of \( l, p, u, y, \) and \( \iota \) may differ across individuals of a given gender. The set of fixed characteristics for a person is denoted by \( x = (l, p, u, y, \beta, \iota) \), which will be called a person’s type.

At the beginning of each period an unattached individual may search for a long-term partner. The odds of finding a partner on the long-term market are denoted by \( \pi_l \). The individual can pick these odds at an increasing cost in terms of lost utility. These search costs are given by \( C_l(\pi_l) = \omega_l[\pi_l/(1 - \pi_l)]^{\kappa_l+1} \), where \( \kappa_l \geq 0 \) and \( \omega_l > 0 \). Observe that \( C(0) = 0 \) and \( C(1) = \infty \). A long-term relationship may break up (at the end of) each period with exit probability \( \xi \). If the person is unsuccessful at finding a long-term mate s/he then enters the short-term market, where s/he can still engage in sexual behavior for this period. Note that an individual who does not want a long-term relationship can set \( \pi_l = 0 \). If the person wants a short-term one, then s/he must decide whether to have one involving protected or unprotected sex. Let \( \pi_p \) and \( \pi_u \) represent the odds of finding a partner in the protected and unprotected markets for short-term relationships, which will be choice variables. The cost of searching in each market is given by \( C_s(\pi_p) \) and \( C_s(\pi_u) \), which have the same functional form as \( C_l(\pi_l) \), but where the parameters \( \kappa_s \) and \( \omega_s \) are allowed to differ from the long-term market. The total cost of searching for a short-term partner will then be \( C_s(\pi_p) + C_s(\pi_u) \). Assume that an individual will not simultaneously draw a partner on both markets. The odds are therefore constrained by \( \pi_p + \pi_u \leq 1 \), and an individual will be abstinent with probability \( \pi_a = 1 - \pi_p - \pi_u \). Also, observe that individuals can choose abstinence by picking \( \pi_p = \pi_u = 0 \).

Given the pervasive evidence on gift giving in the context of sexual relationships (see Section 3), transfers are exchanged for sex. Associated with each market is
a transfer payment, $t$, that is made between the two partners. For the person receiving the transfer, $t$ will be positive, while it will be negative for the individual making it. Think about the people receiving the transfers as supplying relationships on the market, and those paying transfers as demanding them. Interpret the transfer as representing the inputs into a relationship: affection, entertainment, gifts, etc. The magnitude of this transfer is determined in competitive equilibrium. The size of it will depend upon the demand and supply for a given type of relationship by each gender weighted by the effort $\pi$ put into finding a partner for this relationship. This will hinge on the utility that each gender realizes from a partnership in the various markets and the riskiness of participating in them.

People form beliefs about their own infection status. A person enters a period with a prior belief about the likelihood of not being infected with HIV/AIDS. Denote this prior belief by $\phi$, which is private information. The person then may have a relationship involving either protected or unprotected sex. The risk of catching HIV/AIDS from an infected person is different for the two kinds of sex. If the individual has sex with an HIV/AIDS infected person then the virus will get transmitted with probability $1 - \gamma$, where $\gamma$ differs across the types of sex and by gender. The transmission probability is lower for protected sex vis à vis unprotected sex. A person who is inflicted with HIV/AIDS will typically not show symptoms for a while, in which case he cannot distinguish his health state from a person who is not infected. If symptoms occur, they mark the severe part of the illness. The person, and others in his surroundings, will know that he is ill. Assume that an infected person will develop symptoms each period with probability $\alpha$. At the end of a period, a person updates his prior in Bayesian fashion depending upon: (i) the type of relationship he was in; (ii) whether or not he observed symptoms in himself, and (iii) in the case of marriage, whether or not he observed symptoms in his partner.

Let the expected lifetime utility for a person with the symptoms of HIV/AIDS be represented by $A$. Assume that a person stricken with HIV/AIDS symptoms engages in no further relationships. The probability that a person displaying

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5 Alternatively, market clearing could be achieved through different meeting probabilities. This should lead to qualitatively similar results.

6 Note that $A$ cannot be too large to assure people prefer living over dying.
symptoms dies is $\delta_2$. Since a person with HIV/AIDS symptoms engages in no further activity, $\delta_2$ does not appear in the value functions. However, it is relevant for computing the average HIV/AIDS rate in society. Note that in the framework there is an attrition in the population each period both due to natural death and to HIV/AIDS. This loss is replenished by an inflow each period of newly born males and females. Recall that $x$ denotes the set of permanent characteristics for an individual, namely $l, p, u, y, \beta,$ and $\iota$. People also differ by gender. Gender will be suppressed unless it is specifically needed and then it will be represented by the subscript $g$ (for $g = f, m$) attached to a function or variable. Assume that $\mu$ type-$x$ individuals are born at the beginning of each period. Figure 1 portrays the timing of events.

**Figure 1: Timing of Events**

- Indicates search intensity choice at this node.
- Indicates sexual activity.

Before proceeding on to the formal analysis some notation will be defined. An individual will be indexed by his prior that he is healthy, $\phi$, his current discount...
factor, $d$, and his exogenous type $x$. Let $\tilde{V}_r^d(\phi, x)$ denote that lifetime utility for a person with prior $\phi$, a discount factor $d = \iota, \beta$, and an exogenous type $x$ who just found a partner for a relationship of type $r = a, l, p, u$ (abstinent, long-term, short-term protected and short-term unprotected). Similarly, $V_r^d(\phi, x)$ will represent the expected lifetime utility for a person who is currently searching for a partner in a type-$r$ relationship (for $r = l, s$ where $s$ denotes short term), but has not found one yet. Attention will now be directed toward the determination of the functions $\tilde{V}_r^d(\phi, x)$ and $V_r^d(\phi, x)$. The focus will be on studying a stationary equilibrium for this setting.

### 4.1 Short-term Relationships

#### 4.1.1 Abstinence

The case of abstinence is the easiest to analyze. So, start here. To this end, imagine a type-$x$ person with a high discount factor, $\beta$, who has failed to match on the short-term sex markets. Thus, he will be abstinent for the current period. The individual’s discount factor will now remain high forever.

The value function for a type-$x$ individual with prior $\phi$ who is currently abstinent is given by

$$\tilde{V}_a^\beta(\phi, x) = \ln(y) + [1 - (1 - \phi)\alpha]\beta V_l^\beta(\phi', x) + (1 - \phi)\alpha\beta A,$$

with

$$\phi' = \Phi_a(\phi).$$

The first term on the right-hand side of (1) gives the person’s momentary utility from his current consumption. Two things can happen next period, as the next two terms illustrate. Even though the individual doesn’t have sex in the current period, and is symptom free, he may still develop the symptoms of HIV/AIDS next period because of past relationships. He starts the current period with a prior, $\phi$, about his probability of being non-infected. Therefore, he believes that he will develop symptoms next period with probability $(1 - \phi)\alpha$. In this event the person will realize an expected utility level of $A$, which is discounted at rate $\beta$. 

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This explains the third term on the righthand side of (1). Likewise, he will remain healthy next period with odds \(1 - (1 - \phi)\alpha\). In this situation, the person will enter the long-term market next period and search for a mate. The discounted expected utility from searching on the long-term market next period with prior \(\phi'\) is given by \(V_l^\beta(\phi', x)\). This accounts for the second term in the equation. Also, when the individual does not suffer the symptoms of HIV/AIDS, he updates his prior in a Bayesian fashion to \(\phi'\) summarized by the function \(\Phi_a(\phi)\), which will be explained fully in Section 4.3. Note that if one shows the symptoms of HIV/AIDS in the current period then it is known with certainty that one has the virus.

Next, consider the case of an abstinent person with a low discount factor, \(\iota\). The discount factor may switch next period to the high value, \(\beta\), with probability \(\eta\), or remain at the low one, \(\iota\), with probability \(1 - \eta\). It is easy to see that the value function for a type-\(x\) person with a low discount factor, \(\iota\), and a prior \(\phi\), will now read

\[
\tilde{V}_a^\iota(\phi, x) = \ln(y) + [1 - (1 - \phi)\alpha]\iota[\eta V_l^\beta(\phi', x) + (1 - \eta)V_l^\iota(\phi', x)] + (1 - \phi)\alpha \iota A, \tag{2}
\]

with \(\phi' = \Phi_a(\phi)\).

### 4.1.2 Sexual Relationships

Now, suppose that the individual is matched in a short-term relationship. Again, start with the situation where the person has a high discount factor \(\beta\). If \(s = p\) then the person will realize the utility \(p\) from his relationship. If \(s = u\) the individual will enjoy \(u\). Define the indicator function \(I(s)\) to return a value of 1 when \(s = p\), and a value of 0 otherwise. Thus, the joy from a short-term sexual relationship can be written as \(pI(s) + u[1 - I(s)]\). The cost of sex on the two markets differs for two reasons. First, the transmission risk of catching HIV/AIDS from an infected person differs across markets. Specifically, the transmission risk in the protected market, \(1 - \gamma_p\), is lower than in the unprotected one, \(1 - \gamma_u\). Second, the average level of healthiness in the pool of participants in the two markets will in general differ. The fact that a person desires a short-term sexual relationship that
does not use a condom signals something about their past tendencies to engage in risky behavior. In light of this, $\bar{\phi}_s$ gives the odds that a randomly drawn partner on the short-term market $s$ (for $s = p, u$) does not have the HIV/AIDS virus.

Given his prior about his own health status, $\phi$, the individual believes that he will suffer the symptoms of HIV/AIDS next period with probability $\alpha[(1 - \phi) + \phi(1 - \bar{\phi}_s)(1 - \gamma_s)]$. Symptoms can arise from two potential sources. The person could already have the virus and the symptoms materialize. The odds of this event are $\alpha(1 - \phi)$. Or, the person can catch the virus from his current partner, and then the symptoms appear, an event that occurs with probability $\alpha\phi(1 - \bar{\phi}_s)(1 - \gamma_s)$. The odds of not suffering the symptoms of HIV/AIDS next period are then just $1 - \alpha[(1 - \phi) + \phi(1 - \bar{\phi}_s)(1 - \gamma_s)]$.

The value function for a type-$x$ individual with prior $\phi$ who is currently having a short-term relationship in market $s$ is given by

$$\tilde{V}_s^\beta(\phi, x) = \ln(y - t_s) + pI(s) + u[1 - I(s)]$$

$$+ \{1 - \alpha[(1 - \phi) + \phi(1 - \bar{\phi}_s)(1 - \gamma_s)]\} \beta V_l^\beta(\phi', x)$$

$$+ \alpha[(1 - \phi) + \phi(1 - \bar{\phi}_s)(1 - \gamma_s)] \beta A,$$

with

$$\phi' = \Phi_s(\phi), \text{ for } s = p, u.$$ (4)

The function $\Phi_s(\phi)$ specifies how the individual will update his prior after having short-term sex, contingent upon not observing the symptoms of HIV/AIDS in the current period. The form of this function is discussed in Section 4.3. Recall that the person’s exogenous type, $x$, determines his tastes for protected sex, unprotected sex, short-term relationships, and his level of income. The dependence of $p, u$, and $y$ on $x$ in the value functions (1) and (3) is suppressed for convenience. Similarly, the gender of the individual is also omitted. This will be indicated later on by a subscript $g$ for $g = f, m$ (male or female) attached, when relevant, to a variable or function.

Next, consider the case when the person has a low discount factor, $\iota$. The discount factor may switch next period to the high value, $\beta$, with probability $\eta$, or remain
at the low one, \( t \), with probability \( 1 - \eta \). Therefore, the analogue to (3) is

\[
\tilde{V}'_s(\phi, x) = \ln(y - t_s) + pI(s) + u[1 - I(s)] + \{1 - \alpha[(1 - \phi) + \phi(1 - \phi_s)(1 - \gamma_s)]\}t[\eta V_i^\beta(\phi', x) + (1 - \eta)V'_i(\phi', x)]
\]

\[
+ \alpha[(1 - \phi) + \phi(1 - \phi_s)(1 - \gamma_s)]tA,
\]

again with \( \phi' = \Phi_s(\phi) \), for \( s = p, u \).

Last, upon entering the market for short-term relationships a person must decide how much effort to expend searching in each market; that is, he must choose \( \pi_p \) and \( \pi_u \). This is done in accordance with the problem outlined below.

\[
\begin{align*}
V^d_s(\phi, x) &= \max_{0 \leq \pi_p^d, \pi_u^d \leq 1, \pi_u^d + \pi_p^d = 1} \{\pi_p^d \tilde{V}_p^d(\phi, x) + \pi_u^d \tilde{V}_u^d(\phi, x) + (1 - \pi_p^d - \pi_u^d)\tilde{V}_a^d(\phi, x)\} \\
&\quad - C(\pi_p^d) - C(\pi_u^d), \text{ for } d = \iota, \beta.
\end{align*}
\]

The function \( V^d_s(\phi, x) \) gives the ex ante value for a type-\( x \) individual, with prior \( \phi \), of entering the market for short-term sex. The solution for search effort is represented by the function \( \pi^d_s = \Pi^d_s(\phi, x) \), for \( s = p, u \).

### 4.2 Long-term Relationships

Imagine a person who is currently in a long-term relationship. In a long-term relationship there are no choices to make: there are no affairs, all sex is unprotected, and the partnership endures until some form of exogenous breakup occurs. Suppose that the person entered this relationship \( n \) periods ago with prior \( \phi \). At the end of period \( n \) this relationship can breakup for three reasons (besides natural death).\(^7\) First, it could transpire that both people are symptom free and that the relationship terminates due to an exogenous breakup. Second, it could end because the individual’s partner becomes afflicted with HIV/AIDS symptoms. Third, the person may develop symptoms and the relationship stops.

The value of entering into a long-term relationship, \( \tilde{V}_l^d(\phi, x) \), will now be speci-

\(^7\) For simplicity assume that both partners die together; i.e., with probability \( \delta \) the pair dies, and with probability \( (1 - \delta) \) both survive.
fied. Start first with a person who has a high discount factor $\beta$. Note that at the end of each period the relationship either continues or it may break up for one of the three reasons mentioned above. From this observation it follows that $\bar{V}_{i}^{\beta}(\phi, x)$ can be written as

$$
\bar{V}_{i}^{\beta}(\phi, x) = \ln(y - t_i) + u + l \\
+ \sum_{n=1}^{\infty} \beta^n (1 - \xi)^n \Pr[\text{no symptoms in either person at end of period } n|\phi] \\
\times [\ln(y - t_i) + u + l] \\
+ \sum_{n=1}^{\infty} \beta^n (1 - \xi)^{-1} \xi \Pr[\text{no symptoms in either person at end of period } n|\phi] \\
\times V_{i}^{\beta}(\Phi_{i}^{h}(\phi, n), x) \\
+ \sum_{n=1}^{\infty} \beta^n (1 - \xi)^{-1} \Pr[\text{symptoms just in partner at end of period } n|\phi] \\
\times V_{i}^{\beta}(\Phi_{i}^{a}(\phi, n), x) \\
+ \sum_{n=1}^{\infty} \beta^n (1 - \xi)^{-1} \Pr[\text{symptoms in person at end of period } n|\phi] \times A.
$$

This expression looks more complicated than it really is. It will now be explained.

The term on the right-hand side of the first line gives the current utility from the match. The next two lines give the discounted expected utility accruing when the match sustains over the next $n$ periods. For a match to sustain until period $n + 1$ there cannot be an exogenous breakup prior to this date. This occurs with probability $(1 - \xi)^n$. Additionally, neither party can show symptoms before this date. The formula for the odds that neither partner will show symptoms before period $n$ is presented in Section C.2 of the Appendix, as are the formulae for the other probabilities shown. The rest of the expression enumerates the expected utilities associated with breakup events in period $n + 1$. In particular, the fourth and fifth lines reflects separations due to exogenous breakups. An exogenous breakup will occur at the end of period $n$ with probability $(1 - \xi)^{-1}$. When this happens the individual enters into single life again. At that time he updates his prior according to the function $\Phi_{i}^{h}(\phi, n)$ as specified in Appendix C.1. The individual may also enter into single life because his partner develops HIV/AIDS.
symptoms in some period \( n \). This is accounted for by the next two lines, six and seven. In this situation the individual will update using the function \( \Phi^d_n(\phi, n) \), also specified in Appendix C.1. Finally, HIV/AIDS may manifest itself in the individual at the end of some period \( n \). This is captured by the last line. Also recall that the odds of dying from a natural death are incorporated into \( \beta \).

The value of a long-term relationship for a person with a low discount factor \( \iota \) is determined analogously. One now must take into account that the discount factor may switch at some future date from \( \iota \) to \( \beta \). The expression for \( \bar{V}^d_n(\phi, x) \) is now slightly more complicated. It is developed in Section C.3 of the Appendix – see (23). The ex ante value of a long-term relationship for a type-(\( \phi, x \)) person with discount factor \( d \) is given by

\[
V^d(\phi, x) = \max_{\pi^d} \left[ \pi^d \bar{V}^d_n(\phi, x) + (1 - \pi^d)V^d_{a}(\phi, x) - C(\pi^d) \right], \text{ for } d = \iota, \beta \tag{8}
\]

The solution for search effort, \( \pi^d_l \), is represented by the function \( \pi^d_l = \Pi^d_l(\phi, x) \).

### 4.3 Belief Updating

The easiest case for belief updating is a person who was abstinent in a given period. The function for updating the prior, \( \Phi_a(\phi) \), is given by Bayes’ rule:

\[
\Phi_a(\phi) = \frac{\Pr(\text{not being infected this period } \mid \phi)}{\Pr(\text{not observing any symptoms this period } \mid \phi)} \tag{9}
\]

\[
= \frac{\phi}{\phi + (1 - \phi)(1 - \alpha)} = \frac{\phi}{1 - (1 - \phi)\alpha}.
\]

The prior probability of not being infected is \( \phi \). The probability of not observing any symptoms has two components: the odds of not having HIV/AIDS, \( \phi \), and the odds of being infected but not showing symptoms, \( (1 - \phi)(1 - \alpha) \). As will be seen, the formula for updating can become complicated when one has sex. Here, the odds of transmission from the partner must be taken into account. Additionally, in a long-term relationship there is valuable information contained in a partner’s health status.

After engaging in a short-term relationship, Bayes’ rule says that the prior should
be updated according to the formula

\[ \Phi_s(\phi) = \frac{\Pr(\text{not being infected this period} \mid \phi)}{\Pr(\text{not observing any symptoms this period} \mid \phi)}, \text{ for } s = p, u, \]

\[ = \frac{\phi \bar{s}_s + \phi (1 - \bar{s}_s) \gamma_s}{\phi \bar{s}_s + (1 - \phi)(1 - \alpha) + \phi (1 - \bar{s}_s) [(1 - \gamma_s)(1 - \alpha) + \gamma_s]}. \]

There are two reasons why the individual might not have HIV/AIDS. Perhaps neither him nor his partner have it. The odds of this are \( \phi \bar{s}_s \). Or, maybe his partner does have it, but it fails to transmit. This will happen with probability \( \phi (1 - \bar{s}_s) \gamma_s \). This explains the numerator. Turn now to the denominator. The individual will show no symptoms in both of these cases. He may also be symptom free even though he actually has the virus. He could initially have the virus yet no symptoms appear, an event that occurs with probability \( (1 - \phi)(1 - \alpha) \), or he could catch it from his current partner but the symptoms fail to materialize, the odds of which are \( \phi (1 - \bar{s}_s)(1 - \gamma_s)(1 - \alpha) \). Note that this formula presumes that a short-term relationship ends before an individual can observe whether or not his partner is afflicted by the symptoms of HIV/AIDS at the end of the current period.

Finally, updating in long-term relationships is relevant only when the relationship ends. Relationships can end for three reasons: an exogenous breakup, the person develops symptoms, or the partner develops symptoms. Obviously if a person develops symptoms, he knows that he is sick and no further updating is necessary. Denote Bayesian updating for the other two cases by \( \Phi^h(\phi, n) \) and \( \Phi^n(\phi, n) \) respectively. The exact formulae are given in Appendix C.1.

\section{5 Stationary Equilibrium}

A stationary equilibrium for the developed framework will now be formulated. This involves two steps. First, the equilibrium type distributions for singles will be implicitly specified; an explicit characterization is contained in Appendix C.4. Second, the HIV/AIDS non-prevalence rates for each of the three markets are presented. Third, the market-clearing conditions for the various types of rela-
tionships will be cast. After this is done, the analysis will conclude with a formal
definition of the equilibrium.

5.1 Equilibrium Type Distributions

At the beginning of each period there will be a certain number of single type-$x$
individuals that have a prior $\phi$ and discount factor $d$. Let $S^d(\phi; x)$ represent the
non-normalized stationary distribution over the prior $\phi$ for single type-$x$ individ-
uals that holds at the beginning of a period for people with a discount factor $d$.
Some of these people will find a match in the long-term market and exit single
life. Others will find a partner in one of the short-term markets. The rest will
remain abstinent. At the end of the period, all singles who either don’t die or
experience the symptoms of HIV/AIDS will update their priors according to the
sexual experiences they just had. Additionally, there will be a flow in of new
arrivals. The new arrivals will be made up of two groups: the newly born, and
older people whose long-run relationships have broken up. Let $L^d(\phi'; x)$ denote
the distribution over the prior $\phi'$ for type-$x$ individuals with discount factor $d$
who exit long-term relationships at the end of a period.

For simplicity, $S^d(\phi; x)$ and $L^d(\phi'; x)$ will be represented by discrete distributions.
A finite number of values for $x$ will be assumed and the number of values for $\phi$
is countable. The process just described above, which maps this period’s singles
distribution into the next period’s one, will be represented by the transition oper-
ator $T$. In a steady state these distributions will be determined by the fixed point
of this operator:

\[(S^\beta, L^\beta, S'^\beta, L'^\beta) = T(S^\beta, L^\beta, S'^\beta, L'^\beta).\]  \hspace{1cm} (10)

The operator $T$ is characterized fully in Section C.4 of the Appendix—see equa-
tions (24) to (30).

For the probability that a randomly drawn person does not have HIV/AIDS it is
now useful to introduce the subscript $g$ (for $g = f, m$) to a function or variable
to denote the gender of the person in question. For example, the relevant odds

\(\text{They will be countable in equilibrium, as only a finite number of types enter each period, and}
\text{each type has, with probability one, only a finite number of possible experiences before exit due}
\text{to exogeneous death.}\)
for men in the long-term market is the rate of nonprevalence in the associated pool of females, $\bar{\phi}_{f,l}$, while for females the pertinent odds refer to those in the pool of potential male partners, $\bar{\phi}_{m,l}$. It is readily apparent that the odds of a randomly drawn person of gender $g (= f, m)$ not having HIV/AIDS, who entered into relationship of type $r (= l, p, u)$, are given by

$$\bar{\phi}_{g,l} = \frac{\sum_d \sum_x \sum_{\phi} \phi \Pi_{g,l}^d(\phi, x) S_g^d(\phi; x)}{\sum_d \sum_x \sum_{\phi} \Pi_{g,l}^d(\phi, x) S_g^d(\phi; x)},$$ (11)

$$\bar{\phi}_{g,p} = \frac{\sum_d \sum_x \sum_{\phi} \phi \Pi_{g,p}^d(\phi, x)[1 - \Pi_{g,l}^d(\phi, x)] S_g^d(\phi; x)}{\sum_d \sum_x \sum_{\phi} \Pi_{g,p}^d(\phi, x)[1 - \Pi_{g,l}^d(\phi, x)] S_g^d(\phi; x)},$$ (12)

and

$$\bar{\phi}_{g,u} = \frac{\sum_d \sum_x \sum_{\phi} \phi \Pi_{g,u}^d(\phi, x)[1 - \Pi_{g,l}^d(\phi, x)] S_g^d(\phi; x)}{\sum_d \sum_x \sum_{\phi} \Pi_{g,u}^d(\phi, x)[1 - \Pi_{g,l}^d(\phi, x)] S_g^d(\phi; x)}.$$ (13)

Recall that $\Pi_{g,r}^d(\phi, x)$ gives the search intensity (or the probability of finding a mate) in market $r$ for a type-$x$ person of gender $g$ with a discount factor $d$ and a prior $\phi$.

### 5.2 Market-Clearing Conditions

In equilibrium the number of females and males in each type of relationship must exactly balance. Again, using the subscript $g = f, m$ to indicate the gender of the person in question, this implies that the following market-clearing conditions hold:

$$\sum_d \sum_x \sum_{\phi} \Pi_{f,l}^d(\phi, x) S_{f,l}^d(\phi, x) = \sum_d \sum_x \sum_{\phi} \Pi_{m,l}^d(\phi, x) S_{m,l}^d(\phi, x).$$ (14)

$$\sum_d \sum_x \sum_{\phi} \Pi_{f,p}^d(\phi, x)[1 - \Pi_{f,l}^d(\phi, x)] S_{f,p}^d(\phi, x) = \sum_d \sum_x \sum_{\phi} \Pi_{m,p}^d(\phi, x)[1 - \Pi_{m,l}^d(\phi, x)] S_{m,p}^d(\phi, x),$$ (15)

and

$$\sum_d \sum_x \sum_{\phi} \Pi_{f,u}^d(\phi, x)[1 - \Pi_{f,l}^d(\phi, x)] S_{f,u}^d(\phi, x) = \sum_d \sum_x \sum_{\phi} \Pi_{m,u}^d(\phi, x)[1 - \Pi_{m,l}^d(\phi, x)] S_{m,u}^d(\phi, x).$$ (16)
Take the second condition, as an example. The left-hand side represents the number of females in short-term relationships involving protected sex. The right-hand side is the number of males. Additionally, a transfer paid by one gender on a market is a transfer earned by the other so that
\[ t_{f,r} + t_{m,r} = 0, \text{ for } r = l, p, u. \] (17)

5.3 Definition of Equilibrium

Definition 1 A stationary equilibrium is described by a set of decision rules for search effort, \( \Pi_{g,r}^d(\phi, x) \), a set of transfer payments, \( t_{g,r} \), a set of stationary type distributions, \( S_g^d(\phi; x) \) and \( L_g^d(\phi', x) \), and a set of HIV/AIDS (non)prevalence rates for a partner on each market, \( \bar{\phi}_{g,r} \), for all \( d = \{\iota, \beta\} \), \( g \in \{f, m\} \), \( r \in \{l, p, u\} \), such that:

1. The decision rules for search intensities, \( \Pi_{g,r}^d(\phi, x) \), satisfy the appropriately gender subscripted versions of the generic problems (6) and (8), taking as given transfer payments and HIV/AIDS prevalence rates;

2. The stationary type distributions, \( S_g^d(\phi; x) \) and \( L_g^d(\phi, x) \), solve the appropriately gender subscripted version of (10);

3. The prevalence rates for HIV/AIDS on each market, \( \bar{\phi}_{g,r} \), are given by (11) to (13);

4. The transfer payments, \( t_{r,g} \), are such that the markets for all types of relationships clear, or so that equations (14) to (16) hold. Additionally, the flow of transfers across the genders must balance as specified by (17).

6 The Benchmark Simulation

To address the HIV/AIDS epidemic in Malawi, the model is analyzed numerically. This is done in two steps. First, a benchmark simulation is constructed that displays features that are broadly consistent with the Malawian case. In particular, the simulated model has an HIV/AIDS infection rate that corresponds with the Malawian data, a proportion of casual sexual encounters is approximately the
same, and a reasonable fraction of these encounters use a condom. It should be noted upfront, given the computationally difficult nature of the model, that an informal benchmarking procedure is adopted. Second, the benchmark simulation is then used to analyze some topical policy experiments, such as circumcision or the promotion of marriage. As was mentioned already in the introduction, at this early stage in the research program using computational general equilibrium models, the upshot of these experiments cannot be taken as evidence in favor or against any particular policies proposals; to do so on such an important issue would be reckless. They illustrate the complex nature of the HIV/AIDS problem and uncover the more elusive general equilibrium channels and behavioral changes through which countervailing effects may reduce the efficacy of some interventions.

6.1 Parameterization

At this stage in the development cycle, the model is too complicated to estimate. It has many parameters. Plus, for a given set of parameter values the model takes some time to run. The solution process is often a bit temperamental. While theoretically speaking models with externalities can display multiple equilibria, the simulations recover only a single interior equilibrium; although, some additional (uninteresting) boundary equilibria may exist.\(^9\) This and some other algorithmic issues require monitoring when running the computer program and occasionally some intervention is required. This makes an automated estimation process difficult. Therefore, a more informal approach is taken to construct the benchmark simulation. This is done in two steps. First, to the extent possible, parameters with direct data analogs are taken from the literature. Second, the remaining parameters are chosen to match, roughly, some key observations related to the HIV/AIDS epidemic in Malawi. The data mostly obtains from the 2004 Demographic and Health Survey (DHS) that was conducted in Malawi. With the micro data from this survey, a number of statistics are computed regarding HIV prevalence rates, sexual behavior, marital status, etc. The facts garnered from Malawi

\(^9\)Define an equilibrium to be interior if all markets attract a positive measure of agents. Call all others boundary equilibria. Clearly, only interior equilibria have a chance of matching the data.
are often complemented with data from other sources, as necessary.

Even though the model is set up to allow for heterogeneity along many dimensions, such as the utility from sex, income and the discount factor, only two dimensions of heterogeneity are exploited in the application. First, assume that people differ in their discount factor. Second, suppose that men and women have different transmission rates, so that the same level of sexual activity leads to a discrepancy in the HIV rate across genders. This limited degree of heterogeneity economizes on the number of parameters to be specified.

The most important parameter values for the simulation are those concerning HIV/AIDS. Fortunately, for the most part, these can be taken from the medical literature. Take the period to be one quarter. In line with Bracher, Santow, and Watkins (2004) suppose that condoms are not used in marriage; i.e., all marital sex is unprotected. The transmission risk for one-time male unprotected sex is taken to be 2.3 per 1,000. This number falls in the range of estimates reported by a variety of studies. Since couples on average have sex 9 times a month, as reported in Gray et al. (2001), this translates into a quarterly non-transmission risk of $\gamma_u = 0.94$. The transmission risk when condoms are used is obviously lower, but protection is far from perfect—Bracher, Santow, and Watkins (2004). Select $\gamma_p = 0.98$, corresponding to a 67% efficacy rate, which is in line with Weller (1993) who conducted a meta-analysis of condom efficacy. Further, for physiological and anatomical reasons, and in accord with the medical evidence, females are assumed to have a higher risk of contracting HIV than males. Nicolosi et al. (1994) reports a risk that is 2.3 times as high for women. However, the range of estimates is wide. On the one extreme, Gray et al. (2001) find no statistically significant difference between transmission rates by gender. On the other extreme, Padian, Shiboski, and Jewell (1991) calculate a factor as high as 20. Erring on the conservative side, pick $\gamma_p = 0.965$, which corresponds to women being 75%

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10As discussed in Section 3, Bracher, Santow, and Watkins (2004) point out strongly that condoms within marriage are rare and considered as indication of “not a real marriage.”

11For example, the annual report of the UNAIDS Joint United Programme on HIV/AIDS (2007) gives a range of 2 to 6 per 1,000, depending on whether other STDs are present or not. Baeten et al. (2005) also report a transmission risk of 6 per 1,000. Gray et al. (2001) report a somewhat lower number of 1.1 per 1,000 for Uganda; however, free condoms were distributed as part of the study. Wawer et al. (2005) finds transmission rates as high as 82 per 1,000 during the first few months after infection.
more likely to get infected. Using the same gender gap in transmission also for unprotected sex, set $\gamma_u^f = 0.895$.

Note that the higher male-to-female transmission risk is the only exogenous gender difference fed into the benchmark simulation described in Section 6.2. As is discussed in greater detail in Section 6.2, this single exogenous gender difference leads to a number of endogenous gender differences in the outcomes, including a higher rate of HIV prevalence for females and the fact that females contract HIV at younger ages. Last, the average time from infection to the outbreak of symptoms is equal to 10 years (DHS 2004). Therefore, let $\alpha = 0.025$; i.e., 40 quarters. The average time from the outbreak of symptoms to death is 2 years (DHS 2004). Thus, pick $\delta_2 = 0.125$; i.e., 8 quarters.

Some other parameters values can also be pinned down using a priori information. Set the quarterly divorce hazard equal to $\xi = 0.03$. Bracher, Santow, and Watkins (2004) report that 26.4% of all marriages in Malawi end in divorce within the first five years. Assuming a constant annual divorce hazard, this would imply a quarterly risk of 1.56%. $^{12}$ A rate twice this number is used: First, polygyny is fairly common in Malawi, from which the analysis abstracts. Second, extramarital affairs are relatively common as well. Therefore, interpret, for example, a 10-year marriage with one affair as two long-term relationships with a third casual one in between. Section 7.3.2 explores what happens to equilibrium outcomes when the risk of divorce is lower.

The quarterly (non-HIV related) death hazard is taken to be $\delta = 0.006$. A study conducted by the U.S. Census Bureau (2004) reports a life expectancy without HIV of 56.3 years for Malawi. Since the model starts at age 15, the quarterly death hazard is selected to match a life expectancy of 41.3 years. Malawi is a very poor country. Set $y = 320$ which corresponds roughly to quarterly GDP per working age population (note that only about half the population is of working age in Malawi).

Table 1 summarizes the preceding paragraphs by listing all parameters that are set a priori. The remaining parameters have no clear data analogues. These parameters are picked to match (via the eyeball metric) several facts related to sex,

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$^{12}$A similar number is also reported by Reniers (2003).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_u$</td>
<td>0.94</td>
<td>6% quarterly transmission risk, unprotected sex, men</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>0.965</td>
<td>3.5% quarterly transmission risk, protected sex, women</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.025</td>
<td>10 years from infection to symptoms</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.006</td>
<td>6% quarterly death risk</td>
</tr>
<tr>
<td>$\gamma_u$</td>
<td>0.98</td>
<td>2% quarterly transmission risk, protected sex, men</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>0.965</td>
<td>3.5% quarterly transmission risk, protected sex, women</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.025</td>
<td>10 years from infection to symptoms</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.006</td>
<td>6% quarterly death risk</td>
</tr>
<tr>
<td>$\gamma_u$</td>
<td>0.98</td>
<td>2% quarterly transmission risk, protected sex, men</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>0.965</td>
<td>3.5% quarterly transmission risk, protected sex, women</td>
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<td>$\alpha$</td>
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<td>10 years from infection to symptoms</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.006</td>
<td>6% quarterly death risk</td>
</tr>
<tr>
<td>$\gamma_u$</td>
<td>0.98</td>
<td>2% quarterly transmission risk, protected sex, men</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>0.965</td>
<td>3.5% quarterly transmission risk, protected sex, women</td>
</tr>
</tbody>
</table>

marriage and HIV/AIDS in Malawi. For example, utilities from the different types of sexual relationships are free parameters, constrained only by $p \leq u$; i.e., people enjoy unprotected sex more than protected sex. To ensure interior solutions, the model also specifies that people enjoy variety in their sexual partners, and so marriage (sex with one partner) decreases overall utility from unprotected sex ($l < 0$).

To keep things simple, assume that there are only two exogenous types of males and females. The only exogenous heterogeneity (in addition to the gender difference in transmission risk) is the degree of patience people have. Assume that young people are more impatient than old people and that there is a small subset of the population that is extremely impatient. 13 Denote by $\tilde{\beta}$ and $\tilde{\iota}$ the discount factors for the young and the old. Subscript these variables by either 1 or 2 to connote regular agents impatient agents respectively. Note that these are “pure” discount factors, i.e., net of mortality risk. That is, $\beta = \tilde{\beta}(1 - \delta)$ and $\iota = \tilde{\iota}(1 - \delta)$.

In sum then, there are 14 parameters to pin down: $p, u, \ell, \omega^u, \omega^f, \kappa, A, \mu_1, \mu_2, \eta, \tilde{\beta}_1, \tilde{\beta}_2, \tilde{\iota}_1$ and $\tilde{\iota}_2$. Note that the population size is a pure scaling parameter and only the relative sizes of the two types matter: $\mu_2/\mu_1$. To render the structure more parsimionous, let $\tilde{\beta}_2 = \tilde{\iota}_1$ and $\tilde{\iota}_2/\tilde{\beta}_2 = \tilde{\iota}_1/\tilde{\beta}_1$. Thus, the discount factor for an old impatient person is equal to that of a young patient one. Additionally, 13 The rationale for this choice is as follows. A small group of risky people is needed to generate any HIV in equilibrium. Assuming young people to be particularly impatient leads to more risky behavior among them. Both features are discussed in applied works.
the relative difference between the impatient and patient is preserved across age. Thus, there are 11 parameters to choose, see Table 2 for a summary. The next subsection describes the data moments that are used as targets for the calibration.

### 6.2 Benchmark Model vs. The Data: Targeted Observations

So, what are the data targets and how well does the benchmark simulation match them? The main targets are the overall prevalence rate for HIV/AIDS in society, and the prevalence rates for each gender. Some additional targets are selected to discipline the exercise, such as the fraction of sex that is casual as opposed to long-term, the fraction of the population that is single, and the pattern of marriage by age. This ensures that there is not too much reliance on risky short-term interactions. Many people are married, and get married quickly. Moreover, females get married faster. It is important that the model match the observed prevalence rates in light of these facts. Additionally, the fraction of condom users in casual encounters and the number of such encounters are considered. This is important because the model emphasizes the choice of whether to engage in risky activity. Finally, the fraction of people that die of natural causes, as opposed to HIV/AIDS, is included for overall consistency.

The upshot of the analysis is that the benchmark simulation delivers results that are qualitatively in line with some key features of the Malawian HIV/AIDS epidemic. To begin with, as can be seen from Table 3, the HIV/AIDS prevalence rate:

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### Table 2: Calibrated Parameters

<table>
<thead>
<tr>
<th>Interpretation</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow utility unprotected sex</td>
<td>( u = 7.5 )</td>
</tr>
<tr>
<td>Flow utility protected sex</td>
<td>( p = 2.3 )</td>
</tr>
<tr>
<td>Flow utility long-term sex</td>
<td>( l = -5 )</td>
</tr>
<tr>
<td>Discount factors</td>
<td>( \tilde{\beta}_1 = 0.995, \tilde{t}_1 = 0.8955 )</td>
</tr>
<tr>
<td>Value of life with Aids</td>
<td>( A = 5 )</td>
</tr>
<tr>
<td>Prob. of switch to high discount factor</td>
<td>( \eta = 0.06 )</td>
</tr>
<tr>
<td>relative group size</td>
<td>( \mu_1/\mu_2 = 10 )</td>
</tr>
<tr>
<td>Search cost parameters</td>
<td>( \omega_s = 0.5, \omega_l = 30, \kappa = 0.2 )</td>
</tr>
</tbody>
</table>
predicted by the model is 11%, matching the data. Moreover, the experiment captures the gender difference in HIV/AIDS infection rates, with females experiencing an HIV/AIDS rate that is 3 percentage points higher than that for males (13% versus 10%).

In addition to exactly matching moments on HIV/AIDS prevalence, the benchmark model is also broadly consistent with the data on other aspects of sexual activity. In the model casual or short-term sex is a small fraction of all sexual encounters: 24%, relatively close to the 18% of sex that occurs outside of a union that is reported in the data. For those who engage in casual sex, the model predicts that 33% use a condom. This is less than the 39% seen in the data, but is still close. In fact, as people have been found to overstate the amount of protected sex they have [see Allen et al. (2003)], these two numbers may be closer in reality than first meets the eye. The next row in Table 3 reports the fraction of singles who had casual sex in the last year. These statistics are different from the fraction of all sexual activity that is casual both because (all) married people have sex and because some singles are abstinent. Singles in the model have more casual sex than their real-life counterparts, but again, it is possible that people systematically under-report their risky sexual behavior. Finally, the fraction of the population that die from HIV/AIDS is comparable across model and data (23 versus 29%).

What probably is not misreported is people’s marital status. Table 3 lists the fraction of singles in the entire population. The model predicts this fraction to be 38%, close to the 33% observed in the data. Moreover, it captures some of the gender differences in the timing of marriage. Women marry much earlier than men—in the data, 90% of women are married by age 22, whereas only 58% of men are married by this age. The corresponding numbers in the model are 67% and 60%. The model generates the earlier marriage of women (relative to men) via their higher infection risk. This makes the safety of marriage more attractive for women vis-à-vis men. In reality HIV risk is only one reason why women get married earlier than men. Additional considerations such as the risk of pregnancy make casual encounters more risky for women, which might explain the larger wedge between the genders in the data compared to the model. As is only rea-

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14 All data sources for the figures and tables are discussed in the Data Appendix.
### Table 3: Stylized Facts on HIV/AIDS in Malawi

<table>
<thead>
<tr>
<th>Observation</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>HIV/AIDS rate, %</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>–Males</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>–Females</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>Fraction of all sex that is casual, %</td>
<td>18</td>
<td>24</td>
</tr>
<tr>
<td>Condom use for casual sex, %</td>
<td>39</td>
<td>33</td>
</tr>
<tr>
<td>% (of) Singles that had casual sex in past year</td>
<td>37</td>
<td>54</td>
</tr>
<tr>
<td>% Singles</td>
<td>33</td>
<td>38</td>
</tr>
<tr>
<td>% Married by age 22</td>
<td>58</td>
<td>60</td>
</tr>
<tr>
<td>–Males</td>
<td>58</td>
<td>60</td>
</tr>
<tr>
<td>–Females</td>
<td>90</td>
<td>67</td>
</tr>
<tr>
<td>% Married by age 50</td>
<td>100</td>
<td>97</td>
</tr>
<tr>
<td>–Males</td>
<td>100</td>
<td>97</td>
</tr>
<tr>
<td>–Females</td>
<td>100</td>
<td>96</td>
</tr>
<tr>
<td>% of deaths related to HIV</td>
<td>29</td>
<td>23</td>
</tr>
</tbody>
</table>

Reasonable, men eventually “catch up,” and by age 50 almost everyone is married, both in the model and the data. See Figure 2 for a comparison of the fraction of the population that has ever married in the model vs. the data.

#### 6.3 Benchmark Model vs. The Data: Non-Targeted Observations

The benchmark model generates some other predictions that are not targeted when picking values for parameters. Recall that agents in the model have rational beliefs about their HIV/AIDS status. Figure 3 plots the belief distribution for symptom-free people in the model and compares it to that reported by Delavande and Kohler (2009), who elicited responses on beliefs from rural Malawians. In both the data and model, there is a steep drop between the number of people who believe themselves to be almost certainly HIV/AIDS-negative (or healthy)—say less than a 5% probability of being infected—and the people who are only relatively sure that they are negative—say close to a 15% probability of being infected. After this drop, beliefs are flat throughout the rest of the distribution, a pattern confirmed by Delavande and Kohler (2009). Moreover, in the data the prevalence
rates implied by the reported beliefs roughly match the actual prevalence rates, which is supportive of the modeling assumption that agents are rational.

Figure 4 plots HIV/AIDS prevalence by age. Both the model and data agree on a hump-shaped infection pattern, despite the fact that agents in the model become sexually active earlier than is observed in the data, which shifts the model’s life-cycle predictions on HIV/AIDS infections to the left (for the younger cohorts). That is, HIV/AIDS prevalence initially rises as one moves from younger to older cohorts. Prevalence eventually peaks and then drops, however, so that the oldest cohorts have lower HIV/AIDS infection rates than do the younger ones. This pattern is explained by two observations. First, the rise in HIV/AIDS infection is due to the fact that older people have had more time to be sexually active, and so a larger percentage of the older cohort is infected with HIV/AIDS. People who are infected early in life will die before they make it to old age, though. Put differently, people who have made it to old age must be those who have engaged in less risky sexual behavior and so are less likely to be infected with HIV/AIDS. This second fact explains the eventual drop in HIV/AIDS prevalence seen in the oldest cohorts. Figure 4 also illustrates the differentiated patterns of infection between the sexes. The figure shows that women get infected earlier than men, both in the model and the data.

The data is fitted with a third-order polynomial. See Figures 9 and 10 for a comparison of the raw data and the fitted line. The somewhat choppy raw data is due to the small sample sizes.
Figure 3: Distribution of Beliefs – Model vs. Data

Figure 4: HIV Rate – Men vs. Women, Model vs. Data
The model also does a very nice job in matching the decline in risky activity over the life cycle. Older people are less likely to be single, see Figure 5. As people age, they are thus less likely to engage in casual sex; this is reported in Figure 6. Note that a related but different statistic is the fraction of singles that have sex in a given period, since some of them choose to be abstinent. Figure 7 compares the data to the model counterpart. The data exhibits a declining fraction of singles that engage in sex over the life cycle and the model also generates a similar pattern. The reason for this is that people become more patient as they age and, again, the survivors are more likely to be the individuals who prefer safer lifestyles.

An additional prediction of the model is regarding the death causes, since agents may die due to HIV/AIDS or to other natural causes. Figure 8 compares the model prediction over the life cycle with its data counterpart. Both the model and the data exhibit a hump-shaped pattern of HIV/AIDS caused deaths; this is consistent with the hump-shaped pattern of infection rates.

Finally, protected sex in the benchmark simulation is substantially cheaper than unprotected sex. The premium for unprotected casual sex is 67%. Note that such a premium has in fact been documented in the literature. Gertler, Shah, and Bertozzi (2005) use data from commercial sex workers in Mexico to document a 23% premium for unprotected sex. The premium increases to 46% when the sex worker is considered to be very attractive.
The model broadly tracks these data patterns despite the limited degree of heterogeneity and despite the limited state variables that describe the agent’s lifecycle.

7 Policy Experiments

The model is now ready to explore the effectiveness of various policies intended to curb the spread of HIV/AIDS. Equate effectiveness with the reduction of the prevalence rate, as this is the stated goal of many governments and non-profit
organizations. It should be mentioned, however, that a decrease in HIV/AIDS does not necessarily imply an increase in welfare. On the one hand, if people have less sex, an activity they enjoy, their welfare might decrease. On the other hand, the model features an externality as people do not internalize the effect of their own risky behavior on the health of future partners. Moreover, men and women might be differentially affected due to a change in prices.

A caveat is in order before proceeding. Research using computational general equilibrium models to assess the implications that interventions might have on the spread of HIV/AIDS (or other diseases) is in its infancy. The goal of obtaining hard numbers that can be used for policy analysis is still some way down the road. The model does land itself as a useful tool for elaborate thought experiments to discover areas that might need further investigation, however. The simulations do illustrate potential pitfalls in efforts to limit the disease. Specifically, as will be illustrated with an example on condom policy, moderate policy interventions have the potential to backfire due to the shifts in sexual behavior that they induce. These shifts in sexual behavior then feed back on the equilibrium rate of HIV/AIDS. In principle, computational general equilibrium models are well suited to analyze such effects. In practice, the best structure to be
employed needs to be determined. A prototype structure is offered here. This section first gives a (non-exhaustive) overview of the policy-related HIV/AIDS literature and briefly discusses some aspects where equilibrium models can shed additional light. Then, several different policies are explored using the model.

7.1 Literature

There are two broad classes of policies surrounding HIV: treatment and prevention—see Canning (2006) for an excellent survey. Treatment consists mostly of different generations of anti-retroviral drugs. Prevention includes condom distribution, reducing mother-to-child transmission, mass media campaigns, voluntary counseling and testing, and diagnosis and treatment of other sexually transmitted diseases. Overall, prevention is more cost-effective than treatment—Canning (2006). Previous studies that analyze the effectiveness of a given policy are typically based on cross-sectional comparisons (e.g., different countries, different regions, or different ethnic groups), on epidemiological simulations, or on randomized field experiments. The approach taken in this paper, which focuses on simulating a choice-theoretic equilibrium model, may bring new angles to the debate. In particular, it allows the analysis of both behavioral changes in response to an intervention and general equilibrium feedback effects. The findings obtained may be useful for guiding future policy interventions. In order to prepare the ground for the policy analysis, some previous findings on policies related to the reduction of transmission risk as well as the promotion of marriage and faithfulness will now be discussed.

A policy intervention that has received a lot of recent attention is male circumcision. Several studies find a decline in the female-to-male transmission rate for circumcised males. Based on this evidence, UNAIDS now lists male circumcision as one recommended strategy for HIV prevention. The most widely cited evidence comes probably from Gray et al. (2007) and Auvert et al. (2005). Based on a randomized field experiment in Uganda, Gray et al. (2007) find that the incidence of HIV/AIDS (over 24 months) in the treatment group was half the HIV

\[16\text{See http://www.unaids.org/en/PolicyAndPractice/Prevention/MaleCircumcision/ accessed on October 7, 2009.} \]
incidence in the control group. Similarly, Auvert et al. (2005) find a 60% reduction in female-to-male transmission for those who were circumcised in a randomized field experiment in South Africa. One thing that randomized experiments cannot measure are general equilibrium effects. If all men were circumcised, perhaps the entire village would feel safer and engage in riskier behavior, which would crowd out some of the gains. On the other hand, if all men were circumcised and even if there were no behavioral changes between the treated and control groups, there could still be a decrease in HIV prevalence rates due to the lower transmission rates in the overall economy. In fact, Gray et al. (2007) find that alcohol use with sexual intercourse (a measure of risky sexual behavior) before the study was 38%, and increased to more than 50% for both the intervention and control groups at the end of the study. The approach taken in the current paper is complementary to randomized field experiments. As will be seen, it allows researchers to assess the general equilibrium consequences of widespread circumcision, and to analyze the implications for both males and females.

Several other policies involve the reduction in transmission risk, albeit in a more symmetric fashion, i.e. affecting both men and women. For example, results of a new vaccine trial were made public very recently showing a 30% efficacy.17 Another policy is the treatment of other sexually transmitted diseases (STDs) to decrease the transmission of HIV/AIDS. The idea is that the presence of other STDs makes a person more susceptible to contracting HIV. Treating other STDs to reduce transmission risk is in many ways similar to advocating male circumcision, and therefore measuring success is equally problematic. By comparing data from African countries and from the US and Western Europe, Oster (2005) reaches the conclusion that treating other STDs would be an effective policy. Using a “diff-in-diff” approach, she argues that the most likely cause for different HIV rates are STDs—rather than behavioral differences. This conclusion is somewhat problematic. If current sexual behavior in African countries (where the HIV rate and related risks are high) is similar to European countries (where the rate and associated risks are much smaller), then, this may suggest very different at-

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17 Promising results of a new vaccine that would reduce transmission by 30% were recently reported in the media, e.g. in a Wall Street Journal article on September 25, 2009: “Vaccine Shows Promise in Preventing HIV Infections.” However, additional data analysis found that the results are statistically insignificant.
titudes towards risky behavior in the two continents. If this was the case, then reducing the transmission risk by treating other STDs might lead to behavioral adjustments that crowd out the gains from the reduced risk. Simulating people’s behavior in response to changed circumstances is one way of taking such behavioral responses into account and trying to assess the overall impact of such a policy.

Other policies have been aimed directly at social and behavioral change. The so-called ABC approach stands for abstention, being faithful, and the use of condoms—Murphy et al. (2006). It is somewhat less clear how such behavioral changes can be achieved. One avenue that is pursued are large scale media campaigns aimed at disseminating information. More generally, the emphasis is often on information, education, and communication, which is sometimes called the IEC approach. For example Gallant and Maticka-Tyndale (2004) study the effectiveness of education campaigns. One problem with such studies is that success is often measured as changes in attitudes and/or reported behaviors, while it is left open whether actual behaviors changed. In the experiments conducted here the effects of “promoting marriage” are investigated for various interpretations of what this policy might mean.

7.2 Medical Policies

An analysis will now be undertaken of several medical policies that have been recommended recently.

7.2.1 Policies that Reduce Transmission for Both Sexes

A policy that has been advocated recently is the treatment of other sexually transmitted diseases (STDs). STDs often lead to open sores that make a person more susceptible to the transmission of the HIV/AIDS virus. Thus, reducing other STDs will decrease the transmission risk, both for men and women. For example, Grosskurth et al. (1995) finds that improved STD treatment reduced HIV

\[^{18}\text{This point is also made in Dupas (2011).}\]
incidence by about 40% in rural Tanzania. Similarly, much medical research is currently devoted to developing a vaccine with the hope that it will reduce the transmission rate by a substantial fraction.

Table 4 shows the simulation results for this policy. As the transmission risk for both men and women declines, the HIV incidence decreases by 1.5 percentage points from 11.4 to 9.9%. Note that this decrease in HIV prevalence masks the find that, when faced with better odds when having sex, agents engage in riskier behavior. The fraction of sex that is casual increases even though there are less singles around (since unprotected sex is safer and sex within marriage is unprotected, people have an extra incentive to marry). The reason is that the fraction of non-abstinent singles increases from 54% to 57.6%. Moreover, out of the singles having sex, condom usage falls from 33% to 29.7%. The upshot of this experiment is that agents can dramatically change their behavior in response to the policy and that these behavioral changes can have non-trivial effects, which can be seen as follows.

Compare the results with an epidemiological version of the experiment, where by assumption, behavior does not change. In the epidemiological experiment, the decline in HIV prevalence is much larger to 9.4%, a 0.5% difference compared to the benchmark. The reason for this difference is exactly the behavioral changes described above.

Another interesting comparison is with the model version of a field experiment. Here the general equilibrium effects on aggregate prevalence rates and transfers in each market are shut down (because by assumption only a small fraction of the population is treated). The difference here actually goes in the opposite direction: the small field experiment predicts a much smaller decrease in HIV incidence compared to the benchmark (11.1% in the former versus 9.9% in the latter). The reason is that, in the field experiment, the reduced number of infections does not lead to an overall decrease in the population prevalence rate. Therefore, it does not feed back into lower infection rates for the treated population, something that is naturally part of the full model. It is interesting to note that eight of the nine trials of STD treatment for HIV prevention surveyed by Padian et al. (2010) delivered flat results. Even though the authors discuss some potential explanations for these weak results, the simulations presented here highlight a novel mechanism,
<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Lower Risk</th>
<th>Epidemiol.</th>
<th>Small Field</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma^m_u$</td>
<td>0.940</td>
<td>0.948</td>
<td>0.948</td>
<td>0.948</td>
</tr>
<tr>
<td>$\gamma^f_u$</td>
<td>0.895</td>
<td>0.902</td>
<td>0.902</td>
<td>0.902</td>
</tr>
<tr>
<td>HIV/AIDS rate, %</td>
<td>11.4</td>
<td>9.9</td>
<td>9.4</td>
<td>11.1</td>
</tr>
<tr>
<td>Fraction of sex that is casual, %</td>
<td>23.9</td>
<td>24.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% (of) Casual sex with condom</td>
<td>33.0</td>
<td>29.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Singles who have casual sex</td>
<td>54.0</td>
<td>57.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Men who are single</td>
<td>42.8</td>
<td>42.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Women who are single</td>
<td>38.7</td>
<td>37.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: TREATING OTHER STDs

namely the general equilibrium effects of the policy.

Taking stock, treating other STDs seems to decrease the overall HIV prevalence rate even though people engage in riskier behavior in response to the lower likelihood of getting infected. This shift in behavior makes the use of a choice-theoretic model like the one proposed here essential. The differences arising from these behavioral responses seem to be quantitatively important. Moreover, conducting randomized field experiments on a small scale might not be enough to assess the effects of such a policy. General equilibrium effects are present and they might be powerful.

7.2.2 Male Circumcision

Another policy that has received attention is male circumcision, which supposedly lowers the likelihood of infection for men. Table 5 reports the results in the model when men (and only men) are faced with lower odds of infection. The first thing to note is that, even though sex is safer only for men, the prevalence rate for both sexes decrease. The reason for this is that, with the lower likelihood of getting infected, men are on average healthier and thus the spread of the disease slows down.

Two other things are worth noting in Table 5. First, both the small field experiment and the epidemiological study suggest that the policy is less effective than
what the full equilibrium says; and that is even more true for women.\textsuperscript{19} Second, agents’ behavior regarding single life versus marriage, condom usage, casual sex, etc do not seem to change much. Actually, both of these observations can be explained once one looks at the changes in transfers, also reported in the table. Given that the likelihood of infection does not change for women, they demand higher transfers from men; that is particular true, of course, where sex is unprotected: short-term unprotected and long-term relationships. Note that the increase in the transfers in these last two markets is much larger than in the protected sex market. Given these higher transfers, men do not change their behavior as much as they would in say, the small field experiments where general equilibrium effects of this type do not play any role.

7.2.3 Condoms

Suppose one could design more pleasurable condoms (or perhaps raise the psychic pleasure of sex with a condom through a publicity campaign). Would this be desirable? The results are reported in Table 6. It turns out that increasing the

\textsuperscript{19}In the small field experiment, given that there is no change in the odds of infection for women and the absence of general equilibrium effects by design, the HIV prevalence rate for females will not change by construction.
Table 6: Better Condoms

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Better</th>
<th>Better Still</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>2.3</td>
<td>3.8</td>
<td>4.5</td>
</tr>
<tr>
<td>HIV/AIDS rate, %</td>
<td>11.5</td>
<td>13.3</td>
<td>12.7</td>
</tr>
<tr>
<td>Fraction of sex that is casual, %</td>
<td>23.9</td>
<td>39.9</td>
<td>53.9</td>
</tr>
<tr>
<td>% (of) Casual sex with condom</td>
<td>33.0</td>
<td>66.2</td>
<td>73.6</td>
</tr>
<tr>
<td>% Singles who have casual sex</td>
<td>54.0</td>
<td>92.2</td>
<td>95.6</td>
</tr>
<tr>
<td>% Men who are single</td>
<td>42.8</td>
<td>52.5</td>
<td>65.1</td>
</tr>
<tr>
<td>% Women who are single</td>
<td>38.7</td>
<td>48.2</td>
<td>61.6</td>
</tr>
</tbody>
</table>

utility from protected sex, $p$, does not always lead to a decrease in the prevalence rate. Single life now becomes more attractive. So, even though condom usage increases, there are more singles in total and they engage in both protected and unprotected sex. From the table, see that the fraction of single men and women increases substantially as condoms become more attractive. Moreover, the fraction of singles that engage in short-term sex skyrockets. Even though more people use condoms, there is a lot more activity in the casual sex market. These two forces push the prevalence rate in opposite directions and this tension can be seen in Table 6. As condoms first get better, HIV/AIDS prevalence rates initially go up. As more and more people start to use condoms, however, the lower likelihood of getting infected becomes more important and the prevalence rate starts to go back down. This experiment highlights the potential of some policies to backfire and actually increase the overall prevalence rate. Note also that these effects can be quantitatively quite important: the prevalence rate goes up by almost two percentage points before it starts to come down.

7.3 Promoting Marriage

Another typical policy is to encourage change in people’s behavior by promoting faithfulness and marriage. There are various ways to induce behavioral change, ranging from campaigns and subsidies to improving the perceived bliss of marriage, social and church activities to enable people to find a suitable marriage partner, and counselling and harsher social sanctions to increase the duration of marriage. In order to get a first insight regarding the efficacy of these various
Table 7: PROMOTING MARRIAGE

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Better</th>
<th>Better Still</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l + u$</td>
<td>2.5</td>
<td>3.5</td>
<td>4.5</td>
</tr>
<tr>
<td>HIV/AIDS rate, %</td>
<td>11.5</td>
<td>10.0</td>
<td>7.8</td>
</tr>
<tr>
<td>Fraction of sex that is casual, %</td>
<td>23.9</td>
<td>18.0</td>
<td>12.2</td>
</tr>
<tr>
<td>% (of) Casual sex with condom</td>
<td>33.0</td>
<td>33.3</td>
<td>33.2</td>
</tr>
<tr>
<td>% Singles who have casual sex</td>
<td>54.0</td>
<td>56.3</td>
<td>53.1</td>
</tr>
<tr>
<td>% Men who are single</td>
<td>42.8</td>
<td>34.3</td>
<td>26.8</td>
</tr>
<tr>
<td>% Women who are single</td>
<td>38.7</td>
<td>27.7</td>
<td>20.7</td>
</tr>
</tbody>
</table>

policies, in the model three experiments are undertaken. First, the utility from marriage is increased. Second, the search cost of finding a marriage partner is lowered. Third, the divorce hazard is reduced.

7.3.1 Increased Utility from Marriage

Table 7 reports the results of an experiment in which marriage is more attractive. That is, the utility of being married ($l + u$) increases. The first thing to note from the table is that, as expected, the fraction of single people plummets. Consequently, the fraction of sex that is casual also exhibits a marked decrease; there is thus less risky activity taking place in the economy. This adds up to a lower overall HIV prevalence rate. An interesting thing to note is the non-monotonic behavior of the fraction of singles that engage in sex: this fraction first rises and then comes back down.

7.3.2 Marriage: Entry vs. Exit

Making marriage more attractive (as in Section 7.3.1) might be difficult to implement. A more realistic policy might be to either facilitate entry into marriage (by making search easier) or impede exit from marriage (by making divorce harder). For example, social events organized by community or religious groups may facilitate searching for a spouse. Likewise, the provision of marriage counselling services may reduce divorce. Similarly, profamily tax codes could promote marriage and dissuade divorce. The impact effect of both strategies should be an
increase in marriage. This has the potential to lower the HIV/AIDS rate. Concretely, these policies are operationalized by lowering the search cost of finding a marriage partner, $\omega_{LT}$, and decreasing the odds of divorce, $\xi$. The corresponding experiments are depicted in Table 8.

As divorce risk decreases, long-term relationships become longer and people become less promiscuous. The fraction of casual sex thus goes down. This happens of course along with a marked decrease in the number of singles. All this contributes to a lower prevalence rate in equilibrium. However, the decrease of the prevalence rate is limited because singles have a lot more sex now, which counteracts the benefits from more marriage.

When the search costs in the long-term market are lower (last two columns), the HIV prevalence rate actually increases somewhat. The reason for this comes from a worsening of the pool of people searching in the long-term market. As the search costs in this market decrease, riskier types find it more profitable to look for a long-term partner. This worsens the marriage pool and drives the prevalence rate up.

### 7.4 Other Experiments

The framework lends itself to analyzing a host of policies. Space constraints do not permit a detail accounting of all the experiments entertained. Therefore, in this section a brief summary of some of them will be presented. No attempts are made to account for information campaigns and testing. The former requires
the introduction of some systematic biases in beliefs, which can then be reduced through the dissemination of information about HIV/AIDS. The second brings in some deeper issues regarding human nature.\textsuperscript{20} The interventions explored are:

\textit{Higher Incomes}

Developing countries receive substantial financial aid from other countries. Could such financial aid play any direct role in HIV/AIDS prevention? Increasing income, $y$, lowers the HIV/AIDS rate substantially in the model. The reason is that richer people value life more and hence behave in a more risk-averse fashion. There are fewer singles and in addition singles have less sex. If income is increased for women only, then the effects are even larger. Women now have more to lose and hence demand a higher price for sex. When male income is held fixed, men reduce their risky behavior in face of a higher price for sex. This shows up very clearly in the experiments, where the price of all forms of sex goes up substantially when income for both genders is increased, but only very little when only women receive the subsidy. Thus, interestingly, the cheaper policy (which involves payment to 50\% of the population only) is the more effective one.

\textit{Discouraging Casual Sex}

One might imagine a policy that makes meeting people of the opposite sex more difficult, for example by taxing popular meeting venues or the inputs they use such as alcohol. (This is operationalized here by increasing $\omega_s$.) This policy leads to a decrease in the prevalence rates for both genders. This lower prevalence rate comes from two reasons. First, a lower fraction of singles have casual sex, since it is more costly to find a mate in these short-term markets. Second there are less singles in equilibrium. The reason for the second effect is that, since agents know it is more costly to search in the short-term markets, they search more heavily in the long-term market. Both of these forces act to decrease the HIV prevalence

\textsuperscript{20}Currently, it is assumed that in casual relationships people care only about their own risk of becoming infected (and they are committed to faithfulness in marriage). This assumption might be justifiable when the belief about being infected is moderate and agents can argue to themselves and others that they did not know they were infected. Under testing, infected people are told with (near) certainty that they are infected. The consequences for their partners are clearly brought to their attention. Extrapolating the current behavioral assumption to such an extreme setting would imply reckless behavior after being tested positive. Concern for others might yield very different implications. Thus, incorporating testing would bring issues of altruism to the fore. It is unclear what the best way of dealing with this is.
rate even though condom usage does not change (given that it is more costly to search in both of the short-term markets).

8 Conclusions

In Malawi about 11 percent of the population has the HIV/AIDS virus. Roughly 18 percent of sex is casual and a condom is used a quarter of the time. An equilibrium search model is constructed to analyze the Malawian HIV/AIDS epidemic. At the heart of the model is homo economicus. Specifically, it is presumed that the economic man (or woman) searches for the type of sexual activity that (s)he desires to engage in, while rationally taking into consideration the risks of this activity. Some people will select stable long-term relationships, others may choose more fleeting ones. Condoms may or may not be used in these more ephemeral encounters, depending on the participants’ mutual desires. The number of such encounters is partially under people’s control. All these of choices crucially affect the spread of HIV/AIDS in society.

The theoretical model developed is simulated to see whether or not it can capture some of the salient features of the Malawian HIV/AIDS epidemic. It can. For example, the framework can match the fraction of sex that is casual, the number of encounters that use a condom, and the HIV/AIDS prevalence rates for men and women. Furthermore, it can mimic the decline in casual sex by age and the \( \cap \)-shaped pattern of HIV/AIDS prevalence over the lifecycle. The benchmark simulation is then used to undertake some policy interventions that are discussed in the literature. The simulation results suggest that policy analysis of HIV/AIDS interventions may be very complicated. In particular, some policies (such as better condoms) may backfire and actually increase HIV. The aim of the analysis is to provide a toolbox that allows the study of various interventions, identifies where behavioral change might be important, and thereby identifies areas where further and deeper exploration might be most warranted.
A Appendix–Additional Figures

Figure 9: Male HIV Rate – Model vs. Data

![Graph showing Male HIV Rate – Model vs. Data]

Figure 10: Female HIV Rate – Model vs. Data

![Graph showing Female HIV Rate – Model vs. Data]

B Appendix—Data

The empirical moments are based on information from the individual interviews of the Malawi Demographic and Health Survey (MDHS) in 2004, carried out by the Malawi National Statistical Office. Overall 11,698 women aged 15 to 49 and
3,261 men aged 15 to 54 are interviewed. In the calculations all observations are tallied using their sample weights. In order to calculate the HIV rates by age and gender, individual information is matched with the HIV test results for those people who agreed on doing the test. The fraction of ever-married people is derived by dividing the number of people who are currently married or have been formerly married by the number of never married people. Current singles are defined as all people who are not married at the moment. To identify the fraction of people having short-term sex, all men and women are considered who had sex in the last year. Those people are asked with whom they had sex in the first place and whether they had sex with a second and third man or woman. If one of the sex partners was not the spouse or cohabiting partner, then the sex in the last year is categorized as short-term sex. Men in addition are asked whether they have ever paid for sex. Those men who have paid for sex in the last year are also defined as being active in the short-term market. Only consistent answers are used to calculate this fraction.

C Appendix—Theory

C.1 Updating in a Long-term Relationship

Whether a breakup occurs for exogenous reasons or because the partner developed symptoms contains information about one’s own HIV status. Below we develop two separate updating formulae, one for each case. Note that when the person himself develops symptoms, no updating is necessary since in this case the person knows he is HIV positive.

First, consider the case where the individual is exiting a long-term relationship at the end of period \( n \) where both partners are symptom free (or appear to be healthy, \( h \)). Here, the individual should update his prior according to the rule

\[
\Phi_i^h(\phi, n) = \frac{\Pr(\text{not being infected and partner showing no symptoms at end of period } n|\phi)}{\Pr(\text{no symptoms in either person at end of period } n|\phi)} \\
= \frac{\phi \Phi_i + \phi (1 - \Phi_i) \gamma_u^n (1 - \alpha)^n}{\Delta^h},
\]

(18)
with

$$\Delta^h \equiv \phi \bar{\phi}_t + (1 - \phi)(1 - \bar{\phi}_t)(1 - \alpha)2^n + \left[ \phi(1 - \bar{\phi}_t) + (1 - \phi)(1 - \bar{\phi}_t) \right] \{ (1 - \gamma_u) \sum_{j=0}^{n-1} \gamma_u^j (1 - \alpha)^{n-j} + \gamma_u^n \}.$$ 

where $\bar{\phi}_t$ is the (time-invariant) probability that a randomly drawn partner on the long-term market does not have the HIV/AIDS virus.

The first term in the numerator is the chance that neither individual in the relationship initially had the HIV/AIDS virus, which occurs with probability $\phi \bar{\phi}_t$. The second term gives the odds that: (i) the person starts the marriage healthy but that his partner initially had the HIV/AIDS virus, the odds of which are $\phi(1 - \bar{\phi}_t)$; (ii) the virus fails to transmit despite having $n$ periods of unprotected sex, which has a likelihood of $\gamma_u^n$; (iii) the partner never shows symptoms, which occurs with probability $(1 - \alpha)^n$. The denominator includes these two terms in addition to the possibility that either person may have caught the HIV/AIDS virus, but the symptoms have not appeared yet. There are three possibilities to consider here. First, perhaps both partners initially had the virus but no symptoms have occurred yet, which is reflected by the term $(1 - \phi)(1 - \bar{\phi}_t)(1 - \alpha)2^n$. Second, there is the situation where the person catches the virus from his partner in any of the $n$ periods, but the symptoms don’t emerge in either individual. This occurs with probability $\phi(1 - \bar{\phi}_t)(1 - \alpha)^n(1 - \gamma_u) \sum_{j=0}^{n-1} \gamma_u^j (1 - \alpha)^{n-j}$, where $\phi(1 - \bar{\phi}_t)(1 - \alpha)^n(1 - \gamma_u) \gamma_u^j (1 - \alpha)^{n-j}$ is the likelihood that the individual catches the disease exactly $j$ periods after marriage but neither partner shows any symptoms until this period. Third, there is the possibility that the individual did have the virus, initially, while his partner didn’t, and no symptoms have occurred. The odds of this happening are $(1 - \phi)\bar{\phi}_t(1 - \alpha)^n \{ (1 - \gamma_u) \sum_{j=0}^{n-1} \gamma_u^j (1 - \alpha)^{n-j} + \gamma_u^n \}$. No symptoms may occur because the virus didn’t transmit (as reflected by the $\gamma_u^n$ in the brackets) or because it did transmit at some time but remains dormant [the $(1 - \gamma_u) \sum_{j=0}^{n-1} \gamma_u^j (1 - \alpha)^{n-j}$ term].

The second case where the individual’s partner develops HIV/AIDS symptoms at the end of period $n$ is similar. Denote this case with superscript $a$. If a long-term relationship ends because of sickness the individual will obviously observe
this. This information should be used in his updating rule, which now reads

$$\Phi_t^a(n, \phi) = \frac{\Pr(\text{not being infected and partner showing symptoms at end of period } n|\phi)}{\Pr(\text{no symptoms in oneself and symptoms in partner at end of period } n|\phi)} = \frac{\phi(1 - \bar{\phi}_t)(1 - \alpha)^{n-1}\alpha}{\Delta^a}.$$  \hspace{1cm} (19)

with

$$\Delta^a \equiv (1 - \phi)(1 - \bar{\phi}_t)(1 - \alpha)^{2n-1}\alpha + \phi(1 - \bar{\phi}_t)(1 - \alpha)^{n-1}\alpha[(1 - \gamma_u) \sum_{j=0}^{n-1} \gamma_j^u(1 - \alpha)^{n-j} + \gamma_u^n] + (1 - \phi)\bar{\phi}_t(1 - \alpha)^n\alpha(1 - \gamma_u) \sum_{j=0}^{n-1} \gamma_j^u(1 - \alpha)^{n-1-j}.$$  

Once again focus on the numerator first. The individual can only be healthy, while his partner shows symptoms, if the former didn’t have HIV/AIDS initially while the latter did. Furthermore, the virus must have failed to transmit after \(n\) periods of unprotected sex. The odds of this happening are \(\phi(1 - \bar{\phi}_t)(1 - \alpha)^{n-1}\alpha\). The denominator can be explained in similar fashion to the one in equation (18), with due alternation. The first term, \((1 - \phi)(1 - \bar{\phi}_t)(1 - \alpha)^{2n-1}\alpha\), gives the odds that both people initially had the HIV/AIDS virus, but just the partner shows the symptoms after \(n\) periods. It could also happen that the individual doesn’t have the virus initially, but his partner does. The chance of this happening, together with just the partner showing symptoms at the end of period \(n\), are given by second term \(\phi(1 - \bar{\phi}_t)(1 - \alpha)^{n-1}\alpha[(1 - \gamma_u) \sum_{j=0}^{n-1} \gamma_j^u(1 - \alpha)^{n-j} + \gamma_u^n]\). The probability that the partner catches the virus from the individual, and just shows the symptoms at the end of \(n\) periods, is \((1 - \phi)\bar{\phi}_t(1 - \alpha)^n\alpha(1 - \gamma_u) \sum_{j=0}^{n-1} \gamma_j^u(1 - \alpha)^{n-1-j}\). Last, the third case is trivial. If the individual shows the symptoms of HIV/AIDS, then he must have the virus.
C.2 Symptom Probabilities

C.2.1 Pr[no symptoms in either person at end of period $n$|φ]

The odds that neither partner shows the symptoms of HIV/AIDS by the end of period $n$ can occur for three reasons. First, neither party might have the disease at the time of marriage. The likelihood of this is $\phi \bar{\phi}_l$. Second, both parties could have had been infected with the virus when they were married. The chances of neither of them showing symptoms after $n$ periods of marriage is $(1 - \phi)(1 - \bar{\phi}_l)(1 - \alpha)^{2n}$. Last, only one of the parties might have initially been infected, but neither partner shows symptoms by the end of $n$. The probability of this compound event is $[(1 - \phi)\bar{\phi}_l + \phi(1 - \bar{\phi}_l)](1 - \alpha)^n[(1 - \gamma_u)\sum_{j=0}^{n-1}\gamma_u^j(1 - \alpha)^{n-j} + \gamma_u^n]$. This last event can be decomposed in two cases. The term $(1 - \phi)\bar{\phi}_l(1 - \alpha)^n(1 - \gamma_u)\sum_{j=0}^{n-1}\gamma_u^j(1 - \alpha)^{n-j}$ gives the odds that the individual in question initially has the disease, transmits it to his partner in some period $j$, and the symptoms fail to materialize in either person. The expression $(1 - \phi)\bar{\phi}_l(1 - \alpha)^n\gamma_u^n$ gives the likelihood that his partner never catches it. The rest of the formula captures the symmetric case where it was the partner who initially had the virus. Taking stock of all of this gives

$$\Pr[\text{no symptoms in either person at end of period } n|\phi] = \phi \bar{\phi}_l + (1 - \phi)(1 - \bar{\phi}_l)(1 - \alpha)^{2n}$$
$$+[(1 - \phi)\bar{\phi}_l + \phi(1 - \bar{\phi}_l)](1 - \alpha)^n[(1 - \gamma_u)\sum_{j=0}^{n-1}\gamma_u^j(1 - \alpha)^{n-j} + \gamma_u^n]. \quad (20)$$

C.2.2 Pr[symptoms just in partner at end of period $n$|φ]

What are the chances that just the individual’s partner shows the symptoms of HIV/AIDS by the end of $n$ periods of marriage? Once again there are three cases to consider. First, perhaps both parties initially had the virus but the symptoms just appear in the partner at the end of $n$. This will occur with probability $(1 - \phi)(1 - \bar{\phi}_l)(1 - \alpha)^{2n-1} \alpha$. Second, maybe just the partner had the disease initially. It may have transmitted to the individual in some period $j$, yet he never shows any symptoms. The chances of this are $\phi(1 - \bar{\phi}_l)(1 - \alpha)^{n-1} \alpha[(1 - \gamma_u)\sum_{j=0}^{n-1}\gamma_u^j(1 - \alpha)^{n-j} + \gamma_u^n]$. Last, the individual might have been the person who initially had
the sickness and it then spread to his partner in some period $j$. The odds of this happening are

$$(1 - \phi)(1 - \bar{\phi})(1 - \alpha)^{n-1} \alpha(1 - \gamma_u) \sum_{j=0}^{n-1} \gamma_u^j (1 - \alpha)^{n-1-j}.$$ 

It then transpires that

$$\Pr[\text{symptoms just in partner at end of period } n | \phi] = (1 - \phi)(1 - \bar{\phi})(1 - \alpha)^{2n-1} \alpha$$

$$+ \phi(1 - \bar{\phi})(1 - \alpha)^{n-1} \alpha[(1 - \gamma_u) \sum_{j=0}^{n-1} \gamma_u^j (1 - \alpha)^{n-j} + \gamma_u^n]$$

$$+(1 - \phi)\bar{\phi}(1 - \alpha)^n \alpha(1 - \gamma_u) \sum_{j=0}^{n-1} \gamma_u^j (1 - \alpha)^{n-1-j}.$$ 

(21)

### C.2.3 Pr[Symptoms in Person at End of Period $n | \phi$]

The likelihood of the individual exhibiting symptoms in period $n$ is now calculated. His partner does not show any symptoms during the first $n - 1$ periods, but might in the $n$th one. As above there are three cases to consider. First, both parties might have had HIV/AIDS at the time of marriage, an event which occurs with probability $(1 - \phi)(1 - \bar{\phi})(1 - \alpha)^{2n-2} \alpha$. Second, maybe only the partner initially had the virus and the person catches it in some period $j$. The odds of this are $\phi(1 - \bar{\phi})(1 - \alpha)^{n-1} \alpha(1 - \gamma_u) \sum_{j=0}^{n-1} \gamma_u^j (1 - \alpha)^{n-j-1}$. Last, the individual may have been the one who had it at the beginning of marriage. He may transmit it to his partner, who in turn shows no symptoms before period $n - 1$. This occurs with probability $(1 - \phi)\bar{\phi} \alpha(1 - \alpha)^{n-1} [(1 - \gamma_u) \sum_{j=0}^{n-2} \gamma_u^j (1 - \alpha)^{n-j-1} + \gamma_u^{n-1}]$. Therefore,

$$\Pr[\text{symptoms in person at end of period } n | \phi] = (1 - \phi)(1 - \bar{\phi})(1 - \alpha)^{2n-2} \alpha$$

$$+ \phi(1 - \bar{\phi})(1 - \alpha)^{n-1} \alpha(1 - \gamma_u) \sum_{j=0}^{n-1} \gamma_u^j (1 - \alpha)^{n-j-1}$$

$$+(1 - \phi)\bar{\phi} \alpha(1 - \alpha)^{n-1} [(1 - \gamma_u) \sum_{j=0}^{n-2} \gamma_u^j (1 - \alpha)^{n-j-1} + \gamma_u^{n-1}].$$ 

(22)
The value of a long-term relationship for a type-\(x\) person who has a prior of \(\phi\) and low discount factor, \(\tilde{V}_t^x(\phi, x)\), needs to be characterized. Recall that the discount factor may switch from the low value, \(\iota\), to the high one, \(\beta\), with probability \(\eta\). If a switch occurs, the discount factor will remain at the high value thereafter. Now, think about the discount factor that will be applied to the utility \(n > 1\) periods ahead. With probability \((1 - \eta)^{n-1}\) the individual will keep the discount factor \(\iota\)——note that he will use the discount factor \(\iota\) for the next period with certainty. If this event transpires he will discount utility \(n\) periods ahead by \(\iota^{n-1}\). Alternatively, he may draw the discount factor \(\beta\) some \(k < n\) periods down the road. The new discount factor will start to apply to period-\((k + 1)\) utility. This event happens with probability \((1 - \eta)^{k-1} \eta\). He will then discount period-\(n\) utility by \(\iota^{k-1} \beta^{n-k}\).

Define the two new discount factors \(\underline{\beta}(n)\) and \(\overline{\beta}(n)\) by

\[
\underline{\beta}(n) \equiv (1 - \eta)^{n-1} \iota^{n-1} \quad \text{(no switch),}
\]

and

\[
\overline{\beta}(n) \equiv \sum_{k=1}^{n-1} \eta(1 - \eta)^{k-1} \iota^{k-1} \beta^{n-k} \quad \text{(switch at some time \(k\))}
\]

\[
= \eta \beta^{n-1} \frac{1 - [\iota(1 - \eta)/\beta]^{n-1}}{1 - \iota(1 - \eta)/\beta},
\]

with

\[
\overline{\beta}(1) = 0.
\]

Given this, the value of a long-term relationship for the low-discount factor case
is given by

\[
\tilde{V}_i^{\alpha}(\phi, x) = \ln(y - t_1) + u + l
\]

\[
+ \sum_{n=1}^{\infty} [\beta^i(n) + \bar{\beta}^i(n)](1 - \xi)^n \Pr[\text{no symptoms in either person at end of period } n | \phi]
\times [\ln(y - t_1) + u + l]
\]

\[
+ \sum_{n=1}^{\infty} \beta^i(n)(1 - \xi)^{n-1} \xi \Pr[\text{no symptoms in either person at end of period } n | \phi]
\times [\eta V_i^i(\phi^h(n, \phi), x) + (1 - \eta) V_i^i(\Phi^h(n, \phi), x)]
\]

\[
+ \sum_{n=1}^{\infty} \bar{\beta}^i(n)(1 - \xi)^{n-1} \xi \Pr[\text{no symptoms in either person at end of period } n | \phi]
\times V_i^i(\Phi^h(n, \phi), x) x)
\]

\[
+ \sum_{n=1}^{\infty} \beta^i(n)(1 - \xi)^{n-1} \Pr[\text{symptoms just in partner at end of period } n | \phi]
\times [\eta V_i^i(\Phi^a(n, \phi), x) + (1 - \eta) V_i^i(\Phi^a(n, \phi), x)]
\]

\[
+ \sum_{n=1}^{\infty} \bar{\beta}^i(n)(1 - \xi)^{n-1} \Pr[\text{symptoms just in partner at end of period } n | \phi]
\times V_i^i(\Phi^a(n, \phi), x)
\]

\[
+ \sum_{n=1}^{\infty} [\beta^i(n) + \bar{\beta}^i(n)](1 - \xi)^{n-1} \Pr[\text{HIV/AIDS symptoms in person at end of period } n | \phi]
\times A.
\]

Except for the possibility of a switch in the discount factor, the formula resembles (7). In fact, (23) collapses to (7) when \( \eta = 1 \) implying \( \beta(n) = 0 \)—here \( \bar{\beta}(n) \) should be set to \( \beta^a \). As before, the first line reports the current utility from the relationship. Suppose that a match sustains until period \( n + 1 \). The second and third lines give the discounted expected utility accruing over the next \( n \) periods. The next lines handle breakup events for period \( n + 1 \). Lines 4 and 5 cover the situation where the discount factor applying to period \( n + 1 \) remains low and an exogenous breakup occurs. Note that the discount factor may switch upwards with probability \( \eta \) in period \( n + 1 \). Lines 6 and 7 assume an exogenous breakup occurs and that the discount factor has switched sometime before \( n + 1 \). The rest
of the lines should now be self evident.

C.4 Stationary Distributions

Before starting, define the function $J$ by $J(z) = 1$, if $z = 0$, and $J(z) = 0$, if $z \neq 0$.

C.4.1 Singles distributions, $d = \nu$

Consider the case where singles have the low discount factor, $\nu$. Note that a person can only get into the low discount factor state from the low discount factor state. The equilibrium distribution for type-$x$ singles, with a low discount factor, and prior $\phi$ is specified by

$$S^i(\phi', x) = \mu(1 - \eta)J(\phi' - 1) + (1 - \delta)(1 - \eta) \sum_{\phi} [1 - \Pi^i_{\phi}(\phi, x)]S^i(\phi, x)$$

$$\times \{ \left[1 - \alpha[(1 - \phi) + \phi(1 - \bar{\phi}_p)(1 - \gamma_p)]\right]J(\phi' - \Phi_p(\phi))\Pi^i_{\phi}(\phi, x)$$

$$+ \left[1 - \alpha[(1 - \phi) + \phi(1 - \bar{\phi}_u)(1 - \gamma_u)]\right]J(\phi' - \Phi_u(\phi))\Pi^i_{\phi}(\phi, x)$$

$$+ [1 - \alpha(1 - \phi)]J(\phi' - \Phi_u(\phi))\Pi^i_{\phi}(\phi, x)\}$$

$$+ \mathcal{L}^i(\phi', x).$$

At the beginning of any period the pool of singles comes from five sources. First, new ones are born. A fraction of these people will draw a low discount factor in the first period. These people have not had sex yet, so they know with certainty they do not have the HIV/AIDS virus. This inflow is given by the first term on right-hand side. Now, some older singles will fail to find a partner on the market for long-term relationships. The fraction of populace in this situation is specified by the second line. Three things can happen to them. Some of these people will find a mate on the short-term protected sex market. This is second source of singles and is given on the third line. Others will match on the short-term unprotected sex market. This accounts for the third source of inflow and is shown by the fourth line. Still others will fail to match, which gives the fourth influx, as the fifth line indicates. The fifth inflow is from long-term relationships
that have broken up. The last line gives the contribution from this source. This term is explained next.

Now, suppose that a type-\( x \) individual with prior \( \phi \) and a low discount factor enters into a long-term relationship. There are two reasons why this person may exit into single life at a later date: the match dissolves exogenously, or his partner develops HIV/AIDS symptoms. These two events are not mutually exclusive. But, note that if a male enters into single life because his partner develops HIV/AIDS symptoms, then it doesn’t matter whether or not there was a breakup as well, because the probability of the latter two events sum to one. Now, if the match terminates in period \( n \) solely due to a breakup then the male will exit into single life with the prior \( \Phi^h_t(\phi, n) \), while if his partner develops HIV/AIDS symptoms then he will exit with prior \( \Phi^a_t(\phi, n) \). The odds that a type-\( x \) person with a low discount factor will exit into single life at some future date with a low discount factor, \( \eta \), and a prior \( \phi' \), conditional on starting with a prior \( \phi \), will consequently read

\[
L^t(\phi', x|\phi) = \sum_{n=1}^{\infty} (1 - \eta)^n (1 - \xi)^{n-1} (1 - \delta)^n J(\phi' - \Phi^h_t(\phi, n)) \\
\times \Pr[\text{no symptoms in either person at end of period } n|\phi] \\
+ \sum_{n=1}^{\infty} (1 - \eta)^n (1 - \xi)^{n-1} (1 - \delta)^n J(\phi' - \Phi^a_t(\phi, n)) \\
\times \Pr[\text{symptoms just in partner at end of period } n|\phi],
\]

where the probabilities for symptoms are specified by (20) to (22) above. From this it is easy to calculate that the unconditional exit distribution, \( L^t(\phi', x) \), is given by

\[
L^t(\phi', x) = \sum_{\phi} L^t(\phi', x|\phi)\Pi^t(\phi, x)S^t(\phi, x). \tag{26}
\]

### C.4.2 Singles distributions, \( d = \beta \)

Next, the situation where singles have the high discount factor, \( \beta \), will be presented. A person can move into the high discount factor state from either the high or low one. The equilibrium distribution function for type-\( x \) singles, with a
high discount factor, and prior $\phi$ is given by

\[
S^\beta(\phi', x) = \eta \mu J(\phi' - 1) + (1 - \delta) \sum_\phi [1 - \Pi_i^{\beta}(\phi, x)] S^\beta(\phi, x)
\]

\[
\times \{ \{ 1 - \alpha((1 - \phi) + \phi(1 - \bar{\phi}_p)(1 - \gamma_p)) \} J(\phi' - \Phi_p(\phi)) \Pi_p^\beta(\phi, x) + \{ 1 - \alpha((1 - \phi) + \phi(1 - \bar{\phi}_u)(1 - \gamma_u)) \} J(\phi' - \Phi_u(\phi)) \Pi_u^\beta(\phi, x) + \} 1 - \alpha(1 - \phi)) J(\phi' - \Phi_a(\phi)) \Pi_a^\beta(\phi, x) \}
\]

\[
+ \mathcal{L}^\beta(\phi', x)
+ (1 - \delta) \eta \sum_\phi [1 - \Pi_i(\phi, x)] S^\prime(\phi, x)
\]

\[
\times \{ \{ 1 - \alpha((1 - \phi) + \phi(1 - \bar{\phi}_p)(1 - \gamma_p)) \} J(\phi' - \Phi_p(\phi)) \Pi_p^\beta(\phi, x) + \{ 1 - \alpha((1 - \phi) + \phi(1 - \bar{\phi}_u)(1 - \gamma_u)) \} J(\phi' - \Phi_u(\phi)) \Pi_u^\beta(\phi, x)
\]

The first six lines are the direct analogue to equation (24), only with a high discount factor. The exit distribution from married life, $\mathcal{L}^\beta(\phi', x)$, is defined below. It includes married individuals whose own discount factor moved up from $\iota$ to $\beta$ at some time during their marriage. The last four lines reflect the inflow of low discount factor singles who transit to a high discount factor from the low one.

To calculate the exit distribution, $\mathcal{L}^\beta(\phi', x)$, imagine a type-$x$ person who entered married life with a high discount factor and a prior of $\phi$. The probability that he will exit married life with the prior $\phi'$ is given by

\[
\mathcal{L}^\beta(\phi', x|\phi) = \sum_{n=1}^{\infty} \xi(1 - \xi)^{n-1}(1 - \delta)^n J(\phi' - \Phi^h(n, \phi))
\]

\[
\times \Pr[\text{no symptoms in either person at end of period } n|\phi]
\]

\[
+ \sum_{n=1}^{\infty} (1 - \xi)^{n-1}(1 - \delta)^n J(\phi' - \Phi^a(n, \phi))
\]

\[
\times \Pr[\text{symptoms just in partner at end of period } n|\phi].
\]

Likewise, consider those start married life with a low discount factor and a prior
of $\phi$ but who switch to high discount factor. The odds that they will exit marriage with the prior $\phi'$ are

$$
L^{\beta}(\phi', x|\phi) = \sum_{n=1}^{\infty} [1 - (1 - \eta)^n] \xi(1 - \xi)^{n-1} (1 - \delta)^n J(\phi' - \Phi^d_i(n, \phi))
\times \Pr[\text{no symptoms in either person at end of period } n|\phi]
+ \sum_{n=1}^{\infty} [1 - (1 - \eta)^n] (1 - \xi)^{n-1} (1 - \delta)^n J(\phi' - \Phi^i_d(n, \phi))
\times \Pr[\text{symptoms just in partner at end of period } n|\phi].
$$

(29)

Therefore, the unconditional exit distribution for people entering into single life from married life with a high discount factor is

$$
L^{\beta}(\phi', x) = \sum_{\phi} L^{\beta}(\phi', x|\phi) \Pi^\beta_\phi(\phi, x) S^\beta(\phi, x) + \sum_{\phi} L^{i\beta}(\phi', x|\phi) \Pi^i_\phi(\phi, x) S^i(\phi, x).
$$

(30)

Last, equations (24) to (30) fully describe the operator $T$ in (10).

**D Appendix—Algorithm**

**D.1 Computing the Value Functions and Distribution Functions**

The algorithm enters each iteration with a guess for the set of values functions, $\bar{V}^d_{g,r}$ and $V^d_{g,r}$, and stationary distributions, $L^d$ and $S^d$, for each type of individual $x$. It is easy to create a guess for the nonprevalence rates, $\bar{\phi}_{g,r}$, from the distribution functions by using (11) to (13). On each iteration the value functions are updated using equations (1), (2), (3), (5), (6), (7), (8), and (23). At the same time the distribution functions are revised on each iteration using (24) to (30). A grid is constructed for the individual’s prior, $\phi$, over his nonprevalence rate. The above functions are computed at each grid point. Even though by construction $\phi$ is a grid point there is no guarantee that $\phi'$ will be, given the form of the updating functions (9), (4), (19) and (18). This is resolved using an interpolation scheme (MATLAB’s cubic Hermite scheme). For example to compute $\bar{V}^\beta_{a}(\phi', x)$ on the righthand side of (1) for an off-the-grid point $\phi'$ a weighted average of $\bar{V}^\beta_{a}(\phi', x)$
and $\tilde{V}_a^\beta(\phi_{i+1}', x)$ at the two nearest adjacent grid points, $\phi_i \leq \phi' \leq \phi_{i+1}'$, is computed. A similar issue arises when computing the distribution functions. Take the density shown in (24), for example. The updating rules (9) and (4) in general will not map a grid point $\phi$ into a $\phi'$ that lies on the grid. Therefore, $\phi$ is mapped onto the two closest adjacent points, $\phi_i'$ and $\phi_{i+1}'$, such that $\phi_i' \leq \Phi_r(\phi) \leq \phi_{i+1}'$, for $r = a, u, p$, using a linear weighting scheme.

D.2 Pseudo Code for Monte Carlo - number of partners up to age $t$

The task is to simulate the average number of partners for $n$ individuals, say males, over $m$ periods. Create matrices to store the sample paths across individuals for variables such as $\phi$, $n$, $p$, $a$, $h$, $w$ where $n$ is the number of periods in a long-term relationship ($n = 0$ for a single), $p$ is the number of partners to date, $a$ is a variable indicating whether the person is alive ($a = 0$ for a dead person and 1 for a living one), $h$ is a variable indicating whether the person is healthy ($h = 0$ for a person with AIDS symptoms and 1 for a person without symptoms) and $w$ is the number of marriages ("weddings") the person was in to date. For each individual do the following:

1. At the beginning of life draw two random numbers from a uniform distribution on $[0, 1]$. If the first is below $\eta$, then start the person with high discount factor and only use decision rules with superscript $\beta$ throughout his life. Otherwise, in the first period $t = 1$ of a person’s life use decision rule with superscript $\iota$. The probability of switching by the end of period $t$ is $n(t) = [1 - (1 - \eta)^t]$. Consider the realization of the second random variable (call it $r$) and find the integer $\tilde{t} \in \{1, 2, \ldots\}$ such that $n(\tilde{t} - 1) \leq r < n(\tilde{t})$. For every period $t \leq \tilde{t}$ of the simulated persons life use policy rules with the index $\iota$, for every period $t > \tilde{t}$ use policy rule with the index $\beta$. (Note that $\tilde{t}$ might be larger than $T$, the maximum number of periods we want to

---

21The Hermit scheme preserves monotonicity of $\tilde{V}_a^\beta$ in $\phi$, and therefore the extrapolation can be interpreted as a weighted-average of the nearest grid points. The weights themselves are computed based on the entire set of grid points, not just based on the nearest ones, to preserve certain smoothness properties.
simulate, in which only subscript $y$ applies).

2. Start off all people as newly born singles with $\phi = 1, n = 0, p = 0, w = 0, h = 1$, and $a = 1$. For each person $i$ draw a matrix of $m \times 3$ uniformly distributed random variables. The seed for the random number generator should be a function of $i$.

3. Individual $i$ will enter a given period $t$ with some values for $d, \phi, n, p, h$ and $a$, denoted by $d_{i,t}, \phi_{i,t}, n_{i,t}, p_{i,t}, h_{i,t}, w_{i,t}$ and $a_{i,t}$. If $a_{i,t} = 0$ the individual is dead and nothing has to be done. Terminate going down the time path for this individual. Otherwise, if $d_{i,t-1} = 1$ check whether $t > \bar{t}$. If so, set $d_{i,t} = \beta$. Alternatively, when $d_{i,t-1} = 1$ then $d_{i,t} = \beta$. If he is alive $a_{i,t} = 1$ but sick $h_{i,t} = 0$, go to step 4. Otherwise, if he is married, $n_{i,t} \geq 1$, go to step 5; if he is single, $n_{i,t} = 0$, go to step 6.

4. If he is alive $a_{i,t} = 1$ but sick $h_{i,t} = 0$, draw the first random variable from column $t$. If it is below $\delta_2$ the person dies, i.e. $a_{i,t+1} = 0$. If it is above $\delta_2$ then he lives $a_{i,t+1} = 1$ and $h_{i,t+1} = 0$. ($\delta_2$ is the probability of death for a person with AIDS symptoms.)

5. Married. If $a_{i,t} = h_{i,t} = 1$ and $n_{i,t} \geq 1$ then the individual is alive and married. Draw the second random variable from the $t$th row of the matrix of random variables. Define the probabilities of some of the events discussed in Section C.2.

$$N_1 \equiv \Pr[\text{no symptoms in either person at end of period } n|\phi],$$

$$E_1 \equiv (1 - \phi)(1 - \bar{\phi}_l)(1 - \alpha)^{2n-1} \alpha, \text{ cf (21)},$$

$$E_2 \equiv \phi(1 - \bar{\phi}_l)(1 - \alpha)^{n-1} \alpha \left[ (1 - \gamma_u) \sum_{j=0}^{n-1} \gamma_j^u (1 - \alpha)^{n-j} + \gamma_n^u \right], \text{ cf (21)},$$

$$E_3 \equiv (1 - \phi)\bar{\phi}_l(1 - \alpha)^n \alpha (1 - \gamma_u) \sum_{j=0}^{n-1} \gamma_j^u (1 - \alpha)^{n-1-j}, \text{ cf (21)},$$

$$A_1 \equiv (1 - \phi)(1 - \bar{\phi}_l)(1 - \alpha)^{2n-2} \alpha, \text{ cf (22)},$$

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\[ A_2 \equiv \phi(1 - \bar{\phi}_t)(1 - \alpha)^{n-1}\alpha(1 - \gamma_u)\sum_{j=0}^{n-1} \gamma_u^j(1 - \alpha)^{n-j-1}, \text{ cf (22)}, \]

\[ A_3 \equiv (1 - \phi)\bar{\phi}_t\alpha(1 - \alpha)^{n-1}[(1 - \gamma_u)\sum_{j=0}^{n-2} \gamma_u^j(1 - \alpha)^{n-1-j} + \gamma_u^{n-1}], \text{ cf (22)}. \]

(a) Marriage persists. If the random variable is less than \( (1 - \varepsilon)(1 - \delta) N_1(n_{i,t})/\Lambda(n_{i,t}) \),

where \( \Lambda(n_{i,t}) \equiv [N_1(n_{i,t}) + E_1(n_{i,t}) + E_2(n_{i,t}) + E_3(n_{i,t}) + A_1(n_{i,t}) + A_2(n_{i,t}) + A_3(n_{i,t})] \), then the marriage persists. Here, set \( \phi_{i,t+1} = \phi_{i,t} \)

\( n_{i,t+1} = n_{i,t} + 1, p_{i,t+1} = p_{i,t}, w_{i,t+1} = w_{i,t}, a_{i,t+1} = h_{i,t+1} = 1. \)

(b) Exogenous breakup. If the random variable lies between \( (1 - \varepsilon)(1 - \delta) N_1(n_{i,t})/\Lambda(n_{i,t}) \) and \( (1 - \delta) N_1(n_{i,t})/\Lambda(n_{i,t}) \) then the marriage marriage breaks up exogenously and the male enters single life. Here, set \( \phi_{i,t+1} = \Phi^h_t(n_{i,t}, \phi_{i,t}), n_{i,t+1} = 0, p_{i,t+1} = p_{i,t}, w_{i,t+1} = w_{i,t}, a_{i,t+1} = h_{i,t+1} = 1. \)

(c) Partner shows symptoms of AIDS/HIV. Alternatively, when the random variable lies between \( (1 - \delta) N_1(n_{i,t})/\Lambda(n_{i,t}) \) and \( (1 - \delta)(1 - \varepsilon)N_1(n_{i,t})/\Lambda(n_{i,t}) \) then the marriage marriage breaks up because the male’s partner has the symptoms of AIDS/HIV. Again, the male enters single life. Here, \( \phi_{i,t+1} = \Phi^d_t(n_{i,t}, \phi_{i,t}), n_{i,t+1} = 0, p_{i,t+1} = p_{i,t}, w_{i,t+1} = w_{i,t}, a_{i,t+1} = h_{i,t+1} = 1. \)

(d) AIDS symptoms or death. With probability \( Q \equiv 1 - (1 - \delta)N_1(n_{i,t}) + E_1(n_{i,t}) + E_2(n_{i,t}) + E_3(n_{i,t})/\Lambda(n_{i,t}) \) the person either gets AIDS symptoms or dies or both. Draw the third random variable in the \( t \)th column. If it is below \( \delta/Q \) the person dies, i.e. \( a_{i,t+1} = 0 \). If it is above the person survives with AIDS symptoms, i.e. \( a_{i,t+1} = 1 \) and \( h_{i,t+1} = 0. \)

(e) Accounting for newly wedded agents. If \( n_{i,t} = 1 \), then set \( p_{i,t+1} = p_{i,t} + 1, w_{i,t+1} = w_{i,t} + 1. \) This step has to be done no matter which event \( a \) to \( d \) occurred.

6. Single. If \( a_{i,t} = h_{i,t} = 1 \) and \( n_{i,t} = 0 \) then the individual is alive and single. Draw the first random variable from the \( t \)th row of the matrix of random variables. If this random variable lies below \( \pi_t \) then the person is newly wedded and enters period-\( t \) married life (set \( n_{i,t} = 1 \)). Then move to the
marriage state in period \( t \), described in step 5. Otherwise, go through the three cases outlined below for a single person.

(a) Abstinence is first choice. If \( \pi_{a,t} = 1 \) then the individual chooses abstinence. Draw a random variable from the second column of the matrix. If the number is less than \( \delta \) the person dies and \( a_{i,t+1} = 0 \). If the number is between \( \delta \) and \( \delta + (1-\delta)\alpha(1-\phi_{i,t}) \) the person gets AIDS symptoms but continues living so that \( a_{i,t+1} = 1 \) and \( h_{i,t+1} = 0 \). Otherwise the individual lives on to the next period so that \( a_{i,t+1} = \Phi_a(\phi_{i,t}) \), \( n_{i,t+1} = 0, p_{i,t+1} = p_{i,t}, w_{i,t+1} = w_{i,t}, a_{i,t+1} = h_{i,t+1} = 1 \).

(b) Abstinence is second choice. If \( 0 < \pi_{a,t}, \pi_{s,t} < 1 \) then the individual’s first choice is short-term market \( s \). Abstinence is his second choice. Draw a random variable from the second column of the matrix. If the first number is less than \( \pi_{s,t} \) then the person enters the short-term market. Draw a random variable from the third column of the matrix. If the number is less than \( \delta \) the person dies and \( a_{i,t+1} = 0 \). If the number is between \( \delta \) and \( \delta + (1-\delta)\alpha(1-\phi_{i,t}) \) the person gets AIDS symptoms but continues living so that \( a_{i,t+1} = 1 \) and \( h_{i,t+1} = 0 \). Otherwise the individual lives on to the next period so that \( a_{i,t+1} = \Phi_s(\phi_{i,t}) \), \( n_{i,t+1} = 0, p_{i,t+1} = p_{i,t} + 1, w_{i,t+1} = w_{i,t}, a_{i,t+1} = h_{i,t+1} = 1 \).

(c) Abstinence is last choice. If \( 0 < \pi_{u,t}, \pi_{p,t} < 1 \) then the individual’s first choice and second choices are the short-term markets. Abstinence is his third choice. Draw a random variable from the second column of the matrix. If the second number is less than \( \pi_{u,t} \) then the person enters the unprotected short-term market. Draw a random variable from the third column of the matrix. If the third number is less \( \delta \) the person dies and \( a_{i,t+1} = 0 \). If the number is between \( \delta \) and \( \delta + (1-\delta)\alpha(1-\phi_{i,t}) \) the person gets AIDS symptoms but continues living so that \( a_{i,t+1} = 1 \) and \( h_{i,t+1} = 0 \). Otherwise the individual lives on to the next period so that \( a_{i,t+1} = \Phi_u(\phi_{i,t}) \), \( n_{i,t+1} = 0, p_{i,t+1} = p_{i,t} + 1, w_{i,t+1} = w_{i,t}, a_{i,t+1} = h_{i,t+1} = 1 \). If the second number lies in the interval \( (\pi_{u,t}, \pi_{p,t} + \pi_{u,t}) \) then the person enters the protected short-term market. Draw a random variable from the third column of the matrix. If the number is less than \( \delta \) the person dies and \( a_{i,t+1} = 0 \). If the number is
between $\delta$ and $\delta + (1 - \delta)\alpha[(1 - \phi_{i,t}) + \phi_{i,t}(1 - \bar{\phi}_p)(1 - \gamma_p)]$ the person gets AIDS symptoms but continues living so that $a_{i,t+1} = 1$ and $h_{i,t+1} = 0$. Otherwise the individual lives on to the next period so that $\phi_{i,t+1} = \Phi_p(\phi_{i,t}), n_{i,t+1} = 0, p_{i,t+1} = p_{i,t} + 1, w_{i,t+1} = w_{i,t}, a_{i,t+1} = h_{i,t+1} = 1$. If the second number is greater than $\pi_{u,t} + \pi_{p,t}$ then the analysis proceeds as in the abstinence case.

7. The average number of partners during their life for people at age $t$ (counted at the beginning of the period before having sex), who are still alive, is given by $\Sigma_{i,t} h_{i,t} p_{i,t} / \Sigma_{i,t} h_{i,t}$. For the number of agents that are infected, first adjust the beliefs for those who are married. For any observation $(i, t)$ with $n_{i,t} \geq 2$ set $\phi_{i,t} = \phi^*(n_{i,t} - 1, \phi_{i,t})$. The reason for the adjustment is that $n$ is already updated in the period where an agent gets newly wed. Then the prevalence in society for those who do not display severe AIDS symptoms yet is $\Sigma_{i,t} h_{i,t} \phi_{i,t} / \Sigma_{i,t} h_{i,t}$. 
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