Interest Rate Policy and Financial Regulation: How to Control Excessive Risk Taking?\textsuperscript{*}\textsuperscript{†}

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Abstract

This paper characterizes the optimal combination of monetary policy and financial regulation in a quantitative infinite horizon model with a risk taking channel of monetary policy. The model economy is rich enough to match main characteristics of the U.S. economy and its financial sector, yet tractable enough to deliver clear prescriptions regarding the optimal policy mix. The optimal policy mix has a simple state contingent leverage regulation and a small financial contribution tax on profits of financial intermediaries. Revenue derived from this tax helps to secure equity financing to solvent financial institutions during economic downturns. Leverage regulation and monetary policy act as complements when policy deviates from the optimum. Standard capital-adequacy regulation is welfare decreasing though effective at reducing risk taking.

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1 Introduction

The recent financial crisis has renewed interest in the optimal mix between monetary policy and financial regulation. Some of the questions which gained a substantial degree of attention recently include the following: 1) Should monetary policy be used to fight the build-up of imbalances in the financial system (leaning against the wind), or should financial stability objectives be left entirely to regulation? 2) Should monetary policy be coordinated with regulation to achieve some common macroprudential goals? 3) Should financial institutions be taxed to help finance bail outs during financial crises? Our paper evaluates these issues through the lens of a dynamic general equilibrium model which is rich enough to match key characteristics of the US economy, yet tractable enough to deliver clear prescriptions regarding the optimal policy mix.

The model has infinitely lived households who invest their wealth into financial and non-financial equity and bank deposits. Deposits are insured by the government. Bank equity is risky. Households cannot differentiate between high risk and low risk banks. The high risk banks have a higher expected return in good states of the world, but they also have a wider spread of possible returns between the good and the bad state. Due to limited liability, high risk banks may face a moral hazard problem investing too much into risky projects. We assume that the high and low-risk banks trade investment funds among themselves using interbank loans collateralized by government bonds. As a monopoly issuer of government bonds, the government can influence the banks’ risk-taking incentives by varying the price of these bonds. In addition, the government imposes various regulatory constraints on assets or liabilities of banks. Our objective in this paper is a joint characterization of optimal interest rate policy and regulation, which implements the first-best allocation in our economy.
The main findings are: i) Monetary policy coupled with a leverage constraint and a small tax on profits of financial intermediaries can achieve the socially optimal outcome; ii) Capital adequacy constraints are effective at controlling risk-taking but impose a welfare loss; iii) Lower policy rates induce more risk taking under a capital adequacy constraint but less risk taking under a leverage constraint.

The optimal policy mix looks remarkably close to the suggestions discussed in recent policy proposals and suggest the following answers to the above stated questions: 1) Both monetary policy and regulation have an important role in controlling risk-taking, however "leaning against the wind" may not necessarily reduce risk-taking; 2) Yes, optimal monetary policy and the optimal regulation are jointly determined, as for example, capital adequacy ratios and leverage constraints imply different optimal levels of policy rates; 3) Yes, a small state-contingent tax on profits of financial intermediaries helps to alleviate some of the moral hazard problems arising in the financial sector. Revenue from this tax is also used to finance a subsidy to solvent financial intermediaries during economic downturns. This tax/subsidy scheme has stabilizing effect on private equity flows to financial institutions over the business cycle.

The paper is organized as follows. Section 2 presents the model and derives equilibrium properties. Section 3 outlines the methods we use to determine the model parameters. Section 4 explains how we find the optimal policy mix and discusses the properties of the optimal policy mix. Section 5 considers results and Section 6 concludes.

2 Model Economy

The model economy augments the framework in Cociuba, Shukayev, and Ueberfeldt (2012) (henceforth, CSU) to allow for capital and leverage regulations, as well as a tax on the financial firms’ profits. In this section, we present the main elements of our model, and refer
the reader to our earlier work for details.

The economy is populated by households, nonfinancial firms, financial intermediaries and a government. The timing of model events is as follows. At the beginning of each period, the economy is subject to an exogenous aggregate shock which affects the productivity of all operating firms: nonfinancial firms, high-risk and low-risk financial intermediaries. Production takes place and, conditional on not being bankrupt, firms pay returns to their debtholders and equityholders. These returns determine the households’ wealth which is used to purchase consumption and investments that will pay off next period. At this stage, financial intermediaries are identical, receive equal amounts of equity and deposits from households and purchase government bonds in the primary market. The remainder of their funds is earmarked for risky projects. Subsequently, financial intermediaries acquire more information about the riskiness of their projects. With probability \( \pi_j \), a financial intermediary holds a project of type \( j \in \{h,l\} \) where \( h \) denotes a high-risk and \( l \) denotes a low-risk project. Intermediaries adjust their portfolios via collateralized borrowing in secondary bond markets.

The idiosyncratic shock \( j \) is i.i.d. The aggregate state \( s_t \in \{\bar{s}, \bar{s}\} \) follows a first-order Markov process. The history of aggregate shocks up to \( t \) is \( s^t \). A more detailed summary of the timing of events is presented in CSU.

### 2.1 Households

There is a measure one of households. Their wealth, \( w(s^t) \), consists of returns on previous period investments, wage income from working in the financial and nonfinancial sectors and lump-sum taxes or transfers from the government. Households split their wealth into current consumption, \( C(s^t) \), and investments that will pay returns in period \( t + 1 \). Investments take the form of deposits, \( D_h(s^t) \), nonfinancial sector equity, \( M(s^t) \), and financial sector equity, \( Z(s^t) \). Returns to equity are contingent on the realization of the aggregate state in the period when they are paid, while returns to deposits are not. In particular, financial equity returns are bounded below by zero due to the limited liability of intermediaries. In contrast,
returns to deposits are guaranteed by deposit insurance. The households’ problem is:

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \varphi(s^t) \log C(s^t)$$

subject to:

$$w(s^t) = R^m(s^t) M(s^{t-1}) + R^d(s^{t-1}) D_h(s^{t-1}) + R^z(s^t) Z(s^{t-1})$$

$$+ \pi_m W_m(s^t) + (1 - \pi_m) \pi_l W_l(s^t) + (1 - \pi_m) \pi_h W_h(s^t) + T(s^t)$$

where $\beta$ is the discount factor, $\varphi(s^t)$ is the probability of history $s^t$, $\pi_m$ is the fraction of a household’s time spent working in the nonfinancial sector and $(1 - \pi_m) \cdot \pi_j$ with $j \in \{h, l\}$ is the fraction of time spent working for financial intermediary of type $j$, $\pi_h + \pi_l = 1$, $W_m(s^t)$ is the wage rate paid by nonfinancial firms, $W_j(s^t)$ is the wage rate paid by a financial intermediary of type $j$ and $T(s^t)$ are lump-sum transfers if $T(s^t) \geq 0$, or lump-sum taxes otherwise. Notice that households supply labor inelastically and we’ve assumed that labor markets are segmented, hence we can normalize labour supplied to each firm to one unit.

2.2 Firms

Financial firms finance their operations through equity and deposits and invest into safe government bonds and risky projects. The latter are investments into the production technologies of small firms and can be of two types: high-risk projects with productivity $q_h(s_t)$ and low-risk projects with productivity $q_l(s_t)$.

Nonfinancial firms are funded through household equity only. All equity raised is invested into capital whose return depends on the productivity of the production technology in the nonfinancial sector, $q_m(s_t)$. Note that, implicitly,
households in our model invest directly into the risky production technology of nonfinancial firms. However, they need intermediaries to invest into the risky projects of small firms.

We assume that the ordering of firms’ productivities as a function of the aggregate state is as follows: \( q_h (\bar{s}) \geq q_m (\bar{s}) > q_l (\bar{s}) \geq q_l (\bar{s}) > q_m (\bar{s}) > q_h (\bar{s}) \). High risk intermediaries are the most productive during a good aggregate state \((s_t = \bar{s})\) and the least productive during a bad aggregate state \((s_t = \bar{s})\). For details on the parameterization of these relative productivity levels, see section 3.

2.2.1 Financial Sector

There is a measure \(1 - \pi_m\) of financial intermediaries. The problem of an intermediary is to choose a portfolio that maximizes the expected value of its equity. This problem can be split in two stages. Initially, all financial intermediaries are identical, they receive the same amount of deposits and equity from the households and choose the same portfolio by investing in government bonds on the primary market and in risky projects. Subsequently, intermediaries find out their idiosyncratic shock \(j \in \{h,l\}\) and reoptimize their portfolios by trading in secondary bond markets. These transactions can be interpreted as repurchase agreements or repo transactions, as argued in CSU. The investments into the high-risk and low-risk projects that result from the portfolio reallocation will determine production by the financial sector realized at the beginning of the next period.

Portfolio Choice in the Primary Market

Since households own all firms in the economy, firms value profits at history \(s^t\) according to the households’ marginal utility of consumption weighted by the probability of history \(s^t\). In particular, \(\lambda (s^t) = \varphi (s^t) / C (s^t)\). Taking as given \(\lambda (s^t)\), the amount of equity issued by an intermediary, \(z (s^{t-1})\), the future repo market activities and all prices, an intermediary chooses deposit demand, \(d (s^{t-1})\), safe bonds, \(b (s^{t-1})\), risky investments, \(k (s^{t-1})\), and labour,
$l(s^{t-1})$, to maximize the expected profits in $(P1)$:

$$\max \sum_{j \in \{h,l\}} \pi_j \sum_{s^t|s^{t-1}} (1 - \tau(s^t)) \lambda(s^t) V_j(s^t)$$

subject to:

$$z(s^{t-1}) + d(s^{t-1}) = k(s^{t-1}) + p(s^{t-1}) b(s^{t-1})$$

$$V_j(s^t) = \max \begin{cases} 
q_j (s_t) [k(s^{t-1}) + \tilde{p}(s^{t-1}) \tilde{b}_j(s^{t-1})]^{\theta} [l(s^{t-1})]^{1-\theta-\alpha} \\
p_j (s_t) (1-\delta) [k(s^{t-1}) + \tilde{p}(s^{t-1}) \tilde{b}_j(s^{t-1})] \\
+ [b(s^{t-1}) - \tilde{b}_j(s^{t-1})] - R^d(s^{t-1}) d(s^{t-1}) - W_j(s^t) l(s^{t-1}), 0 \end{cases}$$

$$z(s^{t-1}) / k(s^{t-1}) \geq \eta_K(s^{t-1})$$

$$z(s^{t-1}) / d(s^{t-1}) \geq \eta_L(s^{t-1})$$

where $V_j(s^t)$ are profits for intermediary $j \in \{h,l\}$ at history $s^t$ taxed at a state-contingent tax rate $\tau(s^t)$, $p(s^{t-1})$ is the primary market bond price, $\tilde{p}(s^{t-1})$ is the secondary market or repo market price, $\tilde{b}_j(s^{t-1})$ is the amount of bonds sold in the repo market by intermediary $j$, $k_j(s^{t-1}) \equiv k(s^{t-1}) + \tilde{p}(s^{t-1}) \tilde{b}_j(s^{t-1})$ is the amount of resources invested in the risky projects and $l(s^{t-1})$ is normalized to 1. Parameters $\theta$ and $\alpha$ satisfy $\alpha, \theta \in [0,1], 1-\alpha-\theta \geq 0$.

We augment the framework in Cociuba, Shukayev, and Ueberfeldt (2012) by considering two types of regulation. Firstly, there is a capital regulation which requires the amount of equity financial intermediaries hold per unit of risky investment to be larger than $\eta_K \geq 0$. This constraint—given in (3) in a general state-contingent form—captures some aspects of the Basel II accord. Secondly, we consider a leverage regulation (equation (4)) that requires the intermediaries to hold a certain amount of equity per unit of deposit. In effect this limits the amount of deposits financial intermediaries are able to accept by the amount of available equity. We will use the version with capital regulation to calibrate the model economy to match key characteristics of the U.S. economy prior to the recent financial crisis. Then later we show that a state-contingent leverage regulation is capable of implementing the optimal
allocation.

**Portfolio Adjustments via Repo Market**

Once intermediaries find out their type \( j \in \{ h, l \} \), they adjust the riskiness of their portfolios by trading bonds, \( \tilde{b}_j (s^{t-1}) \), amongst themselves. Intermediaries choose \( \tilde{b}_j (s^{t-1}) \) to solve:

\[
\max \sum_{s^t|s^{t-1}} (1 - \tau (s^t)) \lambda (s^t) V_j (s^t) \quad (P2)
\]

where \( V_j (s^t) \) is given in equation (2) and \( \tilde{b}_j (s^{t-1}) \in \left[ -\frac{k(s^{t-1})}{\hat{p}(s^{t-1})}, \hat{b}(s^{t-1}) \right] \).

Here, \( \tilde{b}_j (s^{t-1}) \) can be interpreted either as sales of bonds or, alternatively, as repurchasing agreements.\(^3\) For this reason, we use the terms secondary bond market and repo market interchangeably.

The importance of repo market transactions has been motivated by recent literature (e.g. Adrian and Shin (2010)). In our framework, the intermediaries’ ability to increase their risky investment is limited by their primary market activities. Higher purchases of bonds in the primary market make balance sheets seem safer initially, but may lead to increased risk taking through the repo market.

**2.3 Nonfinancial sector**

There are \( \pi_m \) identical nonfinancial firms which are funded entirely through household equity. Each nonfinancial firm enters period \( t \) with equity \( M (s^{t-1}) / \pi_m \) from households which is invested into capital. Hence, \( M (s^{t-1}) / \pi_m = k_m (s^{t-1}) \). The problem of a nonfinancial firm

\(^3\)While we model \( \tilde{b}_j (s^{t-1}) \) as bond sales, incorporating explicitly the repurchase of bonds—which is typical in a repo agreement—would yield identical results.
is to choose capital and labour to produce output:

$$\max \{ y_m(s^t) + q_m(s_t) (1 - \delta) k_m(s^{t-1}) - R^m(s^t) k_m(s^{t-1}) - W_m(s^t) l_m(s^{t-1}) \}$$

subject to: \( y_m(s^t) = q_m(s_t) (k_m(s^{t-1}))^\theta (l_m(s^{t-1}))^{1-\theta} \).

We introduce this sector in order to bring our model closer to U.S. data. Specifically, this allows our model to be consistent with a high equity to deposit ratio observed for U.S. households, a low equity to deposit ratio in the U.S. financial sector and the relative importance of the two sectors in U.S. production. Moreover, a large nonfinancial sector—as observed in U.S. data—reduces the quantitative importance of the financial intermediation sector for welfare and risk taking in our model. Excluding it, would overstate the impact of policy on our results.

### 2.4 Government

The government issues bonds that financial intermediaries can use either as an asset or as a medium of exchange on the repo market. At the end of period \( t - 1 \), the government sells bonds, \( B(s^{t-1}) \), at price, \( p(s^{t-1}) \). These bonds pay off during period \( t \). The proceeds from the bond sales are deposited with financial intermediaries.\(^4\) Each financial intermediary receives \( D_g(s^{t-1}) / (1 - \pi_m) \) of government deposits, where

\[
D_g(s^{t-1}) = p(s^{t-1}) B(s^{t-1})
\]

To guarantee the fixed return on deposits the government provides deposit insurance at zero price which is financed through household taxation.\(^5\) The government balances its

\(^4\) Alternatively, the proceeds from the bond sales could be handed to the households via transfers. Our results would be little affected by such a change.

\(^5\) See Pennacchi (2006, pg. 14), who documents that since 1996, deposit insurance has been essentially free for U.S. banks. For our calibartion, the assumption of a zero price of deposit insurance is not crucial. What matters is that the insurance is not made contingent on the portfolio decisions of the intermediaries.
budget after the production takes place at the beginning of period $t$: \(^6\)

$$T(s^t) + B(s^{t-1}) + \Delta(s^t) = R^d(s^{t-1}) D_g(s^{t-1}) + \tau(s^t) \sum_{j \in \{h,l\}} \pi_j V_j(s^t)$$

Here, $\Delta(s^t)$ is the amount of deposit insurance necessary to guarantee the fixed return on deposits, $R^d(s^{t-1})$. Given the limited liability of intermediaries, if they are unable to pay $R^d(s^{t-1})$ on deposits, they pay a smaller return on deposits which ensures they break-even. The rest is covered by the deposit insurance.

The main instrument of monetary policy is the price of government bonds on the primary market, $p(s^{t-1})$. The government satisfies any demand for bonds given this price. The interpretation is that the monetary authority uses open market operations (i.e. purchases or sales of government bonds) to control interest rates. The key decision from the government’s perspective is to choose the bond return $1/p(s^{t-1})$ that maximizes the welfare of the households in the decentralized economy.

### 2.5 Market clearing

There are eight market clearing conditions. The *labour market clearing conditions* state that labour demanded by financial intermediaries and nonfinancial firms equals labour supplied by households:

$$(1 - \pi_m) l(s^{t-1}) = 1 - \pi_m; \quad \pi_m l_m(s^{t-1}) = \pi_m$$

The *goods market clearing condition* equates total output produced with aggregate consumption and investment. Output produced by nonfinancial firms is $\pi_m q_m(s^t)(k_m(s^{t-1}))^\theta$, while output produced by financial firms is $(1 - \pi_m) \sum_{j \in \{l,h\}} \pi_j q_j(s^t)(k_j(s^{t-1}))^\theta$, where

\(^6\)We concentrate on new issuance of bonds only and abstract from outstanding bonds for computational reasons. Considering the valuation effects of current policy in the presence of outstanding bonds may be an interesting but challenging extension of the model.
Financial markets clearing conditions ensure that the deposit markets, equity markets and bond markets clear. Deposits demanded by financial intermediaries equal deposits from the households and the government:

\[ D_h(s^{t-1}) + D_g(s^{t-1}) = D(s^{t-1}) = (1 - \pi_m) d(s^{t-1}) \]

In the primary bond market, total bond sales by the government equal the bond purchases by financial intermediaries.

\[ B(s^{t-1}) = (1 - \pi_m) b(s^{t-1}) \]

In the repo market, trades between the different types of intermediaries must balance.

\[ \sum_{j \in \{l,h\}} \pi_j \tilde{b}_j(s^{t-1}) = 0 \quad (5) \]

Total equity invested by households in the financial and nonfinancial sectors are distributed over the firms.

\[ M(s^{t-1}) = \pi_m k_m(s^{t-1}) \]
\[ Z(s^{t-1}) = (1 - \pi_m) z(s^{t-1}) \]

### 2.6 Social Planner Problem

We consider the following social planner’s problem as a reference point for our decentralized economy. For ease of comparison between the two environments, we abuse language and refer
to the existence of financial and nonfinancial sectors even in the context of the social planner’s problem. At the beginning of period $t$, the aggregate state, $s_t$, is revealed and production takes place using capital that the social planner has allocated to the different technologies of production: $k_m(s^{t-1})$ for the nonfinancial sector, $k_h(s^{t-1})$ and $k_l(s^{t-1})$ for the high-risk and low-risk technologies of the financial sector. The resulting wealth is then split between consumption and capital to be used in production at $t + 1$. At the time of this decision, the social planner does not distinguish between the high-risk and low-risk technologies of the financial sector used in production next period, and simply allocates resources, $k(s^t)$, to both of them. Once their type is revealed, the social planner reallocates resources between the two technologies.

The social planner solves:

$$\max E \sum_{t=0}^{\infty} \beta^t \log C(s^t)$$

subject to:

$$C(s^t) + \pi_m k_m(s^t) + (1 - \pi_m) k_l(s^t)$$

$$= \pi_m q_m(s_t) \left[ (k_m(s^{t-1}))^\theta + (1 - \delta) k_m(s^{t-1}) \right]$$

$$+ (1 - \pi_m) \pi_l q_l(s_t) \left[ (k_l(s^{t-1}))^\theta + (1 - \delta) k_l(s^{t-1}) \right]$$

$$+ (1 - \pi_m) \pi_h q_h(s_t) \left[ (k_h(s^{t-1}))^\theta + (1 - \delta) k_h(s^{t-1}) \right]$$

$$k_l(s^t) = k(s^t) - \frac{\pi_h}{\pi_l} n(s^t)$$

$$k_h(s^t) = k(s^t) + n(s^t)$$

where $n(s^t)$ is the amount of resources given to (or taken from) each high-risk production technology. To achieve this reallocation, $\frac{\pi_h}{\pi_l} n(s^t)$ resources need to be taken away from (or given to) each low-risk technology.

From a social planner’s perspective, it is optimal for resources to flow to high-risk intermediaries during expansion periods and to low-risk intermediaries during contractions. To
induce these reallocation flows in the decentralized economy, bond prices, \( p(s^t) \), need to be appropriately chosen by the monetary authority (see Section 5 for details).

### 2.7 Competitive Equilibrium Properties

In this section, we discuss equilibrium properties of our model and present results on the relationship between equilibrium bond prices and the return to deposits. In addition, we define what we mean by risk taking behavior of financial intermediaries and provide intuition for how interest rate changes affect risk taking.

#### 2.7.1 Constrained and Unconstrained Equilibria

Financial intermediaries maximize expected returns to equity, but benefit from limited liability. When a bad aggregate shock has occurred, intermediaries of type \( j \) who are unable to pay the promised rate of return to depositors declare bankruptcy. Equity holders receive no return on their investments, while the returns to depositors are covered by deposit insurance. Limited liability introduces an asymmetry in that it allows the high-risk intermediary to make investment decisions that bring large profits in good times, while being shielded from losses in bad times. In our numerical experiments, only the high-risk intermediaries go bankrupt.

For a given policy, \( p(s^t) \), multiple equilibria exist. A common situation is the coexistence of an equilibrium with positive government bond holdings and one with zero bond holdings. We focus our analysis on the former, since repo market trading is welfare improving.\(^7\) Furthermore, equilibria can be of two types. When financial intermediaries choose to pledge only a fraction of bonds as collateral in the repo market, i.e. \( \tilde{b}_j(s^t) < b(s^t) \), we refer to equilibria as having an *unconstrained repo market*. Equilibria with a *constrained repo market* are ones in which either high-risk or low-risk intermediaries pledge all their bond holdings.

\(^7\)Shutting down the secondary bond market in the competitive equilibrium reduces welfare relative to the social optimum by an amount equivalent to about 1 percent of life-time consumption. We perform this calculation using the parameters calibrated as described in Section 3.
holdings as collateral. When the interest rate policy is chosen optimally, the equilibrium has a constrained repo market. The intuition is that optimal policy aims to restrict risk taking of high-risk financial intermediaries, who otherwise may take advantage of their limited liability and overinvest in risky projects. An effective way to restrict risk taking and potential bankruptcy is to limit the amount of bonds, so that collateral for future trading in the repo market is scarce.

Due to the limited liability of financial intermediaries and the possibility of a constrained repo market, we need to employ non-linear techniques to solve our model. We use a collocation method with occasionally binding non-linear constraints (for details, see Appendix A.1).

2.7.2 Bond Prices and the Return to Deposits

**Proposition 1** Consider an economy with positive government bond holdings. In the absence of capital regulation or if this regulation does not bind, the equilibrium bond prices satisfy: \( p(s^{t-1}) = \tilde{p}(s^{t-1}) \). In an equilibrium with binding capital regulation, bond prices are such that: \( p(s^{t-1}) > \tilde{p}(s^{t-1}) \). Furthermore, if the leverage constraint is not binding, then the returns on deposit must weakly dominate returns on bonds: \( R^d(s^{t-1}) \geq \frac{1}{p(s^{t-1})} \).

**Proof.** These results follow from the first order conditions of the financial intermediaries’ problems. Appendix A.2 outlines the proof.■

The intuition for these results is as follows. In the absence of capital regulation, there are no frictions in the model that would make primary and secondary bond prices different. When capital regulation binds, intermediaries are effectively required to hold a minimum share of safe assets, and they are only willing to acquire additional bonds in the repo market if the price is lower than in the primary market.

In addition, returns to deposits are weakly greater than returns to bonds, since otherwise there would be a profit opportunity. Namely, an intermediary would have incentives to pay a slightly higher deposit return to attract additional deposits and be able to invest more
into bonds. The result $R^d(s^{t-1}) \geq \frac{1}{p(s^{t-1})}$ can also be interpreted in terms of the option value provided by bonds in this economy. Bonds have value to intermediaries because they can be retracted on the repo market. Whenever some intermediaries are constrained in the amount of collateral they hold, bonds carry a discount: $R^d(s^{t-1}) > \frac{1}{p(s^{t-1})}$. However, in an unconstrained equilibrium, both high-risk and low-risk intermediaries have sufficient bonds and the option value of bonds is zero: $R^d(s^{t-1}) = \frac{1}{p(s^{t-1})}$.

When the leverage constraint $z(s^{t-1}) = \eta_L(s^{t-1})$ is binding, the intermediaries cannot increase their demand deposits without bounds, given a fixed amount of equity they receive from households. As a result, the profit opportunity mentioned above is restricted even if $R^d(s^{t-1}) \geq \frac{1}{p(s^{t-1})}$ condition is not met. As we will see later, this property of the leverage constraint will be important for its ability to implement the social planner’s solution.

Proposition (1) is important for two reasons. First, it shows that interest rate policy has a direct effect on the repo market. In fact, the close relationship we obtain between policy, $p(s^{t-1})$, and the repo rate, $\tilde{p}(s^{t-1})$, is supported by U.S. evidence, as shown in Bech, Klee, and Stebunovs (2010). Second, the return to depositors is influenced by regulation and by the interest rate on government bonds. Thus, the policy mix not only affects the choices financial intermediaries make, but also affects the investment choices of households. In quantitative experiments, we find the latter effect to be weaker than the former.

### 2.7.3 Risk Taking: Measurement and Impact of Policy

We use our model to assess whether and how interest rate policy influences risk taking of intermediaries. To this end, we make the notion of risk taking precise. We define risk taking as the percentage deviation in resources invested in the high-risk projects in a competitive equilibrium relative to the social planner. Formally,

$$r(s^{t-1}) = \frac{k^{CE}_h(s^{t-1}) - k^{SP}_h(s^{t-1})}{k^{SP}_h(s^{t-1})} \cdot 100$$  \hspace{1cm} (6)
where superscripts \{CE, SP\} denote whether the variable is part of the solution to the competitive equilibrium for a given interest rate policy or part of the social planner’s problem. Here, $k_h^{SP} (s^t) = k^{SP} (s^t) + n^{SP} (s^t)$ is the capital that the social planner invests in the high-risk technology and $k_h^{CE} (s^{t-1}) \equiv k^{CE} (s^{t-1}) + \tilde{p}^{CE} (s^{t-1}) \tilde{b}^{CE} (s^{t-1})$ is the capital invested in the high-risk projects in the competitive equilibrium.

A positive value of $r (s^{t-1})$ in equation (6) tells us that there is excessive risk taking in the competitive equilibrium, while a negative value indicates too little risk taking. In numerical results, we plot the cyclical behaviour of risk taking, but also report an aggregate measure defined as an average over expansions and contractions, $r \equiv E [r (s^{t-1})]$.

In Cociuba, Shukayev, and Ueberfeldt (2012), we have shown that when financial intermediaries are constrained in their repo market transactions by the amount of collateral they hold, lowering interest rates below the optimal level leads to less risk taking, on average, relative to the social planner problem. This paper shows that, in the presence of binding capital constraints, there is less risk taking compared to the social optimum. In addition, when capital regulation binds, lowering interest rates below the optimum increases aggregate risk taking, but this change is relatively small.

### 3 Calibration

This section outlines our approach for determining the various parameters of the model and describes the data we use. We calibrate the following parameters: $\beta, \theta, \eta$ the aggregate shock transition matrix $\Phi$, and $\pi_h$. We determine $\pi_m, \alpha, \delta, q_h (\bar{s}), q_h (\bar{s})$, $q_m (\bar{s})$, $q_m (\bar{s})$, $q_l (\bar{s})$, $q_l (\bar{s})$ using a minimum distance estimator. All parameter values are summarized in Tables 1 and 2.

The utility discount factor, $\beta$, is calibrated to ensure an annual real interest rate of 4% in our quarterly model. We obtain $\beta = 0.99$. The capital income share is determined using data from the U.S. National Income and Product Account (NIPA) provided by the
Bureau of Economic Analysis (BEA) for the period 1947 to 2009. We find \( \theta = 0.29 \) for the business sector.\(^8\) We choose \( \eta_K = 8\% \), \( \eta_L = 0 \) and \( \tau = 0 \). This captures the capital regulation implemented in the United States in accordance with Basel II, the absence of leverage regulation and the absence of differential profit tax treatment of financial and non-financial businesses, respectively.

To calibrate the transition matrix for the aggregate state of the economy, we use the Harding and Pagan (2002) approach of identifying peaks and troughs in the real value added of the U.S. business sector, from 1947Q1 to 2010Q2.\(^9\) We find 11 contractions with an average duration of 5 quarters. Hence, the probability of switching from a bad realization of the aggregate shock at time \( t - 1 \) to a good realization at time \( t \) is \( \phi(s_t = \bar{s}|s_{t-1} = \bar{s}) = 0.20 \). Moreover, the probability of switching from an expansion period to a contraction is \( \phi(s_t = \bar{s}|s_{t-1} = \bar{s}) = 0.055 \). The calibrated transition matrix is

\[
\Phi = \begin{bmatrix}
\phi(s_t = \bar{s}|s_{t-1} = \bar{s}) & \phi(s_t = \bar{s}|s_{t-1} = \bar{s}) \\
\phi(s_t = \bar{s}|s_{t-1} = \bar{s}) & \phi(s_t = \bar{s}|s_{t-1} = \bar{s})
\end{bmatrix} = \begin{bmatrix} 0.945 & 0.055 \\ 0.2 & 0.8 \end{bmatrix}.
\]

The idiosyncratic shock in the economy—the type of risky projects financial intermediaries invest in—is assumed to be i.i.d. to retain tractability of the numerical solution. The motivation behind the i.i.d. assumption is that the financial sector in the U.S. economy is complex and the subset of financial intermediaries who are considered the most risky changes considerably over time.

For this reason, it is difficult to determine the share of high risk financial intermediaries in the data. We set \( \pi_h \) equal to 15\% and perform sensitivity analysis with respect to this parameter. In the context of the recent financial crisis, one can think of brokers and dealers as

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\(^8\)For the corporate business sector—where income is split into capital and labor by the BEA—we find \( \theta = 0.29 \). For noncorporate businesses which include proprietors, we need to split proprietor’s income into capital and labor income in order to compute the capital income share. We attribute 0.788 percent of proprietor’s income to labor income and find a capital share for the noncorporate sector of 0.29. While 0.788 might seem high, it is not unreasonable.

\(^9\)The business cycles we identify closely mimic those determined by the NBER.
a proxy for high-risk intermediaries in the U.S. Under this assumption and using U.S. Flow of Funds data from 2000 to 2007, we find that financial assets of brokers and dealers were, on average 4% of the financial assets of all financial institutions and 20% of the financial assets of depository institutions.\textsuperscript{10} Our benchmark value of $\pi_h$ is between these two estimates. It should be noted that, while the assumption that brokers and dealers are high-risk intermediaries seems reasonable for the recent crisis, the widespread use of off-balance sheet activities among other institutions suggests that this definition may be too narrow.

Next, we determine the following 9 parameters: the importance of the nonfinancial sector, $\pi_m$, the fixed factor in the production function of the financial sector, $\alpha$, the depreciation rate, $\delta$, and the productivity parameters, $q_h(\bar{z})$, $q_h(\bar{s})$, $q_m(\bar{z})$, $q_m(\bar{s})$, $q_l(\bar{z})$, $q_l(\bar{s})$. The absolute level of productivity is not important in our model. As a result, we normalize the productivity of the high-risk intermediary in the good aggregate state, $q_h(\bar{z}) = 1$. We estimate the remaining eight parameters using eight data moments described below. Unless otherwise noted, we use quarterly data from 1987Q1 to 2010Q2. We focus on this time period because U.S. inflation was low and stable.

1. The first moment we target in our estimation procedure is the share of output produced by the nonfinancial sector. This pins down the value of $\pi_m$ in our model. We identify our model’s total output with the U.S. business sector value added published by the BEA. In addition, we identify the nonfinancial sector in our model with the U.S. corporate nonfinancial sector.\textsuperscript{11} We aim to match the average value added share of the corporate nonfinancial sector of 66.9% observed in the U.S. since 1987.

2. The parameter $\alpha$ influences the returns to equity in our model’s financial sector, which, in turn, depend on the equity to total assets ratio of the intermediaries. We use the equity

\textsuperscript{10}We note that the 20% average masks a large variation, from 18% in early 2000s to 28% in the eve of the recent crisis.

\textsuperscript{11}Note that we treat the remainder of the U.S. business sector, namely the corporate financial businesses and the noncorporate businesses, as the model’s financial intermediation sector. In U.S. data, noncorporate businesses are strongly dependent on the financial sector for funding. In the past three decades, bank loans and mortgages were 60 to 80 percent of noncorporate businesses’ liabilities. For simplicity, we do not model these loans, but rather assume that the financial intermediary is endowed with the technology of production of noncorporate businesses.
to asset ratio for corporate financial businesses as a second data moment to target in our estimation. Using data from the U.S. Flow of Funds from 1987Q1 to 2010Q2, we find this ratio to be, on average, 19.83%.

3. In our model, the depreciation rate is stochastic and is given by:

$$\frac{\pi_{mq,t}\delta k_{m,t} + (1 - \pi_m) (\pi_{hq, t}\delta k_{h, t} + \pi_{lq, t}\delta k_{l, t})}{\pi_{m}k_{m,t} + (1 - \pi_m) (\pi_{h}k_{h,t} + \pi_{l}k_{l,t})}$$

We determine the value of $\delta$ to ensure that the average depreciation rate in the model matches the data, namely 2.5% per quarter.

4. We target the maximum decline in real output in the business sector, averaged across all contraction periods since 1947. We detrend output by a constant growth trend to make it stationary. Then, using the turning points approach in Harding and Pagan (2002), we find the average decline in output to be 6.48%.

5. We aim to match a coefficient of variation for the U.S. business sector output of 3.75%. We calculate this statistic after removing a linear trend from the logarithm of output.

6. We target a coefficient of variation for U.S. household net worth of 8.17%. To obtain this statistic, we use U.S. Flow of Funds data and detrend the logarithm of household net worth using a polynomial of order three. We focus on net-worth because it is closely related to the state variable $w(s^t)$ in our model.

7. We aim to match a ratio of household deposits to total financial assets of 17.2%, as observed in U.S. Flow of Funds data.

8. Finally, we aim to match the recovery rate during bankruptcy. We use an estimate provided by Acharya, Bharath, and Srinivasan (2003), which states that, the average recovery rate on corporate bonds in the United States during 1982 to 1999 was 42 cents on the dollar.

We determine all eight parameters jointly using a minimum distance estimator to match the target moments above. Let $\Omega_i$ be a model moment and $\tilde{\Omega}_i$ be the corresponding data moment. Our procedure makes use of the problems given in (7) and (8) below. Notice that
in (7) we impose restrictions on the ordering of productivity parameters across the different technology types, as discussed in Section 2.2.

\[
Q^* = \arg \min_{Q=(q_m(\bar{s}),q_m(\bar{s}),q_l(\bar{s}),q_h(\bar{s},\delta,\alpha,\pi_m))} \sum_{i=1}^{8} \left( \frac{\Omega_i - \hat{\Omega}_i}{\hat{\Omega}_i} \right)^2
\]

s.t. : 
- \( q_h(\bar{s}) < q_m(\bar{s}) < q_l(\bar{s}) \leq q_l(\pi) < q_m(\bar{s}) \) and
- \( \Omega_i \) is implied in a competitive equilibrium given policy \( p^* \)

\[
p^* = \arg \max_p \mathbb{E} \sum_{t=0}^{\infty} \beta^t \log C(s^t)
\]

s.t. : \( \{ C(s^t) \} \) is part of a competitive equilibrium given \( Q^* \)

We start out with a guess \( Q^*_1 \) and solve the problem in (8) for an optimal policy \( p^* \). Next, we take this optimal policy as given and choose parameters to minimize the distance between our model moments and the corresponding data moments, as shown in (7). This step yields \( Q^*_2 \). We continue the procedure till convergence is achieved. The reason for choosing this two-step procedure is because our model is highly nonlinear and the initial guess is very important in finding a competitive equilibrium solution. The solution guess we start with is the social planner’s solution.

Tables 2 presents the estimated parameters. Table 3 shows that the model matches the data moments well. Notice that despite the assumption that depreciation is stochastic, the model is able to perfectly match the average depreciation observed in the data.

It should be noted that at the optimal monetary policy given the capital regulation, we find that the regulation is binding in good times and not binding in bad times. The effect of this is a suboptimally low average risk taking of \(-25.2\) per cent below the social planner level and a welfare loss of \(-0.077\) per cent relative to the social planner. Interestingly, we find that lower than optimal interest rate levels leads to more risk taking if the capital regulation is present, but to less risk taking in absence of such regulation, as shown in CSU.
4 Implementing SPP solution

In this section we will derive a particular combination of monetary policy and regulation, that is capable of implementing the Social Planner’s allocation. As explained further below we find that such implementation requires us to break the inequality $R^d(s^{t-1}) \geq \frac{1}{p(s^{t-1})}$ which holds in our calibrated economy irrespective of whether the capital adequacy constraint $z(s^{t-1})/k(s^{t-1}) \geq \eta^K$ is binding, or not (see Proposition 1). Recall that the rate of return on deposits must be at least as high as the return on government bonds, since otherwise financial intermediaries will have an incentive to borrow an unlimited quantity of deposits and then lend those funds to the government at a higher (risk-free rate). However, if returns on bonds are lower than returns on deposits, the high-risk financial intermediaries have to default in a low productivity state. Given these prospects of imminent default, they are not willing to buy government bonds from low-risk intermediaries. Instead they are betting on a high productivity state and invest into risky capital themselves, thus aggravating the moral hazard situation. To align incentives of high-risk intermediaries properly, we must somehow assure that in equilibrium $R^d(s^{t-1}) < \frac{1}{p(s^{t-1})}$. One way that works is to impose a leverage constraint $z(s^{t-1})/d(s^{t-1}) \geq \eta_L(s^{t-1})$, which effectively imposes an upper bound on deposits, given a fixed amount of equity.

We construct optimal policies implementing the SPP solution in the following way. Let \( \{C^SP_t, k^SP_{m,t}, k^SP_i, n^SP_i\} \) be a solution to the social planner’s problem. Here, \( k^SP_j \) are resources available for production for financial intermediaries before their type \( j \) is revealed. Let \( \{C_t, M_t, Z_t, D_{h,t}, T_t, l_t, k_{m,t}, k_t, b_t, z_t, d_t, x_t, p_t, \tilde{p}_t, \tilde{b}_{h,t}, \tilde{b}_{l,t}, k_{h,t}, k_{l,t}\} \) and prices \( \{R^m_t, R^z_t, R^l_t, W_{m,t}, W_{l,t}, W_{h,t}\} \) be a competitive equilibrium. Our aim is to find prices and regulatory constraints such that the social planner’s allocation is part of a competitive
equilibrium and satisfies equations (9) through (13).

\[ C_t^{SP} = C_t \]  
(9)  
\[ k_{m,t}^{SP} = k_{m,t} \]  
(10)  
\[ k_t^{SP} = \frac{Z_t + D_{h,t}}{1 - \pi_m} \]  
(11)  
\[ k_{l,t}^{SP} = x_t (z_t + d_t) + \tilde{p}_t \tilde{b}_{l,t} \]  
(12)  
\[ k_{h,t}^{SP} = x_t (z_t + d_t) + \tilde{p}_t \tilde{b}_{h,t}. \]  
(13)

**Proposition 2** An equilibrium that implements the social planner allocation has the following features:

- In a high productivity aggregate state,

\[ p_t = \frac{\sum_{s^{t+1}|s^{t}} (1 - \tau_{t+1}) \lambda_{t+1} l_{t+1} 1_{j,t+1}}{\sum_{s^{t+1}|s^{t}} (1 - \tau_{t+1}) \lambda_{t+1} l_{t+1} q_{t+1} \theta [k_{l,t}^{SP}]^{\theta - 1} + 1 - \delta} \]  
(14)

where \(1_{j,t+1}\) is an indicator function given by

\[ 1_{j,t+1}(\delta) = \begin{cases} 1 & \text{if } V_{j,t+1} > 0 \\ 0 & \text{otherwise} \end{cases} \]

the equilibrium variables determined as:

\[ x_t = \frac{k_t^{SP}}{k_{h,t}^{SP}}; \quad z_t + d_t = k_{h,t}^{SP}; \]
\[ \tilde{p}_t = p_t; \quad b_t = \frac{(1 - x_t) k_{h,t}^{SP}}{p_t} \]
\[ R_{t+1}^{m} (s_1) = q_m^1 \left[ \theta \left( k_{m,t}^{SP} \right)^{\theta-1} + 1 - \delta \right] \]
\[ R_{t+1}^{m} (s_2) = q_m^2 \left[ \theta \left( k_{m,t}^{SP} \right)^{\theta-1} + 1 - \delta \right] \]
\[ R_t^d = \frac{\lambda_{t+1}^1 R_{t+1}^{m} (s_1) + \lambda_{t+1}^2 R_{t+1}^{m} (s_2)}{\lambda_{t+1}^1 + \lambda_{t+1}^2} \]

and the binding leverage constraint determined by

\[ \zeta_t^L \eta_L (s_t) = \left\{ \theta \left[ k_{l,t}^{SP} \right]^{\theta-1} + 1 - \delta \right\} \pi_t \sum_{s_{t+1}\mid s_t} (1 - \tau_{t+1}) \lambda_{t+1} q_{l,t+1} \]
\[ + \left\{ \theta \left[ k_{h,t}^{SP} \right]^{\theta-1} + 1 - \delta \right\} \pi_h \sum_{s_{t+1}\mid s_t} (1 - \tau_{t+1}) \lambda_{t+1} q_{h,t+1} \]
\[ - R_t^d \sum_{j \in \{h,l\}} \pi_j \sum_{s_{t+1}\mid s_t} (1 - \tau_{t+1}) \lambda_{t+1} > 0 \]
\[ \eta_L (s_t) = \frac{z (s_t)}{d (s_t)}. \]

- In a low productivity aggregate state,

\[ p_t = \frac{\sum_{s_{t+1}\mid s_t} (1 - \tau_{t+1}) \lambda_{t+1} q_{h,t+1}}{\sum_{s_{t+1}\mid s_t} (1 - \tau_{t+1}) \lambda_{t+1} q_{h,t+1}} \frac{1}{\theta \left[ k_{h,t}^{SP} \right]^{\theta-1} + 1 - \delta} \]

with the equilibrium variables determined as:

\[ x_t = \frac{k_{l,t}^{SP}}{k_{l,t}^{SP}}; \quad z_t + d_t = k_{l,t}^{SP}; \]
\[ \bar{p}_t = p_t; \quad b_t = \frac{(1 - x_t) k_{l,t}^{SP}}{p_t} \]

\[ R_{t+1}^{m} (s_1) = q_m^1 \left[ \theta \left( k_{m,t}^{SP} \right)^{\theta-1} + 1 - \delta \right] \]
\[ R_{t+1}^{m} (s_2) = q_m^2 \left[ \theta \left( k_{m,t}^{SP} \right)^{\theta-1} + 1 - \delta \right] \]
\[ R_t^d = \frac{\lambda_{t+1}^1 R_{t+1}^{m} (s_1) + \lambda_{t+1}^2 R_{t+1}^{m} (s_2)}{\lambda_{t+1}^1 + \lambda_{t+1}^2} \]
and a non-binding leverage constraint

\[ \zeta^L_t \eta_L (s_t) = 0. \]

**Proof.** The results for the first half of the proposition (for high productivity state) are implied by the first order conditions stated in appendix A.2. Specifically the equation (14) is implied by the arbitrage condition (24) of a low risk intermediary in a high productivity state. The equation (15) is a direct consequence of the first-order condition (28) for optimal deposit demand, re-stated with a relaxed capital regulation condition, \( \zeta^K_t \eta_{K,t} \). The results for the low productivity state are derived analogously. ■

From our numerical simulations we find that the leverage constraint is binding in every high productivity period, \( \zeta^L_t \eta_L (s_t) > 0 \), and relaxed in every low productivity period, \( \zeta^L_t \eta_L (s_t) = 0 \). This pattern of leverage constraint is consistent with the high-risk financial intermediary going bankrupt only in the period when the aggregate state switches from high productivity state to low productivity state. The other equations stated in the Proposition above are direct results of our implementability assumption. They will become clearer as we describe the process by which we back out decentralized equilibrium implementing the SPP solution. We will start with Case 1, a high-productivity aggregate state, when it is optimal to redistribute capital to high-risk intermediaries.

### 4.1 Case 1: constraint equilibrium with redistribution of capital toward high risk FI

We compute equilibrium variables in the following sequence:

1. From (13) we know that in case 1 equilibrium

\[ k_{h,t} = x_t (z_t + d_t) + \tilde{p}_t \tilde{b}_{h,t} = x_t (z_t + d_t) + p_t b_t = z_t + d_t \]
which means that we can find \((z_t + d_t)\) as

\[
z_t + d_t = k_{h,t}^{SP}.
\]

2. From (11) we know the amount of physical capital households transfer to financial intermediaries

\[
\frac{Z_t + D_{h,t}}{(1 - \pi_m)} = k_t^{SP},
\]

which must be consistent with the total amount of resources intermediaries would like to retain in the form of capital

\[
\frac{Z_t + D_{h,t}}{(1 - \pi_m)} = x_t (z_t + d_t)
\]

which yields

\[
x_t = \frac{k_t^{SP}}{k_{h,t}^{SP}}
\]

and

\[
p_t b_t = (1 - x_t) (z_t + d_t) = (1 - x_t) k_{h,t}^{SP}.
\]

3. Then we can compute the conditional wealth values

\[
w_{t+1}^1 = \pi_m q_m^1 \left[ (k_{m,t}^{SP})^\theta + (1 - \delta) k_{m,t}^{SP} \right]
+ (1 - \pi_m) \pi_l q_l^1 \left[ (k_{l,t}^{SP})^\theta + (1 - \delta) (k_{l,t}^{SP}) \right]
+ (1 - \pi_m) \pi_h q_h^1 \left[ (k_{h,t}^{SP})^\theta + (1 - \delta) (k_{h,t}^{SP}) \right]
\]

\[
w_{t+1}^2 = \pi_m q_m^2 \left[ (k_{m,t}^{SP})^\theta + (1 - \delta) k_{m,t}^{SP} \right]
+ (1 - \pi_m) \pi_l q_l^2 \left[ (k_{l,t}^{SP})^\theta + (1 - \delta) (k_{l,t}^{SP}) \right]
+ (1 - \pi_m) \pi_h q_h^2 \left[ (k_{h,t}^{SP})^\theta + (1 - \delta) (k_{h,t}^{SP}) \right],
\]
and use those to evaluate next period conditional consumption levels, from the social planner’s policy functions:

\[ c_{t+1}^1 = c^{SP}(w_{t+1}^1, s_1) \]
\[ c_{t+1}^2 = c^{SP}(w_{t+1}^2, s_2) \]

and associated Lagrange multipliers, i.e. marginal utility weights (prices) on the next period variables:

\[ \lambda_{t+1}^1 = \frac{\phi(s_1|s_2)}{c_{t+1}^1} \]
\[ \lambda_{t+1}^2 = \frac{\phi(s_2|s_2)}{c_{t+1}^2} \]

The determination of the consumption policy functions \( c^{SP}(w_{t+1}^1, s_1) \) generally requires approximation techniques.\(^{12}\)

4. Then we used the arbitrage condition for the low-risk financial intermediary

\[
\frac{\sum_{s^t+1|s^t} (1 - \tau_{t+1}) \lambda_{t+1} 1_{l,t+1}}{\tilde{p}_t \sum_{s^t+1|s^t} (1 - \tau_{t+1}) \lambda_{t+1} 1_{l,t+1} q_{l,t+1}} = \theta \left[ x_t (z_t + d_t) + \tilde{p}_t \delta_{l,t} \right]^\theta + 1 - \delta,
\]

who is unconstrained in bond purchases, to compute the price \( \tilde{p}_t \)

\[
\tilde{p}_t = \frac{\sum_{s^t+1|s^t} (1 - \tau_{t+1}) \lambda_{t+1} 1_{l,t+1}}{\sum_{s^t+1|s^t} (1 - \tau_{t+1}) \lambda_{t+1} 1_{l,t+1} q_{l,t+1}} \frac{1}{\theta \left[ k_{l,t}^{SP} \right]^\theta - 1 + 1 - \delta}
\]

where we guess the indicator function \( 1_{l,t+1} = 1 \) in both states of the nature, which implies that the low-risk intermediary is not going bankrupt in either state. This guess will be verified later, though given the parameter restrictions on \( q \) this guess will be correct.

\(^{12}\)For the special case of \( \delta = 1 \) we are able to characterize the non-linear solution to the SP problem analytically.
5. Once we know $\tilde{p}_t$ we also know $p_t = \tilde{p}_t$, and $b_t = \frac{p_{t+1}}{p_t}$.

6. From the returns on non-financial equity

$$R^m_{t+1} (s_1) = q^1_m \left[ \theta \left( k_{m,t}^{SP} \right)^{\theta-1} + 1 - \delta \right]$$

$$R^m_{t+1} (s_2) = q^2_m \left[ \theta \left( k_{m,t}^{SP} \right)^{\theta-1} + 1 - \delta \right]$$

we can compute the arbitrage-free deposit rate

$$R^d_t = \frac{\lambda^1_{t+1} R^m_{t+1} (s_1) + \lambda^2_{t+1} R^m_{t+1} (s_2)}{\lambda^1_{t+1} + \lambda^2_{t+1}}.$$

7. Using $R^d_t$ value from the previous step we can use the financial intermediaries first-order condition from the primary bond market to calculate the product of the Lagrange multiplier $\zeta^L_t$ and of the constraint on leverage ratio $\eta^L_t$ as follows:

$$\zeta^L_t \eta^L_t = \left\{ \theta \left[ k_{l,t}^{SP} \right]^{\theta-1} + 1 - \delta \right\} \pi_l \sum_{s^{t+1} | s^t} (1 - \tau_{t+1}) 1_{t,t+1} \lambda_{t+1} q_{l,t+1}$$

$$+ \left\{ \theta \left[ k_{h,t}^{SP} \right]^{\theta-1} + 1 - \delta \right\} \pi_h \sum_{s^{t+1} | s^t} (1 - \tau_{t+1}) 1_{h,t+1} \lambda_{t+1} q_{h,t+1}$$

$$- R^d_t \sum_{j \in \{h,l\}} \pi_j \sum_{s^{t+1} | s^t} (1 - \tau_{t+1}) 1_{j,t+1} \lambda_{t+1}$$

where $1_{h,t+1}$ is guessed to be equal one in a good state and 0 in a bad state, for case 1 equilibrium. That is high intermediaries go bankrupt if the next period state is bad, and they do not go bankrupt if the next period state is good.

8. Finally, guessing a split of $Z_t + D_{ht}$ into its two components, we compute financial
equity return as follows:

\[ R_{t+1}^z z_t = \sum_{j \in \{k, l\}} (1 - \tau_{t+1}) \pi_j V_{j,t+1} \]

where

\[
\begin{align*}
V_{l,t+1} &= \max \left\{ (\theta + \alpha) q_{l,t+1} \left[ k_{l,t}^{SP} \right]^{\theta} + q_{l,t+1} (1 - \delta) k_{l,t}^{SP} \right\} + \frac{1}{\pi_t} b_t - R_t^d d_t, 0 \\
V_{h,t+1} &= \max \left\{ (\theta + \alpha) q_{h,t+1} \left[ k_{h,t}^{SP} \right]^{\theta} + q_{h,t+1} (1 - \delta) k_{h,t}^{SP} \right\} - R_t^d d_t, 0
\end{align*}
\]

and

\[
\begin{align*}
z_t &= Z_t / (1 - \pi_m) \\
d_t &= (z_t + d_t) - z_t = b_{h,t}^{SP} - z_t
\end{align*}
\]

where the state-contingent profit tax rate \( \tau_{t+1} \) is set at a constant positive rate \( \tau \) when a high productivity state \( \bar{s} \) is realized and set at a negative rate, \(-\tau\), when low productivity states are realized. This financial contribution tax/subsidy scheme helps to stabilize the inflow of private equity capital into financial intermediaries over the business cycle.

9. We calculate the difference in expected marginal returns:

\[ \sigma = R_t^d (\lambda_{l+1}^1 + \lambda_{l+1}^2) - \lambda_{l+1}^1 R_{t+1}^z (s_1) - \lambda_{l+1}^2 R_{t+1}^z (s_2) \]

If the difference is negative, it means the returns on equity dominate, returns on deposits (and returns on non-financial equity) at the guessed split \( Z_t + D_{ht} \). Searching over all possible splits \( Z_t + D_{ht} \), we find solution at which \( \sigma = 0 \).

10. Note that from the above computed portfolio, we can back out the optimal leverage
constraint: \( \eta_{t} = z_{t} / d_{t} \), which must be binding whenever the product \( \zeta_{t}^{L} \eta_{t}^{L} \) is strictly positive, as found in the step 7 above.

11. We check that our guess regarding bankruptcy patterns is correct.

4.2 **Case 2: constraint equilibrium with redistribution of capital toward low risk FI**

The computation procedure is symmetric to the case 1, except we note that

\[
z_{t} + d_{t} = k_{t, t}^{SP}
\]

in this equilibrium, plus we assume that neither high-risk, nor low-risk intermediaries go bankrupt in either state.

5 **Results**

To better understand the model, we conduct a number of experiments, in which we consider the behavior of the economy over the business cycle.

Our benchmark economy is one with the capital adequacy ratio constrained to be above 0.08 and a monetary policy that is optimized given this regulation. We conduct two main experiments to analyze an environment with capital adequacy constraint. First, we vary the interest rate policy by shifting the policy functions uniformly higher or lower relative to the optimized function. Second, we adjust \( \eta^{CAR} \) between zero and 50 per cent. In the comparisons we use welfare and risk taking from the social planner’s solution as reference points.

5.1 **What does capital adequacy regulation do?**

We find the following:
• A binding CAR lowers the average level of risk taking at the expense of welfare (Figure 1)

• Lowering the interest rate below the optimal level increases risk taking in the presence of a binding CAR (Figure 1)

• The optimal level of $\eta^{CAR}$ is zero, though there exists a local maximum at $\eta^{CAR} = 0.38$ (Figure 2)

5.2 Implementation of the SPP solution with leverage constraints

Next, we consider the properties of the optimal policy mix and how deviations from the optimal mix affect risk taking and welfare in the economy.

• Leverage constraint binds in good states and relaxed in bad states (Figure 5).

• The required equity-to-deposit ratio is high in good state (about 24 per cent), (Figure 5).

• A 2 per cent Financial Contribution Tax on FIs profits, applied only in good productivity states, is needed to implement SPP solution. This tax helps to curb the implicit subsidy to FIs in the form of free deposit insurance. In addition the revenue from this tax is used to finance a 2 per cent subsidy (as a fraction of profits) to solvent financial institutions. This tax/subsidy scheme helps to stabilize the supply of private equity to financial institutions over the business cycle.

• The required leverage ratio is mildly procyclical in good states and completely relaxed in bad states.

• Risk taking goes up with interest rates. Higher than optimal interest rates are more welfare costly than lower interest rates (Figure 3).
• Having a too low level of regulation leads to excessive risk taking and welfare losses (Figure 4).

• Interest rates and leverage constraints complement each other. A higher level of interest rates should also be accompanied by a tighter leverage constraint to alleviate welfare losses.

Comment: The problem can be solved analytically for the case of $\delta = 1$.

6 Conclusion

The paper characterizes the optimal combination of monetary policy and financial regulation in a quantitative infinite horizon model with a risk taking channel of monetary policy. The model economy is rich enough to match main characteristics of the U.S. economy and its financial sector, yet tractable enough to deliver clear prescriptions regarding the optimal policy mix. For the special case of full depreciation of capital the optimal policy mix can be characterized analytically.

The optimal monetary policy is similar to what central bank do in practice through its needs to be augmented by a state contingent leverage regulation and a small financial contribution tax (FCT) on financial intermediaries’ profits. During high productivity periods the FCT is levied to reduce the implicit subsidy to financial intermediaries due to the presence of the free deposit insurance. During economic downturns part of the revenue derived from this tax is used to finance a subsidy to solvent financial intermediaries. This tax/subsidy scheme helps to stabilize the supply of private equity capital to financial institutions over the business cycle. When deviating from the optimal policy, leverage regulation and monetary policy act as complements. Standard capital-adequacy regulation is welfare decreasing though effective at reducing risk taking.
References


A Appendix

A.1 Computation of Equilibrium

We compute a recursive formulation of the model, where the state variables at each time period are the aggregate state, $s_t$, and the household wealth, $w_t$. Our strategy is to solve for consumption as a function of the state variables using a collocation method with linear spline functions. To improve the accuracy and the speed of the computation, we use of the endogenous grid method idea of Carroll (2006).

We separate the household problem into two parts: a portfolio choice problem and an intertemporal problem. The household’s portfolio choice allocates resources to the nonfinancial and financial sectors to equate expected returns of investing in these sectors. Then, given the overall resources allocated to the financial sector, the split between equity and deposits is determined to equalize expected returns from the two types of investments (for details, see Carroll (2011), Section 7 on multiple control variables).

There are two main challenges when solving the financial sector problem. First, some financial intermediaries may be constrained in their secondary market trades and, second, financial intermediaries may go bankrupt when the aggregate state is realized. We consider all the possible combinations in sequence and verify which is an equilibrium. For example, an ex-ante assumption that we may make is that when the bad aggregate state occurs, high risk intermediaries are constrained in their secondary market trade and go bankrupt, while the low risk intermediaries are unconstrained and do not go bankrupt. After solving the financial intermediaries’ problems, we check whether the ex-post outcome is consistent with the assumed ex-ante behavior.

A.2 Sketch of Proof for Proposition 1

To simplify notation in our derivations, we use subscripts as a short hand notation for the entire history, $s^{t-1}$. For example, $\tilde{b}_{j,t-1} \equiv \tilde{b}_j (s^{t-1})$ and $b_{t-1} \equiv b (s^{t-1})$. 
Deriving the relationship between primary and secondary bond prices in our model involves studying three possible outcomes on the secondary bond market. Transactions of bonds either satisfy: (i) $b_{j,t-1} < b_{t-1}$ for both $j \in \{h, l\}$ or (ii) $\tilde{b}_{h,t-1} = b_{t-1}$ and $\tilde{b}_{l,t-1} < b_{t-1}$ or (iii) $\tilde{b}_{l,t-1} = b_{t-1}$ and $\tilde{b}_{h,t-1} < b_{t-1}$. Here, we sketch the proof of Proposition 1 for case (ii). The proof is obtained in an analogous fashion for cases (i) and (iii) and is omitted here for brevity.\(^\text{13}\)

In case (ii), the high-risk intermediary increases the amount of resources allocated to risky investments by selling all bond holdings in the secondary bond market.

**Step 1: Some Key Relationships**

In finding and characterizing the equilibrium, it is useful to define the share of resources a financial intermediary retains for risky investment in the primary market, call it $x_{t-1}$. Then,

\[
\begin{align*}
    k_{t-1} &= x_{t-1} (z_{t-1} + d_{t-1}) \\
    b_{t-1} &= \frac{1 - x_{t-1}}{p_{t-1}} (z_{t-1} + d_{t-1})
\end{align*}
\]

where the second equation was obtained from equation (1).

For the case presented here, high-risk intermediaries use all their bonds as collateral in the secondary market, while low-risk intermediaries give resources against this collateral. We have:

\[
\begin{align*}
    \tilde{b}_{h,t-1} &= b_{t-1} = \frac{1 - x_{t-1}}{p_{t-1}} (z_{t-1} + d_{t-1}) \\
    \tilde{b}_{l,t-1} &= \frac{-\pi_h}{\pi_l} b_{t-1} = \frac{-\pi_h}{\pi_l} \frac{1 - x_{t-1}}{p_{t-1}} (z_{t-1} + d_{t-1})
\end{align*}
\]

Lastly, using equations (17) – (20), the resources allocated to risky investments by high-risk and low-risk intermediaries after the secondary market trades are given by (21) and

\(^{13}\)The full derivation is available upon request from the authors.
(22).

\[ k_{t-1} + \tilde{p}_{t-1} \tilde{b}_{h,t-1} = \left[ x_{t-1} + \frac{\tilde{p}_{t-1}}{p_{t-1}} (1 - x_{t-1}) \right] (z_{t-1} + d_{t-1}) \]  

(21)

\[ k_{t-1} + \tilde{p}_{t-1} \tilde{b}_{t-1} = \left[ x_{t-1} - \frac{\pi_h \tilde{p}_{t-1}}{\pi_t p_{t-1}} (1 - x_{t-1}) \right] (z_{t-1} + d_{t-1}) \]  

(22)
Step 2: Equilibrium Conditions for the Financial Sector

In what follows, we make use of the equilibrium result $l_{t-1} = 1$.

We rewrite the secondary market problem given in $(P2)$ as below:

$$
\max_{\tilde{b}_{j,t-1}} \sum_{s^t|s^{t-1}} (1 - \tau_t) \left( q_{j,t} \left[ \left( k_{t-1} + b_{j,t-1} \tilde{b}_{j,t-1} \right)^{\theta} + (1 - \delta) \left( k_{t-1} + \tilde{b}_{j,t-1} \right) \right] \right)

+ \left( b_{t-1} - \tilde{b}_{j,t-1} \right) - R_{t-1}^{s_{t-1}} d_{t-1} - W_{j,t}

$$

where $\tilde{b}_{j,t-1} \in \left[ \frac{k_{t-1}}{\tilde{p}_{t-1}}, b_{t-1} \right]$ and $1_{j,t}$ is an indicator function given by $1_{j,t} \equiv \begin{cases} 1 & \text{if } V_{j,t} > 0 \\ 0 & \text{otherwise} \end{cases}$.

The first order conditions with respect to bond trades, $\tilde{b}_{h,t-1}$ and $\tilde{b}_{l,t-1}$, are given by:

$$
\sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{j,t} \lambda_t \left\{ q_{j,t} \tilde{p}_{t-1} \left[ \theta \left( k_{t-1} + \tilde{p}_{t-1} \tilde{b}_{j,t-1} \right)^{\theta-1} + 1 - \delta \right] - 1 \right\} - \mu_{j,t-1} = 0 \quad (23)
$$

where $\mu_{j,t-1}$ for $j \in \{h, l\}$ are the Lagrange multipliers on the constraints $\tilde{b}_{j,t-1} \leq b_{t-1}$ and they satisfy the complimentary slackness conditions: $\mu_{j,t-1} \geq 0$, $\mu_{j,t-1} \left( b_{t-1} - \tilde{b}_{j,t-1} \right) = 0$.

Notice that for the case we are analyzing here, $\mu_{l,t-1} = 0$ and $\mu_{h,t-1} \geq 0$. Using this, along with the expressions in (21) and (22), we can rewrite equation (23) for $j \in \{h, l\}$ as (24) and (25) below:

$$
\theta \left[ \left( x_{t-1} - \frac{\pi_h}{\pi_t} \tilde{p}_{t-1} \right) (1 - x_{t-1}) \right] \left( z_{t-1} + d_{t-1} \right)^{\theta-1} + 1 - \delta = \frac{\sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{l,t} \lambda_t}{\sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{l,t} \lambda_t q_{l,t} \tilde{p}_{t-1}} \quad (24)
$$

$$
\theta \left[ \left( x_{t-1} + \frac{\tilde{p}_{t-1}}{p_{t-1}} (1 - x_{t-1}) \right) (z_{t-1} + d_{t-1}) \right]^{\theta-1} + 1 - \delta \geq \frac{\sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{h,t} \lambda_t}{\tilde{p}_{t-1} \sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{h,t} \lambda_t q_{h,t}} \quad (25)
$$

\[14\] In equilibrium, the constraint $\frac{k_{t-1}}{\tilde{p}_{t-1}} \leq \tilde{b}_{j,t-1}$ does not bind as returns to capital invested in risky projects would become infinite.
Notice that equation (24) can be equivalently written as:

\[
\left[ x_{t-1} - \frac{\pi_h \tilde{p}_{t-1}}{\pi_l p_{t-1}} (1 - x_{t-1}) \right] (z_{t-1} + d_{t-1}) = \left[ \frac{1}{\theta} \left( \sum_{s|s^{t-1}} (1 - \tau_t) 1_{t,t} \lambda_t \left( \sum_{s|s^{t-1}} (1 - \tau_t) 1_{t,t} \lambda_t q_{t,t} \tilde{p}_{t-1} - 1 + \delta \right) \right) \right]^{1/\theta}
\]

(26)

Using equations (17) – (22) we rewrite the primary market problem (P1) as below:

\[
\max_{x_{t-1} \in [0,1]} \sum_{j \in \{h,f\}} \sum_{s|s^{t-1}} (1 - \tau_t) \lambda_t V_{j,t} \\
\text{subject to :}
\]

\[
V_{l,t} = \max \left\{ q_{l,t} \left[ \left( x_{t-1} - \frac{\pi_h \tilde{p}_{t-1}}{\pi_l p_{t-1}} (1 - x_{t-1}) \right) (z_{t-1} + d_{t-1}) \right]^\theta + q_{l,t} (1 - \delta) \left( x_{t-1} - \frac{\pi_h \tilde{p}_{t-1}}{\pi_l p_{t-1}} (1 - x_{t-1}) \right) (z_{t-1} + d_{t-1}) + \frac{1}{\pi_l} (1 - x_{t-1}) (z_{t-1} + d_{t-1}) - R_{t-1}^d d_{t-1} - W_{l,t}, \ 0 \right\}
\]

\[
V_{h,t} = \max \left\{ q_{h,t} \left[ \left( x_{t-1} + \frac{\tilde{p}_{t-1}}{p_{t-1}} (1 - x_{t-1}) \right) (z_{t-1} + d_{t-1}) \right]^\theta + q_{h,t} (1 - \delta) \left( x_{t-1} + \frac{\tilde{p}_{t-1}}{p_{t-1}} (1 - x_{t-1}) \right) (z_{t-1} + d_{t-1}) - R_{t-1}^d d_{t-1} - W_{h,t}, \ 0 \right\}
\]

\[
z_{t-1} - \eta_{K,t-1} x_{t-1} (z_{t-1} + d_{t-1}) \geq 0
\]

\[
z_{t-1} - \eta_{L,t-1} d_{t-1} \geq 0
\]

Let \( \zeta^K_{t-1} \) be the Lagrange multiplier on the capital regulation constraint and \( \zeta^L_{t-1} \) be the Lagrange multiplier on the leverage constraint. The first order conditions with respect to
$x_{t-1}$ and $d_{t-1}$ are given by (27) and (28), respectively.\footnote{In order to obtain equation (28), we derive the first order condition with respect to deposits and simplify it by using the expression in (27).}

\begin{equation}
\frac{1}{p_{t-1}} \sum_{s^t|s^{t-1}} (1 - \tau_t) \lambda_t 1_{t,t} \tag{27}
\end{equation}

\begin{align*}
= & \left\{ \theta \left[ (x_{t-1} - \frac{\pi_h \tilde{p}_{t-1}}{\pi_l \tilde{p}_t} (1 - x_{t-1}) \right) (z_{t-1} + d_{t-1}) \right]^{\theta-1} + 1 - \delta \right\} \left( 1 + \frac{\pi_h \tilde{p}_{t-1}}{\pi_l \tilde{p}_t} \right) \pi_l \sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{t,t} \lambda_t \eta_{t} \\
+ & \left\{ \theta \left[ (x_{t-1} + \frac{\tilde{p}_{t-1}}{\tilde{p}_t} (1 - x_{t-1}) \right) (z_{t-1} + d_{t-1}) \right]^{\theta-1} + 1 - \delta \right\} \left( 1 - \frac{\tilde{p}_{t-1}}{\tilde{p}_t} \right) \pi_h \sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{t,t} \lambda_t \eta_{t} \right\} \tag{27} \\
\end{align*}

\begin{equation}
R_{d,t-1} \sum_{j \in \{h,t\}} \pi_j \sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{j,t} \lambda_t \tag{28}
\end{equation}

\begin{align*}
= & \left\{ \theta \left[ (x_{t-1} - \frac{\pi_h \tilde{p}_{t-1}}{\pi_l \tilde{p}_t} (1 - x_{t-1}) \right) (z_{t-1} + d_{t-1}) \right]^{\theta-1} + 1 - \delta \right\} \pi_l \sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{t,t} \lambda_t \eta_{t} \\
+ & \left\{ \theta \left[ (x_{t-1} + \frac{\tilde{p}_{t-1}}{\tilde{p}_t} (1 - x_{t-1}) \right) (z_{t-1} + d_{t-1}) \right]^{\theta-1} + 1 - \delta \right\} \pi_h \sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{t,t} \lambda_t \eta_{t} \right\} \tag{27} \\
\end{align*}

Step 3: Bond Prices

Using (26), we rewrite the equilibrium condition for the choice of $x_{t-1}$, equation (27), as below:

\begin{align*}
\left( \frac{1}{\tilde{p}_{t-1}} - \frac{\pi_l}{\tilde{p}_t} \right) \left( 1 + \frac{\pi_h \tilde{p}_{t-1}}{\pi_l \tilde{p}_t} \right) \sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{t,t} \lambda_t \\
= & \left\{ \theta \left[ (x_{t-1} + \frac{\tilde{p}_{t-1}}{\tilde{p}_t} (1 - x_{t-1}) \right) (z_{t-1} + d_{t-1}) \right]^{\theta-1} + 1 - \delta \right\} \left( 1 - \frac{\tilde{p}_{t-1}}{\tilde{p}_t} \right) \pi_h \sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{t,t} \lambda_t \eta_{t} \\
\end{align*}
Using \( \pi_t + \pi_h = 1 \), we can simplify the left hand side of the above equation and write it equivalently as:

\[
\left( 1 - \frac{\tilde{p}_{t-1}}{p_{t-1}} \right) \cdot \Xi - \zeta^K_{t-1} \eta_{K,t-1} = 0 \tag{29}
\]

\[
\Xi = \left\{ \theta \left[ \left( x_{t-1} + \frac{\tilde{p}_{t-1}}{p_{t-1}} (1 - x_{t-1}) \right) \left( z_{t-1} + d_{t-1} \right) \right]^{\theta-1} + 1 - \delta \right\} \pi_h \sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{h,t}\lambda_t q_{h,t} \\
+ \frac{\pi_l}{\tilde{p}_{t-1}} \sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{l,t}\lambda_t \\
+ \zeta^L_{t-1} \eta_{L,t-1}
\]

Notice that \( \Xi > 0 \), unless all financial intermediaries go broke. Then, equation (29) implies that, in the absence of capital regulation or if the capital regulation does not bind (i.e. \( \eta_{K,t-1} = 0 \) or \( \zeta^K_{t-1} = 0 \), the primary and secondary market bond prices are equated, \( \tilde{p}_{t-1} = p_{t-1} \). However, if \( \eta_{K,t-1} > 0 \) and capital regulation binds \( \zeta^K_{t-1} > 0 \), then equation (29) implies that \( \tilde{p}_{t-1} < p_{t-1} \).

**Step 4: Primary Market Bond Price and Return to Deposits**

We combine equations (27) and (28) to eliminate the term \( \zeta^K_{t-1} \eta_{K,t-1} \). We find:

\[
\frac{1}{p_{t-1}} \sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{j,t}\lambda_t - R^d_{t-1} \sum_{j \in \{h,l\}} \pi_j \sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{j,t}\lambda_t \\
= \left\{ \theta \left[ \left( x_{t-1} - \frac{\pi_h}{\pi_l} \frac{\tilde{p}_{t-1}}{p_{t-1}} (1 - x_{t-1}) \right) \left( z_{t-1} + d_{t-1} \right) \right]^{\theta-1} + 1 - \delta \right\} \pi_h \sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{h,t}\lambda_t q_{h,t} \\
- \left\{ \theta \left[ \left( x_{t-1} + \frac{\tilde{p}_{t-1}}{p_{t-1}} (1 - x_{t-1}) \right) \left( z_{t-1} + d_{t-1} \right) \right]^{\theta-1} + 1 - \delta \right\} \frac{\tilde{p}_{t-1}}{p_{t-1}} \pi_h \sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{h,t}\lambda_t q_{h,t} \\
+ \zeta^L_{t-1} \eta_{L,t-1}
\]

Using (24) and (25), equation (30) becomes

\[
R^d_{t-1} \sum_{j \in \{h,l\}} \pi_j \sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{j,t}\lambda_t \geq \frac{1}{p_{t-1}} \sum_{j \in \{h,l\}} \pi_j \sum_{s^t|s^{t-1}} (1 - \tau_t) 1_{j,t}\lambda_t - \zeta^L_{t-1} \eta_{L,t-1} \tag{31}
\]
Thus, if the leverage constraint is not binding (i.e. $\eta_{t-1} = 0$ or $\zeta_{t-1}^L = 0$), we have $R_{t-1}^d \geq \frac{1}{p_{t-1}}$. However, if $\eta_{t-1} > 0$ and leverage regulation binds $\zeta_{t-1}^L > 0$, then equation (31) does not restrict $R_{t-1}^d$ to be weakly above $\frac{1}{p_{t-1}}$. This completes the proof of Proposition 1 for the case in which the high-risk intermediary sells all bonds in the secondary bond market. The other cases are derived analogously, but are omitted here to keep the exposition short.
## B Tables

**Table 1: Calibrated Parameters**

<table>
<thead>
<tr>
<th>Parameter/Value</th>
<th>Moment$^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = \left(\frac{1}{1.04}\right)^{1/4}$</td>
<td>Real interest rate of 4 percent</td>
</tr>
<tr>
<td>$\theta = 0.29$</td>
<td>Capital income share</td>
</tr>
<tr>
<td>$\eta = 0.08$</td>
<td>Capital adequacy requirement</td>
</tr>
<tr>
<td>$\Phi = \begin{bmatrix} 0.945 &amp; 0.055 \ 0.20 &amp; 0.80 \end{bmatrix}$</td>
<td>Average length of expansions/contractions of business sector</td>
</tr>
<tr>
<td>$\pi_l = 0.85, \pi_h = 1 - \pi_l = 0.15$</td>
<td>Sensitivity analysis</td>
</tr>
</tbody>
</table>

$^1$See Section 3 for details on the sources of data.
Table 2: Estimated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The following parameters are determined jointly to match the moments in Table 3</td>
<td></td>
</tr>
<tr>
<td>Share of nonfinancial firms</td>
<td>$\pi_m = 0.697$</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta = 0.0270$</td>
</tr>
<tr>
<td>Fixed factor income share</td>
<td>$\alpha = 0.0004$</td>
</tr>
<tr>
<td>Productivity parameters</td>
<td></td>
</tr>
<tr>
<td>nonfinancial firms</td>
<td>$q_m(\bar{s}) = 0.9663$</td>
</tr>
<tr>
<td></td>
<td>$q_m(\underline{s}) = 0.9381$</td>
</tr>
<tr>
<td>low-risk financial firms</td>
<td>$q_l(\bar{s}) = 0.9384$</td>
</tr>
<tr>
<td></td>
<td>$q_l(\underline{s}) = 0.9383$</td>
</tr>
<tr>
<td>high-risk financial firms</td>
<td>$q_h(\bar{s}) = 1$ (normalization)</td>
</tr>
<tr>
<td></td>
<td>$q_h(\underline{s}) = 0.3686$</td>
</tr>
<tr>
<td>Moment</td>
<td>Data in %</td>
</tr>
<tr>
<td>--------</td>
<td>-----------</td>
</tr>
<tr>
<td>Coefficient of variation of output(^1)</td>
<td>3.75</td>
</tr>
<tr>
<td>Coefficient of variation of household net worth(^2)</td>
<td>8.17</td>
</tr>
<tr>
<td>Average maximum decline in output during contractions(^3)</td>
<td>6.48</td>
</tr>
<tr>
<td>Average deposits over total household financial assets(^2)</td>
<td>17.2</td>
</tr>
<tr>
<td>Recovery rate in case of bankruptcy(^4)</td>
<td>42.0</td>
</tr>
<tr>
<td>Mean output share of corporate nonfinancial sector</td>
<td>66.9</td>
</tr>
<tr>
<td>Average capital depreciation rate in economy</td>
<td>2.5</td>
</tr>
<tr>
<td>Equity to asset ratio of the financial sector(^2)</td>
<td>19.8</td>
</tr>
</tbody>
</table>

\(^1\)Output is measured as the value added for the business sector from 1987Q1 to 2010Q2. This reference period is used for the other moments as well, unless otherwise stated. \(^2\)Data on household net worth, deposits, equity and financial assets are from the U.S. Flow of Funds accounts. \(^3\)The decline in output during contractions takes the growth trend into account. \(^4\)The recovery rate in bankruptcy is from Acharya, Bharath, and Srinivasan (2003).
Table 4: **Calibration Welfare and Risk Taking Relative to the Social Planner**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>LTCE(^2) in %</th>
<th>Risk taking(^3) in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark calibration ((\eta^{CAR} = 0.08))</td>
<td>-0.108</td>
<td>-27.9</td>
</tr>
<tr>
<td>Benchmark with (\eta^{CAR} = 0)</td>
<td>-0.027</td>
<td>-3.6</td>
</tr>
<tr>
<td>Optimal Policy Mix</td>
<td>0.000</td>
<td>0.0</td>
</tr>
<tr>
<td>Optimal Policy Mix –0.5 percentage points</td>
<td>-0.018</td>
<td>-12.8</td>
</tr>
<tr>
<td>Optimal Policy Mix +0.5 percentage points</td>
<td>-0.040</td>
<td>21.0</td>
</tr>
<tr>
<td>Optimal Policy Mix (\eta^{LEV} = 0)</td>
<td>-0.102</td>
<td>-27.5</td>
</tr>
</tbody>
</table>

\(^1\)The statistics are averages over 5000 simulations of 1000 periods each of the model economy and the social planner’s problem. \(^2\)Lifetime Consumption Equivalents (LTCE) is the percentage decrease in the optimal consumption from the social planner problem needed to generate the same welfare as the competitive equilibrium with a given interest rate policy. \(^3\)Risk taking is the percentage deviation in the amount of resources invested in the high-risk projects in the competitive equilibrium relative to the social planner’s choice. The numbers reported here are averages over expansions and contractions in our calibrated model. A positive number indicates too much risk taking, on average, relative to the social planner, while a negative number indicates less risk taking.
Table 5: Sensitivity Analysis for Fraction of High Risk Intermediaries
Welfare and Risk Taking Results Relative to the Social Planner

<table>
<thead>
<tr>
<th>LTCE(^2) in %</th>
<th>0.13</th>
<th>0.15</th>
<th>0.17</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment / (\pi_h) value</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimal Policy Mix -0.5 percentage points</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimal Policy Mix</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimal Policy Mix +0.5 percentage points</td>
<td></td>
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<th>Risk taking(^3) in %</th>
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<td>Experiment / (\pi_h) value</td>
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<td>Optimal Policy Mix -0.5 percentage points</td>
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<td>Optimal Policy Mix</td>
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<td>Optimal Policy Mix +0.5 percentage points</td>
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\(^1\)The statistics are averages over 500 simulations of 750 periods each of the model economy and the social planner’s problem. \(^2,^3\)See definitions given in notes to Table 4.
C Figures

Figure 1: Simulation Results: Benchmark model compared to pure IRP model
Figure 2: Simulation results: Benchmark model with $\eta^{CAR}$ variation

Figure 3: Simulation results: Optimal Mix compared to BM

Figure 4: Simulation results: Optimal Mix with $\eta^{LEV}$ variation

47
Figure 5: Simulation Results: Model with Optimal Policy Mix
Figure 6: Simulation Results: Model with Lower Than Optimal Interest Rate Policy