The Inheritance of Advantage

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Abstract

Some agents are better treated by the market than others. In our model this arises from statistical discrimination based on the observables on the background of an individual. Advantages thus created increase the intergenerational correlation of income. This has some strong implications. First, it implies that intergenerational mobility and income inequality should correlate negatively. Second, the amplification mechanism generated by advantages may produce a multiplicity of steady states. Third, the introduction of “meritocracy” (informative signals on talent) may actually decrease mobility due to general equilibrium effects: by increasing income dispersion, they also increase the value of background.

1 Introduction

“We know all men are not created equal in the sense some people would have us believe - Some people are smarter than others, some people have more opportunity because they’re born with it, some men make more money than others, some ladies make better cakes than others - some men are born gifted beyond the normal scope of most men.’ ” ~ Atticus Finch, To Kill a Mockingbird (Harper Lee, 1960)

The correlation between intergenerational income mobility and inequality was recently considered by Alan Krueger in a presentation he gave to the Center for American Progress (Krueger (2012)). He discussed the “Great Gatsby Curve,” the literary namesake of which provides an example, of sorts, of upward mobility. Krueger’s illustration drew on data provided by Miles Corak and Figure 1 reproduces Corak’s version of the Great Gatsby Curve (Corak (2012)). Income inequality and the intergenerational earnings elasticity are
on the x- and y-axes respectively. We can see that more unequal countries have a greater correlation between the income of fathers and sons or, put another way, that there is a negative correlation between income inequality and mobility.

There exist several explanations for this negative correlation in the literature. One such explanation is given by the “distance effect” in Hassler et al. (2007). In their model, greater mobility leads to a long-run equilibrium in which there are fewer unskilled workers relative to skilled. This reduces the distance between their two incomes, which is their measure of income inequality, and increases the ability of unskilled parents to pay for the eduction of their children. This feeds back into increased mobility. Another explanation is provided in Solon (2004). In Solon’s model, both intergenerational income elasticity and inequality are a function of the same factors, including the inheritance of income generating traits and more policy-related factors such as the progressivity of public human capital investment. This would again give rise to the upward sloping line illustrated in figure 1. This paper will offer an alternative explanation – the inheritance of advantage.

To be clear, we are not talking, as Solon did, about the greater opportunity of the children of the rich to accumulate talent. We assume that this is true, perhaps through some sort of capital market imperfection allowing the rich to invest more in their children’s development. We will look specifically at the underlying investment process in chapter 3. What we are talking about is the greater efficacy with which the children of the rich inherit the ability to look talented. This relates our paper to the literature on statistical discrimination (such as Coate & Loury (1993) and Moro & Norman (2004)). We differ from these papers in that our firms discriminate based on endogenously determined variables and do so within a dynamic model. This naturally produces the observed negative correlation between mobility and inequality. As income inequality increases, people differ more, and
it becomes easier to identify talented individuals within society. As firms become more certain about who is talented and who is not, this feeds back into income dispersion. This feedback mechanism may lead to multiplicity of steady states. In addition, the better firms get at identifying talent, the lower is mobility, both because the talented tend to be from rich backgrounds and because one of the ways which firms identify talent is through information on family background.

These are the two key points which we wish to highlight in this paper. First, that there exists a multiplier effect whereby increased inequality improves the information available to firms when selecting workers, reducing mobility and encouraging further inequality in the incomes that they pay. When there is strong inheritability, in the sense that talent is closely related to family background, this can lead to multiplicity. Second, that it implies a perverse effect of meritocracy: if a meritocratic society is one in which firms can more readily identify the quality of workers and pay them accordingly, this could decrease mobility. As a result, societies with meritocratic institutions (those in which it is easy to signal talent) should tend to have more inequality and less mobility than those without. This is what we observe in the likes of the US and UK, with a hierarchy of higher education institutions from the Ivy League and Oxbridge downwards, compared to the Scandinavian countries with their more egalitarian educational institutions. We will elaborate further on these points in the models which follow.

2 Labour market sorting where firms receive a signal on parental income

In this section we will consider how firms react to a signal on the parental income of a worker. This is going to be relevant to the firm because parental income and talent will be related in the following way,

\[
\tau = \alpha (y_{-1} - \bar{y}_{-1}) + \epsilon_{\tau}
\]

(1)

where \(\tau\) is a worker’s talent, \(y_{-1}\) is his parent’s income, \(\bar{y}_{-1}\) is the mean of parental income, and \(\epsilon_{\tau} \sim N(0, \sigma_{\epsilon_{\tau}}^2)\). The talent process is a function of parental income and luck, where the extent of the role played by parental income is governed by the parameter \(\alpha > 0\). Talent is the post-education, pre-labour force level of skill of a worker, and its development has a tournament component which will keep average talent centered on zero. We do not consider the factors underlying this relationship in this chapter but you could imagine it being the result of capital market imperfections.
2.1 A firm’s beliefs about talent

There are firms which pay wages according to their beliefs about the talent an individual has. Let $\theta_1$ be the information available to a firm. $\theta_1$ can be broken down into two elements: $\mu$, information on the distribution of income in the parents’ generation; and $s_1$, a signal on $y_{-1}$ given by,

$$s_1 = y_{-1} + \epsilon_{s_1}$$

where $\epsilon_{s_1} \sim N\left(0, \sigma_{\epsilon_{s_1}}^2\right)$. This signal can be thought of as an observation on an individual’s background: where they grew up, how they speak, how they dress, and so on. Firms do not, at this stage, receive a direct signal on talent. We assume that the firm knows the distributions of $\epsilon_{s_1}$ and $\epsilon_\tau$.

Using this information, the firm takes expectations of talent, conditional on the information available, in the following way,

$$E(\tau|\theta_1) = \alpha E(y_{-1} - \bar{y}_{-1}|\theta_1) + E(\epsilon_\tau|\theta_1)$$

Since both $\mu$ and $s_1$ are independent of $\epsilon_\tau$, the firm’s expectation of $\epsilon_\tau$ conditional on $\theta_1$ will be equal to zero and the firm can concentrate on forming beliefs on parental income. To do this they initially form their prior, conditional on $\mu$, and then update it with the information contained in an individual’s signal using Bayesian inference. This leads to an expectation of parental income (as a deviation from its mean) given by,

$$E\left(y_{-1} - \bar{y}_{-1}|\theta_1\right) = \frac{\sigma_{y_{-1}}^2}{\sigma_{y_{-1}}^2 + \sigma_{s_1}^2} (s_1 - \bar{s}_1)$$

Substituting this back into equation 3 gives a posterior belief on talent of,

$$E(\tau|\theta_1) = \frac{\alpha \sigma_{y_{-1}}^2}{\sigma_{y_{-1}}^2 + \sigma_{s_1}^2} (s_1 - \bar{s}_1)$$

$\frac{\alpha \sigma_{y_{-1}}^2}{\sigma_{y_{-1}}^2 + \sigma_{s_1}^2}$ is equal to $\frac{\sigma_{\tau, s_1}}{\sigma_{s_1}^2}$ which is the coefficient from an OLS regression of $\tau$ on $s_1$\textsuperscript{1}. Thus the firm’s adjustment away from their prior belief (zero) and towards the signal is to the extent that a change in the signal implies a change in talent. When parental income variance

\textsuperscript{1}Since $\tau$ and $s_1$ are bivariate normal it can also be read directly from the PDF of the conditional distribution of $\tau$. 

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is high, the prior gives little information as people are very different. The signal is more heavily used in determining the posterior and hence talent. When parental income variance is zero, the prior gives perfect information and the signal is disregarded. The point, as we will see in the next section, is that income variance is endogenous.

### 2.2 Steady State Beliefs

Suppose that firms set income equal to their belief about an individual’s talent having observed their signal. They have no further opportunity to learn about the talent of their worker. We will write this income as a linear function of the deviation of the signal from its mean value.

\[ y = E(\tau|\theta_1) = \beta_1 (s_1 - \bar{s}_1) \]

where \( \beta_1 \) can be read from equation 4. Since mean income is then zero in every generation, this implies that the mean signal is zero. We can then write this equation as

\[ y = E(\tau|\theta_1) = \beta_1 s_1 \] (5)

\( \beta_1 \) measures the optimal reaction of the firm to the signal. The more the firms react to the signals, the more income variance there is. But, as we saw above, the more income variance there is, the more value there is in the signal relative to the prior and so the more the firms react to the signal. \( \beta_1 \) both causes and is a reaction to income variance.

By substitution from equation 2 we can calculate the variance of income as a function of the variance in parental income and, by setting \( \sigma_y^2 = \sigma_{y_{-1}}^2 \), the steady state income variance. This is given by,

\[ \sigma_y^2(\beta_1) = \begin{cases} \frac{\beta_1^2 \sigma_{\epsilon s_{-1}}^2}{1 - \beta_1^2} & \text{if } \beta_1 < 1 \\ \infty & \text{if } \beta_1 \geq 1 \end{cases} \]

We can see that, at least up to a point, \( \sigma_y^2 \) is a function of \( \beta_1 \). We also know from equation 4 that \( \beta_1 = \frac{\alpha \sigma_y^2}{\sigma_y^2 + \sigma_{\epsilon s_{-1}}^2} \) or, more generally, \( \beta_1 \) is a function of \( \sigma_y^2 \) (which we shall call \( F(\cdot) \)). Finding a steady state value of \( \beta_1 \) is thus a matter of finding a fixed point of the equation \( \beta_1 = F(\sigma_y^2(\beta_1)) \) which is given by,
Figure 2: Steady State for $\alpha < 1$

\[
\beta_1 = F\left(\sigma_y^2(\beta_1)\right) = \frac{\alpha \sigma_y^2(\beta_1)}{\sigma_y^2(\beta_1) + \sigma_{\epsilon_1}^2} = \begin{cases} 
\alpha \beta_1^2 & \text{if } \beta_1 < 1 \\
\alpha & \text{if } \beta_1 \geq 1 
\end{cases}
\]

There are a maximum of three steady state values of $\beta_1$: $\beta_1 = 0$ will always be a steady state; $\beta_1 = 1/\alpha$ is a steady state when $\alpha$ is greater than one; and $\beta_1 = \alpha$ is a steady state when $\alpha$ is greater than one.

The simplest case is where $\alpha$ is less than one. In this instance there is only one steady state value of $\beta_1$ equal to zero, corresponding to a steady state income variance of zero. This is very intuitive. If the firm pays every worker the same income then by equation 1 their children’s talent will be distributed entirely by luck. By definition the firm can infer nothing about luck through the signal, and so they ignore it and pay all the children the same income. The case where $\alpha$ is less than one is shown in figure 2 where $F(\beta_1)$ is shorthand for $F(\sigma_y^2(\beta_1))$. The economy will always tend to this steady state.

When $\alpha$ is greater than or equal to one there still exists a steady state with no income variance and no reaction by the firm to an individual’s signal. This is stable in the sense that for small perturbations around zero the economy will return to this steady state. There also exist two other steady states: there is an unstable steady state where $\beta_1$ is equal to $\frac{1}{\alpha}$; and there is a stable steady state with $\beta_1$ equal to $\alpha$. We are more interested in the latter. The situation where $\alpha$ is greater than one is shown in figure 3.

The stable steady state with $\beta_1$ equal to $\alpha$ has income variance growing over time towards infinity. As income variance in the economy becomes very high, incomes will be so different that it will be perfectly evident who is the child of whom. As a result the firm fully uses the signal to the extent that parental income tells them about talent ($\alpha$).
2.3 The feedback mechanism

The fact that two stable steady states may emerge is one of the main findings that we will consider in the rest of this paper. When income variance is decreasing, firms care less about the signal because people are increasingly similar and the signal is known to be uninformative. They use it less and income variance continues to fall. When income variance is increasing, firms care more about the signal because it is becoming more informative and people more different. Increasingly there is an extra meaningful dimension in which people differ. As firms use the signal more, this feeds back into greater income inequality. As a result of this latter process, there emerges of a portion of the population who are rich and give advantages to their children that go beyond their talents - the ability to display their privileged upbringing and be paid for it.

3 Labour market sorting where firms receive a signal on talent

In the model of section 2 the information set of the firm consisted of two parts: μ, prior information about the distribution of income in the parent’s generation; and s₁, a signal on parental income. In this section we consider the actions of a firm when faced with an alternative information set \( \theta₂ \). They receive the same prior information but an alternative signal, \( s₂ \), on an individual’s talent given by,

\[
s₂ = \tau + \epsilon_{s₂}
\]  

They do not receive signal \( s₁ \). Talent is still given by equation 1 and income by
Later, we will model an improvement in meritocracy as a fall in $\sigma_{es}^2$ since this improves the precision of the signal and allows talent to be better identified and rewarded.

We calculate the posterior beliefs of the firm, given this new information, which gives it beliefs about talent of,

$$E(\tau|\theta_2) = \frac{\alpha^2\sigma_{y-1}^2 + \sigma_{\tau}^2}{\alpha^2\sigma_{y-1}^2 + \sigma_{\tau}^2 + \sigma_{es}^2}s_2$$  \hspace{1cm} (7)

The income that the firm pays a worker is a linear function of the new signal,

$$y = E(\tau|\theta_2) = \beta_2s_2$$  \hspace{1cm} (8)

where the value of $\beta_2$ can be read from equation 7. From this we can quite easily calculate the steady state income variance as a function of $\beta_2$, remembering that $\sigma_y^2 = \sigma_{y-1}^2$ in steady state,

$$\sigma_y^2(\beta_2) = \begin{cases} \frac{\beta_2^2(\sigma_{es}^2 + \sigma_{\tau}^2)}{1 - (\alpha\beta_2)^2} & \text{if } \beta_2 < \frac{1}{\alpha} \\ \infty & \text{if } \beta_2 \geq \frac{1}{\alpha} \end{cases}$$

We are now in a position to write the fixed point equation $\beta_2 = G(\sigma_y^2(\beta_2))$ where $G(\cdot)$ is the function of parental income variance given in equation 7. This gives,

$$\beta_2 = G(\sigma_y^2(\beta_2)) = \frac{\alpha^2\sigma_y^2(\beta_2) + \sigma_{\tau}^2}{\alpha^2\sigma_y^2(\beta_2) + \sigma_{\tau}^2 + \sigma_{es}^2} = \begin{cases} \frac{(\alpha\beta_2)^2\sigma_{es}^2 + \sigma_{\tau}^2}{\sigma_{es}^2 + \sigma_{\tau}^2} & \text{if } \beta_2 < \frac{1}{\alpha} \\ 1 & \text{if } \beta_2 \geq \frac{1}{\alpha} \end{cases}$$

This is a similar result to what we found in section 2. There are up to two steady state values of $\beta_2$ less than $\frac{1}{\alpha}$ given by,

$$\beta_2 = \frac{\sigma_{es}^2 + \sigma_{\tau}^2 + \sqrt{(\sigma_{es}^2 + \sigma_{\tau}^2)^2 - 4\alpha^2\sigma_{es}^2\sigma_{\tau}^2}}{2\alpha^2\sigma_{es}^2}$$

We will call the lower of these $\beta_2$ and the higher $\bar{\beta}_2$. There is a third possible steady state: $\beta_2 = 1$ for values of $\beta_2$ greater than $\frac{1}{\alpha}$. We will consider two possible cases: in the first, $\sigma_{\tau}^2 \geq \sigma_{es}^2$; in the second $\sigma_{\tau}^2 < \sigma_{es}^2$. 

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3.1 Steady state solutions where $\sigma_{\epsilon \tau}^2 \geq \sigma_{\epsilon s}^2$

The simplest case is where $\sigma_{\epsilon \tau}^2 \geq \sigma_{\epsilon s}^2$. When this condition holds there is only ever one steady state value of $\beta_2$. When the variance of talent is largely exogenous ($\sigma_{\epsilon \tau}^2$ is high), and the signal is precise ($\sigma_{\epsilon s}^2$ is low), a firm’s decision over how much to use the signal is largely independent of what other firms are doing. Since $G(\beta_2)$ measures the firm’s response to the prevailing $\beta_2$ in society it has little slope in this case. The steady state solutions for $\alpha$ in various ranges are shown in figure 4.

3.2 Steady state solutions where $\sigma_{\epsilon \tau}^2 < \sigma_{\epsilon s}^2$

Where $\sigma_{\epsilon \tau}^2 < \sigma_{\epsilon s}^2$, there is a possibility that multiple steady state values of $\beta_2$ exist. Intuitively when $\sigma_{\epsilon \tau}^2$ is low and $\sigma_{\epsilon s}^2$ is high, the variance in talent and quality of the signal are largely driven by income variance. Since this is endogenously determined by the prevailing $\beta_2$ in society, any firm’s response to changes in the prevailing $\beta_2$ is greater than in the previous section. This is why $G(\beta_2)$ has a steeper slope and why we may reach more than one steady state value of $\beta_2$. This is illustrated in figure 5.

Qualitatively the situation is very similar to what we saw for $\beta_1$ in section 2. There are

\[\beta_2 = 1\] when $\beta_2 \geq 1/\alpha \geq 1$ is also ruled out by contradiction. There is therefore never multiplicity when $\sigma_{\epsilon \tau}^2 \geq \sigma_{\epsilon s}^2$ for any value of $\alpha$. 

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Begin by considering $\alpha = 1$. The three possible steady states are then: $\beta_2 = \beta_2 = 1$ for $\beta_2 < 1$; $\beta_2 = \beta_2 = \sigma_{\epsilon \tau}^2/\sigma_{\epsilon s}^2 > 1$ for $\beta_2 < 1$; and $\beta_2 = 1$ for $\beta_2 \geq 1$. The first two of these are contradictions, leaving only one steady state with $\beta_2 = 1$ and $\sigma_2^2 \to \infty$. An increase in $\alpha$ causes the $G(\beta_2)$ curve to pivot upwards leaving the only steady state as $\beta_2 = 1$. A decrease in $\alpha$ will cause $\beta_2$ to fall below one, confirming it as a valid steady state value. $\beta_2$ is ruled out once we consider that any steady state solution must lie on the $G(\beta_2)$ curve which increases monotonically to a maximum value of one at $\beta_2 = 1/\alpha$. Therefore any solution to $\beta_2 = G(\beta_2)$ must have a solution with a value less than or equal to one. A decrease in $\alpha$ below one would cause $\beta_2$ to increase, and since it was already above one, this rules it out as a steady state. $\beta_2 = 1$ when $\beta_2 \geq 1/\alpha \geq 1$ is also ruled out by contradiction. There is therefore never multiplicity when $\sigma_{\epsilon \tau}^2 \geq \sigma_{\epsilon s}^2$ for any value of $\alpha$. 

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Figure 4: Only one steady state exists for a given $\alpha$ and $\sigma_{\epsilon \tau}^2 \geq \sigma_{\epsilon s}^2$
When $1 < \alpha < \alpha^*$ and $\sigma^2_{\tau} < \sigma^2_{s_2}$ there are 3 steady states, 2 stable two stable steady states: one at $\beta_2 = \beta_2$ with finite income variance; and one at $\beta_2 = 1$ with income variance growing over time (towards infinity). We will refer to these as the “low” and “high” (stable) steady states.

In this case there is an upper and lower bound on $\alpha$, equal to one and
$$\alpha^* = \frac{\sigma^2_{s_2} + \sigma^2_{\tau}}{2\sqrt{\sigma^2_{s_2} \sigma^2_{\tau}}}$$ respectively, for which this multiplicity exists. For values of $\alpha$ less than one there is only one steady state value of $\beta_2$ less than $\sigma^2_{\tau}/\sigma^2_{s_2}$. There is insufficient inheritance (parental income is not an important enough driver of talent) for heavy use of the signal to translate into high enough quality that this heavy use be sustained. For values of $\alpha$ greater than $\alpha^*$ there is only one steady state value of $\beta_2$ equal to one. Here the opposite is true. The additional quality of the talent signal generated by even a small increase in the amount of income variance would be sufficient for the talent signal to be used more and more.

The feedback mechanism works in a similar way to what we saw in section 2.3. More income inequality leads to greater dispersion of talent. This in turn leads to greater dispersion of the signals and more value being given to the signals by firms, the combined effect of which is greater income inequality.

### 3.3 The curse of meritocracy I

We model an increase in meritocracy as a fall in $\sigma^2_{s_2}$. This improves the quality of the information which firms have about talent and makes them better able to pay workers according to their talent. From figure 5 we can see that a fall in $\sigma^2_{s_2}$ causes the intercept of $G(\beta_2)$ to increase and, since it must reach the same point at $\beta_2 = 1/\alpha$, the slope to
fall $\beta_2$ increases. The steady state intergenerational correlation of incomes is given by,

$$\rho_{y,y-1} = \begin{cases} \alpha\beta_2 & \text{if } \beta_2 < 1/\alpha \\ 1 & \text{if } \beta_2 \geq 1/\alpha \end{cases}$$

and so increases in the low steady state as $\sigma_{\epsilon s_2}^2$ falls. Improvements in meritocracy reduce mobility.

If we exogenously improve the quality of the signal, firms use it more, increasing the variance of income and talent and further improving the quality of the signal (the feedback mechanism). As this happens, firms get better at identifying talented individuals and paying them accordingly but these individuals tend to be from rich backgrounds, reducing mobility. We produce the observed negative correlation between mobility and inequality in the low steady state. The high steady state is unaffected.

4 Labour market sorting where firms receive two signals

In the previous sections we have considered the actions of a firm trying to work out the talent of individual workers with the aid of two alternative information sets, $\theta_1$ and $\theta_2$. We are now going to combine these so that the firm’s information set, which we will now call $\theta'$, has three parts; the prior, $\mu$, based on information on the distribution of income in the parents’ generation; signal $s_1$ on parental income given by equation 2 and signal $s_2$ on talent given by equation 6. By reintroducing the signal on parental income, we will show that this makes the curse of meritocracy worse. In addition to firms being better able to pick out the talented, who happen to be predominantly from rich backgrounds, they will also use the signal on parental income more.

We assume that talent is still given by equation 1. As before, the distributions of the errors $\epsilon_{s_1}$, $\epsilon_{s_2}$ and $\epsilon_\tau$ are known to the firm and all are independently distributed.

Using Bayesian inference we can calculate the posterior beliefs of a firm conditional on the new information set $\theta'$. This gives a steady state posterior belief of $^{14}$

$^{14}$Formally, the slope of the $G(\beta_2)$ function is increasing in $\sigma_{s_2}^2$ at any given value of $\beta_2 < \frac{1}{\alpha}$.

$$\frac{\partial}{\partial \sigma_{s_2}^2} \left( \frac{\partial G(\beta_2)}{\partial \beta_2} \right) \bigg|_{\beta_2 = \hat{\beta}_2} = \frac{2 \alpha^2 \beta_2 \sigma_\tau^2}{(\sigma_{s_2}^2 + \sigma_\tau^2)} > 0$$

$^{4}$The firm’s belief, given in equation is a weighted average of three things: a prior on talent, $E(\tau | \mu)$, equal to zero; a belief about talent based solely on signal 1, $E(\tau | s_1)$, equal to $\alpha s_1$; and a belief about talent based solely on signal 2, $E(\tau | s_2)$, equal to $s_2$. As $\sigma_\tau^2$ goes to infinity the prior is useless and the weights given to the other parts sum to one. Similarly, if $\sigma_{s_1}^2$ equals zero then the prior is useless since the first signal perfectly informs the firm about parental income. Again it is thrown away. The weights
\[ E(\tau|\theta') = \frac{\alpha \sigma_{y^2} }{\alpha^2 \sigma_{y^2} \sigma_{\epsilon s_1}^2 + (\sigma_{\epsilon r}^2 + \sigma_{\epsilon s_2}^2) (\sigma_{y^2} + \sigma_{\epsilon s_1}^2)} s_1 + \frac{\alpha^2 \sigma_{y^2} \sigma_{\epsilon s_1}^2 + \sigma_{\epsilon r}^2 (\sigma_{y^2} + \sigma_{\epsilon s_1}^2)}{\alpha^2 \sigma_{y^2} \sigma_{\epsilon s_1}^2 + (\sigma_{\epsilon r}^2 + \sigma_{\epsilon s_2}^2) (\sigma_{y^2} + \sigma_{\epsilon s_1}^2)} s_2 \]

(9)

Since firm’s pay wages according to expected talent we can now write the equation for income as,

\[ y = E(\tau|\theta') = \beta_1 s_1 + \beta_2 s_2 \]

(10)

where \( \beta_1 \) and \( \beta_2 \) can be read from equation 9. They are the coefficients of a multivariate regression of talent on \( s_1 \) and \( s_2 \). This gives steady state income variance of

\[ \sigma_y^2 = \begin{cases} \beta_1^2 \sigma_{\epsilon s_1}^2 + \beta_2^2 (\sigma_{\epsilon r}^2 + \sigma_{\epsilon s_2}^2) & \text{if } \beta_1 + \alpha \beta_2 < 1 \\ 1 - (\beta_1 + \alpha \beta_2)^2 & \text{if } \beta_1 + \alpha \beta_2 \geq 1 \end{cases} \]

We now have three equations in three unknowns: one each for steady state values of \( \beta_1 \) and \( \beta_2 \) as function of the income variance which can be read from equation 9 and one for the steady state income variance as a function of \( \beta_1 \) and \( \beta_2 \). Unfortunately finding analytical solutions to these three unknowns for finite income variance is unmanageable. As such, we will proceed in a slightly different manner to before.

### 4.1 The firm’s choice of \( \beta_1 \) when the value of \( \beta_2 \) is fixed

Since it is very difficult to find an analytical solution for \( \beta_1 \) when \( \beta_2 \) and \( \sigma_y^2 \) are both being endogenously determined, what we do instead is to exogenously impose a value of \( \beta_2 \) and ask the question: If this were the value of \( \beta_2 \) which firms faced, what value(s) of \( \beta_1 \) would they tend towards? This method allows us to draw a reaction correspondence for \( \beta_1 \) as a function of different exogenously imposed values of \( \beta_2 \).

To carry out this method, first we need to know the firms’ choice of \( \beta_1 \) when \( \beta_2 \) is given to other parts then reflect the extent to which parental income and the second signal inform about talent and sum to one. When \( \sigma_{\epsilon s_2}^2 \) is zero, only the second signal is used since it perfectly informs about talent so its weight is equal to one.

\[ \sigma_y^2 = \beta_1^2 (\sigma_{\epsilon s_1}^2 + \sigma_{\epsilon s_2}^2) + \beta_2^2 (\sigma_{\epsilon r}^2 + \sigma_{\epsilon s_2}^2) + 2 \beta_1 \beta_2 (\alpha \sigma_{\epsilon s_1}^2) \]

\[ = (\beta_1 + \alpha \beta_2)^2 \sigma_{\epsilon s_1}^2 + \beta_1^2 \sigma_{\epsilon s_1}^2 + \beta_2^2 (\sigma_{\epsilon r}^2 + \sigma_{\epsilon s_2}^2) \]

Solving for \( \sigma_y^2 = \sigma_{\epsilon s_1}^2 \) gives steady state income variance.
exogenous. In section 2 we noted that the posterior belief in equation 4 gave a value of \( \beta_1 \) equal to the coefficient from a regression of \( \tau \) on \( s \). This worked because the OLS regressor was the value of \( \beta_1 \) which minimised the variance of expected talent around its true value, which also happens to be how a rational agent using Bayes rule behaves. We use this result again here. The value of \( \beta_1 \) which minimises the variance of \( E(\tau|\theta') \) around \( \tau \) is found by solving the minimisation,

\[
\min_{\beta_1} E(\tau - \beta_1 s_1 - \beta_2 s_2)^2
\]

which gives,

\[
\beta_1 = \alpha (1 - \beta_2) \frac{\sigma^2_y}{\sigma^2_y + \sigma^2_{e_1}}
\]

(12)

Fixing the value of \( \beta_2 \) at zero shuts off the talent signal and returns us to the model of section 2. Also, if \( \beta_1 + \alpha \beta_2 \geq 1 \), we can see from above that income variance will tend to infinity and \( \beta_1 = \alpha (1 - \beta_2) \). Since rearranging this gives \( \beta_1 + \alpha \beta_2 = \alpha \) the requirement for the existence of a steady state with growing variance is again \( \alpha \geq 1 \).

By substituting the steady state income variance and solving for \( \beta_1 \), equation 12 gives the \( \tilde{\beta}_1 \) reaction correspondence, where the “hat” indicates that this is the value of \( \beta_1 \) that firms choose given an exogenously imposed value of \( \beta_2 \).

4.2 The firm’s choice of \( \beta_2 \) when the value of \( \beta_1 \) is fixed

We proceed in exactly the same manner as in the previous section. First, we want to solve the minimisation in equation 11 for \( \beta_2 \) to give us the firm’s choice of \( \beta_2 \) given \( \beta_1 \). Solving the minimisation gives,

\[
\beta_2 = \frac{\alpha (\alpha - \beta_1) \sigma^2_y + \sigma^2_{e\tau}}{\alpha^2 \sigma^2_y + \sigma^2_{e\tau} + \sigma^2_{e_2}}
\]

(13)

Substituting in the steady state income variance and solving for \( \beta_2 \) gives us a correspondence \( \tilde{\beta}_2 \) which describes the firm’s choice of \( \beta_2 \) in reaction to a particular exogenously imposed value of \( \beta_1 \). Setting \( \beta_1 \) equal to zero cuts off the signal on parental income and returns us to the model of section 3 where the first signal plays no role. Also, when \( \beta_1 + \alpha \beta_2 \geq 1 \) we again find that there is a steady state at which income variance tends
Towards infinity and $\beta_1 + \alpha \beta_2 = \alpha$.

### 4.3 A numerical example

The response correspondences in sections 4.1 and 4.2 do not easily lend themselves to analytical solutions so a numerical analysis was carried out. The correspondences were drawn for the parameter values: $\alpha = 1.1$; $\sigma^2_{e_1} = 5$; and $\sigma^2_{e_2} = 4$; $\sigma^2_{e_T} = 1$. These were chosen based on what had been learned from the one signal models – that to have multiplicity we require: $\alpha$ to be larger than one but below some upper bound $\alpha^*$; and the variance in the signal errors ($\sigma^2_{e_1}$ and $\sigma^2_{e_2}$) to be large compared to the variance in the error on talent ($\sigma^2_{e_T}$). Figure 6 shows how the reaction correspondences look for these parameter values. We have focussed only on non-negative, non-complex values of $\beta_1$ and $\beta_2$.

Along the x- and y-axis, $\beta_1$ and $\beta_2$ are respectively fixed at zero. Thus the solutions along the axes are as in sections 3 and 2 respectively. The highest of these solutions are connected by the line $\beta_1 + \alpha \beta_2 = \alpha$. From sections 4.2 and 4.3 we can see that the infinite variance solutions to both $\hat{\beta}_1$ and $\hat{\beta}_2$ lie on this line.

Where the two correspondences cross we find a finite variance steady state solution for $\beta_1$ and $\beta_2$ with the firm choosing $\beta_1$ as a best response to $\beta_2$ and $\beta_2$ as a best response to $\beta_1$. We find that there are two of these, one with relatively low values of $\beta_1$ and $\beta_2$ and
one with relatively higher ones. The lower one is stable while the higher one is unstable.

The dynamics of the economy work as follows. Suppose $\beta_1$ and $\beta_2$ are fixed at certain values. This will imply a certain income variance. Then imagine that at some point in time firms are allowed to freely choose $\beta_1$ and $\beta_2$. They will do so taking the current income variance as given (since it is our state variable). As a result they act as if they were on the saddle path with the same variance as was created by the initial $(\beta_1, \beta_2)$ combination. This is given by the point at which the “isovariance” curve through this $(\beta_1, \beta_2)$ cuts the saddle path. They will jump to a new point on the saddle path, after which they continue to choose values of $\beta_1$ and $\beta_2$ which follow the saddle path towards steady state. If the initial $(\beta_1, \beta_2)$ was inside the higher of the two isovariance curves illustrated, the economy will converge towards the lower stable steady state (with finite variance). If it is outside this isovariance curve the economy will converge towards the high stable steady state with growing income variance. The equation of the saddle path is given by,

$$
\beta_1 = -\frac{\sigma^2_{s\tau}}{\alpha\sigma^2_{s1}} + \frac{\sigma^2_{s2} + \sigma^2_{s\tau}}{\alpha\sigma^2_{s1}}\beta_2
$$

(14)

It is worth noting that, where two stable steady states exist, it is always the case that one has a lower level of both $\beta_1$ and $\beta_2$ and so there never exists two steady states which substitute one signal for the other.

4.4 The curse of meritocracy II

In the two signal model, the intergenerational income correlation is given by,

$$
\rho_{y,y-1} = \begin{cases} 
\beta_1 + \alpha\beta_2 & \text{if } \beta_1 + \alpha\beta_2 < 1 \\
1 & \text{if } \beta_1 + \alpha\beta_2 \geq 1 
\end{cases}
$$

Income variance in the finite variance steady state is given by,

$$
\sigma^2_y = \frac{\beta_1^2\sigma^2_{s1} + \beta_2^2(\sigma^2_{s2} + \sigma^2_{s\tau})}{1 - (\beta_1 + \alpha\beta_2)^2}
$$

Since the analytical solutions for $\beta_1$ and $\beta_2$ are unmanageable, and intergenerational mobility and income inequality are functions of $\beta_1$ and $\beta_2$, it is also very difficult to solve analytically the effects of a change in our exogenous parameters on them. We proceeded with a numerical analysis. First we will examine the effects of a fall in $\sigma^2_{s2}$. This is
Parameter values the parameter may take

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values the parameter may take</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5</td>
</tr>
<tr>
<td>$\sigma^2_{\epsilon s_1}$</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12</td>
</tr>
<tr>
<td>$\sigma^2_{\epsilon s_2}$</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12</td>
</tr>
<tr>
<td>$\sigma^2_{\epsilon \tau}$</td>
<td>0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6</td>
</tr>
</tbody>
</table>

Table 1: Parameter values for the numerical exercise investigating the curse of meritocracy again how we define an improvement in meritocracy. There are two forces at work as $\sigma^2_{\epsilon s_2}$ falls: given $\beta_1$ and $\beta_2$ a fall in the noise will decrease income variance which will lead to a reduction in $\beta_1$ and $\beta_2$ as firms respond; but for a given income variance a fall in $\sigma^2_{\epsilon s_2}$ necessarily increases the use of the talent signal, which in turn will increase income variance and the use of both signals.

We first define our parameter sets to be any combination of parameters from table 1. $\alpha$ may take any one of fifteen values, $\sigma^2_{\epsilon s_1}$ any one of twelve, $\sigma^2_{\epsilon s_2}$ one of twelve, and $\sigma^2_{\epsilon \tau}$ one of twelve. This gives 25,920 possible combinations of parameters. However for some of those with $\alpha$ greater than one, the finite variance steady state will not exist. We eliminated these cases and were left with 17,918 parameter combinations which produce a finite variance steady state.

For each of these steady states we decreased $\sigma^2_{\epsilon s_2}$ by 0.001. We interpret this as an exogenous marginal increase in meritocracy. In all 17,918 cases for which a finite variance steady state existed, income variance and the intergenerational correlation of incomes both increased. We graph the effect of increasing meritocracy on inequality and mobility for a small number of different parameter sets in figure 7. Meritocracy was measured as $\frac{1}{\sigma^2_{\epsilon s_2}}$. The graph illustrates the increase in inequality and fall in mobility which accompanies an increase in meritocracy. There are two possible reasons for the fall in mobility: in all cases, the quality of the second signal has increased and so $\beta_2$ increases. This was the effect described in section 3.3 where firms get better at picking out the talented but they tend to be from rich backgrounds; additionally, in some cases (all cases for which $\alpha > 1$) the first signal is also used more, reducing mobility since it directly relates to family background.

We also investigated whether the correlation of an individual’s talent and income increased with a fall in $\sigma^2_{\epsilon s_2}$. If it did not, a fall in $\sigma^2_{\epsilon s_2}$ would not be a useful way of exogenously shocking meritocracy. It occurred in all cases.

In addition to investigating the effects of meritocracy we consider the effect of inherited advantages. An exogenous increase in inherited advantages was defined as a fall in $\sigma^2_{\epsilon s_1}$. With a fall in $\sigma^2_{\epsilon s_1}$, it is easier for firms to identify parental income and, since parental income is correlated with talent, to pay higher incomes to those from richer backgrounds. Thus the advantages of a rich background (and disadvantages of a poor background) are greater. We decreased $\sigma^2_{\epsilon s_1}$ by 0.001 and examined the effects for the parameter sets given
In all 17,981 cases for which a finite variance steady state existed, income variance, the intergenerational correlation of incomes, and the correlation of talent and income increased. Figure 8 illustrates how increases in advantages related to background (defined as $\frac{1}{\sigma_{S_1}^2}$) lead to greater income inequality and lower intergenerational mobility.

In summary, an increase in the precision of either signal (an increase in meritocracy or advantages related to background) will lead to greater income inequality and further increases in both meritocracy and advantages related to background. This is because the endogenous response of firms to the more precise information is to use one or both of the signals more. We might have expected that when firms are better informed about an individual’s talent, they would care less about his background. We show that more often than not (and in all cases where there is sufficient inheritance) this is not true and that even when it is true, mobility still falls due to increased use of the talent signal.

5 An extension

Throughout this chapter we have assumed that firms pay workers a wage equal to their expected talent at the point of entrance into the workforce. There is no further opportu-
Figure 8: The effect of increasing advantages on steady state income variance and mobility

ity for them to learn about their worker’s talent. There is a strand of literature which investigates whether, if firms statistically discriminate when sorting new workers, the effects of this discrimination fade as firms observe their worker’s ability on-the-job (Farber & Gibbons (1996); Altonji & Pierret (2001)). Most of this literature has investigated whether a schooling signal is used by firms in the absence of better information on talent, although a paper by Arcidiacono et al. (2008) also looks at race.

A simple way to include such considerations in our framework would be to have the (lifetime) income of an individual be a weighted average of his talent and expected talent.

\[
y = \theta \tau + (1 - \theta) E(\tau | s_1, s_2)
\]

The role played by expected talent reflects the times when the worker’s ability is unobservable to his firm and so it relies on observable signals. The relative weight given to expected talent, \((1 - \theta)\), would indicate the amount of time that the worker spends in this situation during his lifetime, and hence things such as the speed of learning by employers (the subject of a paper by Lange (2007)). This change would reduce, but not eliminate, the role of signalling. This chapter has investigated the situation where \(\theta = 1\). It would be interesting to see for which other values of \(\theta\) the above conclusions hold.
6 Discussion

In this model, inequality leads to discrimination because when people are more different, the signals they provide are more informative. This discrimination feeds back into inequality. This feedback mechanism may lead to multiplicity of steady states, one with a relatively small amount of discrimination and inequality and one with a relatively large amount of discrimination and ever increasing inequality.

There exists a negative correlation between inequality and mobility, which was observed in Corak’s “Great Gatsby Curve”. We believe this may occur for two reasons: the increased discrimination which accompanies greater inequality results in firms being better able to identify talent, which is correlated with parental income; and it may result in them using family background information more. This move towards greater inequality and lower mobility may be started by an exogenous increase in the degree of meritocracy in society, which improves the quality of information available to firms, feeding into discrimination and inequality.

These conclusions suggest some policy implications. Suppose the government has a choice to make between providing funding for pre-school and university education. We would model the early intervention as a way of relieving the credit constraints on poorer families and ensuring a good platform of basic skills on which to build for all children. This should weaken the relationship between talent development and parental income, which in this model refers to a fall in $\alpha$. Alternatively, the university investment would allow the best and the brightest to go on to university and provide a “Spence-type” signalling opportunity for them. This improved ability to signal their skills is what we modelled above as a fall in $\sigma_{\epsilon}\sigma_{\tau}^2$. 

The implications from the model were that the university funding would lead to increased inequality and lower mobility. Pre-school investment would lead to a weakening of the relationship between talent and parental income, making the parental income signal less useful, lowering discrimination and in turn lowering inequality and raising mobility. As a result, to the extent that equality and intergenerational mobility are valued by society, this chapter would support public funding of early childhood interventions over university education.

7 Inheritance and Investment

In the first part of this paper we considered the effects of statistical discrimination in labour markets. Firms had a limited information set available to them, and they pieced together a belief about an individual’s talent by weighting the information that they had
according to its precision. This statistical discrimination fed through to income inequality, but also fed off past inequality. In order to keep that model simple, and to show in a stark way how that feedback mechanism worked, it was limited to two key variables, income and talent, and three key concepts, inequality, intergenerational mobility, and meritocracy. How these three aggregate measures of the state of our economy interact was the focus of that chapter.

The aim of this chapter is to build on that framework a mechanism which permits forward-looking behaviour. If we look through the more recent statistical discrimination literature we see that it is this forward-looking mechanism which determines equilibrium behaviour. For example, consider the discriminating equilibrium in Coate & Loury (1993). It is the beliefs of black workers that firms will use their race to sort them that prevents them from investing in the skills that would prevent firms from doing so. Up until now we have not considered the forward-looking investment behaviour of optimising agents. This will be the focus of chapter 3. We will build on top of the model from chapter 2, with its potential for multiplicity of steady states, the Coate and Loury mechanism whereby agents’ expectations of the return to investment in skills affect their decision over investment. While this creates the potential for multiple self-fulfilling equilibria, we will see that in this model, that is not observed.

In order to build in investment decisions, I am going to alter the model in one key way. The distribution of income is going to be modelled as lognormal rather than normal. Parents will be the optimising agents, each choosing how much to invest in their child’s talent development (education). They are going to be credit constrained. As a result their investment will be a function of their income, and in order to keep investments confined to the set of non-negative real numbers we want incomes to be non-negative. Hence we use the lognormal income distribution. It has the additional feature of adding more realism to the model. Section 8 will derive the optimal investment rule for parents. It is to invest a fraction of their income. The rule will be discussed in section 9.

This version of the model will also allow for growth in mean income. The model in the second chapter was primarily concerned with the variance and correlation of incomes and so we abstracted from changes in mean income. Again, this will add some additional realism to the model.

This chapter will proceed in the following way: first, we will derive an optimal investment rule which governs parents’ investment in their children’s education; second we will show that this is unique and why this uniqueness comes about; finally, given the investment rule, and the income process which it implies, we will look at some features of the economy and how they change following an exogenous improvement in meritocracy or inherited advantages. A key finding is that, for certain sets of parameters, a risk averse agent may have lower utility on average in a country with higher mean income and greater
meritocracy, if that meritocracy brings with it greater variance, and hence uncertainty, over income.

8 An optimal investment rule for children’s education

We will begin with an equation describing the formation of talent. In this case we are going to consider the log of talent, where talent is lognormally distributed. As in the previous chapters, an individual’s level of talent describes their skill level after education and before entering the labour force, and so is dependent on their human capital formation as a child. Our assumption is that this depends on the income level of their parent when they are growing up and the level of investment which their parent makes in them (we assume that each parent has one child). There is an underlying credit constraint which states that it is impossible to borrow in order to invest in your child. Hence, coming from a rich background is beneficial not only through the environment of your childhood but because it better enables your parent to invest in your development. The equation for the log of talent is given by,

\[
\ln \tau = \phi + \alpha_y \ln y_{-1} + \alpha_x \ln x + \epsilon_{\tau}
\] (15)

where \( \tau \) is an individual’s talent, \( y_{-1} \) is his parent’s income level, \( x \) is the investment which his parent chose to make in him, and \( \epsilon_{\tau} \) is a mean zero normally distributed error with variance \( \sigma_{\epsilon_{\tau}}^2 \). \( \phi \) is a constant term, while \( \alpha_x \) and \( \alpha_y \) are the parameters which govern the advantages which come from being part of a rich family. In sections 11 and 12 we will refer to an economy with “low” values of \( \alpha_x \) and \( \alpha_y \) \((\alpha_x + \alpha_y < 1)\) as a weak inheritance economy and one with “high” values \((\alpha_x + \alpha_y \geq 1)\) as a strong inheritance economy.

We will call \( \hat{x} \) \((y_{-1})\) the equilibrium investment rule but we will not assume any structure on it. Instead we will allow for any investment rule and solve the parent’s investment choice problem to find the structure that it has in equilibrium. The equilibrium investment rule is known to all.

8.1 The pay decisions of firms

As outlined in the introduction, we are going to assume in this chapter that income follows a log-normal distribution,
\[ lny_{-1} \sim N\left(\mu_{y_{-1}}, \sigma_{y_{-1}}^2\right) \]

If there were no additional information available to the firm, they would believe everyone to be of average log income and the precision in their beliefs would be \(1/\sigma_{y_{-1}}^2\).

Suppose there are two signals available to the firm which provide some information on a worker’s talent. The first is a noisy signal of her parent’s income given by \(s_1 = y_{-1} + \epsilon s_1\) where \(\epsilon s_1\) is a mean zero normally distributed error with variance \(\sigma_{\epsilon s_1}^2\). Neither this signal nor the prior provide the firm with any information about the error in talent, \(\epsilon\), but, as in the chapter 2 model, they do allow the firm to form a belief about parental income which is itself a signal of talent. Firms, as before, form a posterior belief about log parental income conditional on the signal \(s_1\). The distribution of these posterior beliefs is given by

\[
lny_{-1}|s_1 \sim N\left(\frac{\sigma_{\epsilon s_1}^2}{\sigma_{y_{-1}}^2 + \sigma_{\epsilon s_1}^2} \mu_{y_{-1}} + \frac{\sigma_{y_{-1}}^2}{\sigma_{y_{-1}}^2 + \sigma_{\epsilon s_1}^2} lns_1, \frac{\sigma_{\epsilon s_1}^2}{\sigma_{y_{-1}}^2 + \sigma_{\epsilon s_1}^2}\right)
\]

We will refer to the mean of this distribution as \(\mu_{y_{-1}|s_1}\). It is the firm’s belief about the parental income of an individual with signal \(s_1\). We will refer to the variance as \(\sigma_{y_{-1}|s_1}^2\). The precision in the firm’s belief is therefore \(1/\sigma_{y_{-1}|s_1}^2\).

Given the human capital development function in (15), firms can form a belief about talent based on their posterior beliefs about parental income. They must consider how much parental investment the individual received and in doing so take the equilibrium investment rule as given. Let \(\mu_{x|\hat{x}}\) be the firm’s belief about an individual’s \(\ln x\), given the investment rule, and \(\sigma_{x|\hat{x}}^2\) be the variance in those beliefs. Although the equilibrium investment rule will be known to the firm, parental income is only known with some error which leads to the variance in their beliefs about investment. \(\mu_{x|\hat{x}}\) and \(\sigma_{x|\hat{x}}^2\) are known to all.

This gives the following conditional distribution of talent,

\[
ln\tau|s_1 \sim N\left(\phi + \alpha_y \mu_{y_{-1}|s_1} + \alpha_x \mu_{x|\hat{x}}, \alpha_y^2 \sigma_{y_{-1}|s_1}^2 + \alpha_x^2 \sigma_{x|\hat{x}}^2 + 2\alpha_y \alpha_x \sigma_{y_{-1}|s_1,x|\hat{x}} + \sigma_{\epsilon \tau}^2\right)
\] (16)

Here \(\sigma_{y_{-1}|s_1,x|\hat{x}}\) is the covariance of the firm’s belief about a parent’s income, conditional on the parental income signal, and their belief about the investment that the parent
has made, given the equilibrium investment rule. There is a statistical advantage to looking like you are from a privileged background, both directly and through the investment you will be believed to have received.

The second signal is a noisy signal on the individual’s talent given by

\[ s_2 = \tau e^{\epsilon_{s2}} \]

where \( \epsilon_{s2} \) is a mean zero normally distributed error with variance \( \sigma^2_{s2} \). Each firm is now in a position to choose what weights to give to their beliefs based on the information they have about an individual’s parental income \((\mu_{y-1}, s_1)\) and the information they have about her talent \(s_2\). This gives the following result:

\[
\ln \tau | s_1, s_2 \sim N \left( [1 - \beta_2] \left[ \phi + \alpha_y \mu_{y-1} | s_1 + \alpha_x \mu_x | \hat{x} \right] + \beta_2 \ln s_2, \beta_2 \sigma^2_{s2} \right) \tag{17}
\]

where

\[
\beta_2 = \frac{\alpha_y^2 \sigma_{y-1}^2 | s_1 + \alpha_x^2 \sigma_{x|x}^2 + 2\alpha_y \alpha_x \sigma_{y-1|x} | s_1 | \hat{x} + \sigma^2_{\epsilon_{s2}}}{\alpha_y^2 \sigma_{y-1}^2 | s_1 + \alpha_x^2 \sigma_{x|x}^2 + 2\alpha_y \alpha_x \sigma_{y-1|x} | s_1 | \hat{x} + \sigma^2_{\epsilon_{s2}} + \sigma^2_{s2}}
\]

The firms’ posterior belief about talent is a weighted average of their beliefs conditional on the first and second signals. The weight is given by \( \beta_2 \). As in the previous chapter, \( \beta_2 \) will measure the extent to which firms reacts to changes in the talent signal. Since in equilibrium the distribution of the firm’s beliefs about investment will be common knowledge, so will be the constant \( \beta_2 \).

We are now in a position to consider a payment rule for the firm. As in chapter 2, the firm will pay a wage based on their expectations of the talent of an individual. For any lognormally distributed variable \( z \) with \( E \left( \ln z \right) = \mu_z \) and \( \text{Var} \left( \ln z \right) = \sigma^2_z \), the expected value of \( z \) is equal to \( e^{\mu_z + \frac{\sigma^2_z}{2}} \). It follows that the conditional expectation of talent is,

\[
E \left( \tau | s_1, s_2 \right) = e^{[1 - \beta_2] \left[ \phi + \alpha_y \mu_{y-1} | s_1 + \alpha_x \mu_x | \hat{x} \right] + \beta_2 \ln s_2 + \frac{\beta_2 \sigma^2_{s2}}{2}} \]

And so, if firms pay incomes equal to their conditional expectation of talent, \( \ln y \) is

\[
\ln y = [1 - \beta_2] \left[ \phi + \alpha_y \mu_{y-1} | s_1 + \alpha_x \mu_x | \hat{x} \right] + \beta_2 \ln s_2 + \frac{\beta_2 \sigma^2_{s2}}{2} \tag{18}
\]

This is the log income which an individual will receive. She will receive a higher
income if the firm believes her parent to have been richer, both because of the direct effect of parental income on human capital development and because, given the equilibrium investment rule, richer parents may be believed to invest more. She will also receive a higher income if she looks more talented via the direct signal on talent, $s_2$. We will now consider how parents invest and hence what the equilibrium investment rule is.

8.2 The investment decisions of parents

Parents choose how much to invest in their child. In order to know how they do that, we must define for a parent a particular value function. This is given by,

$$V(y_{-1}) = \max_{c_{-1}} \left\{ ln c_{-1} + \frac{1}{1+\delta} E[V(y)|y_{-1},x] \right\}$$  \hspace{1cm} (19)

subject to

$$y_{-1} \geq c_{-1} + x$$

Parents gain utility from their own consumption and from the expected value which their child will have, and make their consumption-investment choice optimally given their income. $\delta$ is the discount rate. Parent’s know their own income and investment choice and so condition on them when taking expectations of their child’s value function. We also include a credit constraint whereby parents are unable to borrow. They can consume themselves or invest in their children, but solely through educational investment. Since they are maximising utility we assume that this constraint is binding.

We proceed with a guess and verify strategy. We guess that the value function is of the following form,

$$V(z) = A ln z + B$$

Conditional on this proving to be the case, we can find a parent’s expectation of their child’s value function. Performing the maximisation in equation 19 then allows the Euler equation to be found.
\[
\frac{1}{y_{-1} - x} = A \frac{\partial E[lny|y_{-1}, x]}{1 + \delta} \frac{\partial x}{\partial x}
\]

The last term describes the expected return on investment of the parent. Firms’ take as given that parent’s invest according to the equilibrium investment rule. There is no way for a parent to signal if they have deviated from it and, by definition, no reason to do so without such a signal (when firms take the rule as given, following the rule solves the parent’s maximisation in equation \([19]\)). Hence the marginal effect of investment in a child is independent of the beliefs that firms have on marginal investment. The only effect which investment has on a child’s income is through their talent development which they can signal through \(s_2\). We can see this by taking expectations of the income process described in equation \([18]\)

\[
E[lny_{-1}, x] = [1 - \beta_2] \left[ \phi + \frac{\alpha_y}{\sigma_{y_{-1}}^2 + \sigma_{e_{y_{-1}}}^2} \left[ \sigma_{e_{y_{-1}}}^2 \mu_{y_{-1}} + \sigma_{y_{-1}}^2 lny_{-1} \right] + \alpha_x E(\mu_x|x|y_{-1}) \right] \\
+ \beta_2 \left[ \phi + \alpha_y lny_{-1} + \alpha_x \ln x \right] + \frac{\beta_2 \sigma_{e_{y_{-1}}}^2}{2}
\]

Examining the above expectation of log income, the only place where the chosen investment level plays a role is in the penultimate term. This term is the expected value of \(lns_2\). A parent expects a marginal increase in investment to raise their child’s income only to the extent that they expect it to make them appear more talented. They do not expect it to change the firms’ belief about how much they have invested. Firms take the equilibrium investment rule as given and parents have no way of making themselves look richer.

Solving the Euler equation provides the optimal investment choice of a parent. This is the equilibrium investment rule.

\[
\hat{x}(y_{-1}) = \frac{\alpha_x A \beta_2}{(1 + \delta) + \alpha_x A \beta_2} y_{-1}
\]

(20)

An equilibrium investment rule is one for which the following statement holds: given that firms expect individuals to invest according to the equilibrium investment rule, the optimal choice of investment is in accordance with the equilibrium investment rule. We have shown so far that given any investment rule which firms believe individuals are following, the optimal investment for a parent is a constant fraction of their income. It
then follows that investing a constant fraction of income is an equilibrium investment rule: given that firms believe individuals to be investing a fraction $\lambda$ of their income in their children, the optimal investment for parents to make is a fraction $\lambda$ of their income.

Furthermore, since investing a constant fraction $\lambda$ of their income is the optimal investment strategy irrespective of the rule firms believe they are following, this is a unique investment rule. There is no other rule which firms would ever believe parents to be following, nor is there any other rule which parents would ever choose to follow.

8.3 Solving the equilibrium payment and investment rules

We can see that, if the guess about the value function is correct, parents spend a constant fraction of their income investing in their child. We are calling this fraction $\lambda$. Based on this result,

$$\mu_{x|x} = \ln \lambda + \mu_{y-1|s_1}$$

and

$$\sigma_{x|x}^2 = \sigma_{y-1|s_1,x|x} = \sigma_{y-1|s_1}^2$$

The firms belief about how much you are investing in your child is equal to a constant plus their belief about your income, conditional on the signal it receives about your income. By substitution back into equation 18 we find that,

$$\ln y = \mu_r + \beta_1 (lns_1 - \mu_{s_1}) + \beta_2 (lns_2 - \mu_{s_2}) + \frac{\beta_2 \sigma_{es_2}^2}{2}$$  \hspace{1cm} (21)

where,

$$\beta_1 = \frac{(\alpha_x + \alpha_y) \sigma_{y-1}^2 \sigma_{es_2}^2}{(\alpha_x + \alpha_y)^2 \sigma_{y-1}^2 \sigma_{es_1}^2 + (\sigma_{er}^2 + \sigma_{es_2}^2) \left( \sigma_{y-1}^2 + \sigma_{es_1}^2 \right)}$$  \hspace{1cm} (22)

and,

$$\beta_2 = \frac{(\alpha_x + \alpha_y) \sigma_{y-1}^2 \sigma_{es_2}^2 + \sigma_{er}^2 \left( \sigma_{y-1}^2 + \sigma_{es_1}^2 \right)}{(\alpha_x + \alpha_y)^2 \sigma_{y-1}^2 \sigma_{es_1}^2 + (\sigma_{er}^2 + \sigma_{es_2}^2) \left( \sigma_{y-1}^2 + \sigma_{es_1}^2 \right)}$$  \hspace{1cm} (23)
This defines the optimal payment rule for firms. The mean of $lny$ is given by the mean (log) talent, adjusted by the variance in the conditional distribution of $\tau$. Workers will be believed to be of average (log) income unless they have above or below average signals. An above or below average signal will cause the firm to adjust their beliefs to the extent that the signal is informative. This is captured by the precision of the signals through $\beta_1$ and $\beta_2$.

For example, as $\sigma_{\epsilon s^2}$ tends to infinity, the second signal is of no use to the firms. They disregard it. Alternatively, if $\sigma_{\epsilon s^2}$ were to be zero, the second signal would be perfect and the firms would completely ignore the first signal. In an equivalent fashion to chapter 2, $\beta_1$ and $\beta_2$ are the coefficients from a multivariate regression of $ln\tau$ against $lns_1$ and $lns_2$, the only difference being the use of the log terms. Substituting for $lns_1$ and $lns_2$ we find the income process.

$$lny = \left\{ \mu_\tau + \frac{\beta_2 \sigma_{\epsilon s^2}^2}{2} \right\} + [\beta_1 + (\alpha_x + \alpha_y) \beta_2] (lny_{-1} - \mu_{y_{-1}}) + \beta_1 \epsilon_{s_1} + \beta_2 (\epsilon_{s_2} + \epsilon_{\tau})$$ (24)

This is the income that an individual will earn, given his parents income, and the realisation of the shocks to his talent and signals. Inherent in this is that parents invest according to the equilibrium investment rule. The last thing that we need to do then is check that our value function guess was correct and find the equilibrium investment rule.

From equation 19 we can see that our value function, given the investment rule, will be of the form,

$$V (y_{-1}) = ln (1 - \lambda) + lny_{-1} + \frac{1}{1 + \delta} \left\{ A [\mu_y + [\beta_1 + (\alpha_x + \alpha_y) \beta_2] (lny_{-1} - \mu_{y_{-1}})] + B \right\}$$

This is of the desired form, $V (y_{-1}) = A lny_{-1} + B$, and so verifies that our guess was correct where,

$$A = \frac{1 + \delta}{1 + \delta - [\beta_1 + (\alpha_x + \alpha_y) \beta_2]}$$ (25)

and,
\[
B = \frac{1 + \delta}{\delta} \ln (1 - \lambda) + \frac{A}{\delta} \left\{ \mu_x + \frac{\beta_2 \sigma_{s_1}^2}{2} - [\beta_1 + (\alpha_x + \alpha_y) \beta_2] \mu_{y-1} \right\} 
\] (26)

\(A\) is the marginal value of log income and is constant in income. It is increasing in \(\beta_1 + (\alpha_x + \alpha_y) \beta_2\) which, from equation 24, is the extent to which log income is passed on to the following generation. The equilibrium investment rule is given by equation 20. By substituting for \(A\), we find the investment rule as a function of the parameters of the model and variance of log income in the parents’ generation (a state variable).

\[
\hat{x} (y-1) = \frac{\alpha_x \beta_2}{1 + \delta - (\beta_1 + \alpha_y \beta_2)^{y-1}} 
\] (27)

In order for the model to be a reasonable simplification of how individuals act, we must constrain \([\beta_1 + (\alpha_x + \alpha_y) \beta_2]\) to be less than \((1 + \delta)\). This ensures that: an individual’s income has a positive effect on his value function; and there is an interior solution to the consumption-investment problem whereby the fraction of income invested (or consumed) is between 0 and 1. We can see, to some extent, what effect this constraint will have. \(\delta\) measures the extent to which parents care for their children’s expected wellbeing versus their own, \(\alpha_y\) and \(\alpha_x\) measure the returns to parental income and investment in terms of developing talent, and \(\beta_1\) and \(\beta_2\) the (endogenous) returns to the parental income and talent signals in terms of income. In order to ensure an interior solution to the parent’s consumption-investment problem we constrain their willingness to invest in their child or the advantages of coming from a rich background to be below some upper bound.\(^\text{6}\)

Since we have not established how \(\beta_1\) and \(\beta_2\) change with the various parameters of the model, we will investigate how changes in the parameters feed through to the investment rule in the discussion section of this chapter. The parameter \(\delta\) plays no role in \(\beta_1\) and \(\beta_2\) so we can say at this stage that increases in the discount rate lead to a smaller share of parental income being invested. This is very intuitive.

9 The investment rule

The equilibrium investment rule is one of the novel features of this chapter. In chapter 2 we considered the effects of marginal decreases in \(\sigma_{s_1}^2\) and \(\sigma_{s_2}^2\), exogenously increasing the precision of signals 1 and 2 respectively. The result which we found was that, although \(\beta_1\)

\[\text{6The largest value that } \beta_1 + (\alpha_x + \alpha_y) \beta_2 \text{ can take is } \alpha_x + \alpha_y. \text{ This occurs as } \sigma_{s_2}^2 \text{ grows towards infinity. } \alpha_x + \alpha_y < 1 + \delta \text{ is a sufficient condition to ensure that } A > 0 \text{ and } 0 < x < 1.\]
and $\beta_2$ did not necessarily shift in the same direction for a given change, the overall effect of an increase in the precision of either signal was an increase in inequality and a fall in mobility. We are now going to consider the effects of these same changes on the payment rule of firms in more detail.

Firms’ response to changes in the precision of our signals can be considered in much the same way as consumers respond to a price fall in standard consumer choice theory. According to consumer choice theory with two normal goods, a fall in the price of one good leads to both a substitution effect and an income effect. The good which experiences the price fall unequivocally experiences increased consumption, but there are two offsetting effects on the consumption of the other good: it is now relatively more expensive but the consumer effectively has higher income.

The actions of firms in our model are analogous to that of a consumer in consumer choice theory. The direct effect of an increase in the precision of one signal is a shift toward using that signal at the expense of the other one. This can be seen from equations 22 and 23 where $\sigma^2_{y-1}$ is fixed. This is our “substitution” effect. We know from chapter 2, however, that the long run effects of such a change are that the economy converges to a higher level of steady state income variance (or in this chapter, variance in log income), endogenously increasing the precision of both signals, and increasing the firms’ use of both. This is our “income” effect. The net result is that the signal for which precision exogenously increased should unambiguously be used more by the firm in their payment rule, but the other signal may experience increased or decreased use.

$\beta_1$ and $\beta_2$ both play important roles in the investment decision of parents. First consider equation 20. This illustrates the “direct” role of $\beta_2$. If the firm uses the talent signal more, there is an improved channel through which to influence the income of your child. A parent takes advantage of that channel through investment. There is a further effect of both $\beta_1$ and $\beta_2$ through $A$, the marginal value of log income. $A$ is increasing in the use of both signals. An additional pound of income is more valuable, the more being rich is either directly or indirectly rewarded by firms. As a result parents will invest more in their children as $\beta_1$ and $\beta_2$ increase.

To see how this plays out in the model let us take an example. Suppose that there is an exogenous improvement in the precision of signal one. This must increase the use of the parental income signal in the firms’ payment rule, but suppose that the talent signal is used less. A parent may change their investment choice as a result of a number of forces which come into play. Most directly, as the talent signal is used less by firms, a parent’s ability to influence their child’s income is reduced, reducing their incentive to invest. This also reduces the opportunity to improve the prospects of their grandchildren through increasing the funds available for investment of their parents. On the other hand, since firms are using the parental income signal more, there is an incentive to attempt to raise the income
of your child since the advantages of a privileged background for your grandchildren, great-grandchildren and so on, are improved. Any change in the investment choice thus weighs up the reduced effectiveness of investment in generating talent and income in subsequent generations with the increased ability to pass on advantages through birth if such advantages can be established. It is not clear which of these effects will dominate.

10 Steady states of the economy

As in chapter 2, I will be primarily interested in how the economy is influenced by exogenous changes in meritocracy and advantage. I will examine this issue again within the new framework of chapter 3. In order to do so, I will deal with three separate cases of the economy. In the first, there will be “weak” inheritance, by which I mean that \( \alpha_x + \alpha_y < 1 \). The technology of talent development is such that parent’s income and investments play a relatively limited role and so who your parents are should be relatively less important. As in chapter 2, there will no multiplicity of steady states in this instance. In the second case, inheritance takes on larger values \( (\alpha_x + \alpha_y \geq 1) \). This may lead to a multiplicity of steady states, and we will consider first the steady state where the firms give relatively little weight to the two signals. Our third case will examine when firm’s give relatively more weight to the two signals. As in chapter 2, when inheritance is sufficiently large this will be the only steady state.

10.1 Case 1: The steady state of a weak inheritance economy

As mentioned above, a weak inheritance economy will be defined as one for which \( \alpha_x + \alpha_y < 1 \). We will first focus on two things: the steady state mean log income level; and the steady state value of the variance in log income. These in turn will allow us to calculate the steady state mean income, variance in income, correlation between parent’s and children’s income, and expected value or utility.

We will begin with the variance in the log of income. In order to find the steady state value, we begin with the income process described in equation 24. The variance of this equation provides a relationship between the variance in log income of adjacent generations, and the condition \( \sigma_y^2 = \sigma_{y-1}^2 \) allows this to be solved for the steady state variance of log income. I will use a * to denote steady state values.

\[
\sigma_{y*}^2 = \beta_1^2 \sigma_{t1}^2 + \beta_2^2 \left( \sigma_{t2}^2 + \sigma_{t3}^2 \right) \frac{1}{1 - (\beta_1^2 + (\alpha_x + \alpha_y) \beta_2^2) (\sigma_{t1}^2)}
\]

(28)
\(\beta_1^*\) and \(\beta_2^*\) are the steady state solutions to equations 22 and 23. It should be noted that finding a steady state of this model requires a solution to the same three equations in three unknowns as finding a steady state to the model of chapter 2, but with the parameter \(\alpha\) replaced with \(\alpha_x + \alpha_y\). The results of chapter 2 hold for the variance of the log of income and the correlation of the log of income across generations.

The steady state value of the mean of log income is found by taking expectations of the log income equation (24) and then finding the value \(\mu_y^*\) for which

\[
\mu_y^* = \frac{1}{1 - (\alpha_x + \alpha_y)} \left\{ \phi + \alpha_x \ln \left( \frac{\alpha_x \beta_2^*}{1 + \delta - (\beta_1^* + \alpha_y \beta_2^*)} \right) + \frac{\beta_2^* \sigma^2_{\epsilon s}}{2} \right\}
\]

Equations 28 and 29 give us the steady state variance and mean of the log of income. We are not primarily interested in these values. They do however allow us to calculate the mean and variance in income. From the definition of the mean of a log-normally distributed variable, we can find the steady state mean level of income.

\[
E[y] = e^{\mu_y^* + \frac{\sigma_y^*}{2}}
\]

\(E[\cdot]\) is the expectations operator. The mean of income is increasing in both the mean and variance of log income. For \(\alpha_x + \alpha_y\) less than one, both of these have a constant finite steady state value and so steady state mean income has a constant finite value.

We can use the mean and variance of log income is a similar way to find the variance of income. Again, appealing to the fact that income is a log-normally distributed variable we find,

\[
\text{Var}[y] = \left( e^{\sigma_y^*} - 1 \right) e^{2\mu_y^* + \sigma_y^*}
\]

It is also increasing in both the mean and variance of log income and, for \(\alpha_x + \alpha_y\) less than one, will have a constant finite value.

In order to find the steady state value of the correlation of income across generations

\footnote{Were \(\beta_1^* + (\alpha_x + \alpha_y) \beta_2^*\) to be greater than or equal to one \(\beta_1^* + (\alpha_x + \alpha_y) \beta_2^*\) would equal \(\alpha_x + \alpha_y\). A necessary condition for this is \(\alpha_x + \alpha_y \geq 1\) and so we can be sure that our weak inheritance economy converges to a finite value steady state variance in log income with \(\beta_1^* + (\alpha_x + \alpha_y) \beta_2^* < 1\).}
we need to know not just the mean and variance of log income, but the covariance of the log of a parent’s and child’s income. We can see from equation 24 that this is given by 
\[(\beta_1^* + (\alpha_x + \alpha_y) \beta_2^*) \sigma_y^{2\ast}].\]

Finally, we are interested in the mean level of utility or value in the economy in steady state. This is the utility that an individual could expect to receive from behind the veil of ignorance.

\[E[V(y)]^\ast = A^* \mu_y^\ast + B^*\]  

Prior to knowing who their parents will be, an individual’s expected utility will be increasing in mean log income, \(A^*\) and \(B^*\). Both \(A^*\) and \(B^*\) are endogenous and given by equations 25 and 26 with \(\beta_1 = \beta_1^*\) and \(\beta_2 = \beta_2^*\).

Equations 30 through 33 define the summary statistics of our steady state economy when \(\alpha_x + \alpha_y < 1\). Section 9 provides some intuition for how the steady state values of \(\beta_1\) and \(\beta_2\) will change for an exogenous change in advantage or meritocracy, but we will again leave it to a numerical exercise to investigate these issues further. We will be particularly interested to see if there are cases when mean income increases but expected utility decreases. This could be possible if an exogenous shock caused income variance to increase and hence the risk which a risk averse agent would face from behind the veil of ignorance would be greater. It is possible that given the choice of being born into a country with relatively high or low mean income, an individual may favour the poorer country because income variance, and hence risk, would be reduced.

10.2 Case 2: The low steady state in a strong inheritance economy

If \(\alpha_x + \alpha_y \geq 1\) there are two possible steady states which could emerge. In the “low” steady state, \(\beta_1 + (\alpha_x + \alpha_y) \beta_2 < 1\). In this case the mean of log income will grow over time while the variance of log income will tend towards a constant finite value.

Let us begin with the variance. From equation 24 we can see that the variance in log income is given by,
\[ \sigma_y^2 = \left( \beta_1 + (\alpha_x + \alpha_y) \beta_2 \right)^2 \sigma_{y-1}^2 + \beta_1^2 \sigma_{\epsilon_{s1}}^2 + \beta_2^2 (\sigma_{\epsilon_{s2}}^2 + \sigma_{\epsilon_{\tau}}^2) \]  

(34)

Since \( \beta_1 + (\alpha_x + \alpha_y) \beta_2 < 1 \) this will tend towards the steady state value given by equation 28. In steady state, it will have a constant finite value. We will see, however, that this is not the case for the mean of log income.

The mean of log talent can be found by taking expectations of equation 15. Mean log income is then found by taking expectations of the income process in equation 24. Doing so, it is immediately apparent that \( \alpha_x + \alpha_y \) plays a key role.

\[ \mu_y = \phi + \alpha_x \ln \left( \frac{\alpha_x \beta_2}{1 + \delta - (\beta_1 + \alpha_y) \beta_2} \right) + (\alpha_x + \alpha_y) \mu_{y-1} + \frac{\beta_2 \sigma_{\epsilon_{s2}}^2}{2} \]  

(35)

With \( \alpha_x + \alpha_y > 1 \), the value of \( \mu_y \) will grow forever. There is steady state growth which will tend towards a constant rate of \( \alpha_x + \alpha_y - 1 \). Although the variance in log income does reach a constant finite value, and so log income still has a non-degenerate distribution, we can see from equations 28 and 29 how an ever increasing value of \( \mu_y \) will affect the mean and variance of income in steady state. As both are increasing functions of mean log income, they too will increase over time.

In order to examine the effects of an exogenous change in meritocracy or advantage in an environment where the income distribution is changing we will follow a slightly different method to that of case 1. Instead of examining the effects of a shock on our steady state, we will examine the effects of a shock given the current state of our economy. The state variables for case 2 will be \( \mu_{y-1} \) and \( \sigma_{y-1}^2 \) and these will become additional parameters of our numerical exercise.

One interesting aspect of this change in method is that, since \( \sigma_{y-1}^2 \) is fixed, we will be looking purely at the “substitution effect” of a change in meritocracy or advantage. As one signal is used more, the other will be used less. The effects of an exogenous shock to meritocracy and advantage will be examined for the generation in which the shock occurs. Our economy will be summarised by the same statistics as in case 1: mean income; income variance; the intergenerational correlation of incomes; and expected utility. Other than the fact that we are no longer examining steady state values, the equations for these will be the same as in section 10.1 for all but the intergenerational correlation. It will be given by,
\[
\rho_{y,y-1} = \frac{e^{(\beta_1 + (\alpha_x + \alpha_y) \beta_2) \sigma_y^2} - 1}{\sqrt{e^{\sigma_y^2} - 1} [e^{\sigma_y^2} - 1]}
\]

where \(\sigma_y^2\) is given in equation 34.

10.3 Case 3: The high steady state in a strong inheritance economy

If both \(\alpha_x + \alpha_y \geq 1\) and \(\beta_1 + (\alpha_x + \alpha_y) \beta_2 \geq 1\), it is clear from equations 34 and 35 that both the variance and mean of log income will be increasing over time. It is equally evident from equations 28 and 29 that this will mean that the steady state variance and mean of income will also be increasing. This being the case, the same approach will be followed as in section 10.2. For different values of the state variables, a numerical exercise will examine the effects of an exogenous shock to advantage or meritocracy on our economy at the time of the change.

11 The weak inheritance economy

As noted in the previous section, when \(\alpha_x + \alpha_y < 1\) the mean and variance of the log of income tend towards constant finite values in the long run. This in turn implies constant finite steady state values of all the key statistics which summarise the state of our economy. In what follows, we will examine how shocks to meritocracy and advantage translate into changes in these steady state values. Meritocracy is defined, as in chapter 2, as the precision in the talent signal, while advantage is the precision in the parental income signal. The numerical exercise below considers how marginal increases in both affect the steady state economy given certain values of the parameters. The parameters for this numerical exercise are given in table 2. There are 24,381 combinations of the parameters for which \(\alpha_x + \alpha_y < 1\).

The parameters \(\phi\) and \(\delta\) have been fixed since their role in the determination of investment and mean income is known. They do not affect \(\beta_1\) and \(\beta_2\) and so an increase in \(\delta\), the discount rate, can be seen from equation 27 to decrease investment and from equations 29 and 30 to lead to a decrease in mean income. When parents are investing less in developing talent, the mean income of the economy falls. In addition, since steady state \(A, B\) and mean income are all decreasing in \(\delta\), it must decrease expected value. \(\phi\) has no effect on investment but, since it is the constant term in the talent process, increases mean income.
Table 2: Parameter values for the numerical exercise in the weak inheritance economy

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values the parameter may take</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_y$</td>
<td>0.05, 0.15, 0.25, 0.35, 0.45, 0.55, 0.65, 0.75</td>
</tr>
<tr>
<td>$\alpha_x$</td>
<td>0.05, 0.15, 0.25, 0.35, 0.45, 0.55, 0.65, 0.75</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_1}^2$</td>
<td>2, 4, 6, 8, 10, 12, 14, 16, 18</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_2}^2$</td>
<td>2, 4, 6, 8, 10, 12, 14, 16, 18</td>
</tr>
<tr>
<td>$\sigma_{\tau}^2$</td>
<td>1, 2, 3, 4, 5, 6, 7</td>
</tr>
<tr>
<td>$\phi$</td>
<td>5</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 3: The number and proportion of cases where the steady state values increased with an exogenous increase in meritocracy in the strong inheritance economy. Total number of cases = 24,381

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Number of cases in which it increased</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[y]^*$</td>
<td>14,515</td>
<td>59.5</td>
</tr>
<tr>
<td>$Var[y]^*$</td>
<td>24,381</td>
<td>100.0</td>
</tr>
<tr>
<td>$\rho_{y,y-1}^*$</td>
<td>13,058</td>
<td>53.6</td>
</tr>
<tr>
<td>$E[V(y)]^*$</td>
<td>8,209</td>
<td>33.7</td>
</tr>
<tr>
<td>$E[x]^*$</td>
<td>18,219</td>
<td>74.7</td>
</tr>
</tbody>
</table>

11.1 The effects of an improvement in meritocracy in a weak inheritance economy

For this numerical exercise, the steady state was found for each of our 24,381 parameter combinations, then the value of $\sigma_{\epsilon_2}^2$ was lowered by 0.001, simulating an exogenous marginal improvement in the precision of the talent signal. The proportion of cases for which our key statistics increased is given in table 3.

There are some interesting findings in the table. Improvements in the information on talent do not universally lead to increases in mean income, and only lead to expected utility improvements in around one-third of cases. In the majority of cases, prior to discovering who their parents are, people would choose to be born in the economy with less precise information. Improvements in meritocracy also do not necessarily lead to increased investment. The rest of this section will discuss some of these findings.

11.1.1 The fall in mean income

The fall in mean income may seem surprising because we know that an improvement in meritocracy will cause the firm to place greater weight on the talent signal. This tends to increase the share of income that parents invest, which has a positive effect on average income and average talent. What is more, in over half the cases where mean income is falling, firms’ use of the parental income signal is increased in steady state, providing
further incentive to invest since improvements in children’s income will more easily be passed on to subsequent generations.

The circumstances under which these do not lead to an increase in mean income can be seen in equation 29. When $\alpha_x$ is small, the effect of investment on mean log income is weakened. When investment does not translate into increased talent, there is no way for parents to increase the income or expected utility of their children and so they do not invest, even if $\beta_2$ is high. The driving force of falls in mean income tends to be the last term in equation 29. This comes from the variance in the distribution of talent, conditional on the signals. Additional information provided by the talent signal causes this to fall, and all the more so for large values of $\beta_2$.

An example of an economy likely to exhibit falling steady state mean income would be one with a high value of $\sigma^2_{\epsilon \tau}$ but a low value of $\alpha_x$. The high value of $\sigma^2_{\epsilon \tau}$ leads to a large amount of variance in talent, and a high value of $\beta_2$. However, the low value of $\alpha_x$ means that parents do not invest to any great extent, despite the high value of $\beta_2$, and do not respond to increases in $\beta_2$ with increased investment. This is an economy characterised by low investment, talent distributed to a large extent by luck, and firms using the talent signal a great deal.

It is also the case that mean income may be falling despite increases in mean investment. This happens in close to 6,700 cases. It must be the case in these situations that the share of income being invested in children is increasing but it is not translating to any great extent into improvements in income. Since parents know that this will happen, it is likely that the increases in investment are small. The conditions for which this occurs are the same as for a fall in income in general. A small value of $\alpha_x$ prevents the increased investment share from impacting on income while a large $\beta_2$, induced by a large exogenous variance in talent, causes the falling $\sigma^2_{\epsilon \tau}$ to pull down mean income.

A fall in mean investment itself may seem odd when faced with an environment where talent is more easily displayed. This fall in investment is always associated with a fall in mean income, and so it is the fact that mean income can be lower in more meritocratic societies which may push down mean investment.

11.1.2 The fall in expected value

The fact that individuals may choose to live in a country where their talent is more difficult to display may also seem somewhat surprising. This is especially so since in over 6,000 cases where expected value is falling, mean income is increasing. From behind the veil

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8To provide some evidence of this, there are nearly 3,000 cases of falling mean income when $\sigma^2_{\epsilon \tau}$ is 7 compared to only 19 when it is 1, and over 3,000 cases when $\alpha_x$ is 0.05 compared to 217 when it is 0.75.
of ignorance, people would be happier choosing to be born prior to the improvement in meritocracy into the poorer economy. Why this might occur has to do with the increasing variance in income. The value function displays decreasing marginal utility from income and hence risk or inequality aversion. The increase in mean income has to offset the increased variance in income for an individual to be better off.

The reasons why this might not occur again boil down to the parameter values which tend to accompany it. Once more, the inheritance parameters are relatively low and the exogenous variance in talent high. If these conditions do not cause mean income to fall, they are likely to cause mean income gains to be small. That, coupled with the difficulty in passing those gains onto children and grandchildren, will make them relatively less valuable and more likely to be overshadowed by the increased variance.

11.1.3 The fall in the intergenerational correlation of income

The fall in the correlation on incomes is a new feature to this chapter. The correlation of the log of income and the log of parental income is increasing, as in chapter 2, but this does not necessarily translate into an increase in the correlation of income and parental income. It is given by equation 32. Since a fall in $\sigma^2_{\epsilon s}$ will inevitably lead to the firm using the talent signal more, one simple way to nullify the effect of this on the correlation is to have little or no inheritance (a small value of $\alpha_x + \alpha_y$).

There is a common theme that has emerged over the course of this section: occasions where mean income falls while information is more readily available or investment increasing, expected value falls in the face of increasing income, or mobility increases despite increased use of the talent signal, all feature low inheritance (particularly $\alpha_x$) and high exogenous variance in talent. In some respects these could be thought of as healthy economies. The lack of inheritance will tend to push up intergenerational mobility, and the negative effect which it has on investment will be offset by the high variance in talent propping up $\beta_2$. In other ways they could be thought off as unhealthy. The low value of $\alpha_x$ implies the technology for turning investment into talent (the private education system) is inefficient. In any case, and for various reasons outlined above, these economies display some interesting features.

An interesting case is illustrated in figure. Here $\alpha_x$ and $\alpha_y$ are 0.35 and $\sigma^2_{\epsilon\tau}$ is 2.5. Higher value of the exogenous variance in talent, and lower values of the inheritance parameters, will tend to cause mean income to be falling across the whole range of meritocracy values. We see here that it is only falling at high values of meritocracy, which coincide with higher values of $\beta_2$. This illustrates the role of $\beta_2$ in pulling down mean income as meritocracy increases. The implications of the previous discussion were that, if $\sigma^2_{\epsilon\tau}$ was not large enough, nor inheritance small enough, to cause mean income to be
falling as meritocracy increases, they could still pull down expected value. This is exactly what happens when meritocracy is increasing from a low value.

11.2 The effects of an increase in inherited advantages in a weak inheritance economy

The results of the numerical example investigating the effects of a fall in $\sigma_{e_{s1}}^2$ of 0.001 are shown in table 4. In all cases firms use the parental income signal more, but the talent signal less. Intuitively the share of income invested by parents could go up or down. However, the intergenerational correlation of incomes always increases. We saw in the previous section that increased precision in the talent signal, which tends to be correlated with parental income, was not always enough to induce falls in mobility. Improved information about background is, on every occasion, for this parameter set.

11.2.1 The fall in mean income and variance

We know from equations 30 and 31 that the steady state mean and variance of income are both determined by the steady state mean and variance of the log of income. From
<table>
<thead>
<tr>
<th>Statistic</th>
<th>Number of cases in which it increased</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[y]^*$</td>
<td>16,338</td>
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<td>$\text{Var}[y]^*$</td>
<td>24,076</td>
<td>98.7</td>
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<tr>
<td>$\rho_{y,y-1}^*$</td>
<td>24,381</td>
<td>100.0</td>
</tr>
<tr>
<td>$E[V(y)]^*$</td>
<td>8,890</td>
<td>36.5</td>
</tr>
<tr>
<td>$E[x]^*$</td>
<td>20,616</td>
<td>84.6</td>
</tr>
</tbody>
</table>

Table 4: The number and proportion of cases where the steady state values increased with an exogenous increase in inherited advantages in the weak inheritance economy. Total number of cases = 24,381

In chapter 2 we know that the variance in the log of income will be increasing. A fall in the mean or variance in income then requires a fall in the mean of log income.

If the share of income being invested is increasing, a small value of $\alpha_x$ will nullify that, while a large fall in $\beta_2$ would push down the last term in equation 29. At least for the parameter values we are investigating, a large fall in $\beta_2$ is induced by a large value of $\beta_2$. There are at least two reasons why firms may respond a great deal to the talent signal. A large amount of exogenous variance in talent would create an incentive for firms to use it. Also, a high precision would make it more informative, although as a counter to this a small value of $\sigma_{\epsilon_x}^2$ will tend to nullify the effects of the fall in $\beta_2$. A small value of $\sigma_{\epsilon_x}^2$ does lead to falls in the mean and variance of income, although a large amount of exogenous variance in talent may have an even stronger effect.

As in the previous section, a small value of $\alpha_x$ and large amount of exogenous variance in talent are the conditions under which a fall in mean income can occur in the face of increasing mean investment. The effect of an increased share of income being invested in children is reduced by an inability to turn that into talent. The lack of a strong investment effect will make it more likely that the fall in $\beta_2$ which accompanies an increase in advantage will pull down mean income.

11.2.2 The fall in expected value

There are just under 7,500 cases where expected value falls while mean income is increasing. The conditions for this are the same as in the case of an increase in meritocracy. An example where $\alpha_x$ and $\alpha_y$ are 0.05, and $\sigma_{\epsilon_x}^2$ 7, is illustrated in figure 10. Although the additional information may raise your expected income, it will also raise the uncertainty you face over that income (if mean income is increasing, the variance in income is also increasing). The increase in mean income has to make you sufficiently happier to overcome

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9 The mean and variance of income fall in 18 and zero cases respectively when $\sigma_{\epsilon_x}^2$ is 1 but in 2,410 and 94 cases when it is 7. The mean and variance fall in 1,594 and 259 cases respectively when $\sigma_{\epsilon_x}^2$ is 2, but only 419 and zero when it is 18 (in fact for any value of $\sigma_{\epsilon_x}^2$ greater than 6 we find that variance is always increasing).
Figure 10: The effects of increasing advantage on mean income, income variance, the intergenerational correlation of incomes and expected value: $\alpha_x = 0.05; \alpha_y = 0.05; \sigma_{\epsilon_1}^2 = 12; \text{ and } \sigma_{\epsilon_2}^2 = 7$

the increased risk of low income.

11.3 A discussion of the weak inheritance economy

Some of the more interesting results derived in this section have had at their heart a great deal of similarity in the parameters which bring them about. One of the key things is a lack of inheritability, particularly a low value of $\alpha_x$. This implies that private investments made in children do not pay off. The private education system is inefficient. Even when mean investment increases it may not lead to increases in expected income since that investment is ineffective and, even when it does raise mean income, it may not improve the utility which an individual expects to receive since those income gains cannot easily be passed on to the next generation and the improvements in the information available to firms create greater uncertainty. Large values of $\sigma_{\epsilon_2}^2$ also feed into this. Therefore, in an economy where people display a wide range of talents, but it is difficult to influence the talent of your child and hence pass on advantages, you may prefer a lower mean income if increases in that mean income reflect increases in meritocracy and advantage that bring about greater income variance.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values the parameter may take</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_y$</td>
<td>0.3, 0.4, 0.5, 0.6, 0.7</td>
</tr>
<tr>
<td>$\alpha_x$</td>
<td>0.3, 0.4, 0.5, 0.6, 0.7</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_1}^2$</td>
<td>2, 6, 10, 14, 18</td>
</tr>
<tr>
<td>$\sigma_{\epsilon_2}^2$</td>
<td>2, 6, 10, 14, 18</td>
</tr>
<tr>
<td>$\sigma_{\tau_1}^2$</td>
<td>1, 3, 5, 7</td>
</tr>
<tr>
<td>$\mu_{y-1}$</td>
<td>2, 4, 6, 8, 10</td>
</tr>
<tr>
<td>$\sigma_{y-1}^2$</td>
<td>2, 4, 6, 8, 10</td>
</tr>
<tr>
<td>$\phi$</td>
<td>5</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 5: Parameter values for the numerical exercise in the strong inheritance economy

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Number of cases in which it increased</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[y]$</td>
<td>37,500</td>
<td>100.0</td>
</tr>
<tr>
<td>$Var[y]$</td>
<td>37,500</td>
<td>100.0</td>
</tr>
<tr>
<td>$\rho_{y,y-1}$</td>
<td>34,500</td>
<td>92.0</td>
</tr>
<tr>
<td>$E[V(y)]$</td>
<td>34,510</td>
<td>92.0</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>37,500</td>
<td>100.0</td>
</tr>
<tr>
<td>$E(x)$</td>
<td>37,500</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 6: The number and proportion of cases where the steady state values increased with an exogenous increase in meritocracy in the strong inheritance economy. Total number of cases = 37,500

12 The strong inheritance economy

The numerical exercise which we will undertake for the strong inheritance economy ($\alpha_x + \alpha_y > 1$) is slightly different to what we examined in the weak economy case. This is because, although it may converge to a constant finite value of the variance in log income, the economy will not converge to a steady state finite value of the mean and variance in income. Both will experience growth. What we will do instead of examining the steady states is to look at the shift in the level of our key variables when there is an exogenous shock to meritocracy and advantage, given the current state of the economy. It is the shift at the time of the change for that generation. For this we need two more parameters which are our state variables: $\mu_{y-1}$, the mean of the log of parental income; and $\sigma_{y-1}^2$ the variance of the log of parental income. The parameters used are given in table 5. There are 37,500 combinations of these parameters for which $\alpha_x + \alpha_y$ is greater than one.

12.1 The effects of an improvement in meritocracy in a strong inheritance economy

The results for the exercise with a fall in meritocracy are shown in table 6.
12.1.1 The fall in the intergenerational correlation of income

The fall in the intergenerational correlation of incomes was always more likely to occur in this exercise than that performed on the steady states in the previous section. This is because, in the steady state case, the substitution effect which causes a shift towards the talent signal was accompanied by a feedback effect whereby increased variance of the log of income caused both signals to be used more. This feedback effect was likened to the income effect from a fall in an equilibrium price. As it pushes up the use of both signals by the firm, it decreases mobility. In the current case, where we examine the effect in one generation, it is not present. There is therefore less reason to believe mobility should necessarily fall.

The 3,000 cases where intergenerational mobility increases require $\alpha_x + \alpha_y = 1$ and $\sigma^2_{\epsilon\tau} = 1$ (the lowest value that both can take). These are the only cases for these parameters for which the variance in log income would converge to a finite value. In the language of chapter 2, the economy would converge to the “low” steady state where multiplicity exists. Low values of $(\alpha_x + \alpha_y)$ and $\sigma^2_{\epsilon\tau}$ tend to be associated with relatively little use of the signals by firms and high mobility. Further evidence that such circumstances may be important are provided by the fact that increases in mobility also require $\sigma^2_{\epsilon s_1} > 2$. So in societies where there is growth in income but, given this, little inheritance, little incentive to invest, and high mobility, an improvement in meritocracy may lead to further increases in mobility.

Since there were so few cases of the “low” steady state for the parameter set given in table 6, a further experiment was carried out where $\sigma^2_{\epsilon s_1}$ and $\sigma^2_{\epsilon s_2}$ could take values from \{16,18,20,22,24\} and $\sigma^2_{\epsilon\tau}$ from the range \{1,2,3,4\}. This increased the occurrences for which the economy would converge to a steady state with finite variance in log income to 29,575. In all of these cases, the correlation of incomes across generations was falling. In all other cases it was increasing. So the conditions which bring about convergence to a finite value of variance in log income also bring about improvements in mobility following a shock to meritocracy.

12.1.2 The fall in expected value

A fall in the value which an individual expects to derive from his income is associated with three features of the parameters: there is more exogenous variance in talent ($\sigma^2_{\epsilon\tau}$ is high); the mean of the log of parental income is higher ($\mu_{y-1}$ is high); and the precision in the parental income signal is higher ($\sigma^2_{\epsilon s_1}$ is low).

The exogenous variance of talent is familiar from the weak economy cases. The other two affect income variance. As $\mu_{y-1}$ increases, current income variance increases by more
Table 7: The number and proportion of cases where the steady state values increased with an exogenous increase in advantages in the strong inheritance economy. Total number of cases = 37,500

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Number of cases in which it increased</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[y]$</td>
<td>25,935</td>
<td>69.2</td>
</tr>
<tr>
<td>$Var[y]$</td>
<td>37,500</td>
<td>100.0</td>
</tr>
<tr>
<td>$\rho_{y,y-1}$</td>
<td>34,500</td>
<td>92.0</td>
</tr>
<tr>
<td>$E[V(y)]$</td>
<td>37,500</td>
<td>100.0</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>26,250</td>
<td>70.0</td>
</tr>
<tr>
<td>$E(x)$</td>
<td>25,955</td>
<td>69.2</td>
</tr>
</tbody>
</table>

12.2 The effects of an increase in inherited advantages in a strong inheritance economy

The result of a fall in inherited advantages is shown in table 7

12.2.1 The increase in expected value

There are 11,565 cases where, following an exogenous improvement in the ability to pass on advantages through signalling, people expect to be happier yet to have lower incomes and experience more uncertainty over their income. There are two ways to increase the utility of your descendants: you can invest in them at the expense of your own consumption; or you can pass on advantages through the talent or signalling processes just by having a high income. Since they do not require any reduction in your own consumption, the latter means of creating advantage are particularly valuable and in some cases lead people to prefer a world in which their income will be lower on average but create greater advantages for their children.

This also feeds into investment decisions. A fall in $\sigma^2_{cs1}$ inevitably leads to a fall in $\beta_2$ and hence reduces the returns to investment and the ability of parents to pass on advantages by sacrificing their own consumption. However, any increases in income which can be created for your children will be costlessly passed on to your grandchildren,
great-grandchildren and so on, to a greater extent. This provides an incentive to increase investment, which we see in more cases than not.

12.2.2 The fall in the intergenerational correlation of incomes

The improvement in intergenerational mobility in 3,000 cases is notable because they are the same 3,000 cases for which mobility increased following an improvement in meritocracy. We established then that the parameters values which lead to improvements in mobility following a shock are the same ones which lead to convergence to the “low” steady state. We can add to that the finding that the source of the shock, whether it be from meritocracy or inherited advantages, is irrelevant. This was further investigated using the higher parameter values for $\sigma^2_{\xi_1}$ and $\sigma^2_{\xi_2}$, and lower values for $\sigma^2_{\tau}$, outlined in section 12.1.1. The same 29,575 cases where the correlation increased were found as in the case of a shock to meritocracy.

13 Conclusions

There were two main parts to this chapter. The first was to provide a forward-looking component to the model of chapter 2, and consider the equilibrium investment rule of parents. This brings the model much more closely in line with the existing statistical discrimination literature, although the multiplicity of equilibria present in those models does not feature here. In this model there is a unique equilibrium investment rule, but the multiplicity of steady states founds in chapter 2 remains. The results found in that chapter translate directly into increases in the variance of log income, and correlation of the logs on income and parental income, as meritocracy and advantage increase.

We are not primarily interested in the mean, variance or correlations of the log of income. Our main concern is with the distribution of income itself. The second part of this chapter looked at how the mean and variance of income changed with exogenous shocks to meritocracy and advantage within this new framework. It also considered mobility and expected utility.

In many cases, though not all, we found that variance and mobility still moved in opposite directions. This was particularly true for exogenous increases in inherited advantages. There were also a number of interesting results relating to a particular subset of the parameters. These were when inheritance was weak (particularly $\alpha_{xz}$, such as in the case where the private education system is inefficient) and there was a large amount of exogenous variance in talent. In this case, mean investment might fall despite talent being more easily recognised, mean income may fall despite mean investment increasing,
and expected value might decrease despite expected income going up. It is the last of these that is probably the most interesting. Meritocracy and inherited advantages both increase variance and hence, prior to birth, create greater uncertainty over the income you will receive. A risk averse person might prefer to be born into an economy where they expect to be poorer but avoid this increased uncertainty, and so despite raising incomes, meritocracy and inherited advantages may make agents, on average, more unhappy.

14 Bibliography


