Factor Specificity and Real Rigidities*

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Abstract

We develop a multisector model in which capital and labor are free to move across firms within each sector, but cannot move across sectors. To isolate the role of sectoral specificity, we compare our model with otherwise identical multisector economies with either economy-wide factor markets (as in Chari et al. 2000) or firm-specific factor markets (as in Woodford 2005). Sectoral specificity induces within-sector strategic substitutability and across-sector strategic complementarity in price setting. Our model can produce either more or less monetary non-neutrality than those other two models, depending on the distribution of price rigidity across sectors. Under the empirical distribution for the U.S., our model behaves similarly to an economy with firm-specific factors in the short-run, and later on approaches the dynamics of the model with economy-wide factor markets. This is consistent with the idea that factor price equalization might take place gradually over time, so that firm-specificity might be a reasonable short-run approximation, whereas economy-wide markets might be a better description of how factors of production are allocated in the longer run.

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1 Introduction

Much of the Monetary Economics literature tries to make sense of the extent of monetary non-neutrality that is apparent in the data. An important part of this literature does so by resorting to models in which prices (and sometimes wages) are sticky. A problem with bare bone versions of these models is that the degree of price rigidity required to generate substantial non-neutrality is at odds with the microeconomic evidence on the frequency of price changes. However, since Ball and Romer (1990) and Kimball (1995), it is well-known that large real rigidities – which can induce strategic complementarities in price-setting decisions – can generate substantial endogenous persistence in the real effects of monetary shocks, and thus help bridge this gap.\footnote{Other such mechanisms have to do with information frictions, heterogeneity in price rigidity etc.}

In a series of contributions to our understanding of the sources of real rigidities, Woodford (2003, 2004, 2005) argues forcefully that factor specificity matters. In particular, Woodford (2005) develops a model in which both capital and labor are specific to firms – i.e., they cannot move freely from one firm to another.\footnote{To be precise, Woodford’s (2003, 2005) models features industry-specific labor coupled with assumptions that make it mathematically equivalent to a particular model with firm-specific labor models, as will become clear subsequently.} He shows that factor specificity at the firm level is a powerful source of real rigidities.

The assumption of firm-level specificity contrasts sharply with the (usually unstated) assumption that factors of production can move freely across firms, as in the Real Business Cycle literature. Under standard assumptions about preferences and technology, such economy-wide factor markets tend to induce strategic substitutability in price setting (e.g., Woodford 2003, chap. 3), and thus generate a small degree of monetary non-neutrality (Chari et al. 2000).

The two alternative assumptions about factor markets are, to some extent, unrealistic. It is likely that factor price equalization takes place gradually over time, so that firm-specificity might be a reasonable short-run approximation, whereas economy-wide markets might be a better description of how factors of production are allocated in the longer run.

In this paper, we study whether the nature of factor specificity matters. To that end, we develop a multisector model in which capital and labor are free to move across firms \textit{within} each sector, but cannot move \textit{across} sectors – i.e., factors of production are sector-specific. To isolate the role of sectoral specificity, we compare our model with otherwise identical multisector economies with either economy-wide factor markets (as in Chari et al. 2000) or firm-specific factor markets (as in Woodford 2005).

It turns out that it matters a great deal whether factor markets are specific at the firm or at the sector level. Sectoral factor specificity does not induce real rigidities as in the case of firm-specific factors. In fact, it generates \textit{within-sector strategic substitutability} in pricing decisions. This tends
to reduce the degree of monetary non-neutrality relative to the case of firm-specific factors. At the same time, sectoral relative price movements generate distributional effects that induce strategic complementarity in pricing decisions across sectors, relative to the model with economy-wide factor markets. Our sector-specific factor model can produce either more or less monetary non-neutrality than those other two models, depending on the distribution of price rigidity across sectors. Under the empirical distribution for the U.S., our model behaves similarly to an economy with firm-specific factors in the short-run, and later on approaches the dynamics of the model with economy-wide factor markets.

As shown by Woodford (2005), firm-specific capital is a powerful source of real rigidities, and thus tends to induce strategic complementarities in firms’ pricing decisions. To understand the mechanism at work, consider first the case of economy-wide factor markets. This market structure tends to make firms’ pricing decisions strategic substitutes rather than complements. This is because firms that do not respond to an increase in aggregate demand with a price increase need to employ disproportionately more production inputs. This puts upward pressure on factor prices, and leads firms that do adjust prices in response to the demand increase to set relatively higher prices. This cost pressure transmitted through common factor markets tends to speed up the response of the aggregate price level to an increase in nominal demand, thus decreasing the persistence of its real effects on output and other real variables.

Now let us consider the case of firm-specific factor markets. Under the same circumstances, adjusting firms will have less of an incentive to increase their prices. The reason is that marginal costs no longer depend on common factor prices, but are instead specific to each firm. Consider an increase in nominal aggregate demand as in our analysis of economy-wide factor markets. Let us assume that adjusting firms choose to increase prices by as much as they would in that case. With firm-specific factors, the relative decrease in the quantity demanded from each adjusting firm, which is induced by higher prices, puts downward pressure on (firm-specific) factor prices, relative to the case of economy-wide factor markets. This makes it suboptimal for firms to raise prices by as much as in that case. In other words, firm-specific factors lead firms that change prices to keep them closer to other firms’ unchanged prices. This comparison shows that factor specificity at the firm level induces a complementarity (or weakens the degree of substitutability) in pricing decisions. The different implications of factor market structures for the interactions between firms’ pricing decisions is what leads to higher persistence of the real effects of nominal disturbances in Woodford’s model.

So why is this not the case when factors are sector as opposed to firm specific? Once again, the reason has to do with the implications of factor market structure for the interdependence between firms’ pricing decisions. To understand the mechanism at work when factors are sector specific, consider again an increase in demand. Firms that do not respond with a price increase need to
employ disproportionately more production inputs. This puts upward pressure on sectoral factor prices, and leads firms that do adjust prices in response to the demand increase to set relatively higher prices. This cost pressure transmitted through sectoral factor markets induces within-sector strategic substitutability in pricing decisions, muting the real effects of nominal shocks. Note that the mechanism at work is very similar to the case of economy-wide factor markets, but applies at the sectoral level. Indeed, if all sectors are identical, these two models become indistinguishable. At the same time, consider an increase in prices everywhere in the economy, except in a given sector. This shifts demand in favor of that sector, and puts upward pressure on its marginal cost – thus leading firms that change prices in that sector to increase their prices. This amounts to what we refer to as across-sector strategic complementarities in price setting. With heterogeneous sectors, the associated distributional effects contribute to richer aggregate dynamics, as we discuss in Sections 3 and 4.

While the assumption that factors cannot move across sectors is certainly extreme, our model is motivated by existing empirical evidence that both capital (e.g., Ramey and Shapiro 2001) and labor (e.g., Parent 2000) have an important degree of sector (or industry) specificity. Interestingly, our parameterized model delivers dynamics that are consistent with the idea that firm specificity might be a good short-run approximation, whereas full factor mobility might be a better description for the long run.

Section 2 presents the reference model of our multisector economy with sector-specific factors of production. It also presents the otherwise identical multisector models with either economy-wide or firm-specific factor markets. Section 3 describes the underlying new Keynesian Phillips curves for the three models and discusses their properties. Section 4 follows with a quantitative analysis of the effects of monetary shocks under the three types of factor specificity. The last section concludes.

2 Three models of factor specificity

2.1 Sector-specific factors

In this section, we consider a sticky-price DSGE model in which factor mobility is limited. Identical infinitely-lived consumers supply labor and capital to intermediate firms that they own, invest in a complete set of state-contingent financial claims, and consume a final good. The latter is produced by competitive firms that bundle varieties of intermediate goods. The monopolistically competitive intermediate firms that produce these varieties are divided into sectors that differ in their frequency of price changes. Labor and capital are the only variable inputs in the production of intermediate

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3Woodford sometimes refers to his assumption regarding labor markets as involving industry-specific (e.g., Woodford 2003, chapter 3) or sector-specific (e.g., Woodford 2005) labor markets. This is not the same as our assumption of sector-specificity. In Woodford’s work, an industry (or sector) is characterized by fully synchronized price setting, so that his assumption is mathematically equivalent to firm-specific labor markets – as long as firms behave competitively and do not try to exploit their monopsony power. In contrast, in our model, price changes are asynchronized.
goods and we assume that these inputs can be reallocated freely across firms in the same sector but cannot flow across sectors.

2.1.1 Consumers

The representative consumer maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^{1-\sigma} - 1}{1-\sigma} - \sum_{s=1}^{S} \omega_s N_t^{1+\gamma} \right)$$

subject to the flow budget constraint:

$$P_tC_t + P_tI_t + E_t [\Theta_{t,t+1}B_{t+1}] \leq \sum_{s=1}^{S} W_{s,t}N_{s,t} + B_t + T_t + \sum_{s=1}^{S} Z_{s,t}K_{s,t},$$

the law of motion for the stocks of sector-specific capital:

$$K_{s,t+1} = (1-\delta) K_{s,t} + \Phi (I_{s,t}, K_{s,t}) I_{s,t}, \forall s$$

$$I_{s,t} \geq 0, \forall s$$

and a standard “no-Ponzi” condition:

$$B_{t+1} \geq -\sum_{l=t+1}^{\infty} E_{t+1} \left[ \Theta_{t+1,l} \left( W_tN_t + \sum_{s=1}^{S} Z_{s,t}K_{s,t} + T_t \right) \right] \geq -\infty,$$

where $\Theta_{t,l} \equiv \prod_{t'={t+1}}^{l} \Theta_{t'-1,t'}, E_t$ denotes the time-$t$ expectations operator, $C_t$ is consumption of the final good, $N_{s,t}$ denotes total labor supplied to firms in sector $s$, $W_{s,t}$ is the associated nominal wage rate, and $\omega_s$ is the relative disutility of supplying labor to sector $s$.\(^4\) $I_{s,t}$ denotes investment in sector-$s$ capital, $I_t \equiv \sum_{s=1}^{S} I_{s,t}$, $K_{s,t}$ is capital supplied to firms in sector $s$, and $Z_{s,t}$ is the associated nominal return on capital. The final good can be used for either investment or consumption, and sells at the nominal price $P_t$. $B_{t+1}$ accounts for the state-contingent value of the portfolio of financial securities held by the consumer at the beginning of $t+1$. Under complete financial markets, agents can choose the value of $B_{t+1}$ for each possible state of the world at all times. $T_t$ stands for profits received from intermediate firms. A no-arbitrage condition requires the existence of a nominal stochastic discount factor $\Theta_{t,t+1}$ that prices in period $t$ any financial asset portfolio with state-contingent payoff $B_{t+1}$ at the beginning of period $t+1$.\(^5\) Finally, $\beta$ is the time-discount factor, $\sigma^{-1}$ denotes the intertemporal elasticity of substitution, $\gamma^{-1}$ is the Frisch elasticity of labor supply, $\delta$ is the rate of depreciation, and

\(^4\)As in Carvalho and Lee (2011), this parameter is only used to obtain a symmetric steady state, and simplify the algebra. It does not play a role in any of our findings. For details see the online Appendix.

\(^5\)To avoid cluttering the notation, we omit explicit reference to the different states of nature.
\( \Phi(\cdot) \) is the adjustment-cost function. We follow Chari et al. (2000), and assume \( \Phi(I_{s,t}, K_{s,t}) \) takes the following form:

\[
\Phi(I_{s,t}, K_{s,t}) = \Phi\left(\frac{I_{s,t}}{K_{s,t}}\right) = 1 - \frac{1}{2} \kappa \left(\frac{I_{s,t}}{K_{s,t}} - \delta\right)^2,
\]

which is convex and satisfies \( \Phi(\delta) = 1 \) and \( \Phi'(\delta) = 0 \) and \( \Phi''(\delta) = -\frac{\kappa}{\delta^2} \).

The first-order conditions for consumption and labor are:

\[
\frac{C_t}{C_{t+1}} = \beta^t \frac{P_t}{\Theta_{t,t} P_{t+1}},
\]

\[
W_{s,t} = \omega_s N^\gamma_s C^t_f, \forall s.
\]

For sector-\( s \) investment \( I_{s,t} \) and capital \( K_{s,t+1} \):

\[
Q_{s,t} = \beta E_t \left\{ \frac{C_{t+1}^{\sigma}}{C_t^{\sigma}} \left( \frac{Z_{s,t+1}}{P_{t+1}} + Q_{s,t+1} \left[ (1 - \delta) + \Phi'(\frac{I_{s,t+1}}{K_{s,t+1}}) \left( \frac{I_{s,t+1}}{K_{s,t+1}} \right)^2 \right] \right) \right\},
\]

\[
Q_{s,t} \left( \Phi'\left(\frac{I_{s,t}}{K_{s,t}}\right) \frac{I_{s,t}}{K_{s,t}} + \Phi\left(\frac{I_{s,t}}{K_{s,t}}\right) \right) = 1,
\]

where \( Q_{s,t} \) denotes Tobin’s \( q \) for sector \( s \).

The solution must also satisfy a transversality condition:

\[
\lim_{l \to \infty} E_t [\Theta_{l,t} B_l] = 0.
\]

### 2.1.2 Final goods firms

A representative competitive firm produces the final good, which is a composite of varieties of intermediate goods from both countries. Monopolistically competitive firms produce each variety of intermediate goods. The latter firms are divided into sectors indexed by \( s \in \{1, ..., S\} \), each featuring a continuum of firms. Sectors differ in the degree of price rigidity, as we detail below. Overall, firms are indexed by their sector \( s \), and are further indexed by \( j \in [0, 1] \). The distribution of firms across sectors is given by sectoral weights \( f_s > 0 \), with \( \sum_{s=1}^S f_s = 1 \).

The final good is used for both consumption and investment and is produced by combining the intermediate varieties according to the technology:

\[
Y_t = \left( \sum_{s=1}^S f_s^{\frac{\theta-1}{\sigma}} Y_{s,t}^{\frac{\theta-1}{\sigma}} \right)^{\frac{\theta}{\theta-1}},
\]

\[
Y_{s,t} = \left( f_s^{\frac{\theta-1}{\sigma}} \int_0^1 Y_{s,j,t}^{\frac{\theta-1}{\sigma}} dj \right)^{\frac{\theta}{\theta-1}},
\]
where \( Y_t \) is the final good, \( Y_{s,t} \) is the aggregation of sector-\( s \) intermediate goods, and \( Y_{s,j,t} \) is the variety produced by firm \( j \) in sector \( s \). The parameters \( \eta \geq 0 \), and \( \theta > 1 \) are, respectively, the elasticity of substitution across sectors, and the elasticity of substitution within sectors.

The representative final-good-producing firm solves:

\[
\max \ P_t Y_t - \sum_{s=1}^{S} f_s \int_{0}^{1} P_{s,j,t} Y_{s,j,t} dj
\]

\[ s.t. \quad (1)-(2), \]

which yields as first-order conditions, for \( j \in [0,1] \) and \( s = 1, ..., S \):

\[
Y_{s,j,t} = \omega \left( \frac{P_{s,j,t}}{P_{s,t}} \right)^{-\theta} \left( \frac{P_{s,t}}{P_t} \right)^{-\eta} Y_t.
\]

The price indices are given by:

\[
P_t = \left( \sum_{s=1}^{S} f_s P_{s,t}^{1-\eta} \right)^{\frac{1}{1-\eta}}, \tag{3}
\]

\[
P_{s,t} = \left( \int_{0}^{1} P_{s,j,t} dj \right)^{\frac{1}{1-\theta}}, \tag{4}
\]

where \( P_t \) is the price of the final good, \( P_{s,t} \) is the price index of sector-\( s \) intermediate goods, and \( P_{s,j,t} \) is the price charged by firm \( j \) from sector \( s \).

2.1.3 Intermediate goods firms

Monopolistically competitive firms produce varieties of the intermediate good by employing capital and labor. For analytical tractability, we assume that intermediate firms set prices as in Calvo (1983). The frequency of price changes varies across sectors, and it is the only source of (ex-ante) heterogeneity. In each period, each firm \( j \) in sector \( s \) changes its price independently with probability \( \alpha_s \). At each time a firm \( j \) from sector \( s \) adjusts its price, it chooses \( X_{s,j,t} \) to solve:

\[
\max E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1 - \alpha_s)^l \left[ -W_{s,t+l} N_{s,j,t+l} - Z_{s,t+l} K_{s,j,t+l} \right]
\]

\[ s.t. \quad Y_{s,j,t} = \left( \frac{P_{s,j,t}}{P_{s,t}} \right)^{-\theta} \left( \frac{P_{s,t}}{P_t} \right)^{-\eta} Y_t
\]

\[
Y_{s,j,t} = F (A_t, K_{s,j,t} N_{s,j,t}) = A_t (K_{s,j,t})^{1-\chi} (N_{s,j,t})^\chi.
\]

Optimal price setting implies:

\[
X_{s,j,t} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1 - \alpha_s)^l \Lambda_{s,t+l} \left( \chi K_{s,j,t+l}^{1-\chi} N_{s,j,t+l}^{\chi-1} \right)^{-1} W_{s,t+l}}{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1 - \alpha_s)^l \Lambda_{s,t+l}},
\]
where:

\[ \Lambda_{s,t} = \omega \left( \frac{1}{P_{x,t}} \right)^{\theta} \left( \frac{P_{x,t}}{P_{t}} \right)^{-\eta} Y_t. \]

Given the optimality (cost-minimization) conditions for capital and labor, the real marginal cost can be expressed as:

\[ MC_{s,j,t} = MC_{s,t} = \frac{W_{s,t}/P_t}{\chi \left( \frac{1-\chi}{\chi} Z_{s,t} \right)^{1-\chi}} \left( \frac{W_{s,t}}{P_t} \right)^{\chi} \left( \frac{Z_{s,t}}{P_t} \right)^{(1-\chi)}. \]

Note that marginal costs are equalized only within sectors. This is a direct implication of the assumption of sectoral capital and labor markets.

Finally, under that assumption, the market-clearing condition for capital and investment are:

\[ K_{s,t} = f_s \int_{0}^{1} K_{s,j,t} dj, \quad \forall \ s \]
\[ I_{s,t} = f_s \int_{0}^{1} I_{s,j,t} dj, \quad \forall \ s. \]

### 2.2 Firm-specific factors

We now consider a variant of the previous model in which production inputs are specific at the firm level. This version of the model generalizes Woodford (2005) to a multisector economy. It requires that we reformulate the consumers’ and intermediate firms’ problems. The maximization problem of final goods firms remains the same as in the model with sectoral factor markets.

#### 2.2.1 Consumers

The representative consumer maximizes:

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \sum_{s=1}^{S} f_s \int_{0}^{1} N_{s,j,t}^{1+\gamma} \frac{1}{1 + \gamma} dj \right), \]

subject to the flow budget constraint:

\[ P_tC_t + P_tI_t + E_t [\Theta_{t,t+1} B_{t+1}] \leq \sum_{s=1}^{S} f_s \int_{0}^{1} W_{s,j,t} N_{s,j,t} dj + B_t + T_t + \sum_{s=1}^{S} f_s \int_{0}^{1} Z_{s,j,t} K_{s,j,t} dj, \]

the law of motion for the stocks of each firm-specific capital:

\[ K_{s,j,t+1} = (1 - \delta) K_{s,j,t} + \Phi \left( I_{s,j,t}, K_{s,j,t} \right) I_{s,j,t}, \quad \forall \ s, j \]
\[ I_{s,j,t} \geq 0, \quad \forall \ s, j, \]
and a standard “no-Ponzi” condition:

\[ B_{t+1} \geq -\sum_{l=t+1}^{\infty} E_{t+l} \left[ \Theta_{t+l} \left( \sum_{s=1}^{S} \int_{0}^{1} W_{s,j,t} N_{s,j,t} dj + \sum_{s=1}^{S} \int_{0}^{1} Z_{s,j,t} K_{s,j,t} dj + T_{t} \right) \right] \geq -\infty, \]

where \( \Phi (I_{s,j,t}, K_{s,j,t}) \) now takes the form:

\[ \Phi (I_{s,j,t}, K_{s,j,t}) = \Phi \left( \frac{I_{s,j,t}}{K_{s,j,t}} \right) = 1 - \frac{1}{2} \left( \frac{I_{s,j,t}}{K_{s,j,t}} - \delta \right)^{2}. \]

The notation is the same as before, except that now \( N_{s,j,t} \) denotes total labor supplied to firm \( j \) in sector \( s \), \( W_{s,j,t} \) is the associated nominal wage rate, \( I_{s,j,t} \) denotes investment by firm \( j \) in sector \( s \), \( I_t = \sum_{s=1}^{S} \int_{0}^{1} I_{s,j,t} dj \), \( K_{s,j,t} \) is capital supplied to firm \( j \) in sector \( s \), and \( Z_{s,j,t} \) is the associated nominal return on capital.

The first-order conditions for consumption and labor are now:

\[ \frac{C_{t} - \sigma}{C_{t+1} - \sigma} = \beta \frac{P_t}{\Theta_{t+1} P_{t+1}}, \]

\[ \frac{W_{s,j,t}}{P_t} = N_{s,j,t}^{\gamma} C_{t}^{\sigma}, \forall s,j. \]

For all investment \( I_{s,j,t} \) and capital \( K_{s,j,t+1} \) types:

\[ Q_{s,j,t} = \beta E_t \left\{ \frac{C_{t} - \sigma}{C_{t+1} - \sigma} \left( \frac{Z_{s,j,t+1}}{P_{t+1}} + Q_{s,j,t+1} \left[ (1 - \delta) + \Phi' \left( \frac{I_{s,j,t+1}}{K_{s,j,t+1}} \right) \left( \frac{I_{s,j,t+1}}{K_{s,j,t+1}} \right)^{2} \right] \right) \right\}, \]

\[ Q_{s,j,t} \left( \frac{I_{s,j,t}}{K_{s,j,t}} \right) \frac{I_{s,j,t}}{K_{s,j,t}} + \Phi \left( \frac{I_{s,j,t}}{K_{s,j,t}} \right) = 1, \]

where \( Q_{s,j,t} \) denote Tobin’s \( q \) for firm \( j \) in sector \( s \).

### 2.2.2 Intermediate goods firms

Once we introduce firm-specific factor markets, the intermediate goods producer’s problem also changes since wages and the return on capital will also be determined at the firm level:

\[
\begin{align*}
\max_{\mathbf{X}, \mathbf{Y}} E_t \sum_{l=0}^{\infty} \Theta_{t+l} (1 - \alpha_s)^l \left[ X_{s,j,t} Y_{s,j,t+1} + W_{s,j,t+1} N_{s,j,t+1} - Z_{s,j,t+1} K_{s,j,t+1} \right] \\
st \quad Y_{s,j,t} = \left( \frac{P_{s,j,t}}{P_s} \right)^{-\theta} \left( \frac{P_{s,t}}{P_t} \right)^{-\eta} Y_t \\
Y_{s,j,t} = F (A_t, K_{s,j,t} N_{s,j,t}) = A_t (K_{s,j,t})^{1-\chi} (N_{s,j,t})^{\chi}.
\end{align*}
\]
Optimal price setting implies:

\[
X_{s,j,t} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} \Theta_{t,l+t+l} (1 - \alpha_s)^l \Lambda_{s,l+t+l} \left( \chi \left( 1 - \chi \right) \left( 1 - \chi \right) \right)^{-1} W_{s,j,t+l}}{E_t \sum_{l=0}^{\infty} \Theta_{t,l+t+l} (1 - \alpha_s)^l \Lambda_{s,l+t+l}},
\]

where \( \Lambda_{s,t} \) is defined as before.

Given the optimality (cost-minimization) conditions for capital and labor, the real marginal cost can be expressed as:

\[
MC_{s,j,t} = \frac{W_{s,j,t}/P_t}{\chi \left( \frac{(1 - \chi)}{\chi} \frac{W_{s,j,t}}{Z_{s,j,t}} \right)^{1-\chi}} = \frac{1}{\chi^{1-\chi} \left( 1 - \chi \right)^{1-\chi}} \left( \frac{W_{s,j,t}}{P_t} \right)^{\chi} \left( \frac{Z_{s,j,t}}{P_t} \right)^{1-\chi}.
\]

Note that marginal costs are now firm specific. This is a direct implication of the assumption of firm-specific capital and labor markets.

### 2.3 Economy-wide factors

Finally, we consider a version of the model in which factors can move freely across firms and sectors. This requires that we reformulate the consumers’ and intermediate firms’ problems once again. The maximization problem of final goods’ firms remains the same as before.

#### 2.3.1 Consumer

The representative consumer maximizes:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \frac{N_t^{1+\gamma}}{1 + \gamma} \right),
\]

subject to the flow budget constraint:

\[
P_t C_t + P_t I_t + E_t \left[ \Theta_{t,t+1} B_{t+1} \right] \leq W_t N_t + B_t + T_t + Z_t K_t,
\]

the law of motion for the stocks of each firm-specific capital:

\[
K_{t+1} = (1 - \delta) K_t + \Phi (I_t, K_t) I_t,
\]

\[
I_t \geq 0,
\]

and a standard “no-Ponzi” condition

\[
B_{t+1} \geq - \sum_{l=t+1}^{\infty} E_{t+1} \left[ \Theta_{t+1,l} (W_l N_l + T_l + Z_l K_l) \right] \geq -\infty,
\]
where $\Phi(I_t, K_t)$ now takes the form:

$$
\Phi(I_t, K_t) = \Phi \left( \frac{I_t}{K_t} \right) = 1 - \frac{1}{2} \left( \frac{I_t}{K_t} - \delta \right)^2.
$$

The notation is the same as before, except that $N_t$ is total labor supply, $W_t$ is the corresponding nominal wage rate, $I_t$ denotes investment, $K_t$ stands for capital, $Z_t$ is the associated nominal return on capital is the associated nominal return on capital.

The first-order conditions for consumption and labor are now:

$$
\frac{C_t^{\sigma}}{C_{t+1}^{\sigma}} = \frac{\beta l}{\theta l} \frac{P_t}{P_{t+1}},
$$

$$
\frac{W_t}{P_t} = N_t^\sigma C_t^\sigma.
$$

For all investment $I_t$ and capital $K_{t+1}$ types:

$$
Q_t \left( \Phi' \left( \frac{I_t}{K_t} \right) \frac{I_t}{K_t} + \Phi \left( \frac{I_t}{K_t} \right) \right) = 1,
$$

$$
Q_t = \beta E_t \left\{ \frac{C_{t+1}^{\sigma}}{C_t^{\sigma}} \left( \frac{Z_{t+1}}{P_{t+1}} + Q_{t+1} \left[ (1 - \delta) + \Phi' \left( \frac{I_t}{K_{t+1}} \right) \left( \frac{I_t}{K_{t+1}} \right)^2 \right] \right) \right\},
$$

where $Q_t$ denotes the Tobin’s $q$.

### 2.3.2 Intermediate goods firms

Economy-wide capital and labor imply wages and the return on capital equalizing across firms and sectors, and the intermediate firm’s problem becomes:

$$
\max E_t \sum_{l=0}^{\infty} \Theta_{t+l} (1 - \alpha_s)^l \left[ X_{s,j,t} Y_{s,j,t+l} + \right. \\
\left. -W_{t+l} N_{s,j,t+l} - Z_{t+1} K_{s,j,t+l} \right]
$$

$$
st \quad Y_{s,j,t} = \left( \frac{P_{s,t}}{P_{s,t}} \right)^{-\theta} \left( \frac{P_{s,t}}{P_{t}} \right)^{-\eta} Y_t
$$

$$
Y_{s,j,t} = F(A_t, K_{s,j,t} N_{s,j,t}) = A_t (K_{s,j,t})^{1-\chi} (N_{s,j,t})^{\chi}.
$$

Optimal price setting implies:

$$
X_{s,j,t} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} \Theta_{t+l} (1 - \alpha_s)^l \Lambda_{s,t+l} (\chi K_{s,j,t+l}^{1-\chi} N_{s,j,t+l}^{\chi-1})^{-1} W_{t+l}}{E_t \sum_{l=0}^{\infty} \Theta_{t+l} (1 - \alpha_s)^l \Lambda_{s,t+l}},
$$

where $\Lambda_{s,t}$ is defined as before.
Given the optimality (cost-minimization) conditions for capital and labor, the real marginal cost can be expressed as:

\[ MC_{s,j,t} = \frac{W_t/P_t}{\chi \left( \frac{(1-\chi)}{\chi} \frac{W_t}{Z_t} \right)^{1-\chi}} = \frac{1}{\chi^{\gamma}(1-\chi)^{1-\chi}} \left( \frac{W_t}{P_t} \right)^{\chi} \left( \frac{Z_t}{P_t} \right)^{(1-\chi)}. \]

Note that marginal costs are equalized across firms and sectors.

### 2.4 Monetary policy

In our baseline specification we assume that the growth rate of nominal aggregate demand follows a first-order autoregressive (AR(1)) process, thus leaving monetary policy implicit. This specification can be justified through a cash-in-advance constraint when money growth itself follows an AR(1), or as the result of a monetary policy rule. Denoting nominal aggregate demand by \( M_t \equiv P_tY_t \), we assume:

\[ \Delta m_t = \rho_m \Delta m_{t-1} + \sigma_{\varepsilon_m}\varepsilon_{m,t}, \]

where \( m_t \equiv \log (M_t) \), \( \rho_m \) determines the autocorrelation in nominal aggregate demand growth, and \( \varepsilon_{m,t} \) is a purely monetary, uncorrelated, zero-mean, unit-variance \( i.i.d. \) shock.

We analyze the model using a loglinear approximation around the zero-inflation steady state. While solving the models with sectoral and economy-wide factor markets is trivial, the model with firm-specific capital is more challenging. The reason is that firms can have different capital-accumulation histories. We solve that model by generalizing the approach pioneered by Woodford (2005) to the case of a multisector economy. Details of the solution are available upon request.

### 3 The underlying new Keynesian Phillips curves

After log-linearization around a zero inflation steady state, each model presented in the previous section gives rise to a different New Keynesian Phillips curve (NKPC).

\[ \pi_t = \beta E_t \pi_{t+1} + (\bar{\sigma} + \omega) \left( \sum_{s=1}^{S} f_s \mu_s \right) \tilde{y}_t + \eta^{-1} \sum_{s=1}^{S} f_s \mu_s (\bar{y}_{s,t} - \bar{y}_t) - \bar{\sigma} \left( \sum_{s=1}^{S} f_s \mu_s \right) \tilde{y}_t, \quad (5) \]

where \( \omega = (\gamma + 1 - \chi) / \chi \), \( \bar{\sigma} = (Y/C) \sigma \), \( \mu_s = \alpha_s (1 - \beta (1 - \alpha_s)) / (1 - \alpha_s) \), and variables with a tilde superscript denote deviations from the underlying flexible-price equilibrium.\(^7\) The last term on the

---

\(^6\)Derivations are available upon request.

\(^7\)The flexible-price equilibrium is defined as in Woodford (2003, 2005), i.e., it assumes flexible prices going forward, given the current capital stock.
right-hand-side of (5) is due to endogenous capital accumulation,\(^8\) and the third term is reminiscent of multisector models with heterogeneity in price rigidity (Carvalho 2006). The second term is the standard output gap component of the NKPC.\(^9\)

Note that in a model without capital and with the same frequency of price changes in all sectors \((\alpha, \text{say}),\) equation (5) collapses to the standard NKPC:

\[
\pi_t = \beta E_t \pi_{t+1} + (\sigma + \omega) \mu \tilde{y}_t,
\]

where \(\mu = \alpha (1 - \beta (1 - \alpha)) / (1 - \alpha).\) The coefficient \(\tilde{\sigma} + \omega\) that multiplies the output gap term in (5) is thus seen to correspond to the Ball and Romer (1990) index of real rigidities. As is well known, models with economy-wide factor markets tend to generate strategic substitutability in price setting, which obtains if \(\tilde{\sigma} + \omega > 1.\)\(^{10}\) This is why sticky-price models with economy-wide factor markets have difficulty generating a large amount of monetary non-neutrality with a reasonable amount of nominal price rigidity (e.g., Chari et al. 2000).

Turning to the model with firm-specific factors (Section 2.2), the underlying NKPC is given by:

\[
\pi_t = \beta E_t \pi_{t+1} + \tilde{\sigma} \left( \sum_{s=1}^{S} f_s \frac{\mu_s}{\phi_s} \right) \tilde{y}_t + \omega \sum_{s=1}^{S} f_s \frac{\mu_s}{\phi_s} \tilde{y}_{s,t} + \eta^{-1} \sum_{s=1}^{S} f_s \frac{\mu_s}{\phi_s} (\tilde{y}_{s,t} - \tilde{y}_t) - \tilde{\sigma} \left( \sum_{s=1}^{S} f_s \frac{\mu_s}{\phi_s} \right) \tilde{y}_t, \tag{6}
\]

where \(\phi_s\) is a function of structural parameters that is obtained (numerically) as the solution of a system of nonlinear equations (see the Appendix).

Equation (6) generalizes the NKPC derived in Woodford (2005) to the case of a multisector economy with heterogeneity in price rigidity. The main difference relative to the case of economy-wide factor markets is that all “gaps” are now multiplied by \(\phi_s^{-1}.\) As in Woodford (2005), it is the case that \(\phi_s > 1\) under reasonable parameterizations. Thus, under those parameterizations, firm-specific factors are seen to mute the sensitivity of inflation to aggregate and sectoral output gaps, and the investment gap. Firm-specificity is thus a source of real rigidities. This is most easily seen in a version of this model without capital and with the same frequency of price changes in all sectors. In that case the NKPC simplifies to:

\[
\pi_t = \beta E_t \pi_{t+1} + (\sigma + \omega) \frac{\mu}{\phi} \tilde{y}_t, \tag{7}
\]

where \(\phi = 1 + \theta \gamma.\) The reason why inflation becomes less sensitive to the output gap can be understood by studying how factor specificity at the firm level affects firms’ pricing decisions. Suppose the

---

8Investment is defined as percentage deviation from its steady state.

9Due to heterogeneity in price stickiness, the coefficient associated with nominal price rigidities is a weighted average of the associated sectoral coefficients \(\left( \sum_{s=1}^{S} f_s \mu_s \right).\)

10For a thorough discussion of sources of real rigidities in the canonical New Keynesian model see Woodford (2003, Ch. 3).
economy is hit by a shock that induces a negative output gap, and thus puts downward pressure on prices. With economy-wide factor markets, firms can cut prices as much as they want without any effect on their marginal costs. In contrast, if labor is firm-specific, firms that set prices too low will attract more demand, will have to hire more labor for production, and will end up having to pay higher wages – as long as the marginal disutility of labor is increasing ($\gamma > 0$). Hence, firms will cut their prices by less than they would if there was an economy-wide labor market.

When capital and labor are specific to sectors, the NKPC becomes:

$$\pi_t = \beta E_t \pi_{t+1} + \sigma \left( \sum_{s=1}^{S} f_s \mu_s \right) \bar{y}_t + \omega \sum_{s=1}^{S} f_s \mu_s \bar{y}_{s,t} + \eta^{-1} \sum_{s=1}^{S} f_s \mu_s (\bar{y}_{s,t} - \bar{y}_t) - \bar{\sigma} \left( \sum_{s=1}^{S} f_s \mu_s \right) \vec{\bar{u}}_t. \quad (8)$$

Note that this Phillips curve is almost identical to the one that obtains under firm-specific factor markets (equation 6), with the crucial exception that there are no $\phi^{-1}_s$ coefficients multiplying the output and investment gaps. The reason is that factor prices are now equalized within sectors, so that, in contrast to the model with firm-specific factors, individual firms’ pricing decisions have no impact on their marginal cost. Suppose a sector is hit by a shock that reduces its demand. With economy-wide factor markets, marginal costs are not affected by firms’ pricing decisions. The same is true in the presence of sectoral factor markets. Any given firm can reduce its price without worrying that its relative increase in output will put upward pressure on factor prices. The mechanism at work in the latter model is thus seen to be the same as in the model with economy-wide factor markets, but operating at the sectoral level. Indeed, if sectors are identical in terms of price rigidity, the dynamics of sectoral output gaps in response to an aggregate disturbance will the same in all sectors ($\bar{y}_{s,t} = \bar{y}_t$), and equations (5) and (8) become identical, and simplify to:

$$\pi_t = \beta E_t \pi_{t+1} + (\bar{\sigma} + \omega) \mu \bar{y}_t - \bar{\sigma} \mu \bar{\bar{u}}_t.$$

The same does not happen when factors are firm-specific, because firms still need to internalize the effects of their pricing decisions on their marginal cost, through factor prices. The corresponding equation with homogeneous price rigidity simplifies to:

$$\pi_t = \beta E_t \pi_{t+1} + (\bar{\sigma} + \omega) \frac{\mu}{\phi} \bar{y}_t - \bar{\sigma} \frac{\mu}{\phi} \bar{\bar{u}}_t,$$

where the $\phi$ coefficient embeds the effects of firm specificity.\(^{11}\)

When sectors are heterogeneous, the Phillips curve with sectoral specificities differs from the Phillips curve under economy-wide factor markets only in that $\omega$ now multiplies sectoral output gaps rather than the aggregate output gap. The reason for this difference is that factor prices now also

\(^{11}\)Here we abuse notation because, due to capital accumulation, $\phi$ does not equal that in equation (7).
depend on sectoral conditions. The \( \omega \sum_{s=1}^{S} f_s \mu_s \tilde{y}_{s,t} \) term summarizes the effects of sectoral conditions on marginal costs, keeping aggregate output constant. These effects operate through increasing marginal disutility of labor and decreasing marginal product of capital – which affect sectoral factor prices. The \( \tilde{\sigma} \left( \sum_{s=1}^{S} f_s \mu_s \right) \tilde{y}_t \) term captures the effects of aggregate conditions on marginal costs, which continue to operate through households’ decreasing marginal utility of consumption. In the model with economy-wide factor markets, the first effect hinges on the aggregate output gap, rather than on sectoral gaps.\(^\text{12}\)

Whether the model with sectoral specificity produces larger or smaller non-neutralities than the models with economy-wide or firm-specific factor markets is thus a quantitative question, to which we turn next.

### 4 Quantitative analysis

In this section, we parameterize our three models and analyze their quantitative predictions. We set the intertemporal elasticity of substitution \( \sigma^{-1} \) to 1/2, labor supply elasticity \( \gamma \) at 0.5, and the usual labor share \( (\chi = 2/3) \). The consumer discount factor \( \beta \) implies a time-discount rate of 4% per year.

For the final-good aggregator, we set the elasticity of substitution between varieties of the same sector to \( \theta = 7 \). The elasticity of substitution between varieties of different sectors should arguably be smaller than within sectors. We assume a unit elasticity of substitution across sectors, \( \eta = 1 \) (i.e. the aggregator that converts sectoral into final output is Cobb-Douglas).

To specify the process for nominal aggregate demand, the literature usually relies on estimates based on nominal GDP, or on monetary aggregates such as M1 or M2. With quarterly data, estimates of \( \rho_m \) typically fall in the range of 0.4 to 0.7,\(^\text{13}\) which maps into a range of roughly 0.75 – 0.90 at a monthly frequency. We set \( \rho_z = 0.8 \), and the standard deviation of the shocks \( \sigma_{\varepsilon_m} = 0.6\% \) (roughly 1\% at a quarterly frequency), in line with the same estimation results.\(^\text{14}\)

Finally, to discipline our analysis, we follow an approach that is common in the real business cycle literature (e.g., Chari et al. 2000) and calibrate the investment adjustment-cost parameter \( (\kappa) \) to match the standard deviation of investment in the data relative to the standard deviation of GDP. Whenever we analyze a different version of the model, we redo the calibration.\(^\text{15}\)

It remains to specify the distribution of price rigidity. We start by investigating whether the model with sectoral factor markets is flexible enough to approximate both the model with firm-

---

\(^\text{12}\)Incidentally, note that in equation (6) \( \omega \) also multiplies sectoral output gaps. The reason is that the solution of the model is such that the effects of firm specificity are subsumed in \( \phi_s \), and the model is in effect solved by taking sectoral averages (see the Appendix).

\(^\text{13}\)See, for instance, Mankiw and Reis (2002).

\(^\text{14}\)All results for volatilities scale-up proportionately with \( \sigma_{\varepsilon_z} \).

\(^\text{15}\)It turns out that the calibration of the three versions of the model usually yields very similar values for \( \kappa \).
specific factors and the model with economy-wide factor markets. To that end, we experiment with different arbitrary distributions of price stickiness in economies with 3 sectors.\textsuperscript{16} Subsequently, we use the available microeconomic evidence on price rigidity in the U.S. to discipline the calibration of the three versions of the model.

### 4.1 Arbitrary distributions of price rigidity

We analyze economies with 3 sectors, and entertain two alternative distributions of price rigidity. In both cases the frequencies of price changes ($\alpha_k$) in sectors 1-3 are, respectively, 1, 1/12, and 1/30 – corresponding to price changes of, on average, once a month, once a year, and once every 30 months. We differentiate the two distributions by changing the sectoral weights.

In the first distribution, which we term “flexible distribution”, the weights of sectors 1-3 are, respectively, 85%, 7.5%, and 7.5%. In the second distribution, which we label “sticky distribution”, the weights of sectors 1-3 are, respectively, 7.5%, 7.5%, and 85%.

Figure 1 presents the impulse response functions of real GDP for the two alternative sets of sectoral weights. The panel on the left shows the impulse response functions of GDP under the flexible distribution for the three alternative models of factor markets, while the panel on the right shows the responses for the three models under the sticky distribution. Note that the scales on the two charts are different.

The results show that, under the flexible distribution, the sector-specific factor market economy generates more monetary non-neutrality than the other two models. In contrast, under the sticky distribution that model is much more similar to the model with economy-wide factor markets. This result suggests that, depending on the distribution of price rigidity, the pattern of pricing interactions reflected in the NKPC under sectoral factor markets is rich enough to emulate the dynamics of both an economy with strong real rigidities (such as the one with firm-specific factor markets) and an economy with strategic substitutability in pricing decisions (such as the one with economy-wide factor markets).

### 4.2 Empirical distribution of price rigidity

Having shown that the model with sectoral factor markets can behave similarly to the other two models that we consider, we now discipline our quantitative analysis by focusing on the empirical distribution of price rigidity for the U.S. economy. We use the statistics on the frequency of regular price changes – those that are not due to sales or product substitutions – reported by Nakamura

\textsuperscript{16}We do so motivated by our finding that a 3-sector economy with a suitably chosen distribution of price rigidity provides a very good approximation to the dynamics of a multisector economy calibrated to the empirical distribution of price rigidity in the U.S. See details below.
Figure 1: IRFs of GDP to monetary shocks - arbitrary distributions of price rigidity
and Steinsson (2008). To make the model computationally manageable, we build from the statistics for 271 categories of goods and services, and aggregate those 271 categories into 67 expenditure classes. Each class is identified with a sector in the model. The frequency of price changes for each expenditure class is obtained as the weighted average of the frequencies for the underlying categories, using the expenditure weights provided by Nakamura and Steinsson (2008). Finally, expenditure-class weights are given by the sum of the expenditure weights for those categories. The resulting average monthly frequency of price changes is \( \bar{\alpha} = \sum_{k=1}^{K} f_k \alpha_k = 0.211 \), which implies that prices change on average once every 4.7 months.

Solving, calibrating and simulating the multisector model with 67 sectors is computationally costly. To sidestep this problem we work with a 3-sector approximation to the underlying 67-sector economy. We choose the frequencies of price changes and sectoral weights in the approximating model to match a suitably chosen set of moments of the cross-sectional distribution of price stickiness of the original 67-sector economy. This delivers an extremely good approximation to the dynamics of a 67-sector model, under all three assumptions about factor markets (details are available upon request).

Figure 2 reports our main results. It shows impulse response functions to a monetary shock for the three models, economy-wide, sectoral, and firm-specific factor markets. Focusing first on the IRFs for GDP, the chart shows that the economy-wide factor model implies the smallest monetary non-neutrality. The sectoral and firm-specific factor models are more similar on impact and during the first few months, featuring more sizable non-neutralities than the economy-wide model. In the medium to long run, however, the model with sectoral specificity quickly approaches the one with economy-wide factor markets. At the end of the day, the model with firm-specific factors generates much longer lasting effects of monetary shocks on real GDP than the other two models. Despite the similarities across the three models with respect to their impact on inflation, the model with economy-wide implies a slightly larger initial impact on inflation, which is the flip side of its smaller impact on real GDP. The remaining charts report the impulse response functions for other main variables of the three models and basically replicate the patterns obtained for real GDP.

### 4.3 Robustness

In our previous analyses we left monetary policy unspecified and postulated an exogenous stochastic process for nominal aggregate demand. In this section, we consider a specification with an explicit

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17 Nakamura and Steinsson (2008) report statistics for 272 categories. We discard the category “Girls’ Outerwear”, for which the reported frequency of regular price changes is zero. We renormalize the expenditure weights to sum to unity.

18 As an example of what this aggregation entails, the resulting “New and Used Motor Vehicles” class consists of the categories “Subcompact Cars”, “New Motorcycles”, “Used Cars”, “Vehicle Leasing” and “Automobile Rental”; the “Fresh Fruits” class comprises four categories: “Apples”, “Bananas”, “Oranges, Mandarins etc.” and “Other Fresh Fruits.”
Figure 2: Monetary shocks – impulse response functions
description of monetary policy. We assume that monetary policy is conducted according to an interest-rate rule subject to persistent shocks:

\[ I_t = \beta \left( \frac{P_t}{P_{t-1}} \right)^{\phi_x} \left( \frac{GDP_t}{GDP} \right)^{\phi_Y} e^{v_t}, \]

where \( I_t \) is the short-term nominal interest rate, \( GDP_t \) is gross domestic product, \( GDP \) denotes gross domestic product in steady state, \( \phi_x \) and \( \phi_Y \) are the parameters associated with Taylor-type interest-rate rules (Taylor 1993), and \( v_t \) is a persistent shock with process \( v_t = \rho_v v_{t-1} + \sigma_{v, u} \varepsilon_{v, t} \), where \( \varepsilon_{v, t} \) is a zero-mean, unit-variance \( i.i.d. \) shock, and \( \rho_v \in [0, 1) \). We set \( \phi_x = 1.5, \phi_y = .5/12, \) and \( \rho_v = 0.965. \) The remaining parameter values are unchanged from the baseline specification.

Figure 3 shows that the main findings reported in the previous section are also present in models with an explicit interest-rate rule. In particular, focusing on the impulse response functions for real GDP, the sector-specific factor model initially tracks the response of the firm-specific factor model but over time its adjustment speeds up and the impulse response function approaches that of the model with economy-wide factor markets. The real effects of monetary shocks in the model with firm-specific factors are not only larger but also more persistent. The same patterns hold for the remaining variables depicted in Figure 3.

5 Conclusion

Our results show that it matters a great deal whether specificity in factors of production arises at the firm or sectoral level. In response to nominal shocks, our parameterized model with sector-specific factors yields aggregate dynamics in the short run that resemble those of a model with firm-specific factors. After one to two years, however, the behavior of the model is already more similar to that of a model with economy-wide factor markets.

This result is consistent with the idea that factor price equalization might take place gradually over time, so that firm-specificity might be a reasonable short-run approximation, whereas economy-wide markets might be a better description of how factors of production are allocated in the longer run.

Whether or not this happens in reality is, of course, an empirical question. Existing empirical evidence that both capital (e.g., Ramey and Shapiro 2001) and labor (e.g., Parent 2000) have an important degree of sector (or industry) specificity suggests that this question deserves further investigation.

\[ ^{19} \text{Recall that the parameters are calibrated to the monthly frequency, and so this value for } \rho_v \text{ corresponds to an autoregressive coefficient of roughly } 0.9 \text{ at a quarterly frequency. We specify the size of the shocks to be consistent with the estimates of Justiniano et al. (2010), and thus set the standard deviation to } 0.2\% \text{ at a quarterly frequency.} \]
Figure 3: Monetary shocks under a Taylor rule – impulse response functions
References


A Appendix

A.1 Firm-specific model – $\phi_s$

The $\phi_s$ coefficients in equation (6) are given by:

$$\phi_s = \left(1 + \theta \frac{\gamma - (\chi - 1)}{\chi} - \frac{\kappa_{2,s}(1 - \chi) (1 + \gamma) \beta (1 - \alpha_s)}{\chi (1 - \beta (1 - \alpha_s) \kappa_{1,s})}\right),$$

where $\kappa_{1,s}$ and $\kappa_{2,s}$ are obtained from a nonlinear system of 3 equations in 3 unknowns, $\psi_s, \kappa_{1,s}$ and $\kappa_{2,s}$:

$$\frac{(1 - \chi) (1 + \gamma)}{\chi} \frac{(1 - \beta (1 - \alpha_s))}{1 - (1 - \alpha_s) \beta \kappa_{1,s}} = \phi_s \psi_s$$

$$- [\beta (\kappa_{1,s} + \kappa_{2,s} \alpha_s \psi_s) - A + \alpha_s \psi_s \Xi] \kappa_{1,s} = 1$$

$$- [\beta (\kappa_{1,s} + \kappa_{2,s} \alpha_s \psi_s) - A + \alpha_s \psi_s \Xi] \kappa_{2,s} = [\Xi (1 - \alpha_s) + \kappa_{2,s} (1 - \alpha_s) \beta],$$

where

$$A = \left[\beta + [1 - (1 - \delta) \beta] \left[\frac{\chi + (1 + \gamma) (1 - \chi)}{\kappa \chi}\right] - 1\right]$$

$$\Xi = [1 - (1 - \delta) \beta] \frac{\theta (1 + \gamma)}{\kappa \chi}.$$  

Note that in a version of the model without capital accumulation, $\chi = 1$, $\phi_s$ simplifies to $\phi = (1 + \theta \gamma)$ – which is familiar from new Keynesian DSGE models. Details of the derivation of these equations are available upon request.