Labor Shares and Income Inequality*

Jonathan Adams
University of Chicago

Loukas Karabarbounis
University of Chicago and NBER

Brent Neiman
University of Chicago and NBER

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Abstract

The share of aggregate income paid as compensation to labor is frequently used as a proxy for income inequality. If capital holdings are very concentrated among high income individuals, increasing their share of GDP, all else equal, widens the gap with poorer workers. Indeed, two striking features over the last three decades of many advanced and developing economies are the declining labor shares in income and the rise in income inequality. The relationship between factor shares and inequality, however, is not so simple in a richer world with realistic features such as endogenous portfolio decisions and capital-skill complementarity. In such a world, total inequality will change with (i) the labor share, (ii) the amount of within-labor and within-capital income inequality, and (iii) the degree to which the highest wage earners are also those earning the highest capital incomes. Macroeconomic trends and shocks that impact any one of these three moments are likely to impact simultaneously all of them. We develop a framework where all these terms are jointly determined and estimate the model to clarify the roles of changing technology, policies, and factor proportions on labor shares and total income inequality around the globe.

JEL-Codes: E21, E25, J31.
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1 Introduction

The share of aggregate income paid as compensation to labor is frequently used as a proxy for income inequality. If capital holdings are very concentrated among high income individuals, increasing their share of GDP, all else equal, widens the gap with poorer workers. Indeed, two striking features over the last three decades of many advanced and developing economies are the declining labor shares in income and the rise in income inequality.

The relationship between factor shares and inequality, however, is not so simple in a richer world with realistic features such as endogenous portfolio decisions and capital-skill complementarity. Imagine that each agents' total income y_j equals the sum of labor income y_j^l and capital income y_j^k. The coefficient of variation (CV) of total income in this economy, a standard measure of inequality, can generally be decomposed as:[1]

$$CV\left(y\right) = s_L \rho\left(y^l\right) CV\left(y^l\right) + \left(1 - s_L\right) \rho\left(y^k\right) CV\left(y^k\right). \tag{1}$$

We use s_L to denote labor's share of aggregate income, $\rho(y^l)$ and $\rho(y^k)$ to denote the correlation of labor and capital income with total income, and $CV\left(y^l\right)$ and $CV\left(y^k\right)$ to measure within-labor and within-capital income inequality. Total inequality will change with the labor share, labor or capital income inequality, and the degree to which the highest wage earners are also those earning the highest capital incomes. Macroeconomic trends and shocks that impact any one term in the decomposition (1) are likely to impact simultaneously all the terms.

In this paper, we develop a framework where all these terms are jointly determined in order to evaluate quantitatively the relationship between the changes in labor share and total income inequality. We start with a production function with two types of capital and two types of labor that exhibits capital-skill complementarity, as in Krusell, Ohanian, Rios-Rull, and Violante (2000). Shocks that increase the return to capital also increase the return to skilled labor, raising the skill premium and generating increases in within-labor income inequality. These ingredients, however, are insufficient to analyze capital income inequality and the correlation

[1] See Shorrocks (1982) for a derivation of this decomposition. Under fairly general assumptions such as the non-negativity of income, similar decompositions exist for most measures of inequality. In our model below we will allow for negative capital incomes, but present this decomposition here to build intuition.

of capital and labor incomes. We therefore embed our production function into an Aiyagari (1994) style model with heterogeneous agents, persistent idiosyncratic shocks, and incomplete markets. Agents trade assets with each other in order to smooth consumption. Capital incomes and labor incomes are jointly determined, and the full endogenous distribution of both income sources proves important for understanding the response of overall inequality to many shocks.

We describe an environment with no aggregate uncertainty, calibrate it to resemble the U.S. economy around 1980, and solve for the ergodic distribution or steady state of the model. We consider a number of experiments where we introduce a permanent shock, solve for the new steady state, and evaluate comparative statics relative to the pre-shock steady state. Most shocks will simultaneously influence the within-labor income inequality, the within-capital income inequality, and the correlation between agents' labor and capital income, which allows us to to analyze the joint behavior of labor share and inequality.

For example, we consider a decline in the relative price of IT capital, which is more complementary to skilled than to unskilled labor. The resulting IT capital deepening increases the skill premium and labor income inequality, as in Krusell, Ohanian, Rios-Rull, and Violante (2000). In our baseline calibration, the shock also drives a reduction in the labor share, as in **?**, which increases the primacy of capital income inequality for overall income inequality. Depending on the model's calibration, this can reinforce, mute, or even reverse the conclusions drawn about inequality relative to a framework that ignores capital income. We additionally consider a shock to the distribution of human capital stocks, a shock that relaxes the borrowing constraint, shocks in the tax schedule, and shocks to the depreciation rate, among others. We generally find that the dynamics of capital income inequality are important for understanding overall income inequality when XXX...

[TBD: mapping to the data.]

[TBD: related literature.]

2 Trends in Inequality and its Determinants

[TBD: Trends in overall, labor, and capital inequality.]

[TBD: Declining labor shares. Financial development. Skill stock. Etc.]

3 Model

Our model embeds the production structure of Krusell, Ohanian, Rios-Rull, and Violante (2000) in the Aiyagari (1994) and Huggett (1996) model of heterogeneous agents with incomplete asset markets. On the production side, the economy produces a consumption good C, an information and communication technology (IT) capital good X_s, and a non-IT capital good X_u. We take the consumption good to be the numeraire and set its price equal to one in each period. On the household side, there is a measure one of heterogeneous agents that differ in idiosyncratic labor income shocks, skill endowment, and age. Throughout our analysis, we focus on stationary equilibria in which aggregate variables are constant. We drop time subscripts to denote variables in the current period and we use a prime to denote variables in the next period.

3.1 Households

Households are heterogeneous with respect to their labor income shocks z, their skill endowment h, their age j, and their accumulated assets a. We denote the household type by (a, z, h, j) and the distribution of households across these states by $\lambda(a, z, h, j)$. We assume that the economy is in a stationary state with constant distribution of households across these states and constant aggregate variables. Therefore, when solving their problem households expect all prices to be constant.

Household (a, z, h, j) is endowed with $z\Phi(h, j)(1 - h)$ units of unskilled labor and $z\Phi(h, j)h$ units of skilled labor. The skill level h takes values in the set $H = \{0 \leq h_1, h_2, ..., h_{N_h} \leq 1\}$. Skills are drawn from the distribution $\pi^h(h|j)$ when households are born and are constant over time for each household. The labor income shock z follows a Markov chain defined in the set $Z = \{z_1, z_2..., z_{N_z}\}$ and with a corresponding probability distribution $\pi^z(z'|z, h)$. The dependence of this probability distribution on h allows the process for labor income shocks to vary by skill. The function $\Phi(h, j)$ denotes a skill-age effect on household's labor income. We denote by n the endogenous part of household's labor supply, and by W_u and W_s the wage per unit of unskilled and skilled labor supply. Each household's pre-tax labor income equals $z\Phi(h, j)(W_u(1 - h) + W_s h)n$. Households face a constant marginal tax rate τ_n on their labor income.

Households accumulate financial assets $a \in A$ to smooth consumption over their life-cycle and to insure against shocks. We denote by r the interest rate on these assets. Assets are bounded from below by some exogenous borrowing limit ψ.

We consider an economy with a constant population of measure one. At each point of time, there are λ_{hj} households of skill h and age j, where $\lambda_{hj} = \int_{A\times Z} \lambda(a, z, h, j)dadz$. Households' age j takes values in the set $J = \{1, 2, ..., N_j\}$, where N_j denotes the maximum number of periods that a household lives. We denote by μ_{hj} the probability that a household with skill h and age j survives to period $j + 1$, with $\mu_{hN_j} = 0$. Deaths are realized at the end of each period after households choose assets, labor, and consumption. The death rate in the aggregate economy is $\sum_{hj}(1 - \mu_{hj})\lambda_{hj}$.

For the economy to have constant population over time, the measure of deaths must equal the measure of births in each period. To allow our model to generate a steady state with any arbitrary household age distribution, we allocate newborn households across various generations (instead of only to the youngest generation). Let b_{hj+1} be the measure of newborns allocated to households with skill h and age $j + 1$. For any $j > 0$, we have:

$$\lambda_{hj+1} = \mu_{hj}\lambda_{hj} + b_{hj+1}, \tag{2}$$

and for the first generation we have $\lambda_{h1} = b_{h1}$, with $\sum_{hj} b_{hj} = \sum_{hj}(1 - \mu_{hj})\lambda_{hj}$. We assume that newborn households b_{hj+1} are allocated across labor income shocks z and assets a with the same distribution as households of type h that arrived from the previous period j.

Denoting by β the discount factor, we write in recursive form the problem of the household (whether newborn or not) as:

$$v(a,z,h,j) = \max_{c,n,a'} \left\{ u(c,n) + \beta\mu_{hj} \sum_{z' \in Z} \pi^z(z'|z,h) v(a',z',h,j+1) + (1-\mu_{hj})\bar{v}(a',j) \right\}, \quad (3)$$

subject to the budget constraint:

$$c + a' - a = (1-\tau_n) z \Phi(h,j) \left(W_u (1-h) + W_s h\right) n + ra + T(a,z,h,j) + B(a,z,h,j), \quad (4)$$

and the borrowing constraint:

$$a' \in A = [\psi, \infty). \quad (5)$$

In the value function, $\bar{v}(a',j)$ denotes the value that a dying household of age j derives from leaving a bequest equal to a'. In the budget constraint, $T(a,z,h,j)$ denotes tax revenues that are returned to a household of type (a,z,h,j) through lump-sum transfers and $B(a,z,h,j)$ denotes inherited bequests.

3.2 Production

The economy produces a consumption good C, an IT capital good X_s, and a non-IT capital good X_u. We distinguish IT and non-IT capital goods for three reasons. First, price declines of IT goods have been particularly pronounced in recent decades, leading to significant IT capital deepening. Second, IT goods are often operated by educated workers and therefore would be the type of capital that would most naturally exhibit complementarity with skill in production. Third, as we detail below, we obtain a data source which uses a consistent methodology and definition to distinguish IT from non-IT capital goods for many countries around the world. Our disaggregation is similar qualitatively and quantitatively to splitting capital into structures and equipment as has been done extensively in the literature.

Good $G = \{C, X_s, X_u\}$ is produced by a perfectly competitive representative firm using a constant returns to scale technology. The production of G uses inputs of unskilled labor (U^g), skilled labor (S^g), non-IT capital (K_u^g), and IT capital (K_s^g), where the three elements of $g = \{c, s, u\}$ correspond in order to the three elements of G. The two types of labor are rented from households at prices W_u and W_s respectively. The two types of capital are rented from intermediaries at (before-tax) rates $\tilde{R}_u$ and $\tilde{R}_s$ respectively. We describe the problem of the firms that intermediate capital in Section 3.3.

Producers face a corporate tax τ_c on their sales net of their labor expense. The profit maximization problem of a firm in sector G is given by:

$$\max_{U^g, S^g, K_u^g, K_s^g} \Pi^g = (1-\tau_c)\left(p_g G - W_u U^g - W_s S^g\right) - \tilde{R}_u K_u^g - \tilde{R}_s K_s^g, \tag{6}$$

subject to the production possibility set:

$$G \leq F_G = \frac{1}{\xi_g} F(U^g, S^g, K_u^g, K_s^g). \tag{7}$$

The first-order conditions for profit maximization imply the equalization of the value of the net-of-tax marginal product of each factor to the factor's price.

Note that the only difference across the three production functions F_G is the $1/\xi_g$ term, which denotes sector-specific technology. We normalize $\xi_c = 1$. Perfect competition and constant returns to scale imply that for each good the price equals the marginal cost. Therefore, differences in the prices of the three goods are tied down to differences in the technology of production. The price of non-IT investment relative to consumption is given by $p_u = \xi_u$ and the price of IT investment relative to consumption is given by $p_s = \xi_s$.

Because all producers face the same factor prices and taxes, in equilibrium the marginal revenue products are equalized across sectors. Aggregating across sectors and using the fact that the production functions are constant returns to scale, we show that total production and spending, denominated in units of consumption goods, equal total payments accruing to the factors of production:

$$Y = C + \xi_u X_u + \xi_s X_s = W_u U + W_s S + R_u K_u + R_s K_s. \tag{8}$$

where $R_u = \tilde{R}_u/(1-\tau_c)$ and $R_s = \tilde{R}_s/(1-\tau_c)$ denote the user cost of each type of capital. In equation (8), U and S denote the total supply of unskilled and skilled labor defined as:

$$U = \int_{A\times Z\times H\times J} z\Phi(h,j)(1-h)n(a,z,h,j)d\lambda \quad \text{and} \quad S = \int_{A\times Z\times H\times J} z\Phi(h,j)hn(a,z,h,j)d\lambda, \tag{9}$$

where $n(a, z, h, j)$ denotes the optimal choice of labor supply.

We define the factor income shares as $\alpha_{L_u} = W_u U/Y$, $\alpha_{L_s} = W_s S/Y$, $\alpha_{K_u} = R_u K_u/Y$, and $\alpha_{K_s} = R_s K_s/Y$, with $\alpha_{L_u} + \alpha_{L_s} + \alpha_{K_u} + \alpha_{K_s} = 1$. We define the labor share as $\alpha_L = \alpha_{L_u} + \alpha_{L_s}$ and the capital share as $\alpha_K = \alpha_{K_u} + \alpha_{K_s}$.

3.3 Intermediaries of Capital

Firms that intermediate capital purchase investment goods X_u and X_s from the final goods producers at prices ξ_u and ξ_s respectively to augment the capital stocks:

$$K'_u = (1 - \delta_u)K_u + X_u \qquad \text{and} \qquad K'_s = (1 - \delta_s)K_s + X_s, \tag{10}$$

where δ_u and δ_s denote the depreciation rates. The intermediaries rent the two types of capital to all final goods producers at before-tax rates of $\tilde{R}_u$ and $\tilde{R}_s$ respectively.

In each period, intermediaries maximize the cum-dividend value of the firm W. Let D denote the dividends paid by the intermediary, which are discounted at rate ζ. The value of the firm is:

$$W(K_u, K_s) = \max_{\{K'_u, K'_s\}} D + \zeta W(K'_u, K'_s), \tag{11}$$

where the firm pays out its cash flows as dividends:

$$D = \tilde{R}_u K_u + \tilde{R}_s K_s - \xi_u X_u - \xi_s X_s. \tag{12}$$

Intermediaries maximize the value in (11) subject to the capital accumulation equations in (10). We focus on a stationary state in which the discount factor is $\zeta = 1/(1+r)$. In stationary state, the first-order conditions for value maximization yield the following expressions that relate the before-tax rental rate of each capital stock to the price of investment goods, the interest rate, and the depreciation rate:

$$\tilde{R}_u = \xi_u (r + \delta_u) \qquad \text{and} \qquad \tilde{R}_s = \xi_s (r + \delta_s). \tag{13}$$

Intermediaries are owned by a mutual fund with which households invest all of their financial assets ($a > 0$) or from which households have borrowed and generated all their financial liabilities ($a < 0$). The aggregate of all financial assets is used to purchase the firm that intermediates capital.

Equilibrium in the asset market requires that the value of funds raised by the mutual fund from the household at any given time (new shareholder equity in the mutual fund's balance sheet) equals the ex-dividend value of its newly purchased shares in the intermediaries at that time (new assets on the mutual fund's balance sheet). Using the fact that in stationary state we have $X_u = \delta_u K_u$ and $X_s = \delta_s K_s$ together with the definition of dividends in (12), we show that the ex-dividend value Q of the intermediary firm equals the value of the capital stock:

$$Q = W - D = \xi_u K_u + \xi_s K_s. \tag{14}$$

This leads to the equilibrium condition in the asset market:

$$\int_{A\times Z\times H\times J} a d\lambda = Q = \xi_u K_u + \xi_s K_s. \tag{15}$$

Equation (15) says that the supply of financial assets from households must equal the demand for capital from all final goods producers when expressed in terms of consumption goods. Finally, we note that in stationary state dividends equal $D = rQ$, so that r equals the rate of return on the value of the capital stock.

3.4 Equilibrium

We define a stationary equilibrium for the economy as a household value function $v(a, z, h, j)$, household policy functions $c(a, z, h, j)$, $a'(a, z, h, j)$, and $n(a, z, h, j)$, transfers $T(a, z, h, j)$ and bequests $B(a, z, h, j)$, intermediary value function $W(K_u, K_s)$, a time-invariant distribution of households over states $\lambda(a, z, h, j)$, time-invariant interest rate r, wages W_u and W_s, prices p_u and p_s, and user costs of capital R_u and R_s, time-invariant stocks of capital K_u and K_s and labor U and S, and time-invariant aggregate consumption C, and investments X_u and X_s such that:

1. Taking as given the equilibrium r, W_u, and W_s, and $T(a, z, h, j)$ and $B(a, z, h, j)$, the value function $v(a, z, h, j)$ and the policy functions $c(a, z, h, j)$, $a'(a, z, h, j)$, and $n(a, z, h, j)$, solve the household's problem in equations (3)-(5).

2. In each sector, the price of the final good equals the marginal cost. Normalizing the price of consumption goods to $p_c = 1$ we take:

$$p_u = \xi_u, \tag{16}$$

$$p_s = \xi_s. \tag{17}$$

3. Taking as given the equilibrium prices p_u and p_s, wages W_u and W_s, and user costs R_u and R_s, production decisions maximize profits:

$$\frac{\partial F(U,S,K_u,K_s)}{\partial U} = W_u, \tag{18}$$

$$\frac{\partial F(U,S,K_u,K_s)}{\partial S} = W_s, \tag{19}$$

$$\frac{\partial F(U,S,K_u,K_s)}{\partial K_u} = R_u, \tag{20}$$

$$\frac{\partial F(U,S,K_u,K_s)}{\partial K_s} = R_s. \tag{21}$$

4. Taking as given prices p_u and p_s and the interest rate r, capital accumulation decisions maximize $W(K_u, K_s)$ and imply before-tax rental rates for the capital stocks given by equation (13).

5. Markets for consumption and investment goods clear:

$$C = \int_{A\times Z\times H\times J} c(a,z,h,j)d\lambda, \tag{22}$$

$$X_u = \delta_u K_u, \tag{23}$$

$$X_s = \delta_s K_s. \tag{24}$$

6. The government's budget is balanced:

$$T = \tau_n \left(W_u U + W_s S\right) + \tau_c \left(Y - W_u U - W_s S\right) = \int_{A\times Z\times H\times J} T(a,z,h,j)d\lambda. \tag{25}$$

7. Aggregate bequests are given by:

$$B = \int_{A\times Z\times H\times J} (1-\mu_{hj})a'(a,z,h,j)d\lambda = \int_{A\times Z\times H\times J} B(a,z,h,j)d\lambda. \tag{26}$$

8. The goods market clearing condition (8), the labor market clearing condition (9), and the asset market clearing condition (15) hold.

9. The stationary distribution of households across states, $\lambda(a,z,h,j)$, is consistent with household optimization, the transition probabilities for the labor income shocks, and the evolution of demographics. Given some initial distribution $\lambda(a,z,h,1)$ we have:

$$\lambda(a',z',h,j+1) = \mu_{hj} \sum_{z \in Z} \pi^z(z'|z,h) \int_{a \in A} I_{a'} \lambda(a,z,h,j) da + b_{hj+1} \lambda(a',z',h,j+1), \quad (27)$$

$\forall h \in H$ and for $j > 0$, where $I_{a'} = I\left(a' = a'(a,z,h,j)\right)$ denotes an indicator variable taking the value of one when next period assets equal the assets implied by the optimal policy function under state (a,z,h,j). By multiplying the mass of newborns b_{hj+1} by the probability $\lambda(a',z',h,j+1)$, we assume that newborns are distributed across new assets a' and productivity shocks z' according to the same probability as the continuing households. Note that this implies we can rearrange (27) and write it as:

$$\lambda(a',z',h,j+1) = \frac{\mu_{hj}}{1-b_{hj+1}} \sum_{z \in Z} \pi^z(z'|z,h) \int_{a \in A} I_{a'} \lambda(a,z,h,j) da. \quad (28)$$

3.5 Labor and Capital Income

We now discuss how we measure labor and capital income for each household. Pre-tax labor income is given by:

$$y^l(a,z,h,j) = z\Phi(h,j)\left(W_u(1-h) + W_s h\right) n(a,z,h,j). \quad (29)$$

Using the definition of unskilled and skilled labor in equation (9), it is easy to verify that total labor income is given by:

$$\int_{A \times Z \times H \times J} y^l(a,z,h,j) d\lambda = W_u U + W_s S. \quad (30)$$

We define capital income as:

$$y^k(a,z,h,j) = \frac{r+\iota}{1-\tau_c} a, \quad (31)$$

where ι is the aggregate investment rate:

$$\iota = \frac{\xi_u X_u + \xi_s X_s}{\xi_u K_u + \xi_s K_s}. \tag{32}$$

We choose this definition such that when aggregating capital income across households we obtain aggregate capital income:

$$\int_{A\times Z\times H\times J} y^k(a,z,h,j)d\lambda = R_u K_u + R_s K_s. \tag{33}$$

Note that the degree of inequality across households in capital income y^k is identical to that measured using assets a or interest income ra.

4 Parameterization and Solution

In this section we describe our calibration strategy and characterize the steady state of our baseline economy. We parameterize the model to targets chosen from U.S. data around 1980. In the Appendix we describe the numerical solution of the model.

4.1 Benchmark Parameterization

Some parameter values are taken from other studies and are listed in Panel A of Table 1. The remainder are estimated in the context of our model to match target moments and are listed in Panel B. These internally calibrated parameters are jointly determined but below we associate each parameter with the particular target moment most closely identified by that parameter.

We start by describing the production technology. Following Krusell, Ohanian, Rios-Rull, and Violante (2000), we assume that:

$$F = A(K_u)^{\phi_u}\left(\phi_s\left(\phi_k K_s^{\frac{\epsilon_k-1}{\epsilon_k}} + (1-\phi_k)S^{\frac{\epsilon_k-1}{\epsilon_k}}\right)^{\left(\frac{\epsilon_k}{\epsilon_k-1}\right)\left(\frac{\epsilon_s-1}{\epsilon_s}\right)} + (1-\phi_s)U^{\frac{\epsilon_s-1}{\epsilon_s}}\right)^{\left(\frac{\epsilon_s}{\epsilon_s-1}\right)(1-\phi_u)}. \tag{34}$$

This production function is a Cobb-Douglas aggregator of non-IT capital K_u with the remaining three factors. Therefore, the share of non-IT capital in income is $\alpha_{K_u} = \phi_u$. The other three factors of production enter in the production function according to a nested CES structure.

We set the elasticities ϵ_s and ϵ_k to the values estimated by Krusell, Ohanian, Rios-Rull, and Violante (2000). We then calibrate the share parameters ϕ_u, ϕ_k, and ϕ_s to match the factor income shares α_{K_u}, α_U, and α_S observed in KLEMS for the United States around 1980. (Will later replace "around".)

We normalize overall and sector-specific productivity levels A, ξ_u, and ξ_s to 1, and we choose the ratio of the supply of unskilled to skilled labor U/S to match the skill premium observed in the U.S. data in 1980.[2] We estimate the depreciation rates for IT and non-IT capital from KLEMS data. The borrowing constraint ψ is internally calibrated to target the share of households with negative asset holdings observed in the data (insert references).

Next, we calibrate household preference parameters. We assume a constant relative risk aversion period utility function:

$$u(c) = (c^{1-\gamma} - 1)/(1-\gamma), \tag{35}$$

and follow Heathcote, Storesletten, and Violante (2010) in setting $\gamma = 1.5$. The discount factor β is calibrated such that the equilibrium real interest rate matches that found in the U.S. in 1980.

Finally, we assume that the log labor endowment follows an AR(1) process:

$$\log z' = \rho \log z + \varepsilon, \tag{36}$$

with innovations following a zero-mean normal distribution, $\varepsilon \sim \mathcal{N}(0, \sigma^2)$. We set the persistence parameter ρ equal to the estimate in Guvenen (2007).[3] We then choose the variance of the shocks σ^2 so that the model matches the Gini coefficient on labor income in the United States.

[2] We later will calibrate ξ internally to hit K/Y.

[3] TBD, but: There are two complications in taking these parameters form existing studies. First, most studies that estimate such an income process use longitudinal data for individuals or households, not dynasties as in the model. Accordingly, if lifecycle estimates are used to parameterize dynastic income, the resulting stationary distribution can look very different from the actual cross-sectional income distribution. Secondly, a large literature (see for example Guvenen (2007)) suggests that there is much more known heterogeneity to income growth (e.g. person-specific income growth rates) than implied by a homogenous AR(1) process, and when accounting for these factors, permanent shocks to income appear to have much lower persistence than the near unit root typically estimated.

4.2 Characterizing the Stationary State

Key moments of the steady state of our model are listed in the first column of table 3. The model's values for the interest rate, skill-premium, factor shares, and share of borrowers in rows (i) to (viii) are identical to the corresponding values in the data, as those moments were targets used in our calibration strategy. (Ultimately, we'll include $CV(y^l)$ in the exactly matched category, but not here due to differences in CV and Gini.)

Figure 1 plots the distribution of capital income y^k in the equilibrium of the economy. Households each period experience changes in their particular asset holdings and capital income but in steady state the distribution across these households is stationary. The vertical dashed line to the left of zero corresponds to $r\psi < 0$, the negative capital income associated with interest payments on the largest amount of borrowing allowed. The total mass between that dashed line and the zero value corresponds with the values in row (viii) of Table 3. Note that there is essentially no mass at the borrowing constraint itself. Agents always respond at least a bit to their precautionary saving motive given the potential for a negative shock in the next period.

The modal capital income is below 0.5 in Figure 1, but there is significant mass to the right of that point, with some households earning many multiples of that amount in capital income. Row (xi) of Table 3 shows that capital income inequality in the benchmark model equals XXX, only XXX lower than the empirical counterpart in the U.S. in the early 1980s. (Describe here how we adjust data, i.e. subtracting outliers, etc.)

The equilibrium distribution of labor income y^l in the economy reflects two factors. First, there is a skill premium and workers differ in their skill endowments. Second, within any given skill endowment, labor income varies with the AR1 process (36). Figure 2 plots the labor distributions for each endowment type separately. The domain of both distributions includes only positive labor incomes. The skilled distribution is shifted somewhat to the right of the other distribution as the skill premium is positive. Given the AR1 process applies to the log rather than the level of the labor endowment, this further results in more dispersion and greater mass on very large incomes for the skilled distribution. As shown in Row (x) of Table 3, these forces lead to overall labor income inequality in the benchmark model of XXX, which equals that in the data as it was a targeted moment used in our calibration strategy.

[BN: I would like to add here a Figure with 3 lines – capital, labor, and total income. This would allow us to discuss the intuition behind the ρ values, which we will list in the table. In particular, if $\rho(Y^l, Y^k) = 0$, we'd expect the labor and capital lines to sort of "average" together to form the overal income line. If $\rho(Y^l, Y^k) > 0$, these lines should both be more visually concentrated than the total income. Now, our decomposition does not use $\rho(Y^l, Y^k) = 0$ (it instead uses the related but different $\rho(Y^k, Y)$), so I don't know if this will work. But I'd like to try. I believe John has a note saying this looks bad due to scaling or something, but it'd help me to see the issue.]

5 Experiments

We now consider several experiments in which we introduce a shock which results in a new stationary state of the system. We compare the initial and final stationary states to demonstrate the repercussions of the shock for labor, capital, and overall inequality.

5.1 Changes in the Relative Price of Capital Goods

The first shock we consider decreases the price of IT investment, ξ_s, relative to that of non-IT investment, ξ_u. (SOURCE) suggests that ξ_s/ξ_u declined by 50% from 1980 to 2010. ξ_s and ξ_u are not identified in levels in the model, so we hold $\xi_u = 1$ and consider a decline in ξ_s from 1 (the benchmark value) to 0.5. Key moments of the resulting steady state are listed in the second column of 3. The changes to the capital, labor, and total income distributions are plotted in figures 4, 5, and 6 respectively.

The IT rental rate $R_s = \xi_s(r + \delta_s)$ falls because the decline in ξ_s dominates any change to the interest rate in equilibrium. The steady state value of K_s rises because it has become cheaper, but not by enough to offset the rental rate decline, so the IT share α_{K_s} declines from 0.05 to 0.04.

IT capital and skilled labor are complements, so firms would like to hire more skilled workers relative to unskilled workers. But, the supply of skilled and unskilled labor is fixed, so the wages must adjust: the wage premium w_s/w_u rises from 1.50 to 1.59, and α_s rises from 0.60 to 0.61.

Because the wage premium rises, labor income inequality also rises, with the coefficient of variation growing from 0.70 to 0.72. Without any significant change to the precautionary savings motive, capital income inequality rises just because households with higher labor income save more, and labor income inequality has grown. The correlations $\rho(y^l)$ and $\rho(y^k)$ change very little, so total income inequality increases too.

5.2 Changes in the Stock of Skill

The next shock increases the supply of skilled labor. Specifically, we reduce the ratio of unskilled to skilled labor U/S from 1.5 (the benchmark) to 1, the level suggested by (INSERT SOURCE/MOTIVATION). Key moments of the resulting steady state are listed in the third column of 3. The changes to the capital, labor, and total income distributions are plotted in figures 11, 12, and 13 respectively.

The wage premium w_s/w_u declines from 1.5 to 1.19 because unskilled labor becomes scarcer relative to skilled labor. Yet, the skilled labor share α_S rises and and the unskilled labor share α_u falls; in the benchmark calibration skilled and unskilled labor are complementary, so the wages do not move enough to offset the quantity changes. In sum, the labor share falls form 0.60 to 0.59. α_{K_u} is fixed by the Cobb-Douglas assumption, so it is α_{K_s} that increases. The interest rate doesn't move enough to significantly affect the rental rate of capital, because there is little change to households' precautionary savings motive, so the α_{K_s} increase is driven by quantity. IT capital is complementary to skilled labor, which has increased, so firms substitute towards IT capital. (NOTE: if we pick new elasticities, this may have to be rewritten)

All inequality moments exhibit large changes, and The decreased skill premium drives changes to all five inequality moments. Labor income inequality falls because the skill premium has fallen (note the skilled/unskilled distributions in 12 shifting towards one another). Capital income inequality falls directly because of the decreased labor income inequality - the precautionary savings motive is mostly unaffected, so the relationship between household labor income and their savings decision is also unaffected. The labor/total income correlation is unchanged, but the capital/total income correlation falls. [I NEED INTUITION FOR THIS ONE!]

5.3 Changes in Borrowing Limits

The final shock decreases the borrowing constraint, ψ, relative to that of non-IT investment, xi_u. We reduce the borrowing constraint from -1.08 to -10, which yields an increase in the share of households with nonpositive wealth from 0.12 to 0.17, roughly the increase for 1980 to 2010, measured by the Survey of Consumer Finances (NOTE: We should reproduce ourselves). Key moments of the resulting steady state are listed in the fourth column of 3. The changes to the capital, labor, and total income distributions are plotted in figures 18, 19, and 20 respectively.

When the borrowing constraint is relaxed, households have less incentive to hold assets as precautionary savings, so the interest rate rises in equilibrium. The steady state capital stock must be lower to produce the higher interest rate, but the price and quantity changes offset each other offset each other (perfectly in the case of K_u), so the factor shares are largely unchanged. Labor supplies are constant, so the wage premium doesn't move very much either.

However, there are significant implications for inequality. With greater ability to borrow, a larger share of households hold negative wealth in the stationary distribution. The greater range of assets holdings also increases capital income inequality. Labor income inequality is unchanged because skill allocations, wages, and the labor income process are unchanged. The correlation between capital and total income rises because agents are able to adjust their wealth more to self-insure as their labor income evolves. With improved self-insurance, the correlation between labor and total income falls. However, this correlation decrease is dominated by the increased capital income inequality in the decomposition of total income inequality (equation 1), so total income inequality increases.

6 Extensions

There are two goals of this section. The first is to convince the reader that our main conclusions are robust to allowing for features that move us closer to the frontier. The second is to address shortcomings of the basic model. For example, the basic model may underpredict capital income inequality. So we will introduce heterogeneous discount factors to address this and then ask how our conclusions are affected.

6.1 Heterogeneity in Discount Factors

6.2 Endogenous Labor Supply

6.3 Taxes and Transfers

6.4 Life-Cycle

Introduce a new dimension of heterogeneity, say q, that denotes stochastic movements across different stages of the life-cycle. For example q could be young, middle aged and retired. Pick transitions of q to match age profile of labor force. q should be correlated with z shocks to mimic the hump shape in life-cycle wages. See e.g. Castaneda et al 2003, JPE.

7 Cross-Country Empirics, Shock Estimation, Quantitative Results, Etc.

Clearly, all three of us would like to push in the empirical direction and speak more closely with the data. We feel confident we'll get there in at least some way, but we should completely ignore this for now. Make the above sections tight and compelling, and this will work itself out later.

8 Concluding Remarks

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	Parameter	AKN Value	Cobb-Douglas Value	Target Moment	Target Source
Panel A: Externally Calibrated Parameters					
(i)	ρ	0.820	0.820		Guvenen (2007)
(ii)	γ	1.500	1.500		Heathcote, Storesletten, and Violante (2010)
(iii)	(ϵ_k, ϵ_s)	$(2.000, 1.500)$	$(1.000, 1.000)$		XXXXXXX
(iv)	(δ_u, δ_s)	$(0.037, 0.145)$	$(0.037, 0.145)$		KLEMS (U.S., 1980)
(v)	A	1.000	1.000		Normalization
Panel B: Internally Calibrated Parameters					
(vi)	σ^2	0.129	0.129	$\text{Gini}(y^l)$	To Be Added (0.35 for now)
(vii)	(ϕ_u, ϕ_s, ϕ_k)	$(0.308, 0.509, 0.216)$	$(0.308, 0.302, 0.115)$	$(\alpha_u, \alpha_s, \alpha_{K_u})$	KLEMS (U.S., 1980)
(viii)	(ξ_u, ξ_s)	$(1.397, 3.517)$	$(1.397, 3.517)$	$(K_u/Y, K_s/Y)$	KLEMS (U.S., 1980)
(ix)	U/S	3.974	3.974	W_s/W_u	KLEMS (U.S., 1980)
(x)	ψ	-0.683	-0.847	% of HHs with $a < 0$	SCF (1983)
(xi)	β	0.937	0.937	r	To Be Added

Table 1: Calibration of Model Parameters

Notes: TBD

	Moment	Model Value	Data Value	Data Source
(i)	r	0.050	0.050	XXXX
(ii)	W_s/W_u	1.522	1.522	KLEMS (U.S., 1980)
(iii)	α_u	0.483	0.483	KLEMS (U.S., 1980)
(iv)	α_s	0.185	0.185	KLEMS (U.S., 1980)
(v)	α_{K_s}	0.024	0.024	KLEMS (U.S., 1980)
(vi)	α_{K_u}	0.308	0.308	KLEMS (U.S., 1980)
(vii)	s_L	0.668	0.668	KLEMS (U.S., 1980)
(viii)	K_u/Y	2.534	2.534	KLEMS (U.S., 1980)
(ix)	K_s/Y	0.035	0.035	KLEMS (U.S., 1980)
(x)	Share of HHs with $a < 0$	0.169	0.169	SCF (1983)
(xi)	$CV(y)$	0.664	TBD	SCF (1983), adjusted for outliers
(xii)	$CV(y^l)$	0.696	TBD	SCF (1983), adjusted for outliers
(xiii)	$CV(y^k)$	1.128	TBD	SCF (1983), adjusted for outliers
(xiv)	$\rho(y^l)$	0.938	TBD	SCF (1983), adjusted for outliers
(xv)	$\rho(y^k)$	0.626	TBD	SCF (1983), adjusted for outliers
(xvi)	Correlation(y^l, y^k)	0.318	TBD	SCF (1983), adjusted for outliers
(xvii)	Gini(y)	0.344	TBD	SCF (1983), adjusted for outliers
(xviii)	Gini(y^l)	0.350	TBD	SCF (1983), adjusted for outliers
(xix)	Gini(y^k)	0.586	TBD	SCF (1983), adjusted for outliers

Table 2: Key Steady-State Moments of the Benchmark Model

Notes: TBD

	Moment	Baseline Value	$\xi_s = 0.492$	$U/S = 2.18$	$\psi = -10$	$\tau_c = 0.4$
(i)	r	0.050	0.052	0.050	0.052	0.053
(ii)	W_s/W_u	1.522	1.682	1.122	1.521	1.493
(iii)	α_u	0.483	0.437	0.440	0.483	0.491
(iv)	α_s	0.185	0.185	0.226	0.185	0.185
(v)	α_{K_s}	0.024	0.070	0.027	0.024	0.016
(vi)	α_{K_u}	0.308	0.308	0.308	0.308	0.308
(vii)	s_L	0.668	0.622	0.666	0.668	0.676
(viii)	K_u/Y	2.534	2.488	2.532	2.489	1.466
(ix)	K_s/Y	0.035	0.726	0.038	0.034	0.014
(x)	% of HHs with $a < 0$	0.169	0.139	0.163	0.244	0.244
(xi)	$CV(y)$	0.664	0.678	0.627	0.677	0.681
(xii)	$CV(y^l)$	0.696	0.718	0.661	0.696	0.692
(xiii)	$CV(y^k)$	1.128	1.086	1.086	1.263	1.294
(xiv)	$\rho(y^l)$	0.938	0.931	0.933	0.921	0.921
(xv)	$\rho(y^k)$	0.626	0.650	0.610	0.636	0.650
(xvi)	Correlation(y^l, y^k)	0.318	0.348	0.285	0.287	0.301
(xvii)	Gini(y)	0.344	0.347	0.330	0.352	0.354
(xviii)	Gini(y^l)	0.350	0.357	0.338	0.350	0.349
(xix)	Gini(y^k)	0.586	0.560	0.574	0.667	0.669

Table 3: Key Steady-State Moments in Simulated Experiments - Baseline Specification

Notes: Experiment 1 reduces the price ξ_s of IT capital by 86%, from 3.517 to 0.492. Experiment 2 reduces the ratio of unskilled to skilled labor by 45%, from 3.974 to 2.18. Experiment 3 reduces the borrowing constraint ψ from -0.765 to -10. Experiment 4 increases the corporate income tax from 0 to 0.4.

	Moment	Baseline Value	$\xi_s = 0.492$	$U/S = 2.18$	$\psi = -10$
(i)	r	0.050	0.050	0.050	0.052
(ii)	W_s/W_u	1.522	1.522	0.837	1.522
(iii)	α_u	0.483	0.483	0.483	0.483
(iv)	α_s	0.185	0.185	0.185	0.185
(v)	α_{K_s}	0.024	0.024	0.024	0.024
(vi)	α_{K_u}	0.308	0.308	0.308	0.308
(vii)	s_L	0.668	0.668	0.668	0.668
(viii)	K_u/Y	2.534	2.534	2.534	2.487
(ix)	K_s/Y	0.035	0.250	0.035	0.035
(x)	% of HHs with $a < 0$	0.169	0.163	0.167	0.237
(xi)	$CV(y)$	0.664	0.664	0.632	0.676
(xii)	$CV(y^l)$	0.696	0.696	0.665	0.696
(xiii)	$CV(y^k)$	1.128	1.120	1.096	1.255
(xiv)	$\rho(y^l)$	0.938	0.939	0.934	0.922
(xv)	$\rho(y^k)$	0.627	0.625	0.612	0.636
(xvi)	Correlation(y^l, y^k)	0.318	0.319	0.288	0.288
(xvii)	Gini(y)	0.344	0.343	0.332	0.351
(xviii)	Gini(y^l)	0.350	0.350	0.339	0.350
(xix)	Gini(y^k)	0.587	0.582	0.578	0.662

Table 4: Key Steady-State Moments in Simulated Experiments - Cobb-Douglas Specification

Notes: Experiment 1 reduces the price ξ_s of IT capital by 86%, from 3.517 to 0.492. Experiment 2 reduces the ratio of unskilled to skilled labor by 45%, from 3.974 to 2.18. Experiment 3 reduces the borrowing constraint ψ from -0.765 to -10.

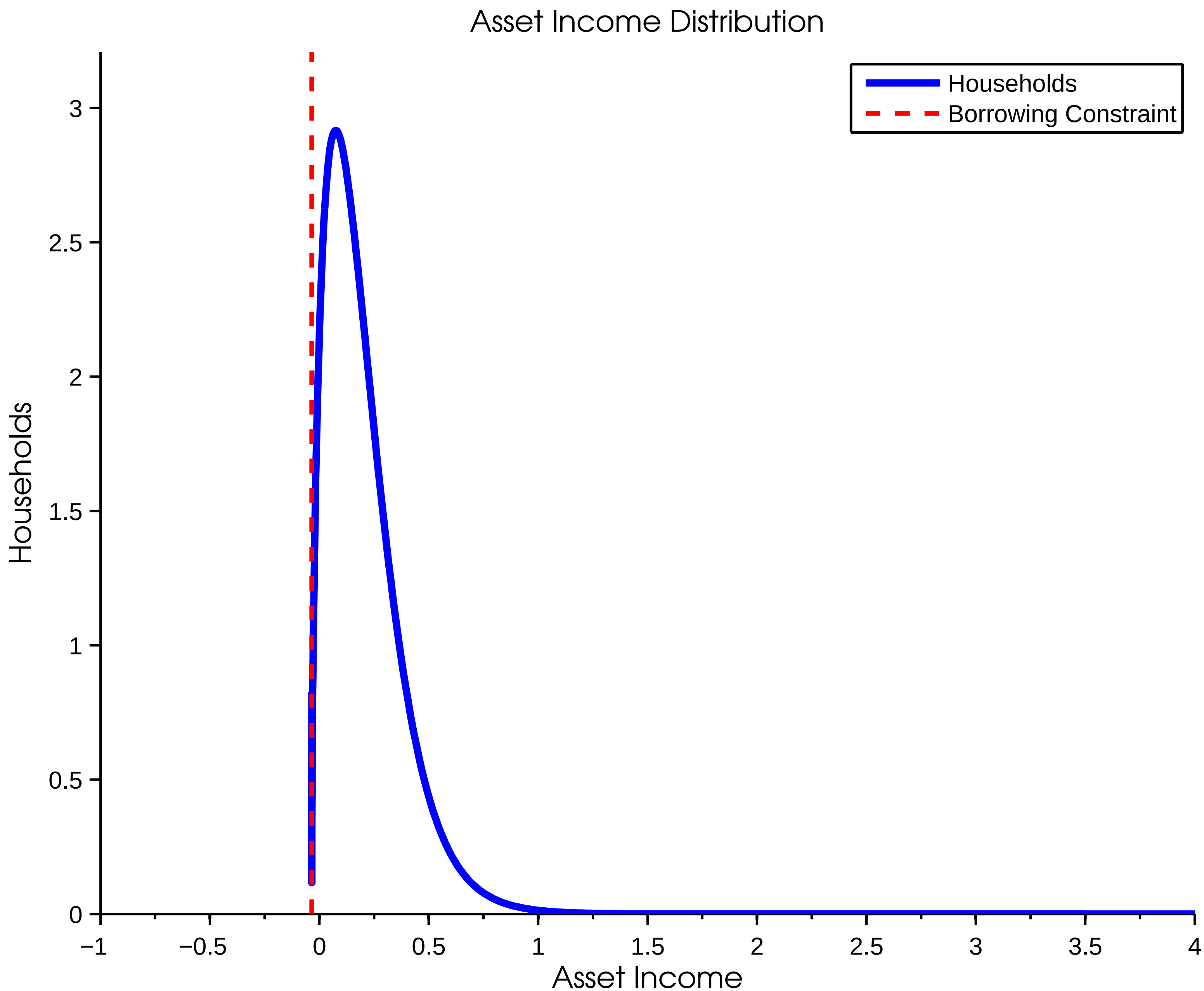

Figure 1: Benchmark Capital Income Distribution

Notes:

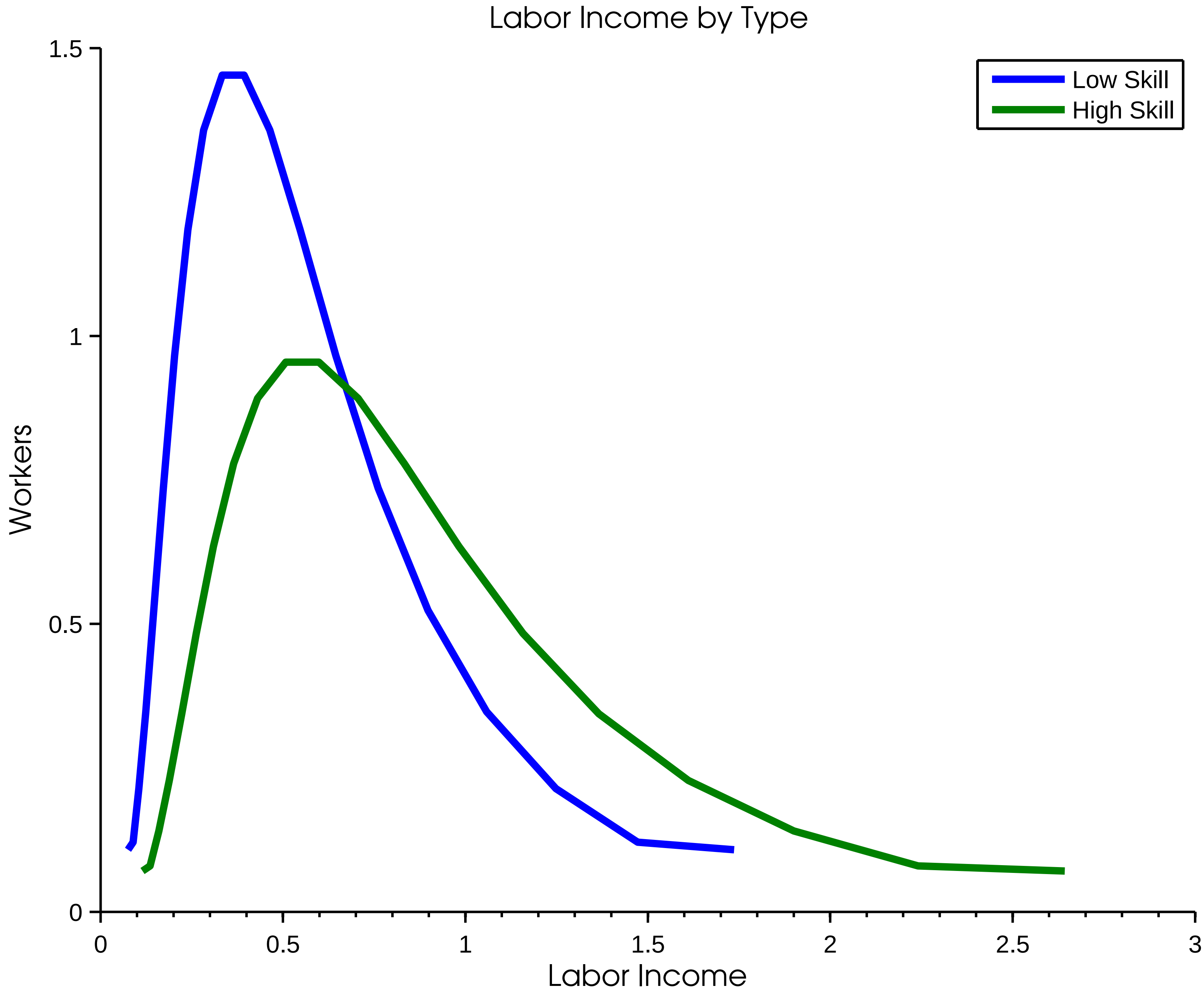

Figure 2: Benchmark Labor Income Distribution by Skill Type

Notes:

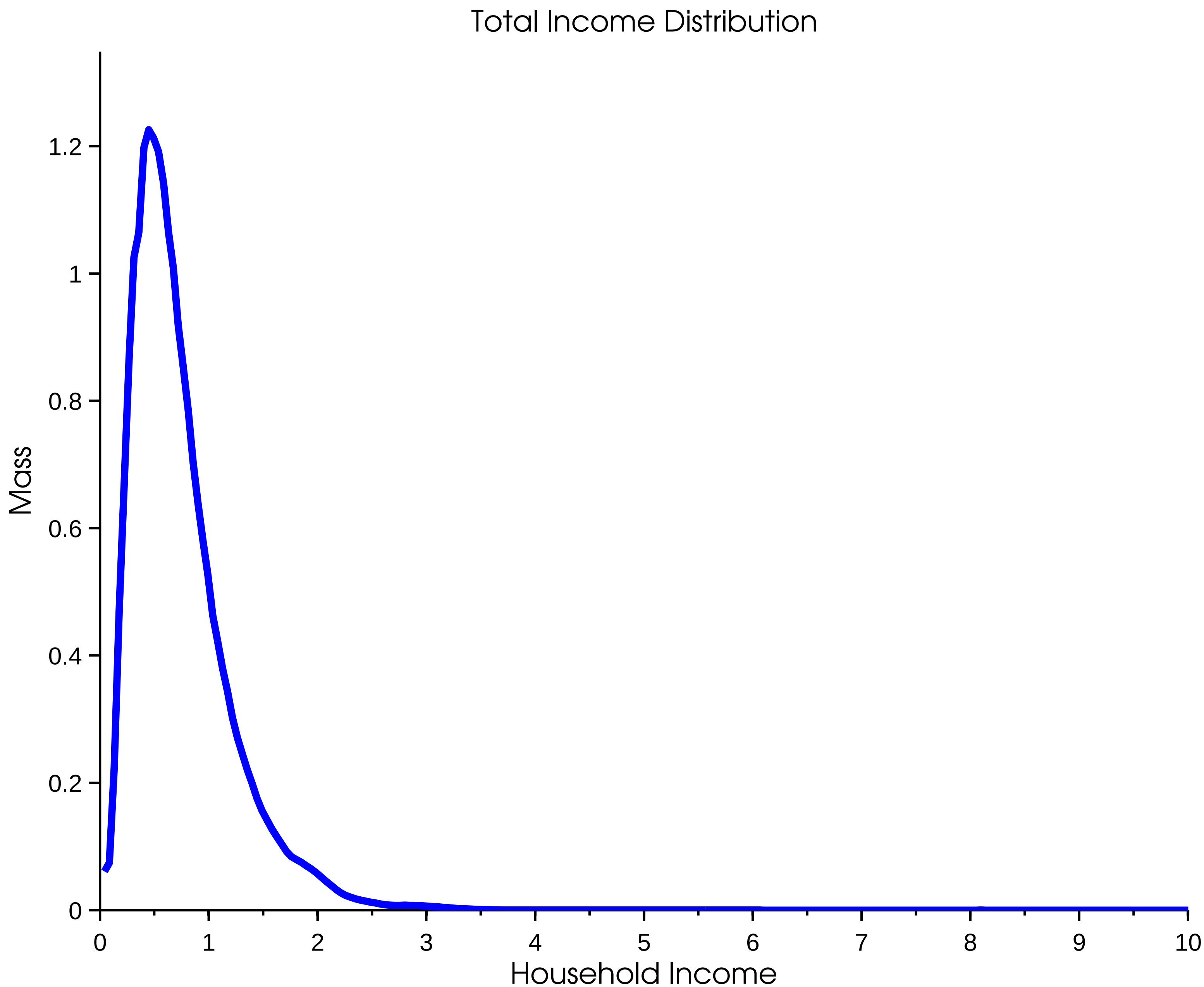

Figure 3: Benchmark Total Income Distribution

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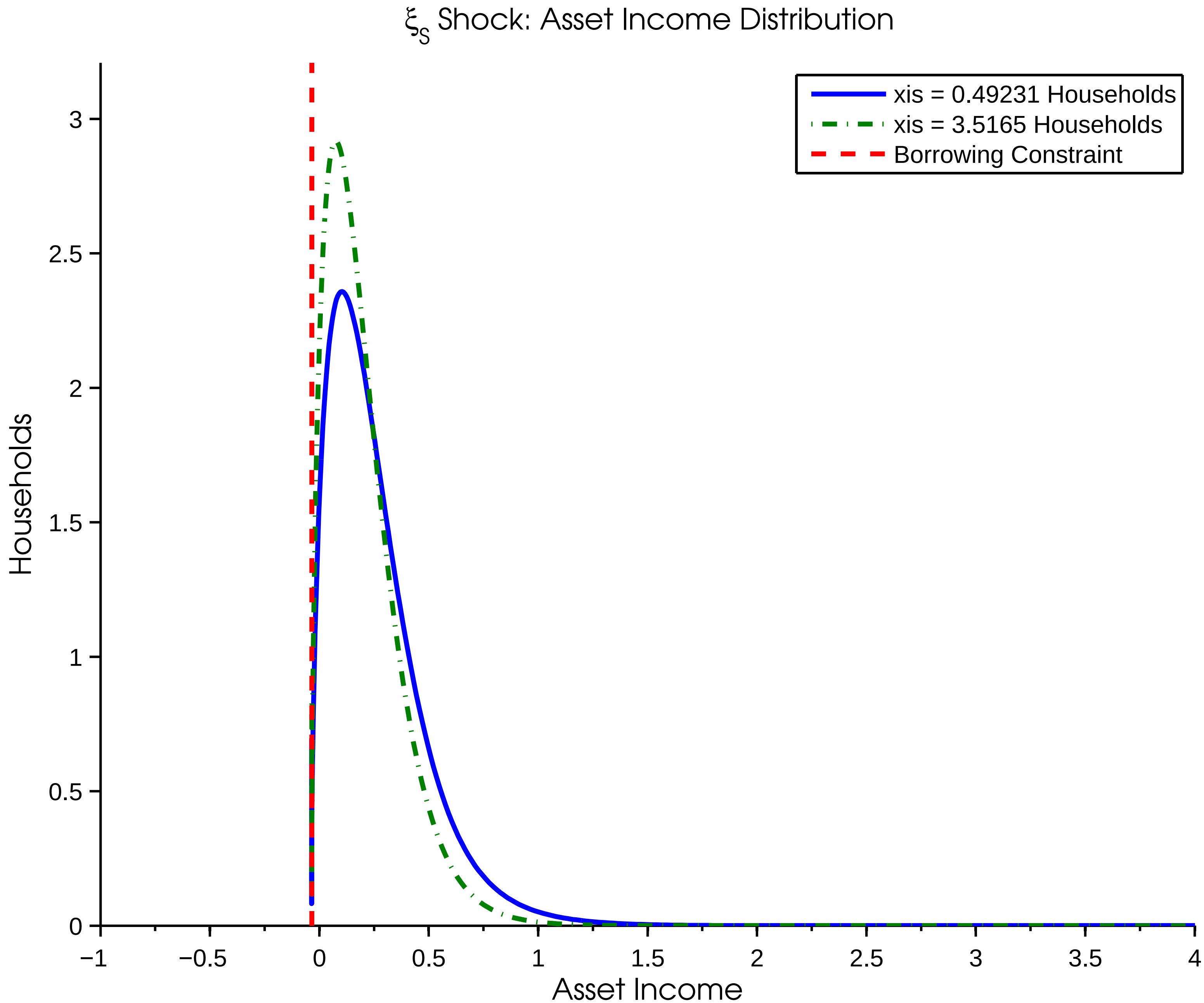

Figure 4: Capital Income Distribution With an IT Price Shock

Notes:

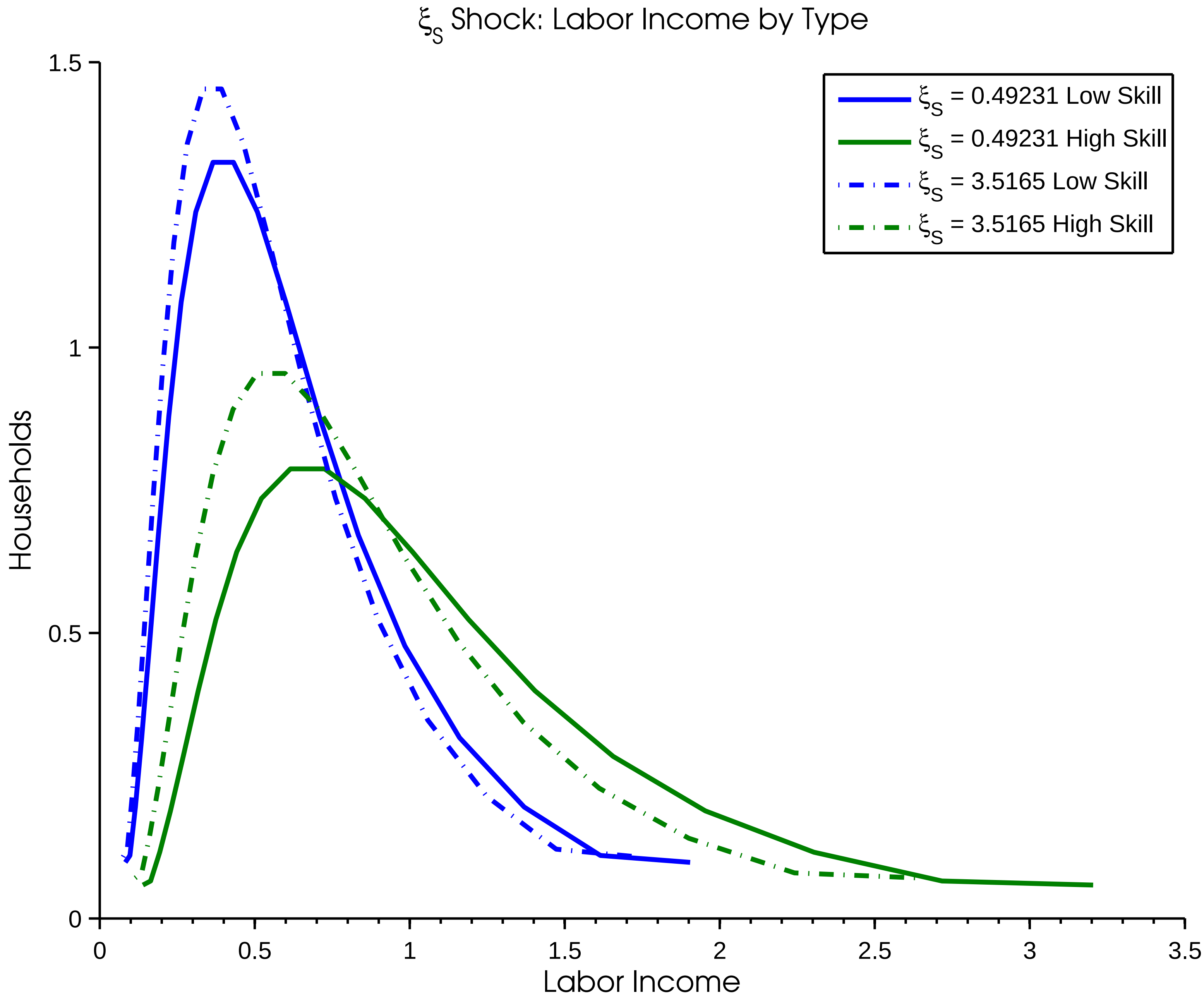

Figure 5: Benchmark Labor Income Distribution by Skill Type With an IT Price Shock

Notes:

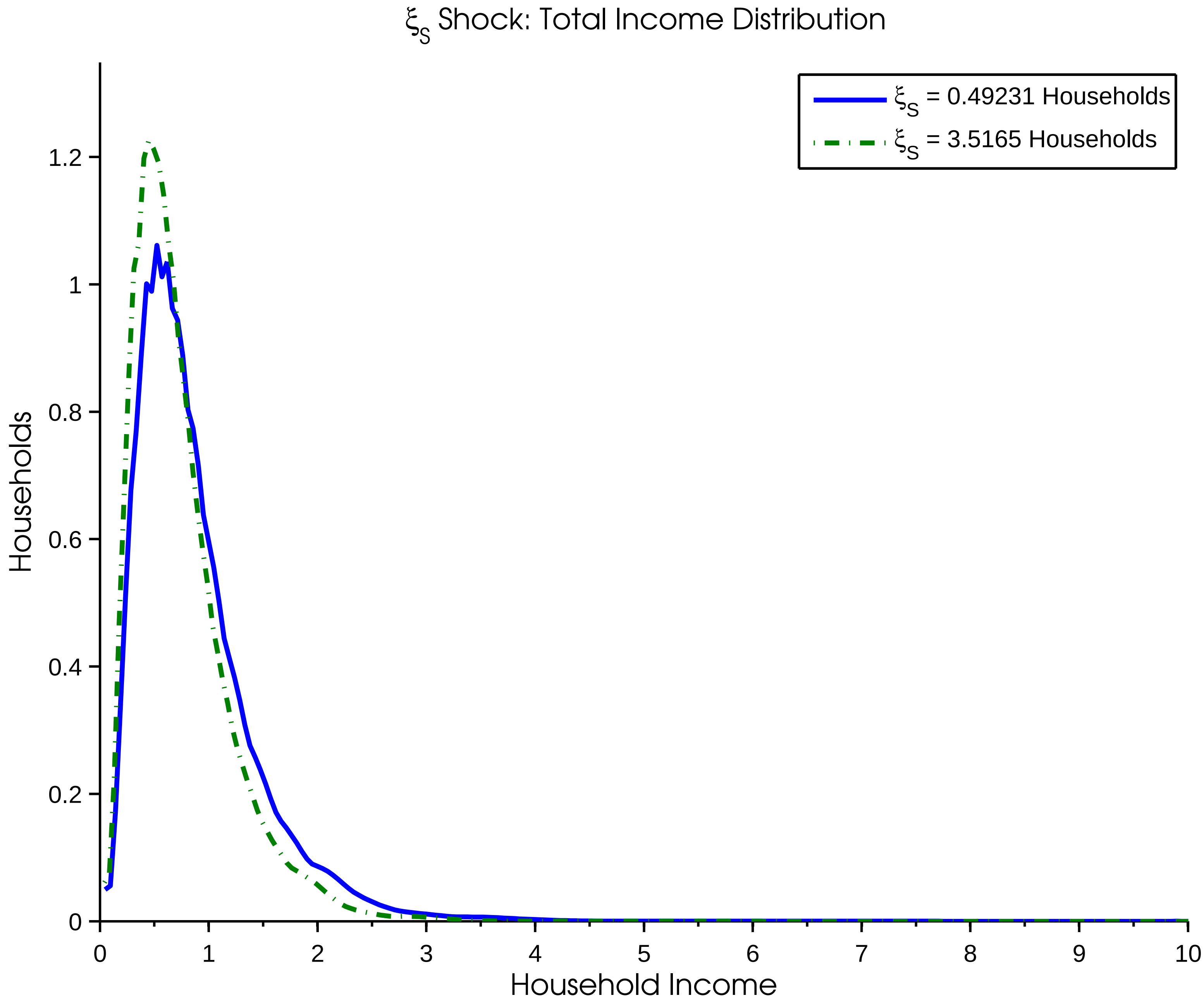

Figure 6: Total Income Distribution With an IT Price Shock

Notes:

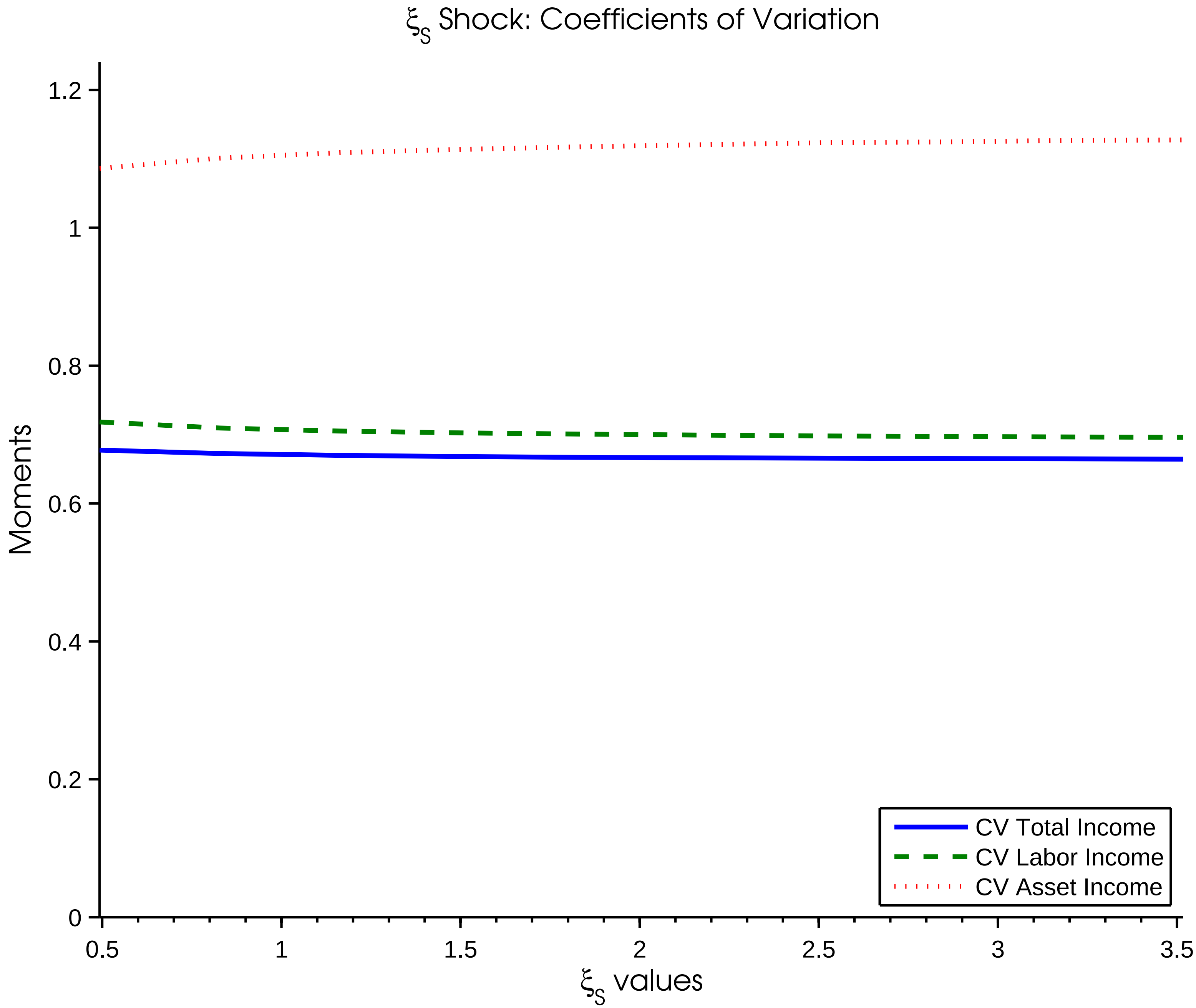

Figure 7: Coefficients of Variation With an IT Price Shock

Notes:

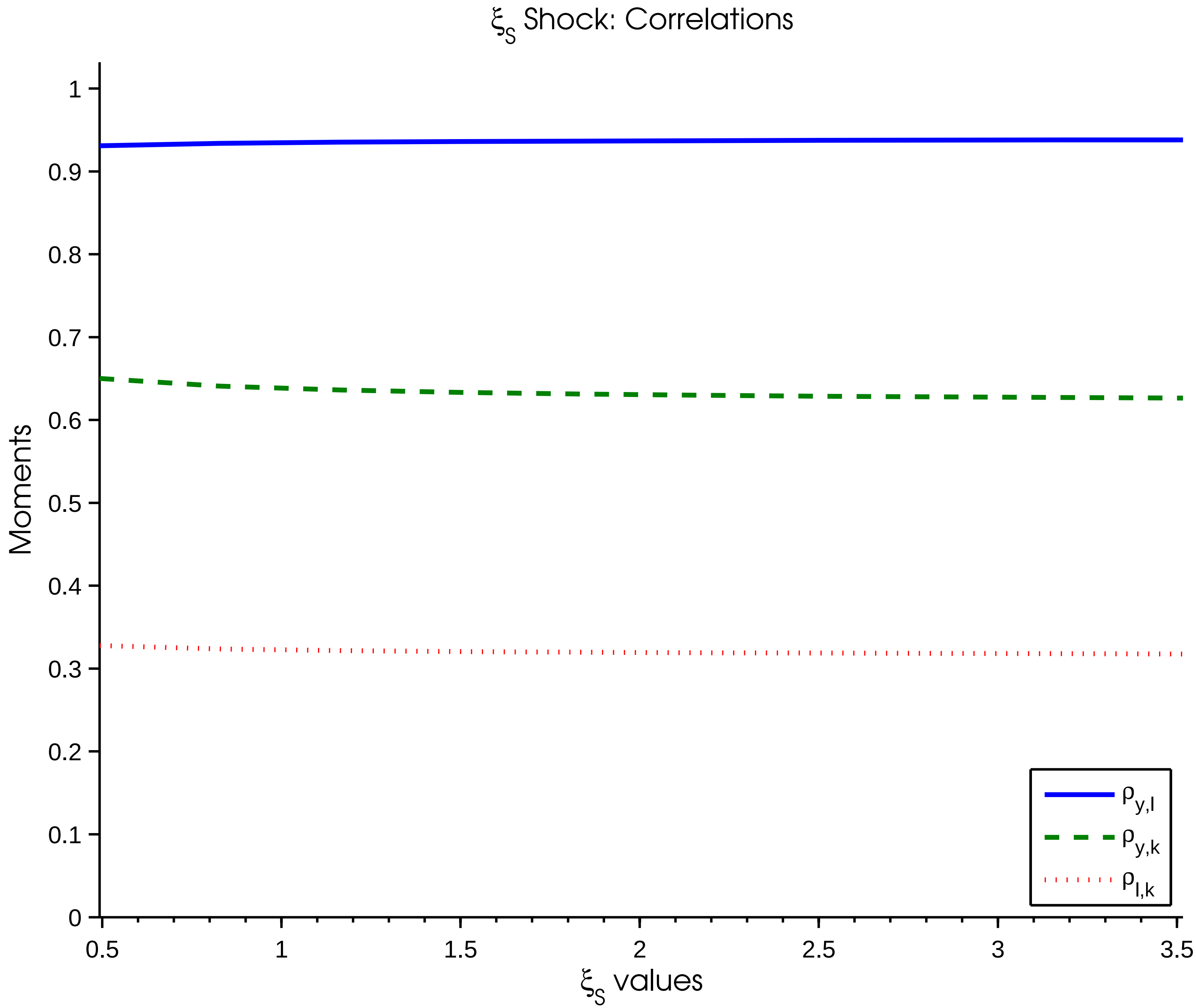

Figure 8: Correlations With an IT Price Shock

Notes:

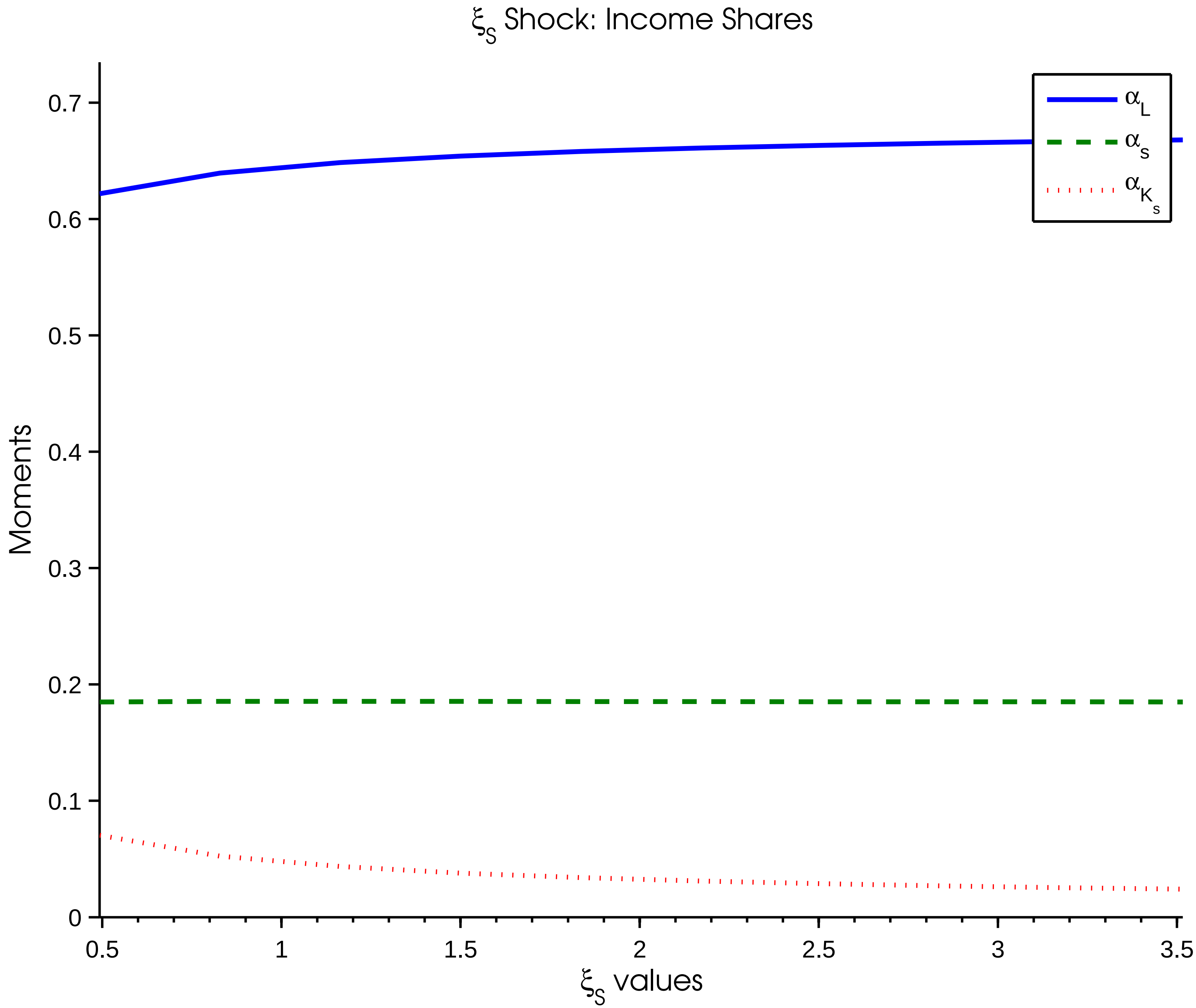

Figure 9: Income Shares With an IT Price Shock

Notes:

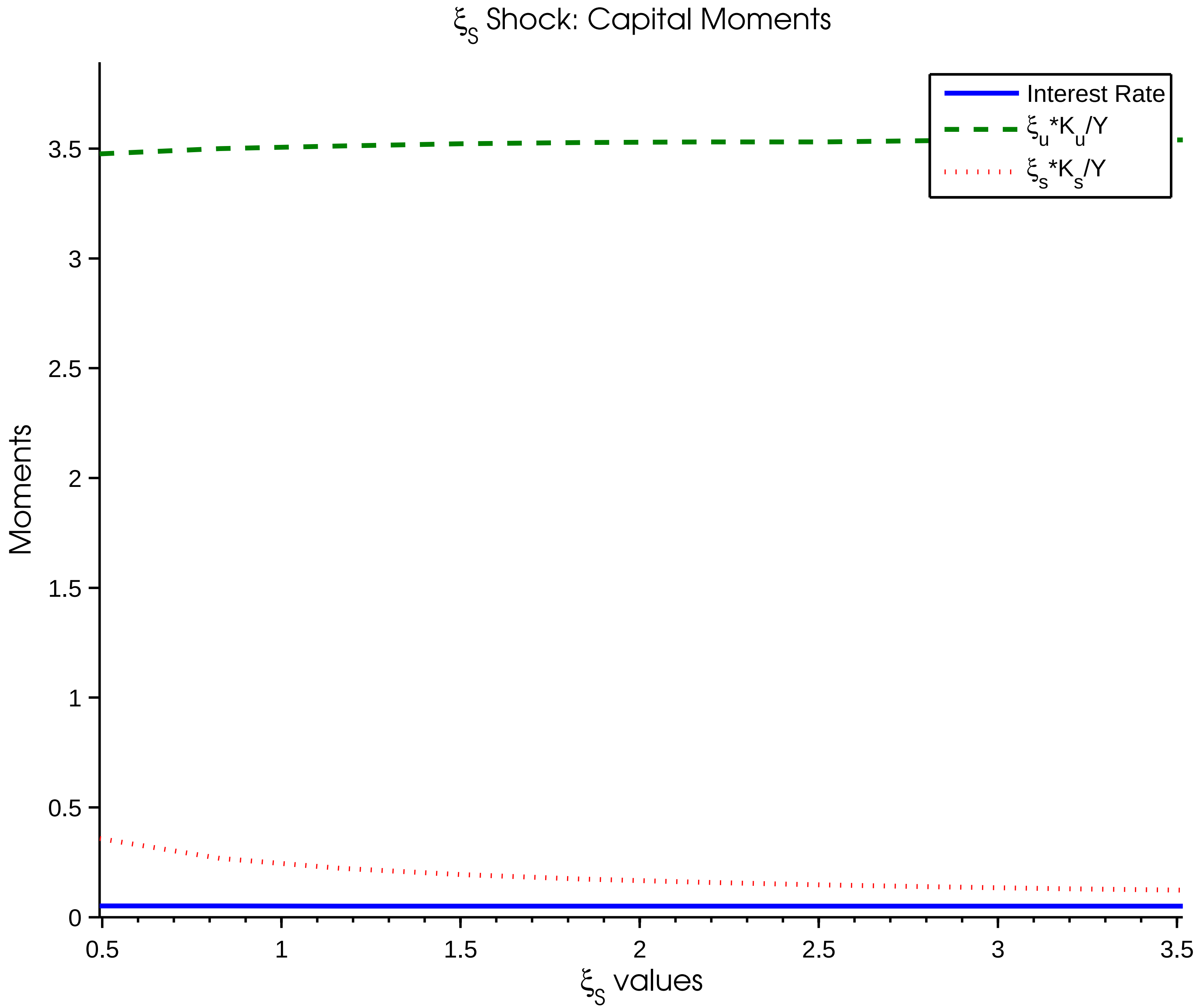

Figure 10: Capital Moments With an IT Price Shock

Notes:

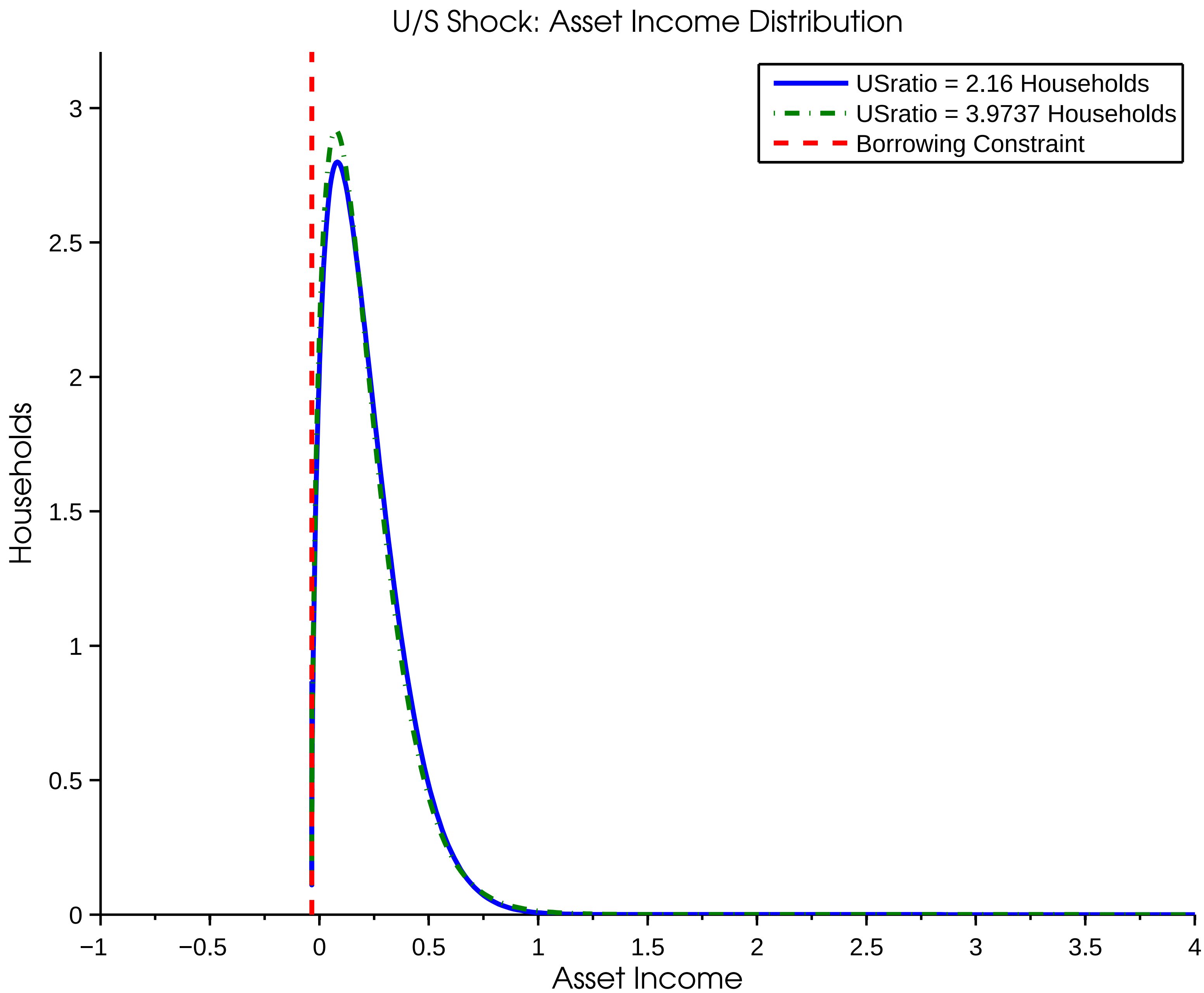

Figure 11: Benchmark Capital Income Distribution With a Change in the Stock of Skill

Notes:

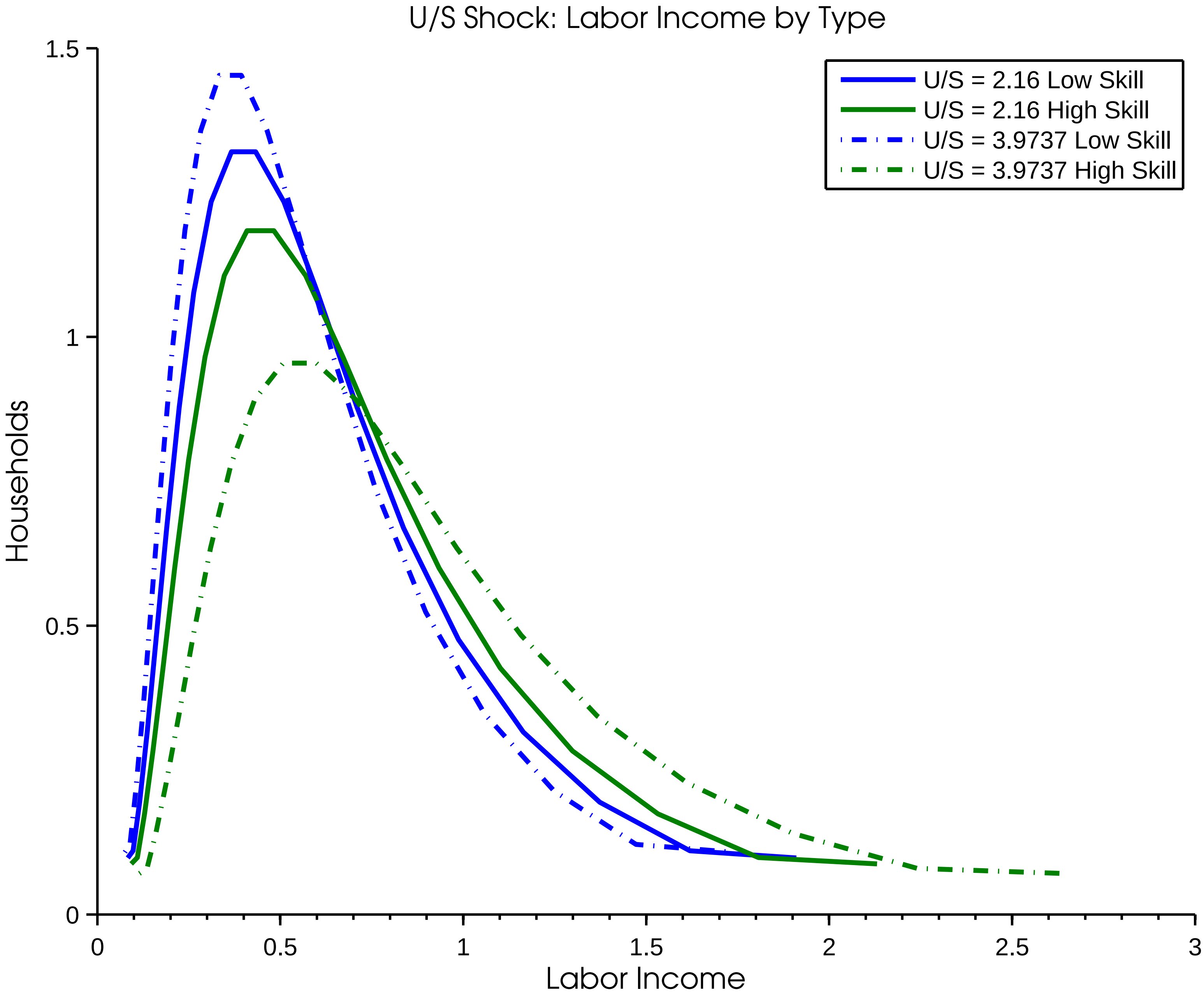

Figure 12: Benchmark Labor Income Distribution by Skill Type With a Change in the Stock of Skill

Notes:

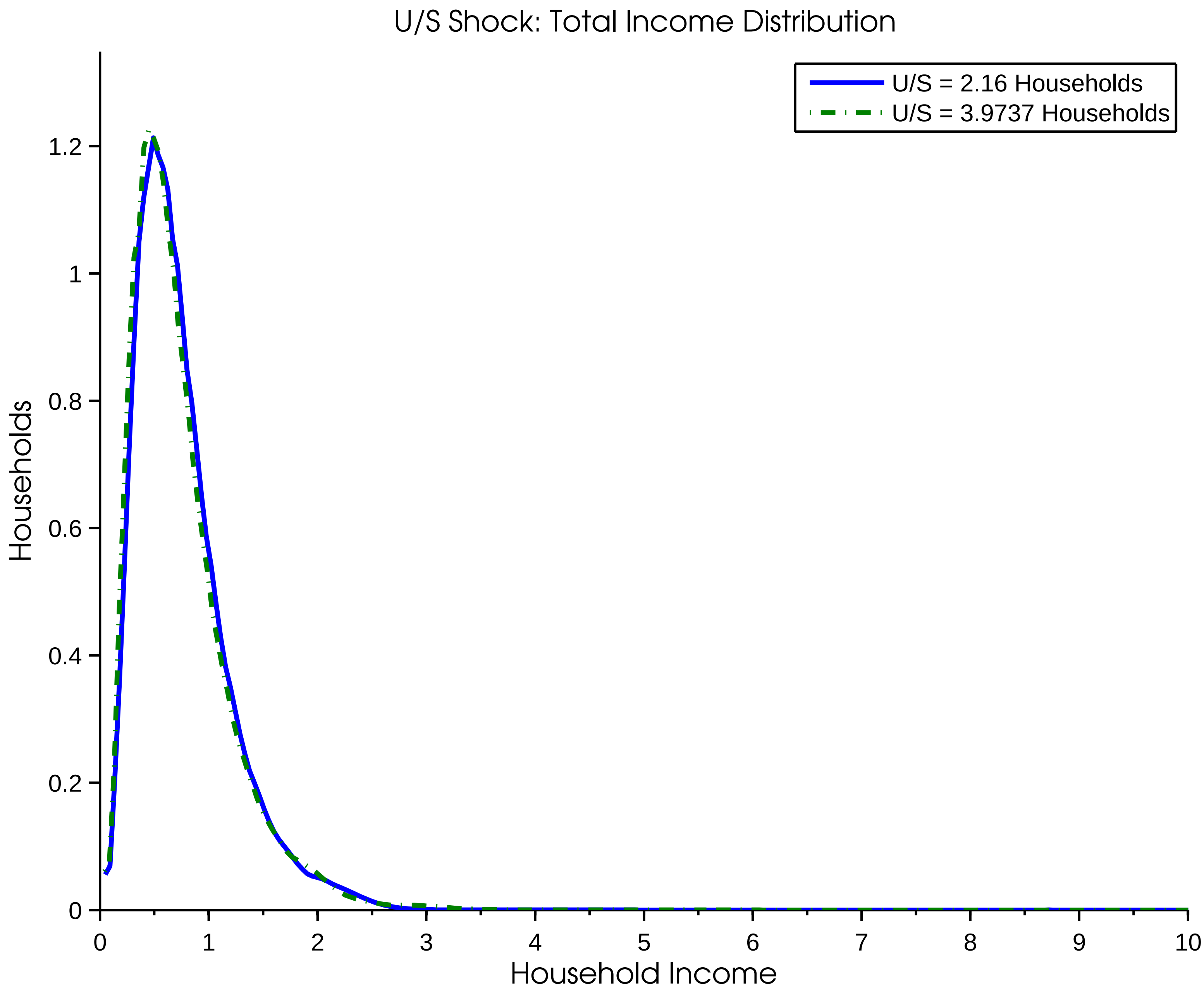

Figure 13: Total Income Distribution With a Change in the Stock of Skill

Notes:

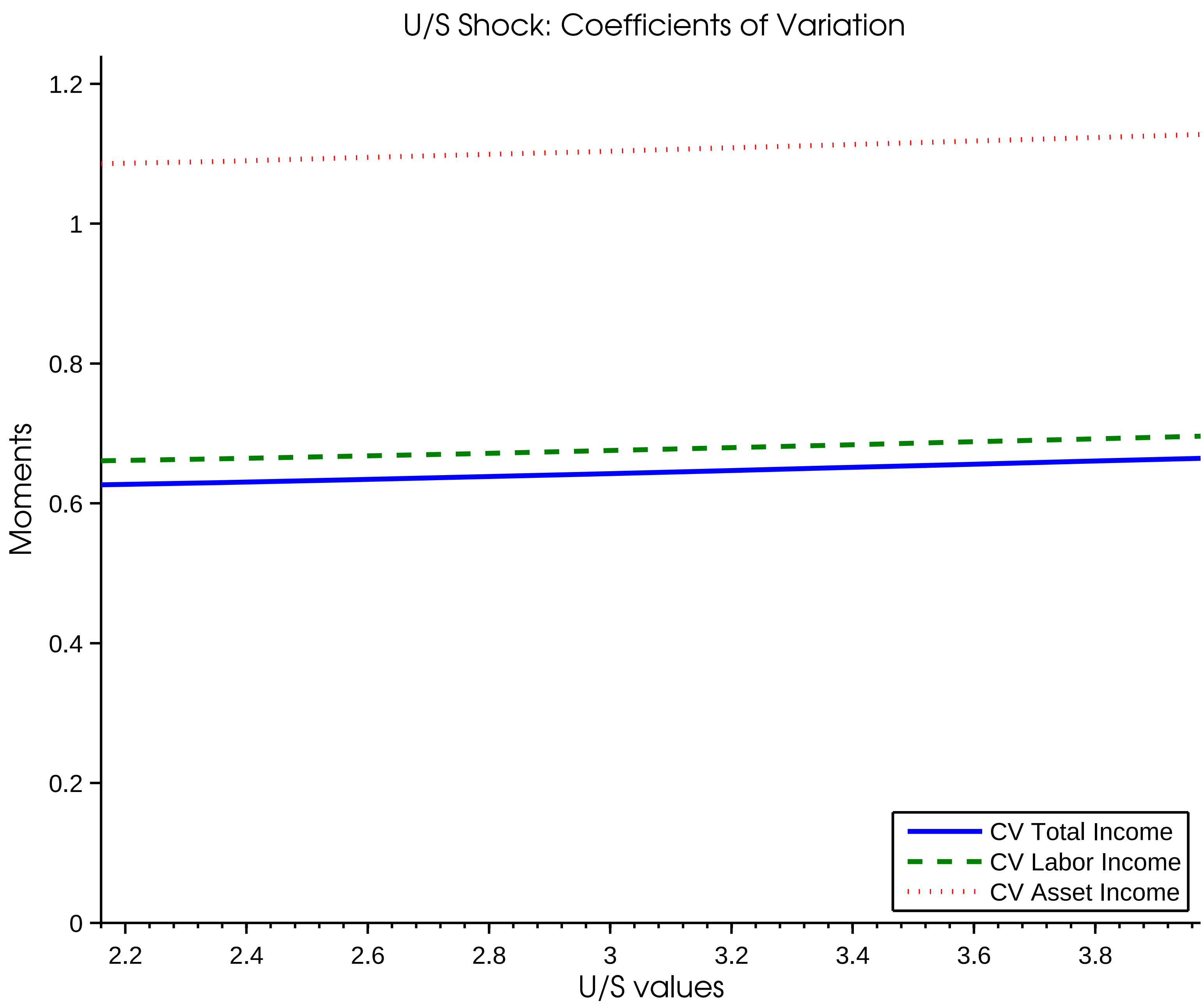

Figure 14: Coefficients of Variation With a Change in the Stock of Skill

Notes:

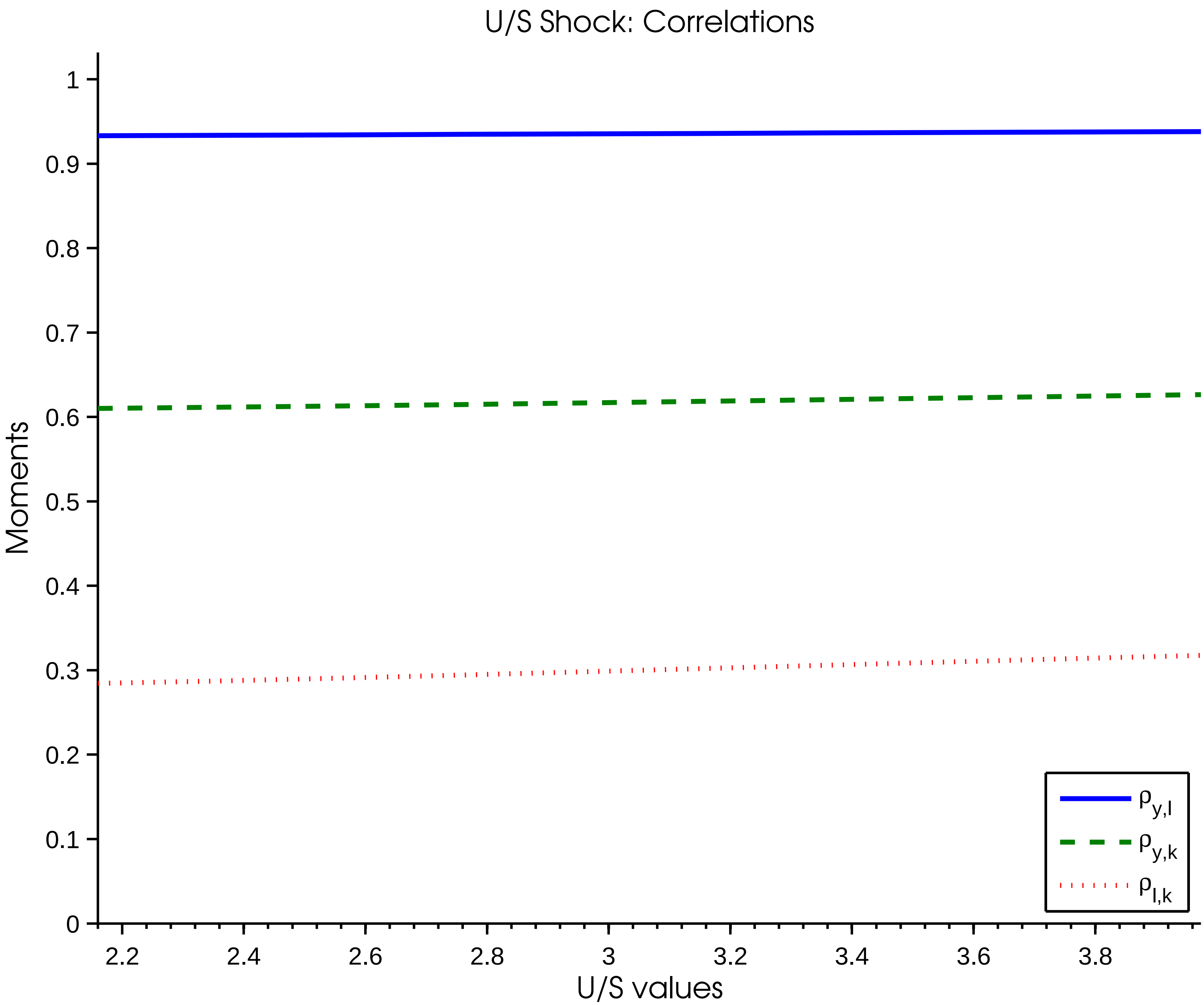

Figure 15: Correlations With a Change in the Stock of Skill

Notes:

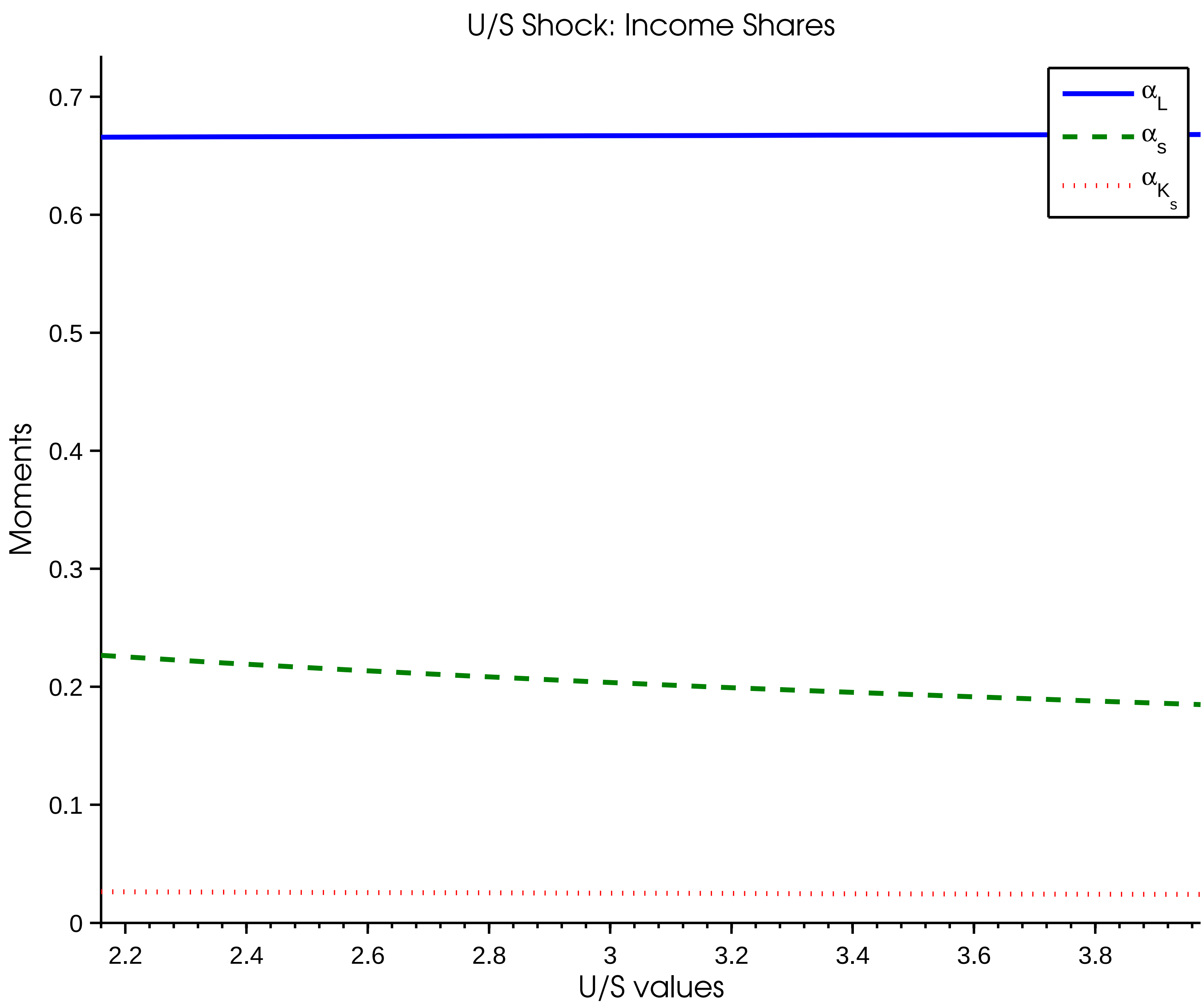

Figure 16: Income Shares With a Change in the Stock of Skill

Notes:

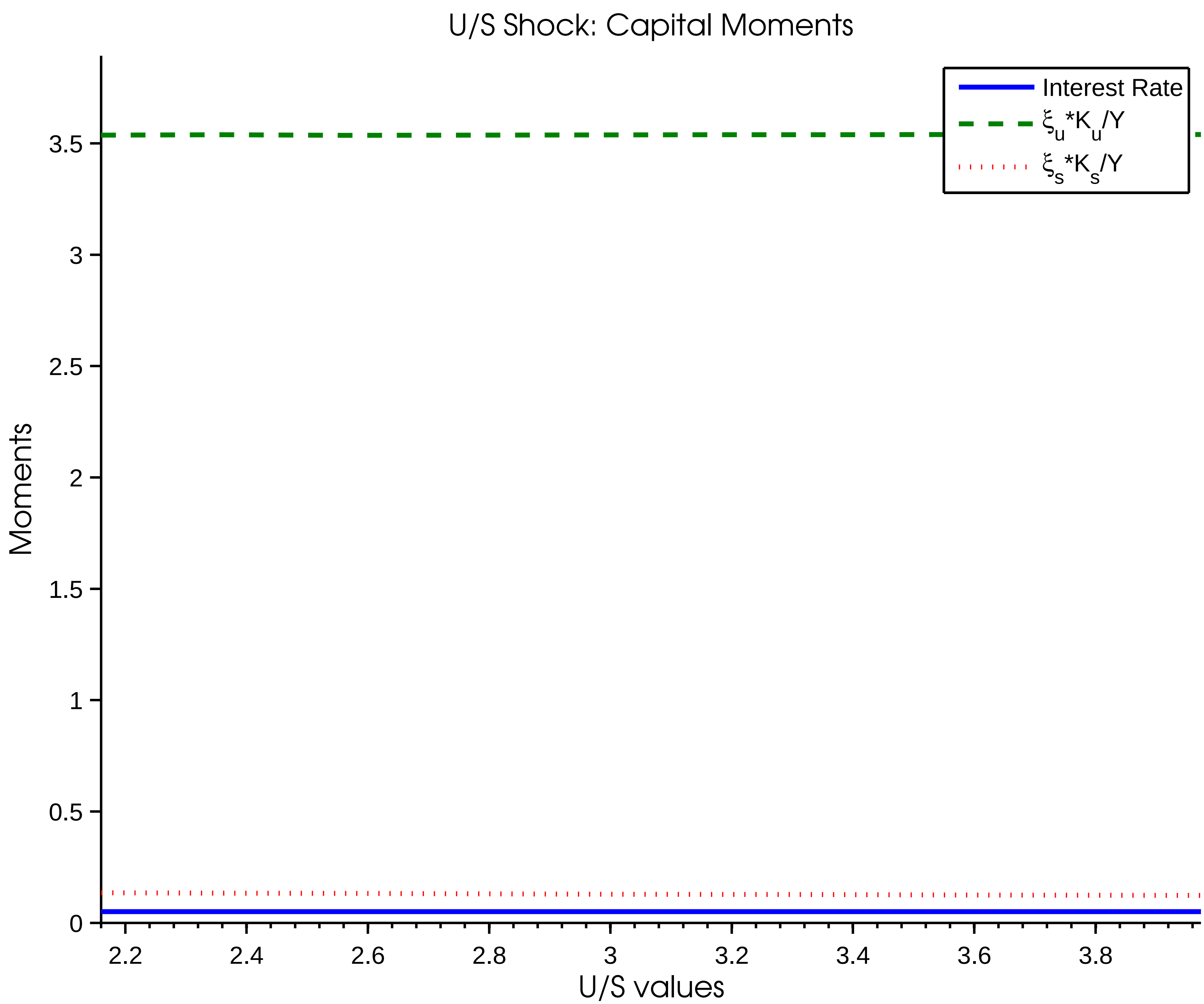

Figure 17: Capital Moments With a Change in the Stock of Skill

Notes:

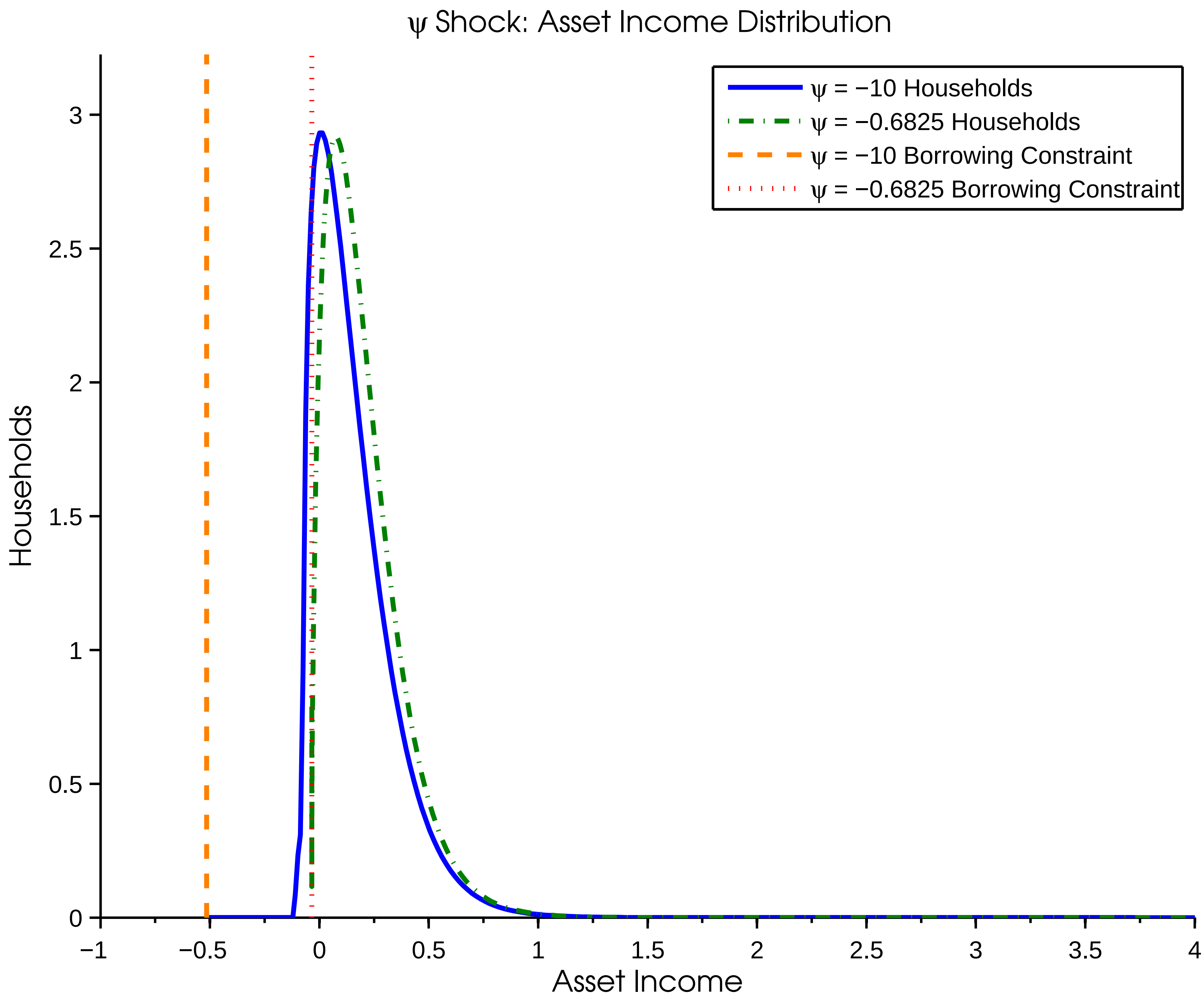

Figure 18: Benchmark Capital Income Distribution With a Change in the Borrowing Constraint

Notes:

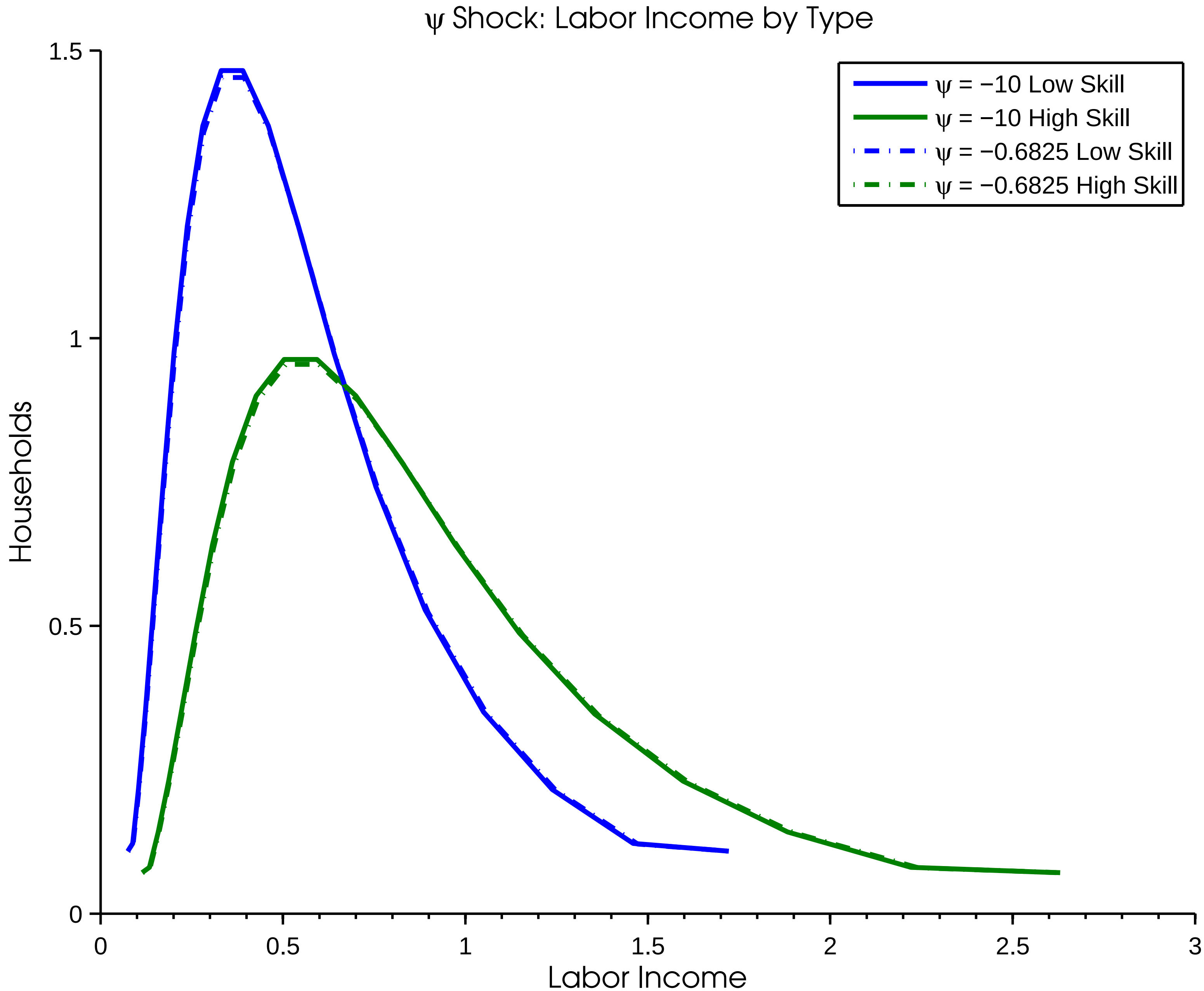

Figure 19: Benchmark Labor Income Distribution by Skill Type With a Change in the Borrowing Constraint

Notes:

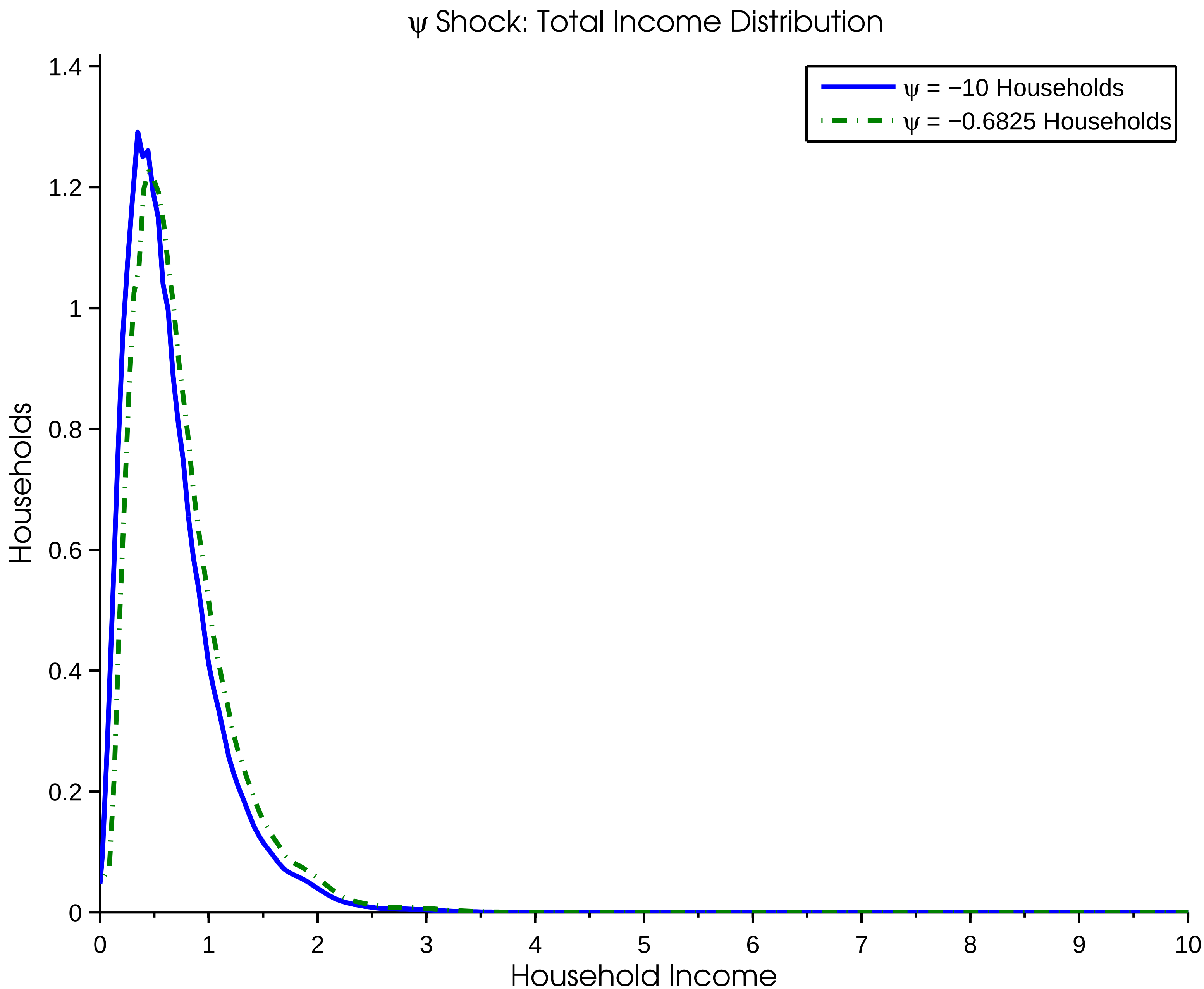

Figure 20: Total Income Distribution With a Change in the Borrowing Constraint

Notes:

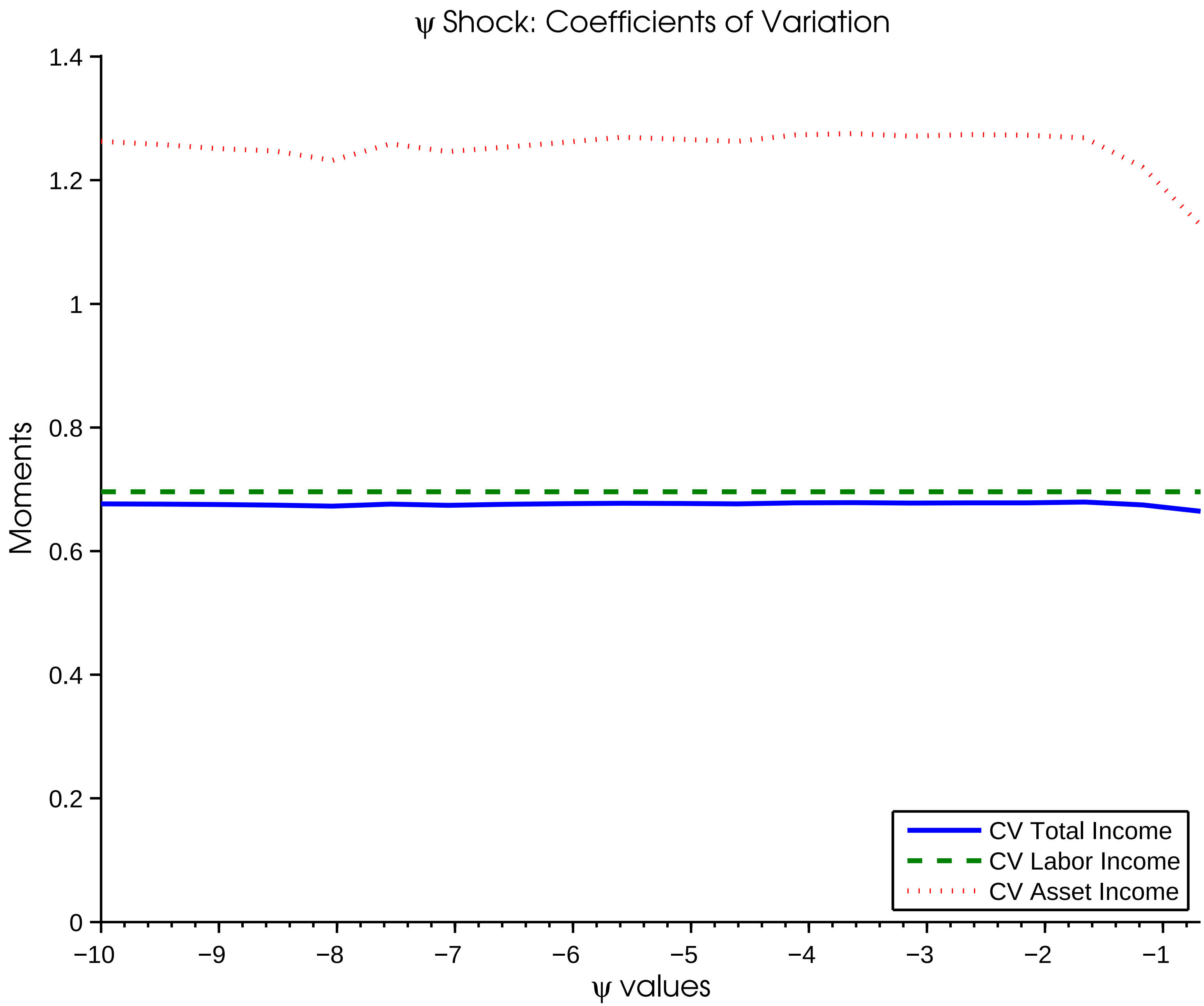

Figure 21: Coefficients of Variation With a Change in the Borrowing Constraint

Notes:

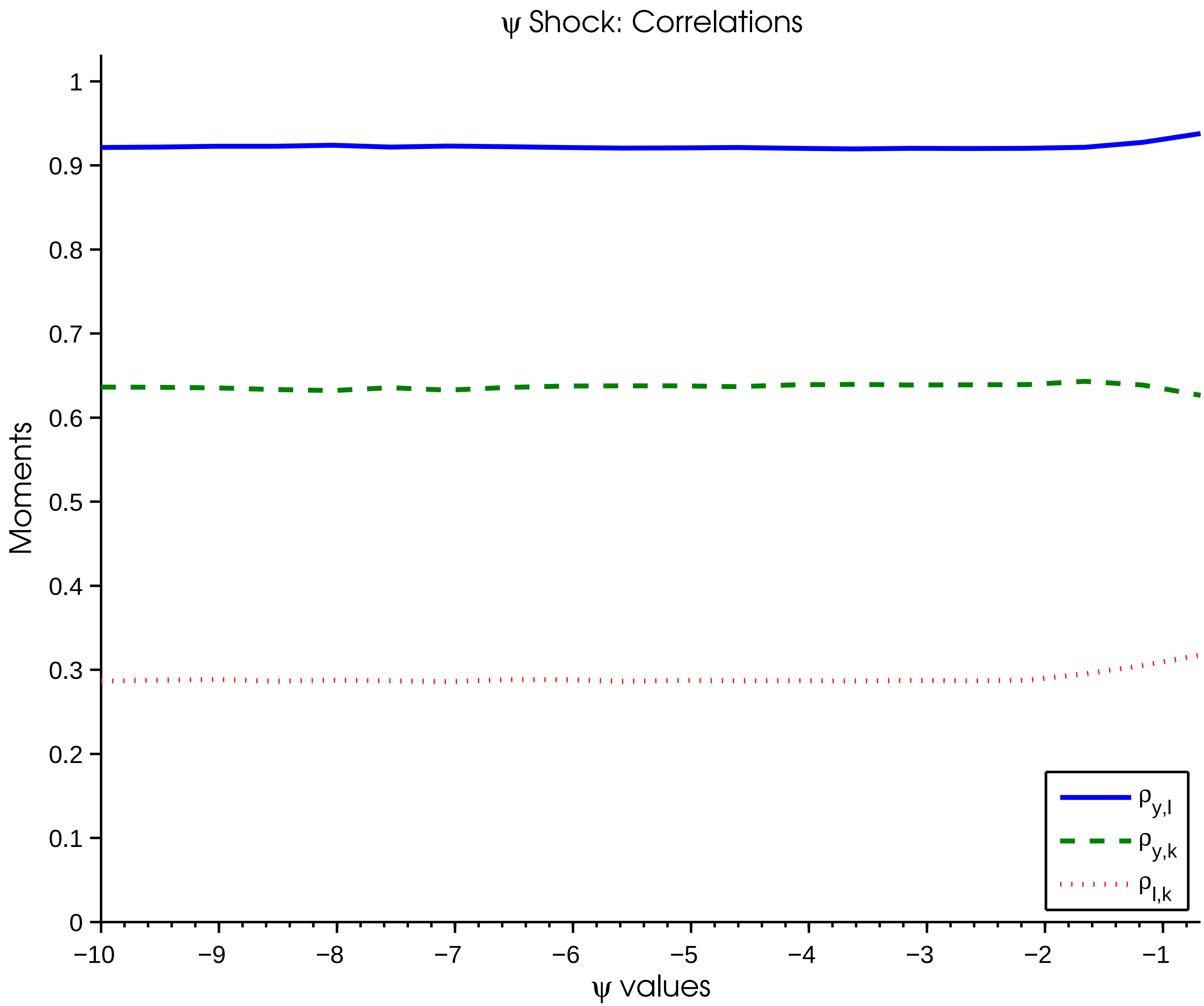

Figure 22: Correlations With a Change in the Borrowing Constraint

Notes:

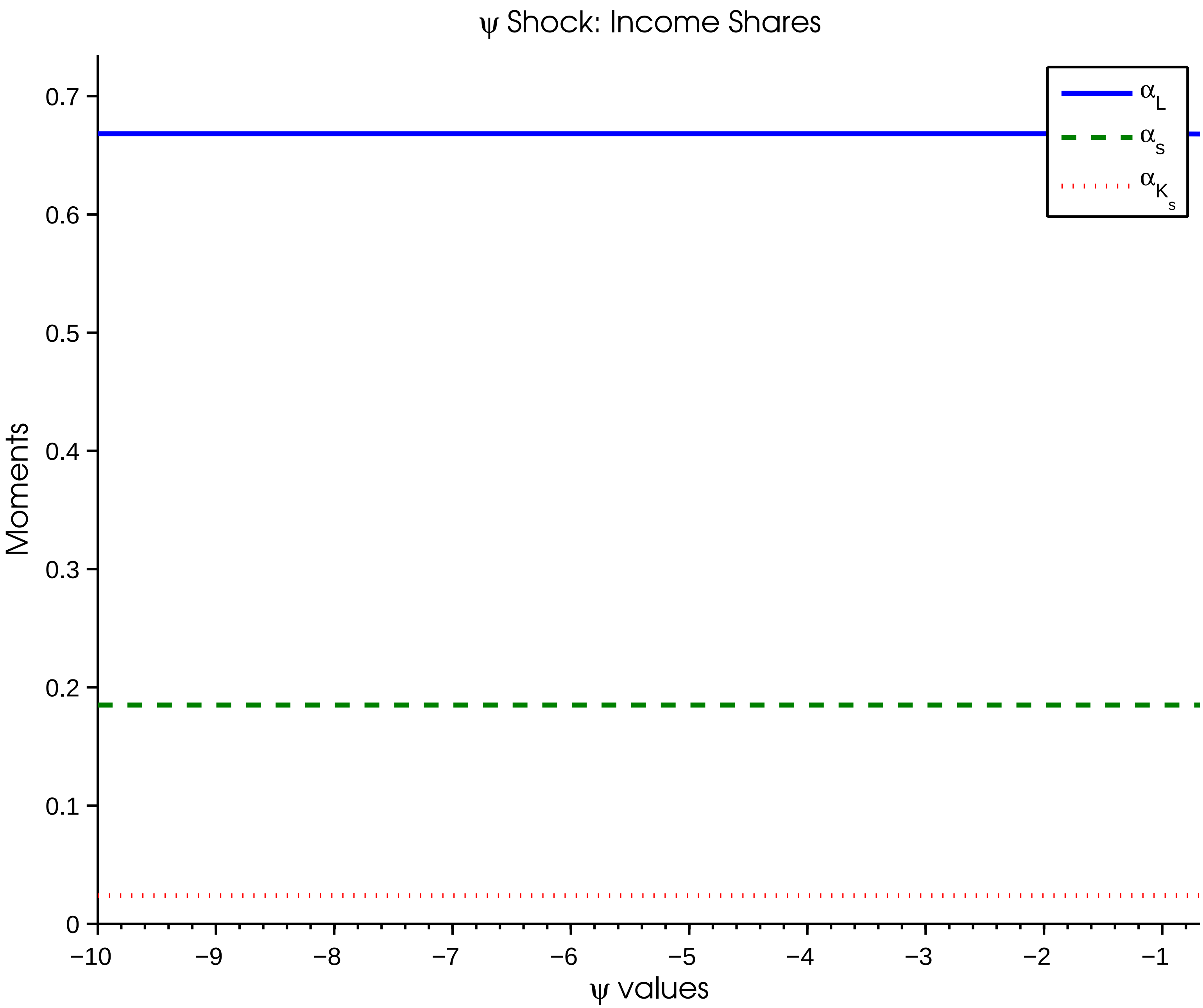

Figure 23: Income Shares With a Change in the Borrowing Constraint

Notes:

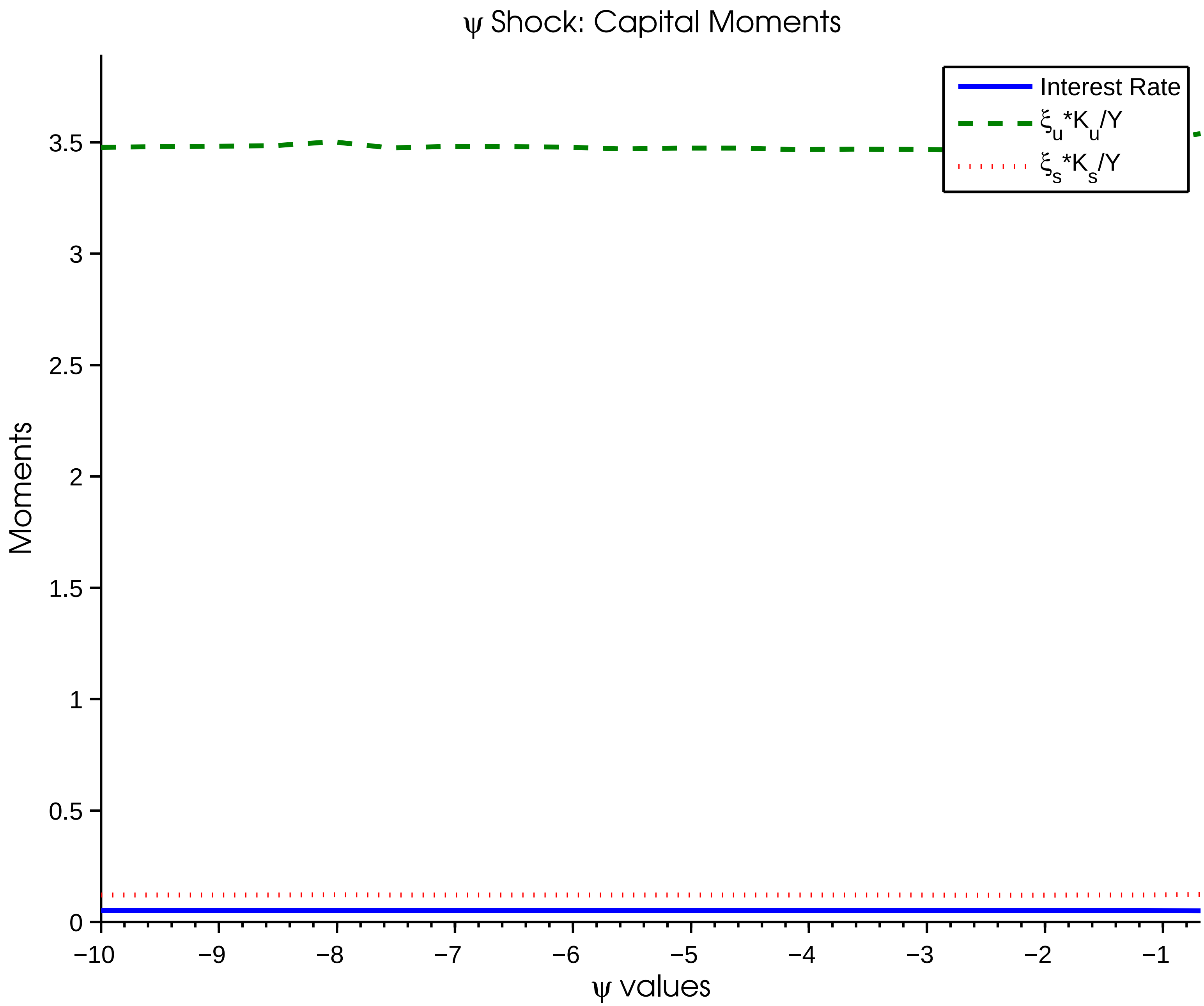

Figure 24: Capital Moments With a Change in the Borrowing Constraint

Notes:

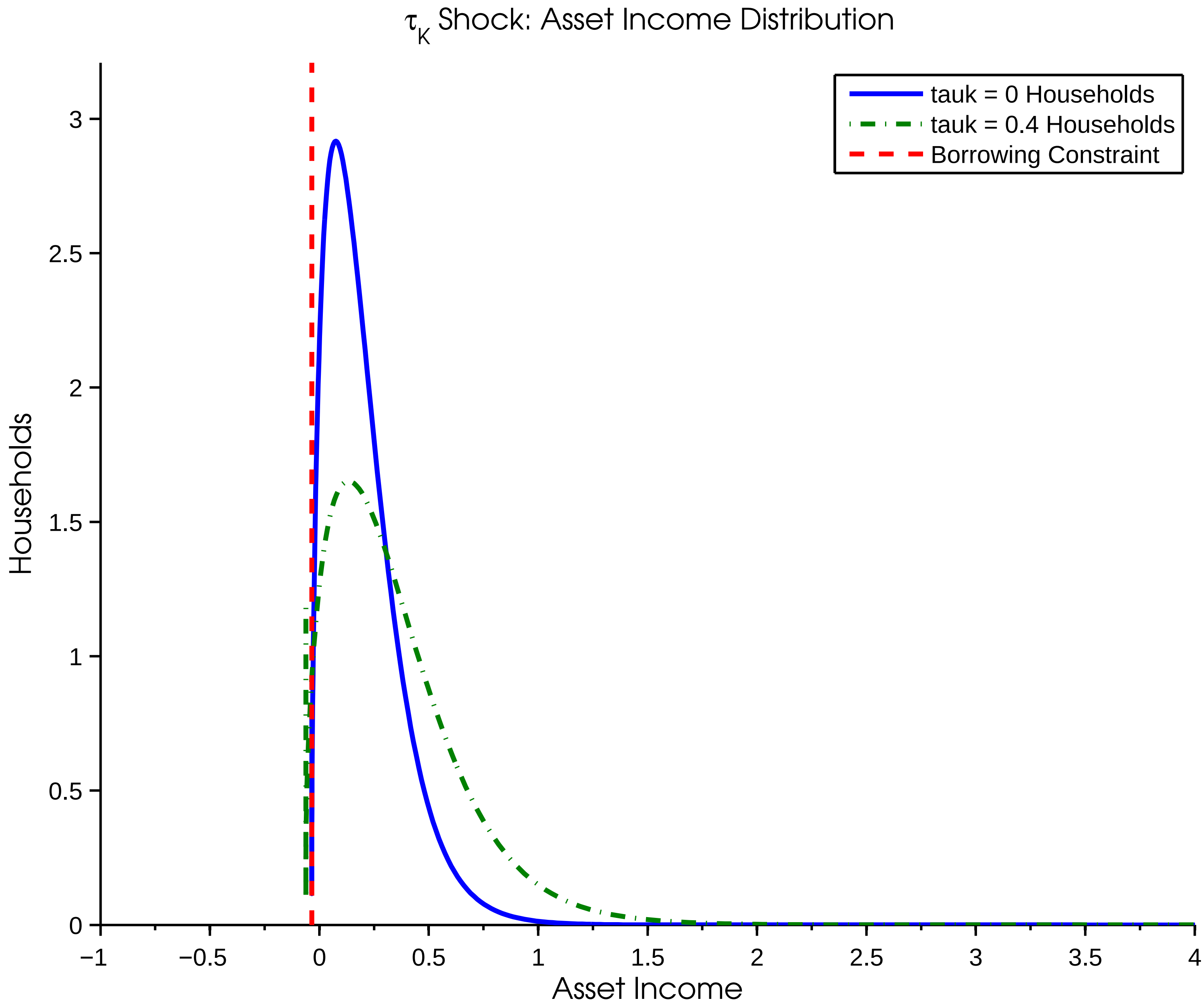

Figure 25: Benchmark Capital Income Distribution With a Change in the Capital Tax

Notes:

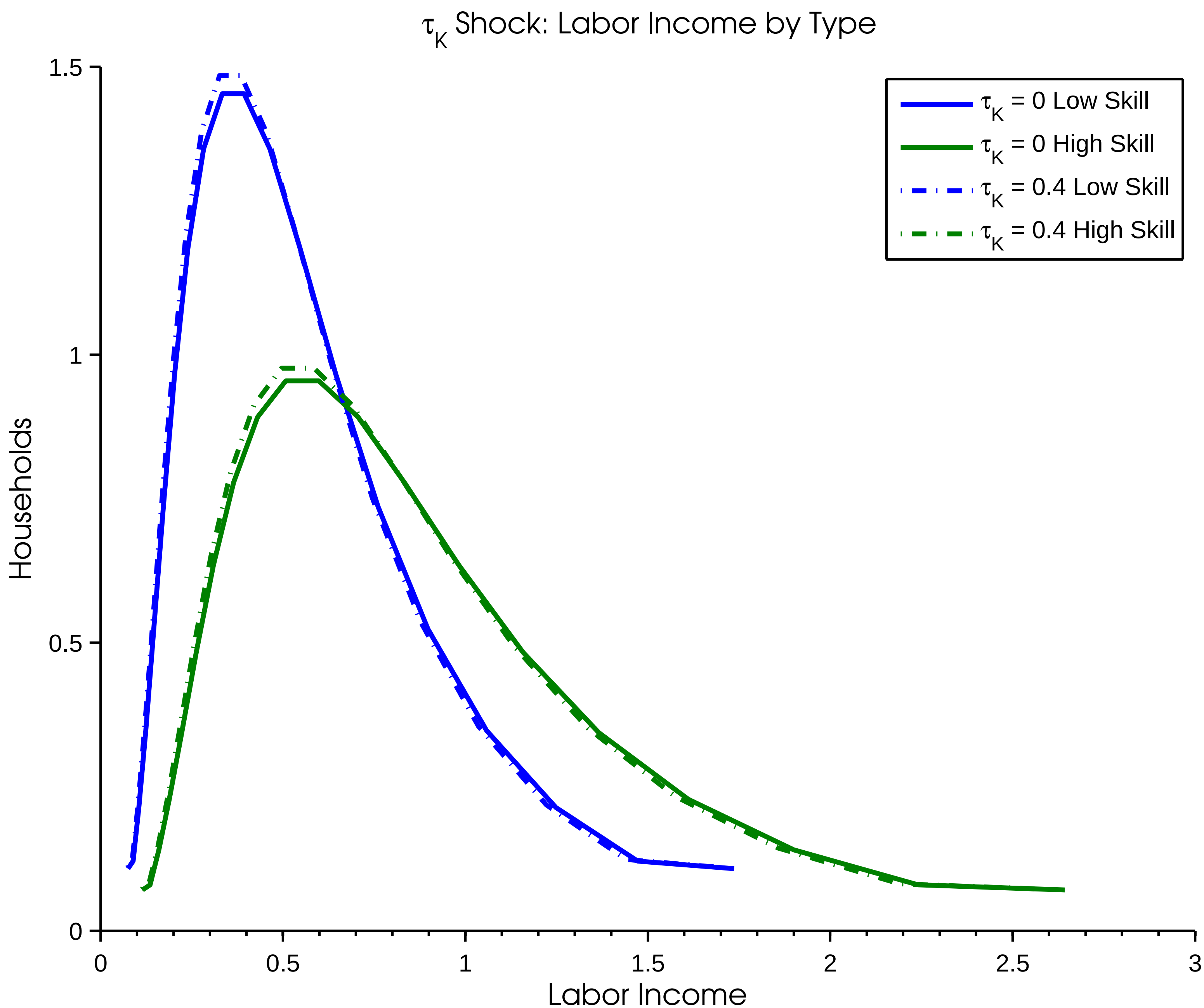

Figure 26: Benchmark Labor Income Distribution by Skill Type With a Change in the Capital Tax

Notes:

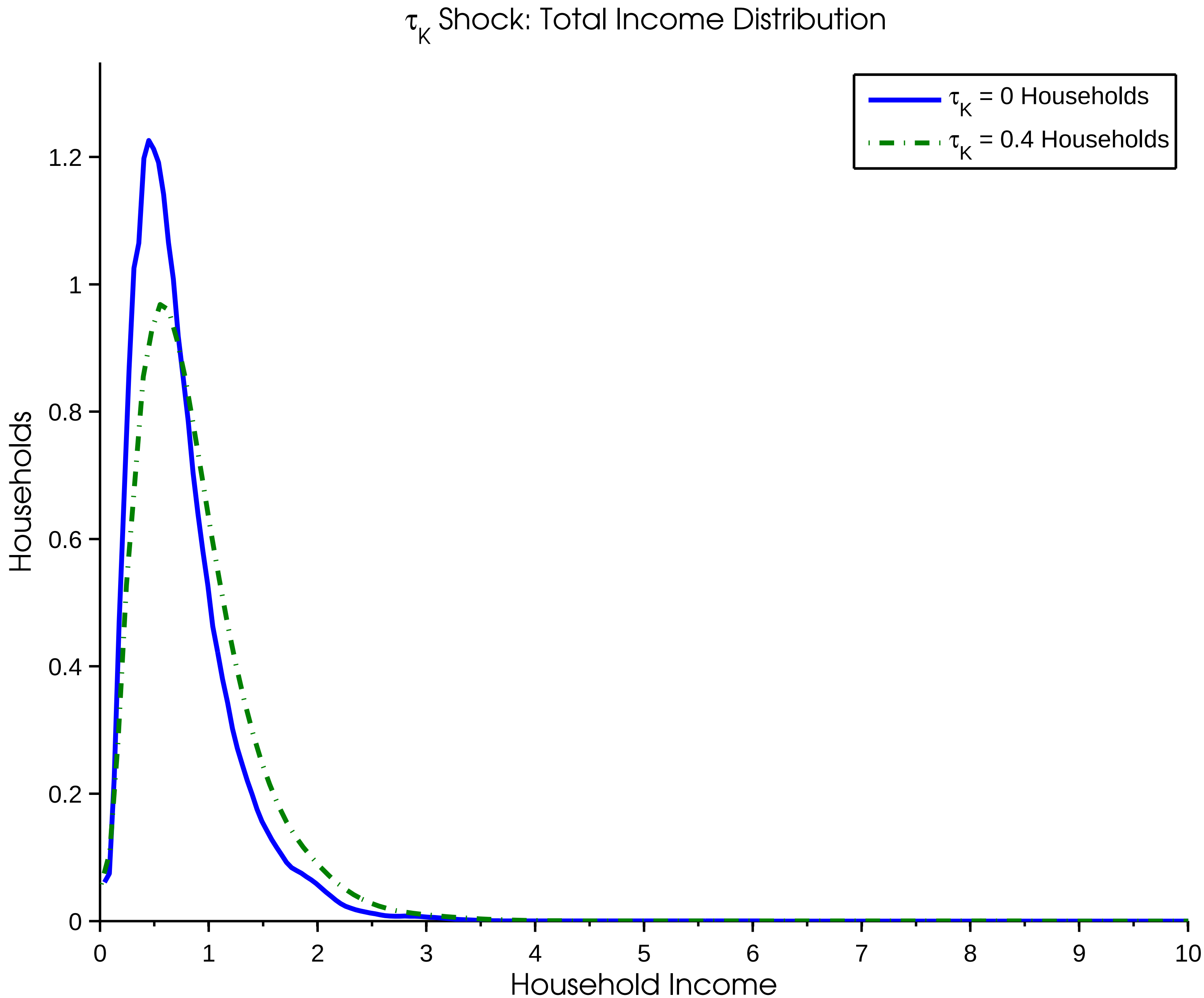

Figure 27: Total Income Distribution With a Change in the Capital Tax

Notes:

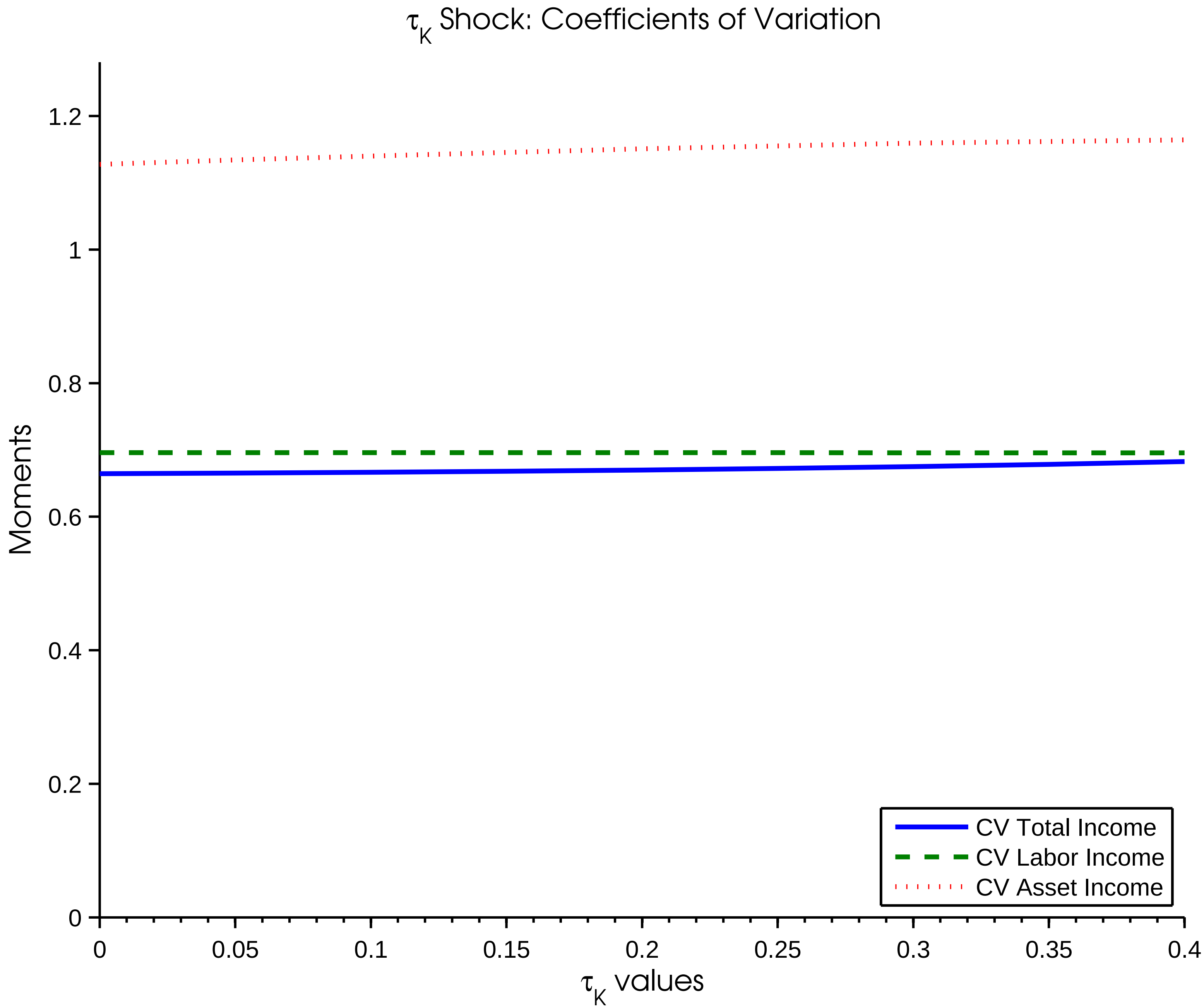

Figure 28: Coefficients of Variation With a Change in the Capital Tax

Notes:

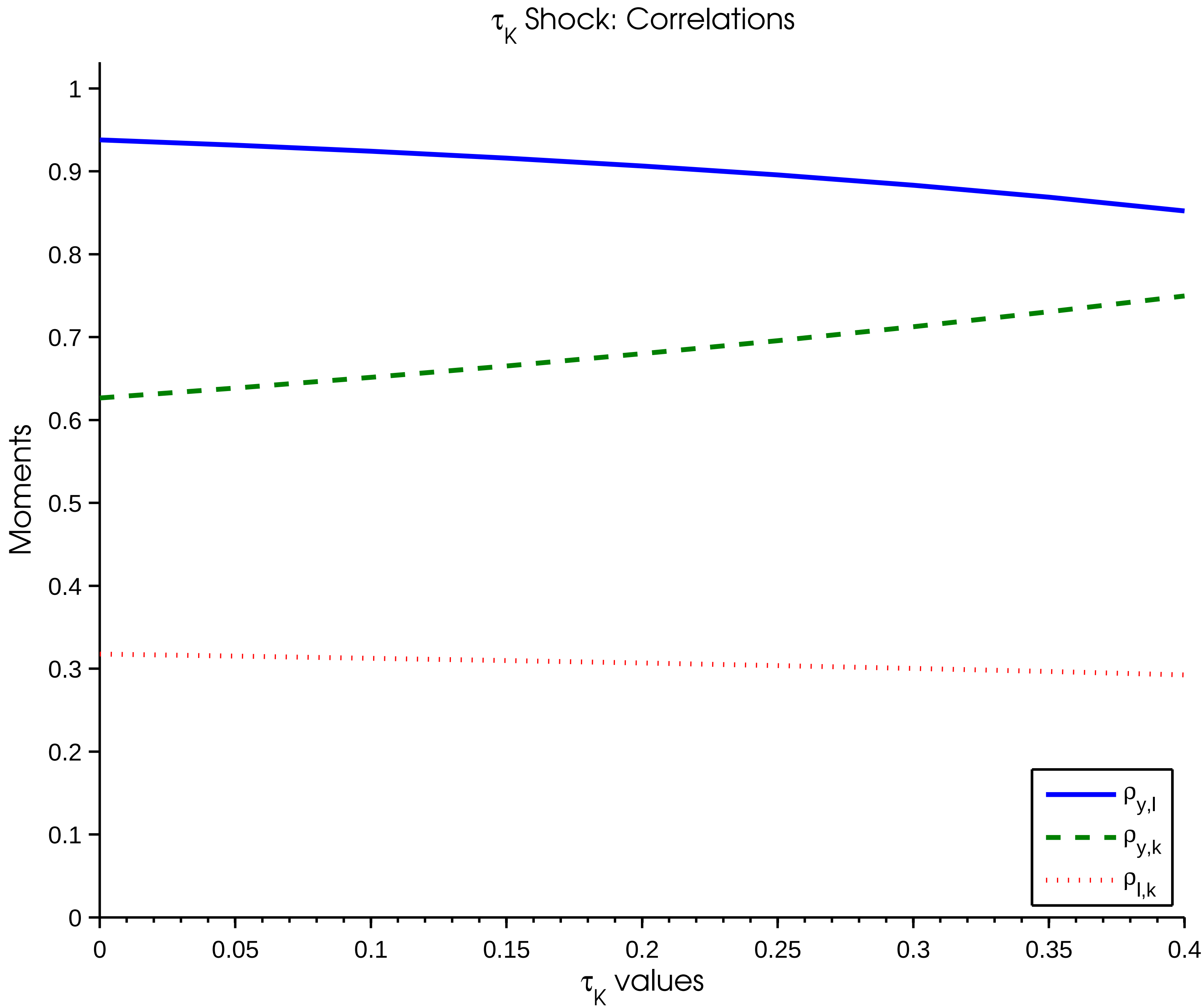

Figure 29: Correlations With a Change in the Capital Tax

Notes:

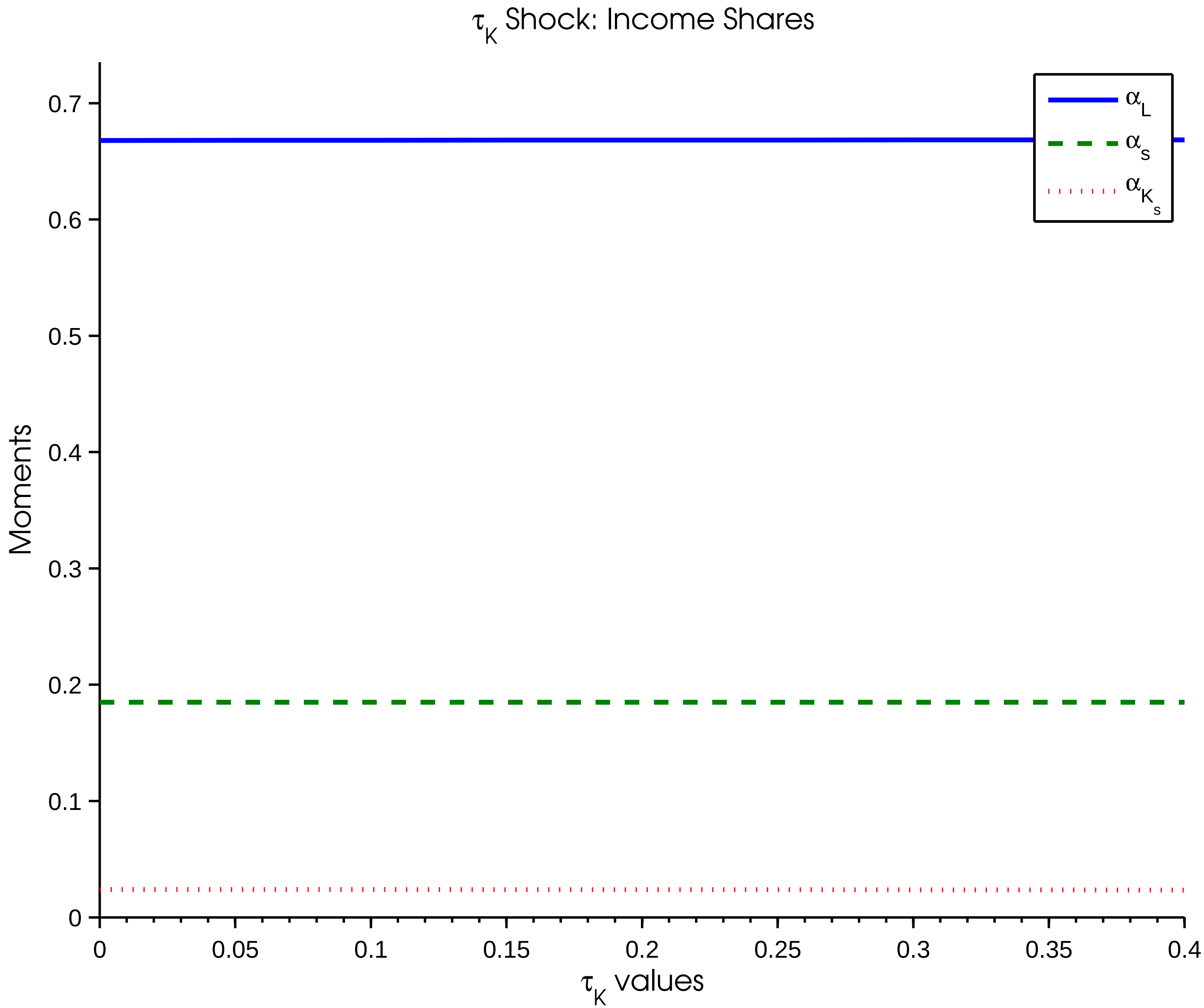

Figure 30: Income Shares With a Change in the Capital Tax

Notes:

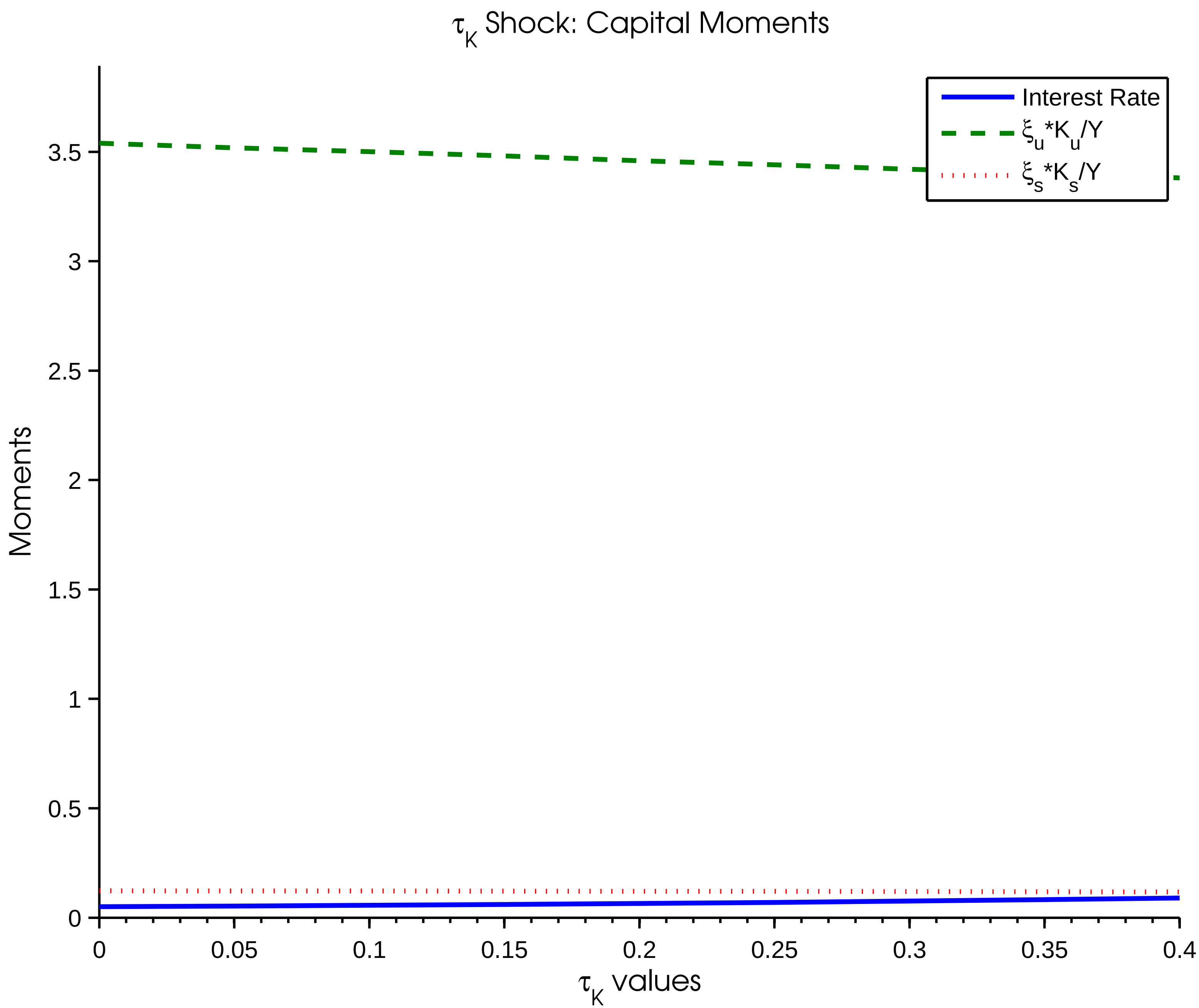

Figure 31: Capital Moments With a Change in the Capital Tax

Notes:

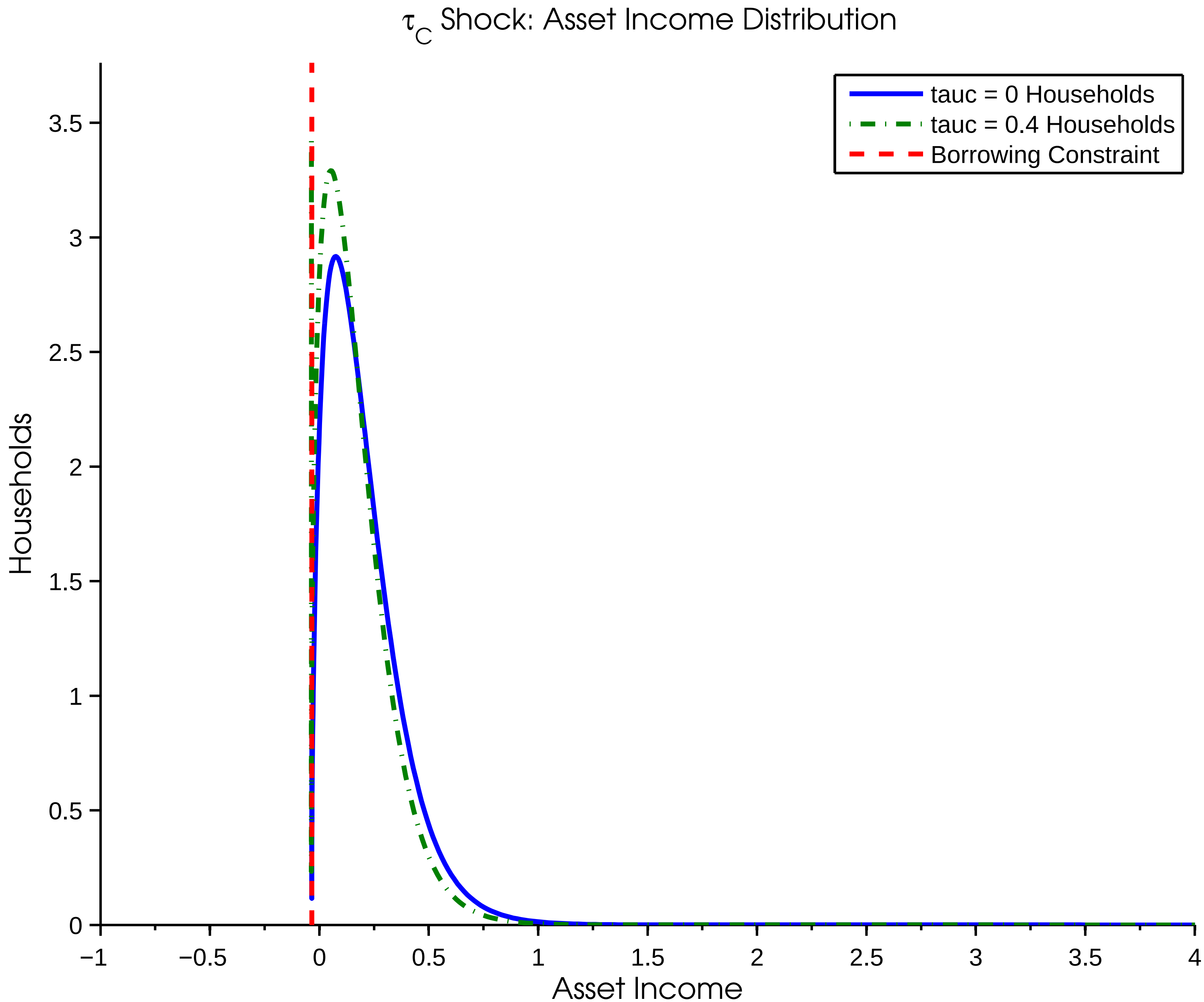

Figure 32: Benchmark Capital Income Distribution With a Change in the Corporate Tax

Notes:

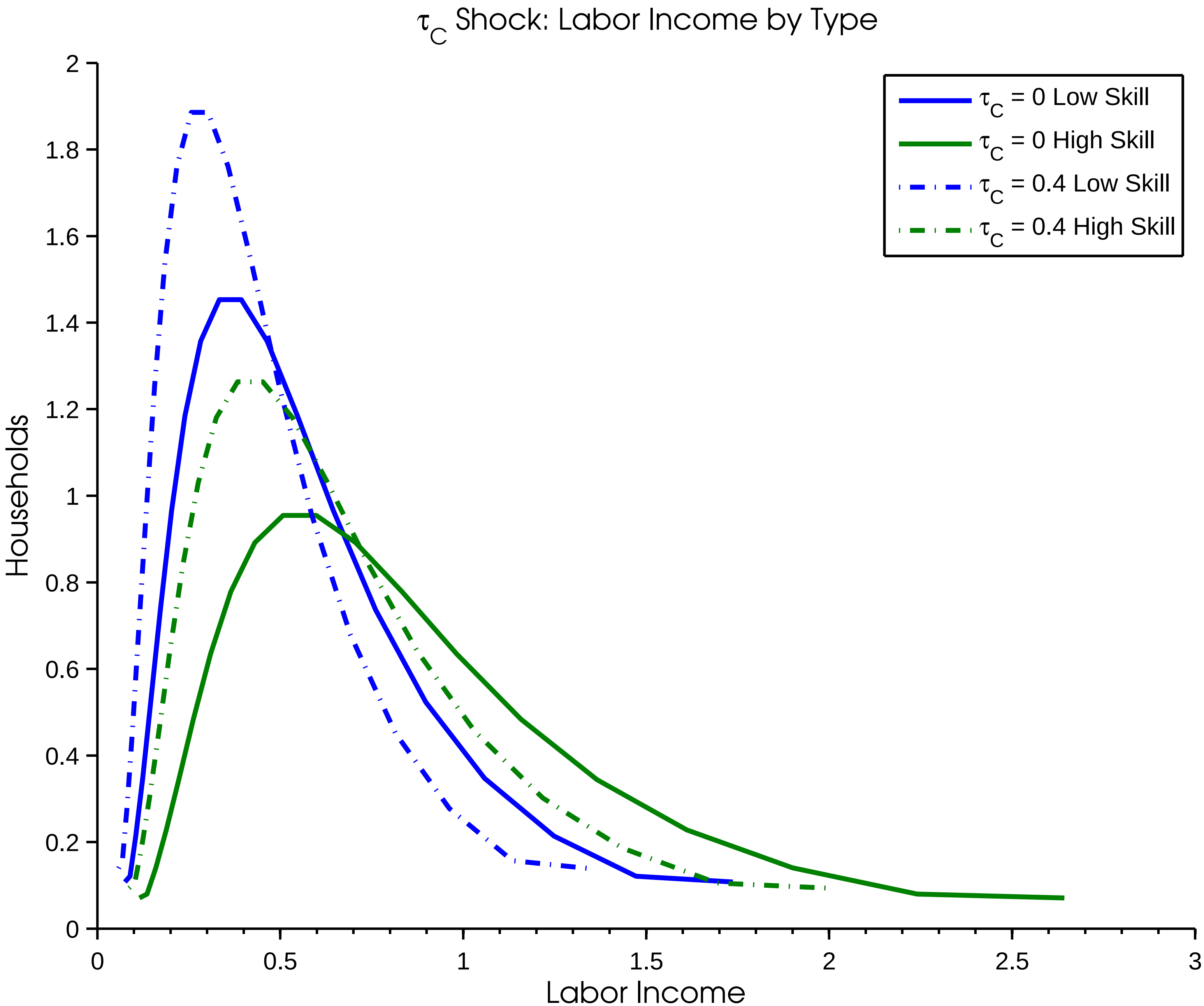

Figure 33: Benchmark Labor Income Distribution by Skill Type With a Change in the Corporate Tax

Notes:

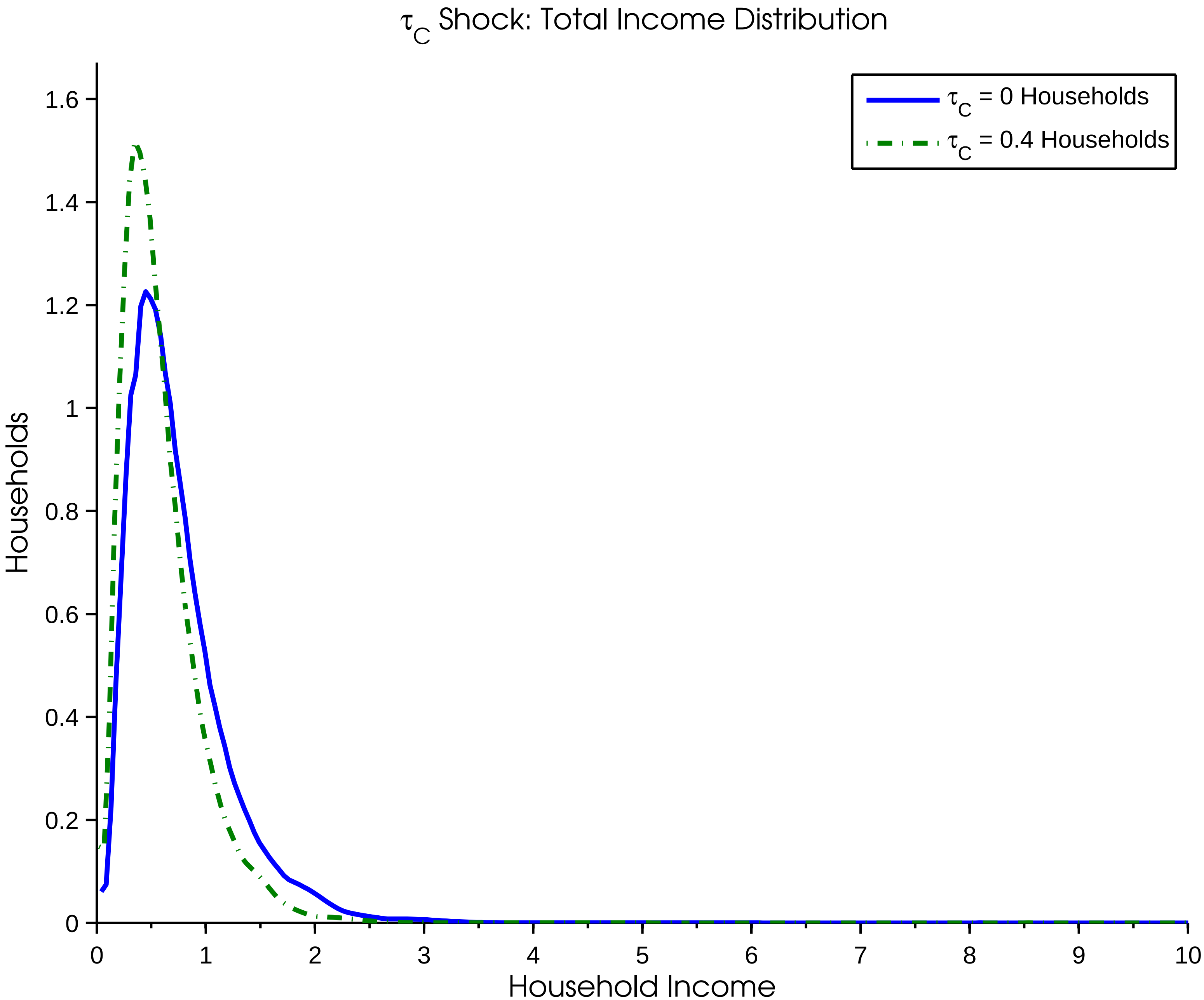

Figure 34: Total Income Distribution With a Change in the Corporate Tax

Notes:

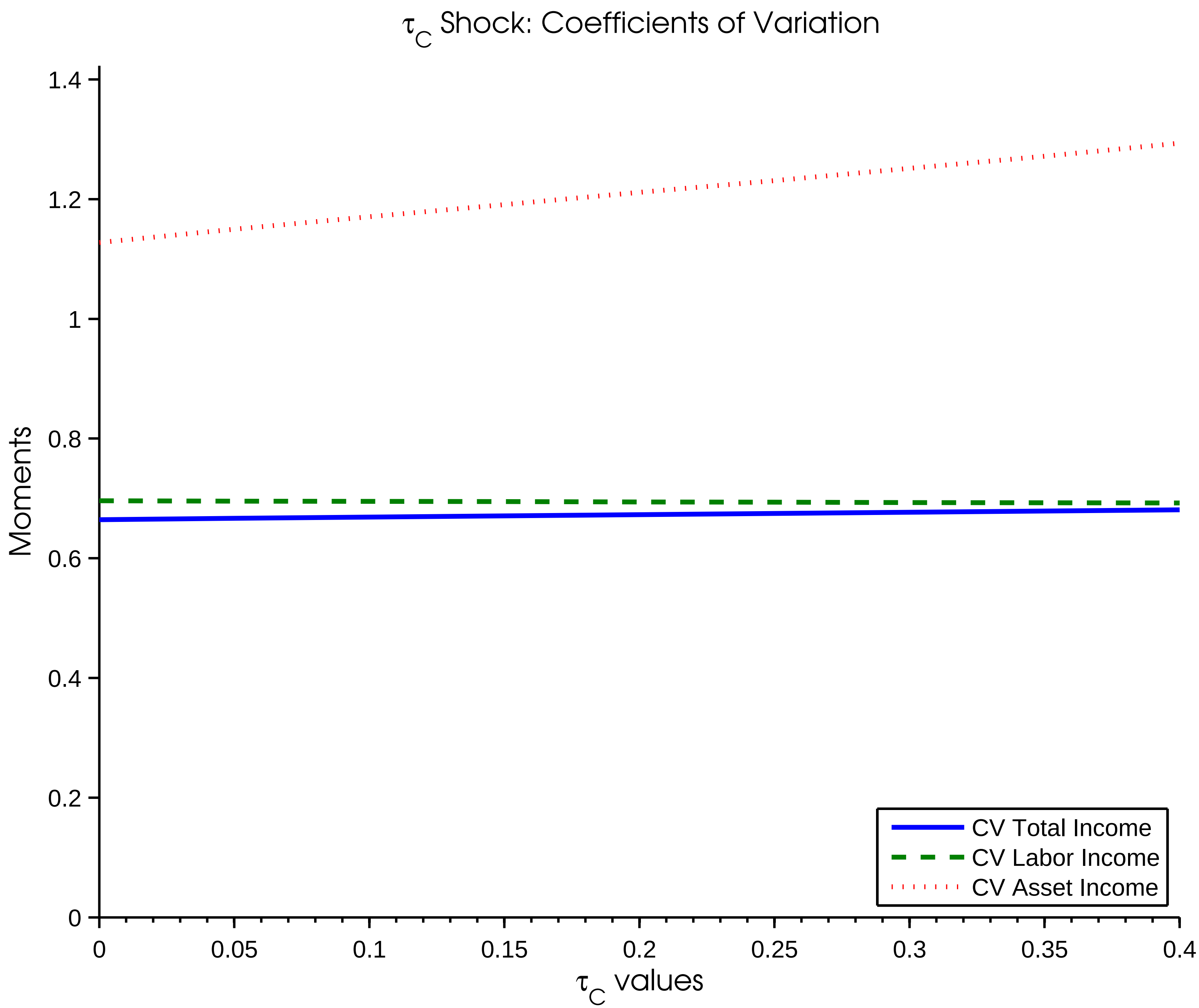

Figure 35: Coefficients of Variation With a Change in the Corporate Tax

Notes:

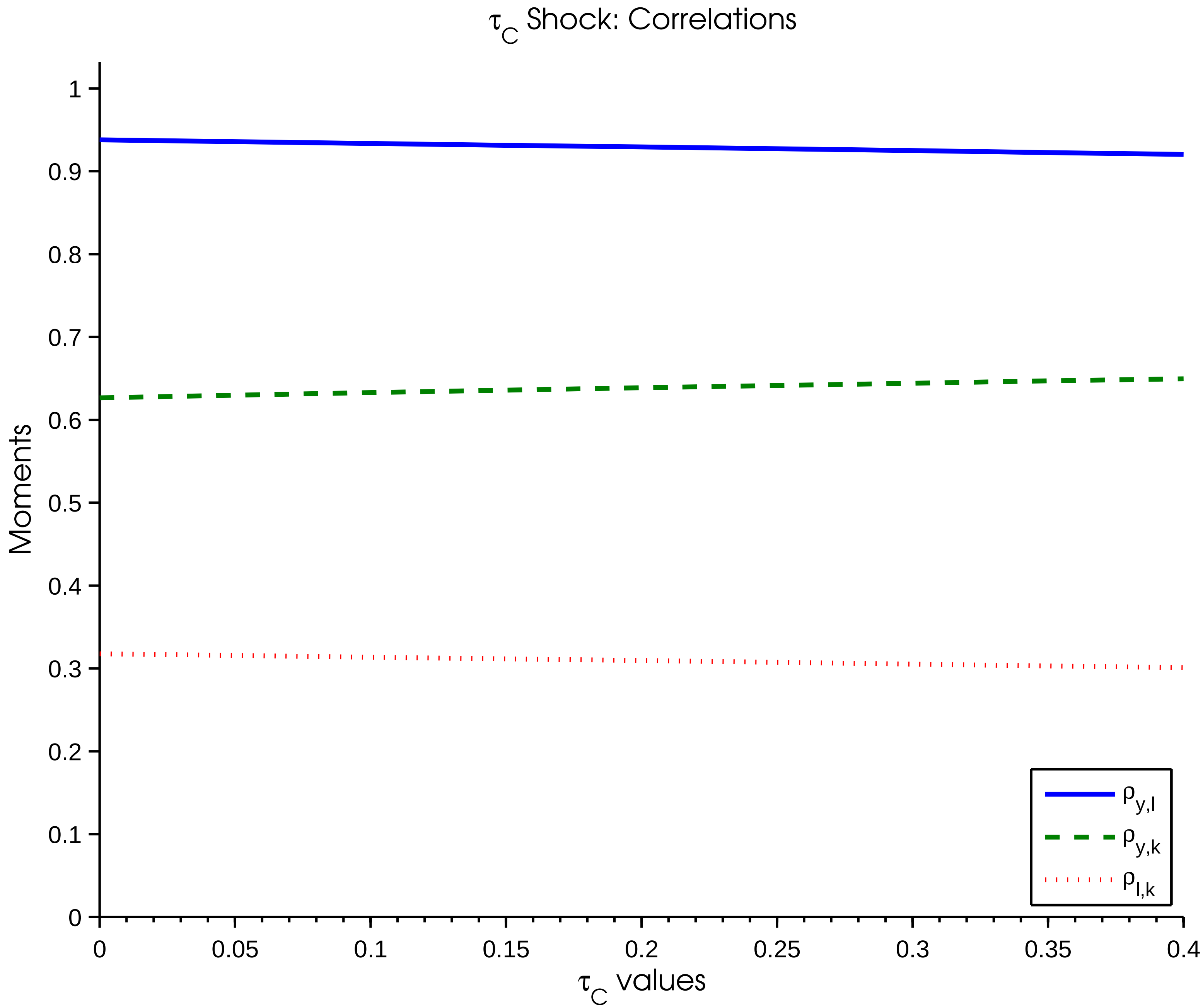

Figure 36: Correlations With a Change in the Corporate Tax

Notes:

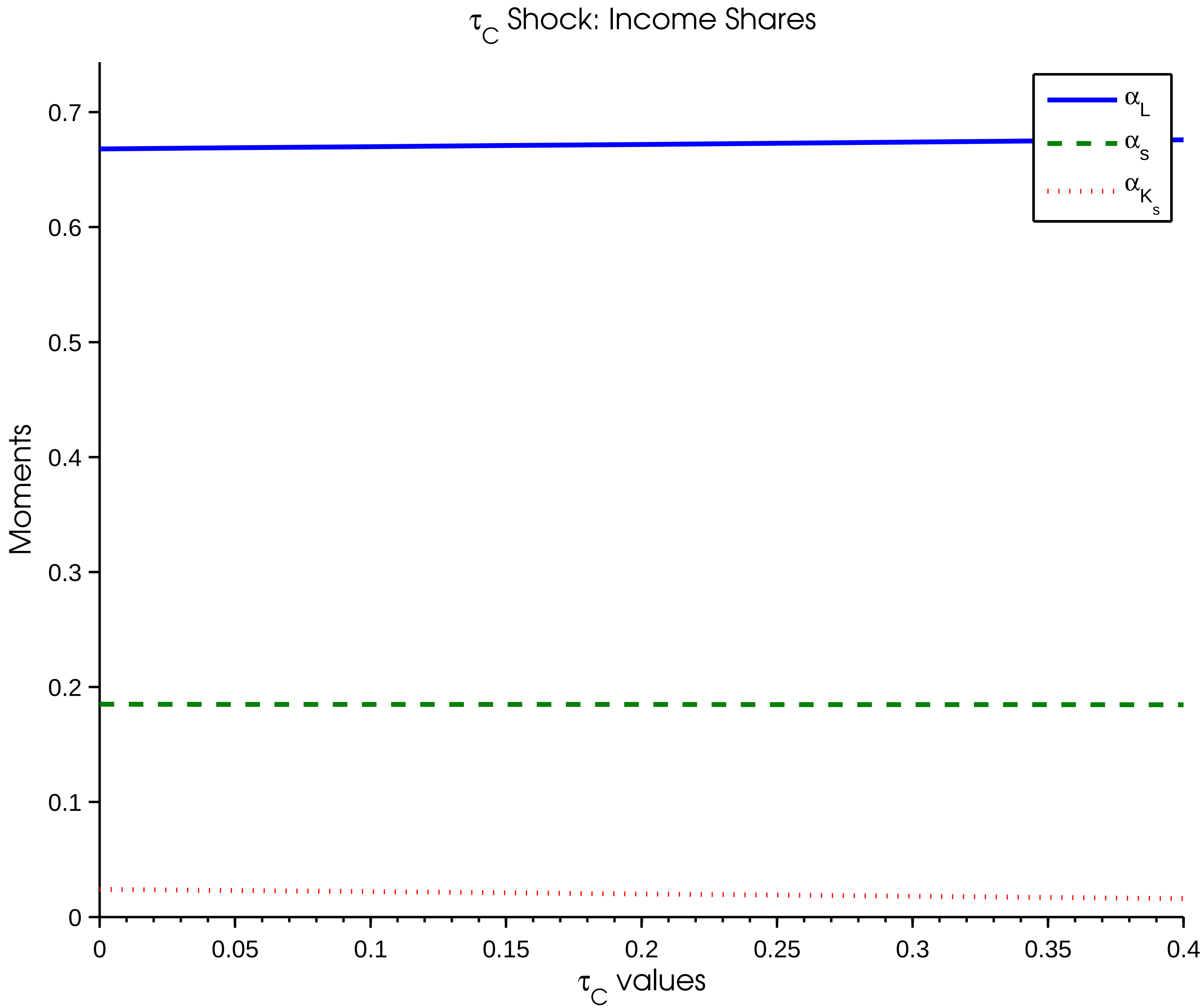

Figure 37: Income Shares With a Change in the Corporate Tax

Notes:

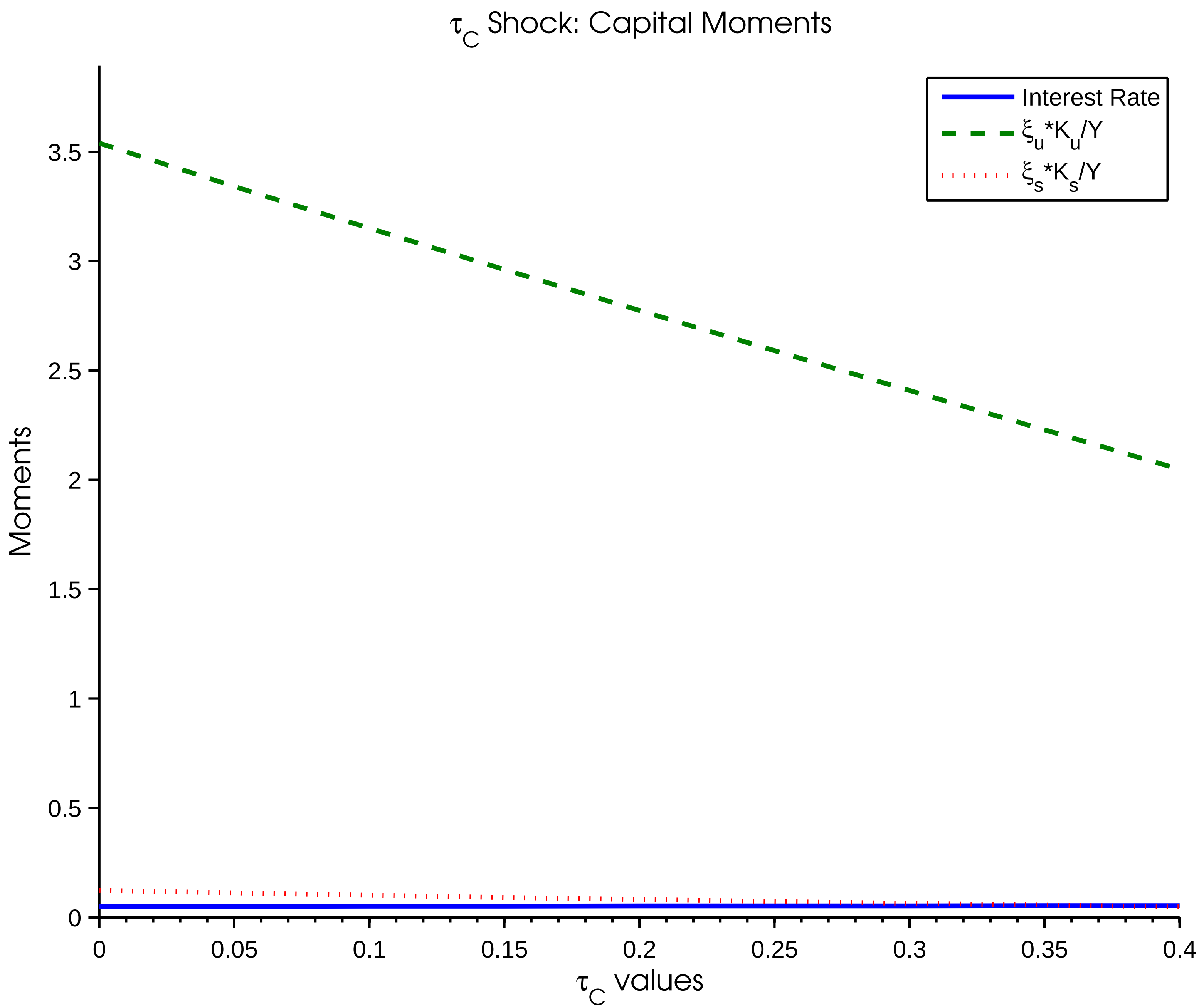

Figure 38: Capital Moments With a Change in the Corporate Tax

Notes:

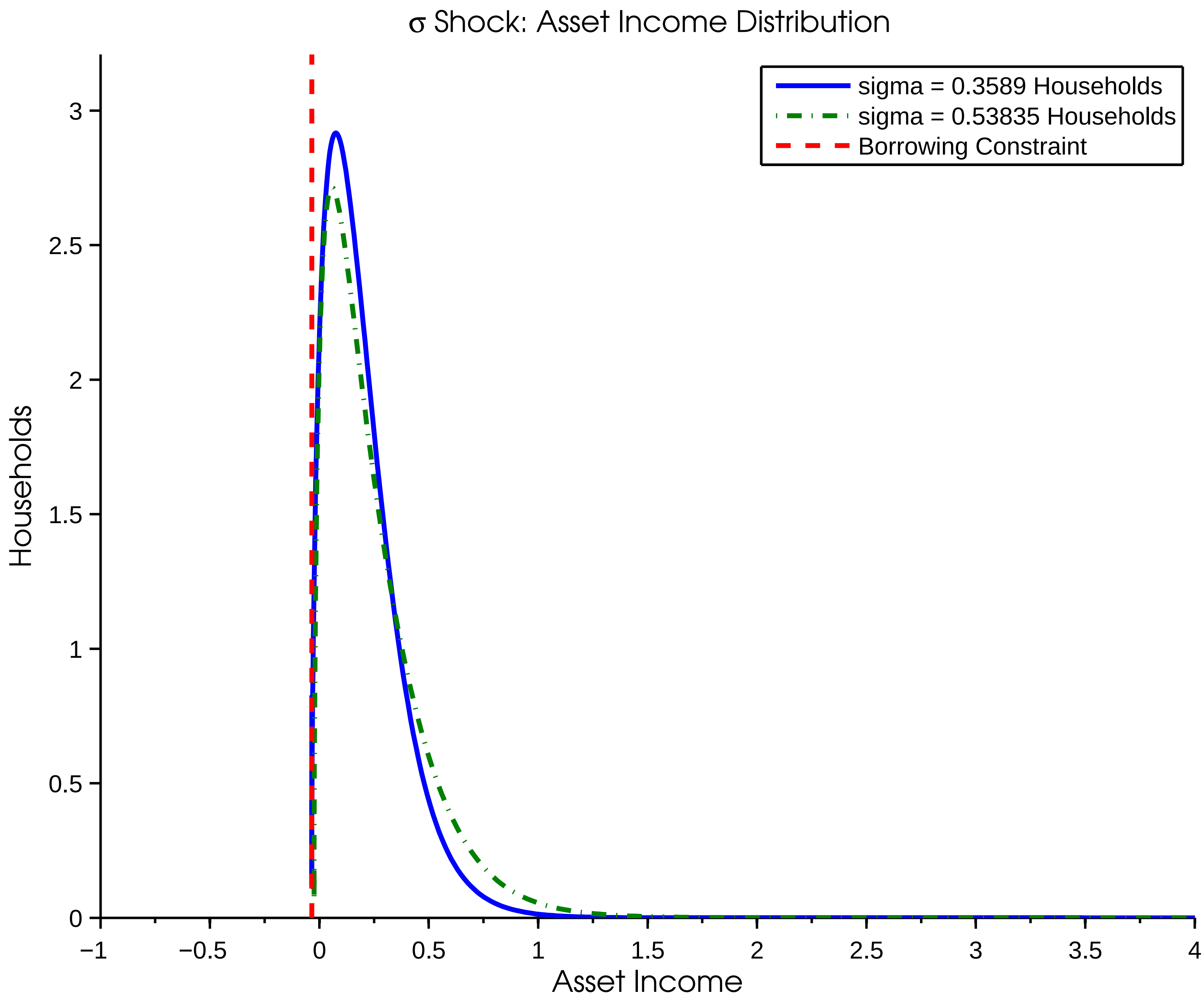

Figure 39: Benchmark Capital Income Distribution With a Change in the Labor Income Variance

Notes:

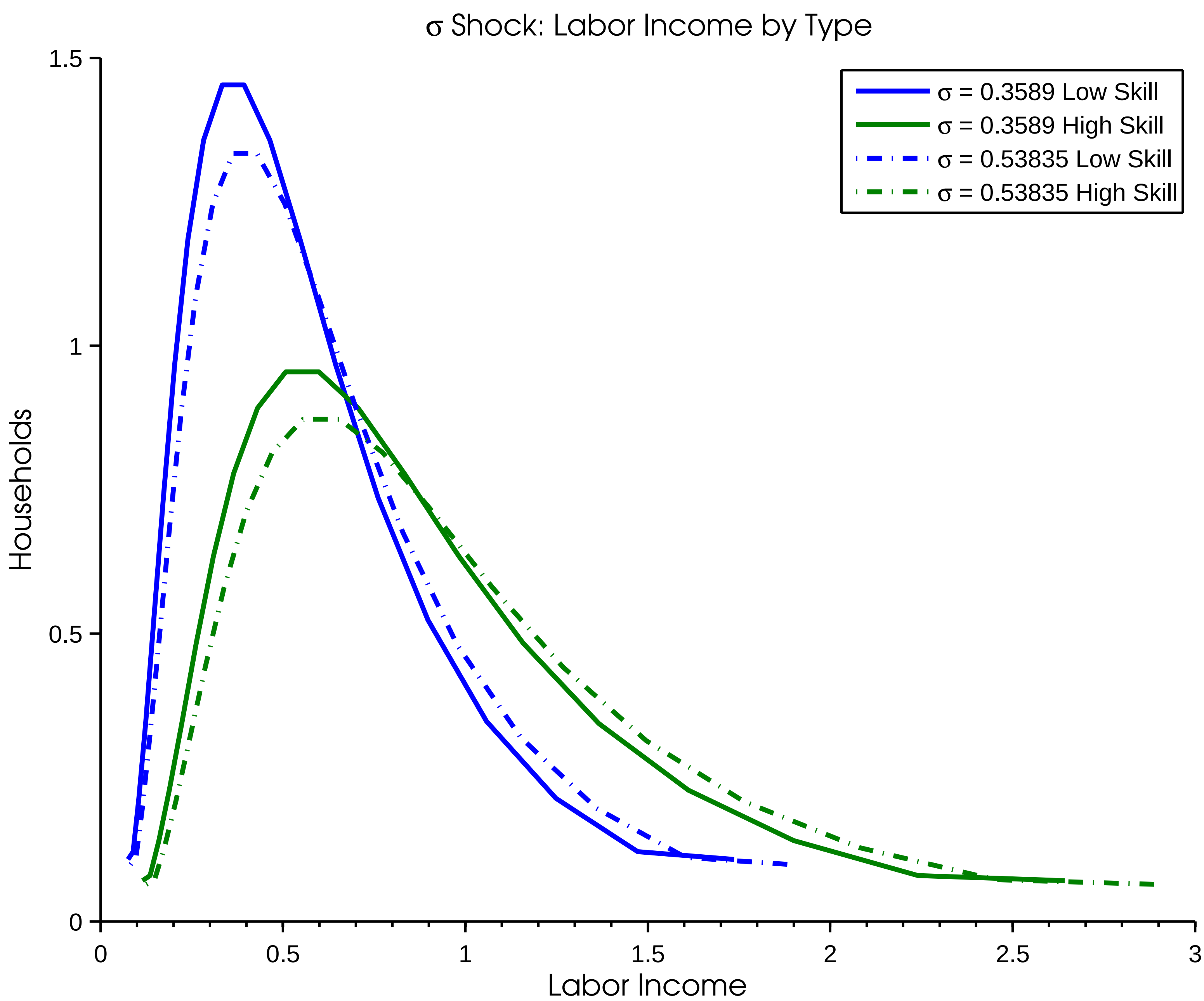

Figure 40: Benchmark Labor Income Distribution by Skill Type With a Change in the Labor Income Variance

Notes:

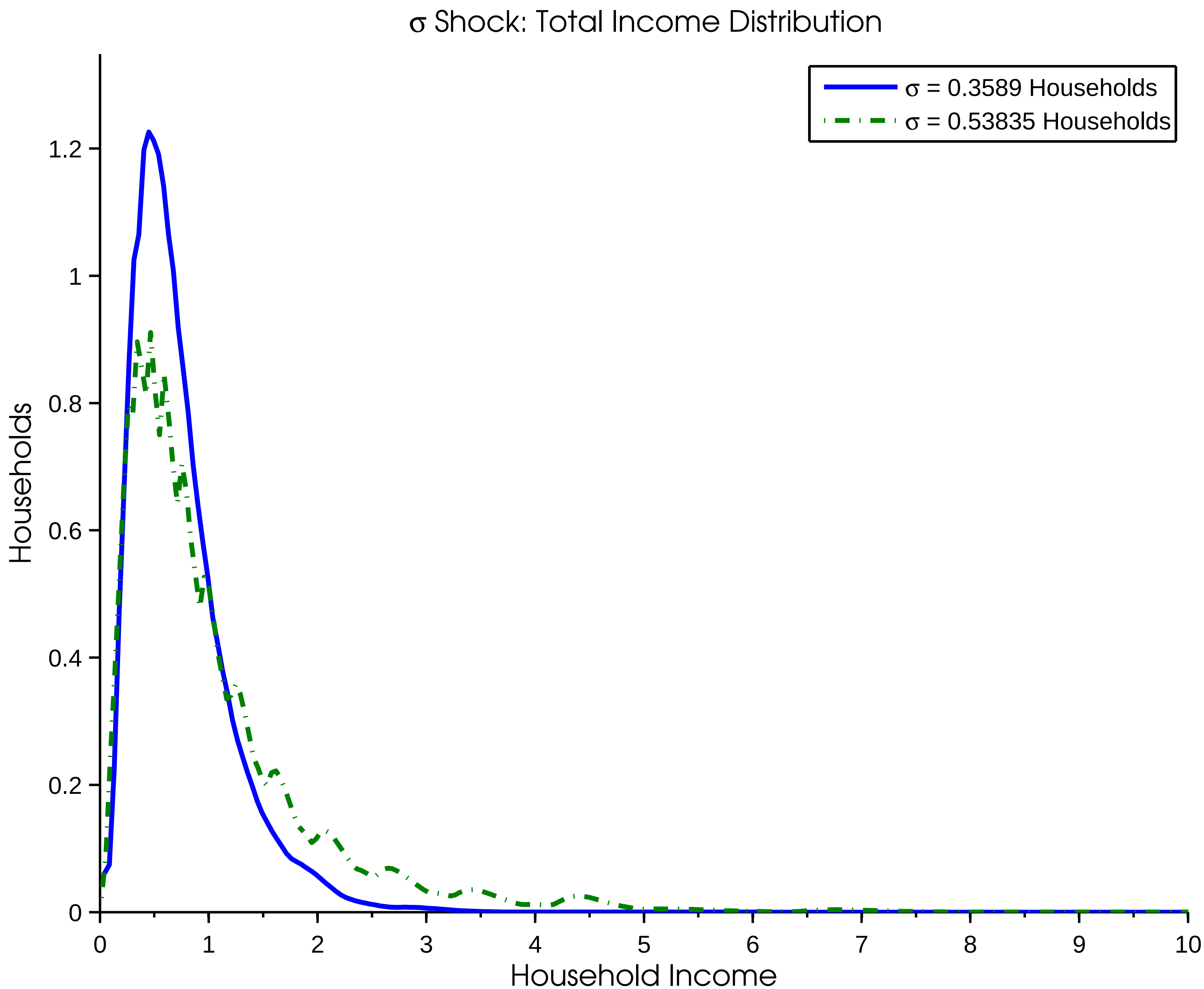

Figure 41: Total Income Distribution With a Change in the Labor Income Variance

Notes:

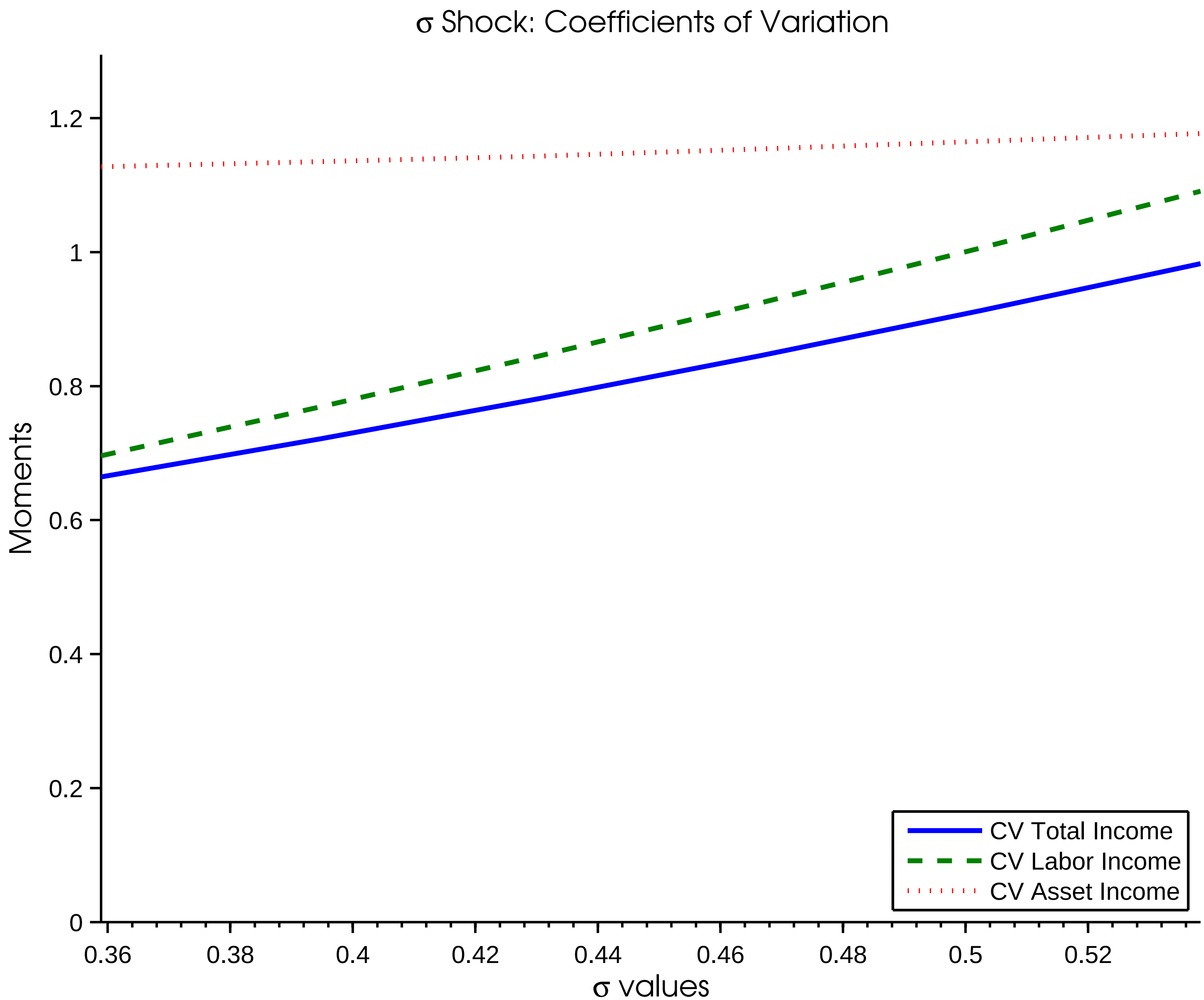

Figure 42: Coefficients of Variation With a Change in the Labor Income Variance

Notes:

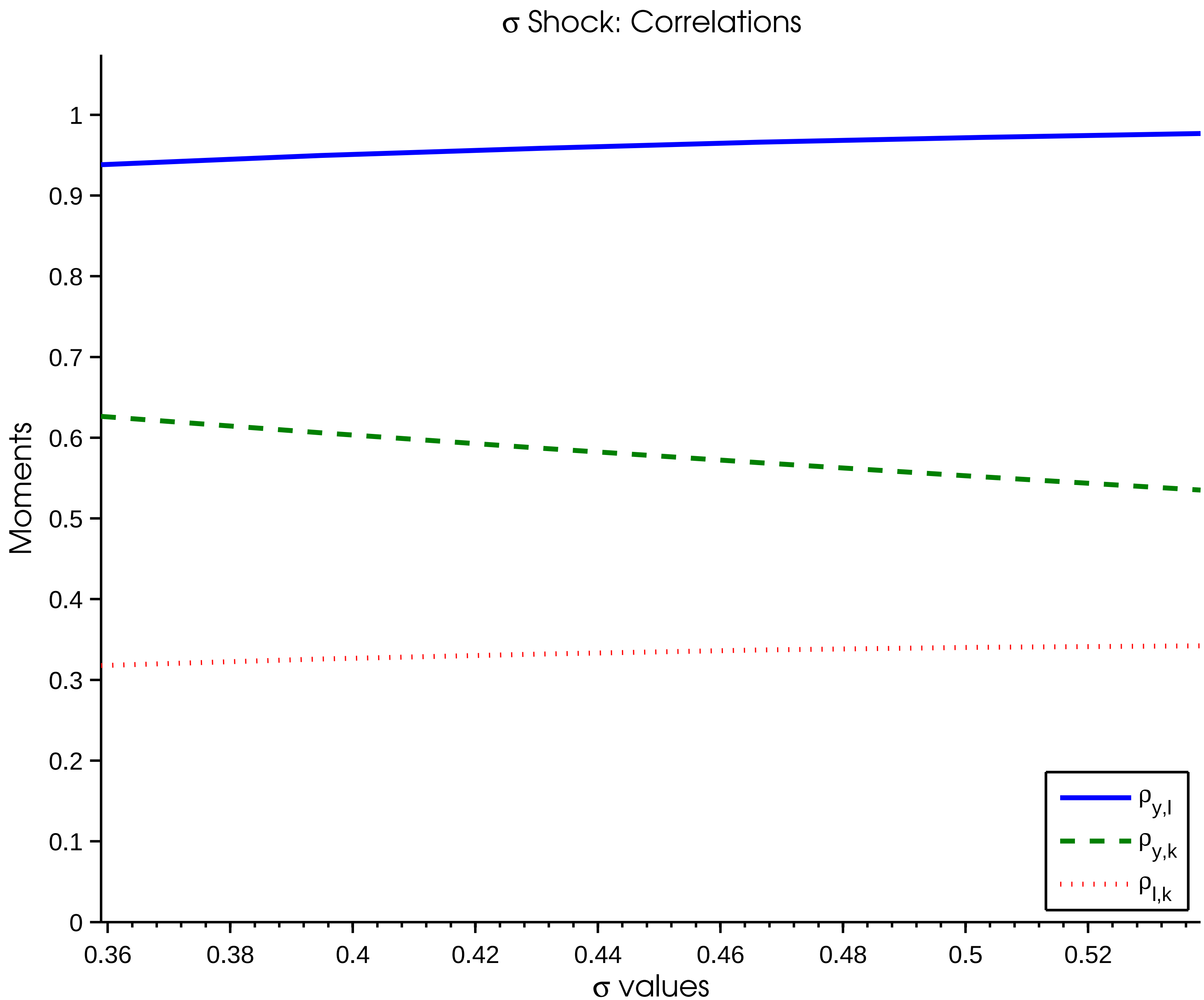

Figure 43: Correlations With a Change in the Labor Income Variance

Notes:

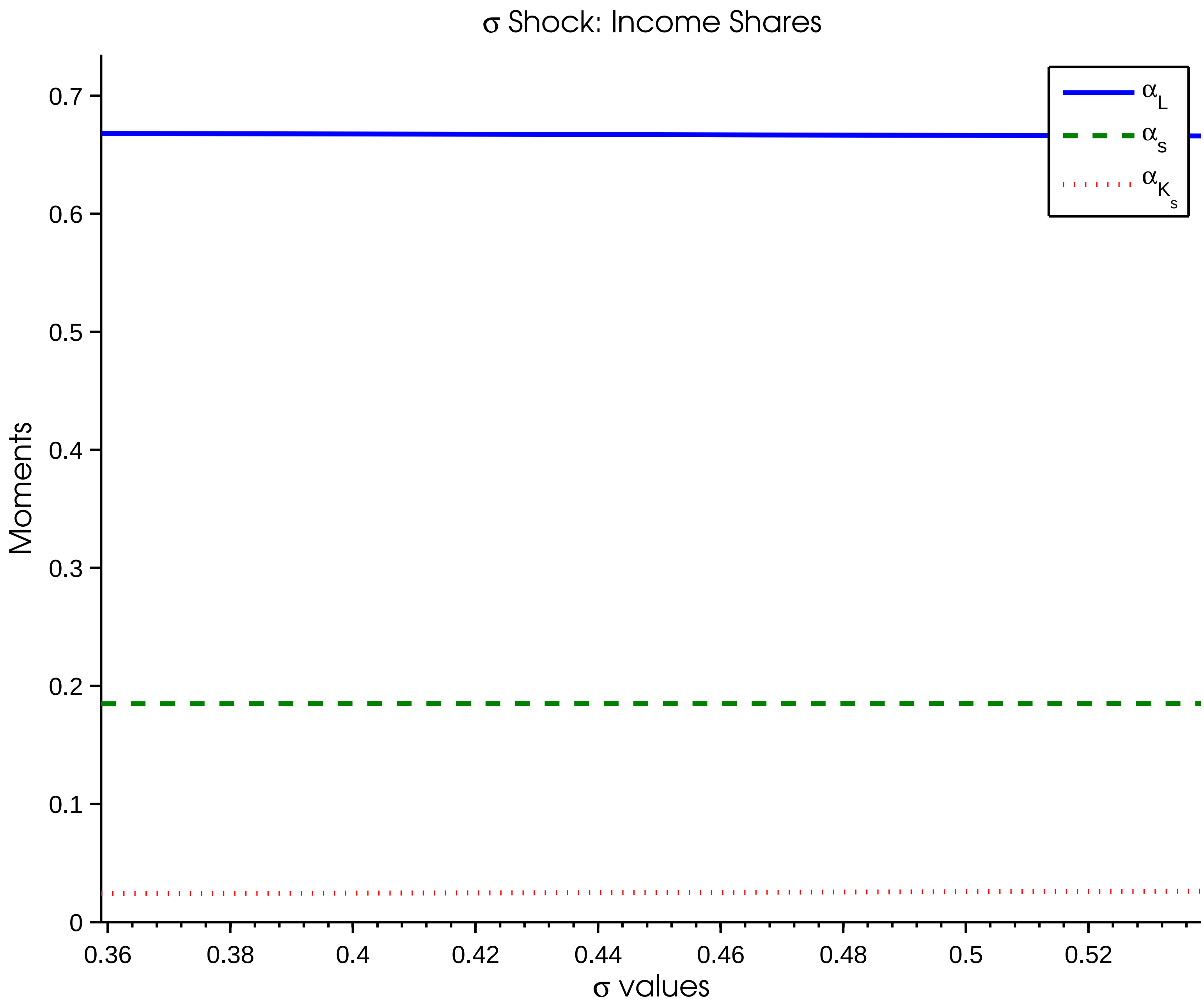

Figure 44: Income Shares With a Change in the Labor Income Variance

Notes:

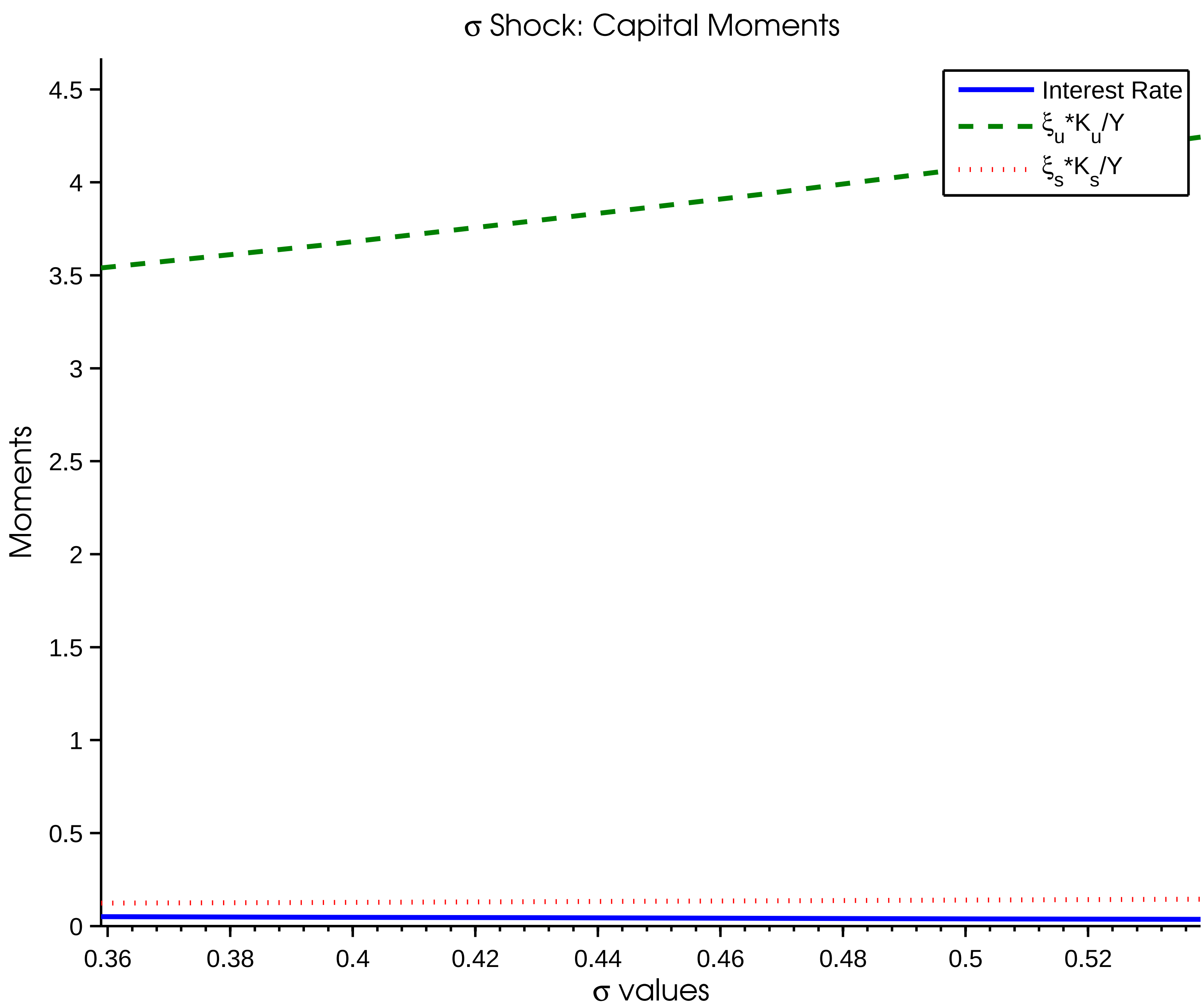

Figure 45: Capital Moments With a Change in the Labor Income Variance

Notes:

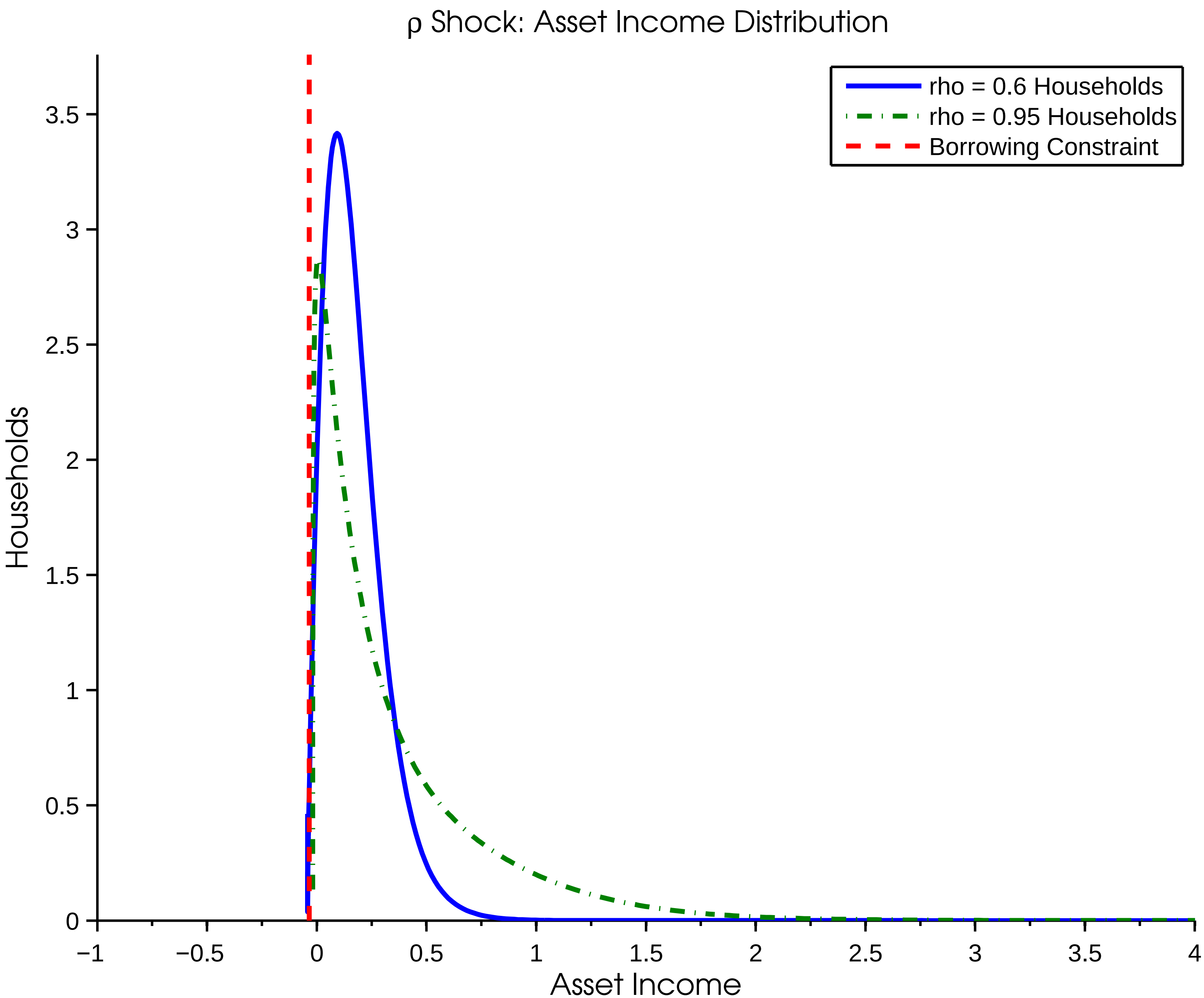

Figure 46: Benchmark Capital Income Distribution With a Change in the Labor Income Persistence

Notes:

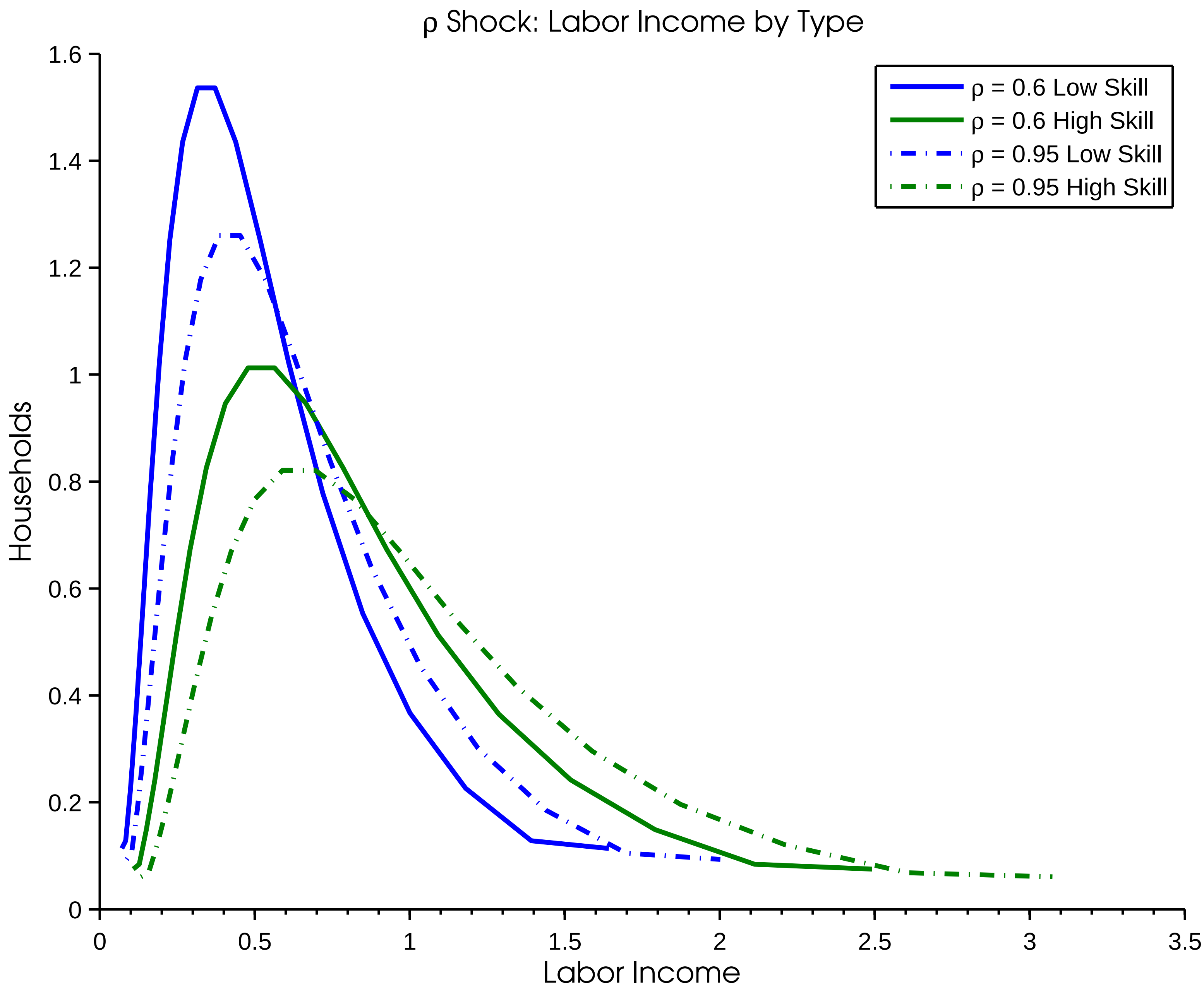

Figure 47: Benchmark Labor Income Distribution by Skill Type With a Change in the Labor Income Persistence

Notes:

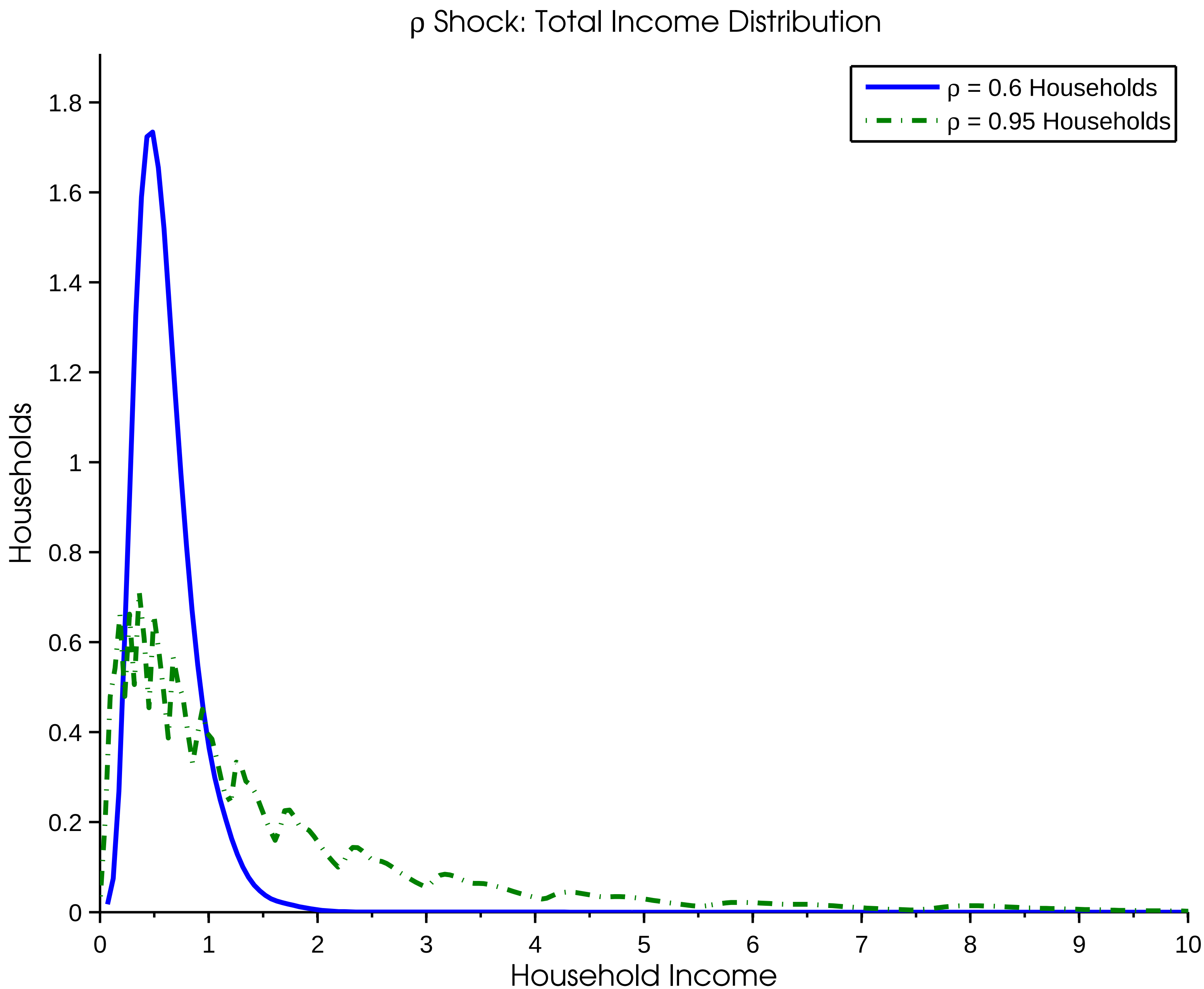

Figure 48: Total Income Distribution With a Change in the Labor Income Persistence

Notes:

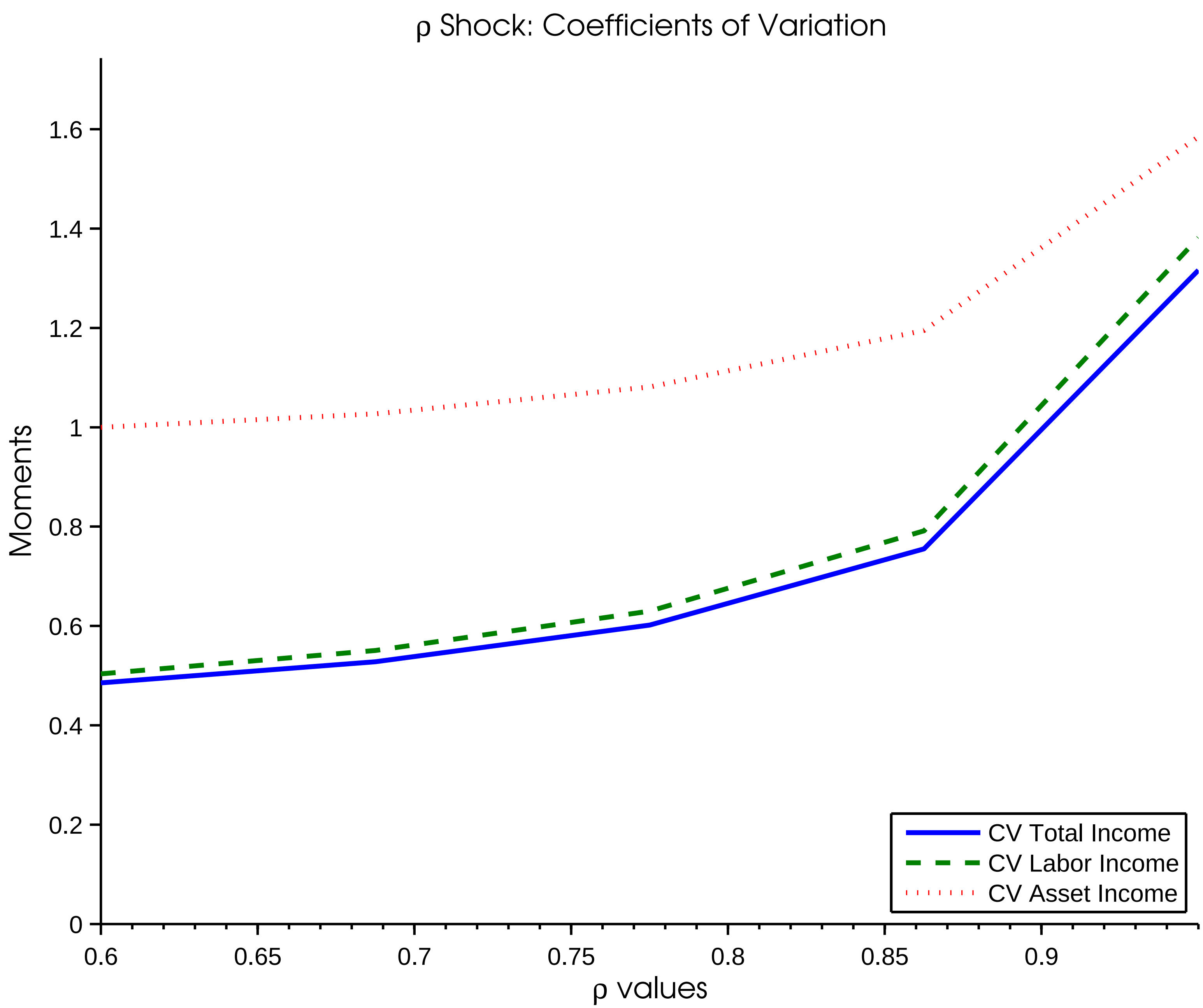

Figure 49: Coefficients of Variation With a Change in the Labor Income Persistence

Notes:

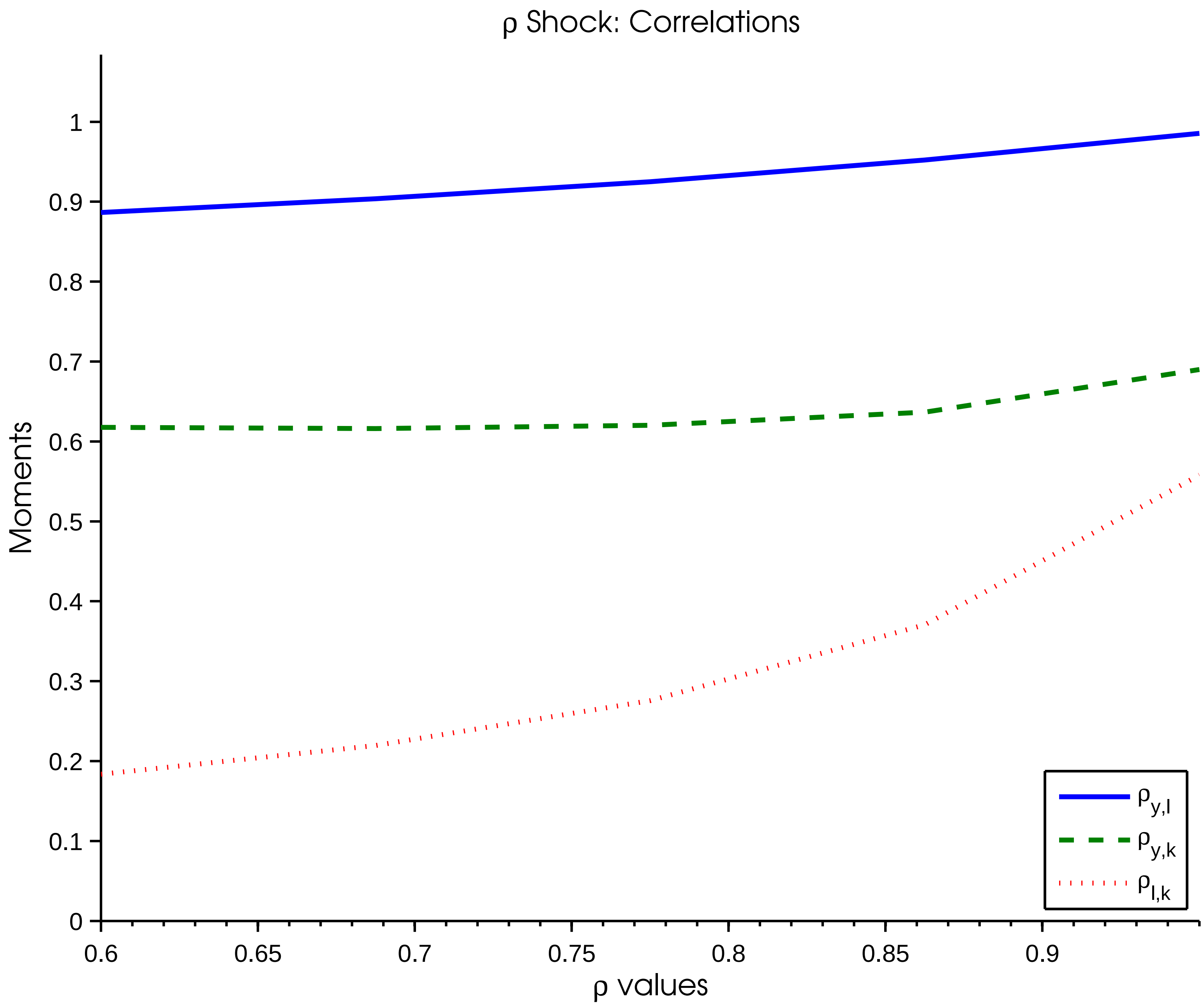

Figure 50: Correlations With a Change in the Labor Income Persistence

Notes:

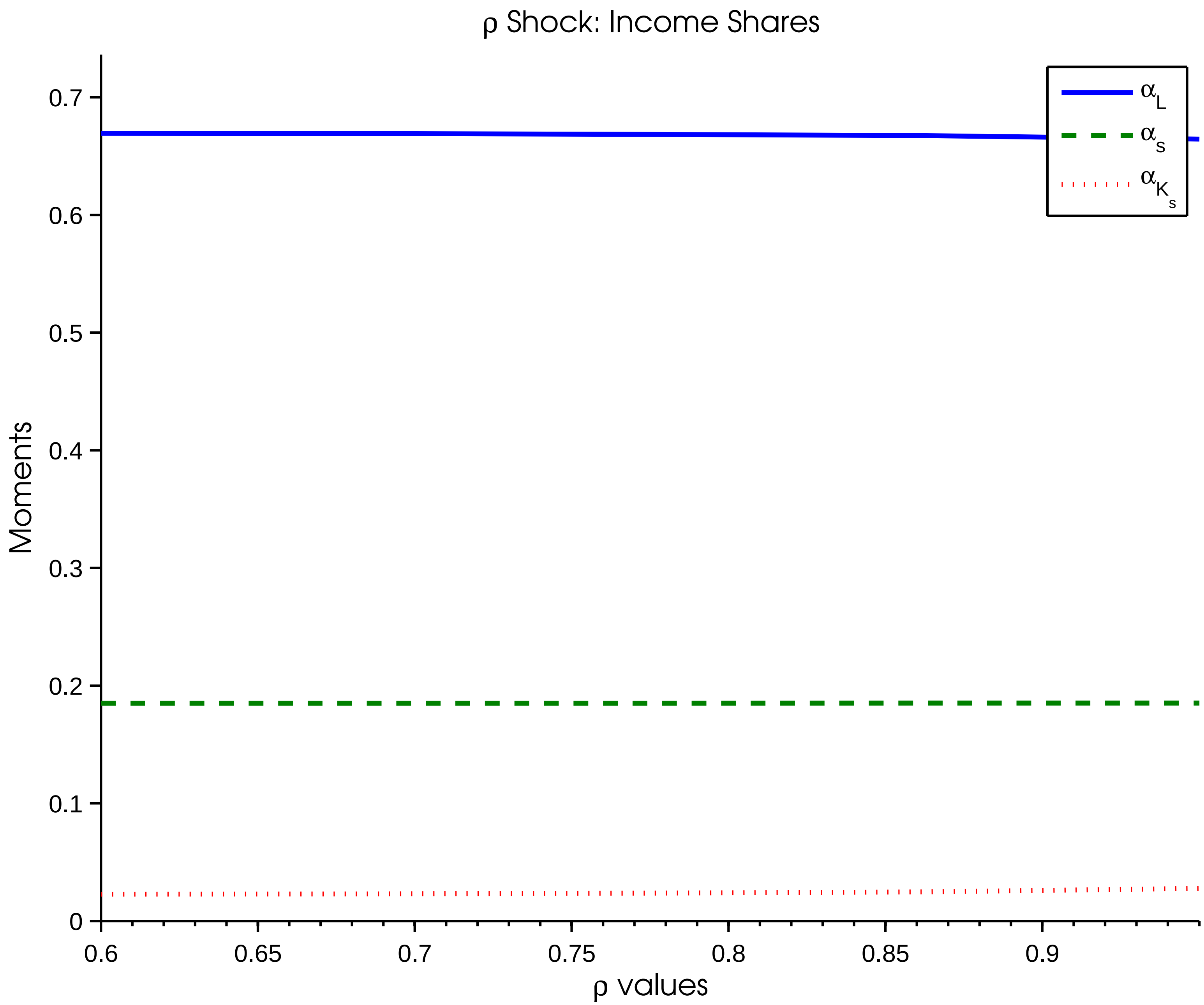

Figure 51: Income Shares With a Change in the Labor Income Persistence

Notes:

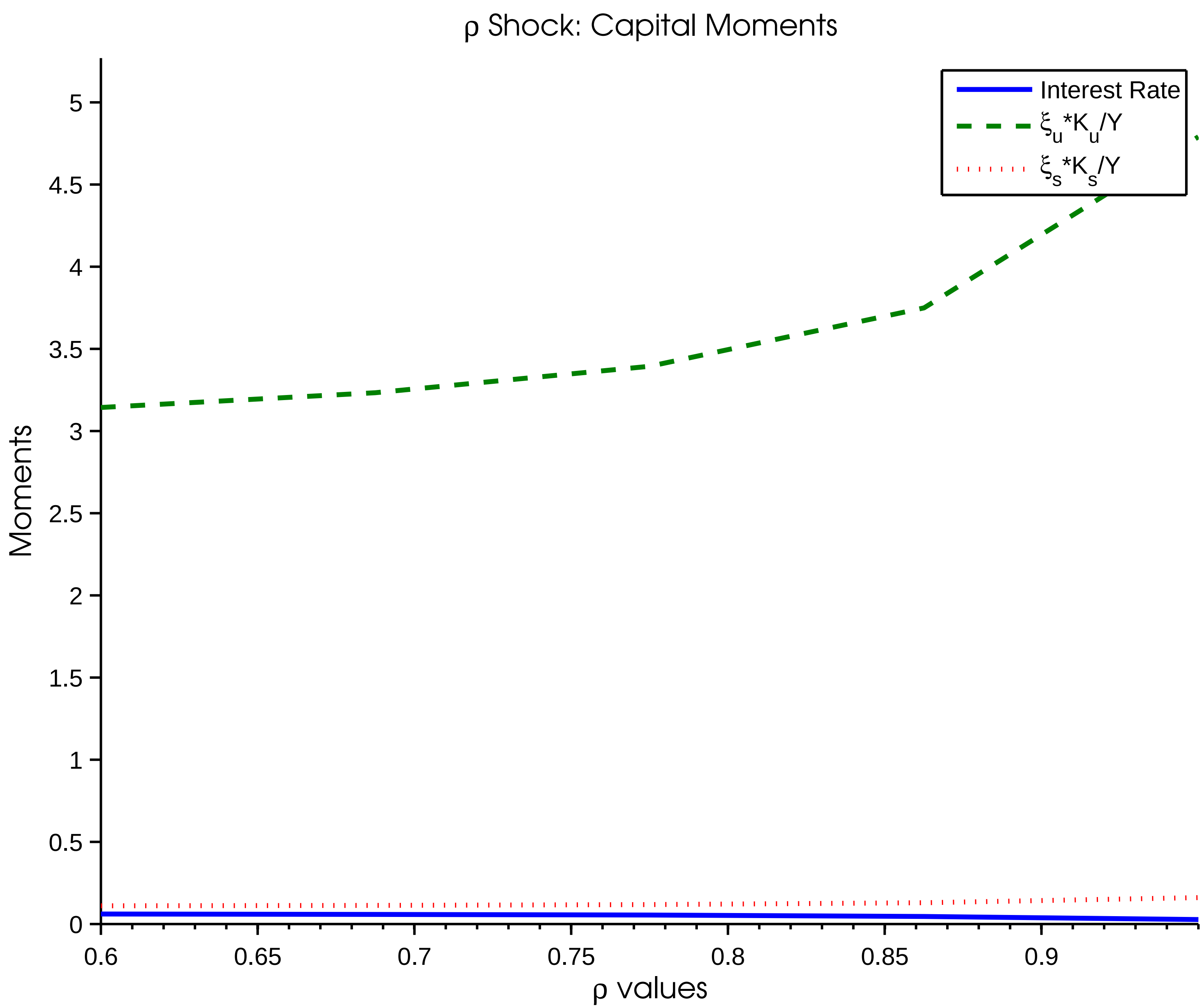

Figure 52: Capital Moments With a Change in the Labor Income Persistence

Notes:

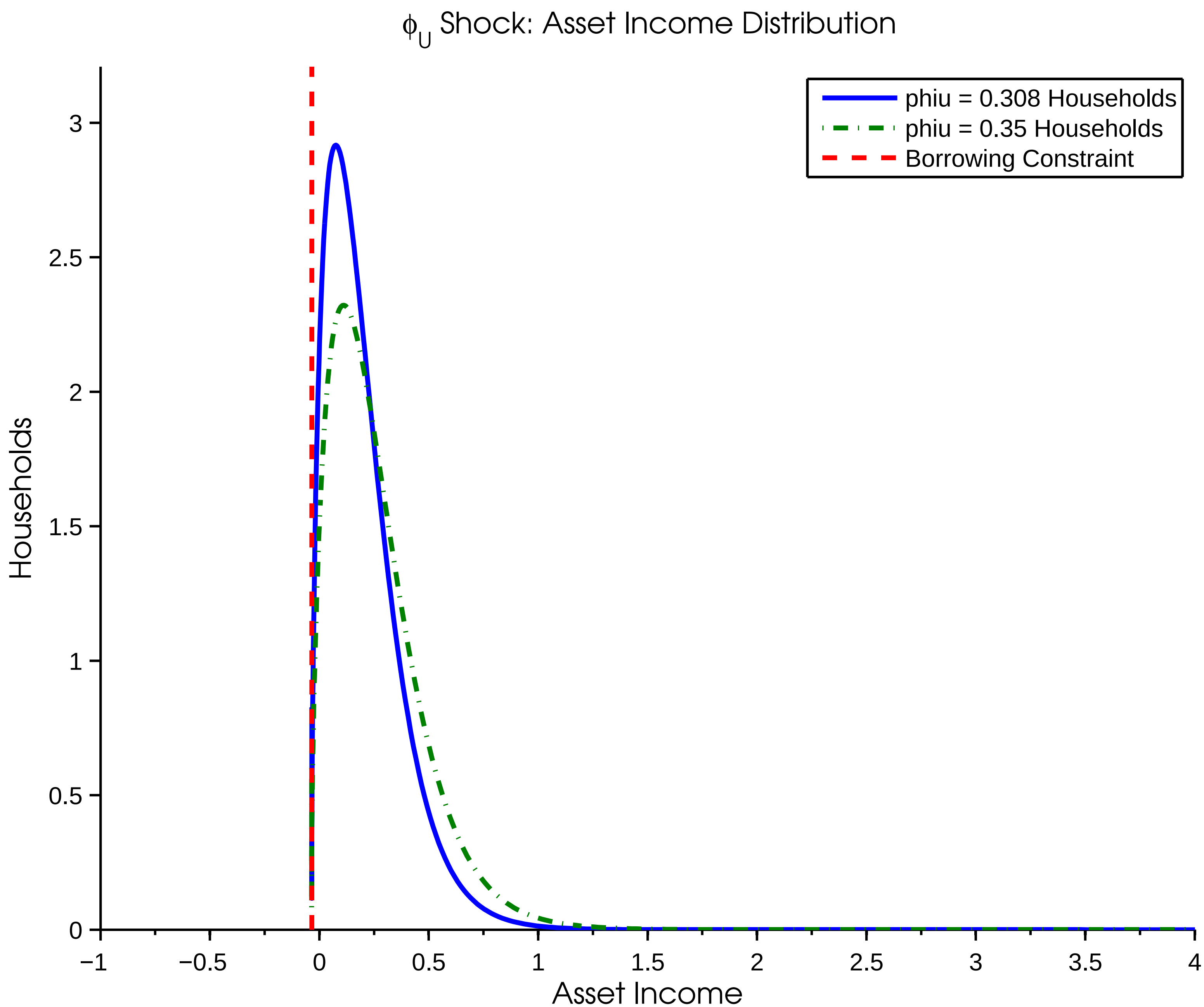

Figure 53: Benchmark Capital Income Distribution With a Change in the Structures Share

Notes:

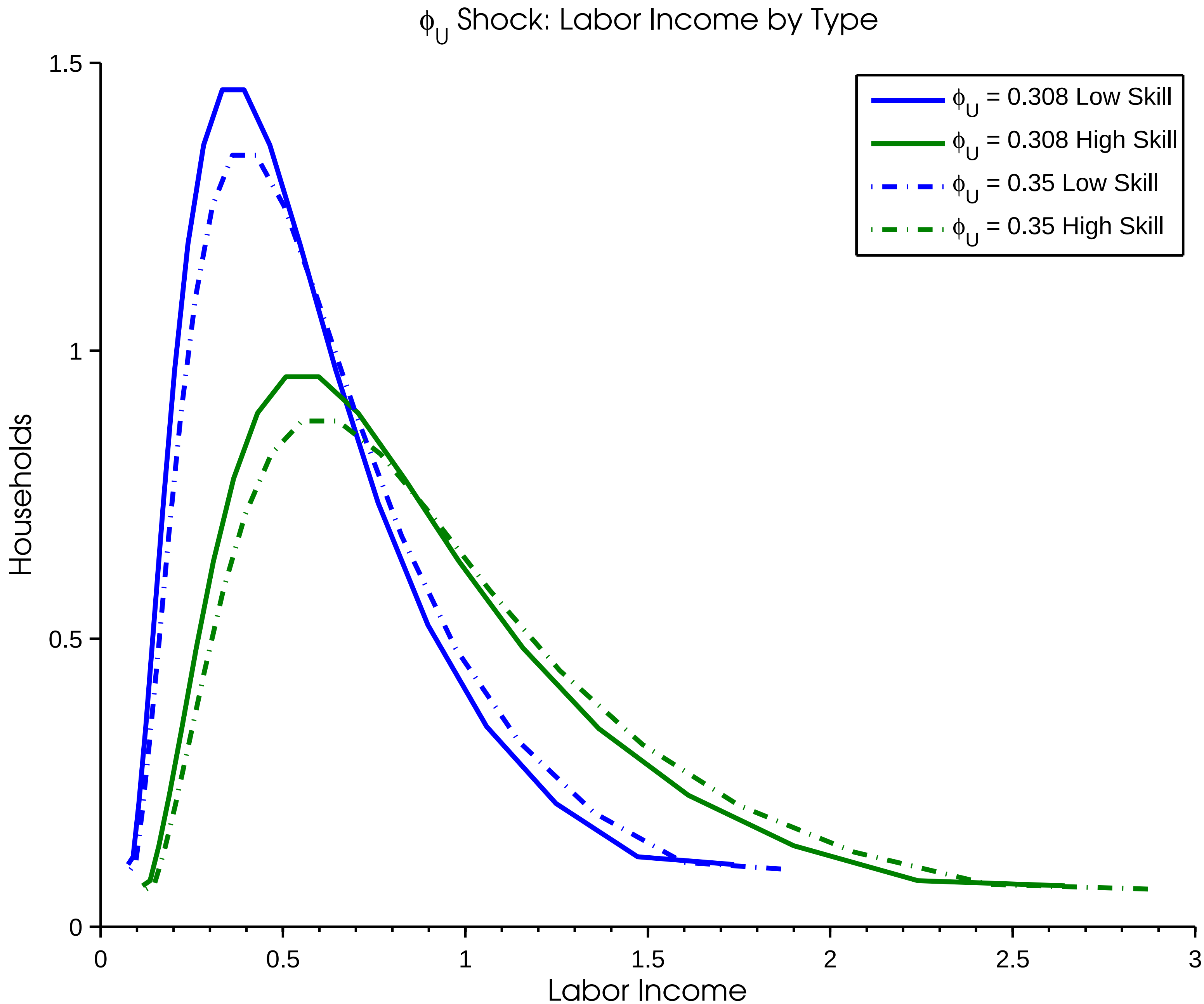

Figure 54: Benchmark Labor Income Distribution by Skill Type With a Change in the Structures Share

Notes:

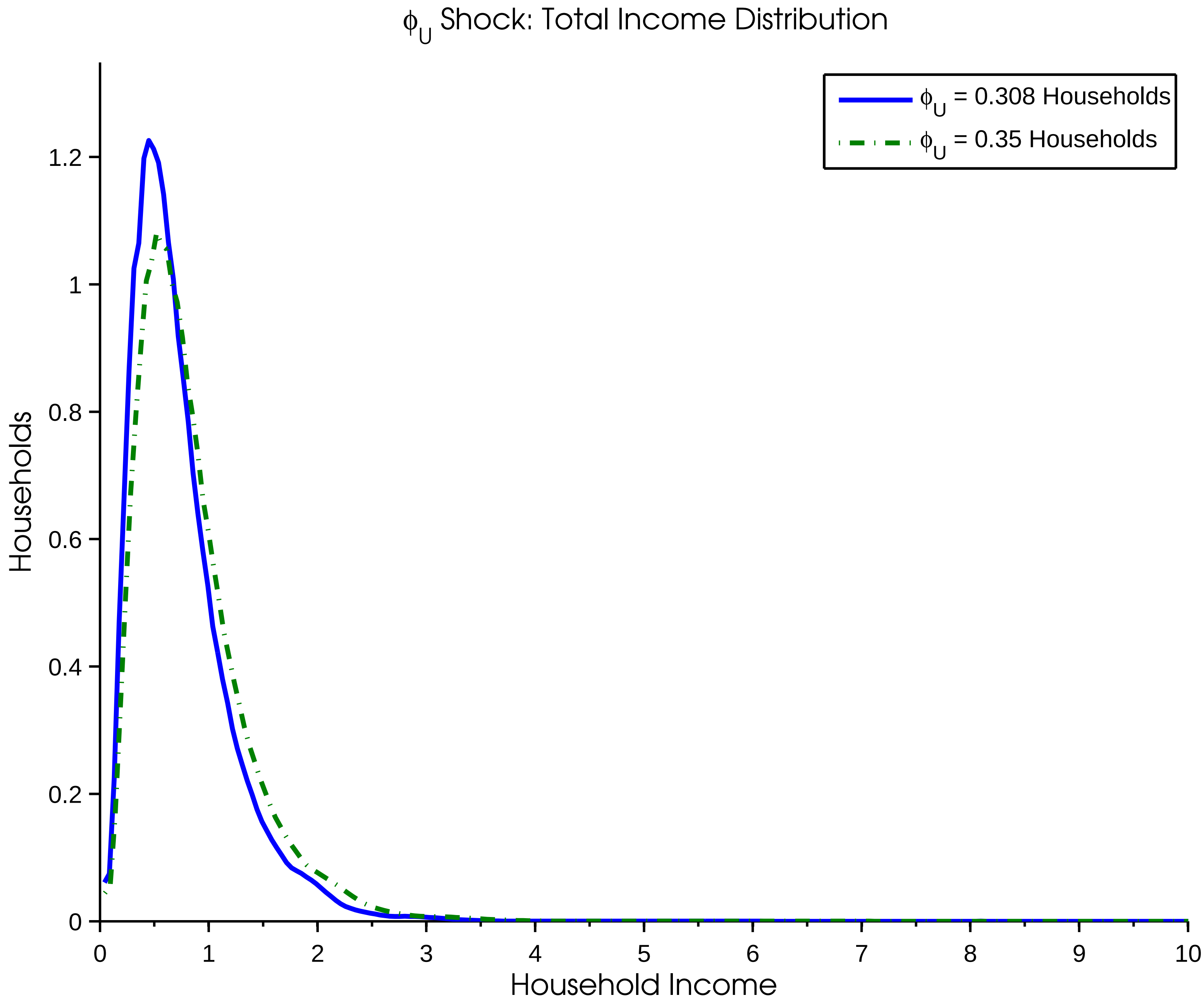

Figure 55: Total Income Distribution With a Change in the Structures Share

Notes:

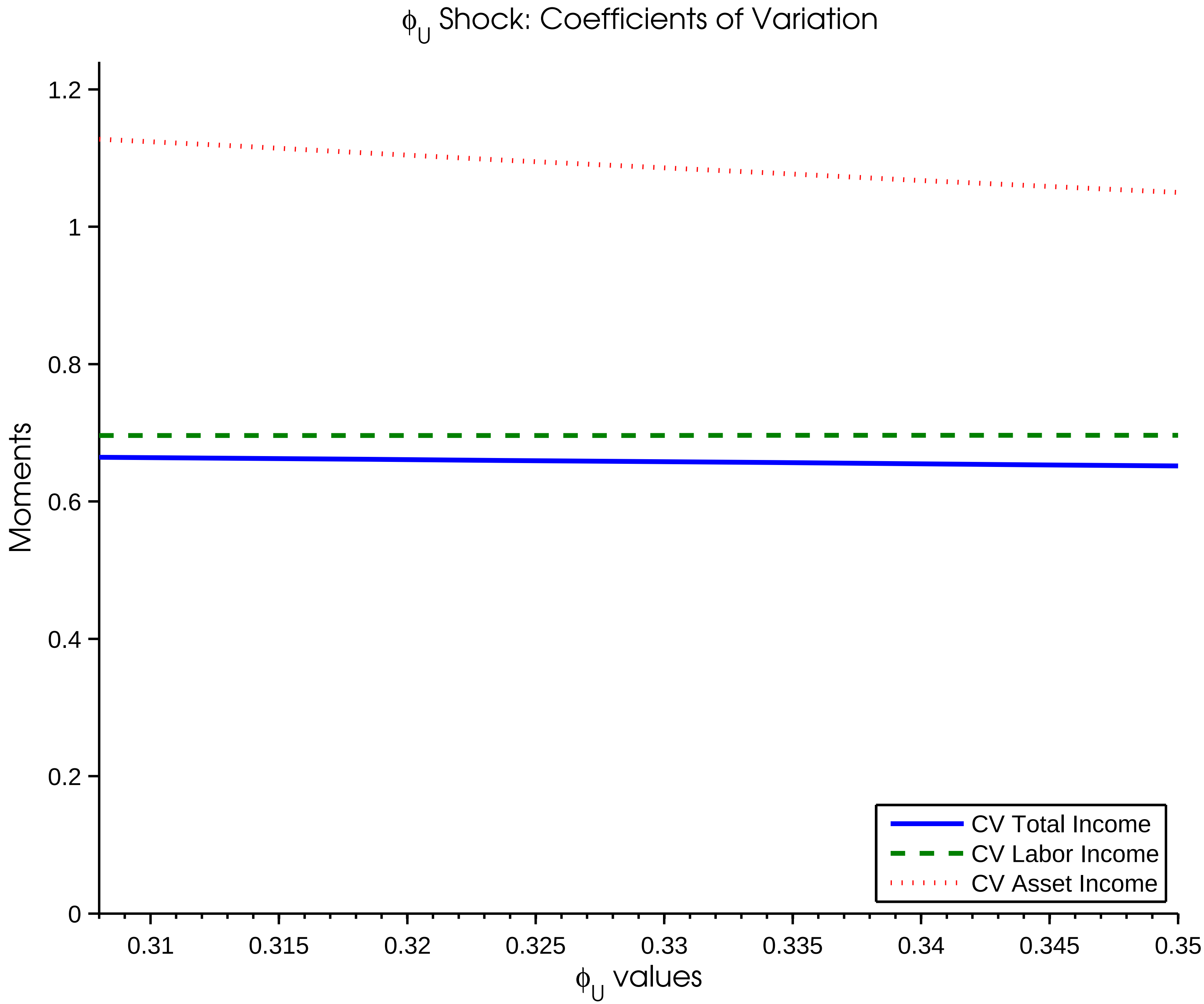

Figure 56: Coefficients of Variation With a Change in the Structures Share

Notes:

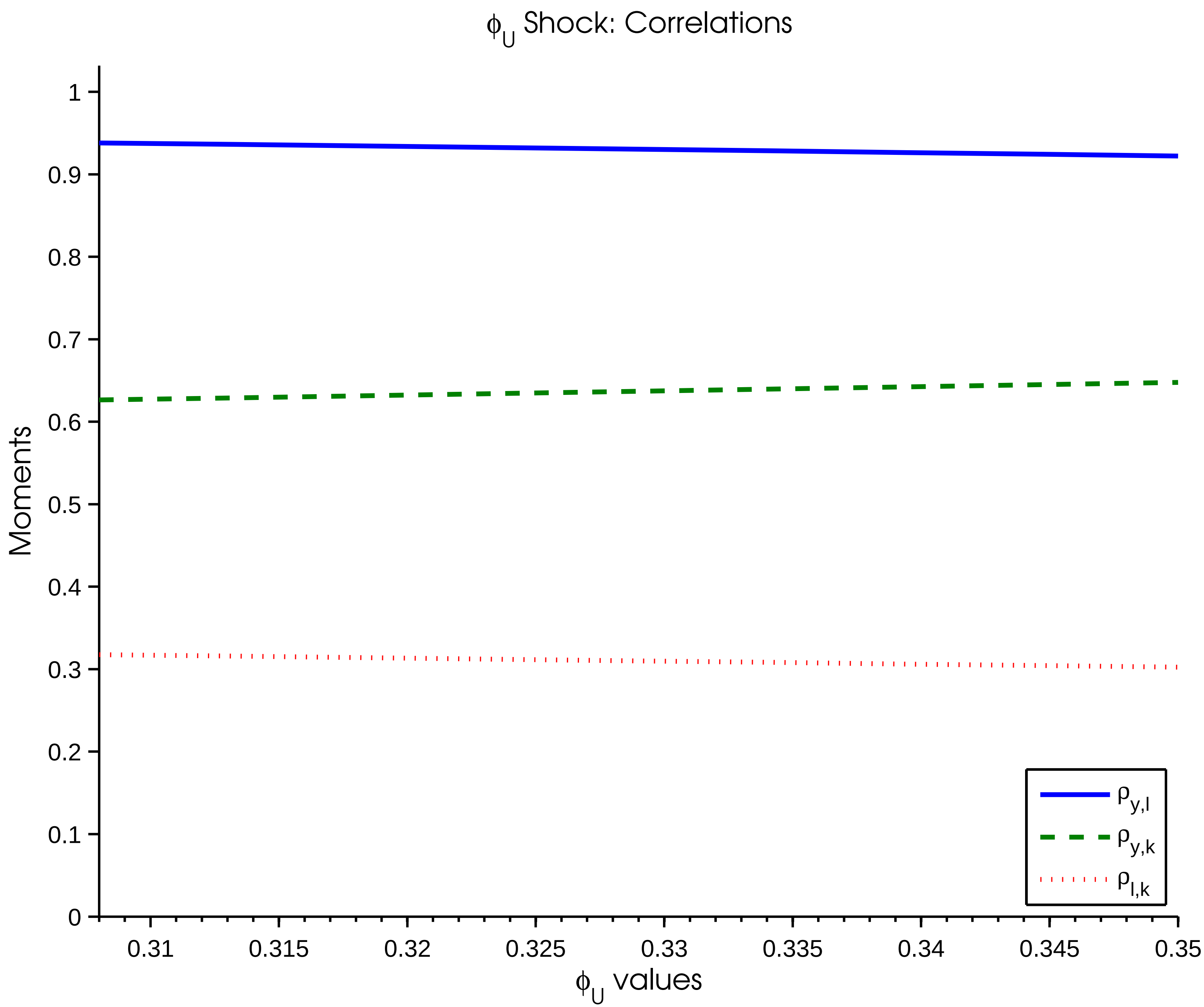

Figure 57: Correlations With a Change in the Structures Share

Notes:

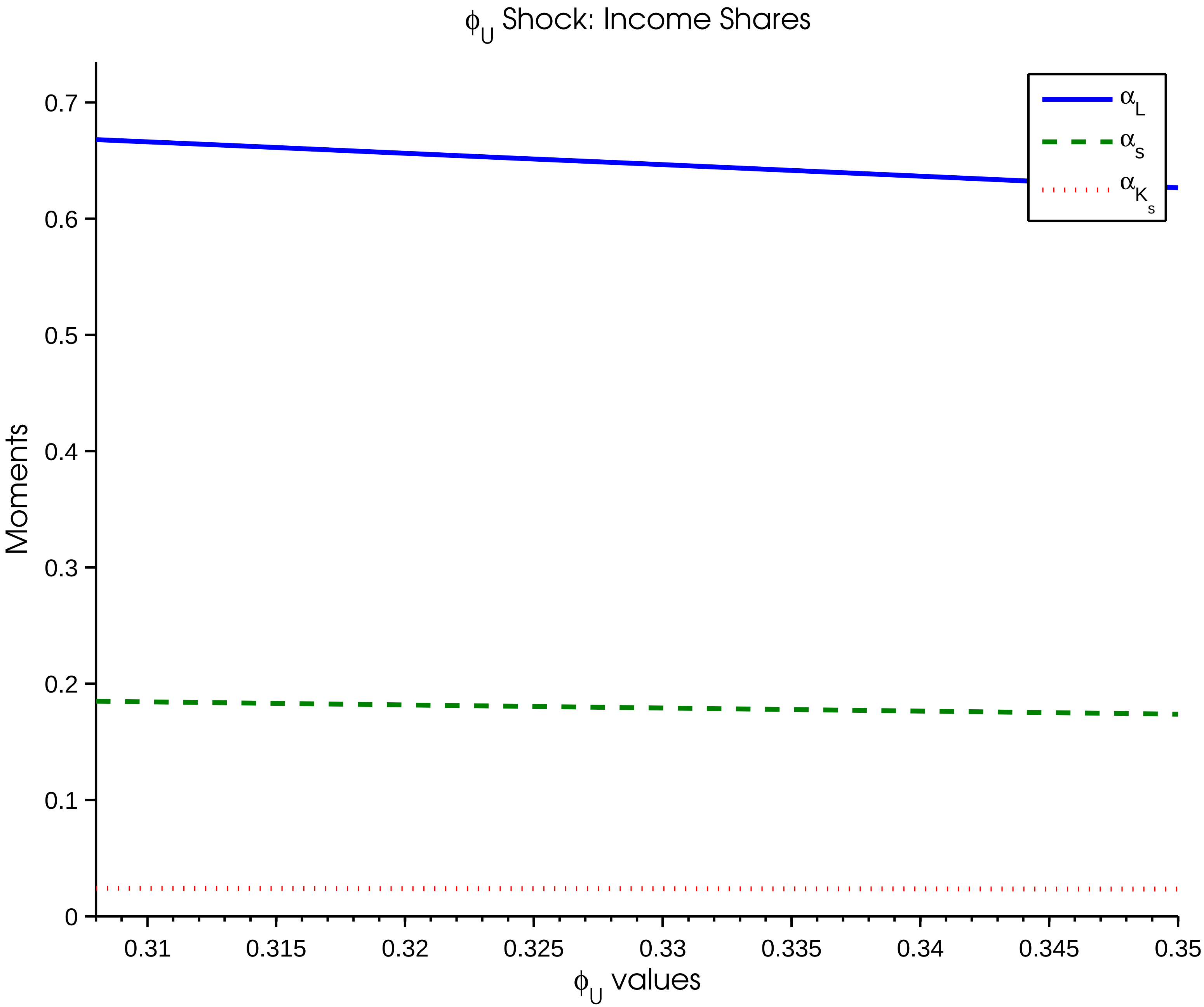

Figure 58: Income Shares With a Change in the Structures Share

Notes:

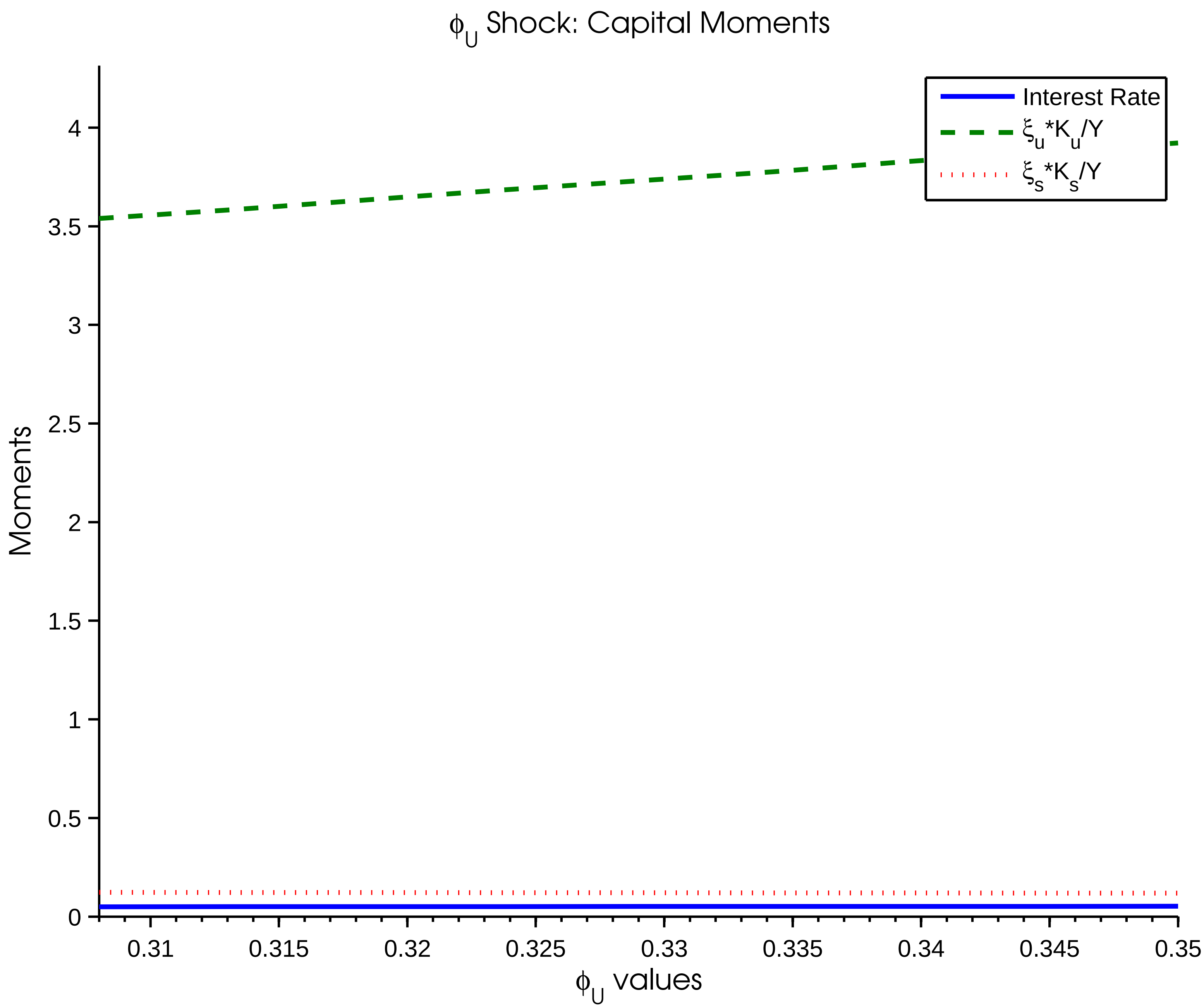

Figure 59: Capital Moments With a Change in the Structures Share

Notes:

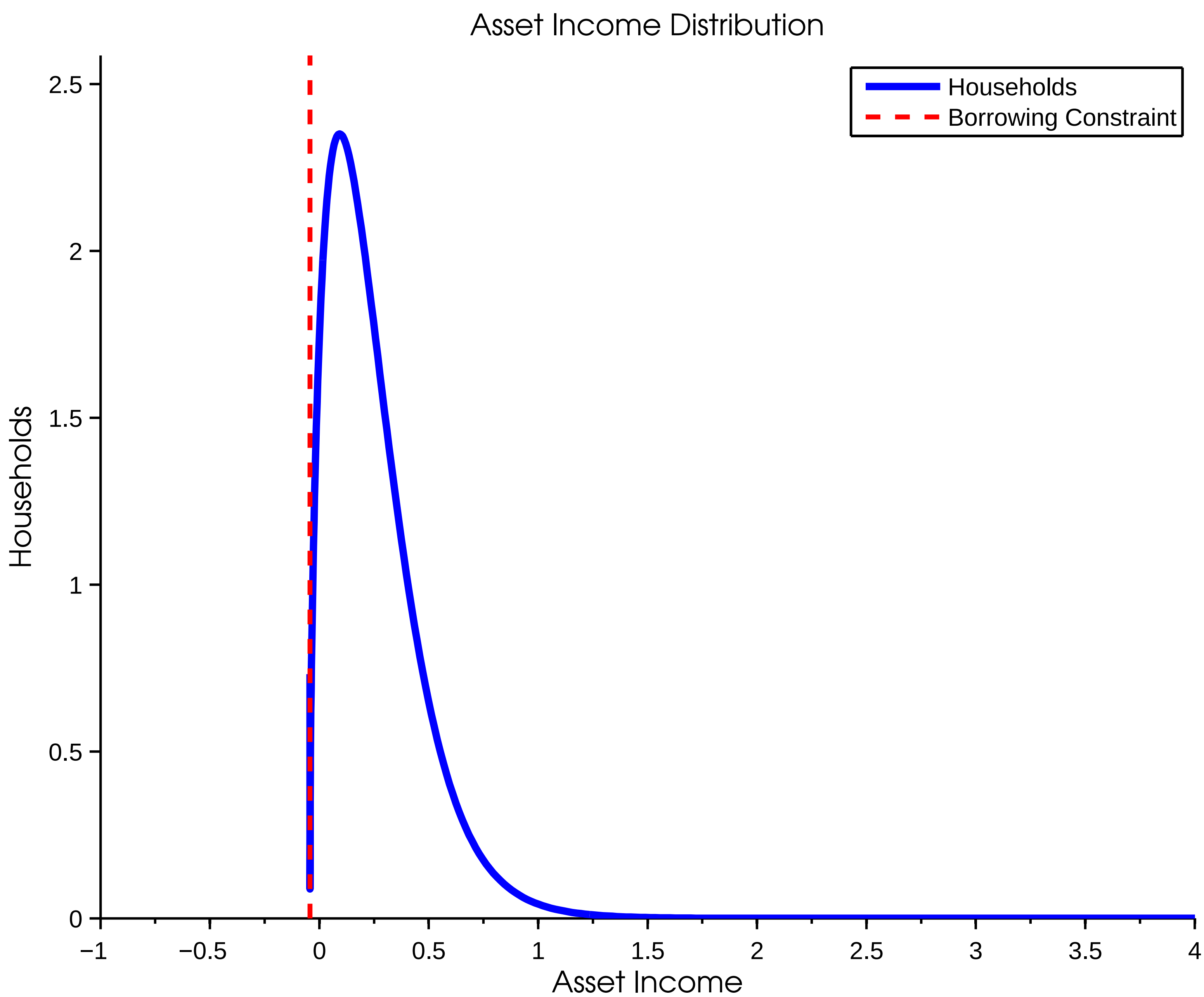

Figure 60: Cobb-Douglas Capital Income Distribution

Notes:

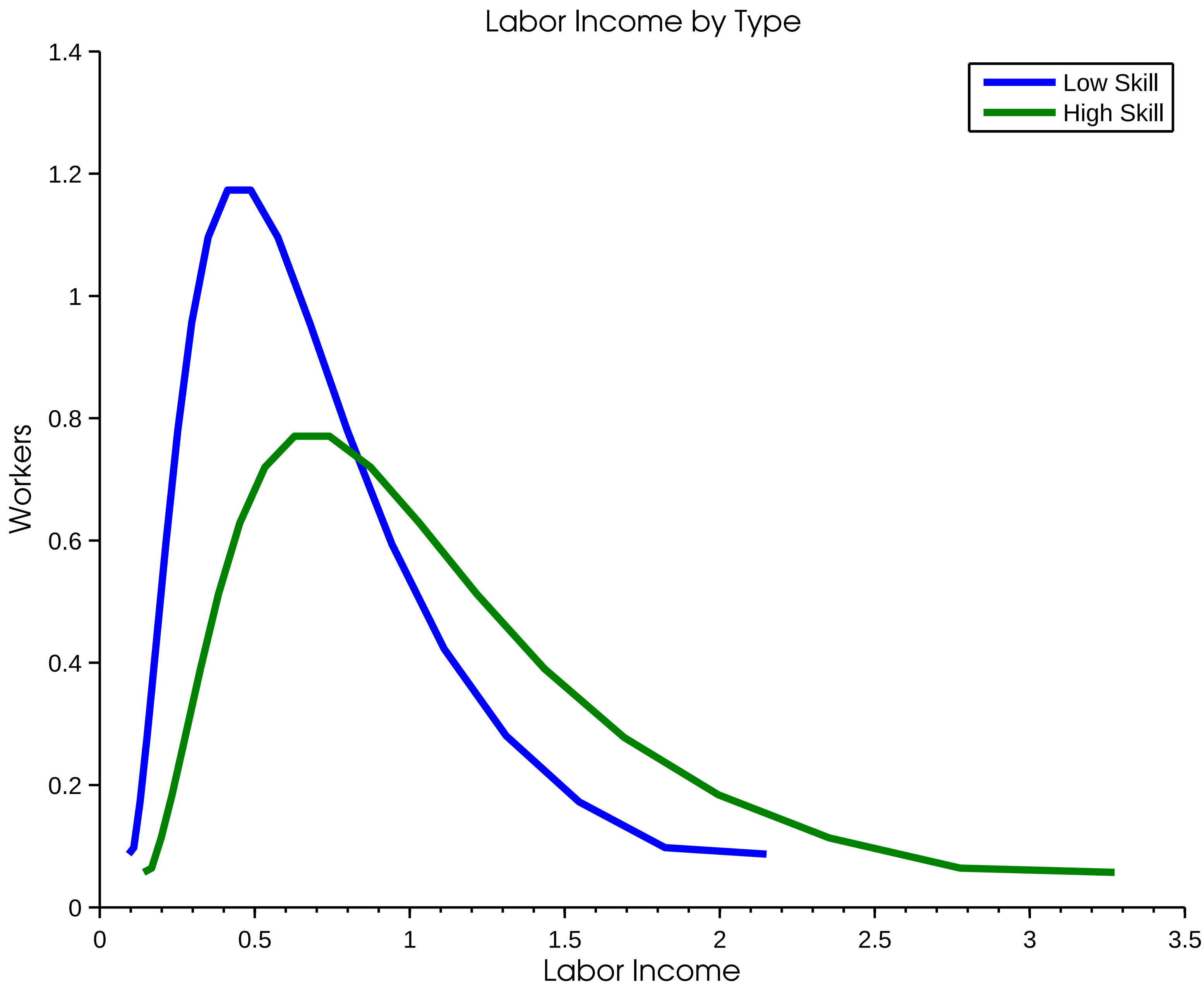

Figure 61: Cobb-Douglas Labor Income Distribution by Skill Type

Notes:

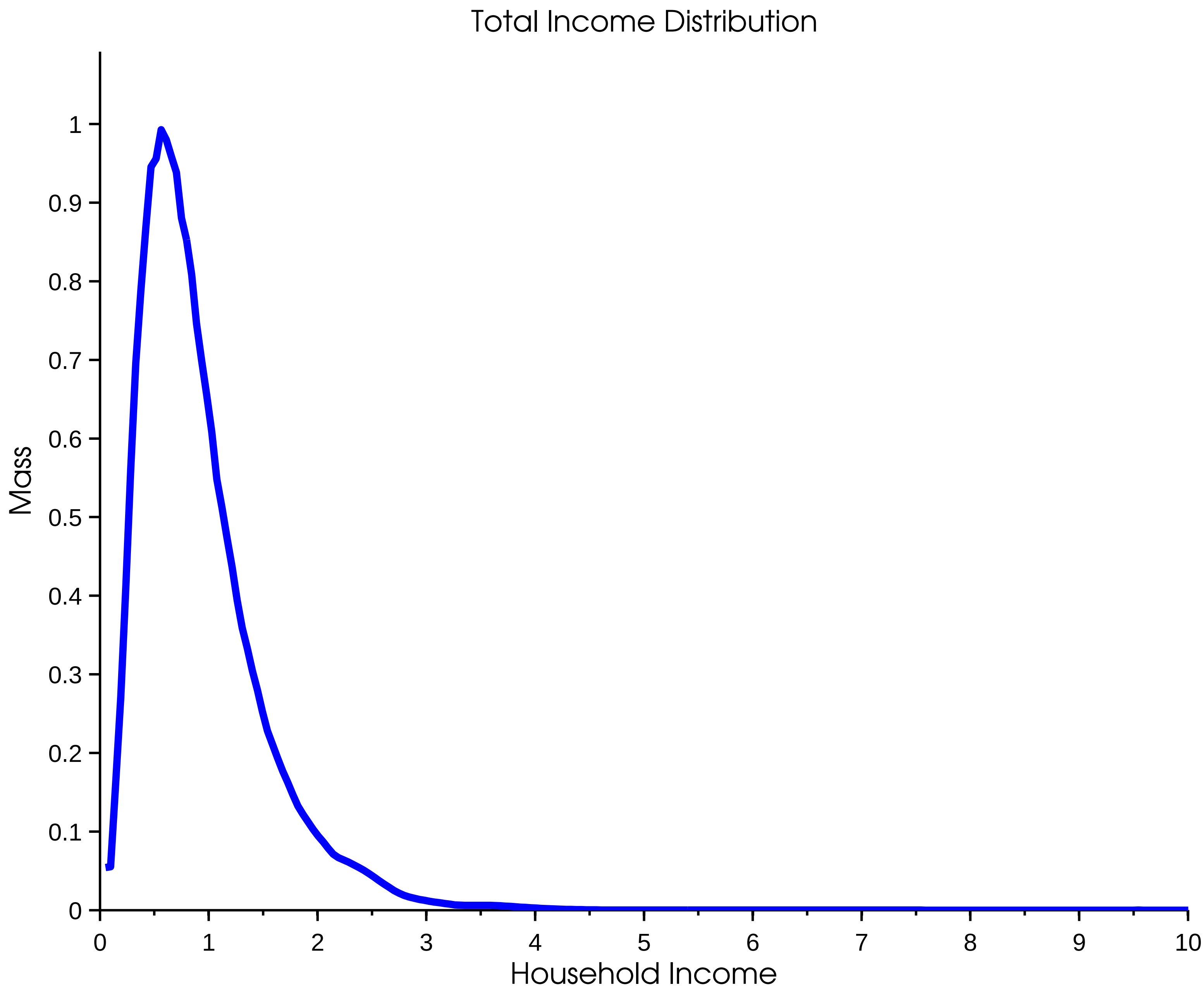

Figure 62: Cobb-Douglas Total Income Distribution

Notes:

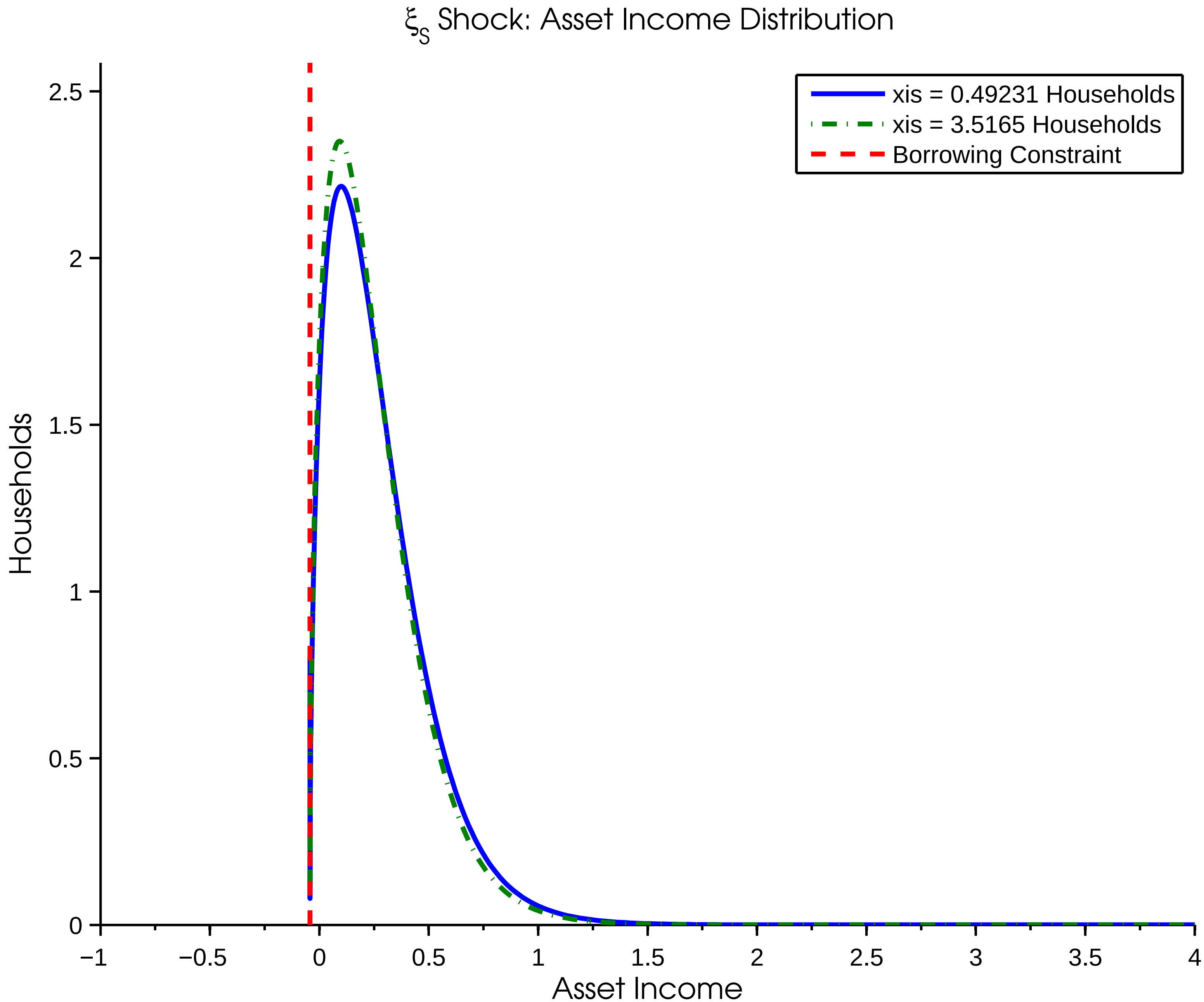

Figure 63: Cobb-Douglas Capital Income Distribution With an IT Price Shock

Notes:

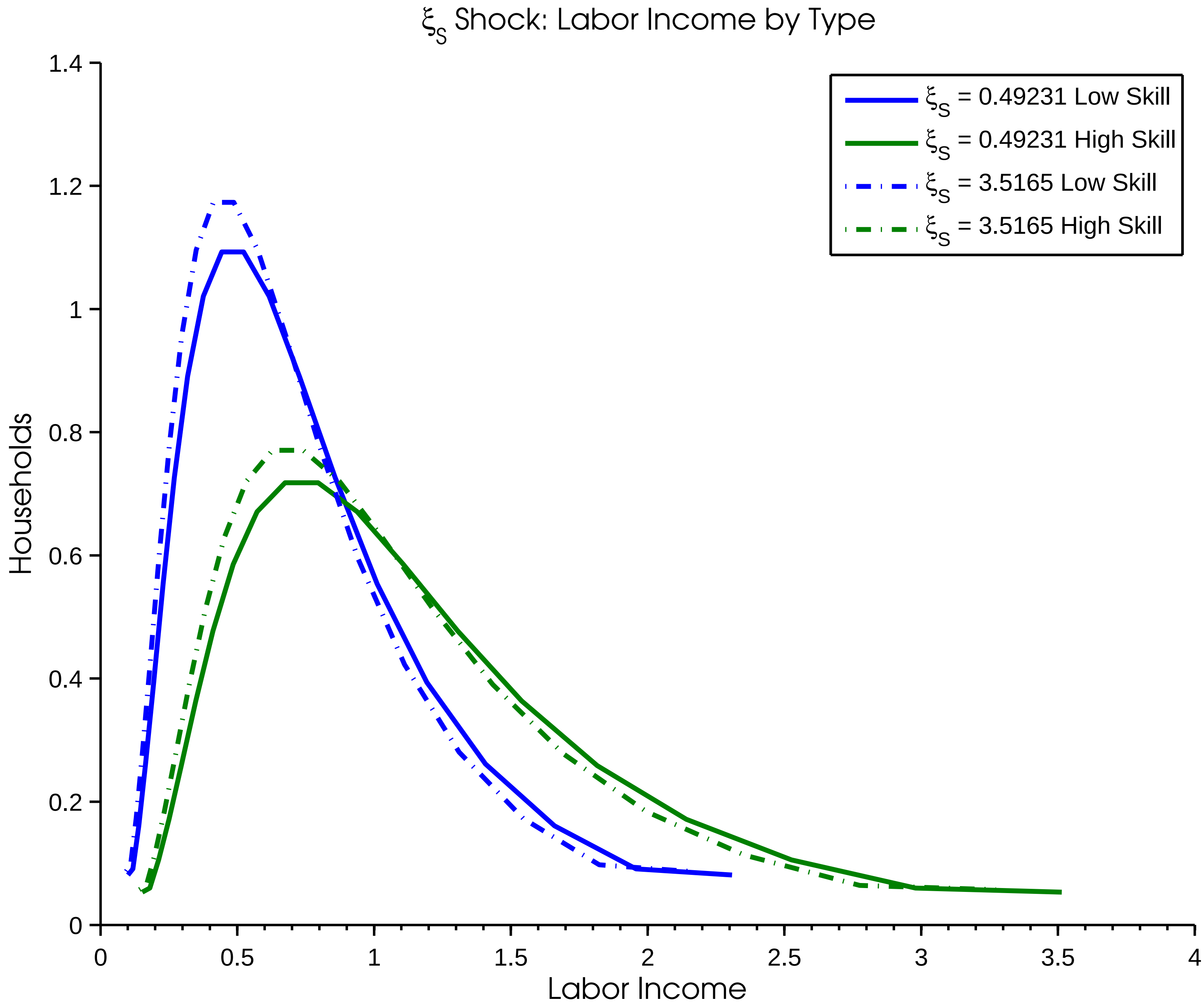

Figure 64: Cobb-Douglas Labor Income Distribution by Skill Type With an IT Price Shock

Notes:

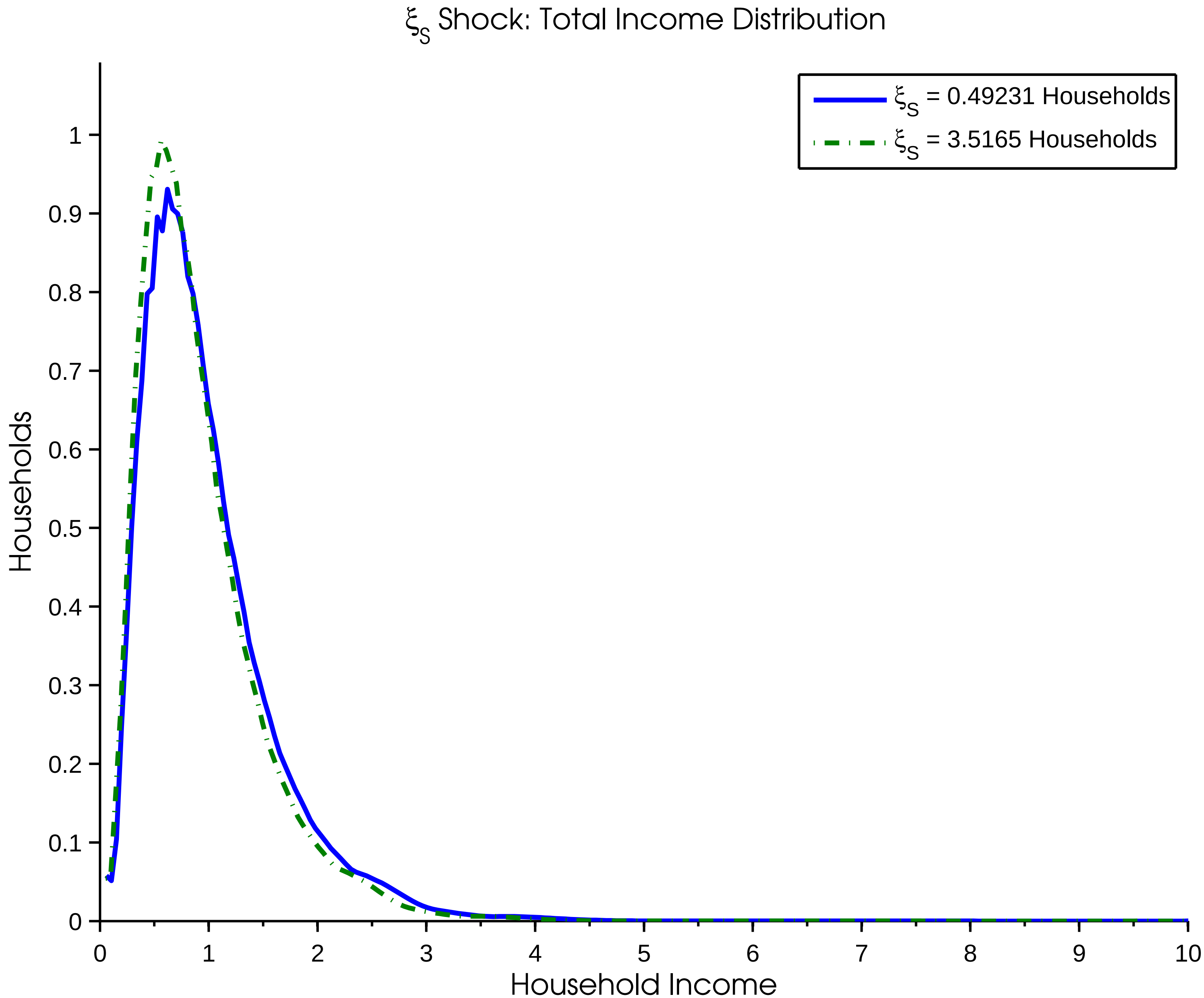

Figure 65: Cobb-Douglas Total Income Distribution With an IT Price Shock

Notes:

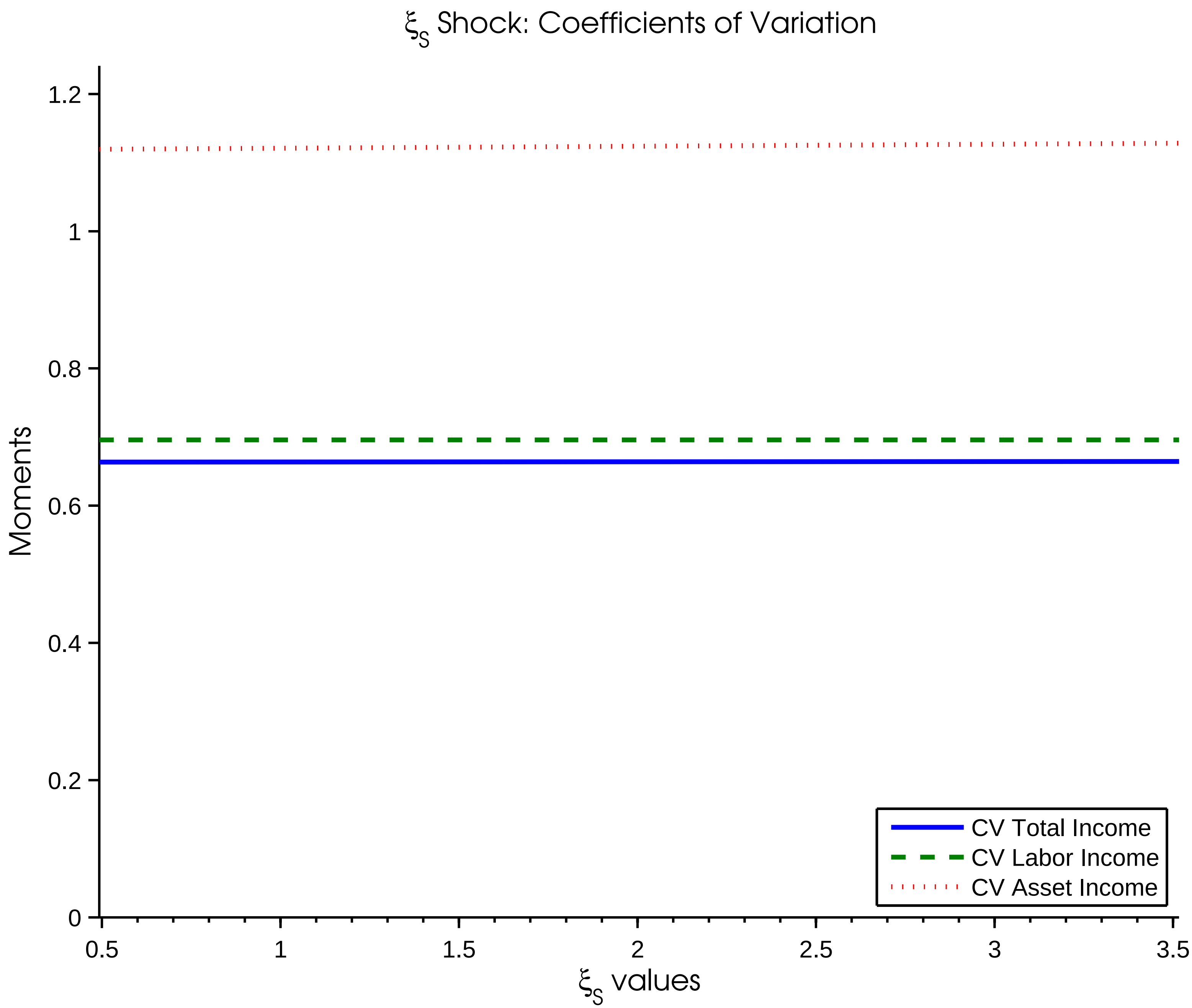

Figure 66: Cobb-Douglas Coefficients of Variation With an IT Price Shock

Notes:

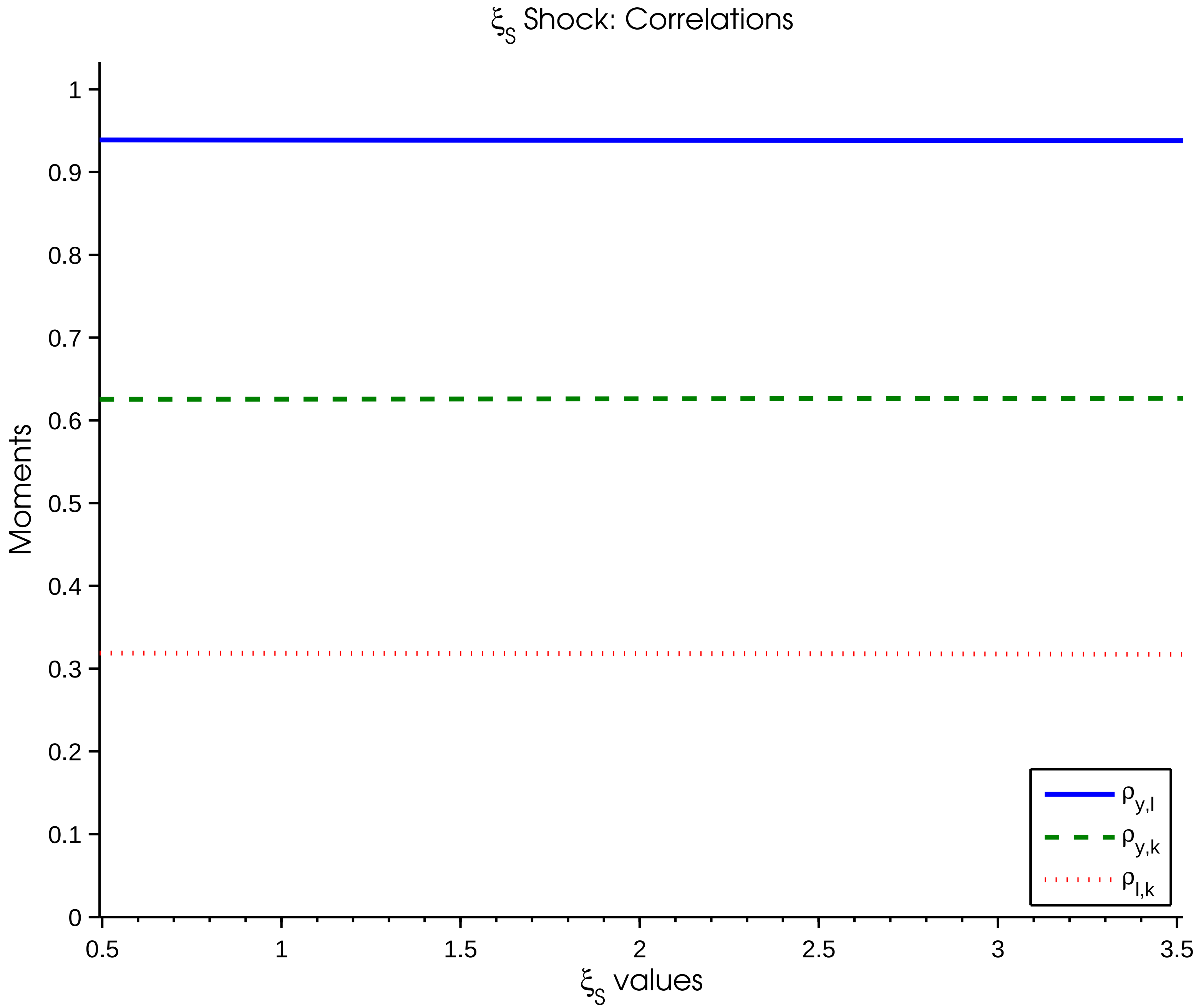

Figure 67: Cobb-Douglas Correlations With an IT Price Shock

Notes:

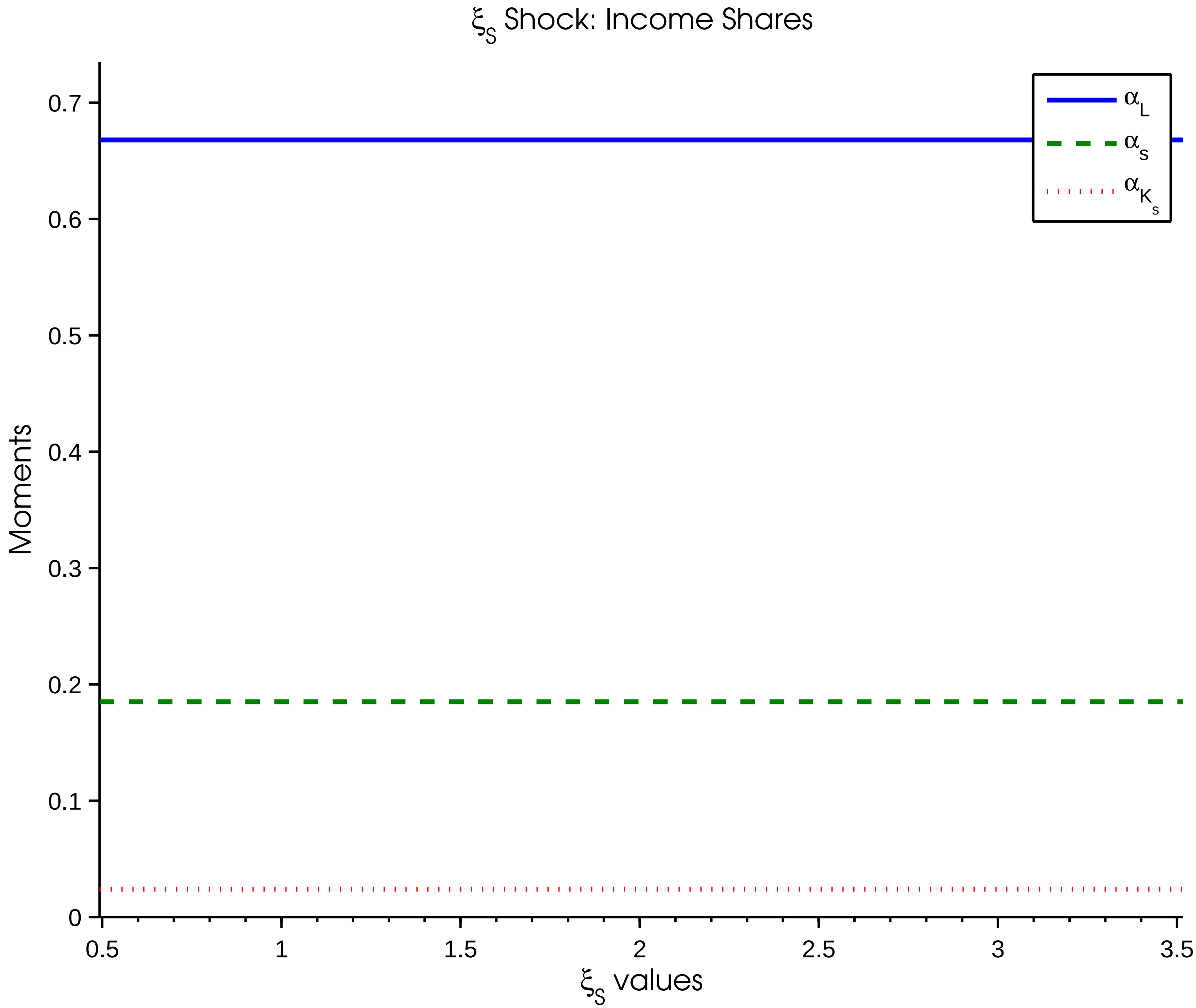

Figure 68: Cobb-Douglas Income Shares With an IT Price Shock

Notes:

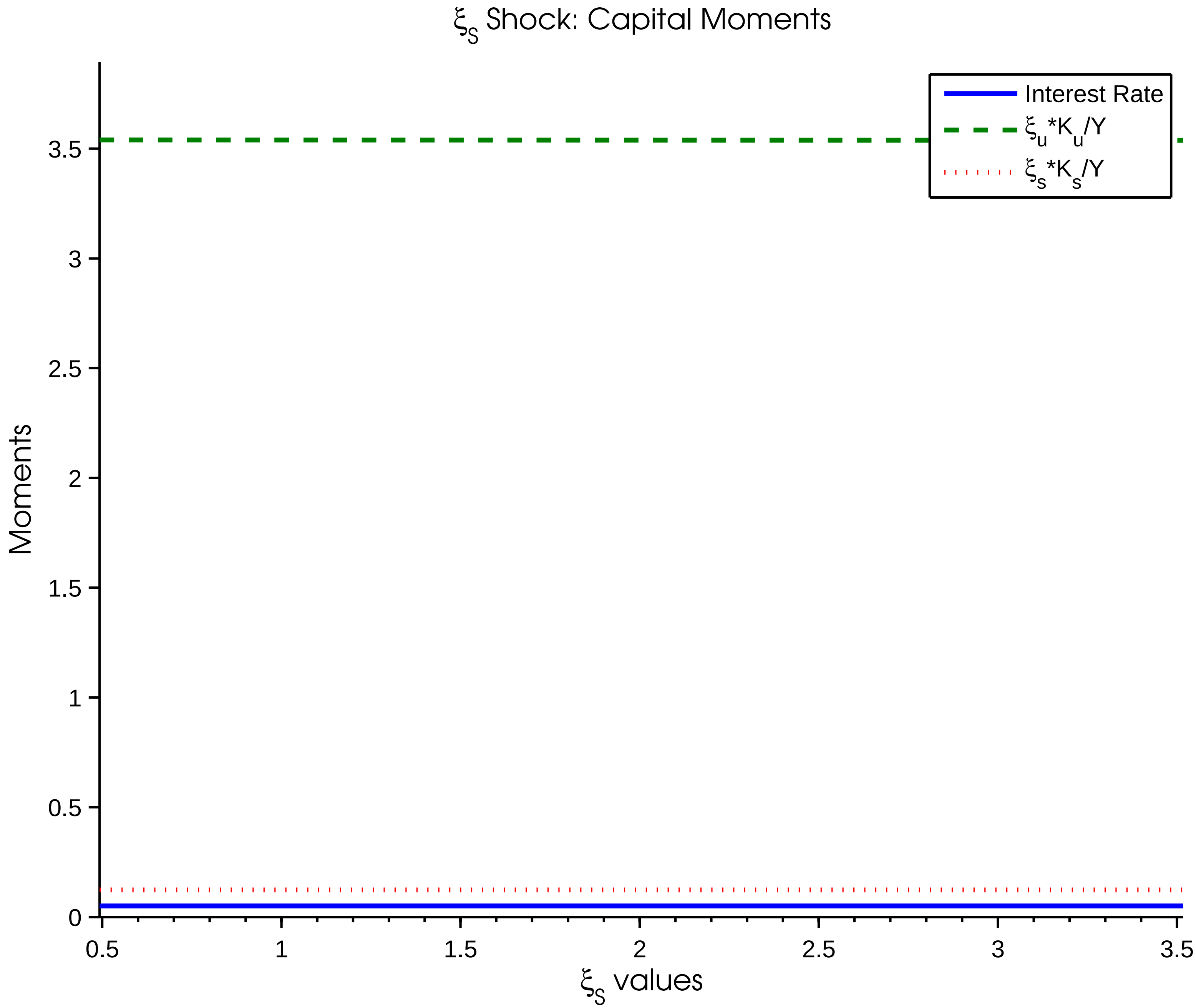

Figure 69: Cobb-Douglas Capital Moments With an IT Price Shock

Notes:

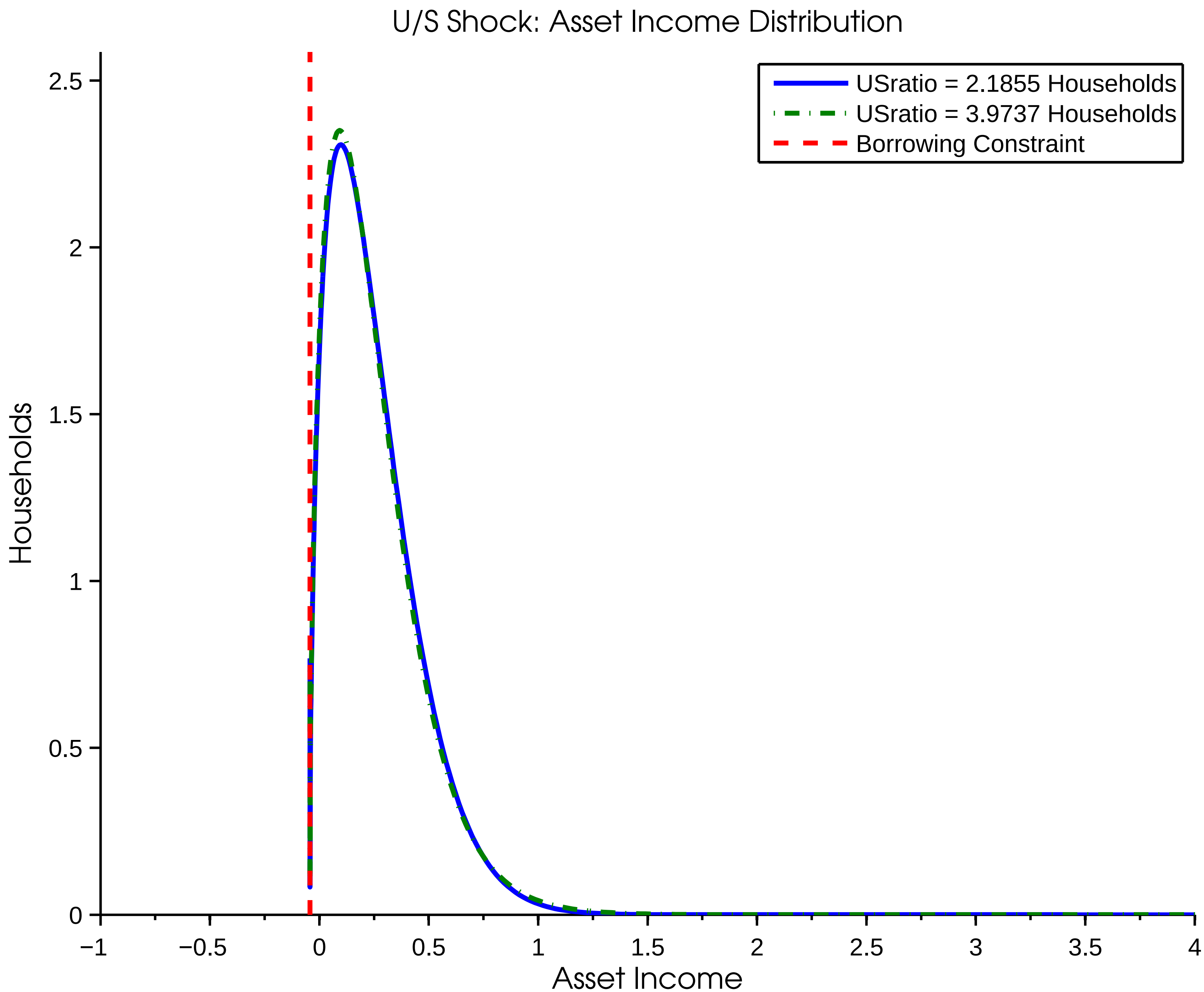

Figure 70: Cobb-Douglas Capital Income Distribution With a Change in the Stock of Skill

Notes:

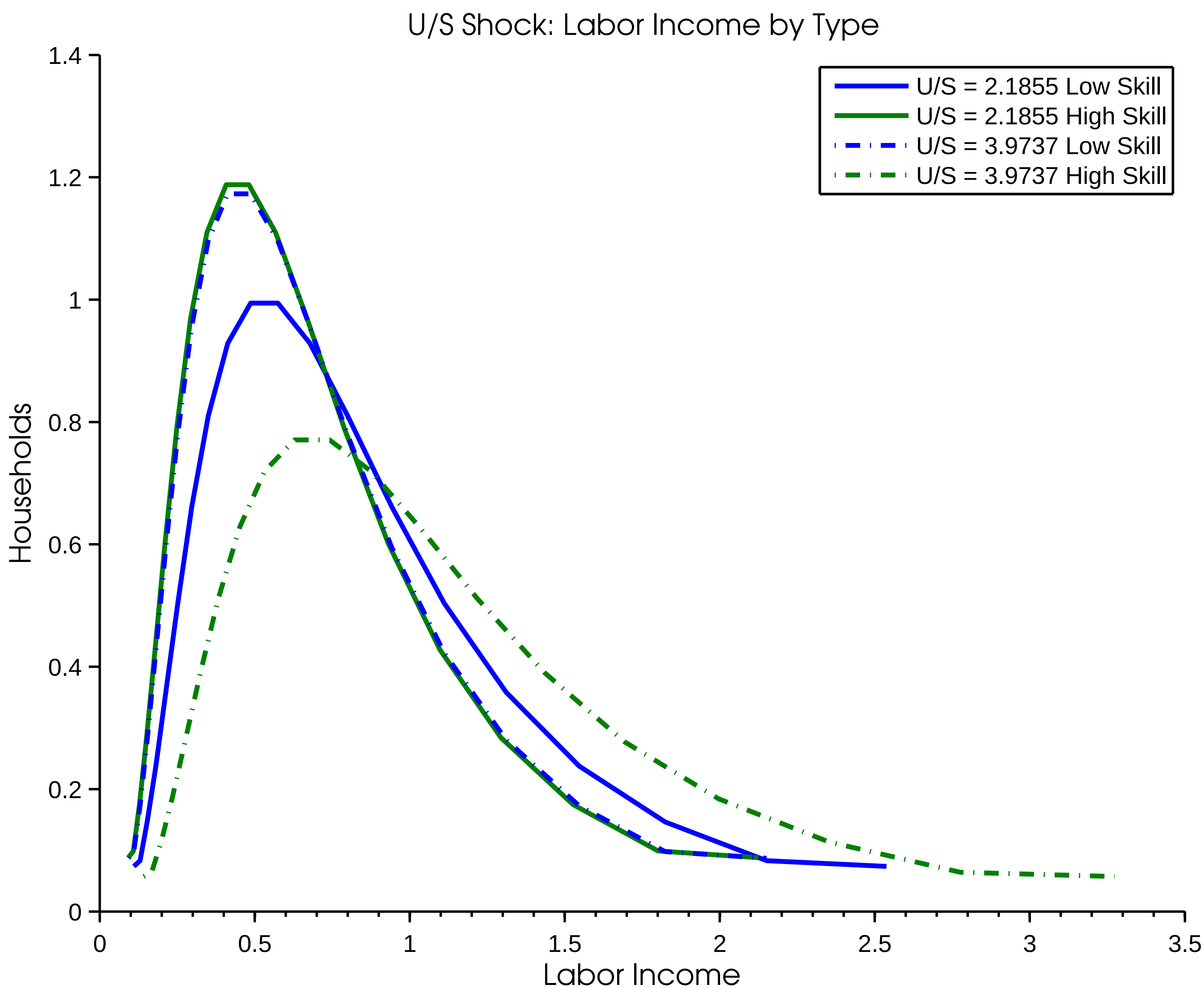

Figure 71: Cobb-Douglas Labor Income Distribution by Skill Type With a Change in the Stock of Skill

Notes:

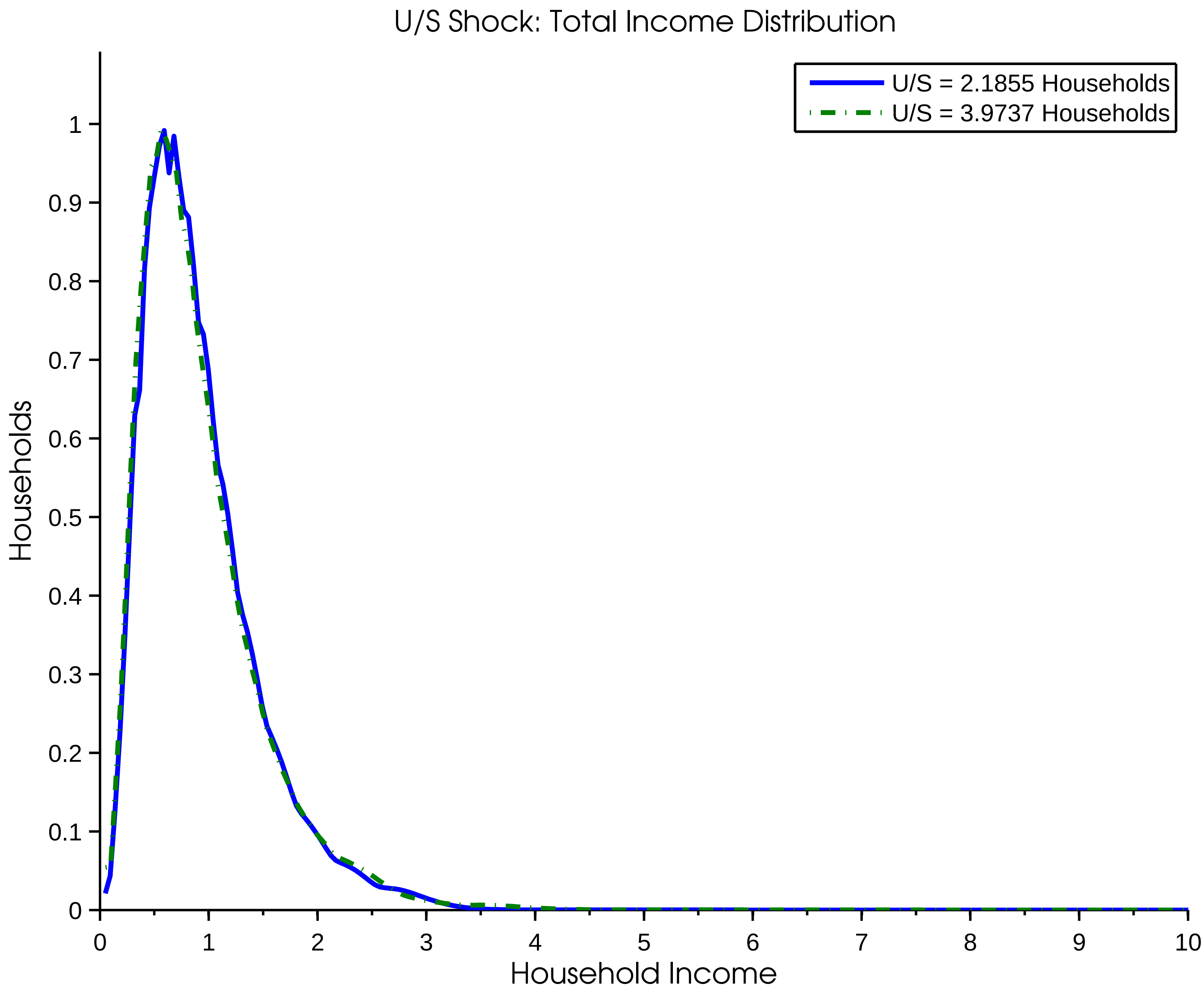

Figure 72: Cobb-Douglas Total Income Distribution With a Change in the Stock of Skill

Notes:

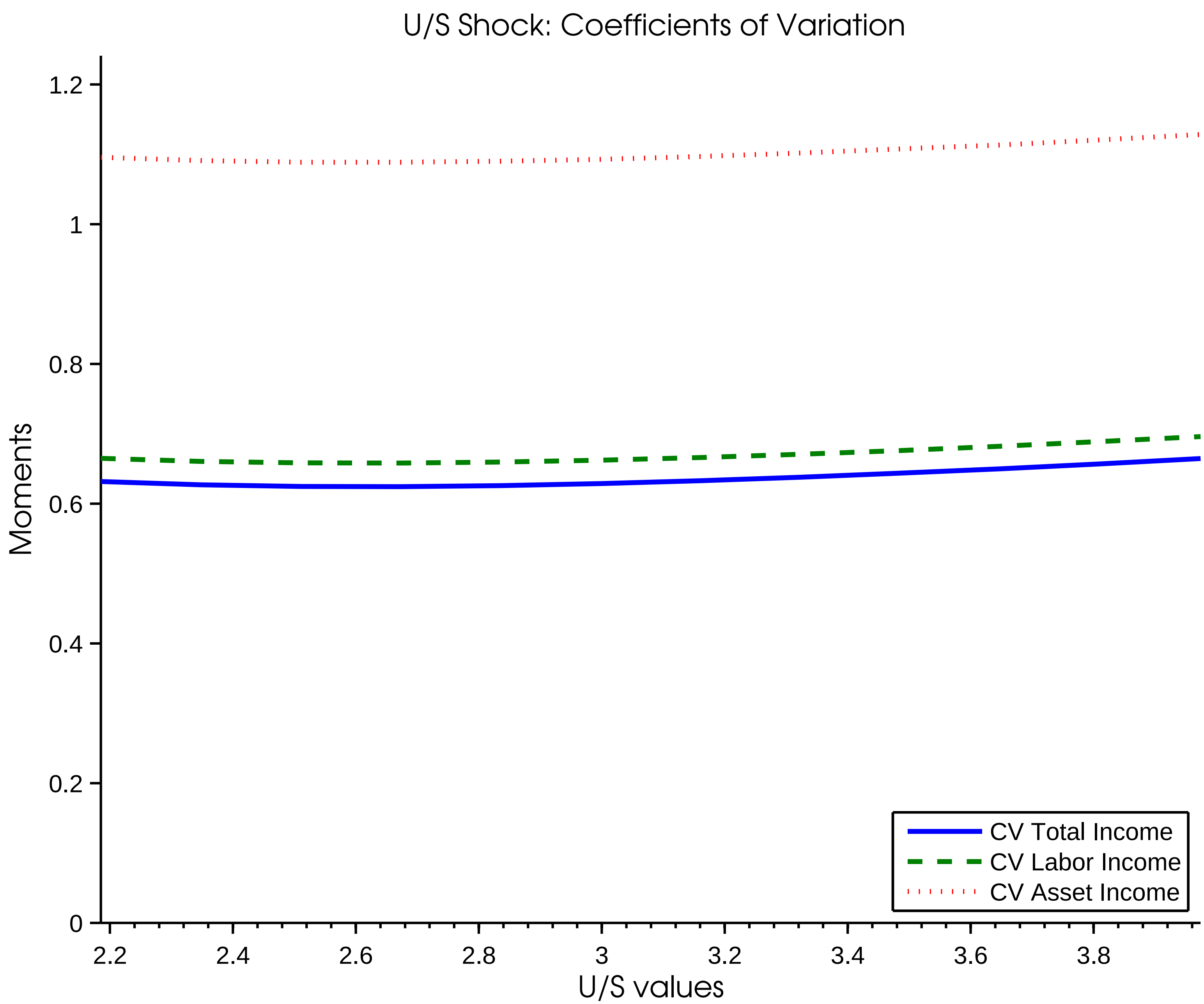

Figure 73: Cobb-Douglas Coefficients of Variation With a Change in the Stock of Skill

Notes:

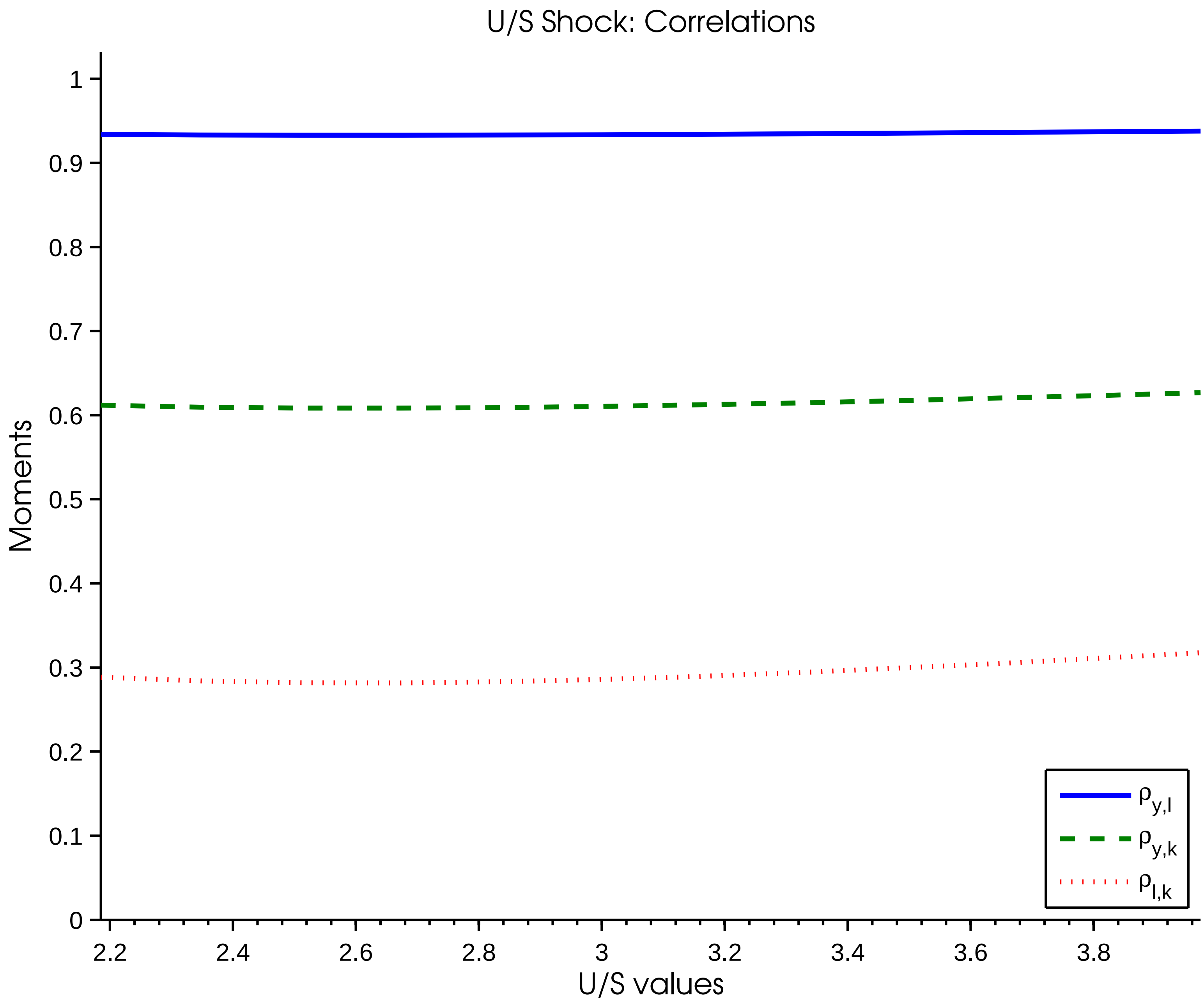

Figure 74: Cobb-Douglas Correlations With a Change in the Stock of Skill

Notes:

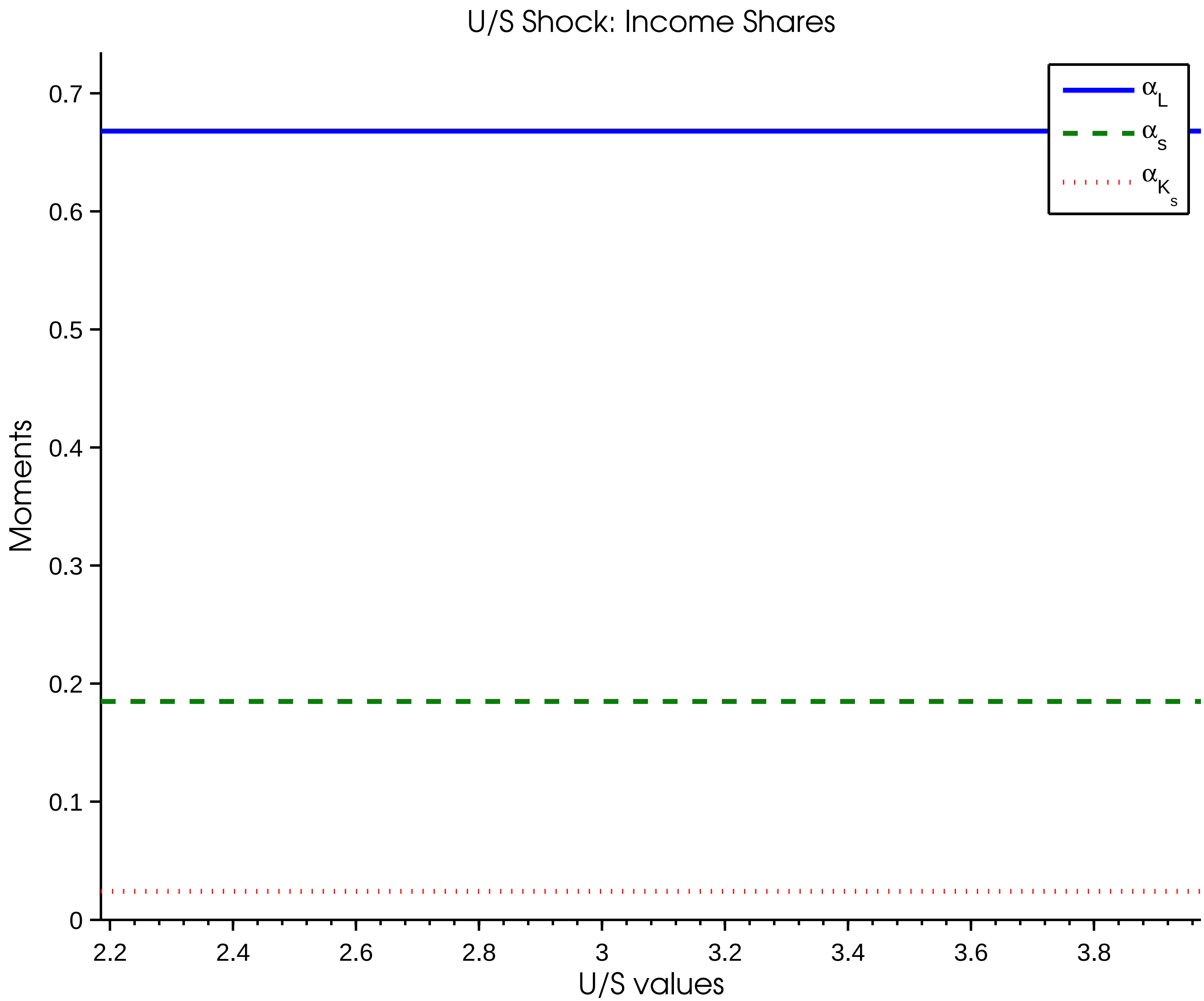

Figure 75: Cobb-Douglas Income Shares With a Change in the Stock of Skill

Notes:

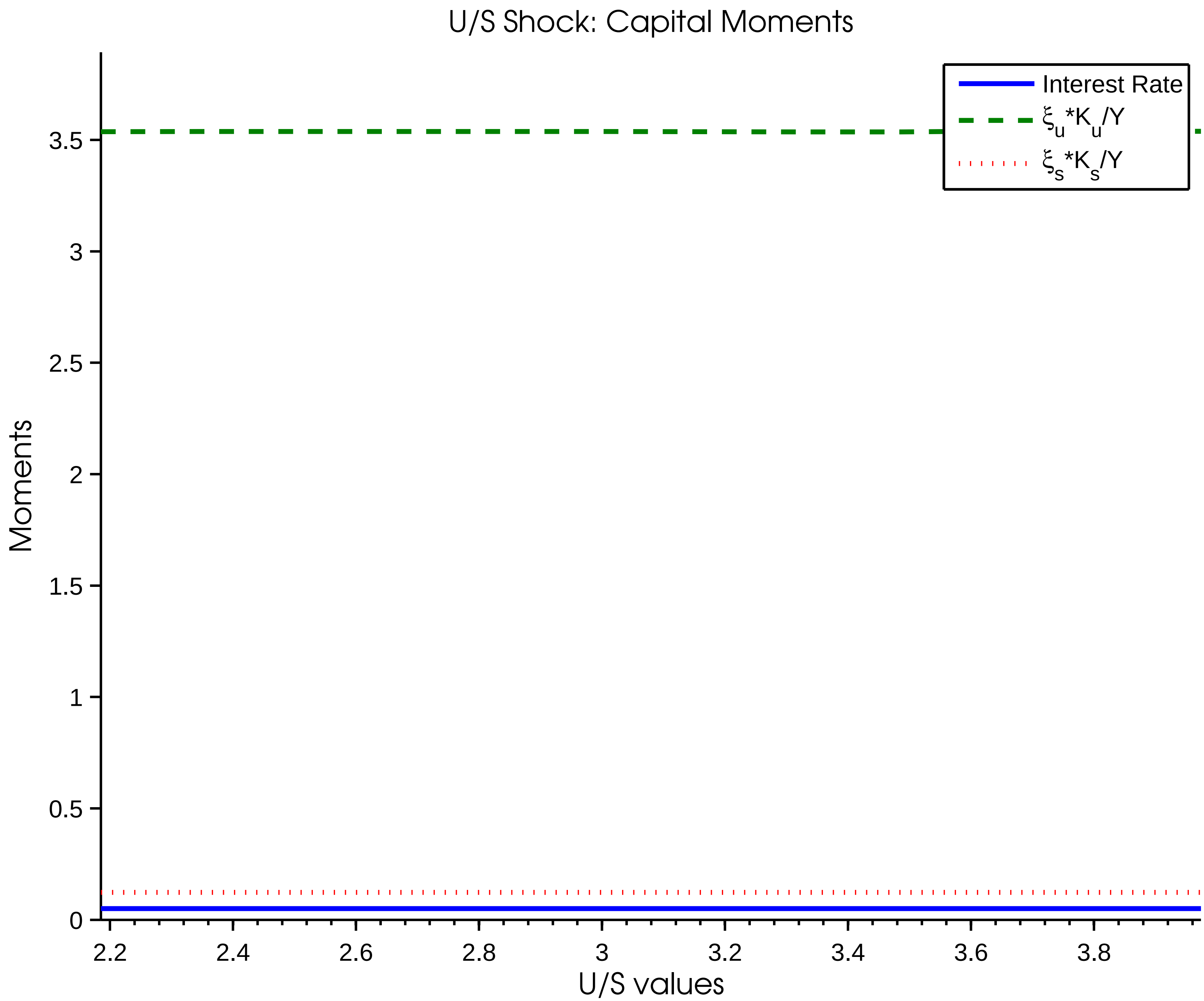

Figure 76: Cobb-Douglas Capital Moments With a Change in the Stock of Skill

Notes:

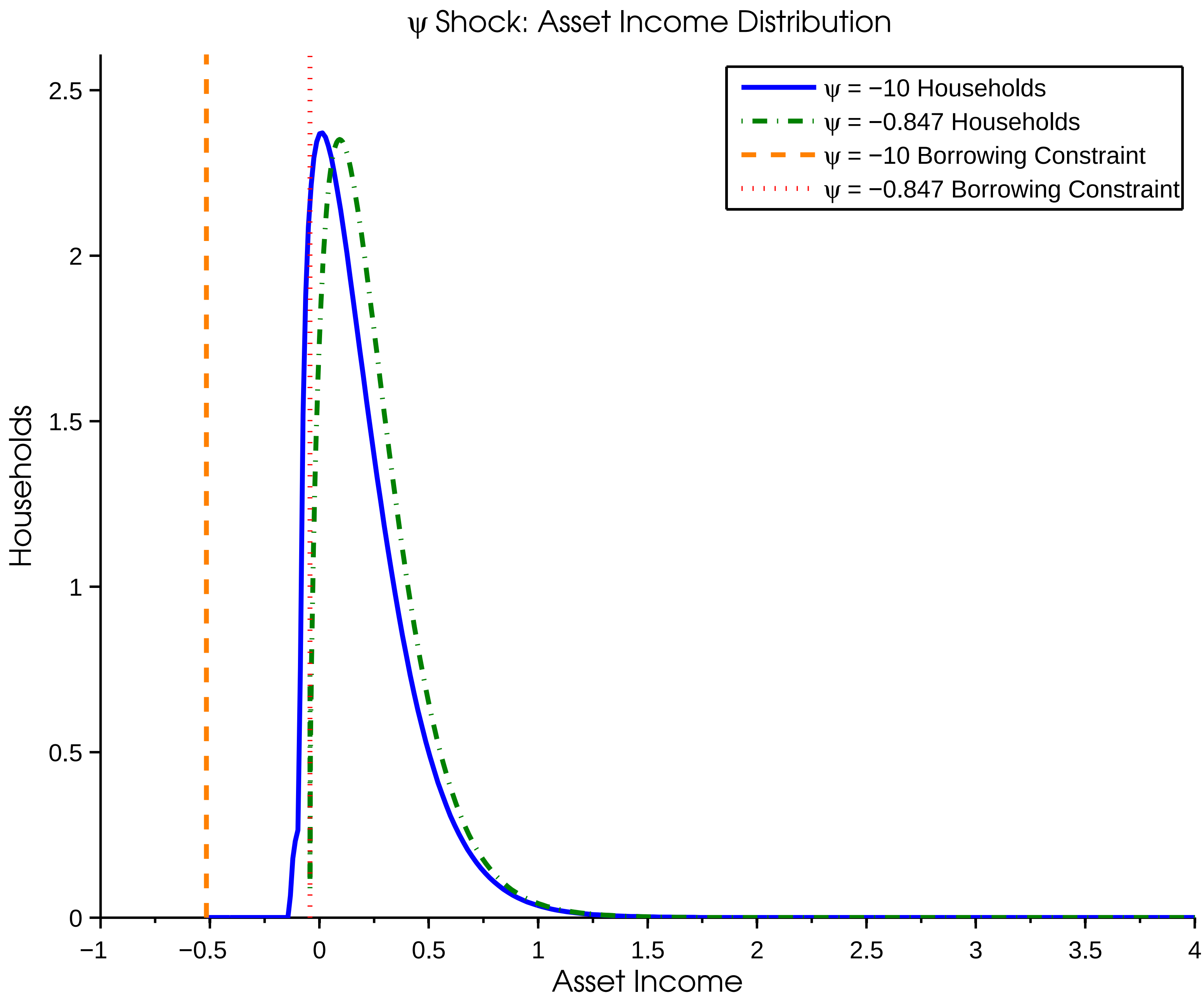

Figure 77: Cobb-Douglas Capital Income Distribution With a Change in the Borrowing Constraint

Notes:

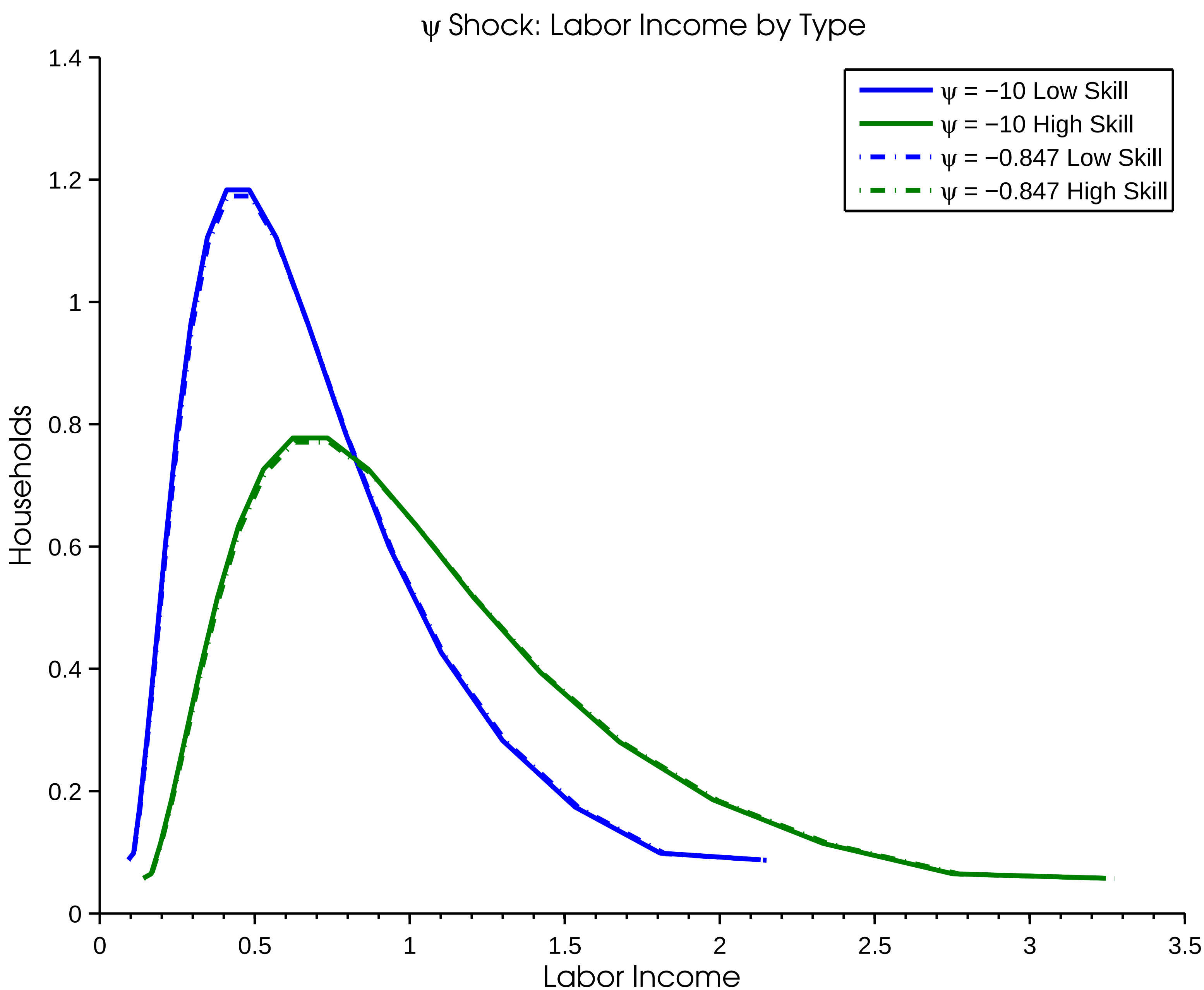

Figure 78: Cobb-Douglas Labor Income Distribution by Skill Type With a Change in the Borrowing Constraint

Notes:

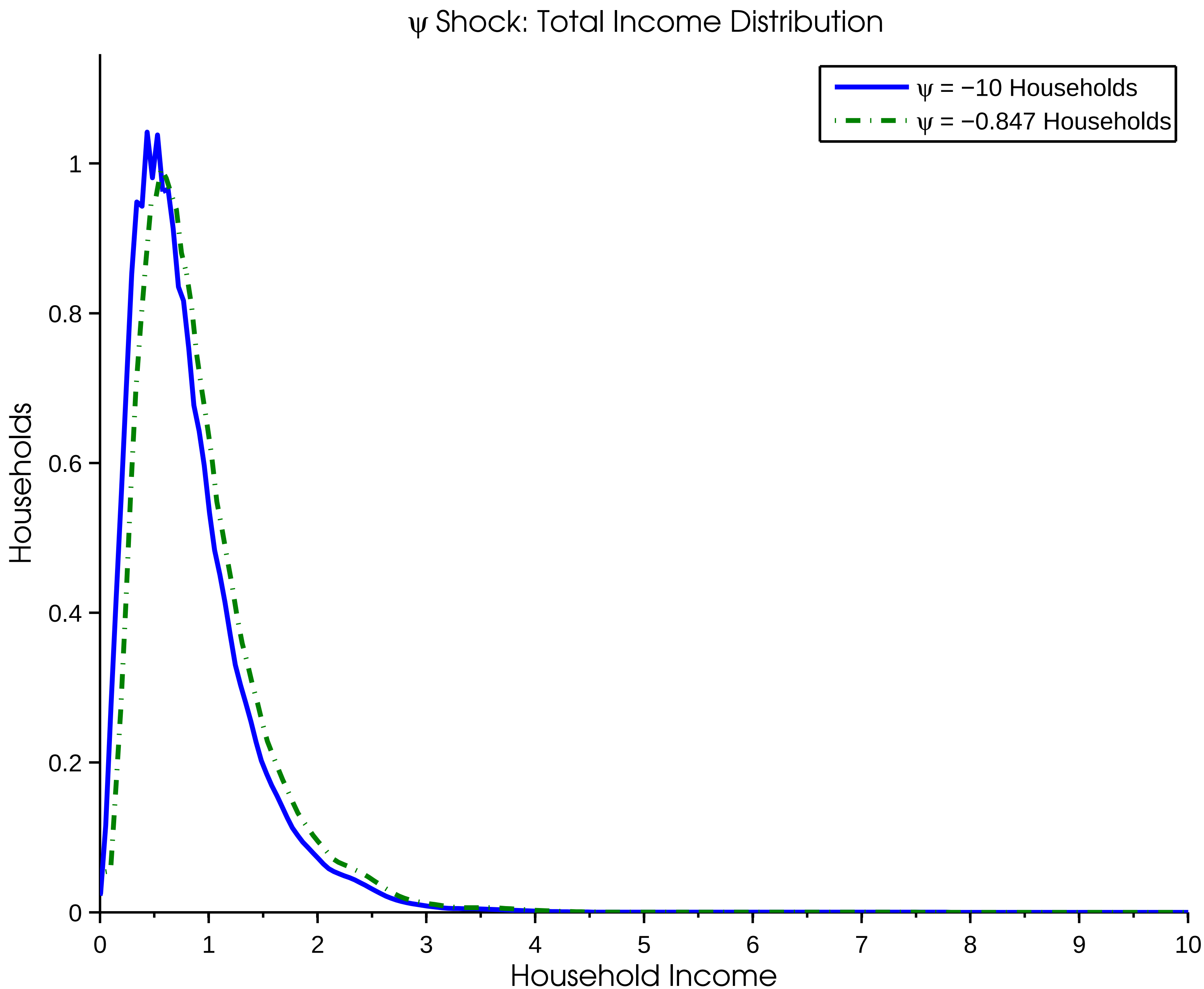

Figure 79: Cobb-Douglas Total Income Distribution With a Change in the Borrowing Constraint

Notes:

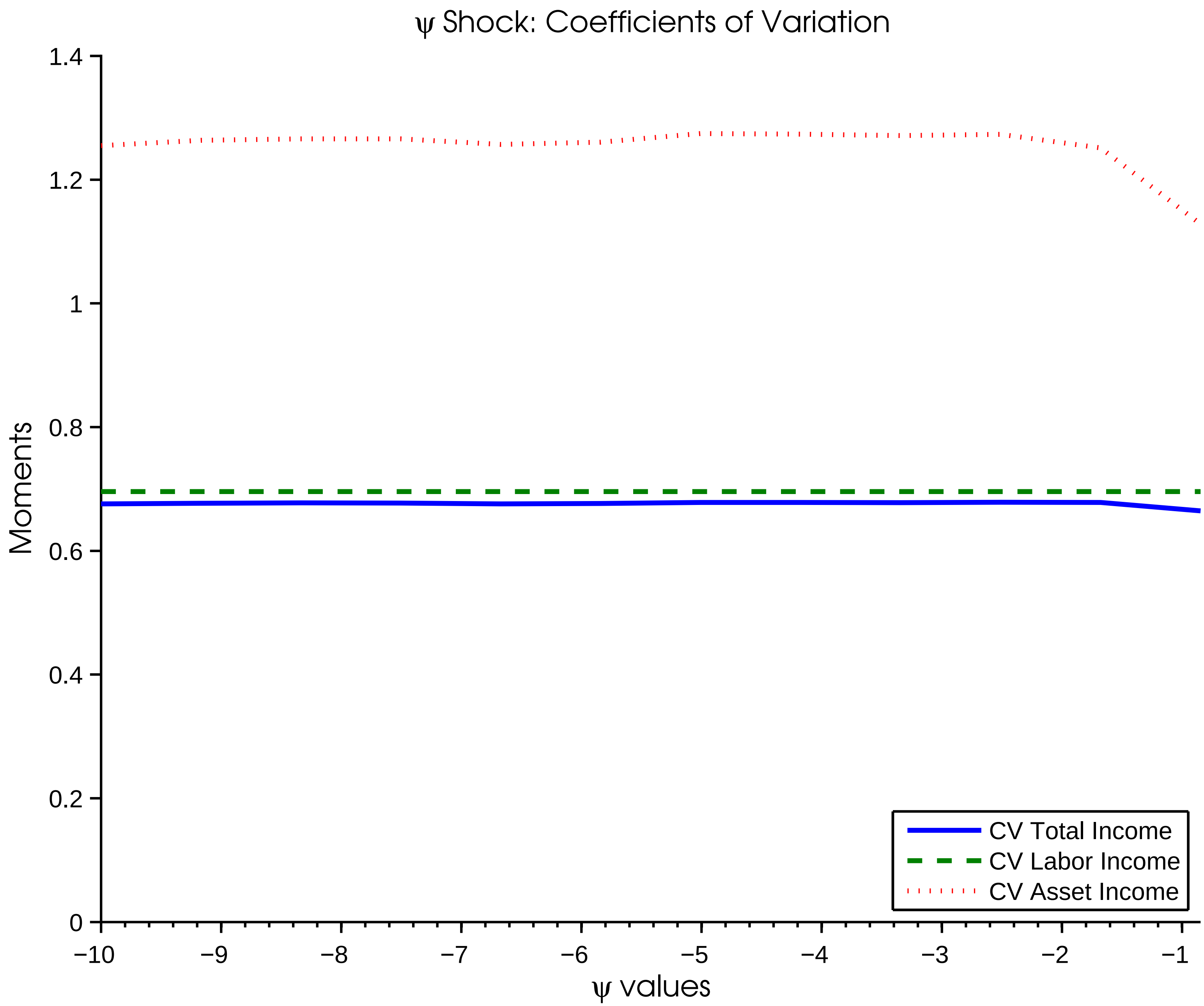

Figure 80: Cobb-Douglas Coefficients of Variation With a Change in the Borrowing Constraint

Notes:

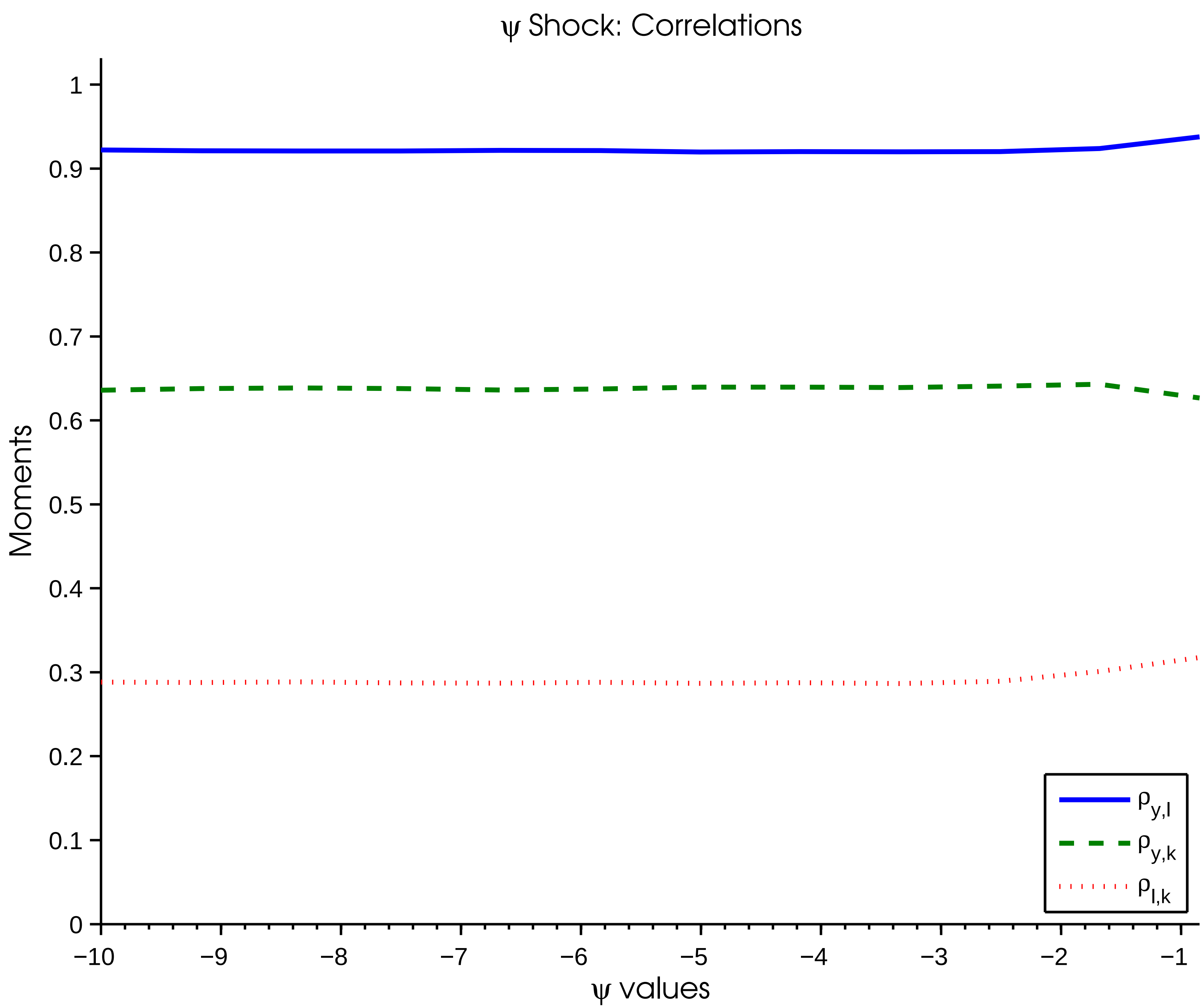

Figure 81: Cobb-Douglas Correlations With a Change in the Borrowing Constraint

Notes:

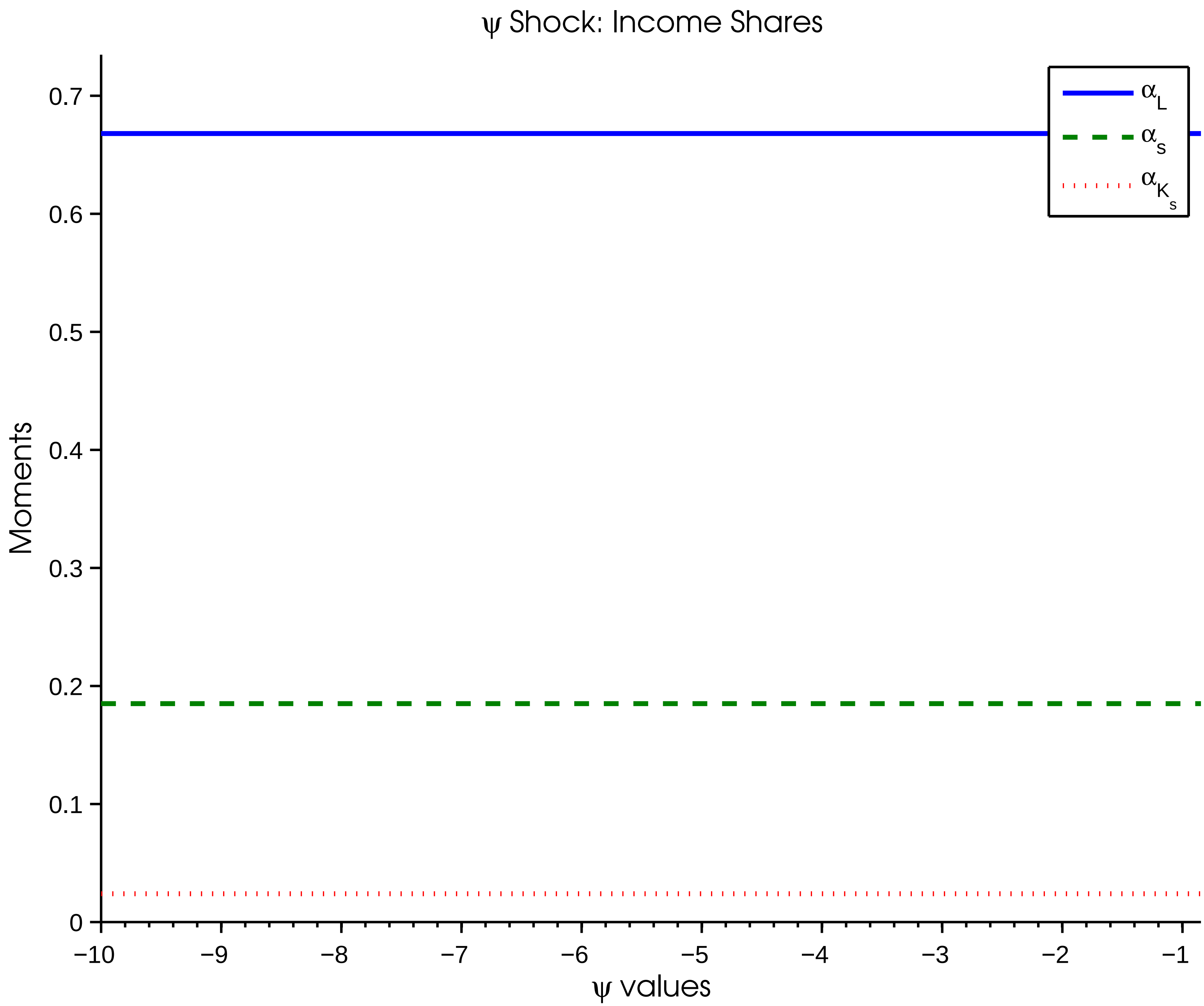

Figure 82: Cobb-Douglas Income Shares With a Change in the Borrowing Constraint

Notes:

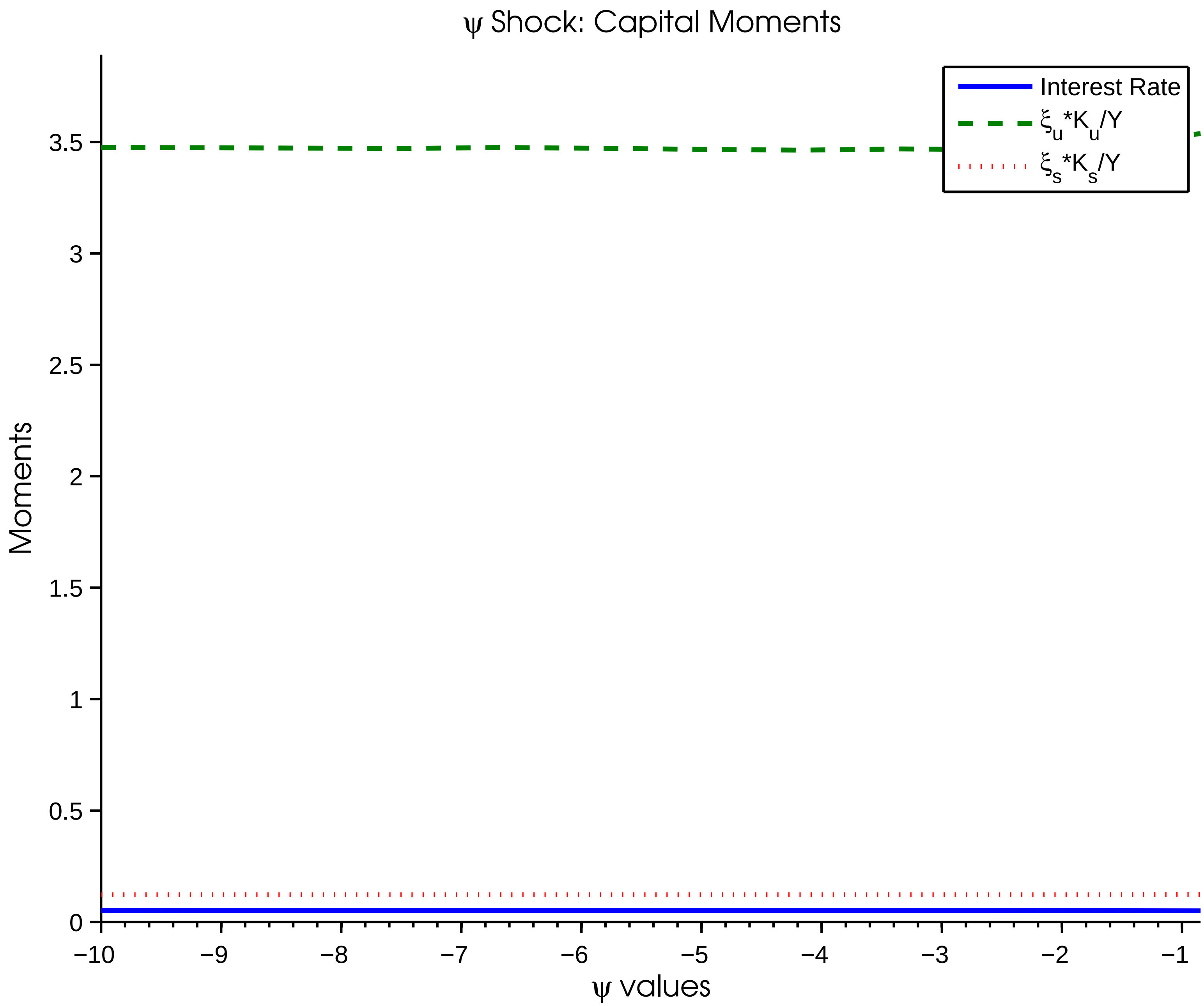

Figure 83: Cobb-Douglas Capital Moments With a Change in the Borrowing Constraint

Notes:

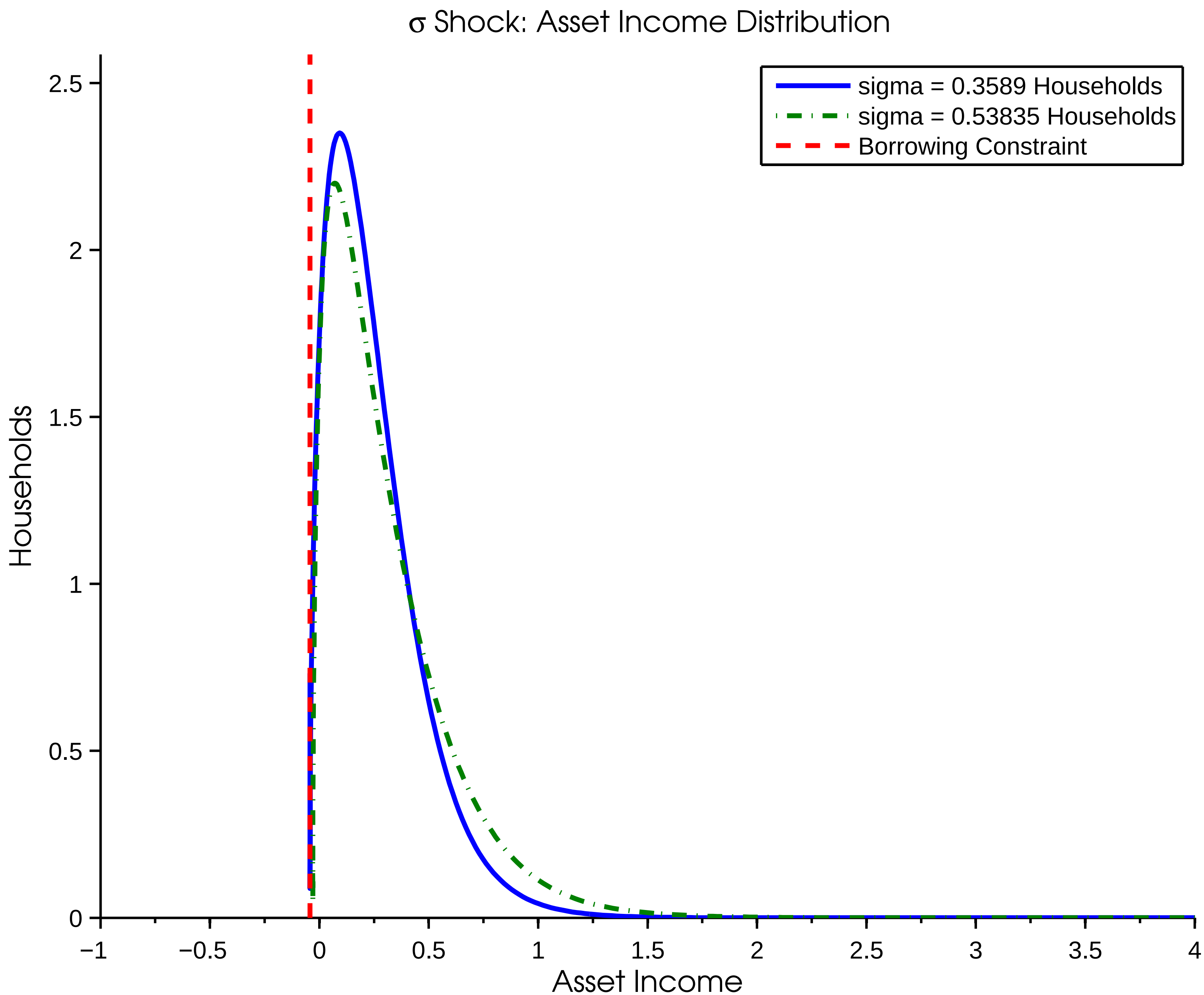

Figure 84: Cobb-Douglas Capital Income Distribution With a Change in the Labor Income Variance

Notes:

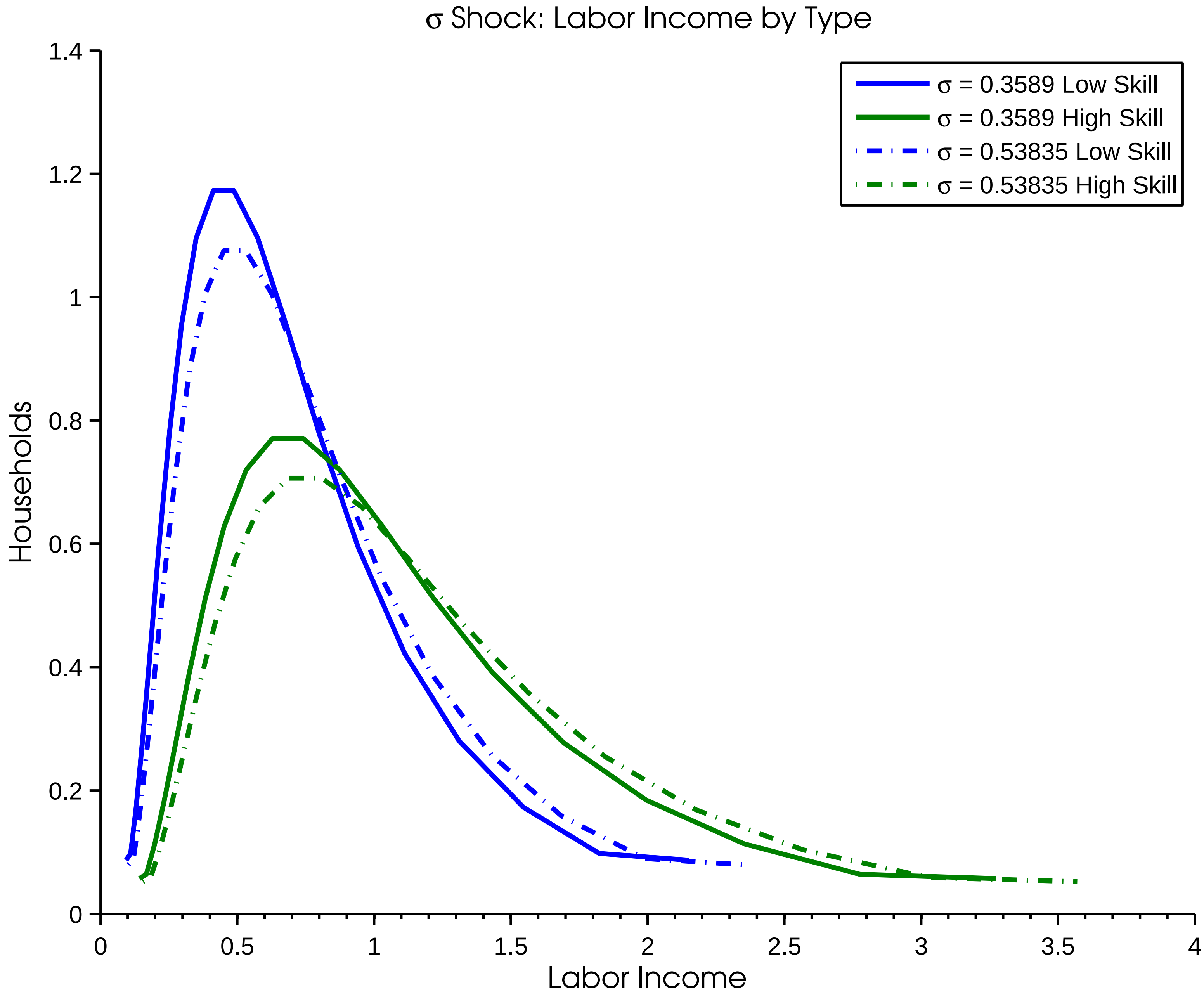

Figure 85: Cobb-Douglas Labor Income Distribution by Skill Type With a Change in the Labor Income Variance

Notes:

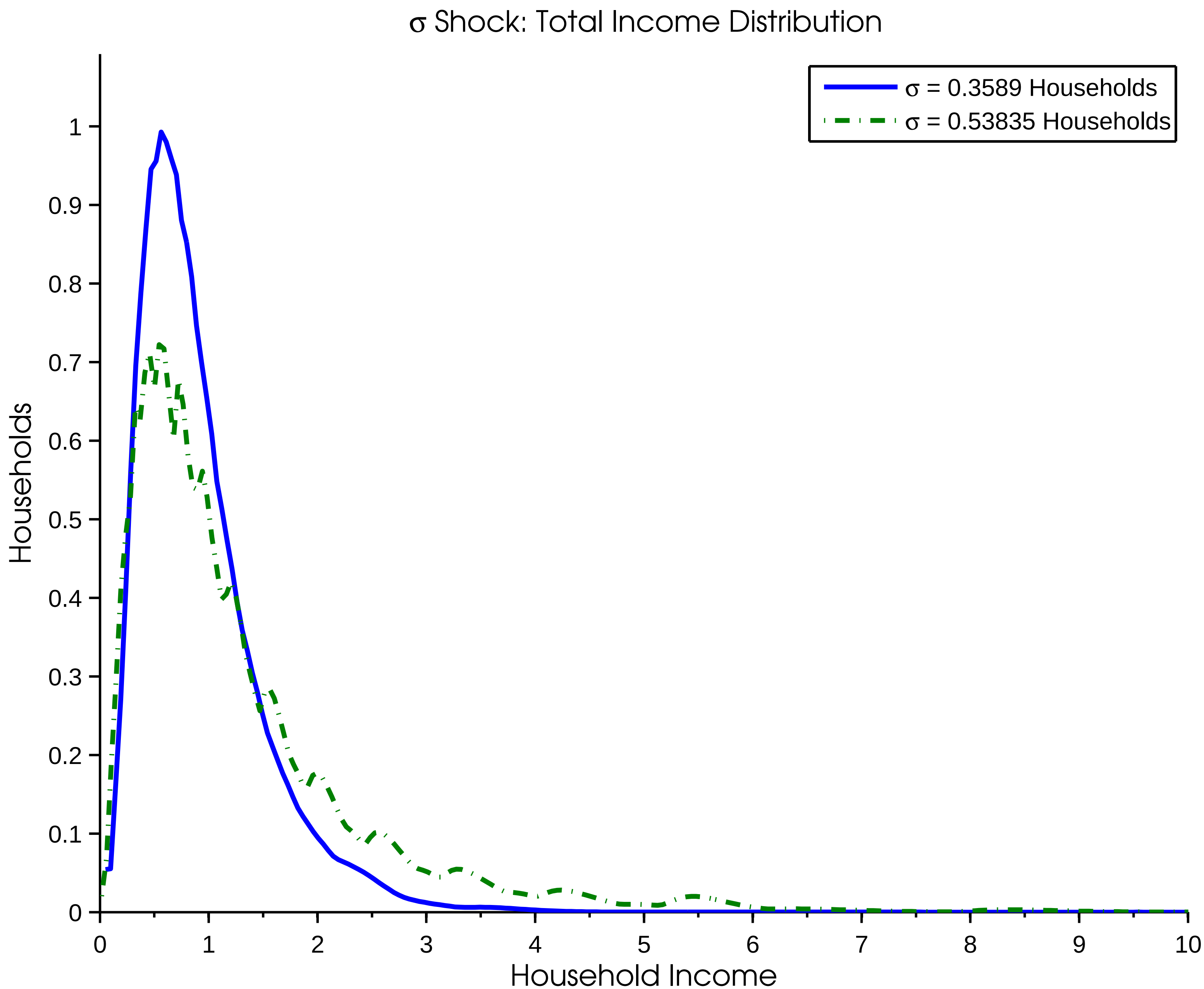

Figure 86: Cobb-Douglas Total Income Distribution With a Change in the Labor Income Variance

Notes:

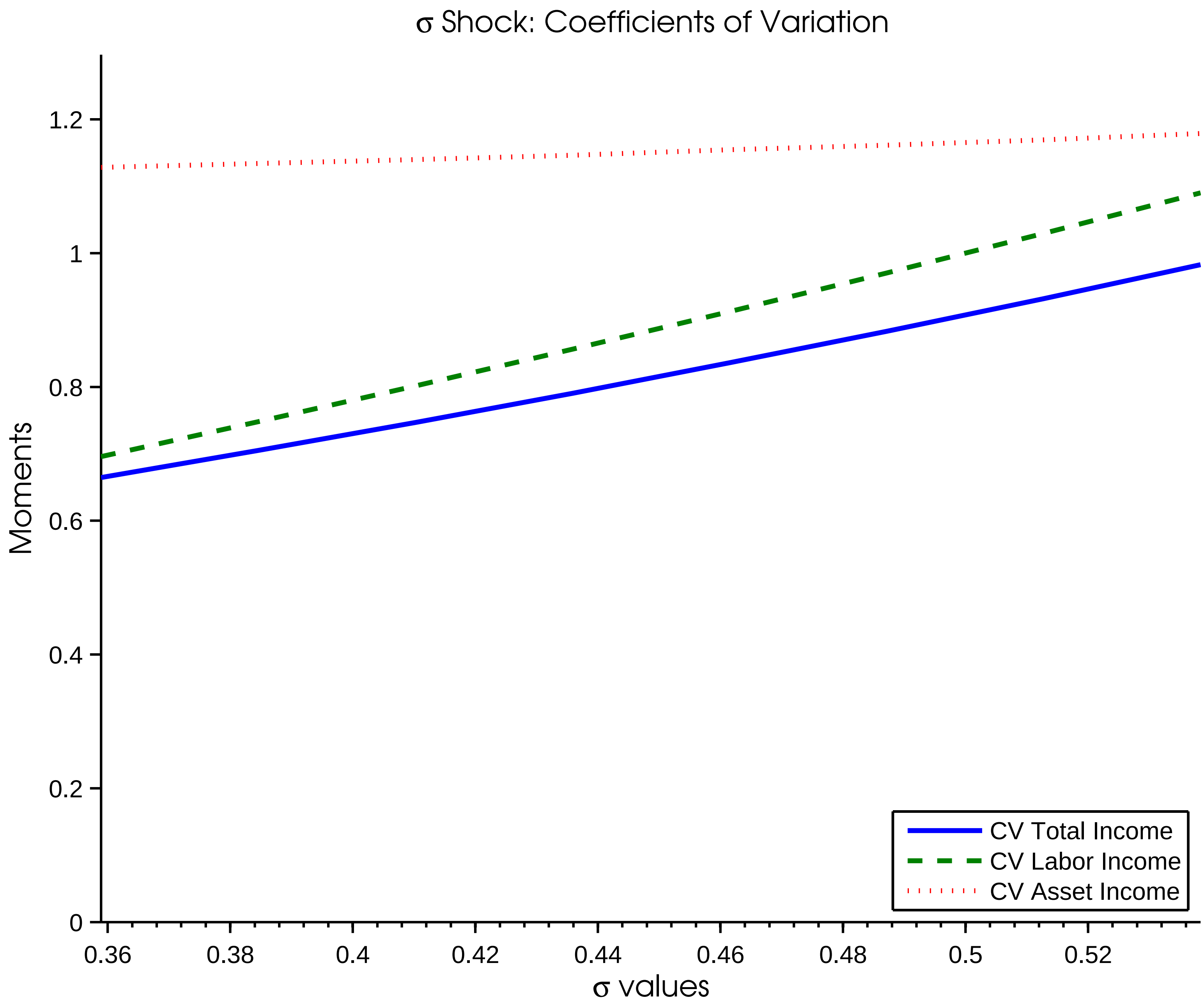

Figure 87: Cobb-Douglas Coefficients of Variation With a Change in the Labor Income Variance

Notes:

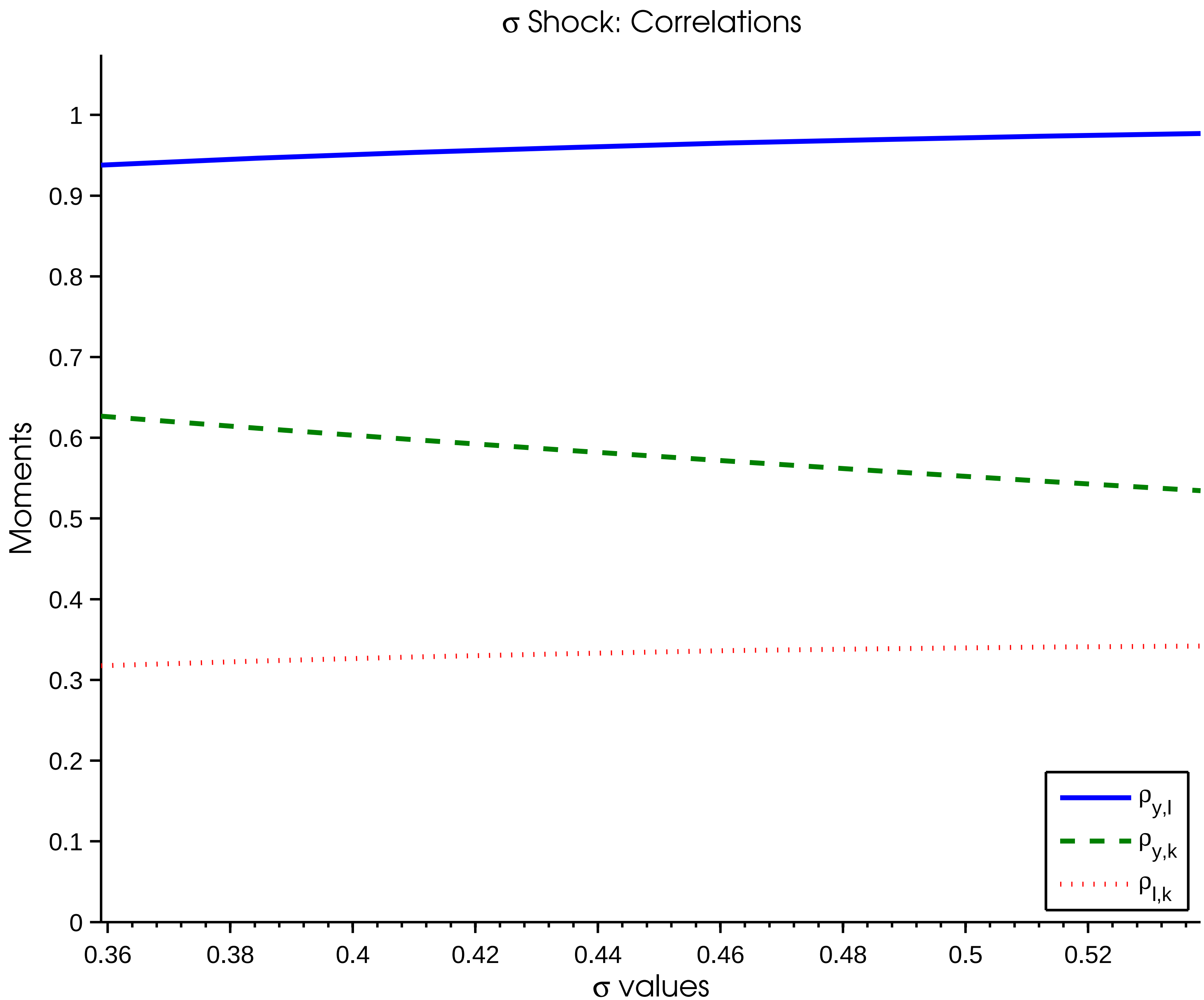

Figure 88: Cobb-Douglas Correlations With a Change in the Labor Income Variance

Notes:

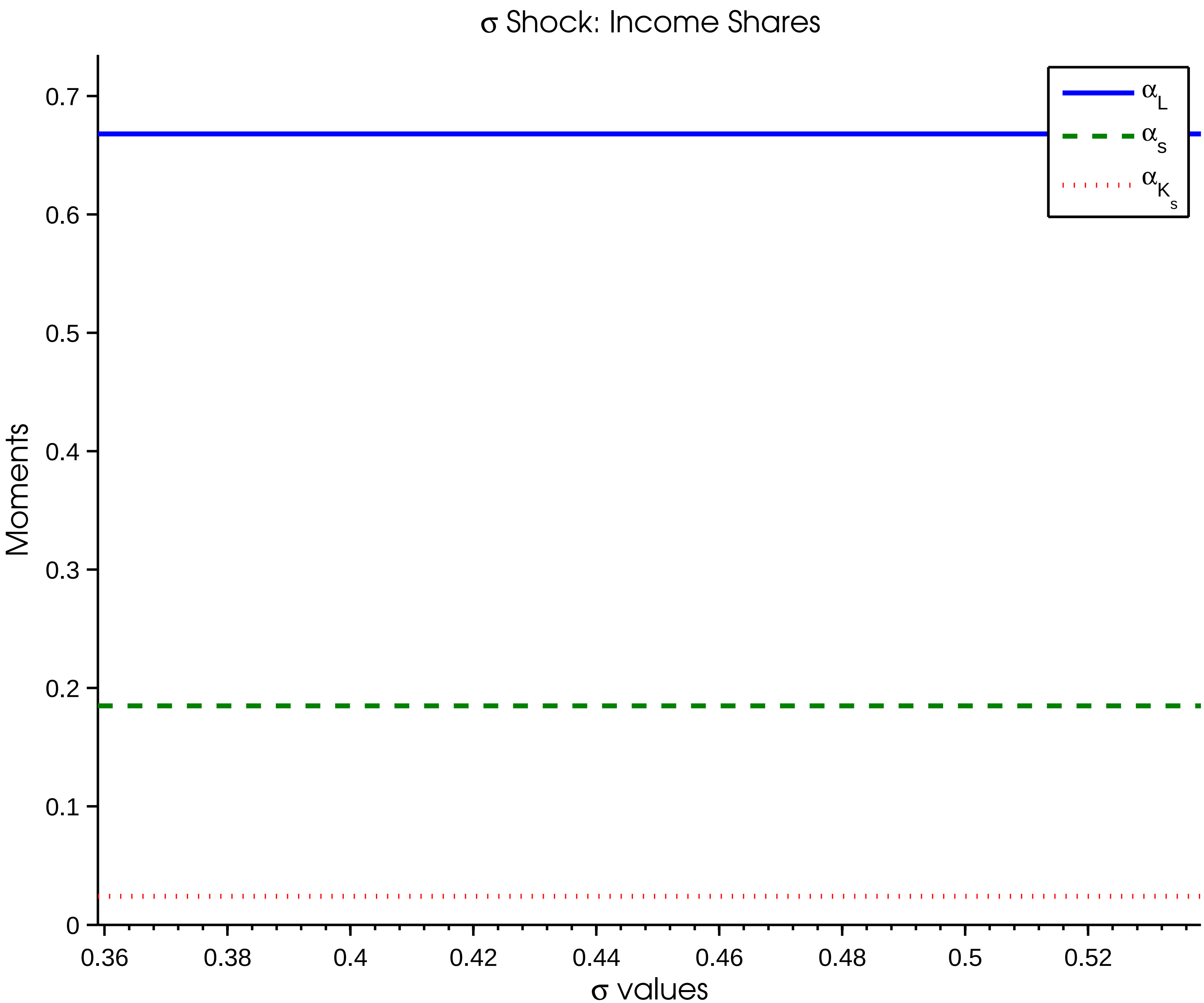

Figure 89: Cobb-Douglas Income Shares With a Change in the Labor Income Variance

Notes:

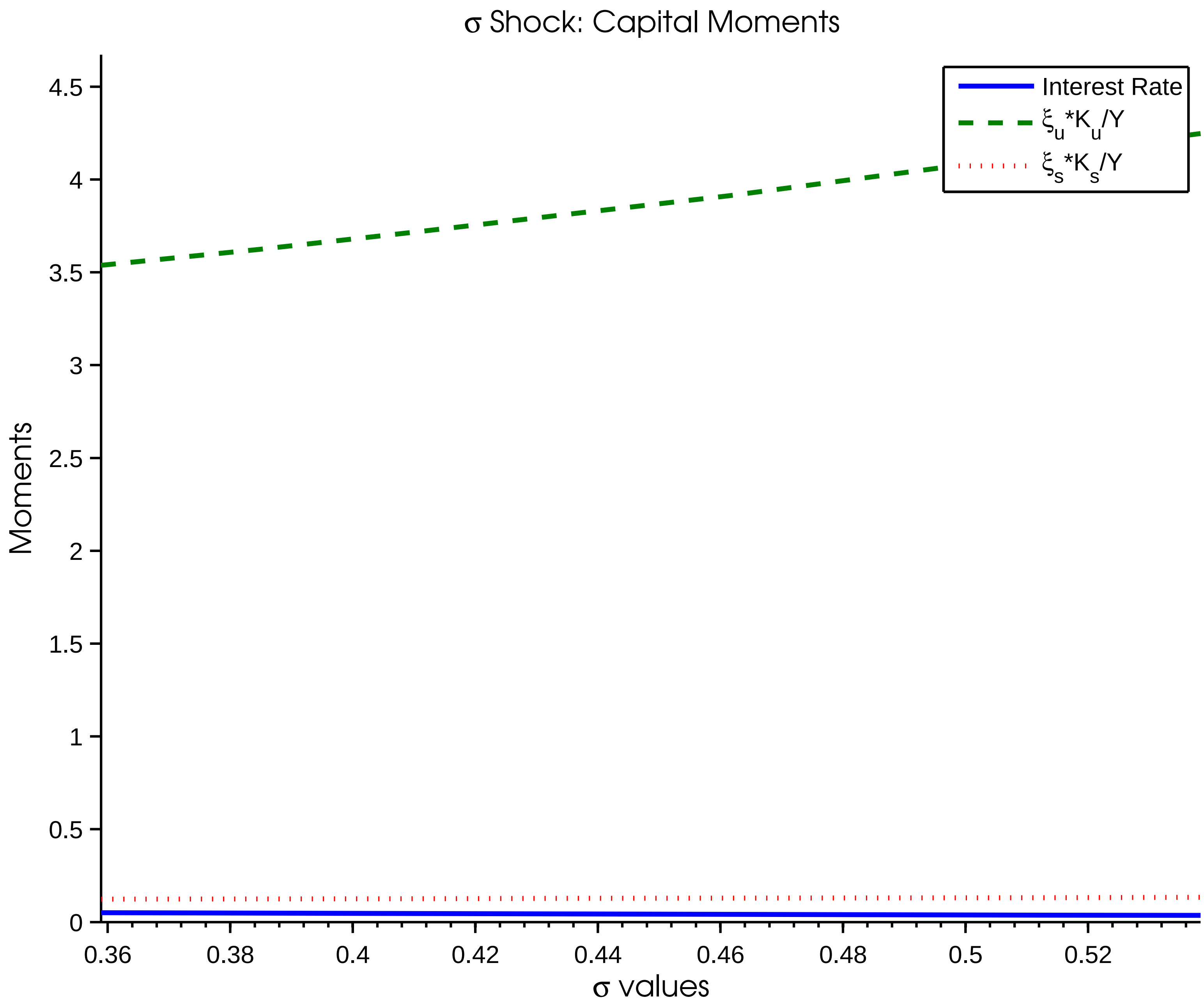

Figure 90: Cobb-Douglas Capital Moments With a Change in the Labor Income Variance

Notes:

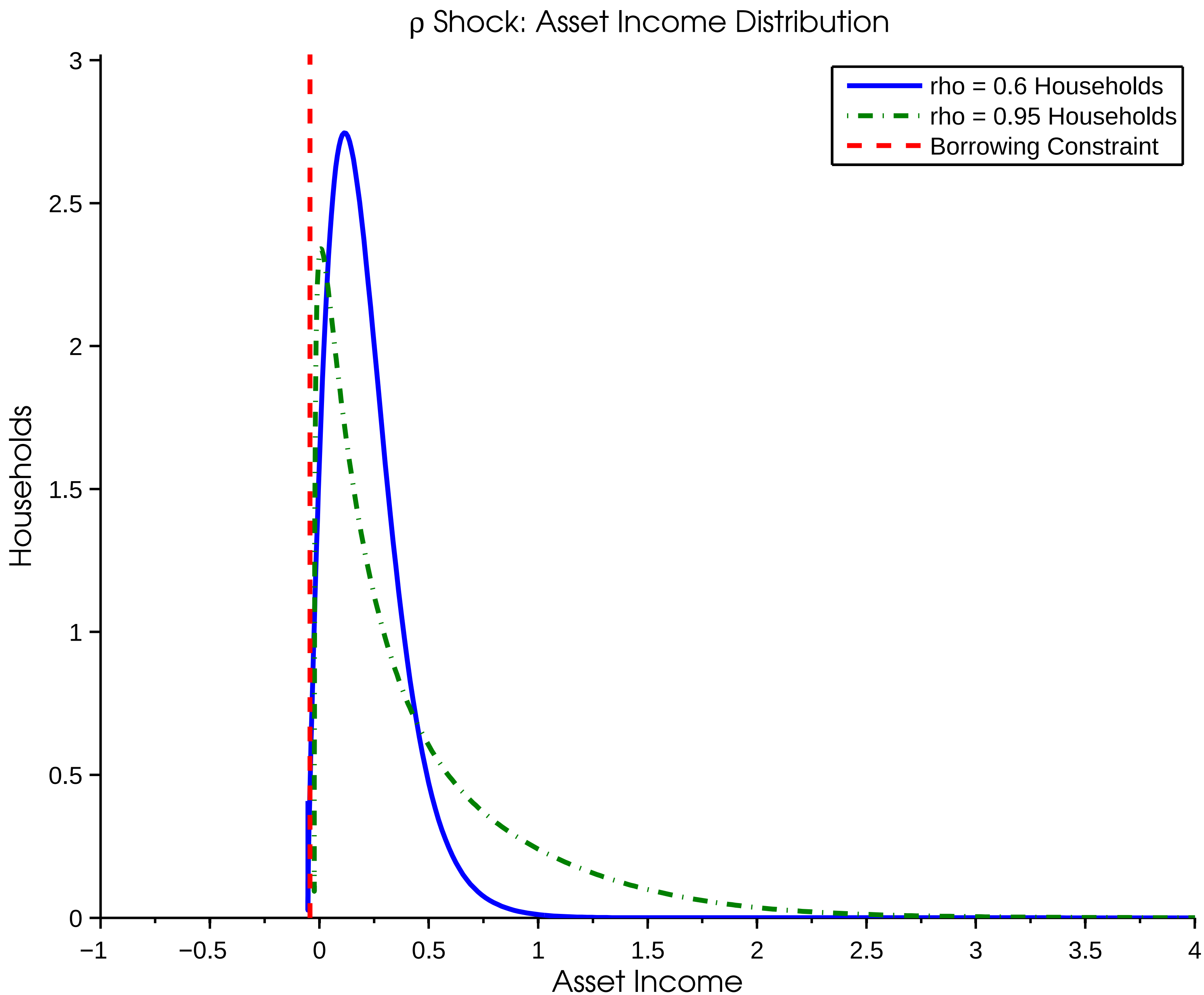

Figure 91: Cobb-Douglas Capital Income Distribution With a Change in the Labor Income Persistence

Notes:

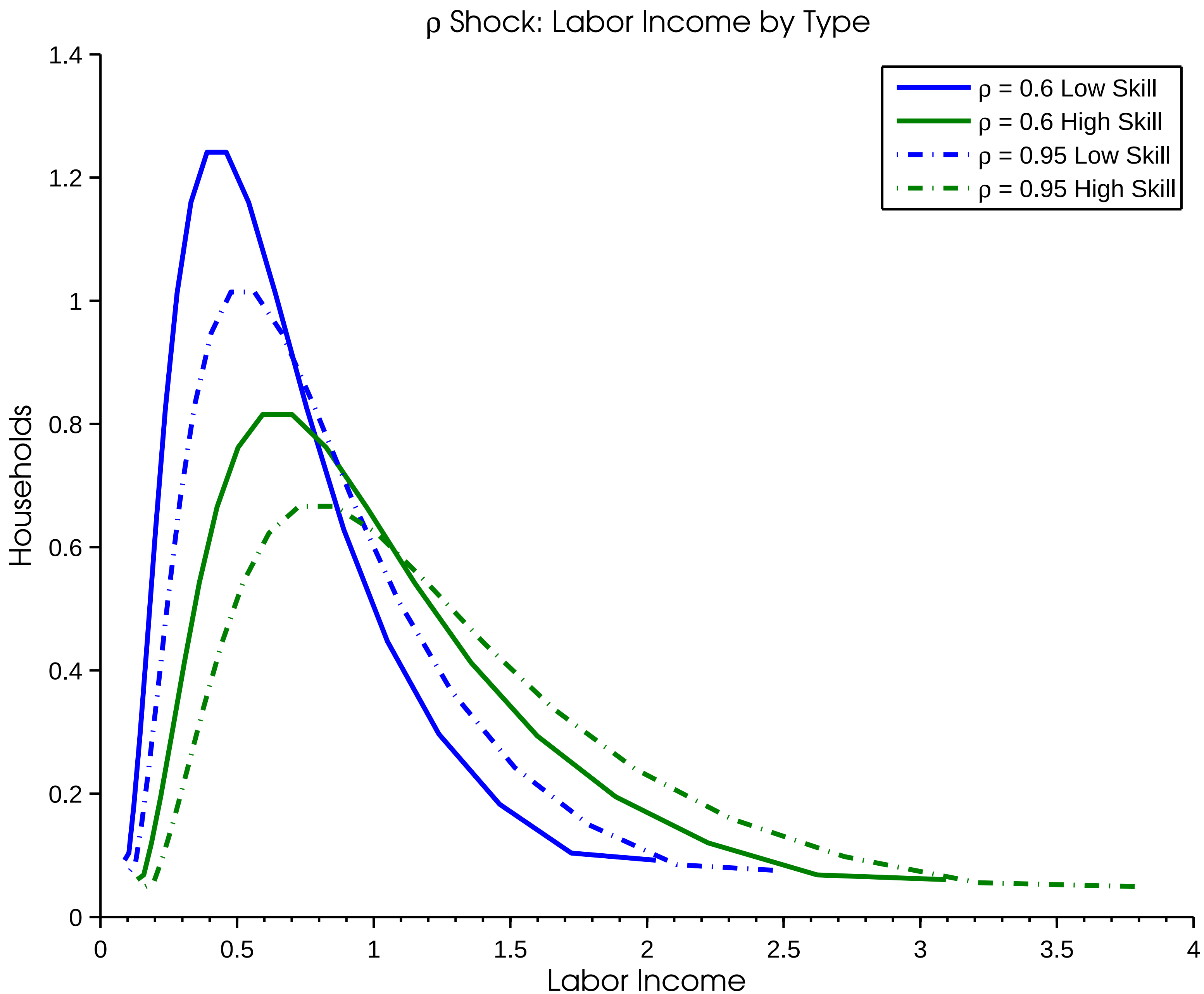

Figure 92: Cobb-Douglas Labor Income Distribution by Skill Type With a Change in the Labor Income Persistence

Notes:

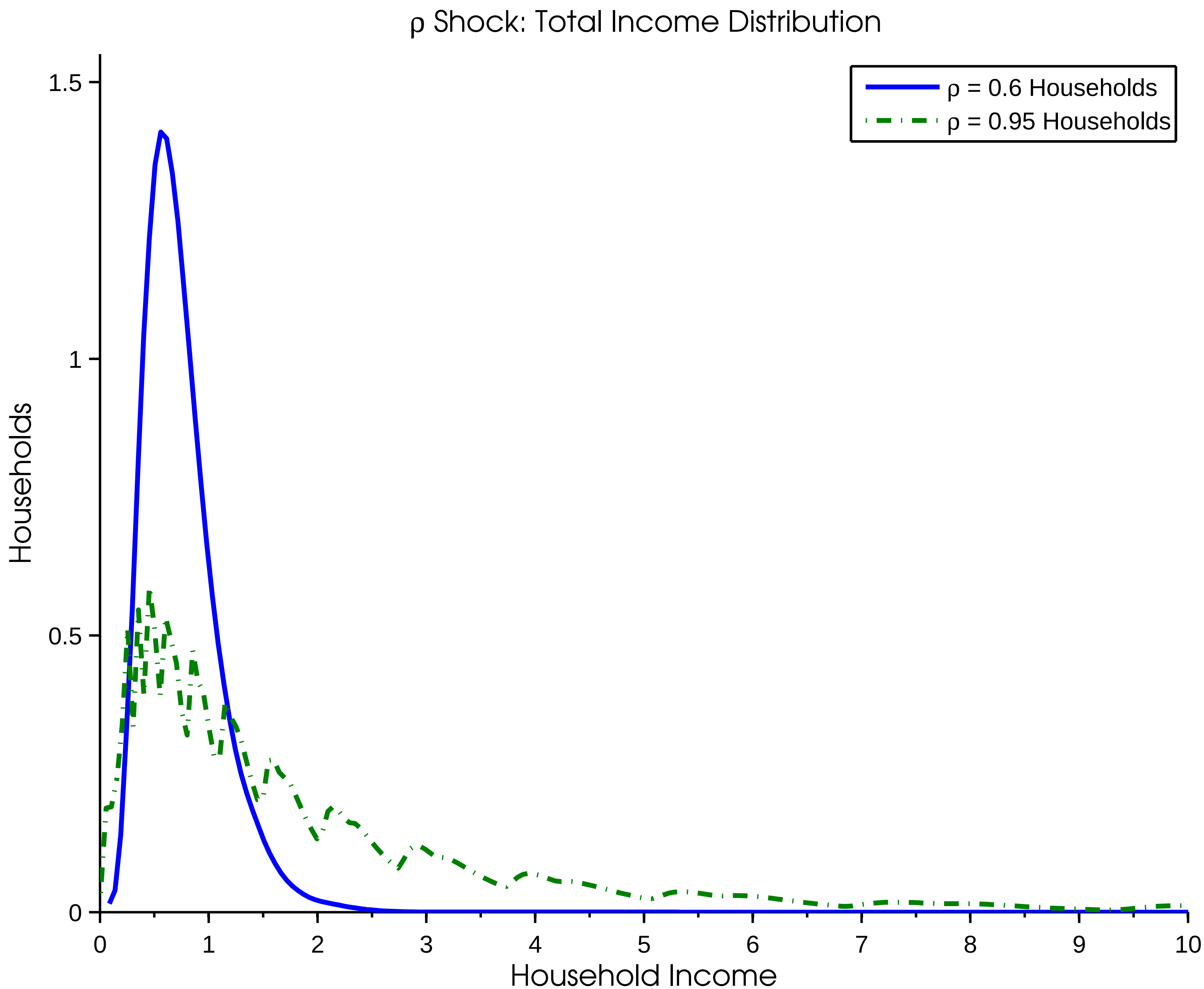

Figure 93: Cobb-Douglas Total Income Distribution With a Change in the Labor Income Persistence

Notes:

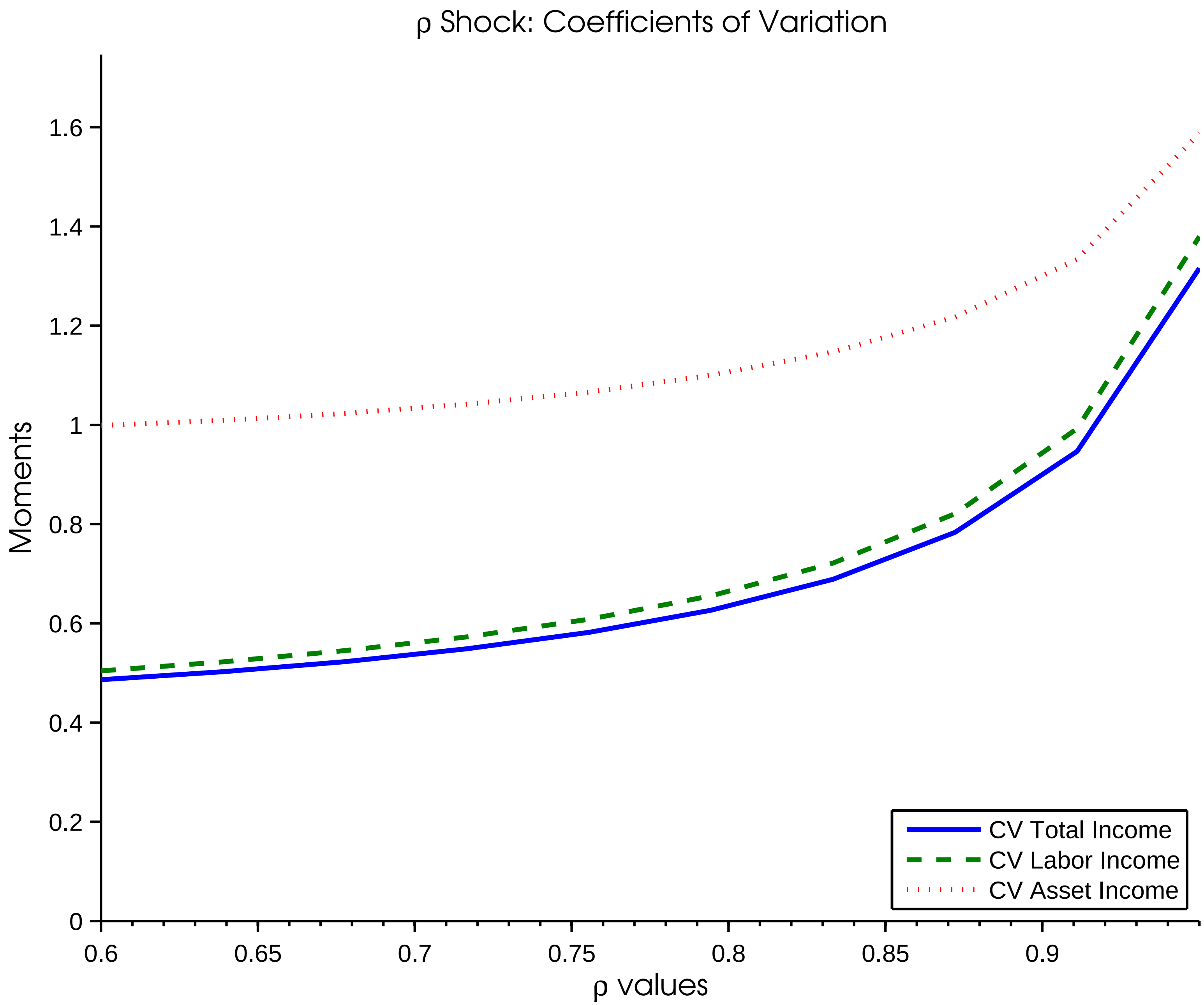

Figure 94: Cobb-Douglas Coefficients of Variation With a Change in the Labor Income Persistence

Notes:

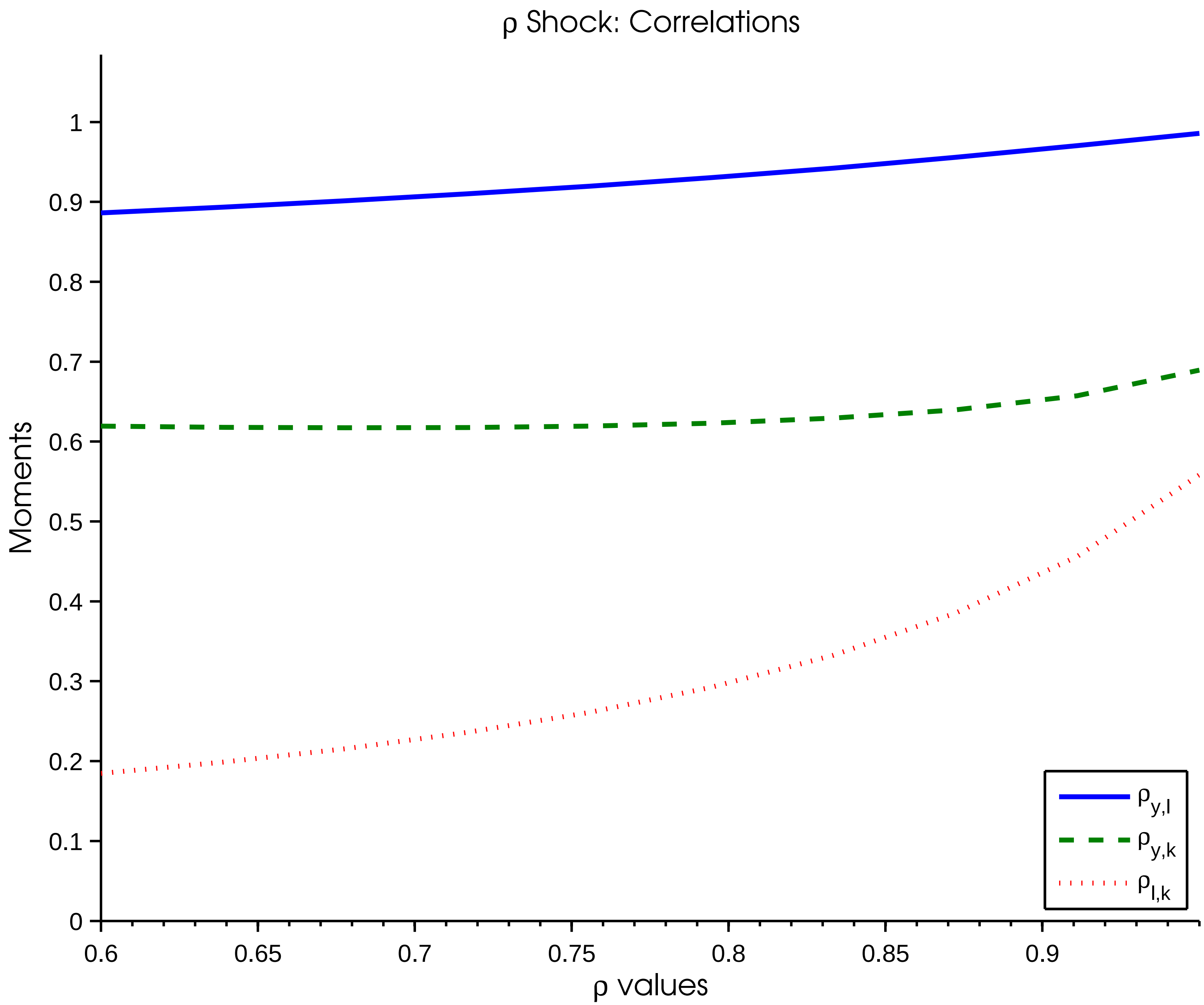

Figure 95: Cobb-Douglas Correlations With a Change in the Labor Income Persistence

Notes:

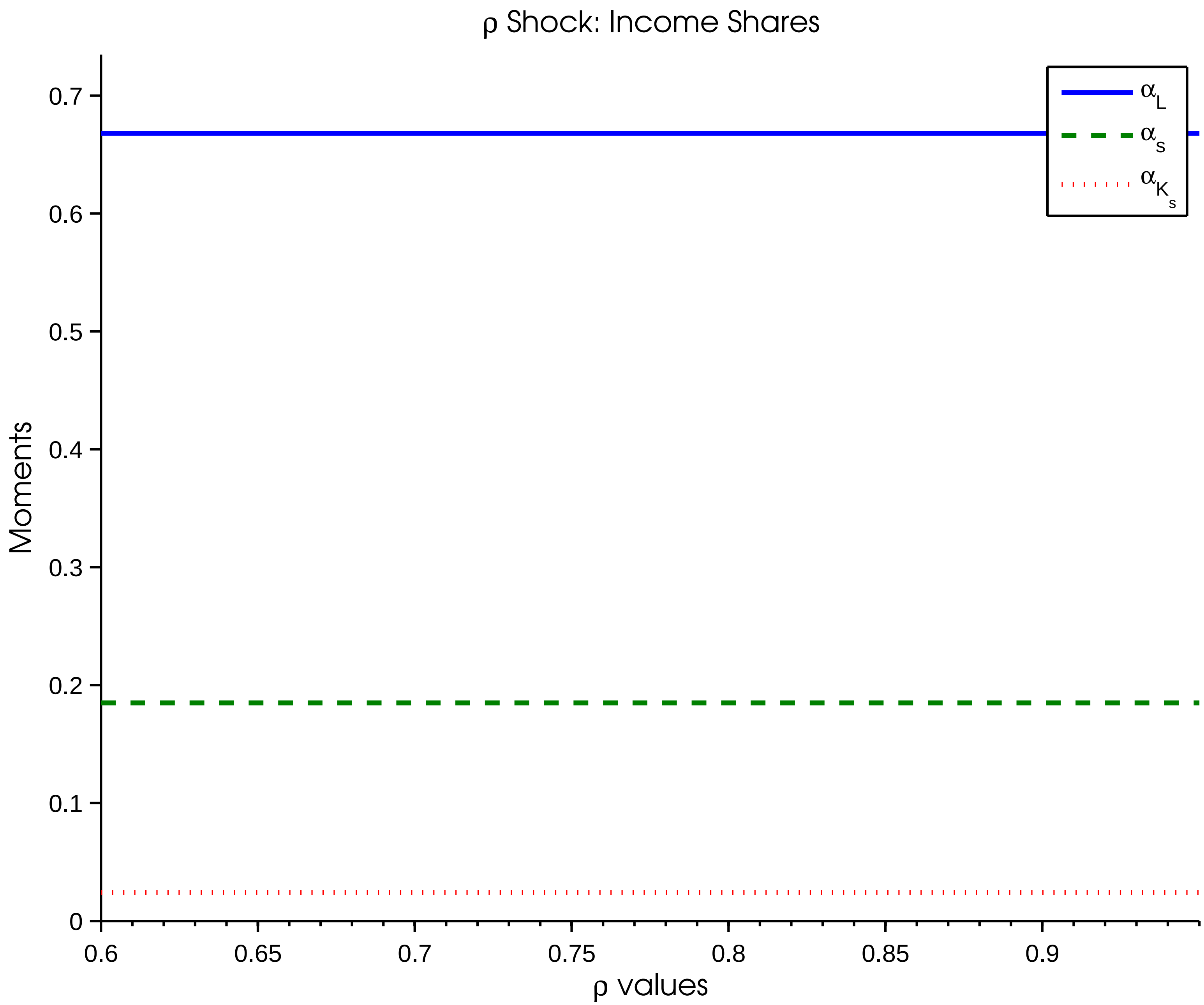

Figure 96: Cobb-Douglas Income Shares With a Change in the Labor Income Persistence

Notes:

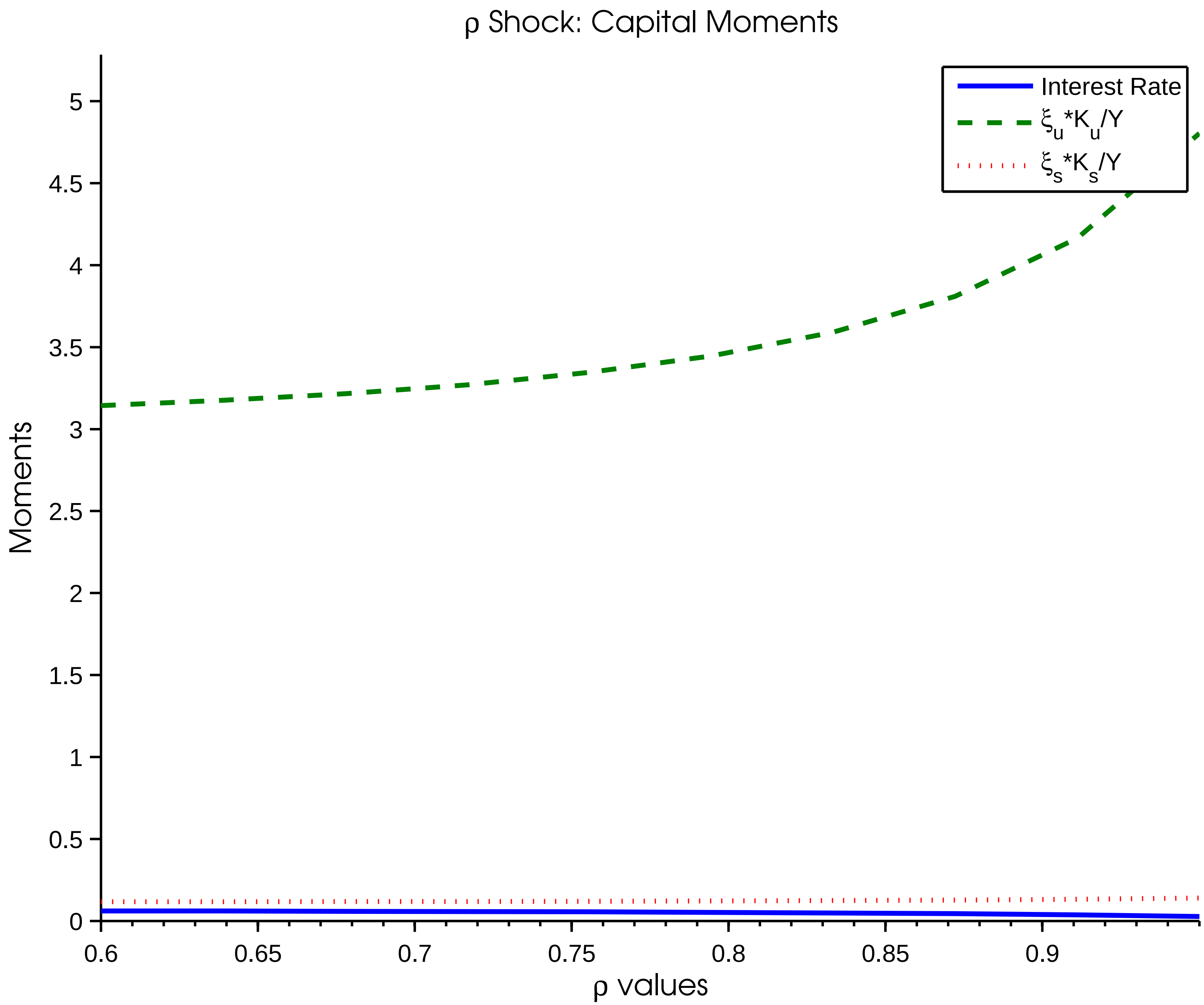

Figure 97: Cobb-Douglas Capital Moments With a Change in the Labor Income Persistence

Notes:

Labor Shares and Income Inequality

Jonathan Adams
University of Chicago

Loukas Karabarbounis
University of Chicago and NBER

Brent Neiman
University of Chicago and NBER

Online Appendix

August 2013

A Numerical Appendix

Several parameter values are chosen from estimates by other studies, and we estimate the remainder. We start by adopting a target real interest rate of $r = 0.05$, following SOURCE. Secondly, we assume the labor supply ratio of $U/S = 1.5$ [NEED TO JUSTIFY - WE DO IT TO HAVE A WAGE PREMIUM OF 1.5 GIVEN OUR SHARES. NEED SOURCE/CALCULATION FOR THAT NUMBER]. With an interest rate and labor supply known, we can solve for aggregate capital stocks and output without running the entire numerical solution.

We solve the model following the numerical approach of Aiyagari (1994), but with some improvements. An agent's state space is three-dimensional, so we must we separately discretize for their asset value a, their current productivity level z, and their skill distribution h. Assets are discretized over 500 points, with a finer grid near the borrowing constraint. The AR(1) process for productivity is approximated as a 8-state Markov chain, using Tauchen (1986)'s method. The allocation of skilled and unskilled labor is discretized into 2 values, set to match the ratio of skilled to unskilled labor in the data.

To solve for the steady state, we guess an interest rate r, which implies quantities for the capital stocks K_u and K_s such that (20) and (21) hold, and calculate the wages and transfers implied by these capital stocks, the labor supplies, the firms' first order conditions and the government's budget constraint. With the vector of prices and transfers, we calculate the household policy function $a'(a, z, h)$ using the endogenous gridpoints method of Carroll (2006), which iterates on the household's Euler equation. To calculate the stationary cumulative distribution $\Lambda(a, z, h)$, let $a(a', z, h)$ denote the inverse of the policy function, and let $\Lambda(a|z, h)_k$

denote the kth iteration of the conditional asset distribution; the distribution is updated by

$$\Lambda(a|z,h)_k = \sum_{z \in Z} \Lambda(a(a',z,h)|z',h)_{k-1}\pi^z(z|z') \tag{A.1}$$

For an asset value a' on the asset grid, its inverse $a(a',z,h)$ is not generally on the grid, so to complete each iteration the asset distribution $\lambda(a|z,h)_k$ is interpolated onto the asset grid.[4]

Finally, we calculate the aggregate wealth implied by the equilibrium asset distribution. If this is sufficiently close to the aggregate value of capital implied by the interest rate guess, we consider the model solved. Otherwise, we guess a new interest rate.

[4]Note that the policy function is not invertible at the borrowing constraint because there are many states from which a household would chose to move to the constraint. But, if we let $\bar{a}$ denote the maximum asset position from which households choose to move to the constraint (i.e. where the constraint just barely holds), and we define $a(\psi,z,h) \equiv \bar{a}$, then (A.1) still holds because $\Lambda(a|z,h)$ is a cdf.