GREAT OPPORTUNITIES OR POOR ALTERNATIVES: SELF-EMPLOYMENT, UNEMPLOYMENT AND PAID EMPLOYMENT OVER THE BUSINESS CYCLE*

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Abstract

In this paper, we study the flows between self-employment, unemployment and paid employment, and how these vary over the business cycle. First, we document these flows in the data, paying particular attention to previous labor market outcomes for workers entering self-employment, and subsequent labor market outcomes for those leaving self-employment, and how these are affected by cyclical conditions. Second, we construct a two-ladder equilibrium model of a frictional labor market capturing these flows: workers search both on and off the job, and receive business ideas while in any of the three states: self-, paid employment and unemployment. We study this model in an environment with aggregate shocks, which affect both the productivity of matches in the paid-employment sector, and the profitability of ideas for the self-employed. Third, we (plan to) calibrate to see how well it can quantitatively account for observed patterns over the business cycle. These allow us to have a notion of entry into self-employment by “opportunity” (highly profitable ideas), and “necessity” (lack of alternatives in paid employment), and how these vary over the business cycle, and to quantify “prosperity pull” of self-employment in good times, and “recession push” in bad times. Finally, we plan to study the impact of labor market policies on self-employment, and on unemployment, taking into account the option to enter self-employment.

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1 Introduction

In this paper, we study the flows between self-employment, unemployment and paid employment, and how these vary over the business cycle. First, we document these flows in the US Survey of Income and Program Participation, paying particular attention to previous labor market outcomes for workers entering self-employment, and subsequent labor market outcomes for those leaving self-employment, and how these are affected by cyclical conditions. These flows are economically important, and vary over the business cycle. We also look at the (cyclical) earnings patterns of the (starting and ending) self-employment spells, relative to paid employment. We also look at the (cyclical) earnings patterns of the (starting and ending) self-employment spells, relative to paid employment. The SIPP data allows us to follow workers who became self-employed after unemployment up to a few years, and hence we can document how self-employed outcomes vary with the origin of the worker when entering self-employment. This gives us an initial sense of how important selection for those becoming self-employment is, and how important the selection of (expected) profitability of self-employed business is.

To get a more formal handle on this issue (and related issues), we construct a model of a frictional labor market capturing these flows. Here, on one hand, workers receive ‘business ideas’ for self-employment, and on the other hand, (regular) firms post vacancies to attract workers into paid employment. Workers search both on and off the job, and receive business ideas while in any of the three states: self-, paid employment and unemployment. The model thus consists of unemployment, a job ladder in employment, and an ‘idea ladder’ in self-employment, and workers flow to and between ladders, and fall off their ladder into unemployment. We study this model in an environment with aggregate shocks, which affect both the productivity of matches in the paid-employment sector, and the profitability of ideas for the self-employed. The model is “block recursive”, which renders the model tractable, even over the business cycle.\(^1\)

Third, we plan to calibrate to see how well it can quantitatively account for observed patterns over the business cycle. These allow us to have a notion of entry into self-employment by “opportunity” (highly profitable ideas), and “necessity” (lack of alternatives in paid employment). This will allow us to quantify the “prosperity pull” of self-employment in good times, and “recession push” in bad times, issues that have been put forward by the empirical literature, using our equilibrium labor market model. Finally, we plan to study the impact of labor market policies on self-employment, and on unemployment, taking into account the option to enter self-employment.

\(^1\)Concretely: we are able to extend the block-recursivity established in Menzio and Shi for the standard on-the-job search model, to the setting with a second, ‘business-’ ladder in self-employment.
1.1 Related literature

[to be expanded]

We build a three-state equilibrium labor market model that is (to our knowledge) most closely related to Rissman (2007), Millán (2012); Narita (2011); Meghir, Narita and Robin (2013); Bradley, Postel-Vinay and Turon (2013); and Poschke (2012). In particular, we build on the two-ladder steady-state setting of Rissman and Millán, which allows for the flows between paid employment, self-employment and unemployment, and firm-to-firm flows within paid employment, which are all relevant and significant for explaining outcomes in the labor market, as will be discussed below. Meghir, Narita and Robin (2013) and Bradley, Postel-Vinay and Turon (2013) use a steady-state two-ladder setup to capture paid employment in the government and private sector, resp. the formal and informal sector. Narita (2011) considers job-to-job transitions in the informal and formal sector, and transitions into (homogenous) self-employment over the life-cycle, and estimates her model on Brazilian data. None of these papers consider business cycle patterns. More generally related, Poschke (2012) considers the choice to become self-employed in a model of occupational choice where –in order to gain paid employment– workers and firms have to overcome search frictions; he derives interesting predictions with respect to the firm size distributions, but lets the occupational choice only be affected by the persistent quality of the worker, and whether the worker is unemployed or not. Albrecht, Navarro, and Vroman (2009) model self-employment as an invariant outside option in an equilibrium search model of the labor market.

Modeling instead an explicit job ladder (and next to it, a business idea ladder) and allowing flows between all labor market states, captures that who start self-employment differ enormously in their previous labor market outcomes. Some enter self-employment because their best alternative is bad, e.g. unemployment; some enter, because there alternative is good, but self-employment is even better. Importantly, the model allows us to get a handle on how the composition of entrants along this dimension shifts over the business cycle.

This means that the model directly speaks to the relation between macroeconomic variables (such as unemployment or economic growth rates) and self-employment, an issue which traditionally has been a source of some controversy. On one side, the prosperity-pull hypothesis suggests that individuals consider entry into self-employment at times when the overall economic climate

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2Finkelstein Shapiro (2013) considers the choice of unemployed to become self-employed in a labor market equilibrium model that incorporates the business cycle, but abstracts from paid-employment to self-employment flows (and the reverse). A main and interesting point of his paper is that the option of self-employment affects the value of unemployment, and hence also the affects the salaried labor demanded by firms. This leads the economy to responds differently than in the absence of self-employment. However, he abstracts from the job ladder, which gives paid-employed workers different outside options when they are considering self-employment, and also abstracts from the flow from self-employment to paid employment, which is relatively large, still about 40% of the firm-to-firm transition rate for paid employed.
is favorable. A range of individual-level studies of occupational choice show that transitions into entrepreneurship among individuals are on average higher in good economic times and lower in bad ones. For example, Audretsch and Acs (1994) show that macroeconomic fluctuations as well as industry-specific elements contribute to startup activity.\(^3\) Koellinger and Thurik (2012) find that the self-employment rate Granger-causes GDP in the U.S. and in the OECD as a whole, which they interpret as evidence for a strong version of the pull theory: entrepreneurs spot good business opportunities from new technologies, which then leads to an expansion. Related but subtly different, Thurik et al. (2008) in a VAR of unemployment and SE in 23 OECD countries find that evidence for both a pull effect (high SE helps to reduce unemployment in the future) and a push effect (unemployment causes higher entry into SE). Their estimates say that the pull effect is stronger, and that it is important allow for time lags of up to 8 years.

The \textit{recession-push} theory builds on the idea that unemployment reduces the opportunities of gaining paid-employment and the expected gains from job search, which \textit{pushes} people into self-employment. In agreement with this, several studies have linked being unemployed with subsequent transitions to self-employment at the individual level. For example, Alba-Ramirez (1994) find that the duration of unemployment increases the probability of becoming self-employed. Evans and Leighton (1989) do not discuss the impact of the business cycle on transition into self-employment, but their numbers (using the NLSY, see their table 4) tell us that transitions from paid-employment to self-employment fell sharply in the 1980-81 recession. Also, their estimates (table 4) implies that the pool of these switchers got tilted more towards workers with recent unemployment spells in the recession, indicating that the motives for the transition changed. At the aggregate level, there is also evidence of a recession-push effect from unemployment to self-employment. (Van Stel et al., 2011).

In our paper we do not restrict ourselves to either recession-push or prosperity-pull channel by itself, and instead allow both to operate. The model capture that the composition of the flows into SE over the business cycle can vary, with individuals with different labor market outcomes becoming self-employed in a boom, vs. in a recession. This will help not only to shed light on the push and pull effects and their relative strength, but also have more implications for subsequent outcomes, both on the level of the individual worker and in the aggregate. Those workers who become self-employed because their alternatives are bad, will be differently attached to self-employment and will have a different future labor market outcomes, than those who leave otherwise good paid employment. This then relates to which individuals become long-term entrepreneurs, hiring work-

\(^3\) Carrasco (1999) studies transitions from and to self-employment in the Spanish Family Expenditure Survey. With respect to the business cycle, she finds that a high unemployment rate is associated with fewer U-SE transitions (consistent with the pull hypothesis) and more PE-SE transitions (consistent with the push hypothesis). However, a potential inference problem is that over her sample period (1985-1991) the Spanish unemployment rate is characterized by a close-to-linear downward trend from 20\% to 15\%.
ers, while for others self-employment is just a temporary state that provides a measure of insurance against otherwise adverse labor market outcomes. A main point of the paper is that we are able to model all this explicitly, in a labor market equilibrium.

We see our microfounded structural approach as complementary to other papers that analyse time series for the self-employment rate and other macroeconomic variables. One recent paper in this vein is Congregado, Golpe & Parker (2012); they find that the cyclical components of self-employment and GDP are negatively correlated in the U.S. and Spain (although the correlation is only significant for Spain). They also distinguish between employers and own-account workers among the self-employed and find that the number of employers is pro-cyclical, whereas the number of own-account workers is counter-cyclical (both significantly). They interpret this as unproductive self-employed exiting into paid employment and productive self-employed expanding their business in good times.

To summarize, in this paper, we want to explicitly model the macroeconomic environment, including aggregate shocks, while in particular we want to incorporate endogenous responses of the demand for labor in paid-employment jobs (responding to the business cycle and, potentially, policy changes), and investigate to what extent a simple two-ladder (a paid-job ladder and a business profitability ladder) model that includes all this can explain the six labor market flows between paid employment, self-employment and unemployment, and how these vary over the business cycle.4

2 Empirical Patterns

2.1 the Data

For our data analysis we use the Survey of Income and Program Participation (SIPP). The SIPP is a longitudinal data set based on a representative sample of the US civilian non-institutionalized population. 5 It is divided into multi-year panels, spanning 2.5 to 4 years. Individuals are interviewed every four months, and are asked, among other things, about their employment status, including whether this was self-employment or for paid employment. 6

There are several advantages of using the SIPP to other data sets like the Current Population

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4Taking the broad view, our paper is also linked to business-cycle models in which varying levels of entrepreneurial activity over the business cycle lead to amplification and propagation of shocks, which have received renewed interest with the recent financial crisis. Bernanke & Gertler (1989) is the seminal contribution: they show that counter-cyclical agency costs in lending to entrepreneurs induce a financial accelerator. In Rampini (2004), entrepreneurs are less risk-averse and need fewer incentives from financial intermediaries to exert high effort, which makes self-employment pro-cyclical.

5For a more detailed description of the data, on which this short description is based, see also Carrillo-Tudela and Visschers (2013).

Survey (CPS) or the Panel Study of Income Dynamics (PSID), which also have been used to measure labor market flows and/or occupational and sectoral mobility. The SIPP’s longitudinal dimension, high frequency interview schedule and explicit aim to collect information on worker turnover allows us to construct reliable measures of flow measures between the three states, and how these vary with recent previous history (the SIPP also has retrospective questions that concern labor market history before the panel started). Further, its panel dimension allows us, compared to the CPS, to subsequently follow workers over time and construct e.g. spells of self-employment that started from unemployment.  

We consider the period 1990 - 2011. To cover this period we use the 1990-1993, 1996, 2001, 2004 and 2008 panels. Although the SIPP started in 1984, firm identities are only reliable since the 1990 panel. Two important differences between the post and pre-1996 panels are worth noting. The pre-1996 panels have an overlapping structure and a smaller sample size. Starting with the 1996 panel the sample size of each panel doubled in size and the overlapping structure was dropped. To overcome these differences and make the sample sizes somehow comparable, we constructed our pre-1996 indicators by obtaining the average value of the indicators obtained from each of the overlapping panels. On the other hand, the SIPP’s sample design implies that in all panels the first and last three months have less than 4 rotation groups and hence a smaller sample size. For this reason we only consider months that have information for all 4 rotation groups. The data also shows the presence of seams effects between waves. To reduce the seam bias we average the value of the indicator over the four months that involve the seam. Our indicators are based on the employment status variable at the second week of each month, “wesr2” for the 1990-1993 panels and “rwkesr2” for the 1996-2008 panels. The choice of the second week is to approximate the CPS reference week when possible. 

For the 1990-2008 panels, we consider a worker employed if he/she was attached to a job. Namely if the individual was (1) with job/business - working, (2) with job/business - not on layoff, absent without pay and (3) with job/business - on layoff, absent without pay. A worker is considered unemployed if he/she was not attached to a job and looking for work. Namely if the individual was with (4) no job/business - looking for work or on layoff. A worker is then considered out of the labor force (non-participant) if he/she was with (5) no job/business - not looking for work and not on layoff.

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8 See Fujita, Nekarda and Ramey (2007) for a similar approach. We have also performed our analysis by constructing the labor market status of a worker based on the employment status monthly recode variable for all panels and our results do not change.
Figure 1: Outflow rate from unemployment into self-employment

2.2 Data Patterns

Using the SIPP, we will first document the overall patterns in terms of stocks and flows, and then proceed to a detailed multivariable statistical analysis of the composition (in terms of observables). This will motivate the model in the next section (which will further help with pinning down selection).

Stocks and Flows – overall

Over the period 1990-2011, about 10% of workers in the labor force are self-employed, about twice as much as there are unemployed workers. The percentage of self-employed workers, starts low for young workers, but grows significantly with age. At age 25 about 4% of the labor force is in self-employment; at age 45 this has risen to 12.5%, and at 55 almost to 15%, rising further to over 20% close to retirement. Self employment also becomes more important as an way out of unemployment as workers grow older. At age 25, about 2.5% of workers exiting unemployment do so to self-employment, but this grows to 8% at age 40, and close to 10% after age 50.

There is persistence in self-employment across unemployment spells, those who have been self-employed will likely return to self-employment after an unemployment spell. Given the increase in the stock of unemployed, this explains a part of this statistic. However, the proportion of those unemployed who were paid employees previously and move to self-employment also goes up until age 40, tapering off afterwards around 5%, as shown in figure 1. This is reflected also in the outflow rate of unemployed workers into self-employment, which increases significantly with age until the early forties, after which it declines – also when looking at those unemployed workers who were
paid before (see figure 2)

Self-employed workers do flow, on average, at a relatively high rate into jobs in paid employment (i.e. this is the business-to-job flow rate), at around 1.3% monthly, a rate a little more than 2/3 of the job-to-job transition rate for paid-employed workers, which is close to 2% monthly (time-averaged after 1996). The reverse transition occurs at a low monthly probability, about 0.15% of workers leave their paid job in a month for self-employment. This means that the job-to-business rate is about 10% of the job-to-job rate. Overall, in the SIPP, for those workers remaining in the labor force, 2.5% of jobs end in a given month; 60% of these jobs are left for another job, a little more than 6% to self-employment, and 33% to unemployment.

Overall, self-employed workers are less likely to fall back into unemployment, at a monthly rate of only 0.35% relative to 0.8% for paid employed workers. However, this masks a lot heterogeneity with self-employed tenure. (In European data, Millán (2012) finds a similar pattern of self-employed workers flowing into unemployment less than paid-employed workers; Taylor (2004) finds this for UK BHPS data). We turn to this next.

Flows as a function of duration in paid employment/self-employment

(to be provided)

Multivariate Statistical Analysis – Probits

(to be provided)
Patterns vary with the business cycle

The proportion of newly self-employed workers among those workers entering employment (from unemployment) is strongly countercyclical. In the boom-times in the late 1990s, for example, only close to 3% of those workers leaving unemployment became self-employed, whereas in 2008-2009 it was more than double, in the range of 7-8%, as can be seen in figure 1.

Overall, the rate at which workers leave self-employment for paid employment does not appear to have a strong cyclical tendency. It is, however, interesting to look at the more disaggregate patterns underneath. For example, to see if an increased outflow to paid employment of self-employed coming from unemployment is offset by decreased outflow of those with longer times spent in self-employment.

2.2.1 Business tenure and termination hazard, over the business cycle

Graph with business tenure on the x-axis and termination hazard on the y-axis, for those quarters in which the economy is in a recession, in a boom, in a good labor market and in a bad labor market. (In a recession the productivity growth is negative, and unemployment is rising; in a boom, productivity growth is positive, and unemployment is dropping; in a bad labor market unemployment is above trend; in a good labor market unemployment is below trend). Same graph with SE exit hazard and business tenure...

SE to U hazard as a function of time, PE to U hazard as function of time. And for different
tenures (in job, and in business).

2.2.2 PE and SE survival as a function of previous PE tenure or U (with PE before)

Here we look at the overall separation hazard, and the exit to unemployment hazard, and the exit to PE hazard, as a function of previous tenure in PE (or unemployment), to get an idea about ex ante selection of ideas. One needs to incorporate too that those PE jobs that separate could be under stress: looking at separation behavior for very high-tenure jobs to get a sense of this.

Motivating Patterns

In the next section, we build a model that captures the following patterns observed in the SIPP data:

- Self-employment and paid employment are not two separate worlds: no once-and-for-all selection of workers in one of the two, rather significant flows in both directions between these.
- Those entering unemployment from paid employment also end up in self-employment to a nonnegligible extent, illustrating that indirect flows between paid and self employment are also important.
- The proportion of unemployed who will start self-employment rises steadily with age then flattens out.
- The proportion of unemployed who were in paid employment before also rises with age until early forties, staying elevated afterwards (declining slightly from peak).
- The rate at which unemployed flow to self-employment rises with age till the early forties, then decreases.
- The rate at which unemployed who were paid employed before flow out to self-employment follows the same pattern, but decreases more sharply after age 40 than the overall proportion of unemployed who will start self-employment.
- The inflow from unemployment and from paid-employment both important in the overall inflow to self-employment.
- Cyclical Behavior: in recessions a higher proportion of unemployed appears to start self-employment.

Hence, we will incorporate into the model the flows between the three states in which workers in the labor force can be: paid employment, self-employment and unemployment. We will model ability heterogeneity and human capital acquisition, with a self-employment-specific component, which will capture the persistence of paid or self-employment at both ends of unemployment spells.
(and the high probability of returning to either paid employment or self-employment, when taking a job in the other employment state). A reason to explicitly add the ability heterogeneity is that we do not want to load all outcomes on realizations of a random search process, but rather allow the model to go between early occupational choice (in terms of employee vs. self-employed), and later occupation outcomes driven by realization in the face of labor market frictions. Furthermore, we will model finitely lived workers, to see if the cost of hiring a worker and starting a business will slow down the outflow rates to self-employment (also in relation to the likewise changing outflow rate to paid employment) in line with the data.

We think that a model that incorporates search frictions on the paid employment side, and self-employment, is not only interesting because of any quantitative correspondence to the empirical in- and outflows of self-employment (or paid employment, for that matter), but also because it can potentially tell us something more about the nature of search frictions. The contrast here is simple: to become paid employed one needs to look for a partner on the other side of the market, the difficulty of which might vary with the extent of the search frictions, which interact with the business cycle to deliver different success rates. Self-employment does not require a partner on the other side of the market; it might however require a fixed investment that ultimately might produce behavior similar, or very different from the investment in vacancies.

## 3 Model

At this early stage, we set the model up in a relatively general manner, to capture a range of forces that could be at work in shaping the stocks of the three labor force states and flows between them: (i) finitely lived workers, (ii) who acquire human capital, (iii) can climb the job ladder, or (iv) climb the business idea ladder, while (v) the position on either ladder might not be fully known at the moment of transition (i.e. there is learning). Below we want to exhibit how in this setup, the model remains tractable, even with aggregate shocks. As such, the model builds on Menzio, Telyukova and Visschers (2012) and Rissman (2007) and Millán (2012).

Time $\tau$ is discrete and goes on forever. The economy is populated with a measure 1 of workers and a large but fixed measure of constant-returns-to-scale firms with labor as their only input. Both workers and firms are risk-neutral, firms live forever but workers do not – they age from $t = 1$ to $t = T$; both discount the future at $\beta < 1$. Workers provide their efficiency units of labor $l$.

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9As this is work in progress, we strive for parsimony, but we do not feel that we are currently at the stage to properly make the trade-off between parsimony and generality. Additionally, we plan to investigate whether we can incorporate a setting in which, (vi), successful business ideas might lead a self-employed worker to become an employer, thus feeding back into labor demand, while keeping tractability – under perfect substitutability of labor in the production function, and restricted now to infinitely-lived workers. (This rules out that heterogeneous workers and heterogeneous firms will sort in such a way that tractability is lost.) This is a route we want to keep open in this investigation.
inelastically to the firms with which they are matched. Workers can be in three employment states: in paid employment, when matched with firms; in self-employment, when being able to produce for the market without being matched to a firm, and in unemployment when not producing for the market. The labor market is frictional: it takes time and resources for firms and workers to find each other. Firms post contracts promising expected income to workers, and workers can direct their search to the submarket where firms promise a specific utility value (after matching).

Self-employed workers, on the other hand, do not need to locate other agents to be productive, but need to come up with a productive and marketable idea. Every period a new idea arrives with a fixed probability. These probabilities can depend on the current employment status, paid employed, unemployed, self-employed, and are denoted by respectively \( \lambda_{PS} \), \( \lambda_{US} \), \( \lambda_{SS} \). Conditional on getting an idea \( z \), its profitability is drawn from a distribution \( F(z) \), the draw being independent of all past draws. Analogously, every period workers can apply, with a probability that likewise depends on his current employment state, to new firms, in the hope of becoming better-paid in a new firm. The productivity in firms and in self-employment is affected by aggregate productivity \( p \).

Let us now delve into the specifics of the model.

**Production** The instantaneous production of a worker in paid employment depends on the aggregate state \( p \), his human capital \( h \), and on the quality of his match \( y \), according to (per-worker) production function \( f(p, h, y) \). Note that \( h \) can be two-dimensional: \((h_p, h_s)\), where the former refers to the human capital relevant (mainly) for paid employment, and \( h_s \) capturing the human capital relevant for self-employment. Likewise, the production of a worker with an idea \( z \) also depends on aggregate productivity and is given by a function \( g(p, h, z) \). Aggregate productivity follows a stationary first-order Markov process \( \Gamma_p \). We assume that matches are experience goods, in the sense that firms and workers do not immediately observe the realized value of \( y \). After the quality of the match is known, it can in principle change over time, but the agents in the match have complete information about this. Concretely, there is a state, with associated ‘average’ productivity \( y_0 \) corresponding to the state where worker and firm do not know the match quality; after learning the match quality \( y \), a well-behaved transition matrix \( \Gamma_y \) controls the evolution of the match quality.

We model that idea quality \( z \) analoguously. The productivity of ideas can be observed before self-employment is started and can be changing over time, with transition matrix \( \Gamma_z \). Salaried workers cannot improve their contract by moving inside the firm (this would already be part of the contract), and since firms are of measure zero, the probability of matching with one’s own firm in the market is zero.

\[\text{For now, we assume that the profitability of a given idea is constant over time; we plan to generalize this latter.}\]
**Timing** We divide each subperiod into different stages (subperiods), corresponding to the decisions that firms and workers make. At the beginning of the period the productivity shocks to $p, y, z$ realize, and human capital is updated. In the first stage (separation stage), an employed worker becomes unemployed with probability $\sigma \in [\delta_P, 1]$ where $\sigma$ is a probability determined by the worker’s employment contract and $\delta_P$ is the probability that the worker has to leave his job for exogenous reasons. Self-employed decide whether to continue their current business, or rather flow into unemployment. Exogenous flows into unemployment from unemployment also take place in this stage with probability $\delta_S$. Subsequently, workers can receive the opportunity to apply for paid employment or, alternatively, receive a business idea; with the remaining probability they do not receive any opportunity to change firm or idea. In the next stage (search stage), workers with a new business idea decide whether to start implementing it in self-employment, and workers with an opportunity to apply for paid employment, decide to which submarket to apply. Self-employed have to pay an initial cost $c$ to open a business. We assume that $c = 0$ for the transitions from self-employment to self-employment. Also, during the search stage, a firm chooses how many vacancies to create in each submarket. The cost of maintaining a vacancy for one period is $k > 0$. Matching then occurs according to the technology laid out below. In the final stage (production stage), self-employed workers produce and salaried worker-firm matches produce, and unemployed workers receive their flow benefit of unemployment, $b$.

**Matching** The market segments (endogenously\(^{11}\)) into submarkets, which are indexed by the type of workers looking in a particular submarket, and the lifetime value promised to these workers. The workers’ type incorporates their current labor market state: an unemployed worker can visit a different market than a self-employed worker with idea quality $x$, or a salaried worker who ‘gets’ lifetime expected value $W$ in his current match. In each submarket, a Pissarides-type Cobb-Douglas matching function determines the number of matches between vacancies and searching workers, $m(u, v) = m_0u^\eta v^{1-\eta}$. As usual, define tightness $\theta = v/u$, and note that workers’ probability of matching $f$ (the job-finding rate) depends only on the tightness, $f(\theta) = m(u, v)/u = m_0\theta^{1-\eta}$, where $p : \mathbb{R}_+ \to [0, 1]$ is a twice-differentiable, strictly increasing and strictly concave function with boundary conditions $p(0) = 0$ and $p(\infty) = 1$. Likewise, firms’ vacancy-filling rate $q$ is a function of tightness in the submarket only, $q(\theta) = m(u, v)/v = m_0\theta^n$, where $f : \mathbb{R}_+ \to [0, 1]$ is a twice-differentiable, strictly decreasing function such that $q(0) = 0$ and $q(\infty) = 1$.

**Workers’ optimization** The worker can be in three distinct labor market states, for which we now write down the value functions that determine their optimal choices, and corresponding ex-

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\(^{11}\)We follow the approach by Menzio and Shi (2011) and Menzio, Telyukova and Visschers (2012) here, more on this below.
pected life-time utility. Unemployed workers receive flow benefits $b$. With probability $\lambda_{UP}$, they they can apply for paid employment, they draw an idea with probability $\lambda_{US}$, and with probability $(1 - \lambda_{UP} - \lambda_{US})$ none of the two occurs and they remain in unemployment with certainty. Let $U(p, h, t)$ the value function of unemployment in aggregate state $p$ with human capital $h$, and age $t$, at the beginning of the period. We are looking for a recursive equilibrium in which this captures all the relevant information for workers. In anticipation of the existence of such an equilibrium, we drop the dependence on variables as time $\tau$ and the distribution of workers over unemployment, paid employment matches, and self-employment ideas. The value function of self-employment of a worker of age $t$ with human capital $h$, business-idea profitability $z$ in aggregate state $p$ is given by $B(p, z, h, t)$; likewise $W(p, y, h, t)$ corresponds to the value of paid employment at match quality $y$ in aggregate state $p$, the latter two in the production stage of the period. Unknown matches are designated match quality $y_0$. Hence, for the unemployed, we have

$$U(p, h, t) = b + \beta \mathbb{E}_{p'} [U(p', h, t + 1) + \lambda_{UP} \left( \max_{\tilde{\theta}, \tilde{W}} \left\{ f(\tilde{\theta}) \left[ \tilde{W}(p', y_0, h, t + 1) - U(p', h, t + 1) \right] \right\} \right] +$$

$$+ \lambda_{US} \int \max \left\{ B(p', z, h, t + 1) - c - U(p', h, t + 1), 0 \right\} dF(z) \right]$$

(1)

Note that when getting the opportunity to apply for a paid-employed job, the worker seeks the sub-market with promised value $\tilde{W}$ that gives him the best prospects taking into account the job-finding rate there.

For paid-employed workers, who are promised life-time expected utility $W$, this should be delivered by a current wage $w$, and contingent continuation values $W(p', y', h', t + 1)$

$$W(p, y, h, t) = w + \mathbb{E}_{p', y'} \left[ \sigma_p U(p', h', t + 1) +$$

$$(1 - \sigma_P) \left\{ \lambda_{PP} \max_{\tilde{\theta}, \tilde{W}} \left\{ 0, f(\tilde{\theta}) \left[ \tilde{W}(p', y_0, h', t + 1) - W(p', y', h', t + 1) \right] \right\} \right] +$$

$$+ \lambda_{PS} \int \max \left\{ 0, B(p', z, h', t + 1) - c - W(p', y', h', t + 1) \right\} dF(z) +$$

$$+ W(p', y', h', t + 1) \right\} \right].$$

(2)

Finally, for the self-employed the Bellman equation is
\[
B(p, z, h, t) = g(p, z, h) + \beta E_{p', z'} \left[ (1 - \delta_s) \max \left\{ U(p', h', t + 1), \lambda_{SP} \max_{\tilde{\theta}, \tilde{W}(p', y_0, h', t + 1)} \left\{ 0, f(\tilde{\theta}) \left[ \tilde{W}(p', y_0, h', t + 1) - B(p', z', h', t + 1) \right] \right\} \right\}
\]

\[
+ \lambda_{SS} \int \max \left\{ 0, B(p', z, h', t + 1) - B(p', z', h', t + 1) \right\} dF(z)
\]

\[
+ B(p', z', h', t + 1) \right\} + \delta_s U(p', h', t + 1).
\]

\[
(3)
\]

Firms’ maximization

**Contracts** Within submarket the meetings between searching workers and firms with vacancies are ‘produced’ by a standard Pissarides-type matching function. We assume that employment contracts are complete, in the sense that they can specify the wage paid by the firm to the worker, \(w\), the probability that the worker and the firm break up at the separation stage, \(\sigma_P\) and the submarket \((\tilde{\theta}, \tilde{W})\) where the worker should search and the ideas \(z\) that the worker might implement while employed by the firm.

We also assume that the contracts offered by firms to workers are bilaterally efficient in the sense that they take the joint interest of both firm and worker into account, maximizing the joint value of the match, that is, the sum of the worker’s lifetime utility and the firm’s lifetime profits.

For paid employment, we can calculate the joint value of the match of worker and firm, \(V(p, y, h, t)\), as

\[
V(p, y, h, t) = f(p, y, h) + \beta E_{p', y'} \left[ \sigma_P U(p', h', t + 1) \right.
\]

\[
(1 - \sigma_P) \left\{ \lambda_{PP} \max_{\tilde{\theta}, \tilde{W}(p', y_0, h', t + 1)} \left\{ 0, f(\tilde{\theta}) \left[ \tilde{W}(p', y_0, h', t + 1) - V(p', y', h', t + 1) \right] \right\} \right\}
\]

\[
+ \lambda_{PS} \int \max \left\{ 0, B(p', z, h', t + 1) - c - V(p', y', h', t + 1) \right\} dF(z)
\]

\[
+ V(p', y', h', t + 1) \right\}.
\]

Finally, the tightness of the submarket is such that

\[
K \geq q(\theta) \left[ V(p, y_0, h, t) - W(p, y_0, h, t) \right]
\]

\[
(6)
\]

and \(\theta \geq 0\) with complementary slackness. The above condition guarantees that the tightness func-
tion $\theta$ is consistent with the firm's incentive to create vacancies. The cost to a firm from opening a vacancy in submarket $(\theta, W)$ is given by $k$. The benefit to a firm from opening a vacancy in submarket $(\theta, W)$ is given by the product between the probability that the firm fills the vacancy, $q(\theta)$, and the value to the firm from filling the vacancy, $V(p, h, y_0, t) - W(p, h, y_0, t)$. Condition (6) states that, if the vacancy-to-applicant ratio in submarket is strictly positive, the cost from opening a vacancy must be equal to the benefit. If the vacancy-to-applicant ratio in submarket is equal to zero, the cost to a firm from opening a vacancy must be smaller or equal to the benefit.

**Lemma 1.** The contract specifies application choices $\tilde{W}(\cdot)$ and breakup decisions such as to maximize the joint value of the match

Menzio and Shi (2009) formally establishes that the profit-maximizing contracts are bilaterally efficient if the contract space is complete in the sense that a contract can specify the wage, $w$, the separation probability, $\sigma_P$, and the submarket where the worker searches while on the job, $(\tilde{\theta}, \tilde{W})$. The firm maximizes its profits by choosing the contingencies for $\sigma_P$ and $(\tilde{\theta}, \tilde{W})$, so as to maximize the joint value of the match and by choosing the contingencies for $w$ so as to deliver the promised value $\tilde{W}$.

In our two-ladder life-cycle model, we are looking for an equilibrium in which value functions and decisions to separate into unemployment, to apply for paid employment, to move to self-employment, only depend on the aggregate state $p$, the individual’s states $h, t$, and the firm-worker match specific quality $y$ or the idea quality $z$. In other words, we define

**Definition 1.** A Block Recursive Equilibrium consists on a market tightness function $\theta(\cdot)$, a value function for the unemployed workers $U(\cdot)$, a policy function for the unemployed workers $(\tilde{\theta}_U, \tilde{W}_U)$, a value function for the paid employed workers $W(\cdot)$, a joint value function for the firm-worker match $V(\cdot)$, policy functions for the firm-worker match $(\tilde{\theta}_P, \tilde{W}_P)$ and $\sigma_P$, a value function for the self-employed workers $P(\cdot)$, a policy function for the self-employed workers $(\tilde{\theta}_S, \tilde{W}_S)$, and allocations, such that

1. $U(\cdot), W(\cdot), B(\cdot), V(\cdot), \theta, (\tilde{\theta}_U, \tilde{W}_U), (\tilde{\theta}_P, \tilde{W}_P, \sigma_P)$ and $(\tilde{\theta}_S, \tilde{W}_S)$ are independent of the distribution of workers over unemployment, paid employment matches, and self-employment ideas.
2. $\theta(\cdot)$ satisfies (6) for every submarket.
3. $U(\cdot)$ and $(\tilde{\theta}_U, \tilde{W}_U)$ satisfy (1)
4. $W(\cdot), (\tilde{\theta}_P, \tilde{W}_P)$ and $\sigma_P$ satisfy (2) and (5)
5. $V(\cdot)$ satisfies (5)
6. $B(\cdot)$ and $(\tilde{\theta}_S, \tilde{W}_S)$ satisfy (4)
7. the allocations follow from using the policy functions, and updating the distribution

Theorem 1. The unique recursive equilibrium is a Block Recursive Equilibrium.

4 Quantitative Investigation

In the following, we provide a preliminary numerical example to illustrate the workings of the model and to roughly gauge its quantitative implications (full calibration to be provided). We focus on a simple version of the model, stripping out the life-cycle and human-capital dimensions.

4.1 Parameters and functional forms

We follow the previous literature and our quantitative results from above to pick the parameters of our numerical example, see Table 1 for an overview. A model period is one month, and all transition rates etc. are given on a monthly basis. We choose the offer distribution of ideas and job-match-quality to be log-normal, and we normalize the median of job-match-quality distribution to 1 (the choice $\mu_F = 0$ is marked by (n) to indicate the normalization in Table 1). Workers die at a monthly rate of $\delta_D = 1/(12 \times (65 - 20))$ to capture the expected length of a work life. New workers enter each period into unemployment, replacing exactly the mass of exiting workers.\footnote{This makes it possible to capture the fact that flow rates and stocks are not in long-run equilibrium in the data due to entry and exit in/from the labor force.}

The business cycle $a$ follows a mean-zero AR(1) process. For the production functions, we set $f(a, y) = e^{ay}, \quad g(a, z) = e^{\alpha a z},$ where $\alpha$ captures how responsive the self-employed’s earnings are to the business cycle in relation-ship to the responsiveness of the paid-employed’s wages to the cycle. The SE learn their idea quality immediately upon drawing an idea, for the PE it is revealed at a constant probability.

4.2 Workings of the model

We will now illustrate the workings of the model. It is first useful to study the optimal decision rules and resulting transition probabilities for different worker types. The upper-left panel of Figure 4 shows the vacancy-posting rates for different worker types in times of high and low aggregate productivity. Moving to the right on the horizontal axis, we see that vacancy-creation stops alto-gether for high levels of PE/SE quality – for these workers, a new job of unknown quality is never attractive, thus job creation is zero and also outflows are zero (as the lower-left panel shows).
<table>
<thead>
<tr>
<th>parameter</th>
<th>symbol</th>
<th>value</th>
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</thead>
<tbody>
<tr>
<td>discount factor</td>
<td>$\beta$</td>
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<td>worker death rate</td>
<td>$\delta_D$</td>
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<td>flow utility unemployment</td>
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<td>arrival rate: ideas in unemployment</td>
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<tr>
<td>arrival rate: ideas in paid employment</td>
<td>$\lambda_{PS}$</td>
<td>0.016</td>
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<tr>
<td>arrival rate: application opportunities in paid employment</td>
<td>$\lambda_{PP}$</td>
<td>0.16</td>
</tr>
<tr>
<td>arrival rate: ideas in self-employment</td>
<td>$\lambda_{SS}$</td>
<td>0.01</td>
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<tr>
<td>arrival rate: application opportunities in self-employment</td>
<td>$\lambda_{SP}$</td>
<td>0.16</td>
</tr>
<tr>
<td>job-destruction rate in paid employment</td>
<td>$\delta_P$</td>
<td>0.0083</td>
</tr>
<tr>
<td>job-destruction rate in self-employment</td>
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<td>mean job match quality distribution</td>
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<td>$\sigma_F$</td>
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<td>mean idea quality distribution</td>
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<tr>
<td>variance idea quality distribution</td>
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<td>Poisson learning about job match quality ex post</td>
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<td>business setup cost</td>
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<tr>
<td>business-cycle sensitivity earnings SE over PE</td>
<td>$\alpha$</td>
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</table>

Table 1: Parameters

Moving towards the left in the upper-left panel, the blue and red crosses indicate the $y$-level below which workers choose to leave PE. The posting rates left of these crosses are only of limited interest since workers have left these positions when the economy comes into the job-creation stage. In Figure 5, we indeed see that the PE density vanishes to the left of these job-quality levels. In a boom, workers prefer some low-quality jobs to unemployment, but they leave those ideas in favor of unemployment in case a recession hits.

As for the unemployed and the paid-employed, Figure 4 shows that there is more job creation in booms than in busts (when fixing the level of worker quality). The reason is that the surplus from created matches in relationship to both the unemployment benefit $\xi$ and the vacancy-posting cost $k_p$ is smaller in a downturn, when job productivity is low. Interestingly, we see that this pattern is reversed in job-creation for the SE: more PE jobs are created for SE workers in busts than in booms (that is, for any given $z$ that has a positive density in a bust). This is because the SE’s earnings suffer more in a recession than the PE’s since $\alpha > 1$. So the SE become more interested in moving to the PE sector in busts (given fixed $z$). This is a force that makes SE-to-PE outflows counter-cyclical.
However, we will now see that there is a countervailing effect: the SE pool changes over the cycle, which makes SE-to-PE flows pro-cyclical. The support of $z$ extends farther to the left in booms, since also inferior ideas are viable when $a$ is high. The job-creation rates for such low-quality SE are higher than average, however, as the dashed blue line in the left panel of Figure 4 indicates.

We now present a decomposition of the business-cycle fluctuations in flows that sheds more light on this issue. Let $f_{ij}(t)$ be the total flow from category $i$ to category $j$ (i.e. the stock in $i$ times the $i$-to-$j$ outflow rate), where $i, j \in \{U, P, S\}$. Let $\bar{f}_{ij}$ be the long-run mean of $f_{ij}$, and let $\hat{f}_{ij}(t)$ be the business-cycle fluctuation, i.e. set $\hat{f}_{ij}(t) \equiv f_{ij}(t) - \bar{f}_{ij}$. Denote by $n_i(t, x)$ the density of workers in category $i$ over state $x$ at time $t$, and denote by $p_{ij}(t, x)$ the probability that a worker in state $x$ in category $i$ moves to category $j$ at $t$. Denoting long-run means of variables with bars and business-cycle deviations with hats, we can then decompose the fluctuations in the flow into two
main effects and a remainder term:

\[
\hat{f}_{ij}(t) = f_{ij}(t) - \bar{f}_{ij} \\
= \int n_i(t,x)p_{ij}(t,x) - \bar{n}_i(x)\bar{p}_{ij}(x)dx \\
= \int [\bar{n}_i(x) + \hat{n}_i(t,x)][\bar{p}_{ij}(x) + \hat{p}_{ij}(t,x)] - \bar{n}_i(x)\bar{p}_{ij}(x)dx \\
= \int \bar{n}_i(x)\hat{p}_{ij}(t,x) + \hat{n}_i(t,x)\bar{p}_{ij}(x) + \hat{n}_i(x)\hat{p}_{ij}(t,x)dx.
\]

Figure 6 shows a time-average of this decomposition for SE-to-PE flows. As mentioned above, the behavioral and the composition effect go into opposite directions.

The first thing that stands out is that only the flows out of low-quality ideas are affected by the cycle. This is because SE with good ideas never accept PE offers of unknown quality, so there is no business-cycle variation stemming from these workers. Note that this is also reflected by the boom and bust densities being almost identical for high-quality workers for both SE and PE.

Figure 7 shows how the two effects play out over a scenario where a 5-year boom is followed abruptly by a 5-year bust. Over the boom, the SE stock rises, average quality of ideas dropping as more bad ideas are accepted. In the moment the bust hits, the behavioral effect on SE-to-PE flows comes into play: low-z SE now move into SE if given the chance to do so, and the SE-to-PE outflow rate shoots up. Over time, the recession cleanses the SE pool in this fashion. The SE stock falls, and average idea quality improves. This brings the composition effect on the SE-to-PE rate to bear, which after a year overrides the behavioral effect and brings the SE-to-PE outflow rate below trend: there are few low-quality SE left, and the high-z SE are less likely to leave.
As for flows into SE (from PE and U), Figure 7 shows that they are pro-cyclical. In this case, the behavioral effect trumps the composition effect. In the right panel of Figure 4, we see that both the unemployed and the paid employed are less picky in accepting business ideas in a boom. As the model stands now, all ideas become more attractive in a boom. Quantitatively, what matters most for aggregates are the flows out low-\(y\) jobs (see the lower-right panel of Figure 4). Again, for high-\(y\) workers the outflow rate to SE does not vary much over the cycle: they only accept ideas of very high quality in both bust and boom, but such ideas are rare. Indeed, we assumed that high-\(y\) workers do not draw better (or more) business ideas than the low-\(y\) workers or the unemployed. Thus, the model in its current form is not suited to capture the notion that a boom opens the door for more successful paid employed to open a high-yield business (pull effect).

4.3 Discussion: to do

As just mentioned, the model so far induces almost no business-cycle fluctuations in the behavior of high-quality SE and PE, which may be seen as a flaw (what do the data say on this?). Also, since low-quality ideas are punished in a recession as any other business in the model, there is currently no push-force into SE in recession. On the contrary, the proportion of U flowing to SE and transitions of low-\(y\) PE into SE are highly pro-cyclical in the current parameterization. Further work is probably needed on these fronts.
5 Conclusion

In this paper, we focus on the labor market flows between self-employment, unemployment and paid employment. These flows economically important and vary over the business cycle. To understand the forces underlying these flows and their response to changes in the environment, due to shocks or due to policy changes, one needs to capture the choice of workers to become self-employed, or leave self-employment. This choice varies with the alternative of the worker: whether he is unemployed or, if employed, how high in the job ladder his current salaried job is.

6 References


Meghir, C., Narita, R., and J-M Robin, Wages and Informality in Developing Countries, Yale University mimeo, 2013.


A Proofs

Proof of Theorem 1: The value functions $U(.,T), W(.,T), V(.,T)$ and $B(.,T)$ satisfy the equilibrium conditions (3)-(6) if and only if

$$U(.,T) = b$$  \hspace{1cm} (7)  

$$W(.,T) = w$$  \hspace{1cm} (8)  

$$V(.,T) = f(.)$$  \hspace{1cm} (9)  

$$B(.,T) = g(.)$$  \hspace{1cm} (10)  

Notice that neither $U(.,T), W(.,T), V(.,T)$ nor $B(.,T)$ depend on the distribution of workers over unemployment, paid employment matches, and self-employment ideas. Thus, they satisfy the equilibrium condition (1).

The market tightness $\theta(.,T)$ satisfies the equilibrium condition (2) if and only if
\[ \theta(.,T) = q^{-1} \left( \frac{K}{V(.,T) - W(.,T)} \right), \quad \text{if } W(.,T) \leq V(.,T) - K \]
\[ \theta(.,T) = 0 \quad \text{else} \quad (11) \]

Notice that \( \theta(.,T) \) satisfies condition (1) since it does not depend on the distribution of workers over unemployment, paid employment matches, and self-employment ideas.

The unemployment policy function \((\tilde{\theta}_U, \tilde{W}_U)\) satisfy the equilibrium condition (3) if and only if it solves the problem

\[ \max_{\tilde{\theta}(.,T), \tilde{W}(.,T)} \left\{ 0, f(\tilde{\theta}(.,T)) \left[ \tilde{W}(.,T) - U(.,T) \right] \right\} \quad (12) \]

From (11), we can rewrite (12) as

\[ \max_{\tilde{\theta}(.,T)} \left\{ 0, -K\tilde{\theta}(.,T) + f(\tilde{\theta}(.,T)) [V(.,T) - U(.,T)] \right\} \quad (13) \]

The objective function in (13) is strictly concave in \( \tilde{\theta} \) and does not depend on the distribution of workers. Thus, (13) admits a unique solution \( \tilde{\theta}_U(.,T) \) which does not depend on the distribution of workers. Given \( \tilde{\theta}_U, \tilde{W}_U \) must be such that

\[ \tilde{W}_U(.,T) = V(.,T) - \frac{K}{q^{-1}(\tilde{\theta}_U(.,T))}, \quad \text{if } \tilde{\theta}(.,T) > 0 \]
\[ \tilde{W}_U(.,T) \geq V(.,T) - K \quad \text{if } \tilde{\theta}(.,T) = 0 \quad (14) \]

The difference among the many policy functions that solve (14) is the \( W \) that the worker chooses when he finds optimal to visit an empty submarket. Thus, without loss in generality we can specialize (14) to

\[ \tilde{W}_U(.,T) = V(.,T) - \frac{K}{q^{-1}(\tilde{\theta}_U(.,T))} \quad (15) \]

which also does not depend on the distribution of workers.

The paid employment policy function \((\tilde{\theta}_P, \tilde{W}_P)\) satisfy the equilibrium condition (4) if and only if it solves the problem

\[ \max_{\tilde{\theta}(.,T)} \left\{ 0, -K\tilde{\theta}(.,T) + f(\tilde{\theta}(.,T)) [V(.,T) - U(.,T)] \right\} \quad (13) \]
\[
\max_{\tilde{\theta}(.,T),\tilde{W}(.,T)} \left\{ 0, f(\tilde{\theta}) \left[ \tilde{W}(.,T) - E_{y'}[W(.,T)] \right] \right\}
\] (16)

From (11), we can rewrite (16) as

\[
\max_{\tilde{\theta}(.,T)} \left\{ 0, -K\tilde{\theta}(.,T) + f(\tilde{\theta}(.,T)) \left[ V(.,T) - E_{y'}[W(.,T)] \right] \right\}
\] (17)

The objective functions in (17) is strictly concave in \(\tilde{\theta}\) and does not depend on the distribution of workers. Thus, (17) admits a unique solution \(\tilde{\theta}_P\) which does not depend on the distribution of workers. Given \(\tilde{\theta}_P\), \(\tilde{W}_P\) must be such that

\[
\tilde{W}_P(.,T) = V(.,T) - \frac{K}{q^{-1}(\tilde{\theta}_P(.,T))}
\] (18)

which does not depend on the distribution of workers.

The employment policy function \(\sigma_P(.,T)\) satisfies the equilibrium condition (4) if and only if it solves the separation problem

\[
\max_{\sigma_P(.,T) \in [\delta_P, 1]} \left\{ \sigma_P U(p', h', t + 1) + (1 - \sigma_P) \left\{ \lambda_{PP} \max_{\tilde{\theta},\tilde{W}(p',y',h',t+1)} \left\{ 0, f(\tilde{\theta}) [\tilde{W}(p',y',h',t+1) - W(p',y',h',t+1)] \right\} \\
+ \lambda_{PS} \int \max \left\{ 0, P(p',z',h',t+1) - c - W(p',y',h',t+1) \right\} dF(z) + W(p',y',h',t+1) \right\} \right\}
\] (19)

The objective function in (19) is linear in \(\sigma_P\) and does not depend on the distribution of workers. Therefore, (19) admits generally a unique solution \(\sigma_P(.,T)\) which does not depend on the distribution of workers.

The self-employment policy function \((\tilde{\theta}_S, \tilde{W}_S)\) satisfy the equilibrium condition (5) if and only if it solves the problem

\[
\max_{\tilde{\theta}(.,T),\tilde{W}(.,T)} \left\{ 0, f(\tilde{\theta}) \left[ \tilde{W}(.,T) - E_{z'}[B(.,T)] \right] \right\}
\] (20)

From (11), we can rewrite (20) as
max_\{0, -K\tilde{\theta}(., T) + f(\tilde{\theta}(., T)) [V(., T) - E_{z'}[B(., T)]]\} \quad (21)

The objective functions in (21) is strictly concave in \(\tilde{\theta}\) and does not depend on the distribution of workers. Thus, (21) admits a unique solution \(\tilde{\theta}_S\) which does not depend on the distribution of workers. Given \(\tilde{\theta}_S\), \(\tilde{W}_S\) must be such that

\[
\tilde{W}_S(., T) = V(., T) - \frac{K}{q^{-1}(\tilde{\theta}_S(., T))} \quad (22)
\]

which does not depend on the distribution of workers.

The value function \(U(., T - 1)\) satisfy the equilibrium condition (3) if and only if

\[
U(., T - 1) = b + \beta \mathbb{E}_{y', y} \left[ U(., T) + \lambda_{U_P} \left( \max_{\tilde{\theta}} \left\{ f(\tilde{\theta}) \left[ \tilde{W}(., T) - U(., T) \right] \right\} \right) \right] + \\
+ \lambda_{U_S} \int \max \left\{ B(., z, T) - c - U(., T), 0 \right\} dF(z) \quad (23)
\]

Notice that

\[
\int \max \left\{ B(., z, T) - c - U(., T), 0 \right\} dF(z) \quad (24)
\]

does not depend on the distribution of workers since neither \(B(., T)\) nor \(U(., T)\) does. Hence, \(U(., T - 1)\) does not depend on the distribution of workers.

The value function \(W(., T - 1)\) satisfy the equilibrium condition (4) if and only if

\[
W(., T - 1) = w + \mathbb{E}_{y', y'} \left[ \sigma_P U(., T) \right] \\
(1 - \sigma_P) \left\{ \lambda_{P_{i, \max_{\tilde{\theta}}} W(., T)} \left\{ 0, f(\tilde{\theta}) \left[ \tilde{W}(., T) - W(., T) \right] \right\} \right\} + \\
+ \lambda_{PS} \int \max \left\{ 0, B(., z, T) - c - W(., T) \right\} dF(z) \\
+ W(., T) \right\} \quad (25)
\]

Notice that

\[
\int \max \left\{ 0, B(., z, T) - c - W(., T) \right\} dF(z) \quad (26)
\]

does not depend on the distribution of workers since neither \(B(., T)\) nor \(W(., T)\) does. Hence,
\( W(., T - 1) \) does not depend on the distribution of workers.

The value function \( V(., T - 1) \) satisfy the equilibrium condition (5) if and only if

\[
V(., T - 1) = f(., h) + \beta \mathbb{E}_{\theta, z'} \left[ \sigma_p U(., T) \right]
\]

\[
(1 - \sigma_p) \left\{ \lambda_{PP} \max_{\tilde{\theta}, \tilde{W}(., t+1)} \left\{ 0, f(\tilde{\theta}) \left[ \tilde{W}(., T) - V(., T) \right] \right\} \right.
\]

\[
+ \lambda_{PS} \int \max \left\{ 0, B(., z, T) - c - V(., T) \right\} dF(z)
\]

\[
+ V(., T) \right\}
\]

(27)

Notice that

\[
\int \max \left\{ 0, B(., z, T) - c - V(., T) \right\} dF(z)
\]

(28)

does not depend on the distribution of workers since neither \( B(., T) \) nor \( V(., T) \) does. Hence, \( V(., T - 1) \) does not depend on the distribution of workers.

The value function \( B(., T - 1) \) satisfy the equilibrium condition (6) if and only if

\[
B(., T - 1) = g(., h) + \beta \mathbb{E}_{\theta, z'} \left[ (1 - \delta_S) \max \left\{ U(., T), \lambda_{SP} \max_{\tilde{\theta}, \tilde{W}(., t+1)} \left\{ 0, f(\tilde{\theta}) \left[ \tilde{W}(., T) - B(., T) \right] \right\} \right. \right.
\]

\[
+ \lambda_{SS} \int \max \left\{ 0, B(., \tilde{z}, T) - B(., T) \right\} dF(\tilde{z})
\]

\[
+ B(., T) \right\}
\]

(29)

Notice that

\[
\int \max \left\{ 0, B(., \tilde{z}, T) - c - B(., T) \right\} dF(\tilde{z})
\]

(30)

does not depend on the distribution of workers since neither \( B(., \tilde{z}, T) \) nor \( B(., T) \) does. Hence, \( V(., T - 1) \) does not depend on the distribution of workers.

By repeating the above steps, it is straightforward to establish that the remaining equilibrium value and policy functions are uniquely determined by the equilibrium conditions (2)-(6) and that they are all independent of the distribution of workers. Hence, an equilibrium exists, is unique and it is block recursive.

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B  Computational Appendix

B.1 Parameters; exogenous shock processes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow utility unemployment</td>
<td>$b$</td>
<td></td>
</tr>
<tr>
<td>Arrival rate: ideas in unemployment</td>
<td>$\lambda_{US}$</td>
<td></td>
</tr>
<tr>
<td>Arrival rate: application opportunities in unemployment</td>
<td>$\lambda_{UP}$</td>
<td></td>
</tr>
<tr>
<td>Arrival rate: ideas in paid employment</td>
<td>$\lambda_{PS}$</td>
<td></td>
</tr>
<tr>
<td>Arrival rate: application opportunities in paid employment</td>
<td>$\lambda_{PP}$</td>
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<tr>
<td>Arrival rate: ideas in self-employment</td>
<td>$\lambda_{SS}$</td>
<td></td>
</tr>
<tr>
<td>Arrival rate: application opportunities in self-employment</td>
<td>$\lambda_{SP}$</td>
<td></td>
</tr>
<tr>
<td>Mean job match quality distribution</td>
<td>$\mu_F$</td>
<td></td>
</tr>
<tr>
<td>Variance job match quality distribution</td>
<td>$\sigma_F^2$</td>
<td></td>
</tr>
<tr>
<td>Mean idea quality distribution</td>
<td>$\mu_G$</td>
<td></td>
</tr>
<tr>
<td>Variance idea quality distribution</td>
<td>$\sigma_G^2$</td>
<td></td>
</tr>
<tr>
<td>Learning about job match quality ex ante</td>
<td>$\alpha_P$</td>
<td></td>
</tr>
<tr>
<td>Learning about idea quality ex ante</td>
<td>$\alpha_S$</td>
<td></td>
</tr>
<tr>
<td>Poisson learning about job match quality ex post</td>
<td>$\zeta_P$</td>
<td></td>
</tr>
<tr>
<td>Poisson learning about idea quality ex post</td>
<td>$\zeta_S$</td>
<td></td>
</tr>
<tr>
<td>Vacancy posting cost</td>
<td>$k_p$</td>
<td></td>
</tr>
<tr>
<td>Business setup cost</td>
<td>$k_s$</td>
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</tr>
<tr>
<td>Elasticity of the matching function</td>
<td>$\eta$</td>
<td></td>
</tr>
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</table>

Table 2: Parameters I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
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<td>Autoregressive parameter $p$ process</td>
<td>$\rho_p$</td>
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<tr>
<td>Innovation variance $p$ process</td>
<td>$\sigma_{sp}$</td>
<td></td>
</tr>
<tr>
<td>Human capital paid employment parameters</td>
<td>$\beta_{hp1}, \beta_{hp2}, \beta_{hp3}, \beta_{hp4}$</td>
<td></td>
</tr>
<tr>
<td>Human capital paid employment parameters</td>
<td>$\beta_{hs1}, \beta_{hs2}, \beta_{hs3}, \beta_{hs4}$</td>
<td></td>
</tr>
<tr>
<td>Conversion paid hc into self hc</td>
<td>$\xi_{PS}$</td>
<td></td>
</tr>
<tr>
<td>Conversion self hc into paid hc</td>
<td>$\xi_{SP}$</td>
<td></td>
</tr>
<tr>
<td>Initial heterogeneity paid hc: mean</td>
<td>$\mu_{0, hp}$</td>
<td></td>
</tr>
<tr>
<td>Initial heterogeneity paid hc: st. dev</td>
<td>$\sigma_{0, hp}$</td>
<td></td>
</tr>
<tr>
<td>Initial heterogeneity self hc: mean</td>
<td>$\mu_{0, hs}$</td>
<td></td>
</tr>
<tr>
<td>Initial heterogeneity self hc: st. dev</td>
<td>$\sigma_{0, hs}$</td>
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</tr>
<tr>
<td>Job match quality shock parameters</td>
<td>$\rho_y, \sigma_y$</td>
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</tr>
<tr>
<td>Business idea quality shock parameters</td>
<td>$\rho_z, \sigma_z$</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Parameters II: human capital + stochastic processes
B.2 Computational Algorithm