

Monetary Policy and Global Equilibria in an Economy with Capital*

Preliminary and incomplete

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Abstract

Short-term interest rates in the United States have been near their lower bound since late 2008. Treasury rates out to a two-year maturity have been close to zero since mid-2011, and over this same period, inflation has been declining. This combination of low interest rates and declining inflation has led some observers to point to the “perils of Taylor rules,” for example, Bullard (2010), when a monetary policy that actively targets a positive inflation rate leads to an outcome with much lower inflation, and possibly even deflation. The possibility of equilibria with persistent deviations of inflation from the target set by the policy maker has been investigated for model economies without state variables. Quantitative representations of the U.S. economy as embodied by DSGE models include as an essential element capital accumulation. In this paper we study the possibility for persistent low inflation outcomes for a monetary model with capital.

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1 Introduction

Short-term interest rates in the United States have been near their lower bound since late 2008. Treasury rates out to a two-year maturity have been close to zero since mid-2011, and over this same period, inflation has been declining.¹ This combination of low interest rates and declining inflation has lead some observers to point to the “perils of Taylor rules,” for example, Bullard (2010), when a monetary policy that actively targets a positive inflation rate leads to an outcome with much lower inflation, and possibly even deflation. The possibility of equilibria with persistent deviations of inflation from the target set by the policy maker has been investigated for model economies without state variables. Quantitative representations of the U.S. economy as embodied by DSGE models include as an essential element capital accumulation. In this paper we study the possibility for persistent low inflation outcomes for a monetary model with capital.

In a series of papers Benhabib, Schmitt-Grohe and Uribe (2001a,b; 2002a,b; hereafter BSU) have shown that in conventional models of money, an oft-prescribed monetary policy rule is consistent with equilibria in which the nominal interest rate and inflation converge to steady-state values below those consistent with the targeted inflation rate. This rule, the “active Taylor rule,” has the nominal interest rate, that is, the policy instrument, respond more than one for one to deviations of inflation from its target. Whereas the previous literature had argued that active policy guarantees a unique equilibrium, BSU showed that uniqueness is only a local property. Because the nominal interest rate is bounded below by zero, an active Taylor rule that satisfies this constraint generally implies the existence of two steady-state equilibria. The existence of the second steady state, at a lower inflation rate and a lower nominal interest rate than the targeted rates, leads to the existence of dynamic equilibria that converge to that low-inflation steady state.

BSU considered a range of models with flexible and sticky prices, but without capital. For the purpose of evaluating whether the “undesirable” behavior associated with active Taylor rules is of practical importance, it is necessary to study models with capital. We analyze global equilibria with Taylor rules in a discrete-time, sticky-price model with capital. We use numerical methods, and for the parameterizations we study, we confirm that the basic results carry over from models without capital: there are two steady-state equilibria, and when policy responds to current inflation there are dynamic equilibria that originate near the targeted steady state but converge to the low-inflation steady state. However, as in Dupor (2001) and Carlstrom and Fuerst (2004), we find that with capital in the model, equilibrium dynamics are sensitive to whether the Taylor rule responds to current or future inflation. If policy is active and responds to future inflation, then local dynamics are indeterminate around *both* steady states.

- road map paragraph

¹We measure inflation by the 12-month change in the PCE price index.

2 Model

The model has a representative household that chooses consumption and savings, and supplies labor to firms. There is a continuum of monopolistically competitive firms that each face quadratic costs of nominal price adjustment, as in Rotemberg (1982). The monetary authority sets the short-term nominal interest rate according to a time-invariant feedback rule.

2.1 Households

The representative household has preferences over consumption (c_t) and (disutility of) labor (h_t) given by

$$\sum_{t=0}^{\infty} \beta^t (\ln(c_t) - \chi h_t). \quad (1)$$

There is a competitive labor market in which the real wage is w_t per unit of time. The consumption good is a composite of a continuum of differentiated products ($c_t(z)$), each of which are produced under monopolistic competition:

$$c_t = \left(\int_0^1 c_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right)^{\frac{\varepsilon}{\varepsilon-1}}. \quad (2)$$

Households own the firms and the capital stock. Firms rent capital from households each period. The household's budget constraint is

$$c_t + R_t^{-1} B_t / P_t + i_t = B_{t-1} / P_t + w_t h_t + q_t k_t + D_t / P_t \quad (3)$$

where Π_t represents nominal dividends from firms, P_t is the price of the composite good, i_t is investment, q_t is the rental price of capital, k_t is the capital stock, B_t is the quantity of one-period nominal discount bonds, D_t is dividends paid by firms and R_t is the gross nominal interest rate. The capital stock evolves according to

$$i_t = k_{t+1} - (1 - \delta)k_t, \quad (4)$$

where δ is the constant rate of depreciation. The household's intratemporal first order conditions representing optimal choice of labor input and consumption are given by

$$\lambda_t w_t = \chi, \quad (5)$$

and

$$\lambda_t = 1/c_t, \quad (6)$$

and the intertemporal first order conditions representing optimal choice of bond-holdings and investment (or next period's capital stock) are given by

$$\frac{\lambda_t}{P_t} R_t^{-1} = \beta \cdot E_t \frac{\lambda_{t+1}}{P_{t+1}} \quad (7)$$

and

$$\lambda_t = \beta \lambda_{t+1} (q_{t+1} + 1 - \delta) \quad (8)$$

In these equations, the variable λ_t is the Lagrange multiplier on the budget constraint for period t – it can also be thought of as the marginal utility of an additional unit of consumption at time t .

2.2 Firms

Firms face a cost (ξ_t) in terms of final goods of changing the nominal price of the good they produce (z):

$$\xi_t(z) = \frac{\theta}{2} \left(\frac{P_t(z)}{P_{t-1}(z)} - 1 \right)^2. \quad (9)$$

There is a Cobb-Douglas technology in labor and capital.

Given the price they charge, firms choose labor and capital services to minimize the cost of meeting demand. Factor demands are thus given by

$$\psi_t (1 - \gamma) \left(\frac{k_t}{h_t} \right)^\gamma = w_t, \quad (10)$$

and

$$\psi_t \gamma \left(\frac{h_t}{k_t} \right)^{1-\gamma} = q_t, \quad (11)$$

where ψ_t is real marginal cost.

An individual firm chooses its price each period to maximize the expected present value of profits, where profits in any single period are given by revenue minus costs of production minus costs of price adjustment. The demand curve facing each firm is $y_t(z) = (P_t(z)/P_t)^{-\varepsilon} y_t$, so the profit maximization problem for firm z is

$$\begin{aligned} \max_{P_{t+j}(z)} \sum_{j=0}^{\infty} \beta^j \left(\frac{\lambda_{t+j}}{\lambda_t} \right) & \left[\frac{P_{t+j}(z)}{P_{t+j}} \left(\frac{P_{t+j}(z)}{P_{t+j}} \right)^{-\varepsilon} y_{t+j} \right. \\ & \left. - \psi_{t+j} \left(\frac{P_{t+j}(z)}{P_{t+j}} \right)^{-\varepsilon} y_{t+j} - \frac{\theta}{2} \left(\frac{P_{t+j}(z)}{P_{t+j-1}(z)} - 1 \right)^2 \right]. \end{aligned}$$

The first term in the square brackets is the real revenue a firm earns charging a price $P_{t+j}(z)$ in period $t+j$; it sells $(P_{t+j}(z)/P_{t+j})^{-\varepsilon} y_{t+j}$ units of goods for relative price $P_{t+j}(z)/P_{t+j}$. The second term in square brackets (in the second line of the expression) is the real costs the firm incurs in period $t+j$, number of goods sold multiplied by marginal cost, which is equal to average cost. Finally, the third term in square brackets is the real cost of adjusting the nominal price from $P_{t+j-1}(z)$ to $P_{t+j}(z)$. Note that the price chosen in any period shows up

only in two periods of the infinite sum. Thus, the part of the objective function relevant for the choice of a price in period t is

$$\begin{aligned} & \frac{P_t(z)}{P_t} \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} y_t - \psi_t \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} y_t - \\ & \frac{\theta}{2} \left(\frac{P_t(z)}{P_{t-1}(z)} - 1 \right)^2 - \beta \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \frac{\theta}{2} \left(\frac{P_{t+1}(z)}{P_t(z)} - 1 \right)^2. \end{aligned}$$

The first order condition is:

$$\begin{aligned} & (1 - \varepsilon) \frac{1}{P_t} \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} y_t + \varepsilon \psi_t \frac{1}{P_t} \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon-1} y_t \\ & - \theta \frac{1}{P_{t-1}(z)} \left(\frac{P_t(z)}{P_{t-1}(z)} - 1 \right) + \beta \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \theta \frac{P_{t+1}(z)}{P_t(z)^2} \left(\frac{P_{t+1}(z)}{P_t(z)} - 1 \right) = 0. \end{aligned}$$

If we multiply both sides by P_t and impose symmetry – that is, assume that all firms choose the same price in any given period, the expression simplifies to

$$\begin{aligned} & (1 - \varepsilon) y_t + \varepsilon \psi_t y_t \\ & - \theta \pi_t (\pi_t - 1) + \beta \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \theta \pi_{t+1} (\pi_{t+1} - 1) = 0. \end{aligned}$$

2.3 Market clearing, monetary policy and equilibrium

Because goods are produced for consumption and investment as well as for accomplishing price adjustment, the market clearing condition is

$$y_t = k_t^\gamma h_t^{1-\gamma} = c_t + i_t + \frac{\theta}{2} (\pi_t - 1)^2, \quad (12)$$

where y_t denotes total output of the composite good, π_t denotes the gross inflation rate (P_t/P_{t-1}), and we have imposed symmetry across firms, meaning that all firms choose the same price.

Using the output definition (12), labor market demand (10), and the household's optimality conditions, (5) and (6), this equation simplifies to a form that we will refer to as the New Keynesian Phillips Curve:²

$$\frac{1}{\theta} \left[\varepsilon - 1 - \varepsilon \left(\frac{\chi c_t}{1 - \gamma} \left(\frac{h_t}{k_t} \right)^\gamma \right) \right] k_t^\gamma h_t^{1-\gamma} + \pi_t (\pi_t - 1) = \beta \frac{c_t}{c_{t+1}} \pi_{t+1} (\pi_{t+1} - 1) \quad (13)$$

where π_t is the gross inflation rate.

Finally, monetary policy is given by a nominal interest rate rule, with the current nominal interest rate responding to the current inflation rate:

$$R_t = 1 + (\pi^*/\beta - 1) (\pi_t/\pi^*)^b. \quad (14)$$

²We should note that the term “New Keynesian Phillips Curve” typically refers to the linearized version of (13).

Combining the policy rule with the household's intertemporal first order condition (7), using the definition of the inflation rate to eliminate the price level, and using the household's intratemporal first order condition (6) to eliminate λ , we have

$$\pi_{t+1} \frac{c_{t+1}}{c_t} = \beta \left[1 + \left(\frac{\pi^*}{\beta} - 1 \right) \left(\frac{\pi_t}{\pi^*} \right)^b \right] \quad (15)$$

The model has now been reduced to four first order nonlinear difference equations in the variables c_t, k_t, π_t, h_t and the equations are (13) and (15) and

$$\frac{c_{t+1}}{c_t} = \beta \left(\chi c_{t+1} \frac{\gamma}{1-\gamma} \frac{h_{t+1}}{k_{t+1}} + 1 - \delta \right), \text{ and} \quad (16)$$

$$k_{t+1} = k_t^\gamma h_t^{1-\gamma} - c_t + (1-\delta)k_t - \frac{\theta}{2}(\pi_t - 1)^2. \quad (17)$$

Equation (16) is the FOC for capital accumulation (8) after solving (10) and (11) for the capital rental rate in terms of the real wage, and then substituting the household optimal labor supply conditions (5) and (6) for the real wage. Equation (17) is the resource constraint.

The system simplifies even further if we write it in terms of the labor/capital ratio instead of labor. That is, define $g_t \equiv h_t/k_t$ and then we have the following four equation system that is linear in capital:

$$\frac{1}{\theta} \left[\varepsilon - 1 - \varepsilon \left(\frac{\chi c_t}{1-\gamma} g_t^\gamma \right) \right] k_t g_t^{1-\gamma} + \pi_t(\pi_t - 1) = \beta \frac{c_t}{c_{t+1}} \pi_{t+1}(\pi_{t+1} - 1) \quad (18)$$

$$\pi_{t+1} \frac{c_{t+1}}{c_t} = \beta \left[1 + \left(\frac{\pi^*}{\beta} - 1 \right) \left(\frac{\pi_t}{\pi^*} \right)^b \right] \quad (19)$$

$$\frac{c_{t+1}}{c_t} = \beta \left(\chi c_{t+1} \frac{\gamma}{1-\gamma} g_{t+1} + 1 - \delta \right) \quad (20)$$

$$k_{t+1} = k_t g_t^{1-\gamma} - c_t + (1-\delta)k_t - \frac{\theta}{2}(\pi_t - 1)^2. \quad (21)$$

3 Dynamics without capital

As a warm-up for the full model, it is useful to look at the version without capital.³ In this case, output is produced with labor only. It is much simpler to study the dynamics without capital because there are no predetermined variables in the dynamic system. As in Hursey and Wolman (2010), the dynamic system describing equilibrium can be written in the form of two equations in

³The basic ideas in this section were first described in BSU (2001a).

inflation and consumption⁴:

$$(\pi_t - 1) \pi_t = \left(\frac{c_t}{\theta} + \frac{(\pi_t - 1)^2}{2} \right) (1 - \varepsilon + \chi \varepsilon c_t) + \beta \left(\frac{c_t}{c_{t+1}} (\pi_{t+1} - 1) \pi_{t+1} \right), \quad (22)$$

$$\left(\frac{c_t}{\pi_{t+1} c_{t+1}} \right)^{-1} = \beta \left(1 + (\pi^*/\beta - 1) (\pi_t/\pi^*)^b \right). \quad (23)$$

There is a steady state equilibrium in which inflation is equal to its target value, π^* , and consumption is determined by solving (22) for c , given $\pi_t = \pi_{t+1} = \pi^*$. If we impose the parameter restriction that $b(\pi^* - \beta) > \pi^*$, then local to the targeted steady state the policy rule has the form of an active Taylor rule.⁵

$$R_t = (\pi^*/\beta) + b \left(\frac{\pi^* - \beta}{\beta \pi^*} \right) (\pi_t - \pi^*).$$

Assuming that the restriction on b holds, both roots of the linearized system have modulus greater than one, and thus the only equilibrium that is not locally explosive is the targeted steady state itself. This kind of result underlies the appeal of active Taylor rules: most applied monetary policy analysis relies on local approximations and thus local properties must be used to select equilibria. {but footnote about Cochrane et al}.

Although the local nonexplosive dynamics are unique around the targeted steady state, it turns out that there is a second steady-state equilibrium with a lower inflation rate. To see this, impose a steady state on the second equation, (23):

$$\pi = \beta \left(1 + (\pi^*/\beta - 1) (\pi/\pi^*)^b \right).$$

As a function of π , the right hand side is increasing, strictly convex, and equal to β when $\pi = 1$. There are two solutions to the equation, and thus two steady state equilibria. One of the steady states is of course the targeted steady state ($\pi = \pi^*$). Given the restriction we imposed on β , the second steady state inflation rate is lower than π^* . Again, consumption in the steady state is determined by (22), given inflation.

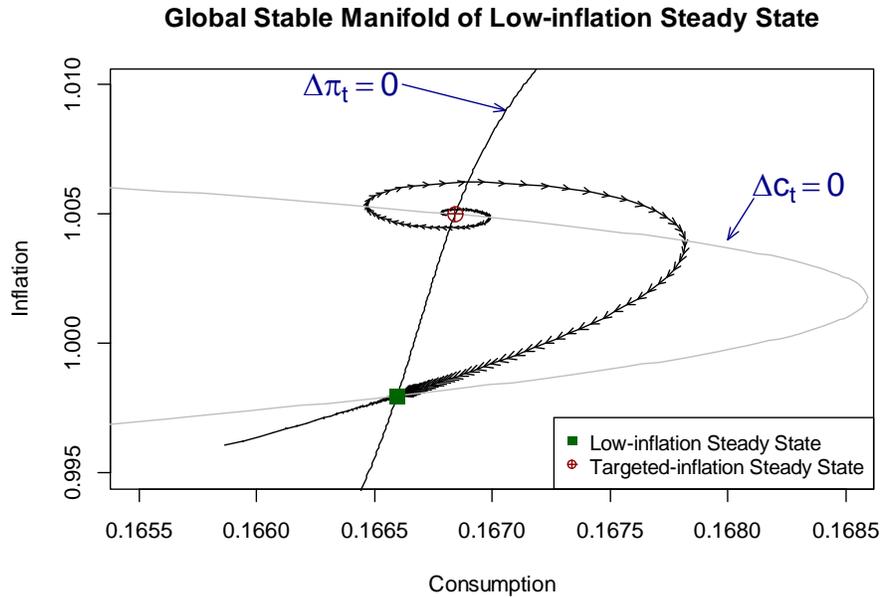
If we linearize the dynamic system (22)-(23) around the low-inflation steady state, one of the eigenvalues has modulus less than one, meaning that locally there are multiple stable equilibria. In other words, as long as we choose from within a small enough neighborhood of the steady state, we can choose an arbitrary initial condition for either consumption or inflation; the corresponding

⁴In Hursey and Wolman (2010), the policy rule responds to future inflation, so the second equation has future inflation rather than current inflation on the right hand side. Qualitatively the dynamics are the same in the two cases. The similarity does not carry over to the model with capital, which is why we use the current-inflation rule here.

⁵Note that the policy rule is active (coefficient on inflation greater than one) if $b(\pi^* - \beta) > \pi^*$. We impose the slightly stronger restriction in the text in order to guarantee that there is a second steady state with a lower inflation rate.

initial condition for the other variable will be determined by a particular linear restriction, and from that initial condition consumption and inflation will converge to the low inflation steady state.

Summing up then, local to the targeted steady state all paths diverge, whereas local to the low-inflation steady state there are a continuum of paths that converge. These contrasting local dynamics lead naturally to the question, what are the global dynamics? Are there paths that travel from a neighborhood of the targeted steady state to the low steady state? Figure 1 provides the answer in the affirmative, displaying the unique path with the conjectured property.



We compute this path in two steps. First, using the linearized system around the low steady state, we pick a point local to the steady state that lies on the path converging to the steady state. Then, using the nonlinear system (22)-(23) we iterate backwards from that point. Each backward iteration requires the numerical solution of one nonlinear equation: given c_{t+1} and π_{t+1} , (23) yields an explicit expression for c_t as a function of π_t , and using that expression in (22) results in an implicit function for π_t in terms of c_{t+1} and π_{t+1} .

What is the economics behind Figure 1? At the targeted steady state, since the inflation rate is at its target, the monetary policy rule sets the nominal interest rate such that the ex ante real interest rate is equal to β^{-1} . Consumers are thus happy to have constant consumption, and firms choose price increases that exactly produce the targeted inflation rate. Along the path traveling to the low-inflation steady state, consumption and inflation are both changing. Consider

the “final” segment of this path, in which inflation begins at the targeted steady state, and consumption is first rising and then falling. In the initial period of this segment, with inflation on target, how can it be an equilibrium for consumption to be above its targeted steady state level? With inflation on target, the policy rule sets the nominal interest rate at its targeted steady state level. However, because consumption is expected to rise, the ex-ante real interest rate is above its steady state level. This means that expected inflation must be below target. Subsequently, because of the active Taylor rule, realized declines in inflation prompt larger decreases in the nominal interest rate, meaning that consumption growth or inflation must decline. Consumption growth indeed becomes negative and inflation continues to decline. As the path approaches the lower steady state, the policy rule effectively becomes less active – local to the low-inflation steady state (π_L), the policy rule is given by

$$R_t = 1 + (\pi^*/\beta - 1) (\pi_L/\pi^*)^b + b \left(\frac{\pi^* - \beta}{\beta\pi^*} \right) (\pi/\pi^*)^{b-1} (\pi_t - \pi_L),$$

which has slope less than one. Thus, as the path approaches the steady state, the effects described above weaken, until at the steady state the system is at rest.

4 Dynamics with capital

We find that similar dynamics are present in the model with capital. That is, the targeted steady state has local dynamics characterized by determinacy; the low-inflation steady state has local dynamics characterized by indeterminacy (one too-many small roots); and there is at least one equilibrium connection leading from a neighborhood of the targeted steady state to the low-inflation steady state. Conceptually, the dynamics are complicated by the endogenous state variable.

Recall that the equilibrium conditions for the model with capital are given by (18) - (21). Again, there is a steady state with inflation equal to its targeted value, π^* . In this steady state, consumption (c^*) is implicitly given by the following equation:

$$\frac{1}{\theta} \left[\varepsilon - 1 - \varepsilon \left(\frac{\chi c^*}{1 - \gamma} \Gamma(c^*)^\gamma \right) \right] \left(\frac{c^* + \frac{\theta}{2}(\pi^* - 1)^2}{1 - \frac{\delta}{\Gamma(c^*)^{1-\gamma}}} \right) + (1 - \beta) \pi^* (\pi^* - 1) = 0, \quad (24)$$

where $\Gamma(c)$ is the steady-state labor to capital ratio:

$$g^* = \Gamma(c^*) = \frac{\beta^{-1} - (1 - \delta)}{\chi(\gamma/(1 - \gamma))} \left(\frac{1}{c^*} \right). \quad (25)$$

The capital stock in the targeted steady state is

$$k^* = \frac{c^* + \frac{\theta}{2}(\pi^* - 1)^2}{\Gamma(c^*)^{1-\gamma} - \delta}. \quad (26)$$

Note that the presence of capital has no effect on the policy rule equation in a steady state equilibrium:

$$\pi = \beta \left[1 + \left(\frac{\pi^*}{\beta} - 1 \right) \left(\frac{\pi}{\pi^*} \right)^b \right]. \quad (27)$$

From invariance of the steady-state policy rule to capital, three properties follow directly:

if $b(\pi^* - \beta) > \pi^*$ then

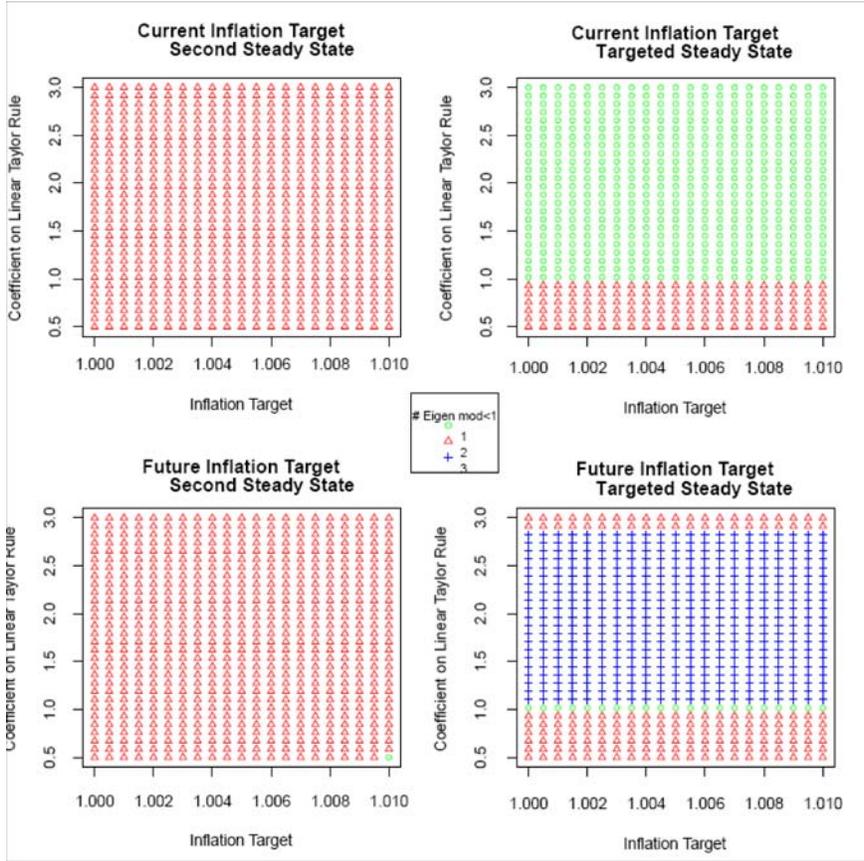
1. policy is active local to the targeted steady state.
2. there is a second steady state, with $\pi = \pi_L < \pi^*$ such that

$$\pi_L = \beta \left(1 + (\pi^*/\beta - 1) (\pi_L/\pi^*)^b \right).$$

3. policy is passive local to the low-inflation steady state.

From property 1, we can compute c, g and k in the low-inflation steady-state with (24)-(26), replacing the $*$ superscript in those equations with an L subscript. Although the local properties of the policy rule do not prove anything about the local properties of the dynamic system, the fact that the policy rule's properties are the same as in the model without capital suggests that the dynamic system may also have similar properties.

We study the dynamic system numerically, beginning with the local properties of the two steady states. The top panels of Figure 2 describe the local stability properties around the lower and targeted (upper) steady states for a range of π^* and b . The horizontal axis plots π^* and the vertical axis plots $f \equiv b \left(\frac{\pi^* - \beta}{\beta \pi^*} \right)$, which is the coefficient on inflation in the linear approximation to the policy rule around the targeted steady state. For each point on the grid of values for π^* and f , the symbol in the plot indicates the number of stable eigenvalues in the linear approximation to the difference equation system around the relevant steady state. A circle indicates one stable eigenvalue, a Δ indicates two stable eigenvalues, and a $+$ indicates three. Because there is one predetermined variable (capital), the locally nonexplosive dynamics are unique if there is one stable eigenvalue.



We focus here on the top two panels, which cover the local dynamics around the low and high-inflation steady states when the policy rule responds to current inflation. More narrowly, consider the regions of these figures corresponding to a policy rule which is active around the targeted steady state, the region above 1.0 on the y-axis. In every case, the model has a unique locally nonexplosive equilibrium around the targeted steady state, with the same number of stable eigenvalues as state variables (i.e., one). Around the lower steady state, in every case there are multiple locally nonexplosive equilibria, with two stable eigenvalues against the one state variable. These local determinacy properties mimic the model without capital, so we proceed to the next step, which involves numerical analysis of global equilibria. The main question we wish to answer is whether the local dynamics around the targeted steady state accurately describe global equilibria: are the only equilibria that start near the targeted steady state the locally unique equilibria that remain near the targeted steady state?

To determine whether the local dynamics are misleading, we will describe particular equilibrium paths for the nonlinear system (18) - (21), using the parameter values in Table 1. In some cases, finding such paths is straightforward: simply pick an initial condition that lies on the local stable manifold of the

low-inflation steady state, use the local linear dynamics as a starting value for a nonlinear equation solver, and watch the solver quickly converge to a solution to the nonlinear system. Once we have such a solution, we can also use the initial conditions as *terminal* conditions, and run the model *backwards* to get an extension of the same path. Given terminal conditions c_T, π_T, k_T and g_T , it is straightforward to compute the backward dynamics:

1. From (20),

$$c_{T-1} = \frac{c_T}{\beta \left(\chi c_T \frac{\gamma}{1-\gamma} g_T + 1 - \delta \right)} \quad (28)$$

2. From (19),

$$\pi_{T-1} = \pi^* \left(\frac{\beta^{-1} \pi_T \frac{c_T}{c_{T-1}} - 1}{(\pi^*/\beta) - 1} \right)^{1/b}, \quad (29)$$

which leaves us with two equations to solve for k_{T-1} and g_{T-1} .

3. Use (21) to eliminate k_{T-1} :

$$k_{T-1} = \Theta(g_{T-1}) \equiv \frac{k_T + c_{T-1} + \frac{\theta}{2}(\pi_{T-1} - 1)^2}{g_{T-1}^{1-\gamma} + 1 - \delta},$$

and then from (18) have one equation in one unknown (g_{T-1}):

$$\frac{1}{\theta} \left[\varepsilon - 1 - \varepsilon \left(\frac{\chi c_{T-1}}{1-\gamma} g_{T-1}^\gamma \right) \right] \Theta(g_{T-1}) g_{T-1}^{1-\gamma} + \pi_{T-1}(\pi_{T-1} - 1) = \beta \frac{c_{T-1}}{c_T} \pi_T (\pi_T - 1), \quad (30)$$

which can be solved numerically for g_{T-1} .

4. Set $k_T = k_{T-1}$, $c_T = c_{T-1}$, $\pi_T = \pi_{T-1}$ and $g_T = g_{T-1}$ and return to step 1.

Benchmark Parameters

β	0.99
θ	17.5
ε	6
χ	5
b	90
δ	0.025
γ	1/3
π^*	1.005

Table 1.

Applying this approach in an ad-hoc manner to a large number of initial conditions teaches us two things. First, in one sense, the answer to the question posed above is no: the local dynamics around the targeted steady state do not accurately describe global equilibria. According to the local dynamics, for

an initial capital stock close to the targeted steady state there is a unique nonexplosive equilibrium path, and it leads to the targeted steady state. To the contrary, we find that there are many other nonexplosive equilibrium paths (presumably a continuum, though we have not proved this) that originate with the same capital stock, close to or even equal to its level in the targeted steady state. Figure 3a and b plot inflation, consumption, capital and the labor/capital ratio for two equilibrium paths, both originating from the capital stock in the targeted steady state. In both cases, the equilibrium path converges to the low-inflation steady state. In Figure 3.a the initial inflation rate is equal to the inflation target, and in Figure 3.b the initial inflation rate is equal to its value in the low steady state.⁶

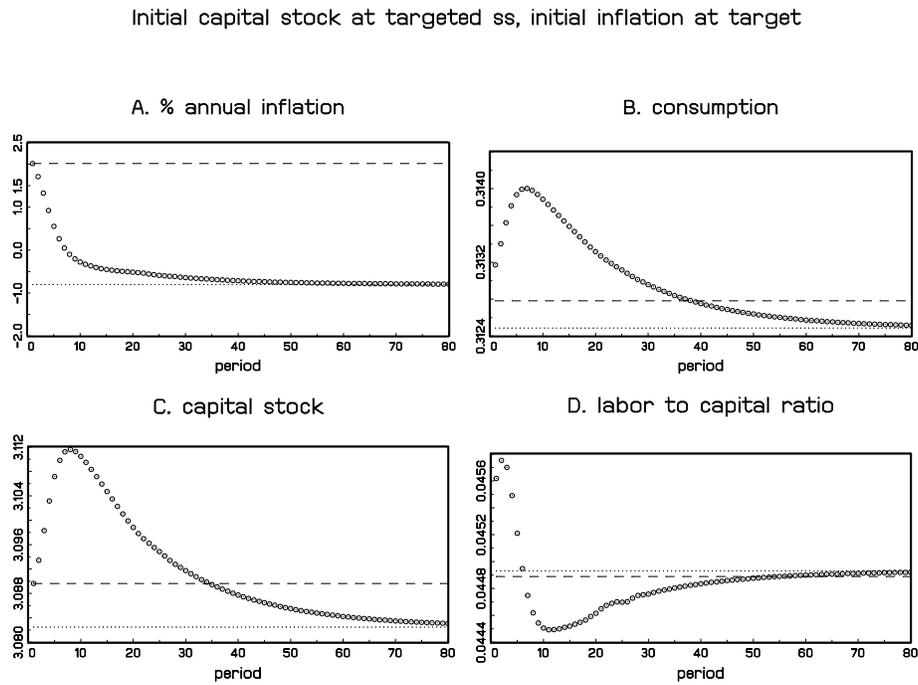


Figure 3.a

⁶That path satisfies the true nonlinear system of difference equations. However, its properties are essentially those of the local dynamics.

Initial capital stock at targeted ss, initial inflation at low ss

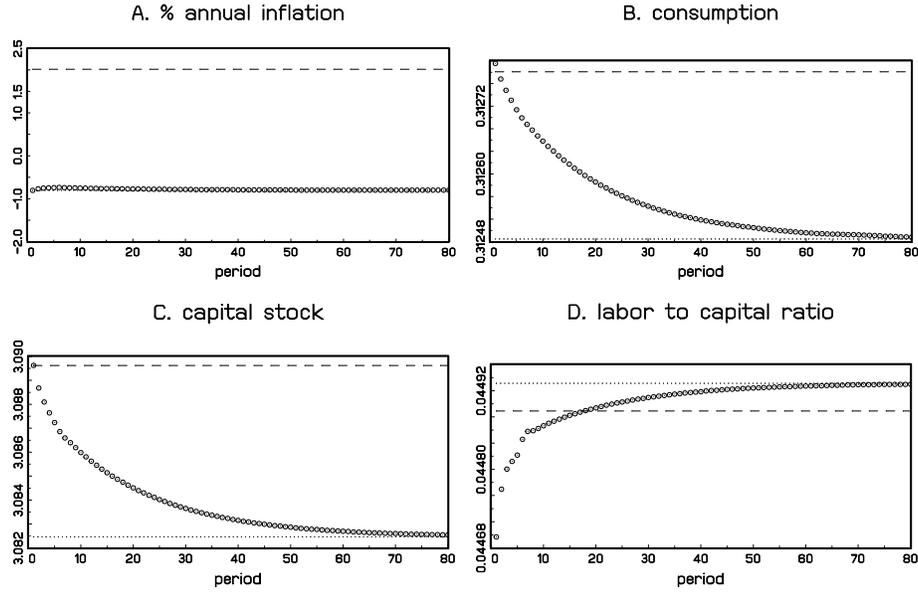


Figure 3.b

The second lesson from ad-hoc numerical analysis is that unlike the model without capital, a path that converges to the low-inflation steady state does not necessarily have its source at the targeted steady state. In fact, there is just one such path, and it is displayed in Figure 4. This path is the analogue for the model with capital to the c, π locus in Figure 1. There are two features that mark the path in Figure 4 as qualitatively different than the paths in Figure 3. First, the path in Figure 4 starts within a small neighborhood of the targeted steady state for all variables, whereas the paths in Figure 3 start only with the steady state capital stock. Second, the path in Figure 4 displays oscillations that first are explosive and then dampened, whereas the paths in Figure 3 display at most an initial hump shape.

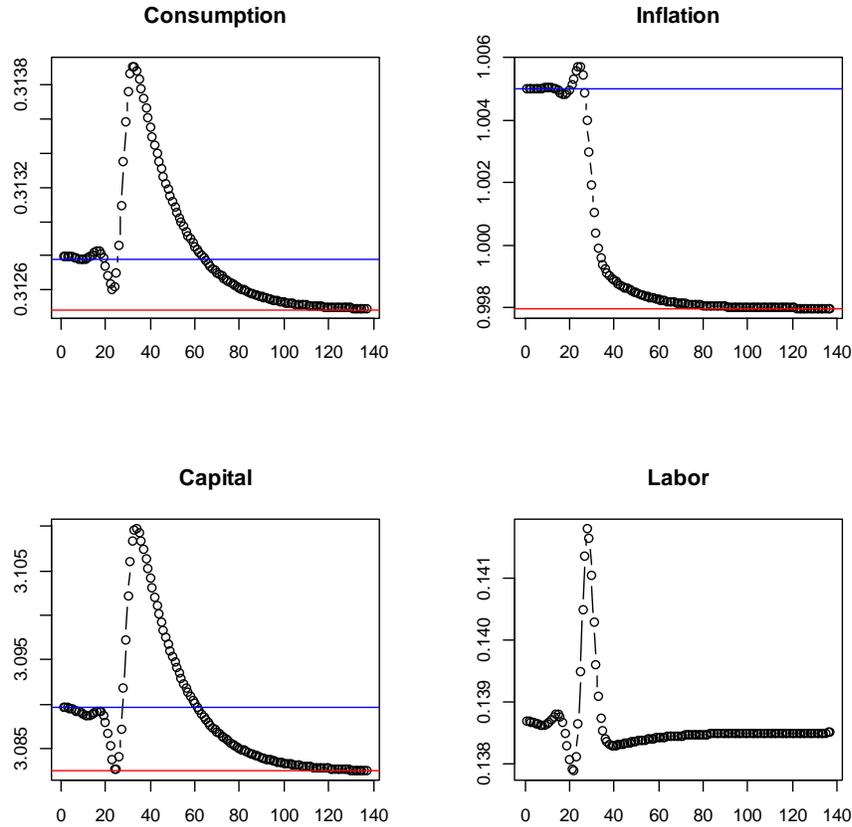


Figure 4. Equilibrium connection between steady states.

- discuss dynamics for real and nominal variables in context of U.S. data for last 5 years.
- discuss behavior with alternative timing in the policy rule.
- relate the results to other papers on determinacy etc. with capital (Carlstrom-Fuerst, Dupor, Benhabib-Eusepi)
- relate to other papers on nonlinear dynamics, ZLB in DSGE models (Aruoba & Schorfheide; Fernandez-Villaverde et al.; Braun, Koerber & Waki)

5 Conclusion

Because capital, as a stand-in for all real endogenous state variables, is a central element in the economy, it is important that lessons from economic theory for

policy analysis be tested on models that include capital. One lesson for monetary policy that has received much recent attention is that active Taylor rules may not guarantee uniqueness of equilibrium, and further that these rules may put the economy at risk of converging to a deflationary equilibrium when the central bank instead wishes to achieve a positive targeted inflation rate. Until now this lesson has not been tested on models with capital. We show that in particular numerical examples (natural benchmarks, in our view) the basic conclusion from models without capital is verified: local dynamics suggest that equilibrium is unique around the targeted inflation steady state, but the global dynamics reveal that paths that originate near the high-inflation steady state in fact converge to the low inflation steady state.

Our paper stops far short of arguing that the U.S. economy's dynamics over the last three years represent a path from the Federal Reserve's targeted inflation rate to a steady state inflation rate associated with the lower bound on nominal interest rates. However, interest rates have been near their lower bound over that period, while inflation has shown signs of drifting downward from the Federal Reserve's 2 percent target. It is therefore important to investigate whether *conventional* policy – an active Taylor rule together with the zero bound, is pushing the economy away from the inflation target, despite the Fed's efforts with unconventional policy. Our paper represents an initial step in that investigative process.

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