We analyze investment incentives and risk-taking by firms when equity markets aggregate information with noise. Noisy information aggregation drives a wedge between the expected social value and the market value of investments, inducing inefficient rent-seeking by incumbent shareholders and corporate short-termism. Excessive risk taking is particularly severe if upside risks are coupled with near constant returns to scale, in which case even small market frictions lead to negative social value, but large shareholder rents. The optimal contract design incentivizes excessive risk-taking through the grant of stock options, or discourages risk-taking through through the use of compensation ceilings. Finally we compare different regulatory or policy interventions and argue that limiting executive incentives to restricted equity contracts may be the most effective way to eliminate investment distortions.

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1 Introduction

Asset prices play a central role in aggregating information about the value of firms. By pooling together the dispersed knowledge of individual actors, prices provide information that shapes investor expectations about a firm’s future earnings. If financial markets are efficient (in the sense that the firm’s share price equals expected future dividends conditional on all available information) and there are no direct externalities from firm decisions, the share price also aligns shareholder returns to investment with social returns, even if the firms’ shareholders sell their shares before the returns from such investments are realized. What’s more, in publicly traded companies where decision-making is delegated to managers, shareholders can align the managers’ preferences with their own, and by extension, society’s, through appropriately linking managerial compensation to share prices. Under this worldview, unregulated financial markets will induce socially efficient firm decisions and channel available financial resources to their most productive uses. Any attempt to regulate the firms’ investment behavior, restrict contracting with managers, or intervene in financial markets can only have negative welfare effects.

Critics of this view argue that if left unregulated, firms and managers engage in rent-seeking behavior, sacrificing long-term fundamental values to maximize short-term market value. This corporate short-termism is seen as socially harmful if it results in sometimes dramatic failures of major corporations, coupled with costly interventions by governments to mitigate the economic consequences of such failures. This raises important questions regarding the optimal regulation of firm behavior. What departures from market efficiency induce corporate short-termism and inefficient risk-taking? If corporate decisions expose taxpayers and society to excessive financial or technological risks, is there scope for society to intervene and regulate corporate decision-making? If so, what form should such regulation take? How does it depend on the firm’s characteristics, the market environment, and the risks to which it exposes society? And how robust are the lessons of the laisser-faire approach to small departures from market efficiency?

To answer these questions, we propose a theory of corporate investment and risk-taking when financial markets aggregate information with noise. Consider a setting in which the incumbent shareholders of a firm take an investment decision, or delegate this decision to a manager, before selling a fraction of their shares in a financial market populated by informed and noise traders. The share price then emerges as a noisy signal pooling the dispersed information investors have about the firm’s value. We explore how the information aggregation friction in the financial market affects the initial shareholder’s ability to capture the returns from their investment, under what
conditions the firm’s decisions depart from socially efficient levels, and how this is reflected in managerial incentive contracts. Finally, we return to the regulatory questions and argue that even small frictions in financial markets provide an important rationale for regulating firm behavior.\footnote{Although our model does not focus on financial risks and financial corporations per se, our results suggest that the incentive problems we highlight are particularly pronounced in this sector, whose activity is motivated by the very frictions that are at the heart of our analysis.}

We formulate the firm’s investment decision as the outcome of an optimal contracting problem between initial shareholders and the firm’s managers, which internalizes the effects of investment on the subsequent share price. Our results all follow from a simple but central observation: with noisy information aggregation and limits to arbitrage, the price is not just a noisy but also a biased estimate of the firm’s dividends. This bias generates a wedge between the expected market returns to investment and its social value, which induces rent-seeking by initial shareholders.

More specifically, market-clearing implies that the price must adjust to stochastic shifts in demand and supply of shares, whenever there are limits to arbitrage, i.e. whenever informed traders have limited ability or willingness to adjust asset holdings to absorb shocks. These shifts appear on top of the information conveyed through the price, and thus imply that the share price generically reacts too much to the information that is aggregated through the market, relative to the shares’ expected dividend value. What’s more, the magnitude of this distortion is linked to uncertainty about firm dividends and scales with the firm’s investment choice.

From an ex ante perspective, the incumbent shareholders’ marginal return from investing is no longer aligned with the firm’s expected dividend value. This introduces a rent-seeking motive to their incumbent shareholder’s incentives, i.e. the expected gap between the share price and the dividend value is a pure transfer from final to initial shareholders, whose sign and magnitude depend on the characteristics of the underlying dividend return and on the investment scale. If the firm’s investment is characterized by upside risk, the expected market return exceeds expected dividend return, and the firm invests more than the efficient level. If instead the investment is characterized by downside risk, the expected market return falls short of dividend values and the firm invests too little. These inefficiencies become particularly pronounced if the firm’s technology has nearly constant returns to scale, in which case (i) the surplus from the investment is small, and (ii) the firm’s investment choice is highly sensitive to return perceptions, so that the scope for rent-seeking becomes very large. If near constant returns are coupled with upside risks, even small frictions in financial markets can have very large efficiency consequences – so large in fact that the firm generates negative expected surplus, while incumbent shareholders take excessive risks purely
in an attempt to maximize the financial market rent they capture from selling their shares.

We then turn to the optimal design of managerial contracts. Following standard arguments, agency costs disappear and any investment level is implementable, if the manager is risk-neutral. Inefficiencies therefore stem solely from a misalignment between incumbent shareholder preferences and social surplus that is caused by market frictions, and not from agency frictions with managers. In other words, giving incumbent shareholders full discretion to design performance contracts merely gives them all the tools they need to optimize their rents. If incumbent shareholders benefit from excessive risk-taking (i.e. the case of upside risks or informational feedbacks), the implemented contract will take the form of stock option compensation. Generally speaking, the incumbent shareholders will exploit any discretion in contract design to distort incentives to their advantage, which completely overturns the laisser-faire argument against regulation of performance pay.

Finally, we discuss the potential for regulatory or policy interventions to mitigate rent-seeking incentives. As our main result, we show that a simple restriction on executive pay, namely ruling out anything other than untraded restricted equity contracts which implement the socially optimal investment decision, is sufficient and (generically) necessary for implementing socially efficient investment decisions, without requiring any further knowledge of the firm’s characteristics or the financial market structure. Other policies, such as direct regulation of investment activities, financial transaction taxes and market interventions such as asset purchasing programs can play a similar role in correcting the investment inefficiencies, but in those cases the optimal intervention requires far more knowledge of firm or market characteristics.

Related Literature. Our modeling of the financial market builds on the literature on noisy information aggregation through stock prices (Grossman and Stiglitz, 1980, Hellwig, 1980, Diamond and Verrecchia, 1981). We make use of the formulation in our earlier paper (Albagli, Hellwig and Tsyvinski, 2011) which enables us to offer return implications for arbitrary securities in a non-linear noisy REE model. Here this model serves as the baseline ingredient for analyzing the link between investment and market returns. Non-linear, non-normal returns are important for defining the notions of upside and downside risks, and the corresponding investment distortions.

An extensive literature has studied the link between managerial incentives and financial markets. The classic paper by Holmstrom and Tirole (1993) emphasizes the positive aspects of tying compensation to market performance when prices accurately reflect future dividend expectations.

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2 In Albagli, Hellwig, and Tsyvinski (2011 and 2013), we further argue that our model can account for important stylized facts on asset returns such as a negative relation between skewness and asset returns, or the so-called credit spread puzzle for corporate bonds. Further quantitative investigations are planned or in progress.
and therefore align private and social costs and benefits from investing. In our analysis instead, market signals contain valuable information, but they are not unbiased. What’s more, these biases in turn feed back into ex ante incentives to invest, thus resulting in investment distortions.

Some papers emphasize the limitations of market-based incentive schemes. Closest to our work are Bolton, Scheinkman, and Xiong (2006). As in our paper, corporate short-termism is in the interest of incumbent shareholders who design contracts that induce inefficient managerial decisions aimed at boosting short-term prices. Different from us, they model information frictions by considering traders that “agree to disagree” about the informativeness of profit signals, which creates a speculative component in prices that can be exploited by managerial decisions. In our model instead mispricing depends on the distribution of cash-flow risks, allowing the possibility of both systematic over- and under-valuation of securities, and over- and under- investment. Moreover, our model features a common prior and imposes no restrictions on contracts which enables us to go much further in discussing contract design, welfare and optimal regulatory interventions.

Other literature instead emphasizes distortions resulting from agency conflicts between shareholders and manager, when managers have superior information and they may take costly actions to manipulate the firms’ earnings. Stein (1988) argues that earnings manipulation may result even if there is no conflict of interest, if the asymmetry leads to temporary price dips that invite takeovers at disadvantageous prices. Stein (1989) argues that earnings manipulation is pervasive if price-based compensation induces short-termism by managers, even when in equilibrium shareholders are not fooled on a systematic basis. Benmelech, Kandel, and Veronesi (2010) take a longer-term view by studying managerial compensation throughout the lifecycle of firms. While early on share price compensation leads to high effort by managers, it creates the potential for earnings manipulation when firms mature and growth opportunities decline. While these papers study the link between stock markets and managerial decisions through priced-based compensation, the analysis differs from ours by placing the emphasis on the managers’ ability to manipulate information. In our model instead, the information structure cannot be manipulated by managers or initial shareholders, and managerial short-termism is in the interest of incumbent shareholders.

Finally, several recent empirical studies provide evidence consistent with some of our central predictions. In our model, upside cash-flow risk will systematically lead to excessive investment, the more so the larger the information frictions and the bias towards short-term results by incumbent shareholder. Gilchrist, Himmelberg and Huberman (2005) find that an increase in belief heterogeneity (proxied by analyst forecast dispersion) leads to an increase in new equity issuance, Tobin’s Q, and real investment. Polk and Sapienza (2008) find a positive relation between abnormal invest-
ment and stock overpricing (proxied by discretionary accruals), a pattern that is more prevalent in firms they identify as being more opaque (higher R&D intensity), as well as in firms with shorter shareholders’ horizons (proxied by share turnover). Moreover, excessive investment is typically followed by abnormally low returns, especially in firms with the aforementioned characteristics.

Section 2 introduces our general model and derives the equilibrium characterization in the financial market. Section 3 analyzes rent-seeking behavior by incumbent shareholders, investment distortions and the associated welfare losses, as well as the design of executive compensation contracts. Section 4 discusses and compares regulatory and policy interventions. Section 5 considers various extensions. Section 6 discusses applications of our model to leverage and risk-taking, welfare effects of public disclosures, and informational feedbacks and stock price sensitivity of investment. Section 6 concludes.

2 The model

Our model has three stages. In the first stage, a firm with a unit measure of shareholders contracts with a manager. The manager takes an unobservable investment decision $k \geq 0$ for the firm. In the second stage, the initial shareholders sell a fraction $\alpha \in (0, 1]$ of the shares to outside investors. At the final stage, the firm’s cash flow is realized as a function of $k$ and a stochastic fundamental $\theta \in \mathbb{R}$, and paid to the final shareholders. This cash flow takes the form $\Pi(\theta, k) = R(\theta) k - C(k)$, where $R(\cdot)$ is a positive, increasing function of the firm’s fundamental (interpreted as the return to the investment $k$), and $C(\cdot)$ is an increasing, convex function of $k$, interpreted as the firm’s cost of investment. For normative results, we use the functional form $C(k) = \frac{1}{1+\gamma} k^{1+\gamma}$, where $\gamma$ indexes the firm’s degree of return to scale. The fundamental $\theta$ is distributed according to $\theta \sim \mathcal{N}(0, \lambda^{-1})$. The manager’s compensation $W(\cdot)$ is a function of the final dividend $\Pi$.

We define efficiency from the perspective of the final shareholders, who have to live with the consequences of the decisions taken in stage 1. The ex post efficient investment equals $k^{FB}(\theta) = \psi(R(\theta))$, where $\psi(\cdot) = (C')^{-1}(\cdot)$ is the inverse marginal cost function. The ex ante efficient investment $k^* = \psi(\mathbb{E}(R(\theta)))$ maximizes $\mathbb{E}(\Pi(\theta, k))$, given the information available at stage 1.

Our analysis focuses on the impact of frictions in the financial market at stage 2 on managerial incentives, investment, and contract design at stage 1. Our next subsection introduces the financial market environment and characterizes the market equilibrium, taking as given the outcome of the first stage contract and investment decision. Our financial market model is a variant of the non-linear noisy rational expectations model we introduced in AHT, so this sub-section will closely
follow our earlier work. Afterwards, we will turn to the initial contract design problem.

2.1 Stage 2 – Financial Market

Description of the Financial Market. As a convention, the manager is paid by initial shareholders, so outside investors care only about $\Pi(\theta, k)$. Moreover, investors can infer the equilibrium choice of $k$ from the contract design, so only $\theta$ remains uncertain. There are two types of outside investors: a unit measure of risk-neutral informed traders, who are indexed by $i$, and noise traders.

Informed traders observe a private signal $x_i \sim \mathcal{N}(\theta, \beta^{-1})$, which is i.i.d. across traders (conditional on $\theta$). After observing $x_i$, an informed trader submits a price-contingent demand schedule $d_i(\cdot) : \mathbb{R} \to [0, \alpha]$, to maximize expected wealth $w_i = d_i \cdot (\Pi(\theta, k) - P)$. That is, informed traders cannot short-sell, and can buy at most $\alpha$ units of the shares. Individual trading strategies are then a mapping $d$ from triplets $(x_i, P)$ into $[0, \alpha]$. Aggregating traders’ decisions leads to the aggregate demand $D(\theta, P) = \int d(x, P) d\Phi(\sqrt{3}(x - \theta))$, where $\Phi(\sqrt{3}(x - \theta))$ represents the cross-sectional distribution of private signals $x_i$ conditional on the realization of $\theta$, and $\Phi(\cdot)$ denotes the cdf of a standard normal distribution.

Noise traders place an order to purchase a random quantity $\alpha \Phi(u)$ of shares, where $u \sim \mathcal{N}(0, \delta^{-1})$ is independent of $\theta$, and $\delta^{-1}$ measures the informativeness of the share price.

Once informed and noise traders have submitted their orders, these orders are executed at a price $P$ that satisfies the market-clearing condition $\alpha = D(\theta, P) + \alpha \Phi(u)$ at which the total demand of shares equals the supply $\alpha$.

Let $H(\cdot|x, P)$ denote the traders’ posterior cdf of $\theta$, conditional on observing a private signal $x$, and a market-clearing price $P$. A noisy Rational Expectations Equilibrium at stage 2 consists of a demand function $d(x, P)$, a price function $P(\theta, u; k)$, and posterior beliefs $H(\cdot|x, P)$, such that $d(x, P)$ is optimal given the shareholder’s beliefs $H(\cdot|x, P)$; $P(\theta, u; k)$ clears the market for all $(\theta, u)$ and $k$; and $H(\cdot|x, P)$ satisfies Bayes’ Rule whenever applicable.

Equilibrium Characterization. Suppose that the prior at stage 2 is that $\theta \sim \mathcal{N}(\mu, \lambda^{-1})$, and traders expect an investment level $k$. Our first result characterizes the equilibrium share price in a noisy Rational Expectations Equilibrium. Moreover this is the unique equilibrium in which

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3We assume that the Law of Large Numbers applies to the continuum of traders, so that conditional on $\theta$ the cross-sectional distribution of signal realizations ex post is the same as the ex ante distribution of traders’ signals.

4In our basic model, no new information emerges between the contracting and the financial market stages, so this is the same as the stage 1 prior ($\theta \sim \mathcal{N}(0, \lambda^{-1})$). By keeping a more flexible prior, our equilibrium characterization will serve as a reference for later sections in which we depart from this benchmark.
demand schedules are non-increasing in $P$. Monotonicity restrictions arise naturally if trading takes place through limit orders.

**Proposition 1 Equilibrium Characterization and Uniqueness**

Define $z \equiv \theta + 1/\sqrt{\beta} \cdot u$. In the unique equilibrium in which the informed traders’ demand $d(x, P; k)$ is non-increasing in $P$, the market-clearing price function is

$$P(z, k) = \mathbb{E}(R(\theta) | x = z, z) \cdot k - C(k).$$

(1)

The variable $z$ is normally distributed with mean $\theta$ and precision $\beta \delta$, and serves as a sufficient statistic for the information conveyed through the share price. This characterization of $P(z, k)$ gains its significance from the comparison with the share’s expected dividend value $V(z, k)$:

$$V(z, k) = \mathbb{E}(R(\theta) | z) \cdot k - C(k).$$

(2)

The equilibrium share price differs systematically from the expected dividend value: Both are characterized as expected dividends conditional on the information contained in $z$, but the share price treats the signal $z$ as if it had precision $\beta + \beta \delta$ (equal to the sum of the private and the price signal precision), while in reality its precision is only equal to $\beta \delta$. Hence the market price is based on an expectation of the marginal return to the investment level $k$ that conditions too much on the market signal $z$, relative to its objective information content.

We label the difference between $P(z, k)$ and $V(z, k)$ the *information aggregation wedge* $\Omega(z, k)$:

$$\Omega(z, k) = k \cdot \{\mathbb{E}(R(\theta) | x = z, z) - \mathbb{E}(R(\theta) | z)\}.$$  

(3)

The wedge results from market clearing with heterogeneous information. Both the price and the expected dividend reflect the public information $z$ that is conveyed through the price. However in addition, the price must shift to equate demand and supply: if $\theta$ increases, all traders become more optimistic through the observation of private signals, and hence demand more. Of course demand also increases when $u$ rises, due to the order by noise traders. To compensate for this increase in demand that arises with a shift in $z$, the market price must respond to these shocks by more than what is consistent merely with the information provided through the price. That is why $P(z, k)$ reacts more strongly to the realization of $z$ than $V(z, k)$.

We discussed properties of this wedge at length in our preceding paper.\(^5\) Here we highlight only the interaction of $\Omega(z, k)$ with $k$: since the wedge results from how traders in the market treat

\(^5\)In AHT, we also show that the equilibrium characterization and the core properties of this wedge are robust to general, non-normal distributional assumptions, risk aversion and arbitrary position limits. The functional form assumptions only invite convenient closed-form solutions, but are not important for the economic forces in play.

\(^6\)The only (minor) modification relative to AHT is the normalization of demand by $a$ instead of $1$. This insures that
uncertainty regarding cash-flows, the magnitude of the wedge scales with the firm’s expected risk exposure. Hence the inferred investment level implemented at the first stage influences the extent to which share prices depart from expected dividend values later on. This is the channel through which the initial contract design imposes externalities on final shareholders.

### 2.2 Stage 1 – Contract Design

**Problem Statement.** At the first stage, the incumbent shareholders and the manager agree on a contract, which specifies a targeted investment level \( \hat{k} \) and a compensation scheme \( W(\cdot) \) that is defined as a function of \( \Pi \). The incumbent shareholders and the manager are all risk neutral. The contract has to offer the manager an expected wage payment at least as high as his outside offer \( \overline{\omega} \), and give the manager the incentives to implement the targeted investment level \( \hat{k} \). In addition, incumbent shareholders and managers take as given the market-clearing price function \( P(\theta, u; k) \) that will result at the financial market stage, when designing the initial contract. We start from the case in which the manager has no additional information when choosing \( k \), and will later discuss how the optimal contract design makes use of additional information.

A triplet \((k, W(\cdot), P(\theta, u; k))\) is implementable, if (i) \( P(\theta, u; k) \) is the market-clearing price function of a noisy rational expectations equilibrium in the financial market, and (ii) the manager’s individual rationality and incentive compatibility constraints hold: \( \mathbb{E}\{W(\Pi(\theta, k))\} \geq \overline{\omega} \) and \( k \in \arg \max_{k \geq 0} \mathbb{E}\{W(\Pi(\theta, k))\} \). Since it’s always possible to hold the manager to \( \overline{\omega} \) by rescaling the contract, we ignore the former from now on. The initial shareholders maximize the expected proceeds from the shares (both from the sale at stage 1, and from the dividends at stage 2), net of the wage payments to the manager

\[
\mathbb{E}\{\alpha P(\theta, u; k) + (1 - \alpha) \Pi(\theta, k) - W(\Pi(\theta, k))\}
\]

with respect to the set of triplets \((k, W(\Pi), P(\theta, u; k))\) that are implementable. Proposition 1 greatly simplifies our task. Treating \( z \sim \mathcal{N}(\theta, (\beta \delta)^{-1}) \) as our state variable at the financial market stage, we replace the implementability condition that \( P(\cdot) \) be a market-clearing price function with the unique equilibrium characterization given by 1. We determine optimal contract design in two steps. First, we determine the investment level that maximizes the initial shareholders’ expected payoffs before wage payments, disregarding the incentive compatibility constraint. Then we characterize the compensation scheme that implements the optimal investment level.

\( \alpha \) only isolates changes in the incentive problem, not in the market setting. We could have equivalently normalized the total amount of shares for sale to 1 (along with the position limits and demand by noise traders), along with a total supply of shares to \( \alpha^{-1} > 1 \).
Our efficiency benchmark. Any discussion of welfare and efficiency is complicated by potential distributional effects between initial shareholders, informed traders and noise traders, and remains necessarily incomplete without fully accounting for the noise traders’ motives for buying the shares. Here we are primarily interested in the externalities that the contract design imposes on final shareholders who have no say in the investment decision, but fully suffer its consequences. Hence we have chosen an efficiency benchmark, in which expected dividends are maximized.

Departures from this benchmark are caused by the combination of two elements. First, the initial shareholder’s plan to sell part of their shares. If initial shareholders hold on to all their shares, i.e. in the limiting case where $\alpha \to 0$, they have no interest in deviating from efficiency.

The second element is the wedge between share prices and expected dividends. Consider the alternative efficient markets scenario in which the share price is automatically equated to expected dividend value, i.e. $P(z; k) = V(z; k)$. Efficient markets eliminate the conflict of interest between initial shareholders and managers on the one hand, and final shareholders on the other. The share price motivates initial shareholders to implement the efficient investment level, even though they plan to sell a fraction of their shares. Moreover, the efficient investment level is implemented if $W$ is linear in $\Pi$, i.e. managers are given a participation in the dividend value of the firm. Thus, when market prices align initial and final shareholder incentives, executive compensation that is tied to share price performance aligns shareholder and manager incentives, thereby aligning privately optimal and socially efficient investment choices.

Once prices depart from expected dividend values, initial and final shareholder incentives are no longer aligned. While our analysis emphasizes the role of noisy information aggregation and limits to arbitrage, these are by no means the only potential sources of mis-pricing. Yet, noisy information aggregation has two properties that are key for distorting incentives: First, the information aggregation wedge doesn’t just add noise to stock prices, which would average out from an ex ante perspective, but it responds systematically to the price realization (in other words, $P$ and $P - V$ are not orthogonal to each other). Second, the investment decision taken at the first stage influence its magnitude. Initial shareholders can therefore use the investment decision to influence the rents they extract through mis-pricing of their shares in the market.

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7 This could result for example with free entry of uninformed arbitrageurs as in Kyle (1987). Alternatively, consider the case where (i) there was no private information, and (ii) $z$ simply represented an exogenous public signal with precision $\beta \delta$. Because in this scenario, all informed traders are identical, and their collective wealth exceeds the available supply, it must be that these traders are indifferent between purchasing and not purchasing the shares, which requires that at equilibrium $P(z, k) = V(z, k)$. Efficient markets also emerge as the limiting case of our private information model, when $\beta \to 0$, while $\beta \delta$ is held constant.
3 Investment with Financial Market Frictions

Assume for now that \( C'(0) = 0 \) so that the optimal investment is always positive, and that the market prior is the same as the initial prior, \( \mu = 0 \), and \( \lambda = \lambda' \). From the initial shareholders’ perspective, the optimal value \( \hat{k} \) maximizes \( \mathbb{E} \{ \Pi(\theta, k) + \alpha \Omega(z, k) \} \). The optimal contract implements an investment level \( \hat{k} \) that satisfies

\[
C' \left( \hat{k} \right) = \alpha \mathbb{E} \{ \mathbb{E} (R(\theta) | x = z, z) \} + (1 - \alpha) \mathbb{E} (R(\theta)).
\]

**Proposition 2** Market frictions cause investment distortions.

\( \hat{k} \gtrless k^* \) whenever \( \mathbb{E} \{ \mathbb{E} (R(\theta) | x = z, z) \} \gtrless \mathbb{E} (R(\theta)). \) \( |\hat{k}/k^* - 1| \) is increasing in \( \alpha \).

The optimal contract equates the marginal cost of investment to a weighted average of the expected market return, \( \mathbb{E} \{ \mathbb{E} (R(\theta) | x = z, z) \} \) and the expected dividend return \( \mathbb{E} (R(\theta)) \). The optimal investment level \( \hat{k} \) departs from the efficient investment level \( k^* \) if the expected market return differs from the expected dividend return. The difference between these two expected marginal values corresponds to the marginal impact of \( k \) on the expected information aggregation wedge, \( \mathbb{E} \{ \mathbb{E} (R(\theta) | x = z, z) \} - \mathbb{E} (R(\theta)) \). Whenever the expected wedge is positive, the expected market price exceeds the expected dividend value, hence on average the shares are over-valued. Moreover, since the expected market return to investment exceeds the dividend return the initial shareholders find it optimal to over-invest, to enhance the over-valuation of their shares. When instead the expected wedge is negative, the initial shareholders want to under-invest in order to limit the under-valuation of their shares. Thus through the investment level, initial shareholders influence the expected magnitude of the information aggregation wedge. This expected wedge enters the initial shareholders’ private returns to investing, but not the social returns, because the share price is a pure transfer from the final to the initial shareholders.

**Information aggregation and return characteristics.** Our next result links the extent of over- or under-investment to the return distribution \( R(\cdot) \) and the parameters determining how much noisy information aggregation affects average prices. Drawing on AHT (Section 3), we define

\[
\gamma_P = \frac{\beta (1 + \delta)}{\lambda + \beta + \beta \delta} \quad \text{and} \quad \gamma_N = \frac{\beta \delta}{\lambda + \beta \delta}.
\]

The market’s posterior of \( \theta | z \) is normal with mean \( \gamma_P z \) and variance \( (1 - \gamma_P) \cdot \lambda^{-1} \), while the prior over \( z \) is normal with mean 0 and variance \( \lambda^{-1} + (\beta \delta)^{-1} = 1/\gamma_N \cdot \lambda^{-1} \). Compounding the two distributions, we compute \( \mathbb{E} \{ \mathbb{E} (R(\theta) | x = z, z) \} \) as a "market-implied" prior expectation of \( R(\theta) \), i.e. as the expectation of \( R(\theta) \) with respect to a distribution over \( \theta \) that is normal with
mean zero, and variance \( \lambda^{-1}_p \), where \( \lambda^{-1}_p = \gamma^2_P / \gamma_Y \cdot \lambda^{-1} + (1 - \gamma_P) \cdot \lambda^{-1} > \lambda^{-1} \). From an ex ante perspective the market thus attributes too much weight to the tail realizations of the \( \theta \), relative to the objective prior distribution. The ratio \( \lambda^{-1}_p / \lambda^{-1} \) governs by how much the market overweights the tail realizations, and thereby governs the impact of information aggregation frictions on equilibrium share prices and investment levels. The direction of the distortion then depends on which of the two tail risks is more important. This is formalized by the following partial ordering of return risks \( R(\cdot) \) and the subsequent proposition.

**Definition 1** *Upside and Downside Risk*

(i) A return \( R(\cdot) \) has symmetric risks if \( R'(\theta) = R'(-\theta) \) for all \( \theta > 0 \).

(ii) A return \( R(\cdot) \) is dominated by upside risks, if \( R'(\theta) \geq R'(-\theta) \) for all \( \theta > 0 \). A return \( R(\cdot) \) is dominated by downside risks, if \( R'(\theta) \leq R'(-\theta) \) for all \( \theta > 0 \).

(iii) Consider two returns \( R_1 \) and \( R_2 \) such that \( \mathbb{E}(R_1(\theta)) = \mathbb{E}(R_2(\theta)) \). Then \( R_1 \) has more upside (less downside) risk than \( R_2 \) if \( R_1'(\theta) - R_1'(-\theta) \geq R_2'(\theta) - R_2'(-\theta) \) for all \( \theta > 0 \).

This definition classifies returns by comparing marginal gains and losses at fixed distances from the prior mean to determine whether returns are steeper on the upside or on the downside. The following result is a direct corollary of Theorem 2 in AHT.

**Proposition 3** *Upside (downside) risk induces over- (under-)investment*

(i) **Sign:** If \( R \) has symmetric risk, then \( \hat{k} = k^* \). If \( R \) is dominated by upside risk, then \( \hat{k} > k^* \). If \( R \) is dominated by downside risk, then \( \hat{k} < k^* \).

(ii) **Comparative Statics w.r.t. \( \lambda^{-1}_p \):** If \( R \) is dominated by upside or downside risk, then \( |\hat{k}/k^* - 1| \) is increasing in \( \lambda^{-1}_p \).

(iii) **Comparative Statics w.r.t. \( R \):** If two returns \( R_1 \) and \( R_2 \) have the same expectation, but \( R_1 \) has more upside risk than \( R_2 \), then the investment level \( \hat{k}_1 \) that is associated with \( R_1 \) is strictly larger than the investment level \( \hat{k}_2 \) that is associated with \( R_2 \).

This result classifies firms according to their dividend risk. Returns that have a significant upside component (for example, if \( R(\theta) \) is increasing and convex) will lead to over-investment, while returns that are dominated by the downside risk (for example, if \( R(\theta) \) incorporates mainly the risk of a failure if the fundamentals are low) will lead to under-investment. From an ex ante perspective, the market expectations place too much weight on tail realizations of \( \theta \), relative to their objective probabilities, so if the tail events are dominated by the upside (as in the first scenario), the firms optimal investment is distorted up, while when the tail events are dominated by the
downside (as in the second scenario), the firms’ optimal investment is distorted down. Moreover, for a given expected dividend return, the distortion are larger if the returns are more asymmetric.

**Rent Extraction and Efficiency Losses.** The initial shareholders’ rents from market frictions scale with the investment choice, are positive when $\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} > \mathbb{E}(R(\theta))$, and negative when $\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} < \mathbb{E}(R(\theta))$. Therefore, $\hat{k}/k^*$ measures the ratio between the rents obtained at the optimal contract, and the rents consistent with efficient investment decisions, and $|\hat{k}/k^* - 1|$ measures the amount of rent manipulation. We measure the resulting efficiency loss by the percentage loss in expected dividends, relative to the ex ante efficient benchmark. If

$$V^* = \mathbb{E}(R(\theta)) \cdot k^* - C(k^*)$$

denotes the maximal expected dividend, and $\hat{V} = \mathbb{E}(R(\theta)) \cdot \hat{k} - C(\hat{k})$ the firm dividend expected under the optimal contract, then the efficiency loss is denoted by

$$\Delta = 1 - \frac{\hat{V}}{V^*}.$$ 

If $\Delta > 1$, the distortion is so severe that the investment cost exceeds the expected returns, i.e. expected dividends are negative. After some algebra, $\Delta$ takes the following form:

$$\Delta = 1 - \left(1 + \alpha \left(\frac{\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} - \mathbb{E}(R(\theta))}{\mathbb{E}(R(\theta))} - 1\right)\right)^{1/\gamma} \cdot \left\{1 - \frac{\alpha}{\gamma} \left(\frac{\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} - \mathbb{E}(R(\theta))}{\mathbb{E}(R(\theta))} - 1\right)\right\}.$$ 

Our next result provides comparative statics of $|\hat{k}/k^* - 1|$ and $\Delta$ with respect to the returns to scale $\gamma$ and the ratio $\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} / \mathbb{E}(R(\theta))$ of expected market to dividend returns.

**Proposition 4:** Rent manipulation and efficiency losses increase with market frictions and returns to scale.

(i) **Comparative Statics:** $|\hat{k}/k^* - 1| = \Delta = 0$ only if $\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} = \mathbb{E}(R(\theta))$ or $\gamma \to \infty$. $|\hat{k}/k^* - 1|$ and $\Delta$ are decreasing in $\gamma$ and increasing in $|\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} / \mathbb{E}(R(\theta)) - 1|$. (ii) **Bounded Distortions on the Downside:** If $\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} < \mathbb{E}(R(\theta))$, then $

\lim_{\gamma \to 0} \hat{k}/k^* = 0$ and $\lim_{\gamma \to 0} \Delta = 1$.

(iii) **Unbounded Distortions on the Upside:** $\hat{k}/k^* - 1|$ and $\Delta$ become infinitely large, if either $\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} / \mathbb{E}(R(\theta)) \to \infty$, or $\gamma \to 0$ and $\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} > \mathbb{E}(R(\theta))$.

(iv) **Negative Expected Dividends:** The implemented investment level leads to negative expected dividends whenever

$$\alpha \left(\frac{\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} - \mathbb{E}(R(\theta))}{\mathbb{E}(R(\theta))} - 1\right) > \gamma.$$ 

The ratio of expected market to dividend returns governs the initial shareholders’ incentive to distort their marginal costs. The returns to scale parameter $\gamma$ in turn translates these marginal cost distortions into investment distortions. When $\gamma$ is very low, the firm operates close to constant returns ($\gamma$ is close to 0), the optimal investment level is very sensitive to changes in the marginal
return expectations, or to incentives that put more weight on market values. In that case, the incentive to manipulate the rent, as well as the associated efficiency losses can become very large. At the other extreme, investment distortions and welfare losses are small if marginal costs are very sensitive to the investment level.

In extreme cases, welfare losses exceed 100% of the first-best welfare level, i.e. the firm’s investment ends up with negative expected cash flows, and thus destroys value. This occurs, whenever the elasticity of marginal costs $\gamma$ is less than the return distortion, which is given by the distance of the return ratio from 1, multiplied by the fraction of shares sold. Even a small departure from the efficient markets benchmark (in terms of $\mathbb{E} \{ \mathbb{E} (R(\theta)|x=z,z) / \mathbb{E} (R(\theta)) \}$) can thus have very large efficiency consequences for firms that operate near constant returns, and with investments that are characterized by upside risk. On the other hand, with under-investment the firm’s expected dividends always remain positive.

**Designing the compensation scheme.** Next we consider the design of the manager’s compensation. Clearly, the efficient investment $k^*$ is implemented if $W^*(\Pi) = \omega \Pi$, i.e. if the manager’s compensation is linear in the final dividend. We will call this a "restricted equity" contract, since the manager cannot sell the equity share in the market. Compensation floors (i.e. stock options) and ceilings can be used to modify incentives, relative to this baseline. Let $k = \lim_{\theta \to -\infty} k^{FB}(\theta)$ and $\bar{k} = \lim_{\theta \to -\infty} k^{FB}(\theta)$. The interval $(k, \bar{k})$ contains all investment levels that are efficient for some $\theta$; investment levels outside cannot be optimal for the initial or final shareholders under any circumstances. Any $k \in (k, \bar{k})$ is implementable by a contract that adds either a floor or a ceiling to this baseline contract.

**Proposition 5** *(Almost) anything is implementable with equity, options, and caps.*

(i) Any $k \in (k^*, \bar{k})$ can be implemented with a contract of the form $W(\Pi) = \max \{ W, \omega \Pi \}$. Any $k \in (k, k^*)$ can be implemented with a contract of the form $W(\Pi) = \min \{ \bar{W}, \omega \Pi \}$.

(ii) Any $k \notin [k, \bar{k}]$ cannot be implemented through a $\Pi$-contingent contract $W(\cdot)$.

Thus, introducing a minimal compensation level into the benchmark contract $W^*(\Pi)$ strengthens incentives by making manager payoffs more convex, thus increasing investment from the benchmark level of $k^*$. Introducing a cap on total compensation on the other hand concavifies payoffs and weakens incentives, thus lowering investment from the benchmark levels.

When $\alpha > 0$, two possibilities arise. If $\hat{k} > k^*$, $W^*(\Pi)$ induces too little investment relative to the optimal level. An optimal contract will add a compensation floor to $W^*(\Pi)$. If instead, $\hat{k} < k^*$, the incentives provided by $W^*(\Pi)$ are too strong and need to be modified with a cap on total
compensation. In our model, $\alpha$ governs not only the extent to which there is a conflict of interest between initial and final shareholders, but also the misalignment between initial shareholders and manager incentives at the two baseline contracts. Under $W^*$ (II), initial shareholders are more focused on the short-term share price than managers, so wage floors and ceilings are used to strengthen incentives and increase distortions.

Thus, to summarize, the investment distortions do not result from an artificial restriction on the contract space, since any targeted investment level can be implemented without agency costs through a $W^*$-contingent contract. Even if the contract design could condition arbitrarily on other variables such as the share price, the investment level or even the realized $\theta$, initial shareholders would still want to distort investment to influence the market price. The implementation possibilities only expand with a richer contract space.\(^8\)

4 Regulation and Policy Interventions

In this section, we discuss policy or regulatory interventions to correct the inefficiencies resulting from rent extraction by incumbent shareholders. We group these policies into two categories: policies that take the market outcome as given and target firm behavior to reduce the initial shareholders’ ability to extract rents, such as regulation of executive compensation, caps on investment size, or other direct regulation of risk-taking behavior, and policies that target the market outcome and affect incentives by modifying investment returns, such as financial transaction taxes or direct market interventions. At the efficient markets benchmark, any intervention of either kind only leads to distortions in investment behavior and thus reduces welfare. Our model offers a clear efficiency rationale for these interventions, and provides some additional insights into their respective practical advantages and drawbacks.

**Regulation of Executive Pay.** First, we consider restrictions on the contracts that incumbent shareholders are allowed to sign with the company’s CEO’s. Our previous discussion has shown that certain types of compensation schemes that enhance or mute risk-taking incentives can be used to incentivize rent-seeking behavior. This suggests that limits on the use of such contracts, especially in relation to a restricted equity component, can be welfare-enhancing. We formalize this intuition in the next result.

\(^8\)Notice however that in our model, price contingent contracts would have no effects on incentives, unless the investment $k$ was publicly and perfectly observable by the market. With noise in the observation of $k$, the market would always attribute any randomness in the observation of investment to noise.
Suppose that the initial shareholders can use $N+1$ "securities" or "instruments" to compensate the manager. Each security $n = 0, \ldots, N$ defines an expected compensation $T_n(k)$ to the manager as a function of the chosen $k$. Each $T_n(k)$ is assumed to be bounded, concave and continuously differentiable. We let $T_0(k) = \mathbb{E}(R(\theta))k-C(k)$ stand for the transfer associated with a restricted equity claim, and assume that restricted equity compensation is always included in the set of available contracts.

**Proposition 6 Efficient investment requires sharp restrictions on incentive pay.**

The initial shareholders implement $k = k^*$ if and only if $\left(\hat{k} - k^*\right)T_n'(k^*) \leq 0$ for all $n$.

Proposition 6 suggests that regulation of executive pay – if done correctly – offers a simple solution to rent-seeking by incumbent shareholders: just ban any form of performance pay other than restricted equity shares. This eliminates the incumbent shareholders’ discretion and fully implements the efficient investment levels. On the other hand, as soon as initial shareholders have access to an instrument that allows them to distort the CEO’s incentives in the desired direction they will find it optimal to include such an incentive component in the compensation contract. Even the minimal qualification above can be dispensed with, if negative positions are allowed by the contract (i.e. a contract that leverages different securities against each other), so that any security can be used to distort in any direction. In practical terms, a ban on the use of any compensation contract other than restricted equity is sufficient to guarantee efficient investments, and generically such a ban is also necessary if one wishes to avoid investment distortions.

This policy has two other important advantages: First, the regulator does not need to acquire any knowledge of firm technologies or return distributions to fine-tune optimal policy. It is a simple "one-size-fits-all" policy that is not easily manipulated by incumbent shareholders, and doesn’t require fine-tuning or direct regulatory oversight, as long as executive compensation is publicly observable. This is particularly relevant in cases where the return characteristics are either not directly observable, or where they can be tailored by the initial shareholders or the manager. Second, limiting executive compensation to restricted equity induces no inefficiency when there are no market frictions, and initial shareholder are aligned with social surplus. Thus, the regulatory policy does not introduce any new inefficiency if it were implemented at the efficient markets benchmark. More generally this policy is robust not only to firm but also to market characteristics.\footnote{In it’s simplest form our analysis abstracts from CEO risk aversion or additional incentive problems which may cause mis-alignment between the manager’s and final shareholder preferences. If these additional elements are important, they offer a rationale to allow for some additional discretion in the design of executive compensation.}
The proposition also suggests that a partial approach to regulation that focuses on specific instruments (for example, a ban on stock option compensation, participation in share sales or limits on bonuses) will not have any impact as long as incumbent shareholders can create the same incentive schemes through other means, for which the possibilities are endless. A successful regulation must thus take a wholesale view and tightly specify the margins along which initial shareholders are given discretion to select the contracting terms.

**Size Caps and Direct Regulation.** Another type of regulation consists in restricting the set of investment levels a firm can implement – in essence this amounts to direct regulatory oversight of the firm’s risk-taking activities. For example, consider the effects of introducing a cap \( \bar{k} \) on investment, such that only \( k \leq \bar{k} \) can be implemented. Because it is a one-sided policy, it only works to limit over-investment, and can never be optimal to correct under-investment (which would require imposing a minimum investment level). In our baseline model without additional information, the optimal size cap sets \( \bar{k} = k^* \), and simply prevents the firm from over-investing. Such a cap depends on the firm’s return characteristics and technologies – an important limitation if these firm characteristics are not directly observable to the regulator, or manipulable by the firm. Direct regulatory oversight of firm investment thus appears as a crude way to limit the initial shareholder’s and managers discretion to engage in excess risk-taking, but it requires inside knowledge of the firms’ characteristics. The informational burden of implementing direct regulatory oversight is thus far greater than implementing restrictions on executive pay.

**Financial Transaction Tax.** A tax on share sales modifies the incentives to distort investments, because it shifts a part of the shareholder rents to the tax authorities. With an uncontingent tax \( \tau \) on the proceeds of share sales, the shareholders maximize \( \mathbb{E}[(1 - \tau) \alpha P(z; k) + (1 - \alpha) V(z, k)] \). A tax on share sales thus reduces the relative weight on the share price from \( \alpha \) to \( \alpha (1 - \tau) / (1 - \alpha \tau) \). This reduces the externality, but unless the tax is completely confiscatory (\( \tau = 1 \)), such a tax can never fully correct for the externality. A small tax has little effect on shareholder incentives.\(^{10}\)

Taxes have to be state-contingent in order to be effective. Consider a tax scheme \( \tau(z) \) that

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\(^{10}\)All this would change if costs \( C(k) \) didn’t come out of dividends, but were paid upfront by initial shareholders. In that case a fixed tax on the share price changes the distortion by shifting the ratio of marginal costs to expected returns. The same would apply with an investment tax or a subsidy on investment costs.
is contingent on the market price. If \( \tau (z) = 1 - V(z,k)/P(z;k) = \Omega(z,k)/P(z;k) \), the tax implements the efficient benchmark by directly compensating for the mispricing. The expected tax revenue \( \alpha \mathbb{E}(\Omega(z,k)) \) captures the entire expected rent from incumbent shareholders. The optimal intervention thus taxes on the upside to subsidize on the downside. But to implement the right taxes and subsidies, the regulator needs to be able to decompose the firm’s share price into a fundamental value component \( V(z,k) \) and a shareholder rent component \( \Omega(z,k) \). This requires detailed knowledge not only of the firm’s technology and return prospects, but also the magnitude of information frictions in the equity market.\(^{11}\)

**Direct Market Interventions.** Market interventions, i.e. policies to purchase or short-sell assets to influence market returns, are another channel through which a policy maker can influence investment incentives. Consider a program such as the Fed’s TARP, or the ECB’s OMT, in which the policy maker commits to buy shares if the share price falls below a threshold level. Since a price support increases expected market returns, such a policy reduces investment distortions, whenever the return distribution represents a downside risk.\(^{12}\)

To analyze the effects of such interventions in our model, suppose that \( R(\cdot) \) is a downside risk. Suppose that a policy maker announces a share purchasing program at a guaranteed price \( \hat{P} > 0 \). For a given \( k \), and for any \( z \) such that \( P(z,k) = \mathbb{E}(R(\theta)|x = z,z)k - C(k) > \hat{P} \), this intervention has no effect. However, if \( P(z,k) < \hat{P} \), the policy maker buys a positive level of shares. This occurs whenever \( z \) falls below some threshold \( \hat{z} \). Thus, given \( k \), this policy automatically limits the investment’s downside risk to \( \hat{R} \), s.t. \( \hat{P} = \hat{R}k - C(k) \). The investment first-order condition now takes the form

\[
C' \left( \hat{k} \right) = \alpha \mathbb{E} \left( \max \{ \hat{R}, \mathbb{E}(R(\theta)|x = z,z) \} \right) + (1 - \alpha) \mathbb{E}(R(\theta))
\]

Thus, a support price \( \hat{P} \) for shares then leads to an implicit return guarantee \( \hat{R} \) and implemented investment level \( \hat{k} \) that satisfy these two conditions. Notice that the price support policy always strengthens investment incentives, which reduces the downside risk distortions: \( \hat{k} \) is strictly increasing in the level of the price support \( \hat{R} \). Our next proposition however shows that such policy have a fiscal cost, i.e. the expected revenues from the purchased shares are less than the cost of purchasing them, and therefore the policy provides an implicit subsidy to the initial shareholders.

\(^{11}\)We can also discuss dividend taxes. Uncontingent taxes on final dividends have no effect on incentives, since the tax is fully passed through into the share price, so that it doesn’t affect the initial shareholders’ weights attached to share price vs. final dividends. Contingent dividend taxes on the other hand can be used just like contingent transaction taxes to modify expected returns to investment to align initial and final shareholder incentives.

\(^{12}\)For upside risks, one would have to consider short-selling policies that seek to limit over-pricing.
A revenue-neutral form of interventions requires to offset the fiscal cost of the intervention through a transactions or dividend tax which always entails distributional consequences.

**Proposition 7** With downside risk, direct market interventions improve efficiency but come at a cost.

(i) Any direct market intervention generates negative expected revenues for the policy maker and increases rents to initial (and sometimes final) shareholders.

(ii) The policy maker can achieve the same efficiency gains in a revenue-neutral fashion by combining a market intervention with a tax on transactions or final dividends, but this unambiguously reduces the welfare of initial shareholders.

Thus, a pure TARP-like price-support intervention is impossible without subsidizing the initial shareholders. The policy maker’s revenues trade off gains from arbitraging the under-pricing against exposure to a winner’s curse problem when the informed traders offload their shares. Unfortunately, from the policy makers’ perspective the winner’s curse always dominates. The second part of the proposition states that it is possible to offset the fiscal burden of the market intervention through a transaction or dividend tax. But this has important distributional effects, because the tax eliminates the subsidy to initial shareholders and in addition correcting the investment distortion shifts the rents from initial to final shareholders (due to downside risk). As a result, initial shareholders unambiguously prefer to avoid revenue-neutral market interventions.\(^{13}\)

What’s more, efficient market interventions need to fine-tune the price support policy to target the efficient investment level, and therefore have to rely once again on inside knowledge of the market frictions, and the firm’s characteristics.

To summarize, regulation of executive pay appears to be by far the most robust and effective policy to limit rent-seeking incentives. In our model, the optimal regulation is admittedly stark, i.e. requiring a complete ban on anything other than untraded equity shares, and in particular, stock options or bonuses tied to market performance. While the optimal scheme may differ from untraded equity with risk aversion or other agency frictions, our more general message should be robust: initial shareholders will try to exploit the flexibility they are given to extract rents, so restrictions imposed on executive compensation are socially desirable.

\(^{13}\)In fact, exactly the same arguments can be made about "bailout" interventions that amount to guarantees on ex post returns, which would for example shift returns from \(R(\theta)\) to \(\max\{\bar{R}, R(\theta)\}\) but require funding either from outside or through dividend taxation.
Size caps, transaction taxes or market interventions can also help to reduce distortions, but they either require far more complex regulatory oversight (in terms of the firm-specific information about return distributions, costs, market frictions etc.). What’s more, incumbent shareholders have a clear preference against regulation if this reduces the rents they can capture by distorting investments, and doesn’t offer a compensating subsidy. In fact, among the proposals discussed here, direct interventions without tax offsets are the only policy that incumbent shareholders will potentially support, since this policy directly subsidizes their shareholdings, while other policies just alter incentives without offering compensation for losing shareholder rents.

Regulation of executive compensation is the most effective way to address the incentive problem, because it tackles the problem at its root by directly limiting the scope for rent-seeking incentives. The other policy proposals all offer remedies that introduce a new distortion to exactly offset the distortions generated by rent-seeking, thus requiring detailed knowledge of the respective magnitudes of these distortions. Direct Regulation of firm behavior requires knowledge of firm characteristics, while market interventions and tax policies require knowledge of both firm and market characteristics to finely balance out incentive distortions for shareholders against return distortions in the market. Hence these policies require far more knowledge, a problem that only gets compounded when the decision problem of the firm is more complex and the firm possibly bases its decisions on proprietary information that is not directly accessible to the regulator.

5 Extensions

In this section we discuss the role of several assumptions made along the way, and we provide extensions to illustrate the robustness of our results to different market environments, disagreement among incumbent shareholder interests, a richer treatment of the firms’ decision problem, agency frictions between the firm and its manager, and a richer informational environment. These extensions also shed light on the potential relevance of our model for applications.

5.1 Financial Market Structure

In the baseline model, we have assumed that initial shareholders passively sell a fraction $\alpha$ of their shares to informed and noise traders. This allowed us to focus on the gap between $P(z,k)$ and $V(z,k)$ as the source of distortions. Abstracting from proprietary information by incumbent shareholders, their possible exposure to liquidity shocks, or even heterogeneity and potential conflicts of interests among incumbent shareholders is an expositional simplification. Changing these as-
assumptions may alter the nature and magnitude of shareholder rents, as well as the direction and magnitude of distortions, but they do not affect the basic insights stemming from our model. In our baseline model, we can decompose expected cash-flows of initial shareholders, informed traders and noise traders into three terms. First, the term $V(z,k)$ measures the expected final dividend from the firm. Second, the initial shareholders earn the price $P(z,k)$ for each share that they sell. The wedge $\Omega(z,k) = P(z,k) - V(z,k)$ thus represents a pure transfer from the buyers to the sellers of equity shares. Third, we can compute the expected payoffs to informed traders in the financial market. Let $F(x|z)$ denote the distribution of the private signal $x$, conditional on observing a market signal $z$; $x|z$ is normally distributed with mean $\frac{\beta \delta}{\lambda + \beta \delta} \cdot z$ and variance $\left(\frac{1}{\lambda + \beta \delta} + \beta^{-1}\right)^{-1}$. The expected payoff (information rent) to an informed buyer is

$I(z,k) = \int \max \{0, E(\Pi(\theta,k)|x',z) - P(z)\} dF(x'|z)$

$I(z,k) = k \cdot \int_{-\infty}^{\infty} (E(R(\theta)|x',z) - E(R(\theta)|x = z,z)) dF(x'|z)$

$I(z,k)$ is strictly positive, and corresponds to the option value of trading on private information. Since the aggregate payoff to buyers is $-\Omega(z,k)$ and informed buyers’ expected payoff is $I(z,k) > 0$, the noise trader’s expected payoff is $-\Omega(z,k) - I(z,k)$. Since by the Law of Iterated Expectations, $V(z,k) = \int E(\Pi(\theta,k)|x',z) dF(x'|z)$, it follows that

$-\Omega(z,k) = k \cdot \int_{-\infty}^{\infty} (E(R(\theta)|x',z) - E(R(\theta)|x = z,z)) dF(x'|z) < I(z,k)$,

and therefore noise trader’s expected payoffs is unambiguously negative. With this decomposition, we can directly extend our analysis to alternative scenarios that may allow for private information or liquidity shocks by incumbent shareholders:\footnote{This decomposition also motivates our focus on $V(z,k)$ as the measure of total surplus. This welfare measure can be justified on utilitarian grounds, if the noise traders are risk-neutral, and their preference for buying or selling the security stems from a preference shock as in Duffie, Garleanu and Pedersen (2005) that is sufficiently large to dominate expected returns in the noise traders considerations for holding the share.}

1. Private information by incumbent shareholders: Suppose incumbent shareholders receive a private signal before deciding whether to sell their equity share. Shares are bought by a random measure of noise traders, given by $\Phi(u)$, as in the benchmark model. In this case, the supply of shares is endogenous and given by $\Pr(x < \hat{x}(P)|\theta) = \Phi\left(\sqrt{\beta} \cdot (\hat{x}(P) - \theta)\right)$, while demand is $\Phi(u)$. The equilibrium characterization remains the same, with $\hat{x}(P) = z = \theta + 1/\sqrt{\beta} \cdot u$. But the initial shareholder objective changes, and is now given by $P(z,k) + I(z,k) = V(z,k) + \Omega(z,k) + I(z,k) > V(z,k)$, while the buyers’ (noise traders’) payoff is $-\Omega(z,k) - I(z,k) < 0$. In other words, the...
initial shareholder rents are no longer given by the gap between expected price and dividend value, but by the option value of trading on private information. Since this value is strictly positive and scales with $k$, we obtain that the initial shareholders now have a strict incentive to over-invest in order to capture a larger information rent regardless of the firm’s risk characteristics.

2. **Liquidity shocks to incumbent shareholders:** Suppose next that incumbent shareholders sell passively, but the fraction sold, given by $1 - \Phi(u)$, is random, while potential buyers of equity shares observe a private signals (i.e. noise traders are only on the seller’s side). Now the demand for shares is given by $\Pr(x > \hat{x}(P)|\theta) = 1 - \Phi(\sqrt{\beta}(\hat{x}(P) - \theta))$, while supply is random. The equilibrium price characterization remains the same, with $\hat{x}(P) = z = \theta + 1/\sqrt{\beta} \cdot u$. But the initial shareholder objective changes to $V(z, k) - I(z, k)$, while the informed buyers earn $I(z, k)$. In this case, the initial shareholders have an unambiguous interest to under-invest so as to reduce information rents conceded to informed buyers.

3. **Disagreement among shareholders:** The discussion also suggests that if shareholders differ ex ante in their investment horizons, their access to private information, or their potential exposure to liquidity shocks, then this heterogeneity leads to disagreement among shareholders about the desired investment decisions. With heterogeneity, each incumbent shareholder will have their own preferred value of $k$. From a voting perspective, this suggests the implementation of the value of $k$ that is preferred by the median shareholder. The departure from efficient markets is crucial for the emergence of shareholder disagreement. Under efficient markets, the shareholder rents, which are the source of disagreement, disappear, and hence shareholder incentives are once more aligned with social surplus. This suggests that financial market frictions may also play an important role in shaping conflicts of interest between existing shareholders of a firm.

### 5.2 Manager Risk Aversion and Agency Frictions

Next, we embed risk aversion and a richer formulation of the agency conflict between shareholders and the manager. The main argument favoring flexibility in contract design and departures from stock-based compensation through options and other high-powered incentive schemes is that this eliminates inefficiencies due to the manager’s exposure to risk. It is straightforward to introduce these elements into our analysis.

Concretely, suppose that in addition to choosing $k$, the manager makes a hidden effort choice $e \in \{0, 1\}$, which affects the return distribution $R(\theta, e)$. The choice of $e = 0$ entails a pecuniary private benefit $B > 0$.\textsuperscript{15} Let $U(\cdot)$ (strictly increasing and concave) denote the manager’s utility

\textsuperscript{15}This is perhaps best interpreted as a cash-flow diversion friction. The analogous arguments apply if the hidden
function, and let \( \bar{U} \) denote the manager’s outside option. Then, the contract design problem can be analyzed in the usual two-step procedure. First, for each pair of choices \((k, e)\), let \( W(k, e) \) denote the minimal expected wage cost (overall possible contracts) that implements the choice of \((k, e)\), subject to the manager’s incentive compatibility and participation constraints:

\[
W(k, e) = \min_{W(\cdot)} \mathbb{E}\left\{ W\left(R(\theta, e) k - C(k)\right)\right\}
\]

subject to:

\[
(k, e) \in \arg \max_{(k', e')} \mathbb{E}\left\{ U\left(W\left(R(\theta, e') k' - C(k')\right) + (1 - e') B\right)\right\}
\]

\[
\bar{U} \leq \mathbb{E}\left\{ U\left(W\left(R(\theta, e) k - C(k)\right) + b(1 - e) k\right)\right\}
\]

Second, we determine the pair \((k, e)\) that maximizes the initial shareholder’s expected payoffs:

\[
(k, e) \in \arg \max_{(k', e')} \mathbb{E}\left\{ \alpha P(z; k', e') + (1 - \alpha) \Pi(\theta; k', e') - W(k', e')\right\},
\]

where \( P(z; k, e) = \mathbb{E}(R(\theta, e)|x = z, z) \cdot k - C(k) \)

is the equilibrium price function that clears the equity market. The optimal investment level \( \hat{k} \) satisfies the first order condition

\[
\alpha \mathbb{E}\left\{ \mathbb{E}(R(\theta, e)|x = z, z)\right\} + (1 - \alpha) \mathbb{E}\left\{ R(\theta, e)\right\} = C'(\hat{k}) + W_k(\hat{k}, e)
\]

and therefore modifies the marginal cost of investment by the marginal wage cost associated with higher investment. This formulation highlights once again that investment distortions are entirely due to distortions in the incumbent shareholders objective function, and the friction in the market price. All our previous results are robust, once the marginal cost of investment is adjusted to include the agency cost component: Upside risk results in over-investment, while downside risk results in under-investment, and the easier it is to scale investment, the more pronounced the distortions are.

In terms of compensation contracts, when the manager is risk-neutral, a restricted equity contract with a sufficient equity share will always incentivize the manager to maximize the expected total surplus from the investment. Thus, our argument favoring restriction of executive compensation to restricted equity remains applicable whenever the manager is risk-neutral. Under risk aversion, this contract will induce the CEO to under-invest relative to the socially optimal level, but this does not over-turn the point that restrictions on executive compensation contracts can be used to curb the incumbent shareholders’ ability to distort incentives in the desired direction.

**Remark on project selection.** This extension also illustrates how shareholders benefit from distorting CEO actions, even when those actions are not observable or contractable. Let \( B = 0 \) effort had a utility cost, or if the cash-flow diversion scales with \( k \).
(no private benefits), and let the CEO be risk-neutral (to abstract from agency frictions). In this case, the choice of $e$ can be interpreted as a choice of project selection, i.e. the selection between different return profiles. While a social planner will want the firm to select the project with the highest expected return $\mathbb{E}\{R(\theta, e)\}$, the initial shareholders have a preference for maximizing a weighted average of the expected fundamental and market returns $\alpha \mathbb{E}\{\mathbb{E}(R(\theta, e) | x = z, z)\} + (1 - \alpha) \mathbb{E}\{R(\theta, e)\}$. This skews the selection of investment projects in favor of projects that are dominated by upside risk, and against projects with downside risk. Thus, the disagreement between initial shareholder objectives and social surplus can also be the cause of distortions in the evaluation of returns and project selection. From the perspective of a regulator, this is especially problematic in situations where the choice of risk profiles is not directly observable.

Remark on the separation of ownership and control. Since rent-seeking incentives result from the mis-alignment of initial shareholder preferences, the separation of ownership from control and the associated agency conflicts that characterize large modern corporations may even be socially beneficial if this serves to temper the shareholders’ ability to extract rents. This benefit is distinct from the "usual" comparative advantage arguments that the person best placed to own a firm may not be the person best placed to manage it.

To illustrate this point, consider the problem of scaling $k$, but introduce a parameter $\eta > 0$ to scale the wage function:

$$\hat{k} \in \arg \max_{k'} \mathbb{E}\{\alpha P(z; k') + (1 - \alpha) \Pi(\theta; k') - \eta W(k')\}$$

For simplicity, here we abstract from the hidden effort margin, but the wage function is strictly increasing and convex in $k$. Then $\eta$ can be seen as a reduced-form measure of the severity of the agency frictions (coming from, e.g. the manager’s outside options or risk aversion). Formally, we show that the social surplus, defined as $\mathbb{E}\left\{\Pi(\theta; \hat{k}) - \eta W(\hat{k})\right\}$, is increasing in $\eta$ if $\eta$ is sufficiently low, and upwards distortion incentives sufficiently severe, i.e. $C(\cdot)$ is sufficiently close to linear, and $\mathbb{E}(R(\theta) | x = z, z) > \mathbb{E}\{R(\theta)\}$. Formally, social surplus is increasing in $\eta$ if the following inequality holds:

$$\frac{\alpha (\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} - \mathbb{E}(R(\theta))) \alpha (\mathbb{E}\{\mathbb{E}(R(\theta) | x = z, z)\} + (1 - \alpha) \mathbb{E}(R(\theta)))}{C'(\hat{k}) + \eta W'(k')} > \frac{C''(\hat{k}) + \eta W''(k')}{C''(\hat{k}) + \eta W''(k')}.$$

The left-hand-side of this condition represents the fraction of incumbent shareholder returns on the investment that come from market rents, and measures in percentage terms the distortion in

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16 For simplicity, here we abstract from the hidden effort margin.

17 For example this would be the case if the manager is risk averse, and simply must be prevented from investing in a low-return project with sufficient private benefits.
returns. The right-hand-side represents the semi-elasticity of costs to investment scale, accounting for the agency friction.

While for a given value of $\hat{k}$, an increase in $\eta$ imposes larger wage costs, and therefore worsens the surplus, at the same time, the increase in $\eta$ also raises the shareholders’ marginal agency costs w.r.t. $k$, and therefore reduces over-investment, which improves welfare. If manipulation incentives and over-investment are particularly severe, the second effect dominates.

5.3 Dynamics and Time Consistency

To be added

5.4 Other information structures

To be added

6 Applications

6.1 Leverage and Risk-Taking

To be added

6.2 Welfare effects of public information provision

In this application, we enrich our model to consider the response of market prices, expected market returns and investment to public information disclosures. As our main result, we show that public information disclosures may have adverse welfare effects: while more accurate information should in principle enable firms to make better investment decisions, here this information can also be misused as an additional margin along which incumbent shareholders optimize their rent. If the rent-seeking motive dominates incentives, the provision of information with limited precision is at first welfare reducing. Enhanced transparency is welfare increasing only once the information provision is sufficiently precise to crowd the market-generated signal $z$ out from investor expectations. This is similar to results established by Morris and Shin (2002) and the subsequent literature for beauty contest games, but here the externalities are linked to rent-seeking in asset markets and thus quite different from the original beauty contest formulation, even if asset markets may have been one of their leading motivations.
Formally, suppose that the manager can condition the investment on a public signal \( y \sim \mathcal{N}(\theta, \kappa^{-1}) \), which is also available to investors in the market. To simplify, we further assume that \( \alpha = 1 \), i.e., initial shareholders sell their entire share and only care about expected market returns, and we suppose for simplicity that \( R(\theta) = e^\theta \), i.e., the return distribution is log-normal.

The contract design problem implements an investment function \( k(y) \) instead of just a single investment level, and the share price is conditioned on \( y \) as well as \( z \). At the efficient markets benchmark, the price equals the expected dividend value \( V(y,z;k) \). Incentivizing the manager with shares yields an investment decision \( k^*(y) = \psi(\mathbb{E}(R(\theta) | y)) \) that maximizes \( \mathbb{E}(V(y,z;k)) \) and incorporates the information contained in \( y \) optimally according to Bayes’ Rule. As the signal becomes infinitely precise, \( k^*(y) \) converges in probability to the first-best decision rule \( k^{FB}(\theta) \).

Conditional on the realization of \( y \), the contracting problem and investment decisions are characterized exactly as in the previous section, with the minor modification that the prior at the market stage is adjusted to reflect the information contained in the public signal. That is, we now set \( \mu = \frac{\kappa}{\lambda + \kappa} y \) (instead of 0) for the expectation and \( \lambda = \lambda + \kappa \) (instead of \( \lambda \)) for the precision at the market stage. With these adjustments, the \( P(y,z;k) \) and \( V(y,z;k) \) take the form

\[
P(y,z;k) = \int R(\theta) \, d\mathcal{F}\left( \sqrt{\lambda + \kappa + \beta + \beta\delta} \left( \theta - \frac{\kappa y + (\beta + \beta\delta) z}{\lambda + \kappa + \beta + \beta\delta} \right) \right) \cdot k - C(k)
\]

\[
V(y,z;k) = \int R(\theta) \, d\mathcal{F}\left( \sqrt{\lambda + \kappa + \beta + \beta\delta} \left( \theta - \frac{\kappa y + \beta\delta z}{\lambda + \kappa + \beta\delta} \right) \right) \cdot k - C(k).
\]

The market price thus overweighs the market information \( z \), but reduces the weight attached to the public signal \( y \). The optimal investment is obtained from the same first-order condition after conditioning the expected market returns on \( y \): \( C'(\hat{k}(y)) = \mathbb{E}\{\mathbb{E}(R(\theta) | x = z, y)\} \). Conditional on \( y \), the market-implied uncertainty about \( \theta \) is \( 1/\lambda_P(\kappa) = (\lambda + \kappa)^{-1} (1 + \gamma_P^2/\gamma_\nu - \gamma_P) \), where

\[ \gamma_P(\kappa) = \frac{\beta + \beta\delta}{\lambda + \kappa + \beta + \beta\delta} \text{ and } \gamma_\nu(\kappa) = \frac{\beta\delta}{\lambda + \kappa + \beta\delta} \]

are adjusted for \( \kappa \). Solving the first-order conditions for \( \hat{k}(y) \) and \( k^*(y) \) yields

\[
C'(\hat{k}(y)) = \hat{k}(y)^\gamma = \mathbb{E}\{\mathbb{E}(R(\theta) | x = z, y) | y\} = e^{\frac{\kappa}{\lambda + \kappa} y + \frac{1}{2} \lambda_P^{-1}}, \text{ and}
\]

\[
C'(k^*(y)) = k^*(y)^\gamma = \mathbb{E}(R(\theta) | y) = e^{\frac{\kappa}{\lambda + \kappa} y + \frac{1}{2} (\lambda + \kappa)^{-1}}.
\]

Therefore, the investment distortion is independent of \( y \): \( \hat{k}(y)/k^*(y) = e^{\frac{1}{2\kappa}(\lambda_P^{-1} - (\lambda + \kappa)^{-1})} \) is constant and depends only on the gap between market uncertainty \( \lambda_P^{-1} \) and objective uncertainty \( (\lambda + \kappa)^{-1} \). Although \( \lambda_P^{-1} - (\lambda + \kappa)^{-1} \) is decreasing in \( \kappa \), our next proposition shows that if the frictions are sufficiently severe to cause expected dividends to be negative, provisions of noisy public information may lead to further reduction in welfare.
Proposition 8 Noisy public news may reduce welfare.

For any \( \lambda, \beta, \) and \( \delta \), if \( \gamma \) is sufficiently small, then there exists \( \kappa \) such that expected welfare is negative and decreasing in \( \kappa \) for \( \kappa \in [0, \bar{\kappa}] \).

This result illustrates that, in the exponential example, information disclosures can have potentially adverse effects. Using the same notation as in the previous section, let \( \hat{V}(y) \) the expected dividend at the equilibrium investment, \( V^*(y) \) the welfare level at the efficient investment, and \( \Delta(y) = 1 - \hat{V}(y)/V^*(y) \) the reduction in welfare at the equilibrium, relative to the efficient welfare level, all conditional on \( y \). Then the ex ante welfare can be written as

\[
E \left( \hat{V}(y) \right) = E \left( V^*(y) \right) - E \left( V^*(y) \cdot \Delta(y) \right).
\]

The first term here measures the welfare level at the efficient benchmark, while the second term measures the welfare loss due to investment distortions. An improvement in information unambiguously increases the first term, but this is not sufficient to guarantee an overall welfare increase if at the same time the distortions get worse. In fact, this will typically be the case if the public information offers the initial shareholders an additional dimension along which they are able to influence the rents.

This is exactly what happens in the exponential example. Notice that here, \( \Delta(y) \) is independent of \( y \) and only depends on \( \kappa \). Taking derivatives w.r.t. \( \kappa \), we have

\[
\frac{\partial E \left( \hat{V}(y) \right)}{\partial \kappa} = E \left( \frac{\partial}{\partial \kappa} (V^*(y)) \right) (1 - \Delta) - \frac{\partial \Delta}{\partial \kappa} E \left( V^*(y) \right).
\]

It’s straightforward to check that in the exponential example \( \frac{\partial \Delta}{\partial \kappa} < 0 \), so the second term is positive, but if \( 1 - \Delta < 0 \), the first term will be negative. Moreover, as \( \gamma \to 0 \), \( E \left( V^*(y) \right) \to 0 \) and \( \Delta \to \infty \), so the increase in distortion ends up unambiguously dominating the welfare effect of information provision. The end result is that public information worsens rent extraction and reduces welfare. Information is welfare-improving only once it is sufficiently precise to crowd out the rent-seeking motive, so that the firm returns to positive expected dividends.

Proposition 8 resembles Morris and Shin (2002)’s result that noisy public information reduces welfare in beauty contest games. While both results build on a similar distortion in the use of public information, our set-up and intuition are different. Here, the frictions in financial markets are key in generating the rent-seeking activities that cause information processing to be distorted – information provides initial shareholders with one further dimension of flexibility along which they can optimize their rent. In Morris and Shin instead, a rent-dissipation game motivated the zero-sum reduced-form coordination motives, but financial markets were not modeled explicitly.
6.3 Stock-Price Sensitivity of Investment

In this application, we consider how investment decisions distort the use of information aggregated through share prices, and how this leads to excess sensitivity of investment to share prices. We modify our benchmark model by assuming that the investment decision is implemented after the price is realized, allowing for informational feedback effects. The initial shareholders incentivize the manager to implement a price-contingent investment rule \( k(\cdot) \) that internalizes the impact of its decisions on the share price. The wage contract is then designed to align the manager’s ex post incentives with the shareholders’ ex ante objective, so that the managers have no incentive to alter their behavior ex post (what’s more, if \( \alpha > 0 \), such commitment is valuable to the initial shareholders). Effectively the model implies that the firm pre-commits to an investment rule before the market opens, and the market can perfectly anticipate the investment level that will realize given the price.\(^{18}\)

We depart from the analysis in section 3 by assuming that the investment rule is a price-contingent, or equivalently \( z \)-contingent function \( k(z) \). Since the market can still perfectly anticipate the investment level that will realize at a given price, nothing changes from the characterization in proposition 1, except that \( k \) is replaced by \( k(z) \). We further assume that there exists a unique \( \hat{z} \) s.t. \( \mathbb{E}(R(\theta) | x = z, z) \gtrless \mathbb{E}\{R(\theta) | z\} \) for \( z \gtrless \hat{z} \).\(^{19}\) For a given \( k(z) \), the equilibrium share price, is

\[
P(z) = P(z, k(z)) = \mathbb{E}(R(\theta) | x = z, z) \cdot k(z) - C(k(z)),
\]

The expected dividend value takes the form

\[
V(z) = V(z, k(z)) = \mathbb{E}(R(\theta) | z) \cdot k(z) - C(k(z)).
\]

To constitute an equilibrium, it must be the case that \( P(z) \) is strictly monotonic in \( z \), a condition that is no longer automatically satisfied due to the endogenous feedback from \( z \) to \( k \).

**Lemma 1** Suppose that

\[
1 \geq \frac{k'(z)/k(z)}{\partial \mathbb{E}(R(\theta) | x = z, z)/\partial z/\mathbb{E}(R(\theta) | x = z, z)} \left( \frac{C'(k(z))}{\mathbb{E}(R(\theta) | x = z, z)} - 1 \right).
\]

\(^{18}\) Instead of thinking of the pre-commitment to a rule as the result of the manager’s incentive contract, we can also interpret this as a model of explicit agreement to a rule before the market opens, and the internal reporting and decision procedures in place generate the commitment that make it difficult to revisit the decision afterwards. The important feature here is that final shareholders are unable to renegotiate the contract terms with the manager once they have taken control, and before the investment is implemented.

\(^{19}\) This condition is satisfied for many return distributions, and is completely unrelated to our concepts of upside vs. downside risks: For any bounded return \( R_1(\theta) \) with upside risk that satisfies the condition, the same condition also holds for \( R_2(\theta) = R - R_1(-\theta) \).

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Then, \( P(z, k(z)) \) is strictly increasing in \( z \).

The condition in this lemma highlights that monotonicity of \( P(z) \) requires the investment rule to be not too responsive to \( z \) in states where the firm invests more than what the market would like to see. Below, we will assume that monotonicity is always satisfied. Due to an envelope condition, this holds automatically for \( \alpha = 1 \), i.e. the chosen investment rule maximizes the share price conditional on \( z \). For \( \alpha < 1 \), monotonicity is not automatic unless one imposes additional restrictions on the shape of returns, since monotonicity may fail when the return \( \mathbb{E}(R(\theta)|x = z, z) \) that is expected by the market is significantly below the objective return, \( \mathbb{E}(R(\theta)|z) \), and the initial shareholders put sufficient weight on the latter (i.e. \( \alpha \) is close to 0).\(^{20}\)

The optimal investment rule \( \hat{k}(z) \) maximizes the initial shareholder’s objective function conditional on \( z \), \((1 - \alpha)V(z, k) + \alpha P(z, k)\). With market frictions, \( \hat{k}(z) \) satisfies

\[
C'(\hat{k}(z)) = \alpha \mathbb{E}(R(\theta)|x = z, z) + (1 - \alpha) \mathbb{E}(R(\theta)|z).
\]

If markets were efficient \((V(z, k) = P(z, k))\), the implemented decision rule \( k^* (z) = \psi(\mathbb{E}\{R(\theta)|z\}) \) incorporates the information contained in the price optimally according to Bayes’ Rule. Relative to \( k^* (z) \), the implemented investment tilts investment in the direction dictated by the market expectations of returns.

**Information Feedback.** Our first result shows that the informational feedback from share prices to investment creates an endogenous element of upside risk. This effect comes from the value of aligning investment with the information contained in the share price and is present even with the efficient investment rule.

**Proposition 9 Informational Feedback creates endogenous upside risk and increases initial shareholder rents.**

(i) **Increased Shareholder Rents:** For any strictly increasing investment function \( k(z) \),
\[
\mathbb{E}(\Omega(z, k(z))) > \mathbb{E}(\Omega(z, \hat{k}(z))).
\]

(ii) **Endogenous Upside Risk:** \( \mathbb{E}(\Omega(z, k(z))) > 0 \) if either \( \mathbb{E}(\mathbb{E}(R(\theta)|x = z, z)) \geq \mathbb{E}(R(\theta)) \), or \( \mathbb{E}(\mathbb{E}(R(\theta)|x = z, z)) < \mathbb{E}(R(\theta)) \) and \( \inf_z k'(z)/k(z) \) is sufficiently large.

(iii) **Unbounded Rents:** If \( \inf_z k'(z)/k(z) \to \infty \), then \( \mathbb{E}(\Omega(z, k(z))) \to \infty \), for any \( R(\cdot) \).

Therefore, any investment strategy that responds positively to the news in \( z \) also tilts the initial shareholder’s expected rents to the upside. If the underlying risk was (weakly) upside dominated,\(^\text{20}\) For example, it suffices to assume that \( \lim_{\theta \to -\infty} R(\theta) \geq \mathbb{E}\{R(\theta)|\hat{z}\}/(1 + \gamma) \), or to assume that the firms’ dividend includes an additional component that is increasing in \( \theta \) but not affected by investment.
the feedback effect strengthens the upside bias and increases initial shareholder rents. If instead the underlying risk was dominated by the downside, then the endogenous investment response mitigates the downside exposure, and if investment is sufficiently responsive it overturns it. In the limit where investment becomes infinitely responsive, the initial shareholder rents are positive and arbitrarily large regardless of the underlying $R(\cdot)$.

We can write the expected wedge as

$$\mathbb{E}(\Omega(z, k(z))) = \mathbb{E}(\mathbb{E}(\Omega(z, k(z)))) + \text{cov}(k(z); \mathbb{E}(R(\theta) | x = z, z) - \mathbb{E}\{R(\theta) | z\})$$

The expected wedge consists of two terms. The first term, $\mathbb{E}(\mathbb{E}(\Omega(z, k(z))))$, corresponds to the expected wedge when investment is fixed at its unconditional expectation $\mathbb{E}(k(z))$. The second term, $\text{cov}(\mathbb{E}(R(\theta) | x = z, z) - \mathbb{E}\{R(\theta) | z\}; k(z))$, captures the fact that investment responds to the information contained in $z$. Therefore, the feedback from prices to investment creates endogenous upside risk, and increases rents to initial shareholders. The response of investment leads to more risk-taking on the upside, when the realization of $z$ is positive, and to less risk-taking on the downside. Thus, endogenously the amount of exposure increases with expected returns. This result arises whenever investment responds positively to the news in $z$.

**Investment Distortions with Informational Feedback.** Next, we discuss how the implemented investment rule distorts the information coming from $z$. The implemented investment rule $\hat{k}(z)$ equates marginal cost to a weighted average of the market’s expected return and the expected dividend return, conditional on $z$. The signal $z$ thus receives more weight in forming posterior expectations about $\theta$ than would be justified from an objective point of view.

At signal realization $\hat{z}$, $\mathbb{E}(R(\theta) | x = z, z) = \mathbb{E}(R(\theta) | z)$, so regardless of $\alpha$, the implemented investment level will be the efficient one, $\hat{k}(z) = k^*(z)$. Away from $\hat{z}$, the response to market prices is distorted in the direction of market expectations, and the distortions become larger the larger is $\alpha$. Therefore, for $z > \hat{z}$, $\hat{k}(z) > k^*(z)$, while for $z < \hat{z}$, $\hat{k}(z) < k^*(z)$. Thus, by shifting the weight given to $z$ in the direction of the market’s expectations, the implemented investment rule magnifies the response of investment to the information contained in the price. These results are summarized in the next proposition:

**Proposition 10 Market noise causes excess investment volatility**

(i) **Excess investment sensitivity:** The investment distortion $|\hat{k}(z) / k^*(z) - 1|$ is increasing in $\alpha$, and increasing in $z$ whenever $\mathbb{E}(R(\theta) | x = z, z) / \mathbb{E}(R(\theta) | z)$ is increasing in $z$.

(ii) **Fundamentals vs. Market Noise:** If market noise is sufficiently important, then volatility of investment is high, but the correlation of investment with future returns is low.
(iii) **Unbounded rents and welfare losses:** If the market friction is sufficiently important or \(\gamma\) sufficiently close to 0, \(\mathbb{E}(\Omega(z, \hat{k}(z)))\) becomes arbitrarily large, and \(\mathbb{E}(V(z, \hat{k}(z)))\) lower than if investment was determined before the market opens.

The rent manipulation logic of the two previous sections extends to the setting in which investment is implemented after the market opens. Here, the initial shareholders can manipulate the rents not just through the average level of investment, but also through its response to the information \(z\). Effectively, by conditioning investment on the share price, initial shareholders have access to an additional marginal along which they can maximize their rents. Since initial shareholder rents are increasing in the sensitivity of investment to \(z\), shareholders take advantage by incentivizing an investment rule that is excessively responsive to market information: the signal \(z\) receives too much weight relative to the information it conveys. This causes excess volatility in investment: on the upside, shareholders encourage too much investment in order to maximize the positive rents they extract from inflated share prices. On the downside, they encourage under-investment in order to limit the losses they incur from the market price being below the fundamental value. If \(\alpha = 1\), the investment equates marginal costs to the expected market return conditional on \(z\), but the latter can be far more volatile than the fundamental return expectation, if information frictions are sufficiently important.

This channel not only increases the volatility of investment, but it also reduces the connection of investment decision from information about fundamentals. A high price translates into a higher investment level, but the increase in investment and marginal costs exceeds the increase in fundamental return expectation. If the market is sufficiently noisy, i.e. \(\beta \delta\) is small relative to \(\lambda\) and \(\beta\), it may be that prices are almost exclusively driven by market noise, and carry little information about fundamental returns, yet investment responds aggressively in order to capture rents on the upside or limit losses on the downside. In the limiting case where investment is orthogonal to fundamentals, the feedback from prices to investment leads to strictly lower efficiency than a fixed ex ante investment decision. In this case, investment volatility purely serves to increase upside risk and extract rents form final shareholders, and is strictly welfare reducing, even though an efficient use of the same information could entail large welfare gains.

### 7 Conclusion

The efficient markets paradigm has proven highly influential for the analysis of asset valuation, firm-decisions, risk-taking and agency conflicts between shareholders and managers. A cornerstone
insight of this paradigm is that the no arbitrage principle aligns private and social objectives across
time and states of nature: in this view, the market provides discipline by offering the right valuation
signals, and incentivizing firms and managers to take decisions that are in the social interest. It is
because of these disciplinining effects of financial prices that firms and managers can be trusted to
take decisions that are in the collective interest without further oversight and regulation.

At the same time the ideal of no arbitrage is rarely met in practice: arbitrage activities are risky
and costly – markets can stay "irrational" for longer than arbitrageurs can stay solvent. Here,
we have therefore taken a different view of price formation in asset markets, one that is based on
limits to information and limits to arbitrage. The resulting price patterns depart from the efficient
markets benchmark, and they induce a rent-seeking motive into shareholder objectives, since any
deviation of share prices from fundamental values amounts to a zero-sum transfer from final to
initial shareholders (or vice versa). Because of this rent-seeking element, initial shareholders can
no longer be trusted to defend the interests of future shareholders or society. In other words,
market discipline no longer works, and even has detrimental effects, if distorted valuations lead
to distorted incentives. Rent-seeking becomes especially problematic for firms whose risks are
tilted to the upside (limited liability) and easily scaleable, i.e. exactly those types of financial
institutions and arbitrageurs who specialize in the arbitrage activities required to align market and
fundamental valuations. According to our theory, such corporations mainly exist to transfer rents to
their incumbent shareholders, while the social surplus they provide may well be negative. Extended
to a dynamic context, shareholder rent-seeking induced by market frictions offers a rationale for
corporate short-termism, or time-inconsistency of corporate decision-making, since at each point in
time the current shareholders have an incentive to take decisions that are designed to inflict harm
(by extracting rents) on their successors, and thus not in the long-term interest of the corporation.

At the same time, our theory also offers a rationale for regulating corporate behavior or for
policy interventions in the market, in the same vein as time-inconsistency of individual preferences
offers a rationale for regulating consumer choices. Among possible interventions, we show that
regulation of executive compensation is most effective at limiting rent-seeking distortions, because
such restrictions limit the shareholders’ ability to incentivize rent-seeking. Here, agency conflicts
resulting from the separation of ownership and control have the additional social benefit of limiting
the shareholders/owners ability to influence market rents through firm decisions.

For expositional reasons, we have limited the applications of our set-up to only a few simple and
particularly stark examples: distortions in risk-taking, incentives for excessive leverage, sensitivity
of investment to stock prices, and potentially harmful side-effects of transparency. But our set-up
suggests a theory of distortions, short-termism and time inconsistency which should be generally applicable to corporate decision making with market frictions, and one can therefore think of numerous other applications, such as capital structure choices, dividend payout policies, collective action problems between shareholders with different investment horizons, or real effects of credit distortions etc. All these applications, and many others, are interesting in their own right, and would deserve to be fully developed on their own. But this is left for future research, since it would go far beyond the scope of this paper.

Another important direction for future work is the empirical assessment of the importance of these departures from market efficiency for asset valuation, and corporate decision-making. In other papers (Albagli, Hellwig and Tsyvinski, 2011, 2014), we argue that our model is consistent with some asset pricing anomalies that appear difficult to reconcile with no arbitrage principles, such as the large volatility in equity prices, or the very large spreads on corporate default risk. On the corporate side, there is ample suggestive evidence of excessive risk-taking by financial institutions, investment banks or hedge funds, and more generally of patterns of corporate short-termism , which is more pronounced in companies without long-term shareholders, or with a stronger focus on shareholder value as measured by market prices. While many of these elements are consistent with the basic story told by our paper, one would require a far more systematic approach to measure and confirm the causal impact of market frictions on firm behavior. This is another important topic left for future research.

8 Appendix: Proofs

Proof of Propositions 1. Characterization. Let \( \hat{x}(P) \) denote the private signal of a trader who is just indifferent between buying and not buying the share at a given price \( P \), so that \( P = \int \Pi(\theta, k) dH(\theta|\hat{x}(P), P) \). Since \( R(\cdot) \) is increasing in \( \theta \), \( \int \Pi(\theta, k) dH(\theta|x, P) \) must be monotone in \( x \), and any trader whose private signal exceeds \( \hat{x}(P) \) must strictly prefer to purchase a share, while any trader whose signal is less than \( \hat{x}(P) \) prefers not to buy. Thus, the total demand by the informed traders is \( \alpha (1 - \Phi(\sqrt{3}(\hat{x}(P) - \theta))) \). Equating demand and supply, a price \( P \) clears the market in state \( (\theta, u) \) if and only if \( \hat{x}(P) = \theta + 1/\sqrt{3} \cdot u \equiv z \). Therefore, in any equilibrium, it must be the case that \( \hat{x}(P(\theta, u; k)) = z \), and if \( P \) is a function of \( z \) only, then it must be invertible. But if \( P(\cdot) \) is invertible, observing \( P \) is informationally equivalent to observing \( \hat{x}(P) = z \sim \mathcal{N}(\theta, (\beta\delta)^{-1}) \). Along the equilibrium path, the traders thus treat the signals \( \hat{x}(P) \sim \mathcal{N}(\theta, (\beta\delta)^{-1}) \) and \( x \sim \mathcal{N}(\theta, \beta^{-1}) \) as
mutually independent normal signals and their posterior beliefs $H(\cdot | x, P)$ are given by

$$\theta | x, P \sim \mathcal{N} \left( \frac{\hat{\lambda}\mu + \beta x + \beta\delta \hat{x}(P)}{\lambda + \beta + \beta\delta}, \left( \frac{\lambda + \beta + \beta\delta}{\hat{\lambda} + \beta} \right)^{-1} \right).$$

Substitute $\hat{x}(P) = z$, we restate the informed traders’ indifference condition in terms of $z$: $P(z, k) = \mathbb{E}(\Pi(\theta, k) | x = z, z)$. The expression for $V(z, k) = \mathbb{E}(\Pi(\theta, k) | z)$ is derived analogously using only the information from the market signal, $\theta | P \sim \mathcal{N}(\hat{\lambda}\mu + \beta \delta \hat{x}(P), (\hat{\lambda} + \beta)^{-1}).$

**Uniqueness.** If demand is restricted to be non-increasing in $P$, $\hat{x}(P)$ must be non-decreasing. If $\hat{x}(P)$ is strictly monotone everywhere, then it is invertible, $P$ is informationally equivalent to $\hat{x}(P) = z$, and we arrive at the equilibrium characterized above. Suppose therefore that $\hat{x}(P)$ is flat over some range, i.e. $\hat{x}(P) = \hat{x}$ for $P \in (P', P'')$. Suppose further that for sufficiently low $\varepsilon > 0$, $\hat{x}(P)$ is strictly increasing over $(P' - \varepsilon, P')$ and $(P'', P'' + \varepsilon)$, and hence uniquely invertible.\footnote{It cannot be flat everywhere, because then informed demand would be completely inelastic, and there would be no way to absorb noise trader shocks.}

But then for $z \in (\hat{x}(P' - \varepsilon), \hat{x})$ and $z \in (\hat{x}, \hat{x}(P'' + \varepsilon))$, $P(z)$ is uniquely defined, and must be characterized as above, from the indifference condition for $\hat{x}(P) = z$. But since the function $P(z, k)$ defined above is continuous and strictly monotonic in $z$, it must be the case that $P' = P''$, contradicting the existence of an interval for which $\hat{x}(P)$ is flat. ■

**Proof of Propositions 2 and 3.** Proposition 2 follows directly from arguments in the text. Proposition 3 is a direct corollary of THM 2 in Albagli, Hellwig and Tsyvinski (2011). ■

**Proof of Proposition 4.** To simplify notation, let $\Upsilon = \alpha \left( \mathbb{E} \left( R(\theta) | x = z, z \right) / \mathbb{E}(R(\theta)) - 1 \right)$. We begin with the results concerning $\hat{k}/k^*$. Since $\hat{k}/k^* = (1 + \Upsilon)^{1/\gamma}$, it is immediate that $\hat{k}/k^*$ is increasing in $\Upsilon$, equal to 1 if and only if $\Upsilon = 0$, and unbounded as $\Upsilon \to \infty$. Moreover, $\partial (\hat{k}/k^*) / \partial \gamma^{-1} = \log (1 + \Upsilon)(1 + \Upsilon)^{1/\gamma}$, which is positive if and only if $\Upsilon > 0$. Hence $\hat{k}/k^*$ is decreasing in $\gamma$, if $\Upsilon < 0$ and increasing in $\gamma$, if $\Upsilon > 0$, which proves that investment distortions are worse, the lower is $\gamma$. Finally, if $\Upsilon > 0$, then clearly $\hat{k}/k^*$ is unbounded as $\gamma \to 0$, while if $\Upsilon < 0$, $\hat{k}/k^* \geq (1 - \alpha)^{1/\gamma} > 0$.

Next we consider comparative statics w.r.t. $\Delta$. Since $\Delta = 1 + (1 + \Upsilon)^{1/\gamma}(\Upsilon/\gamma - 1)$, we have

$$\frac{\partial \Delta}{\partial \Upsilon} = \frac{1 + \gamma}{\gamma} (1 + \Upsilon)^{1/\gamma} \frac{\Upsilon}{1 + \Upsilon}$$

and

$$\frac{\partial \Delta}{\partial \gamma^{-1}} = (1 + \Upsilon)^{1/\gamma} \left( \Upsilon - \log (1 + \Upsilon)(1 - \Upsilon/\gamma) \right),$$

and one therefore obtains that $\Delta = 0$ iff $\Upsilon = 0$, $\Delta$ is increasing in $\Upsilon$ (and therefore positive) if $\Upsilon > 0$, and $\Delta$ is decreasing in $\Upsilon$ (and therefore again positive) if $\Upsilon < 0$. Furthermore, if $\Upsilon \geq \gamma > 0$, ...
it is clear that $\frac{\partial \Delta}{\partial \gamma} > 0$, while if $\gamma < \gamma$, $\gamma - \log(1 + \gamma) (1 - \gamma^2/\gamma) > \gamma - \gamma (1 - \gamma^2/\gamma) > \gamma^2/\gamma$ and so once again $\frac{\partial \Delta}{\partial \gamma} > 0$. The limiting behavior, the bounds, and the result that $\Delta > 1$ if $\gamma > \gamma$ also follow immediately. ■

Proof of Proposition 5. A continuous, differentiable contract $W(\Pi)$ implements investment level $k$ if and only if it satisfies the manager’s first-order condition:

$$\mathbb{E} \left\{ W'(\Pi(\theta, k)) \{ R(\theta) - C'(k) \} \right\} = 0 \text{ or } C'(k) = \frac{\mathbb{E} \left\{ W'(\Pi(\theta, k)) R(\theta) \right\}}{\mathbb{E} \left\{ W'(\Pi(\theta, k)) \right\}}.$$  

Clearly, if $W'(\Pi)$ is constant, the implemented investment level is $k^*$. Now, for each $\tilde{\theta}$, there exists a unique $k\left(\tilde{\theta}\right) > k^*$ such that $C'(k) = \mathbb{E} \left\{ R(\theta) | \theta > \tilde{\theta} \right\}$, and a unique $W\left(\tilde{\theta}\right) = R\left(\tilde{\theta}\right) k\left(\tilde{\theta}\right) - C\left(k\left(\tilde{\theta}\right)\right)$. Therefore, by construction the contract $W(\Pi) = \max \left\{ W\left(\tilde{\theta}\right), \Pi \right\}$ implements effort level $k\left(\tilde{\theta}\right)$. As $\tilde{\theta} \to \infty$, $k\left(\tilde{\theta}\right) \to \psi(\bar{R})$, while as $\tilde{\theta} \to -\infty$, $k\left(\tilde{\theta}\right) \to k^*$. Likewise, for each $\tilde{\theta}$, there exists a unique $k\left(\tilde{\theta}\right) < k^*$ such that $C'(k) = \mathbb{E} \left\{ R(\theta) | \theta < \tilde{\theta} \right\}$, and a unique $W\left(\tilde{\theta}\right) = R\left(\tilde{\theta}\right) k\left(\tilde{\theta}\right) - C\left(k\left(\tilde{\theta}\right)\right)$. Therefore, by construction the contract $W(\Pi) = \min \left\{ \Pi, W\left(\tilde{\theta}\right) \right\}$ implements effort level $k\left(\tilde{\theta}\right)$. As $\tilde{\theta} \to \infty$, $k\left(\tilde{\theta}\right) \to k^*$, while as $\tilde{\theta} \to -\infty$, $k\left(\tilde{\theta}\right) \to \psi(\bar{R})$.

Finally, $\Pi(\theta, k)$ is strictly increasing in $k$ for $k < \psi(\bar{R})$, for all $\theta$, so that for any non-decreasing contract $W(\cdot)$, $\mathbb{E} \left\{ W(\Pi(\theta, k)) \right\} \leq \mathbb{E} \left\{ W(\Pi(\theta, \psi(\bar{R}))) \right\}$ for $k < \psi(\bar{R})$, and the inequality is strict if $W(\cdot)$ is strictly increasing for some $\Pi$. Likewise since $\Pi(\theta, k)$ is strictly decreasing in $k$ for $k > \psi(\bar{R})$, for all $\theta$, it follows that $\mathbb{E} \left\{ W(\Pi(\theta, k)) \right\} \leq \mathbb{E} \left\{ W(\Pi(\theta, \psi(\bar{R}))) \right\}$ for $k > \psi(\bar{R})$, for any non-decreasing contract $W(\cdot)$, that is strictly increasing for some $\theta$. ■

Proof of Proposition 6. We need to show that whenever $(\hat{k} - k^*) T'_n(k^*) > 0$ for some $T_n(\cdot)$, the initial shareholders can use $T_n(\cdot)$ to distort investment in their desired direction.

By construction, for any $\eta > 0$, there exist $\varepsilon_1 > 0$, and $\varepsilon_2 > 0$, such that $T_0(k^*) - \eta = T_0(k^* - \varepsilon_1) = T_0(k^* + \varepsilon_2)$, and $T_0(k^*) - \eta \leq T_0(k)$ iff $k \in [k^* - \varepsilon_1, k^* + \varepsilon_2]$, and $T_0(k) > 0$ for $k \in [k^* - \varepsilon_1, k^*]$, and $T_0(k) < 0$ for $k \in [k^* + \varepsilon_2]$.  

Suppose now that $\hat{k} > k^*$ and there exists $T_n(\cdot)$, such that $T'_n(k^*) > 0$. Choose $\eta$ sufficiently small, so that $T'_n(k) > 0$ for all $k \in [k^* - \varepsilon_1, k^* + \varepsilon_2]$, and $T_n(\cdot)$ is bounded, so we can choose $\xi \in (0, \eta/(2 \max_k ||T_n(k)||))$. Then, for $k \notin [k^* - \varepsilon_1, k^* + \varepsilon_2]$, $T_0(k) + \xi T_n(k) - T_0(k^*) - \xi T_n(k^*) < -\eta + 2\xi \max_k ||T_n(k)|| \leq 0$, so a contract paying $T_0(k) + \xi T_n(k)$ must implement an investment level $k \in [k^* - \varepsilon_1, k^* + \varepsilon_2]$. Moreover, $T_0(k) + \xi T_n(k) > 0$ for $k \in [k^* - \varepsilon_1, k^*]$, implying that the contract $T_0(k) + \xi T_n(k)$ is maximized at $k \in (k^*, k^* + \varepsilon_2)$.

By the same argument, if $\hat{k} < k^*$ and there exists $T_n(\cdot)$, such that $T'_n(k^*) < 0$, then the contract $T_0(k) + \xi T_n(k)$ implements $k < k^*$, for $\xi$ sufficiently small. ■
Proof of Proposition 7. Let \( k \) denote the implemented investment level. For each share bought, the policy maker earn a realized return \( \Pi(\theta, k) - \hat{R}k - C(k) = (R(\theta) - \hat{R})k \). Let \( \hat{x}(\hat{R}) \) denote the investor threshold that prevails when the support price is active. Then the total number of units purchased, given a realization of \( \theta \) and \( u \), is \( \Phi(\sqrt{3}(\hat{x}(\hat{R}) - \theta)) - \Phi(u) = \Phi(\sqrt{3}(\hat{x}(\hat{R}) - \theta)) - \Phi(\sqrt{3}(z - \theta)) = \Pr(x \in [z, \hat{x}(\hat{R})] | \theta) \). The support price in turn is active, whenever \( z < \hat{z} \), where \( \hat{R} = \mathbb{E}(R(\theta) | x = \hat{z}, \hat{z}) \). The threshold \( \hat{x}(\hat{R}) \) is larger than \( \hat{z} \) and satisfies \( \hat{R} = \mathbb{E}(R(\theta) | \hat{x}(\hat{R}), z \leq \hat{z}) \). The expected revenue from the policy is then

\[
k \int_{-\infty}^{\hat{z}} \int_{-\infty}^{\hat{z}} (R(\theta) - \hat{R}) \left( \Phi(\sqrt{3}(\hat{x}(\hat{R}) - \theta)) - \Phi(\sqrt{3}(z - \theta)) \right) d\Phi(\sqrt{3}(\hat{x}(\hat{R}) - \theta)) d\Phi(\sqrt{3}(z - \theta))
= k \left( \mathbb{E}(R(\theta) | x \in [z, \hat{x}(\hat{R})], z \leq \hat{z}) \cdot \Pr(x \in [z, \hat{x}(\hat{R})], z \leq \hat{z}) \right)
= k \left( \mathbb{E}(R(\theta) | x \in [z, \hat{x}(\hat{R})], z \leq \hat{z}) - \mathbb{E}(R(\theta) | \hat{x}(\hat{R}), z \leq \hat{z}) \cdot \Pr(x \in [z, \hat{x}(\hat{R})], z \leq \hat{z}) \right)
< 0.
\]

Proof of Proposition 9. The ex ante expectation of \( \Omega(z, k(z)) \) is

\[
\mathbb{E}\{\Omega(z, k^*(z))\} = k(\hat{z}) \int \left( \mathbb{E}(R(\theta) | x = z, z) - \mathbb{E}(R(\theta) | z) \right) k(z) \frac{d\Phi(\sqrt{\lambda z})}{k(\hat{z})}
> k(\hat{z}) \left( \mathbb{E}(\mathbb{E}(R(\theta) | x = z, z)) - \mathbb{E}(R(\theta)) \right) = \mathbb{E}\{\Omega(z, k^*(\hat{z}))\}
\]

Moreover, if \( k'(z)/k(z) \geq \chi \), then \( k(z) \geq k(\hat{z}) e^{\chi(z-\hat{z})} \) if \( z > \hat{z} \) and \( k(z) \leq k(\hat{z}) e^{\chi(z-\hat{z})} \) if \( z < \hat{z} \).

Thus, if \( \chi \to \infty \), \( \mathbb{E}\{\Omega(z, k^*(z))\} \to \infty \), and \( \mathbb{E}\{\Omega(z, k^*(z))\} > 0 \) for \( \chi \) sufficiently large. \( \blacksquare \)