Sovereign Default and Information Frictions

DEEQA Defense*

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1 Introduction

The financial crisis that erupted in 2007, followed by the Eurozone debt crisis in 2009, has entailed renewed attention on sovereign bond spreads, raising the question of why the market seems to systematically overestimate the probabilities of default of sovereigns.
Building a dataset containing CDS spreads on government bonds of 69 countries over the period 2007/2008-2014, we observe empirically that sovereign bond CDS spreads are very high relative to historical default data. This discrepancy is more severe when a sovereign is given a good ranking by rating agencies, and for sovereign bonds with short maturities. The market seems to overestimate more default risks in these cases.
Furthermore, accounting for the observed sovereign bond spreads is a challenge for standard risk-based no arbitrage models. Such models would require that spreads account for default risk not observed in the data, and that the utility costs of such default risk are very large. In this paper, we depart from standard no arbitrage models and propose a continuous-time NREE model of sovereign bond pricing with information frictions. On the theoretical side, the new ingredient of information frictions is able to explain part of the puzzle we highlighted in the data. It can be interpreted as investors having different expectations and objectives concerning their investment in sovereign bonds. These disagreements explain a part of the spreads we observed in the data. On the methodological side, the continuous-time analysis is a novel and practical tool to study sovereign bond pricing.

The model is a sovereign debt model with a structure close to Arellano (2008). A sovereign receives a stochastic stream of income. Its government is benevolent, and would like to smooth its households' consumption intertemporally. Debt contracts are not enforceable, and the government can choose to default at any time. Furthermore, we add some simplifying assumptions (a CRRA utility and an income growth process following a random walk) in order to improve the tractability of the model. Finally, we extend it to

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introduce information frictions. We assume that the pool of competitive and risk neutral traders who buy the bonds from the sovereign is divided into two parts: informed traders who have access to a noisy private signal about the future income growth and face limits to their asset positions, and noise traders who buy a random fraction of the bonds. Based on their information, both types of traders submit their bids on the market. Hence, the market-clearing price contains some information about the private signals and the noise trading shock, so that it constitutes itself a signal about the future income growth, which is public as it is observable by everyone. With the assumptions of our model, we can sum up all the information contained in the market-clearing price into one random variable, which can be interpreted as a public signal about the future income growth.
As we want to study the impact of dispersed information across traders only (and disentangle it from the impact of the information coming through the market-clearing price), we construct a benchmark setup without information friction, and with an exogenous public signal about future income growth, being the counterpart of the endogenous public signal in the setup with information frictions.

To conclude, in the setup with information frictions, the response of bond prices to fundamental or noise trading shocks differs from the response of bond expected payoffs. In fact, market-clearing causes the price to react more strongly to those shocks than the expectation of fundamentals. Hence, it generates government bond spreads further from the default probabilities, and can explain part of the puzzle we highlighted in the data.

2 Empirical Motivation

2.1 Stylized Facts

In this subsection, we document the main stylized facts regarding CDS spreads on sovereign bonds[1]. Using datastream, we first construct a dataset containing the CDS spreads on sovereign bonds of 69 countries at four maturities (2, 3, 5, and 10 years). We take the maximum time span available, which starts in most of the cases in the years 2007 or 2008, and finishes at the end of our sample in May 2014[2]. Then, we compute CDS spreads net

[1]A Credit Default Swap (CDS) is a credit derivative contract in which a "protection buyer" agrees to pay a quarterly premium to the seller who, in exchange, commits to cover the losses in case of a credit event, be it due to a default or a restructuring. Most CDSs are documented using standard forms promulgated by the International Swaps and Derivatives Association (ISDA). Theoretically, CDS spreads on government bonds and government bond spreads should be equal, as they both depend on the sovereign's probability of default, the expected recovery rate of investors and risk aversion factors. In practice, some factors may create a discrepancy between the two, such as accrued interest, the cheapest-to-delivery option, counterparty risk, or market liquidity. Still, we focus our analysis on CDSs, because the comparison of CDS spreads across countries is more reliable (as we are comparing the same type of instruments).

[2]More information on the content of the dataset is to be found in Appendix A. More generally, we extracted from datastream all the data of CDS on sovereign bonds available, which leaves us with 69 countries over a maximum time interval 2007-2014.

of the US spread for each country, at the four maturities. Second, we use the history of sovereign credit ratings released by Moody's (from Aaa to Caa-C), and build new datasets where we sort all the data points by rating and by maturity[3]. More precisely, each dataset i (accounting for the rating and the maturity) contains CDS spread observations (R_{ij}) indexed by j. We denote the US CDS spread R_f, which is assumed to be the risk-free spread. Table 1 reports a detailed statistical analysis of the data we have on sovereign bond CDS spreads, reporting the average, the maximum, and the minimum of CDS spreads net of the US spread, as well as the standard deviation of CDS spreads, rated from Aaa to Caa-C, at the 2, 3, 5, and 10-year maturities. We finally compute the Sharpe ratio, defined as the ratio of the average CDS spread net of US spread to the standard deviation of CDS spreads.

For credit ratings from Aaa to Caa-C and bond maturities of 2, 3, 5, and 10 years, table 2 reports average CDS spreads on government bonds (net of the US), sovereign cumulative default probabilities, and average annualized loss rates. We use the statistics released by Moody's about the sovereign cumulative default probabilities[4] and impute the average annualized loss rates using their computation that creditors recover on average 63% of the face value of the debt[5]. Finally, the table reports the spread ratios, defined as the ratio of average CDS spreads to annualized loss rates. They measure the excess return on sovereign bonds, and, more precisely, by how much the market over-estimates actual default risks.

As is documented in the literature, the yield curve is generally increasing for high-rating sovereign bonds, and decreasing for low-rating bonds. In fact, the possibility of a downturn makes high-rating bonds riskier in the long run than in the short run, which is at the origin of an increasing yield curve. On the contrary, the possibility of an upturn improves the average perspective of low-rating bonds, and reverses the yield curve. As an example, the Aa bond spread averages are respectively 47.52, 55.54, 73.38, and 79.11 bps at the 2, 3, 5, and 10-year maturities, which pictures an increasing yield curve. On the contrary, the B bond spread averages are 977.91, 959.56, 907.97, and 892.71 bps at the same maturities,

[3]The history of sovereign credit ratings precedes the first CDS spread observation for all the countries in the dataset, except one (Serbia, which Moody's started to rate in 2013 only). For this sovereign, we take into account CDS spreads data only when Moody's' rating starts being available.

[4]As computed using a dataset of rating and default histories of 119 countries from 1983 to 2013 (Moody's investors service report (2014)). Notice that the dataset Moody's used for its computation extends over a longer horizon and for a larger number of countries than the dataset of CDSs we have constructed. Moody's does not release computations of cumulative default probabilities over our period of interest. Nevertheless, we are confident that it does not affect our results, as cumulative default probabilities have been approximately stable over the last years (comparing Moody's computations over 1983-2007 and over 1983-2013). Only bonds rated Caa-C have seen their cumulative default probabilities increase significantly over time by an average of 20.4% points, while A bonds of 10-year maturity's cumulative default probability has risen from 0% to 6.1%. Two noticeable exceptions are 10-year maturity bonds rated Ba and B, which have had their cumulative default probabilities decrease over time, from 18.4% to 13.5% for the first one, and from 20.8% to 18.1% for the second one.

[5]The recovery rate for creditors, is defined in Moody's investors service report (2014) as the ratio of the present value of cash flows received as a result of the distressed exchange versus those initially promised, discounted using yield to maturity immediately prior to default. The average of recovery rates is weighted by issuer, and this computation is based on 20 sovereign defaults that occurred from 1998 to 2013.

2 yr	Aaa	Aa	A	Baa	Ba	B	Caa-C
Average spread (%) $\overline{R_i} - R_f$	0.13	0.48	1.12	1.74	2.44	9.78	17.52
Max Spread (%) $\max_j (R_{ij} - R_f)$	3.24	11.25	14.03	17.68	19.97	81.02	532.29
Min Spread (%) $\min_j (R_{ij} - R_f)$	-2.07	-2.03	-2.05	-1.66	-1.57	-0.75	2.87
St. Dev. Spread (%) $\sigma (R_{ij})$	0.39	0.76	1.60	2.06	2.94	10.04	39.62
Sharpe Ratio $(\overline{R_i} - R_f)/\sigma(R_{ij})$	0.32	0.63	0.70	0.85	0.83	0.97	0.44

3 yr	Aaa	Aa	A	Baa	Ba	B	Caa-C
Average (%) $\overline{R_i} - R_f$	0.14	0.56	1.21	1.90	2.59	9.60	15.83
Max (%) $\max_j (R_{ij} - R_f)$	3.18	10.21	12.73	16.89	18.87	63.97	447.42
Min (%) $\min_j (R_{ij} - R_f)$	-2.50	-2.44	-2.46	-1.91	-1.81	-0.76	3.17
St. Dev. Spread (%) $\sigma (R_{ij})$	0.41	0.77	1.52	1.94	2.80	8.90	32.43
Sharpe Ratio $(\overline{R_i} - R_f)/\sigma(R_{ij})$	0.35	0.72	0.80	0.98	0.93	1.08	0.49

5 yr	Aaa	Aa	A	Baa	Ba	B	Caa-C
Average (%) $\overline{R_i} - R_f$	0.28	0.73	1.38	2.12	2.78	9.08	14.17
Max (%) $\max_j (R_{ij} - R_f)$	3.08	9.94	10.68	15.44	16.39	52.25	370.58
Min (%) $\min_j (R_{ij} - R_f)$	-0.45	-0.36	-0.17	0.43	0.57	-0.31	3.38
St. Dev. Spread (%) $\sigma (R_{ij})$	0.43	0.72	1.37	1.68	2.37	7.87	25.97
Sharpe Ratio $(\overline{R_i} - R_f)/\sigma(R_{ij})$	0.65	1.02	1.01	1.27	1.18	1.15	0.55

10 yr	Aaa	Aa	A	Baa	Ba	B	Caa-C
Average (%) $\overline{R_i} - R_f$	0.26	0.79	1.39	2.22	2.81	8.93	12.35
Max (%) $\max_j (R_{ij} - R_f)$	2.74	7.85	9.59	13.37	14.14	49.98	290.81
Min (%) $\min_j (R_{ij} - R_f)$	-2.97	-2.83	-2.85	-1.93	-1.43	-0.37	2.52
St. Dev. Spread (%) $\sigma (R_{ij})$	0.44	0.65	1.26	1.43	2.00	7.30	20.25
Sharpe Ratio $(\overline{R_i} - R_f)/\sigma(R_{ij})$	0.60	1.21	1.10	1.55	1.41	1.22	0.61

Table 1: CDS on government bonds statistics

	Average CDS Spreads (bps)[a]				**Cumulative Default Rates (%)**[b]			
	2 yr	3 yr	5 yr	10 yr	2 yr	3 yr	5 yr	10 yr
Aaa	12.52	14.45	27.71	26.05	0.00	0.00	0.00	0.00
Aa	47.52	55.54	73.38	79.11	0.00	0.21	0.97	1.18
A	112.23	120.99	137.85	139.05	0.08	0.57	1.32	6.14
Baa	174.47	189.63	212.26	222.43	0.54	0.88	1.65	2.04
Ba	244.48	258.92	278.30	281.07	1.75	3.30	6.12	13.52
B	977.91	959.56	907.97	892.71	6.43	8.54	12.44	18.11
Caa-C	1752.14	1583.39	1417.06	1235.32	46.29	52.78	52.78	52.78

	Annualized Loss Rates (bps)[c]				**Spread Ratio**[d]			
	2 yr	3 yr	5 yr	10 yr	2 yr	3 yr	5 yr	10 yr
Aaa	0.00	0.00	0.00	0.00	X	X	X	X
Aa	0.00	2.55	7.20	4.36	X	21.75	10.19	18.14
A	1.55	7.04	9.77	22.99	72.44	17.19	14.11	6.05
Baa	9.95	10.91	10.26	7.56	17.54	17.38	17.32	29.42
Ba	32.41	40.98	45.83	51.31	7.54	6.32	6.07	5.48
B	120.43	107.03	94.25	69.34	8.12	8.97	9.63	12.87
Caa-C	939.29	724.24	434.55	217.27	1.87	2.19	3.26	5.69

[a] CDS spreads are from Datastream. More information on the dataset is to be found in Appendix A.
[b] Cumulative default rates are from Moody's Investor service report (2014).
[c] Annualized loss rates are computed using the average recovery rate of 63% (as computed in Moody's Investor Service report (2014)), and are calculated as $-(1/T)\ln(1-0.37*CDR)$, where CDR is the cumulative default rate at maturity T.
[d] The spread ratio is the ratio between the average CDS spread and the annualized loss rate.

Table 2: CDS spreads on government bonds and historical default probabilities

which clearly pictures a decreasing yield curve.
Also, average bond spreads are increasing when ratings are deteriorating. As an example, 5-year bond spread averages are respectively 27.71, 73.38, 137.85, 212.26, 278.30, 907.97, and 1417.06 at Aaa, Aa, A, Baa, Ba, B, and C-Caa ratings. The spread curve is clearly increasing, becoming very steep as ratings become low.

Comparing the expected probability of default expressed by CDS spreads on government bonds with the historical one as calculated by Moody's, two observations are particularly striking:

Stylized fact #1: Annualized loss rates do not account much for the average level of CDS spreads on sovereign bonds.

Stylized fact #2: The fraction of CDS spreads explained by default risk is lower for higher-rating sovereign bonds, and for sovereign bonds with shorter maturities.

For CDSs on government bonds of all maturities, default risk can account for only a small fraction of the observed spreads. As an example, for A bonds with a 2-year maturity, the annualized loss rate predicted by historical data is 1.55 bps, whereas the observed average CDS spread is 112.23 bps. The puzzle is more significant for sovereign bonds with higher ratings, as shown by a spread ratio of 72.44 for A bonds with a 2-year maturity. For lower ratings, the default risk accounts for a much larger fraction of the observed CDS spreads, as expressed by a spread ratio of 1.87 for Caa-C bonds of a 2-year maturity. Furthermore, this puzzle is more significant for bonds with shorter maturities. As an example, A bonds have a spread ratio of 72.44 at a 2-year maturity, whereas the ratio falls at 6.05 at a 10-year maturity. To conclude, the size and variation of the spread ratios cannot be explained by risk premia on credit default risk alone.

2.2 Using a no-arbitrage perspective

In this subsection, we discuss the challenge of accounting for the observed CDS spreads on sovereign bonds in risk-based no arbitrage models. We conduct two simple exercises. First, we bound the worst-case scenario for sovereign bond CDS spreads, and show that this worst case scenario is outside the range of realizations, in terms of default losses, that are observed in the last 20 years of data. Second, we compute the Sharpe ratios for the sovereign bond CDS spreads, and argue that the stochastic discount factor required to price these bonds must be very volatile.

Let m denote a stochastic aggregate state at maturity (it will be identified with the stochastic discount factor). Let $R_i(m)$ denote the average return on sovereign bond CDSs with characteristic i (accounting for the rating and the maturity), when aggregate state m is realized. More precisely, $R_i(m)$ can be written as $R_i(m) = \overline{R_i} * (1 - L_i(m))$, where $\overline{R_i}$ is the initial yield and $L_i(m)$ is the aggregate loss rate on bonds with characteristic i in state m. The CDS spreads on government bonds must satisfy the moment condition $\mathbb{E}\left(R_i(m) * m\right) = 1$. The annualized yield spread s_i of a bond with T years to maturity is $s_i = 1/T * log\left(\overline{R_i}/R_f\right)$ where $R_f = 1/\mathbb{E}(m)$ is the risk-free return. Similarly, the annualized loss rate is $d_i = -1/T * log\left(1 - \mathbb{E}(L_i(m))\right)$. The difference $s_i - d_i$ then measures the annualized excess return of CDS spreads.

Bounding the worst case scenario: Restate the moment condition as $\mathbb{E}\left(R_i(m)/R_f * m/\mathbb{E}(m)\right) = 1$. Since $R_i(m)/R_f = \overline{R_i}/R_f * (1 - L_i(m))$, one obtains

$$\max_m L_i(m) \geq \mathbb{E}\left(L_i(m) * \frac{m}{\mathbb{E}(m)}\right) = \frac{\overline{R_i} - R_f}{\overline{R_i}} = 1 - e^{s_i T} \approx s_i T$$

More intuitively, the average sovereign bond CDS spread by category (rating and maturity) must be lower than the maximum loss rate possible on CDSs of the same category. Otherwise, they would offer a return in excess of the risk-free rate, and hence an arbitrage opportunity. Again, using Moody's data on sovereign default losses (over the period 1998-

2013)[6], the worst loss rate realization was 71% for a country with a Caa-C rating (Greece in 2012), and 40% for a country with a B rating (Ecuador in 1999)[7]. In comparison, the government bond CDS spreads reported in table 1 suggest that the maximum loss rate realization for Caa-C bonds should be bounded by $1 - e^{-0.1752*2}$ or 30%, 38%, 51%, and 71% at respectively the 2, 3, 5, and 10-year maturities. As for B bonds, the maximum loss rate realization should be bounded by 18%, 25%, 37%, and 59% at the same maturities. In both cases, there is one maturity at which the bound is (weakly) not respected, the 10-year maturity.

Notice that the comparison relies heavily on how we compute the loss rate realization. Using the cut in face value agreed upon after the post-default debt restructuring, the results would be more striking: as computed by Cruces and Trebesch (2010), the loss rate for Ecuador's default in 1999 was 34%[8]. In this case, the lower bound is not respected at the 5 and 10-year maturities for B bonds.

To summarize, the excess return on sovereign bonds CDS' was enough to cover the worst default loss rates at some maturities, over the last 20 years of data. This means that under a no arbitrage view, the market must put some weight on outcomes that are worse than historical realizations. This highlights the challenge of accounting for sovereign bond CDS spreads with risk-based no arbitrage models.

Sharpe Ratios for sovereign bonds: Now, we compute a lower bound on the variance of m. We have $\mathbb{E}(R_i(m)) = R_f * e^{(s_i-d_i)T}$, $\mathbb{E}(L_i(m)) = 1-e^{-d_iT}$, and $\sigma(R_i(m)) = \overline{R_i} * \sigma(L_i(m))$. Hence, we can re-write the Hansen-Jaganathan inequality as follows:

$$\frac{\sigma(m)}{\mathbb{E}(m)} \geq \frac{\mathbb{E}(R_i(m)) - R_f}{\sigma(R_i(m))} = \frac{e^{(s_i-d_i)T} - 1}{e^{s_iT} - e^{(s_i-d_i)T}} \frac{\mathbb{E}(L_i(m))}{\sigma(L_i(m))} \approx \frac{s_i - d_i}{d_i} \frac{\mathbb{E}(L_i(m))}{\sigma(L_i(m))}$$

Hence, the RHS of the inequality is a product of the Sharpe ratio for the sovereign bonds' loss rate, and of a term that depends on the average CDS spread and the loss rate. The RHS is a measure of the excess return per "unit" of default risk at various maturities by bond category, and gives lower bounds on $\sigma(m)/\mathbb{E}(m)$, and more precisely, on $\sigma(m)$ as $\mathbb{E}(m) \approx 1$. Table 2 reports the average spread and the loss rate to compute the ratio $(s_i - d_i)/d_i$. As we do not have measures for the standard deviation of sovereign bond losses, we need to bound the Sharpe ratio for the sovereign bonds' losses further. Using the fact that $\sigma^2(L_i(m)) \leq \mathbb{E}(L_i(m))(\max_m L_i(m) - \mathbb{E}(L_i(m)))$, we obtain that $\mathbb{E}(L_i(m))/\sigma(L_i(m)) \geq 1/\sqrt{\max_m L_i(m)/\mathbb{E}(L_i(m)) - 1}$.

Again, using Moody's estimates about sovereign default loss rates, we know that the worst loss rate realization was $\max_m L_i(m) = 0.71$ for a country with a Caa-C rating and 0.40 for

[6] Moody's loss rate data differentiate across ratings, but not across maturities.

[7] As in table 2, we use Moody's computation of the recovery rate for creditors, defined as the ratio of the present value of cash flows received as a result of the distressed exchange versus those initially promised, discounted using yield to maturity immediately (see Moody's Investor service report (2014)).

[8] It is still the worst loss rate realization for a country with a B rating in their dataset, containing sovereign defaults over the period 1998-2010

a country with a B rating. Hence, for Caa-C bonds, we obtain the following lower bounds on $\sigma(m)$: 0.49, 0.73, 1.39, and 2.89 at the 2, 3, 5, and 10-year maturities. Similarly, for B bonds, we obtain: 1.79, 2.33, 3.11, and 5.33 at the same maturities. This suggests that the discount factor used to price these bonds must be very volatile, and this is due to the large wedge between spreads and default rates. Hence, it must attach a very high relative value to consumption in very unlikely states of nature where there are defaults. In these states, the default rate must be large (and even larger than any historical default rate observed in the data), and the marginal value of consumption must be high as well. This suggests that the observed sovereign bonds CDS spreads in the context of no-arbitrage models account for some aggregate default risk not observed in the data, where the utility costs of such risk are very large.
Notice that this puzzle is even more striking when looking at corporate bond spreads, as can be seen in Albagli, Hellwig and Tsyvinski (2013).

To conclude, these observations motivate the following of the paper, where we present a theoretical sovereign default model where we introduce information frictions and limits to arbitrage in an attempt to account for the observed sovereign bond CDS spreads.

2.3 Decomposition of spreads using a factor analysis

[In progress]
In the literature on sovereign bond spreads, there is a wide consensus on four main determinants for the spreads (yet, there is no agreement on their relative importance): macroeconomic and fiscal fundamentals, monetary policy, liquidity, and risk aversion. We will carry out a factor analysis in order to extract from the spread data the component that cannot be explained by these determinants. Then, we can make a statistical analysis of this component.
In other words, we will use the spread databases we have constructed, and run regressions for every rating and maturity on the explained variable $s_{c,t}$ (c denoting the country and t the date) on the following explaining variables:

- Macroeconomic variables $M_{c,t}$: Real GDP growth, Inflation rate, Industrial Production growth, Current Account balance
- Fiscal variables $F_{c,t}$: Government Balance ratio, Government Debt ratio
- A common financial variable V_t: VIX index (to account for international risk aversion),
- Other financial variables $X_{c,t}$: the Bid-Ask spread (to account for liquidity), the Central Bank's Policy rate (to account for monetary policy)
- Information Frictions variable: a measure of forecast dispersion about the future state of the economy

$$s_{c,t} = \beta_0 + \beta_1 M_{c,t} + \beta_2 F_{c,t} + \beta_3 V_t + \beta_4 X_{c,t} + \epsilon_{c,t}$$

Finally, we can carry out a statistical analysis on the unexplained component $\widehat{\epsilon_{c,t}}$ for each rating and maturity. It will also be interesting to analyze the potentially differentiated impact of the information frictions variable on CDS spreads.

3 The Model Economy

In this section, we start by presenting a sovereign debt model whose structure is close to Arellano (2008). We add some simplifying assumptions (a CRRA utility and an income growth process following a random walk) in order to improve its tractability.

First, we study a benchmark setup without information friction, where the traders who buy the sovereign bonds have all access to an exogenous public signal about the future income growth. Second, we extend the model further to add dispersed information across traders, which gives rise to a new setup with information frictions. As the information is dispersed, the market-clearing price contains some additional information and constitutes itself a public signal that is observable by all the traders. The public signal is key in the two setups: it is an exogenous signal in the setup without information friction, and an endogenous signal coming from the price in the setup with information frictions. It enables us to study the impact of dispersed information only, when we compare the two setups.

To begin with, we present the model in discrete time, over small time intervals Δ. Then, we take the continuous-time limit of the model to study the implications of the presence of information frictions.

3.1 Model environment

Households and technology Consider a small open economy with identical and risk averse households. They have the following preferences:

$$\mathbb{E}_0 \int_{t=0}^{\infty} e^{-\rho t} U(c_t) dt$$

where $0 < \rho < 1$ is the discount factor, c_t is consumption, and $U(\cdot)$ is increasing and strictly concave. Households receive a stochastic stream of income y_t. The income growth process is assumed to be Markov with a compact support such that $\ln(y_{t+\Delta}/y_t) \sim \mathcal{N}(0, \sigma^2\Delta)$.

The government The government is benevolent and its objective is to maximize the utility of households. Hence, it uses borrowing to smooth the households' consumption path. Debt contracts are not enforceable, and the sovereign can choose to default on its debt at any time. If it defaults, then all its debt is erased, it is once and for all excluded from financial markets, and it incurs direct output costs.

The timing Let ζ_t be a public signal about the future growth of the economy's income. Over each time period Δ, the government starts with an initial level of assets a_t, and receives its current income y_t. It can decide to default on its debt a_t, or to adjust its stock of debt to $a_{t+\Delta}$. The price eventually paid on the debt will be $q(a_{t+\Delta}, y_t, y_{t+\Delta}, \zeta_t)$. As the government does not know the public signal ζ_t, nor its future income $y_{t+\Delta}$, it has to form expectations about the price in order to decide its new stock of debt. Then, the public signal about the future income growth ζ_t is revealed, traders observe it, and submit their bids. Finally, the market clears at price $q(a_{t+\Delta}, y_t, y_{t+\Delta}, \zeta_t)$, and households consume.

Assets The government has access to financial markets, where it can buy only one type of bonds, a perpetuity. It pays $\{\lambda\Delta, \lambda\Delta\,(1-\lambda\Delta)\,, \lambda\Delta\,(1-\lambda\Delta)^2\,, ...\}$ where $\lambda\Delta$ is the size of the coupon, while the principal depreciates at a (normalized) rate of $\lambda\Delta$. The bond price function $q(a_{t+\Delta}, y_t, y_{t+\Delta}, \zeta_t)$ is endogenous to the government's incentives to default, and depends on the size of stock of debt $a_{t+\Delta}$, on the current income y_t, on the future income $y_{t+\Delta}$, and on the public signal ζ_t.

Notice that the idiosyncratic income uncertainty induced by $y_{t+\Delta}$ cannot be insured away with the perpetuities available, which pay a time and state invariant amount. Hence, asset markets are incomplete, because of both the endogenous default risk and the set of assets available.

The sovereign's value function The sovereign's value function at time t is the maximum between the value of defaulting and the value of repaying its debt at time t. On the one hand, we denote $\overline{V}(y_t)$ the value of defaulting, which is a function of current output only. On the other hand, the value of repaying is the maximum of the expected flow utility over the period Δ, and of the expected future continuation value (before the next default decision), discounted at rate ρ.
Hence, the sovereign's value function writes:

$$V\left(a_t, y_t\right) = \max\left\{\overline{V}(y_t), \max_{c_t, a_{t+\Delta}} \mathbb{E}_\Delta\left\{U\left(c_t\right)\Delta + \left(1-\rho\Delta\right)V\left(a_{t+\Delta}, y_{t+\Delta}\right)\right\}\right\}$$

under the (flow) budget constraint, over a time period Δ:

$$c_t\Delta + q\left(a_{t+\Delta}, y_t, y_{t+\Delta}, \zeta_t\right)\left(a_{t+\Delta} - \left(1-\lambda\Delta\right)a_t\right) = y_t\Delta + \lambda\Delta a_t$$

We denote $\mathcal{R}(a_{t+\Delta}, y_t)$ the sovereign's repayment set where $\mathcal{R}(a_{t+\Delta}, y_t) = \{y_{t+\Delta} : \overline{V}(y_{t+\Delta}) \leq \max_{c_{t+\Delta}, a_{t+2\Delta}} \mathbb{E}_\Delta\left\{U\left(c_{t+\Delta}\right)\Delta + \left(1-\rho\Delta\right)V\left(a_{t+2\Delta}, y_{t+2\Delta}\right)\right\}\}$. There is a decision to default, after the new income is realized, if $y_{t+\Delta} \notin \mathcal{R}(a_{t+\Delta}, y_t)$.

Traders There are a large number of competitive and risk neutral traders who have access to an international credit market in which they can borrow or lend at a constant international interest rate $r > 0$. They live for one period, and can decide to buy bonds in period t to

re-sell them in period $t+1$. Their bond holdings are restricted to the interval $[0, -a_{t+\Delta}]$. They have perfect information regarding the economy's income process, and can observe the current level of income. They also have access to the public signal $\zeta_t \sim F_\Delta\left(\cdot \mid \ln\left(y_{t+\Delta}/y_t\right)\right)$ about the future income growth before submitting their bids on the market, which are functions of the public signal ζ_t. Finally, if the country defaults on its debt, traders recover a small part of their initial investment $\underline{q} < q(a_{t+\Delta}, y_t, y_{t+\Delta}\zeta_t)\ \forall a_{t+\Delta}, y_t, y_{t+\Delta}, \zeta_t$.
In the following, we first present the benchmark setup without information friction between traders, and second, we add to this setup dispersed information between traders, which gives rise to the setup with information frictions.

3.2 Setup without information friction

Traders - Specific assumptions The traders have all access to the same information, an exogenous public signal ζ_t, before submitting their bids on the market. We assume that $\zeta_t \sim \mathcal{N}(\ln(y_{t+\Delta}/y_t), (\beta\delta)^{-1})$. After observing the public signal ζ_t, they submit their bids, which are functions of the public signal ζ_t. As traders are assumed to be risk neutral, they break even in expected value in every bond contract they offer. Then, the market clears at price $q(a_{t+\Delta}, y_t, y_{t+\Delta}, \zeta_t)$.
In the setup without information friction, we denote with a hat the specific variables affected by the hypotheses of the setup: the sovereign's policy rule $\hat{\alpha}(\cdot) = a_{t+\Delta}$, and the bond price $\hat{q}(\cdot)$.

Equilibrium definition We focus on a Recursive Bayesian Equilibrium, in which the sovereign only condition on the current $\{a_t, y_t, \hat{q}(a_{t+\Delta}, y_t)\}$, while traders condition on $\{a_{t+\Delta}, y_t, \hat{q}_t = \hat{q}(a_{t+\Delta}, y_t, y_{t+\Delta}, \zeta_t)\}$ and the public signal ζ_t. Given the knowledge of y_t and a_t, the past history of $\{a_{t-s\Delta}, y_{t-s\Delta}, \hat{q}_{t-s\Delta}\}_{s=1}^{t/\Delta}$ contains no further information on current and future prices. A Recursive Bayesian Equilibrium consists of a bond price function $\hat{q}_t$, a value function $V(\cdot)$, a policy rule $\hat{\alpha}(\cdot)$, a repayment set $\mathcal{R}(a_{t+\Delta}, y_t)$, a schedule for the traders $a_i(a_{t+\Delta}, y_t, \hat{q}_t, \zeta_t) \in [0, -a_{t+\Delta}]$, and traders' beliefs $H(\cdot \mid a_{t+\Delta}, y_t, \hat{q}_t, \zeta_t)$ such that: (i) given the repayment set, $\hat{q}_t$ reflects the sovereign's default probabilities and is consistent with traders' expected zero profits, (ii) the value function and the policy rule, combined with the repayment set, solve the Bellman equation, (iii) the schedule for the traders is optimal given their beliefs, (iv) traders' beliefs are consistent with Bayes' rule, and (v) the price function clears the market for all $(a_{t+\Delta}, y_t, y_{t+\Delta}, \zeta_t)$.

Recursive equilibrium characterization For notation ease, let $\hat{q} = \hat{q}(a', y, \zeta)$ denote the market-clearing price of the bond, as a function of the public signal ζ[9]. From now on, we will call it the traders' bond price. Denoting $\hat{\alpha}(a, y) = a'$ the sovereign's policy rule, we

[9]We dropped the variable y' for simplicity, but notice that the public signal ζ is itself a function of y', as its distribution is centered in $\ln(y'/y)$.

can write the traders' bond price function recursively:

$$\hat{q}\left(a',y,\zeta\right) = (1-r\Delta)\left\{\underline{q} + \int_{\mathcal{R}(a',y)} \left(\lambda\Delta + (1-\lambda\Delta)\,\hat{q}\left(\hat{\alpha}\left(a',y'\right),y',\zeta'\right) - \underline{q}\right) d\Psi_{\Delta}\left(y',\zeta' \mid y,\zeta\right)\right\}$$

where $\Psi_{\Delta}(\cdot)$ denotes the joint cdf for the income process y' and the public signal ζ, conditional on their previous realizations, over a time interval Δ.

Proposition 1 (Simplifying assumptions) *Let us assume that $\overline{V}(y) = Dy^{1-\psi}$ and $U(c) = c^{1-\psi}/(1-\psi)$ where D is a constant characterizing the default cost, and ψ is the constant relative risk aversion parameter in the utility function. With the income process following a random walk, we can re-write the model using a single state representation. Hence, $V(a,y) = y^{1-\psi}v(a/y)$ where $v(\cdot)$ is a new value function. Similarly, the traders' bond price can be written as $\hat{q}(a',y,\zeta) = \hat{q}(a'/y,\zeta)$ respectively. Finally, the sovereign's policy rule can be written as $\hat{\alpha}(a,y) = \hat{\alpha}(\alpha)y$ where $\alpha = a/y$ denotes the debt to income ratio, $\hat{\alpha}(\alpha) = \alpha + \mu\Delta = a'/y$ the new debt to income ratio, and $\mu = (a'-a)/y\Delta$ the asset choice to income ratio.*

Letting $y'/y = \epsilon'$, the sovereign's value function simplifies to:

$$v(\alpha) = \max\left\{D, \max_{c,\mu} \mathbb{E}_{\Delta}\left\{U\left(\frac{c}{y}\right)\Delta + (1-\rho\Delta)\left(\alpha+\mu\Delta\right)^{1-\psi}\left(\alpha+d\alpha\right)^{\psi-1} v\left((\alpha+d\alpha)\right)\right\}\right\}$$

under the budget constraint:

$$\frac{c}{y} = 1 + \lambda\alpha\left(1-\hat{q}(\alpha+\mu\Delta,\zeta)\right) - \mu\hat{q}(\alpha+\mu\Delta,\zeta)$$

In other words, we use the debt to income ratio α as a state variable to reduce the dimensionality of the state space. The sovereign decides to default whenever $\alpha \leq \underline{\alpha} = v^{-1}(D)$ where $\underline{\alpha} \leq 0$. That is, when the debt to income ratio, after the income is revealed, exceeds the threshold $-\underline{\alpha}$, the sovereign is better off defaulting rather than repaying its debt. Notice that this characterization holds for any $\hat{q}(\alpha+\mu\Delta,\zeta)$. If the country chooses to repay its debt, we can write its policy rule as $\hat{\mu}(\alpha)$, the asset choice to income ratio.

We obtain an expression for the sovereign's expected bond price at the beginning of the time period Δ:

$$\hat{q}\left(\alpha+\mu\Delta\right) = (1-r\Delta)\left\{\underline{q} + \int_{a'/y\underline{\alpha}}^{\infty} \left(\lambda\Delta + (1-\lambda\Delta)\,\hat{q}\left(\hat{\alpha}\left(\alpha'\right)\right) - \underline{q}\right) d\Phi_{\Delta}\left(\frac{\ln\epsilon'}{\sigma}\right)\right\}$$

where $\alpha' = a'/(y\epsilon')$ and $\Phi_{\Delta}(\cdot)$ denotes the cdf of a standard normal distribution over a time interval Δ. There is no dependence on the public signal distribution.

Once the public signal ζ is revealed, the traders' bond price depends on the value of the signal, and can be written as follows:

$$\hat{q}(\alpha+\mu\Delta,\zeta)=(1-r\Delta)\left\{\underline{q}+\int_{a'/y\underline{\alpha}}^{\infty}\left(\lambda\Delta+(1-\lambda\Delta)\,\hat{q}\left(\hat{\alpha}\left(\alpha'\right)\right)-\underline{q}\right)d\Phi_{\Delta}\left(\frac{\ln\epsilon'}{\sigma}\mid\zeta\right)\right\}$$

The proofs are in Appendix B.

Continuous time approximation By taking the continuous time limit, we can simplify the resolution of the model. Intuitively, as $\Delta\to 0$, the debt to income ratio α evolves continuously as a function of income realizations, and of the sovereign's borrowing choices. Hence, the probability of defaulting is zero as long as $\alpha>\underline{\alpha}$, until α hits the boundary $\underline{\alpha}$. By taking the continuous time limit, we can solve for the equations of the model, taking into account the fact that the probability of default on the interval $]\underline{\alpha},0]$ is zero, and fixing the boundary conditions such that the sovereign chooses to default when α hits $\underline{\alpha}$.

Over the interval $]\underline{\alpha},0]$, the probability of a default being zero as $\Delta\to 0$, we need to solve the following Hamilton-Jacobi-Bellman equation in order to find the ordinary differential equation (ODE) for the sovereign's value function:

$$v(\alpha)=\max_{\mu}\mathbb{E}_{\Delta}\left\{U\left(\frac{c}{y}\right)\Delta+(1-\rho\Delta)\left(\alpha+\mu\Delta\right)^{1-\psi}\left(\alpha+d\alpha\right)^{\psi-1}v\left(\left(\alpha+d\alpha\right)\right)\right\}$$

such that

$$\frac{c}{y}=1+\lambda\alpha-\hat{q}(\alpha)\left(\mu+\lambda\alpha\right)$$

Proposition 2 (ODE for the sovereign's value function) *The sovereign's value function takes the form:*

$$\left(\rho+(\psi-1)\,(2-\psi)\,\frac{1}{2}\sigma^{2}\right)v\left(\alpha\right)=U\left(\frac{c}{y}\right)+v'\left(\alpha\right)\left(\hat{\mu}(\alpha)+(\psi-1)\,\alpha\sigma^{2}\right)+\frac{1}{2}\sigma^{2}\alpha^{2}v''\left(\alpha\right)$$

where

$$\frac{c}{y}=1+\lambda\alpha-\hat{q}(\alpha)\left(\hat{\mu}(\alpha)+\lambda\alpha\right)$$

$$\hat{\mu}(\alpha)=\frac{1+\lambda\alpha}{\hat{q}(\alpha)}-\frac{v'(\alpha)^{-\frac{1}{\psi}}}{\hat{q}(\alpha)^{1-\frac{1}{\psi}}}-\lambda\alpha$$

with the boundary conditions $v(\underline{\alpha})=D$ *and* $v'(\underline{\alpha})=0$.

The proof is in appendix B. When the sovereign's debt to income ratio hits the boundary $\underline{\alpha}$, it is exactly indifferent between defaulting and repaying its debt. Hence, its value function at the boundary is exactly $v(\underline{\alpha})=D$. Furthermore, a smooth-pasting condition requires that the derivative of the value function at the boundary is zero.

Proposition 3 (ODE for the sovereign's expected bond price) *Let us define* $\gamma_v = \frac{\beta\delta}{1/\sigma^2+\beta\delta}$. *Using Bayesian updating, we can write the ODE for the sovereign's expected bond price* $\hat{q}(\alpha)$ *as following:*

$$(r+\lambda)\,\hat{q}(\alpha) = \lambda + \hat{q}'(\alpha)\hat{\mu}(\alpha) + \frac{\alpha^2\sigma^2}{2}\hat{q}''(\alpha)$$

with the boundary conditions $\hat{q}(\underline{\alpha}) = \underline{q}$, $\hat{q}(0) = \lambda/(\lambda + r)$, *and* $\hat{q}'(0) = 0$.

As traders recover $\underline{q}$ if the sovereign decides to default, the bond price at the boundary is $\hat{q}(\underline{\alpha}) = \underline{q}$. When the sovereign has no debt yet, borrowing is risk-free. Hence, its price exactly equals the price of the perpetuity whose all payments are honored $\hat{q}(0) = \lambda/(\lambda+r)$. Finally, a smooth-pasting condition requires that $\hat{q}'(0) = 0$.

Proposition 4 (Equation for the traders' bond price) *Using Bayesian updating, the equation for the traders' bond price* $\hat{q}(\alpha, \zeta)$ *writes:*

$$(r+\lambda)\,\hat{q}(\alpha,\zeta) = \lambda + \hat{q}'(\alpha)\left(\hat{\mu}(\alpha) - \alpha\gamma_v\zeta\right) + \frac{\alpha^2(1-\gamma_v)\sigma^2}{2}\hat{q}''(\alpha)$$

Comparing the traders' bond price with the sovereign's expected one as $\Delta \to 0$, we observe that $\hat{q}(\alpha,\zeta) - \hat{q}(\alpha) = -\hat{q}'(\alpha)\alpha\gamma_v\zeta$.

We end up with 2 ordinary differential equations, one for the sovereign's value function, and one for the sovereign's expected bond price. Once we know these two functions, we can obtain the traders' bond price as a function of the public signal ζ.

3.3 Setup with information frictions

Traders - Specific assumptions Within the pool of traders, there is a unit measure of informed traders. They receive a noisy private signal $x_t \sim \mathcal{N}\left(\ln(y_{t+\Delta}/y_t), \beta^{-1}\right)$ before deciding whether to purchase bonds in period t to re-sell them in period $t+1$. As they perfectly observe the current income y_t at the beginning of each period, the dispersion of beliefs resolves at the end of each period. The remaining traders are noise traders who buy a fraction $\Phi(u_t)$ of the bonds, where $u_t \sim \mathcal{N}(0, \sigma_u^2\Delta)$, and $\Phi(\cdot)$ denotes the cdf of a standard normal distribution. Both types of traders have access to the public signal ζ_t about the future income growth before submitting their bids on the market. In each period, informed traders submit price-contingent orders to purchase the available bonds, while noise traders bid for a fraction $\Phi(u_t)$ of the bonds. Then, the market-clearing price $q(a_{t+\Delta}, y_t, y_{t+\Delta}, \zeta_t, u_t)$ is selected so that the total demand by informed and noise traders equals the available supply of $-a_{t+\Delta}$.
In the setup with information frictions, we denote with a tilde the specific variables affected by the hypotheses of the setup: the policy function $\tilde{\alpha}(\cdot)$, and the bond price $\tilde{q}(\cdot)$.

Equilibrium definition We focus on a Recursive Bayesian Equilibrium, in which the sovereign only condition on the current $\{a_t, y_t, \tilde{q}(a_{t+\Delta}, y_t)\}$, while informed traders condition on $\{a_{t+\Delta}, y_t, \tilde{q}_t = \tilde{q}(a_{t+\Delta}, y_t, y_{t+\Delta}, \zeta_t, u_t)\}$, the public signal ζ_t, and their private signal x_t. Given the knowledge of y_t, the past history of $\{a_{t-s\Delta}, y_{t-s\Delta}, \tilde{q}_{t-s\Delta}\}_{s=1}^{t/\Delta}$ contains no further information on current and future prices. A Recursive Bayesian Equilibrium consists of a bond price function $\tilde{q}_t$, a value function $V(\cdot)$, a policy rule $\tilde{\alpha}(\cdot)$, a repayment set $\mathcal{R}(a_{t+\Delta}, y_t)$, a schedule for the informed traders $a_i(a_{t+\Delta}, y_t, \tilde{q}_t, \zeta_t, x_t) \in [0, -a_{t+\Delta}]$, and informed traders' beliefs $H(\cdot \mid a_{t+\Delta}, y_t, \tilde{q}_t, \zeta_t, x_t)$ such that: (i) given the repayment set, $\tilde{q}_t$ satisfies the bond price equation, (ii) the value function and the policy rule, combined with the repayment set, solve the Bellman equation, (iii) the schedule for the traders is optimal given their beliefs, (iv) traders' beliefs are consistent with Bayes' rule, and (v) the bond price function clears the market for all $(a_{t+\Delta}, y_t, y_{t+\Delta}, \zeta_t, u_t)$.

Recursive equilibrium characterization The results of Proposition 1 carry through in the setup with information frictions. Again, we denote $\tilde{q} = \tilde{q}(\alpha+\mu\Delta, \zeta)$ the market-clearing price of the bond, as a function of the public signal ζ[10]. In the following, we will call it the traders' bond price. The value function is the same as in the setup without information friction, with a budget constraint that now depends on the bond price $\tilde{q}(\cdot)$ instead of $\hat{q}(\cdot)$.

$$v(\alpha) = \max\left\{D, \max_{c,\mu} \mathbb{E}_\Delta \left\{U\left(\frac{c}{y}\right)\Delta + (1-\rho\Delta)\left(\alpha+\mu\Delta\right)^{1-\psi}\left(\alpha+d\alpha\right)^{\psi-1} v\left((\alpha+d\alpha)\right)\right\}\right\}$$

under the budget constraint:

$$\frac{c}{y} = 1 + \lambda\alpha\left(1 - \tilde{q}(\alpha+\mu\Delta, \zeta)\right) - \mu\tilde{q}(\alpha+\mu\Delta, \zeta)$$

If the country chooses to repay its debt, we can write its decision rule as $\tilde{\mu}(\alpha)$ which is the asset choice to income ratio.

The informed trader submits an order of $-a'$, whenever his expected bond price conditional on his private signal x and the public signal ζ is higher than the market-clearing bond price, and 0 otherwise. To begin with, let us suppose that the informed trader's expected bond price conditional on x and ζ is an increasing function of x. With risk-neutrality, position bounds and log-concavity of the private signals, the informed trader's demand is characterized by a threshold function:

$$\begin{aligned}&\tilde{q}\left(\alpha+\mu\Delta,\zeta\right) \leq (1-r\Delta)\left\{\underline{q} + \int_{a'/y\underline{\alpha}}^{\infty}\left(\lambda\Delta + (1-\lambda\Delta)\,\tilde{q}\left(\tilde{\alpha}\left(\alpha'\right)\right) - \underline{q}\right) d\Psi_\Delta\left(\epsilon' \mid x,\zeta\right)\right\}\\ &\Leftrightarrow x \geq \overline{x}(\alpha+\mu\Delta, \tilde{q})\end{aligned}$$

[10]We dropped the variables y' and u for notation ease, but the public signal ζ will be itself a function of y' and u, as will be shown in the next paragraphs of the paper.

where $\Psi_\Delta(\cdot)$ denotes the cdf of the future income growth conditional on the public signal ζ and the private signal x over a time interval Δ.

Therefore, the informed traders' demand for the bond is $-a'\left(1-\Phi(\sqrt{\beta}(\overline{x}(\alpha+\mu\Delta,\tilde{q})-\ln\epsilon'))\right)$, and the market clears if and only if

$$a'\left(1-\Phi\left(\sqrt{\beta}\left(\overline{x}(\alpha+\mu\Delta,\tilde{q})-\ln\epsilon'\right)\right)\right)+a'\Phi(u)=a'$$

or if and only if $\overline{x}(\alpha+\mu\Delta,\tilde{q})=\ln\epsilon'+u/\sqrt{\beta}$. Let us choose $\zeta=\overline{x}(\alpha+\mu\Delta,\tilde{q})$. This random variable summarizes the information conveyed through the market price $\tilde{q}$. We further assume that $\zeta\sim\mathcal{N}\left(\ln\epsilon',(\beta\delta)^{-1}\right)$. Using this fact, and $\zeta=\overline{x}(\alpha+\mu\Delta,\tilde{q})$ (by market-clearing), we obtain $\tilde{q}$ as a function of ζ and $\alpha+\mu\Delta$. Hence, the indifference condition for the signal threshold defines the traders' bond price:

$$\tilde{q}\left(\alpha+\mu\Delta,\zeta\right)=(1-r\Delta)\left\{\underline{q}+\int_{a'/y\underline{\alpha}}^{\infty}\left(\lambda\Delta+(1-\lambda\Delta)\,\tilde{q}\left(\tilde{\alpha}\left(\alpha'\right)\right)-\underline{q}\right)d\Psi_\Delta\left(\epsilon'\mid x=\zeta,\zeta\right)\right\}$$

Integrating over the public signal ζ, we obtain a new expression for the sovereign's expected bond price at the beginning of the time period Δ:

$$\tilde{q}\left(\alpha+\mu\Delta\right)=(1-r\Delta)\left\{\underline{q}+\int_{a'/y\underline{\alpha}}^{\infty}\left(\lambda\Delta+(1-\lambda\Delta)\,\tilde{q}\left(\tilde{\alpha}\left(\alpha'\right)\right)-\underline{q}\right)d\tilde{\Psi}_\Delta\left(\epsilon'\right)\right\}$$

where

$$\tilde{\Psi}_\Delta\left(\epsilon'\right)=\int\Psi_\Delta\left(\epsilon'\mid x=\zeta,\zeta\right)dF_\Delta(\zeta)$$

Continuous time approximation Taking the continuous time limit as $\Delta\to0$, we obtain the same expression for the sovereign's value function, with the only difference being the sovereign's expected bond price, which is $\tilde{q}(\alpha)$ instead of $\hat{q}(\alpha)$.

Proposition 5 (ODE for the sovereign's value function) *The sovereign's value function takes the form:*

$$\left(\rho+(\psi-1)\,(2-\psi)\,\frac{1}{2}\sigma^2\right)v\left(\alpha\right)=U\left(\frac{c}{y}\right)+v'\left(\alpha\right)\left(\tilde{\mu}(\alpha)+(\psi-1)\,\alpha\sigma^2\right)+\frac{1}{2}\sigma^2\alpha^2v''\left(\alpha\right)$$

where

$$\frac{c}{y}=1+\lambda\alpha-\tilde{q}(\alpha)\left(\tilde{\mu}(\alpha)+\lambda\alpha\right)$$

$$\tilde{\mu}(\alpha)=\frac{1+\lambda\alpha}{\tilde{q}(\alpha)}-\frac{v'(\alpha)^{-\frac{1}{\psi}}}{\tilde{q}(\alpha)^{1-\frac{1}{\psi}}}-\lambda\alpha$$

with the same boundary conditions $v(\underline{\alpha})=D$ *and* $v'(\underline{\alpha})=0$.

The proof is in appendix B.

Proposition 6 (ODE for the sovereign's expected bond price) *Let us define* $\gamma_p = \frac{\beta(1+\delta)}{1/\sigma^2+\beta(1+\delta)} > \gamma_v$ *and* $\sigma_p^2 = \left(1 - \gamma_p + \frac{\gamma_p^2}{\gamma_v}\right)\sigma^2 > \sigma^2$. *Using Bayesian updating, we can write the ODE for the sovereign's expected bond price* $\tilde{q}(\alpha)$ *as following:*

$$(r+\lambda)\,\tilde{q}(\alpha) = \lambda + \tilde{q}'(\alpha)\tilde{\mu}(\alpha) + \frac{\alpha^2\sigma_p^2}{2}\tilde{q}''(\alpha)$$

with the boundary conditions $\tilde{q}(\underline{\alpha}) = \underline{q}$, $\tilde{q}(0) = \lambda/(\lambda+r)$, *and* $\tilde{q}'(0) = 0$.

We observe that the implied variance of the sovereign's expected bond price in the information frictions setup equals σ_p^2, which is strictly larger than the variance in the setup without information friction σ^2. There is more volatility in the information frictions setup, as more weight is assigned to tail realizations than the objective distribution with variance σ^2. The implied variance σ_p^2 depends on the variance of fundamental shocks σ^2, as well as on $\beta\delta$, the precision of the public signal, and on β, the precision of the private signal.

Proposition 7 (Equation for the traders' bond price) *Using Bayesian updating, the equation for the traders' bond price* $\hat{q}(\alpha, \zeta)$ *writes:*

$$(r+\lambda)\,\tilde{q}(\alpha,\zeta) = \lambda + \tilde{q}'(\alpha)\left(\tilde{\mu}(\alpha) - \alpha\gamma_p\zeta\right) + \frac{\alpha^2(1-\gamma_p)\sigma^2}{2}\tilde{q}''(\alpha)$$

As for the traders' bond price, the implied variance in the setup with information frictions $(1-\gamma_p)\sigma^2$ is strictly smaller than $(1-\gamma_v)\sigma^2$, in the setup without information friction. Furthermore, the change in drift compared to the sovereign's expected bond price is larger, ceteris paribus, in the information frictions setup ($\gamma_p > \gamma_v$).
Comparing the sovereign's expected bond price with the traders' one as $\Delta \to 0$, we observe that $\tilde{q}(\alpha,\zeta) - \tilde{q}(\alpha) = -\tilde{q}'(\alpha)\alpha\gamma_p\zeta$.

As in the setup without information friction, we end up with 2 ordinary differential equations, one for the sovereign's value function, and one for the sovereign's expected bond price. Once we know these two functions, we can obtain the traders' bond price as a function of the public signal ζ.

4 Comparative Statics

4.1 The price of debt

wrt. the lack of information of the sovereign The fact that the sovereign has less information about the future income of its economy than the market is at the origin of a discrepancy between the price expected by the sovereign (before the releasing of the public signal), and the price that clears the market (after the revelation of the public signal). This discrepancy exists in both setups, with and without information frictions, but its size varies.

$$\left|\frac{\partial\left(\hat{q}(\alpha,\zeta)-\hat{q}(\alpha)\right)}{\partial\zeta}\right| = |-\hat{q}'(\alpha)\alpha\gamma_v| > 0$$
$$\left|\frac{\partial\left(\tilde{q}(\alpha,\zeta)-\tilde{q}(\alpha)\right)}{\partial\zeta}\right| = |-\tilde{q}'(\alpha)\alpha\gamma_p| > 0$$

The further the public signal is from its unconditional mean (zero), the larger the discrepancy between the two prices. This works in the two directions, whether the signal is a good or a bad news. Furthermore, the size of the discrepancy is different between the two setups, and depends on the size of $\mid \hat{q}'(\alpha)\gamma_v \mid$ vs $\mid \tilde{q}'(\alpha)\gamma_p \mid$, keeping α constant.

wrt. the maturity λ The size of the coupon λ enables us to carry out some comparative statics wrt. the maturity. The larger λ, the more the sovereign has to repay in a close future, so that the average maturity of the available bond is shorter.
Interestingly, λ shows up similarly in the equation for the price $q(\alpha)$ expected by the sovereign, as well as in the equation for the traders' bond price $q(\alpha, \zeta)$ in the two setups. Hence, λ does not appear in the expression for the difference between the two prices $q(\alpha, \zeta) - q(\alpha)$, which indicates that maturity has no impact on the discrepancy between the two prices[11].

4.2 The sovereign's utility

Keeping the expected price $q(\alpha)$ constant, the expression of the ODE for the value function is the same in both setups, with and without information frictions. Similarly, the expression for the policy function $\mu(\alpha)$ is the same. Yet, both the value function and the policy function are function of the expected price $q(\alpha)$, which varies from one setup to the other. Overall, the sovereign's value function is different in the two setups, as a consequence of different expected prices, $\hat{q}(\alpha)$ in the setup without information friction vs $\tilde{q}(\alpha)$ in the setup with information frictions. We will carry out this analysis numerically in the next section of the paper.

5 Numerical Resolution

[In progress]
The continuous-time theoretical model leaves us with a system of two ODEs to solve (one for the sovereign's value function, and one for the sovereign's expected bond price, accompanied by their boundary conditions) in each setup, with and without information frictions. Then, we can impute the traders' bond price in each setup[12]. In the following, we briefly present

[11] Notice though that we are interested in the effect of λ on the difference of traders' bond prices between the two setups $\hat{q}(\alpha, \zeta) - \tilde{q}(\alpha, \zeta)$, which we can study only numerically.

[12] The ODEs, as well as their associated boundary conditions, and the equations for the traders' bond price are reported in Appendix C.

the general method to solve numerically the model, and analyze the results by comparing the two setups.

5.1 Resolution method

First, we focus on the system of two ODEs, in order to find the value function and the expected bond price function. To this end, we assume a parametric form for the two functions (second-order polynomial in α), taking into account the boundary conditions. Then, we make an initial guess about the parameters of the two functions, compute what the guess implies as new functions using the system of ODEs. By solving for new parameter values using the two new functions, we can adjust our initial guess. Finally, we iterate the procedure until there is convergence.

5.2 Results

Using the method explained above, we have to assign values to the parameters of the model. Table 3 reports them, as well as the implied values of three parameters that are specific to our model.

Parameter	**Value**	**Explanation**
ρ	0.01	Discount factor
ψ	8	Coefficient of relative risk aversion
σ	0.05	Standard deviation of output growth
λ	0.9	Size of the perpetuity's coupon
r	0.03	Constant risk-free interest rate on international markets
D	2.3	Default value
β	4000	Precision of the private signal x, conditional on output growth y'/y
$\beta\delta$	40	Precision of the public signal ζ, conditional on output growth y'/y
$\gamma_v = \frac{\beta\delta}{1/\sigma^2+\beta\delta}$	0.09	Parameter in the No Arbitrage setup
$\gamma_p = \frac{\beta(1+\delta)}{1/\sigma^2+\beta(1+\delta)}$	0.91	Parameter in the Information Frictions setup
$\frac{\sigma_p^2}{\sigma^2} = \left(1-\gamma_p+\frac{\gamma_p^2}{\gamma_v}\right)^2$	60	Variance wedge between the two setups

Table 3: Parameter values

Figure 1 pictures the traders' bond price of the two setups as a function of α, the debt to income ratio for a public signal $\zeta = -0.1$. Such a ζ signals to the traders a bad news about the future income growth, indicating a potentially negative income growth of 10%. In this case, the traders' bond price function is lower in the setup with information frictions than in the setup without information friction. In the setup with information frictions, the fact that the information is dispersed across traders and that the price is uninformative makes the traders' bond price react more to a "bad" public signal. Hence, the existence of information frictions reduces the price at which the sovereign can borrow, and hence its borrowing capacity. It can thus explain part of the gap that we observed in the data, between the sovereign bond spread and the default probability implied by historical data.

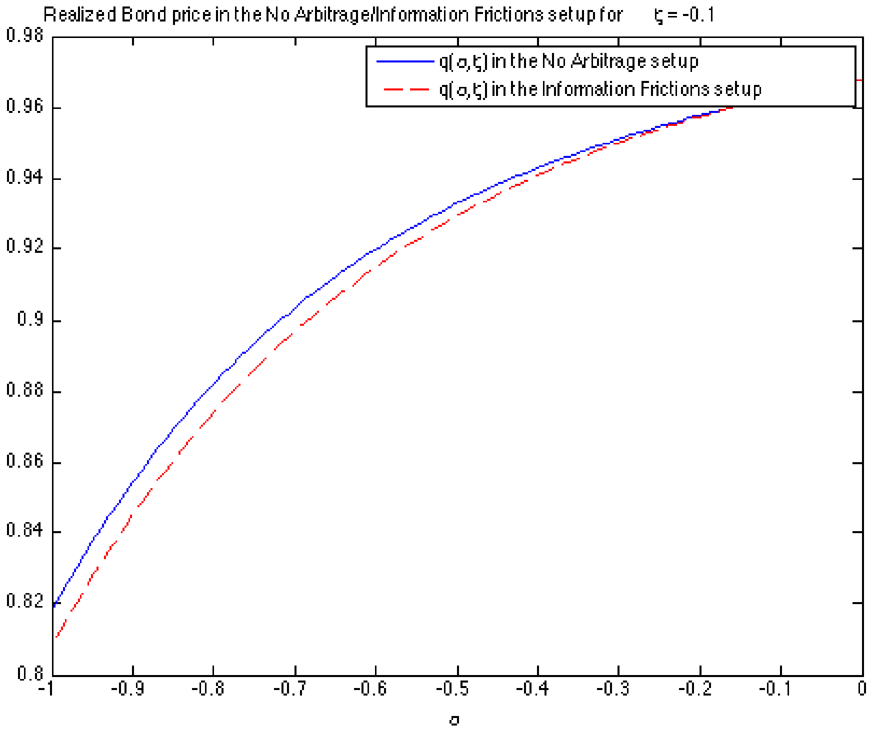

Figure 1: The traders' bond price function

Furthermore, the gap is widening as the level of debt to income increases.
In other simulations, we observed that as the public signal becomes more negative, the gap between the two traders' bond prices widens.

6 Conclusion

In this paper, we developed a theory of sovereign bond pricing based on dispersed information and limits to arbitrage. Quantitatively, the model explains a portion of the spreads that we observe in the data. In therms of method, the continuous-time structure is a practical and convenient way to derive the pricing implications of the model.

References Albagli, Elias, Christian Hellwig and Aleh Tsyvinski. 2013. "Dynamic Dispersed Information and the Credit Spread Puzzle" TSE Working Paper.
Arellano, Cristina. 2008. "Default Risk and Income Fluctuations in Emerging Economies" The American Economic Review, Vol. 98, No. 3 (Jun 2008), pp. 690-712.
Bortoli, Clement, Louis Harreau and Cyril Pouvelle. 2014. "Determinants of OECD countries' sovereign yields: safe havens, purgatory, and the damned" Bank of France Working Paper.
Cruces, Juan and Christoph Trebesch. 2011. "Sovereign Defaults: The Price of Haircuts."

CESifo Working Paper, NO. 3604.
Moody's. 2014. "Sovereign Default and Recovery Rates 1983-2013", Report of April 11, 2014.

Appendix A

[See at the end of the paper]

Appendix B

Proof of Proposition 1:

Let us guess and verify that $V(a,y) = y^{1-\psi}v(a/y)$, $\hat{q}(a',y,\zeta) = \hat{q}(a'/y,\zeta)$, and $\hat{\alpha}(a,y) = \hat{\alpha}(\alpha)y$.
Using the definition of $\mu = (a'-a)/y\Delta \Leftrightarrow a'/y = \alpha + \mu\Delta$, the sovereign's problem simplifies to:

$$V(a,y) = \max\left\{\overline{V}(y), \max_{c,\mu}\mathbb{E}_\Delta\left\{U(c)\Delta + (1-\rho\Delta)V(a+y\mu\Delta,y')\right\}\right\}$$

where

$$\frac{c}{y} = 1 + \lambda\alpha\left(1-\hat{q}(a',y,\zeta)\right) - \hat{q}(a',y,\zeta)\mu$$

$$\begin{aligned}
&\max_{c,\mu}\mathbb{E}_\Delta\left\{U(c)\Delta + (1-\rho\Delta)V(a+y\mu\Delta,y')\right\}\\
=\;& y^{1-\psi}\max_{c,\mu}U\left(\frac{c}{y}\right)\Delta + (1-\rho\Delta)\mathbb{E}_\Delta\left\{\left(\frac{y'}{y}\right)^{1-\psi}v\left((\alpha+\mu\Delta)\frac{y}{y'}\right)\right\}\\
=\;& y^{1-\psi}\max_{c,\mu}U\left(\frac{c}{y}\right)\Delta + (1-\rho\Delta)\left(\frac{a'}{y}\right)^{1-\psi}\mathbb{E}_\Delta\left\{\left(\frac{y'}{a'}\right)^{1-\psi}v\left((\alpha+\mu\Delta)\frac{y}{y'}\right)\right\}\\
=\;& y^{1-\psi}\max_{c,\mu}U\left(\frac{c}{y}\right)\Delta + (1-\rho\Delta)(\alpha+\mu\Delta)^{1-\psi}\mathbb{E}_\Delta\left\{(\alpha+d\alpha)^{\psi-1}v\left((\alpha+d\alpha)\right)\right\}
\end{aligned}$$

where we use the random walk assumption implying that $a'/y' = \alpha + d\alpha$. Hence,

$$V(a,y) = y^{1-\psi}\max\left\{D, \max_{c,\mu}U\left(\frac{c}{y}\right)\Delta + (1-\rho\Delta)(\alpha+\mu\Delta)^{1-\psi}\mathbb{E}_\Delta\left\{(\alpha+d\alpha)^{\psi-1}v\left((\alpha+d\alpha)\right)\right\}\right\}$$

There is a decision to default, after the new endowment is revealed, if $y' \notin \mathcal{R}(a',y) \Leftrightarrow a'/y' \leq \underline{\alpha} \leq 0 \Leftrightarrow y' \leq a'/\underline{\alpha}$. Notice also that $\hat{\alpha}(a,y) = a'/y * y$, so that under our guess,

$\hat{\alpha}(\alpha) = a'/y$. Let us write the bond price function:

$$\begin{aligned}
\hat{q}\left(a', y, \zeta\right) &= (1 - r\Delta)\left\{\underline{q} + \int_{a'/\underline{\alpha}}^{\infty} \left(\lambda\Delta + (1-\lambda\Delta)\,\hat{q}\left(\hat{\alpha}\left(a', y'\right), \zeta'\right) - \underline{q}\right) d\Psi_{\Delta}\left(y', \zeta' \mid y, \zeta\right)\right\} \\
\hat{q}\left(a', y, \zeta\right) &= (1 - r\Delta)\left\{\underline{q} + \int_{a'/y\underline{\alpha}}^{\infty} \left(\lambda\Delta + (1-\lambda\Delta)\,\hat{q}\left(\hat{\alpha}\left(\frac{a'}{y\epsilon'}\right), \zeta'\right) - \underline{q}\right) d\Psi_{\Delta}\left(\epsilon', \zeta' \mid y, \zeta\right)\right\} \\
\hat{q}\left(a', y, \zeta\right) &= \hat{q}\left(\frac{a'}{y}, \zeta\right)
\end{aligned}$$

As intended to be shown, the bond price is a function of $a'/y = \alpha + \mu\Delta$ and ζ. The sovereign's policy function is a function of α as well. Finally, c/y from the sovereign's budget constraint is a function of α as well, so that the sovereign's value function is a function of α. Finally, we obtain:

$$v(\alpha) = \max\left\{D, \max_{c,\mu} U\left(\frac{c}{y}\right)\Delta + (1-\rho\Delta)\left(\alpha + \mu\Delta\right)^{1-\psi}\mathbb{E}_{\Delta}\left\{\left(\alpha + d\alpha\right)^{\psi-1} v\left(\left(\alpha + d\alpha\right)\right)\right\}\right\}$$

such that:

$$\frac{c}{y} = 1 + \lambda\alpha\left(1 - \hat{q}(\alpha + \mu\Delta, \zeta)\right) - \mu\hat{q}(\alpha + \mu\Delta, \zeta)$$

where we verify our guesses.

Now, we can integrate the traders' bond price equation of the setup without information friction on both sides wrt. ζ':

$$\hat{q}\left(\frac{a'}{y}, \zeta\right) = (1 - r\Delta)\left\{\underline{q} + \int_{a'/y\underline{\alpha}}^{\infty} \left(\lambda\Delta + (1-\lambda\Delta)\,\hat{q}\left(\hat{\alpha}\left(\frac{a'}{y\epsilon'}\right), \zeta'\right) - \underline{q}\right) d\Psi_{\Delta}\left(\epsilon', \zeta' \mid y, \zeta\right)\right\}$$

$$\hat{q}\left(\frac{a'}{y}, \zeta\right) = (1 - r\Delta)\left\{\underline{q} + \int_{a'/y\underline{\alpha}}^{\infty} \left(\lambda\Delta + (1-\lambda\Delta)\int_{\zeta'} \hat{q}\left(\hat{\alpha}\left(\frac{a'}{y\epsilon'}\right), \zeta'\right) dF_{\Delta}(\zeta') - \underline{q}\right) d\Phi_{\Delta}\left(\frac{\ln\epsilon'}{\sigma} \mid \zeta\right)\right\}$$

Let us define $\hat{q}(a'/y)$ the sovereign's expected bond price:

$$\hat{q}\left(\frac{a'}{y}\right) = \int_{\zeta} \hat{q}\left(\frac{a'}{y}, \zeta\right) dF_{\Delta}(\zeta)$$

We obtain the following equation:

$$\hat{q}\left(\frac{a'}{y}\right) = (1 - r\Delta)\left\{\underline{q} + \int_{a'/y\underline{\alpha}}^{\infty} \left(\lambda\Delta + (1-\lambda\Delta)\,\hat{q}\left(\hat{\alpha}\left(\frac{a'}{y\epsilon'}\right)\right) - \underline{q}\right) d\Phi_{\Delta}\left(\frac{\ln\epsilon'}{\sigma}\right)\right\}$$

Proof of Proposition 2:

Let us derive the ordinary differential equation for the sovereign's value function in the setup without information friction. We start by taking the continuous-time limit as $\Delta \to 0$ to obtain the Hamilton-Jacobi-Bellman equation, and then, we solve the maximization problem in order to derive the ODE.

$$v(\alpha) = \max_{\mu} \left\{ U\left(\frac{c}{y}\right)\Delta + (1-\rho\Delta)(\alpha+\mu\Delta)^{1-\psi}\,\mathbb{E}\Big((\alpha+d\alpha)^{\psi-1} v(\alpha+d\alpha)\Big) \right\}$$

where

$$\begin{aligned} \frac{c}{y} &= 1+\lambda\alpha - \hat{q}(\alpha)(\mu+\lambda\alpha) \\ \frac{a'}{y} &= \alpha+\mu\Delta \end{aligned}$$

Now, using Ito's Lemma,

$$\begin{aligned} \mathbb{E}\Big((\alpha+d\alpha)^{\psi-1} v(\alpha+d\alpha)\Big) &= (\alpha)^{\psi-1} v(\alpha) + \Big[(\psi-1)(\alpha)^{\psi-2} v(\alpha) + (\alpha)^{\psi-1} v'(\alpha)\Big]\mu\Delta \\ &\quad + \Big[(\psi-1)(\psi-2)(\alpha)^{\psi-3} v(\alpha) + 2(\psi-1)(\alpha)^{\psi-2} v'(\alpha) \\ &\quad + (\alpha)^{\psi-1} v''(\alpha)\Big]\frac{1}{2}\alpha^2\sigma^2\Delta \\ &= (\alpha)^{\psi-1}\left\{ v(\alpha)\left(1+(\psi-1)\frac{\mu}{\alpha}\Delta + (\psi-1)(\psi-2)\frac{1}{2}\sigma^2\Delta\right)\right. \\ &\quad \left. + v'(\alpha)\left(\mu+(\psi-1)\alpha\sigma^2\right)\Delta + \frac{1}{2}\sigma^2\alpha^2 v''(\alpha)\Delta \right\} \end{aligned}$$

As $\Delta \to 0$, $(1+(\mu/\alpha\cdot\Delta))^{1-\psi} \to (1+(1-\psi)(\mu/\alpha\cdot\Delta))$, and

$$\begin{aligned} & (1-\rho\Delta)(\alpha+\mu\Delta)^{1-\psi}\left(1+(\psi-1)\frac{\mu}{\alpha}\Delta+(\psi-1)(\psi-2)\frac{1}{2}\sigma^2\Delta\right) \\ \to\ & 1-\rho\Delta+(\psi-1)(\psi-2)\frac{1}{2}\sigma^2\Delta \end{aligned}$$

Therefore, the Hamilton-Jacobi-Bellman equation takes the form

$$\left(\rho+(\psi-1)(2-\psi)\frac{1}{2}\sigma^2\right)v(\alpha) = \max_{\mu}\left\{ U\left(\frac{c}{y}\right) + v'(\alpha)\left(\mu+(\psi-1)\alpha\sigma^2\right) + \frac{1}{2}\sigma^2\alpha^2 v''(\alpha) \right\}$$

where

$$\frac{c}{y} = 1+\lambda\alpha-\hat{q}(\alpha)(\mu+\lambda\alpha)$$

We can now solve for the optimal $\hat{\mu}(\alpha)$. Taking the FOC wrt. μ, we obtain:

$$\hat{\mu}(\alpha) = \frac{1+\lambda\alpha}{\hat{q}(\alpha)} - \frac{v'(\alpha)^{-\frac{1}{\psi}}}{\hat{q}(\alpha)^{1-\frac{1}{\psi}}} - \lambda\alpha$$

Incorporating $\hat{\mu}(\alpha)$ into the ODE of the Value function, we obtain:

$$\left(\rho+(\psi-1)(2-\psi)\frac{\sigma^2}{2}\right)v(\alpha) = \frac{\psi}{1-\psi}\left(\frac{v'(\alpha)}{\hat{q}(\alpha)}\right)^{1-\frac{1}{\psi}} + v'(\alpha)\left(\frac{1+\lambda\alpha}{\hat{q}(\alpha)} - \lambda\alpha + (\psi-1)\alpha\sigma^2\right) + \frac{\sigma^2\alpha^2}{2}v''(\alpha)$$

with $\frac{c}{y}$ as above, and boundary conditions $D = v\left(\underline{\alpha}\right)$ and $v'\left(\underline{\alpha}\right) = 0$.

Proof of Proposition 3:

In the setup without information friction, using Bayesian updating,

$$\begin{aligned}
\ln(\epsilon_{t+\Delta}) \mid \zeta_t &\sim \mathcal{N}\left(\frac{\beta\delta}{\frac{1}{\sigma^2}+\beta\delta}\zeta_t, \frac{1}{\frac{1}{\sigma^2}+\beta\delta}\Delta\right) = \mathcal{N}\left(\gamma_v\zeta_t, (1-\gamma_v)\sigma^2\Delta\right) \\
\zeta_t &\sim \mathcal{N}\left(0, \left(\sigma^2 + (\beta\delta)^{-1}\right)\Delta\right) = \mathcal{N}\left(0, \frac{1}{\gamma_v}\sigma^2\Delta\right) \\
\ln(\epsilon_{t+\Delta}) &\sim \mathcal{N}\left(0, \sigma^2\Delta\right)
\end{aligned}$$

with $\gamma_v = \frac{\beta\delta}{\frac{1}{\sigma^2}+\beta\delta}$.

We can derive the ODE for the sovereign's expected bond price $\hat{q}(\alpha)$:

$$\hat{q}(\alpha) = (1 - rdt)\left\{\lambda dt + (1-\lambda dt)\mathbb{E}\left(\hat{q}(\alpha + d\alpha)\right)\right\}$$

with $d\alpha = \hat{\mu}dt - \alpha dZ_t$, dZ_t being the limit of $\ln(\epsilon_t)$ as $\Delta \to 0$.

By Ito's lemma, using $\mathbb{E}(d\alpha) = \hat{\mu}(\alpha)\Delta$ and $\mathbb{E}(d\alpha^2) = \alpha^2\sigma^2\Delta$, and taking the limit as $\Delta \to 0$,

$$\begin{aligned}
\hat{q}(\alpha) &= (1-rdt)\left\{\lambda dt + (1-\lambda dt)\left(\hat{q}(\alpha) + \hat{q}'(\alpha)\hat{\mu}dt + \frac{\alpha^2\sigma^2}{2}\hat{q}''(\alpha)dt\right)\right\} \\
(r+\lambda)\,\hat{q}(\alpha) &= \lambda + \hat{q}'(\alpha)\hat{\mu}(\alpha) + \frac{\alpha^2\sigma^2}{2}\hat{q}''(\alpha)
\end{aligned}$$

This ODE must hold for any α, with boundary conditions $\lim_{\alpha\to\infty}\hat{q}(\alpha) = \lambda/(r+\lambda)$, and $\hat{q}(\underline{\alpha}) = \underline{q} < \lambda/(r+\lambda)$.

Proof of Proposition 4:

We can now derive the equation for the traders' bond price $\hat{q}(\alpha, \zeta)$:

$$\hat{q}(\alpha,\zeta) = (1-rdt)\left\{\lambda dt + (1-\lambda dt)\mathbb{E}\left(\hat{q}(\alpha + d\alpha) \mid \zeta\right)\right\}$$

By Ito's lemma,

$$\hat{q}(\alpha,\zeta) = (1-rdt)\left\{\lambda dt + (1-\lambda dt)\left(\hat{q}(\alpha,\zeta) + \hat{q}'(\alpha)\mathbb{E}\left(d\alpha \mid \zeta\right) + \frac{1}{2}\hat{q}''(\alpha)\mathbb{E}\left(d\alpha^2 \mid \zeta\right)\right)\right\}$$

with

$$\begin{aligned}\mathbb{E}\left(d\alpha \mid \zeta\right) &= \hat{\mu}(\alpha)\Delta - \alpha\gamma_v\zeta\Delta \\ \mathbb{E}\left(d\alpha^2 \mid \zeta\right) &= \alpha^2(1-\gamma_v)\sigma^2\Delta\end{aligned}$$

Taking the limit as $\Delta \to 0$,

$$(r+\lambda)\,\hat{q}(\alpha,\zeta) = \lambda + \hat{q}'(\alpha)\left(\hat{\mu}(\alpha) - \alpha\gamma_v\zeta\right) + \frac{\alpha^2(1-\gamma_v)\sigma^2}{2}\hat{q}''(\alpha)$$

Proof of Proposition 5:

Using the same method as in the proof for Proposition 2, we can derive the ODE for the sovereign's value function in the setup with information frictions:

$$\left(\rho + (\psi-1)(2-\psi)\frac{1}{2}\sigma^2\right)v(\alpha) = U\left(\frac{c}{y}\right) + v'(\alpha)\left(\tilde{\mu}(\alpha) + (\psi-1)\,\alpha\sigma^2\right) + \frac{1}{2}\sigma^2\alpha^2 v''(\alpha)$$

where

$$\frac{c}{y} = 1 + \lambda\alpha - \tilde{q}(\alpha)\left(\tilde{\mu}(\alpha) + \lambda\alpha\right)$$

$$\tilde{\mu}(\alpha) = \frac{1+\lambda\alpha}{\tilde{q}(\alpha)} - \frac{v'(\alpha)^{-\frac{1}{\psi}}}{\tilde{q}(\alpha)^{1-\frac{1}{\psi}}} - \lambda\alpha$$

with the same boundary conditions $v(\underline{\alpha}) = D$ and $v'(\underline{\alpha}) = 0$.

Proof of Proposition 6:

In the setup with information frictions, using Bayesian updating,

$$\begin{aligned}\ln(\epsilon_{t+\Delta}) \mid \zeta_t, x_t = \zeta_t &\sim \mathcal{N}\left(\frac{\beta(1+\delta)}{\frac{1}{\sigma^2} + \beta(1+\delta)}\zeta_t, \frac{1}{\frac{1}{\sigma^2} + \beta(1+\delta)}\Delta\right) = \mathcal{N}\left(\gamma_p\zeta, (1-\gamma_p)\sigma^2\Delta\right) \\ \zeta_t &\sim \mathcal{N}\left(0, (\sigma^2 + (\beta\delta)^{-1})\Delta\right) = \mathcal{N}\left(0, \frac{1}{\gamma_v}\sigma^2\Delta\right) \\ \ln(\epsilon_{t+\Delta}) &\sim \mathcal{N}\left(0, \left(1 - \gamma_p + \frac{\gamma_p^2}{\gamma_v}\right)\sigma^2\Delta\right)\end{aligned}$$

with $\gamma_p = \frac{\beta(1+\delta)}{\frac{1}{\sigma^2}+\beta(1+\delta)}$.

Using the fact that $\mathbb{E}(d\alpha) = \tilde{\mu}(\alpha)\Delta$ and $\mathbb{E}(d\alpha^2) = \alpha^2\sigma_p^2\Delta$, we can derive the ODE for the sovereign's expected bond price $\tilde{q}(\alpha)$:

$$(r+\lambda)\,\tilde{q}(\alpha) = \lambda + \tilde{q}'(\alpha)\tilde{\mu}(\alpha) + \frac{\alpha^2\sigma_p^2}{2}\tilde{q}''(\alpha)$$

with the boundary conditions $\tilde{q}(\underline{\alpha}) = \underline{q}$, $\tilde{q}(0) = \lambda/(\lambda+r)$, and $\tilde{q}'(0) = 0$. We obtain the same equation as in the setup without information friction, except for the sovereign's policy function $\tilde{\mu}(\alpha)$ and the variance $\sigma_p^2 > \sigma^2$.

Proof of Proposition 7:

We can derive the equation for the traders' bond price $\tilde{q}(\alpha, \zeta)$:

$$\tilde{q}(\alpha,\zeta) = (1 - rdt)\left\{\lambda dt + (1-\lambda dt)\mathbb{E}\left(\tilde{q}(\alpha + d\alpha) \mid \zeta\right)\right\}$$

By Ito's lemma,

$$\tilde{q}(\alpha,\zeta) = (1 - rdt)\left\{\lambda dt + (1-\lambda dt)\left(\tilde{q}(\alpha,\zeta) + \tilde{q}'(\alpha)\mathbb{E}\left(d\alpha \mid \zeta\right) + \frac{1}{2}\tilde{q}''(\alpha)\mathbb{E}\left(d\alpha^2 \mid \zeta\right)\right)\right\}$$

with

$$\begin{aligned}
\mathbb{E}\left(d\alpha \mid \zeta\right) &= \tilde{\mu}(\alpha)\Delta - \alpha\gamma_p\zeta\Delta \\
\mathbb{E}\left(d\alpha^2 \mid \zeta\right) &= \alpha^2(1-\gamma_p)\sigma^2\Delta
\end{aligned}$$

$$(r+\lambda)\,\tilde{q}(\alpha,\zeta) = \lambda + \tilde{q}'(\alpha)\left(\tilde{\mu}(\alpha) - \alpha\gamma_p\zeta\right) + \frac{\alpha^2(1-\gamma_p)\sigma_p^2}{2}\tilde{q}''(\alpha)$$

Additional proof:

$$\begin{aligned}
\mathbb{E}\left(f(\zeta)\right) &= f(0) + f'(0)\mathbb{E}(\zeta) + \frac{1}{2}f''(0)\mathbb{E}(\zeta^2) \\
\mathbb{E}\left(f(\zeta)\right) &= f(0) + \frac{1}{2}f''(0)\frac{1}{\gamma_v}\sigma^2 dt \\
\Rightarrow \mathbb{E}\left(f(\zeta)\right)dt &\to f(0)dt
\end{aligned}$$

We can write the expression for the value function of the sovereign in the two setups:

- In the setup with no arbitrage:

$$\begin{aligned}
\frac{1}{1-\psi}\mathbb{E}\left(\left(\frac{c}{y}\right)^{1-\psi}\right)dt &= \frac{1}{1-\psi}\left(1 + \lambda\alpha - \hat{q}(\alpha)\left(\hat{\mu}(\alpha) + \lambda\alpha\right)\right)^{1-\psi}dt \\
&= \frac{1}{1-\psi}\left(\frac{v'(\alpha)}{\hat{q}(\alpha)}\right)^{\frac{\psi-1}{\psi}}dt
\end{aligned}$$

- In the setup with information frictions:

$$\frac{1}{1-\psi}\mathbb{E}\left(\left(\frac{c}{y}\right)^{1-\psi}\right)dt = \frac{1}{1-\psi}\left(1+\lambda\alpha-\tilde{q}(\alpha)\left(\tilde{\mu}(\alpha)+\lambda\alpha\right)\right)^{1-\psi}dt$$
$$= \frac{1}{1-\psi}\left(\frac{v'(\alpha)}{\tilde{q}(\alpha)}\right)^{\frac{\psi-1}{\psi}}dt$$

Appendix C

In the setup without information friction, we have the following system of two ODEs, the first one for the value function, and the second one for the sovereign's expected bond price. The third equation is the traders' bond price. Notice that we can easily obtain the third equation once we have solved for the sovereign's expected bond price function.

$$\left(\rho+(\psi-1)(2-\psi)\frac{\sigma^2}{2}\right)v(\alpha)=\frac{\psi}{1-\psi}\left(\frac{v'(\alpha)}{\hat{q}(\alpha)}\right)^{\frac{\psi-1}{\psi}}+v'(\alpha)\left(\frac{1+\lambda\alpha}{\hat{q}(\alpha)}-\lambda\alpha+(\psi-1)\alpha\sigma^2\right)+\frac{\sigma^2\alpha^2}{2}v''(\alpha)$$

$$(r+\lambda)\hat{q}(\alpha)=\lambda+\hat{q}'(\alpha)\left(\frac{1+\lambda\alpha}{\hat{q}(\alpha)}-\frac{v'(\alpha)^{-\frac{1}{\psi}}}{\hat{q}(\alpha)^{\frac{\psi-1}{\psi}}}-\lambda\alpha\right)+\frac{1}{2}\alpha^2\sigma^2\hat{q}''(\alpha)$$

$$(r+\lambda)\hat{q}(\alpha,\zeta)=\lambda+\hat{q}'(\alpha)\left(\frac{1+\lambda\alpha}{\hat{q}(\alpha)}-\frac{v'(\alpha)^{-\frac{1}{\psi}}}{\hat{q}(\alpha)^{\frac{\psi-1}{\psi}}}-\lambda\alpha-\alpha\gamma_v\zeta\right)+\frac{\alpha^2(1-\gamma_v)\sigma^2}{2}\hat{q}''(\alpha)$$

In the setup with information frictions, we have similarly:

$$\left(\rho+(\psi-1)(2-\psi)\frac{\sigma^2}{2}\right)v(\alpha)=\frac{\psi}{1-\psi}\left(\frac{v'(\alpha)}{\tilde{q}(\alpha)}\right)^{\frac{\psi-1}{\psi}}+v'(\alpha)\left(\frac{1+\lambda\alpha}{\tilde{q}(\alpha)}-\lambda\alpha+(\psi-1)\alpha\sigma^2\right)+\frac{\sigma^2\alpha^2}{2}v''(\alpha)$$

$$(r+\lambda)\tilde{q}(\alpha)=\lambda+\tilde{q}'(\alpha)\left(\frac{1+\lambda\alpha}{\tilde{q}(\alpha)}-\frac{v'(\alpha)^{-\frac{1}{\psi}}}{\tilde{q}(\alpha)^{\frac{\psi-1}{\psi}}}-\lambda\alpha\right)+\frac{1}{2}\alpha^2\sigma_p^2\tilde{q}''(\alpha)$$

$$(r+\lambda)\tilde{q}(\alpha,\zeta)=\lambda+\tilde{q}'(\alpha)\left(\frac{1+\lambda\alpha}{\tilde{q}(\alpha)}-\frac{v'(\alpha)^{-\frac{1}{\psi}}}{\tilde{q}(\alpha)^{\frac{\psi-1}{\psi}}}-\lambda\alpha-\alpha\gamma_p\zeta\right)+\frac{\alpha^2(1-\gamma_p)\sigma^2}{2}\tilde{q}''(\alpha)$$

For each setup, we solve first the system of two ODEs (the value function and the sovereign's expected bond price function), and second, we compute the implied traders' bond price function.
We assume that the two functions are second-order polynomial in α:

$$v(\alpha)=\beta_0+\beta_1\alpha+\beta_2\alpha^2$$
$$q(\alpha)=\gamma_0+\gamma_1\alpha+\gamma_2\alpha^2$$

Using the boundary conditions of the ODEs, we can reduce the number of degrees of freedom:

$$\begin{aligned} q(0) &= \frac{\lambda}{\lambda + r} \\ q'(0) &= 0 \\ q(\underline{\alpha}) &= \underline{q} \\ v(\underline{\alpha}) &= D \\ v'(\underline{\alpha}) &= 0 \end{aligned}$$

Hence, we obtain the following expression for the parametric guesses of the 2 functions, along with their associated first and second derivatives:

$$\begin{aligned} v(\alpha) &= \beta_0 + \beta_1\alpha + \beta_2\alpha^2 \\ v'(\alpha) &= \beta_1 + 2\beta_2\alpha \\ v''(\alpha) &= 2\beta_2 \end{aligned}$$

$$\begin{aligned} q(\alpha) &= \frac{\lambda}{\lambda + r} + \gamma_2\alpha^2 \\ q'(\alpha) &= 2\gamma_2\alpha \\ q''(\alpha) &= 2\gamma_2 \end{aligned}$$

We rewrite the system more conveniently before solving it:

$$v(\alpha) = \frac{1}{\left(\rho + (\psi - 1)(2 - \psi)\frac{\sigma^2}{2}\right)} \left\{ \frac{\psi}{1-\psi} \left(\frac{v'(\alpha)}{q(\alpha)}\right)^{\frac{\psi-1}{\psi}} + v'(\alpha)\left(\frac{1+\lambda\alpha}{q(\alpha)} - \lambda\alpha + (\psi - 1)\alpha\sigma^2\right) + \frac{\sigma^2\alpha^2}{2} v''(\alpha) \right\}$$

$$q'(\alpha) = \frac{1}{\left(\frac{1+\lambda\alpha}{q(\alpha)} - \frac{v'(\alpha)^{-\frac{1}{\psi}}}{q(\alpha)^{\frac{\psi-1}{\psi}}} - \lambda\alpha\right)} \left\{ (r+\lambda)\, q(\alpha) - \lambda - \frac{1}{2}\alpha^2\sigma^2 q''(\alpha) \right\}$$

We can gather all the parameters in a vector $\Theta = \{\beta_0, \beta_1, \beta_2, \gamma_2\}$. Notice that for any value of α, the two ODEs imply a value for $v(\alpha)$ and $q'(\alpha)$. However, given the assumed functional form, that implies that the two ODEs must satisfy:

$$\begin{aligned} \beta_0 + \beta_1\alpha + \beta_2\alpha^2 &= f(\Theta, \alpha) \\ \gamma_1\alpha + 2\beta_2\alpha &= g(\Theta, \alpha) \end{aligned}$$

Then, the iterative procedure works as follows.

1. Start with a guess for the value of the parameters $\Theta^0 = \{\beta_0^0, \beta_1^0, \beta_2^0, \gamma_2^0\}$.

2. Consider N values for α, $\{\alpha_1, ..., \alpha_N\}$, in the interval $\{\underline{\alpha}, 0\}$ where $\underline{\alpha}$ is a chosen parameter.

3. For each value α_i, $i = 1, ..N$, we can compute the RHS using the two ODEs as follows:

$$f\left(\Theta^0, \alpha_i\right) = \frac{1}{\left(\rho + (\psi - 1)(2 - \psi)\frac{\sigma^2}{2}\right)} \left\{ \frac{\psi}{1 - \psi} \left(\frac{\beta_1^0 + 2\beta_2^0 \alpha_i}{\frac{\lambda}{\lambda + r} + \gamma_2^0 \alpha_i^2} \right)^{\frac{\psi - 1}{\psi}} \right.$$
$$\left. + (\beta_1^0 + 2\beta_2^0 \alpha_i) \left(\frac{1 + \lambda \alpha_i}{(\frac{\lambda}{\lambda + r} + \gamma_2^0 \alpha_i^2)} - \lambda \alpha_i + (\psi - 1)\alpha_i \sigma^2 \right) + \frac{\sigma^2 \alpha^2}{2} (2\beta_2^0)(\alpha_i) \right\}$$

$$g\left(\Theta^0, \alpha_i\right) = \frac{1}{\left(\frac{1 + \lambda \alpha_i}{(\frac{\lambda}{\lambda + r} + \gamma_2^0 \alpha_i^2)} - \frac{(\beta_1^0 + 2\beta_2^0 \alpha_i)^{-\frac{1}{\psi}}}{(\frac{\lambda}{\lambda + r} + \gamma_2^0 \alpha_i^2)^{\frac{\psi - 1}{\psi}}} - \lambda \alpha_i \right)} \left\{ (r + \lambda) \left(\frac{\lambda}{\lambda + r} + \gamma_2^0 \alpha_i^2 \right) - \lambda - \frac{1}{2} \alpha_i^2 \sigma^2 (2\gamma_2^0) \right\}$$

Therefore, for each α_i, we obtain a number $f\left(\Theta^0, \alpha_i\right)$ and $g\left(\Theta^0, \alpha_i\right)$. Let's gather in two vectors B_1 and B_2 the N elements of $f\left(\Theta^0, \alpha_i\right)$ and $g\left(\Theta^0, \alpha_i\right)$ for each α_i. Each vector has dimension $(N \times 1)$.

4. We use the theory to update the value of the parameters Θ in a new set of values Θ^1 . In particular, the ODEs and the assumed functional forms imply, for any α_i, that :

$$\beta_0^1 + \beta_1^1 \alpha_i + \beta_2^1 \alpha_i^2 = f\left(\Theta^0, \alpha_i\right)$$
$$2\gamma_2^1 \alpha_i = g\left(\Theta^0, \alpha_i\right)$$

5. Now for any α_i, we can write the LHS of the equations above in a vector form as follows: for the first ODE we have:

$$\begin{bmatrix} 1 & \alpha_i & \alpha_i^2 & 2\alpha_i & 0 \end{bmatrix} \Theta^1.$$

Denote with

$$A_1(i, :) = \begin{bmatrix} 1 & \alpha_i & \alpha_i^2 & 2\alpha_i & 0 \end{bmatrix}.$$

Similarly, for the second ODE we have:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 2\alpha_i \end{bmatrix} \Theta^1.$$

Denote with

$$A_2(i, :) = \begin{bmatrix} 0 & 0 & 0 & 0 & 2\alpha_i \end{bmatrix}.$$

6. We can construct now two matrices A_1 and A_2 by considering the N values of α_i, we then obtain two matrix of dimension $(N \times 4)$.

7. We now have two set of linear equations:

$$A_1 \Theta^1 = B_1$$

and

$$A_2 \Theta^1 = B_2$$

. Since the two set of equations should hold simultaneously we can merge the systems as follow:

$$A \Theta^1 = B,$$

where $A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$ and $B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$. This system can be solve using the QR factorization with column pivoting as done with matlab function *linsolve*.

8. We obtain then an new value Θ^1. We can than iterate the procedure until convergence.

Country	Moody's rating	CDS on government bonds denominated in $			
	Data: start date	Data: start date - end date			
		2 yr	3 yr	5 yr	10 yr
Abu Dhabi	11/07/07	10/04/08-01/05/14	10/04/08-01/05/14	10/04/08-01/05/14	10/04/08-01/05/14
Argentina	18/11/86	18/12/07-01/05/14	18/12/07-01/05/14	18/12/07-01/05/14	18/12/07-01/05/14
Australia	15/01/62	07/11/08-01/05/14	07/11/08-01/05/14	07/11/08-01/05/14	07/11/08-01/05/14
Austria	26/06/77	10/04/08-01/05/14	10/04/08-01/05/14	21/07/08-01/05/14	10/04/08-01/05/14
Bahrain	29/01/96	29/04/08-01/05/14	29/04/08-01/05/14	29/04/08-01/05/14	29/04/08-01/05/14
Belgium	22/03/88	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14
Brazil	18/11/86	18/12/07-01/05/14	18/12/07-01/05/14	18/12/07-01/05/14	18/12/07-01/05/14
Bulgaria	27/09/96	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14
Chile	17/02/96	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14
China	18/05/88	04/01/08-01/05/14	04/01/08-01/05/14	04/01/08-01/05/14	04/01/08-01/05/14
Colombia	04/08/93	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14
Costa Rica	08/05/97	07/10/08-01/05/14	07/10/08-01/05/14	07/10/08-01/05/14	07/10/08-01/05/14
Croatia	17/01/97	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14
Cyprus	28/02/96	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14
Czech Rep.	10/03/93	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14
Denmark	06/09/67	26/05/08-01/05/14	26/05/08-01/05/14	26/05/08-01/05/14	26/05/08-01/05/14
Dominican Rep.	30/05/97	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14
Egypt	09/10/96	20/08/09-01/05/14	20/08/09-01/05/14	08/10/08-01/05/14	20/08/09-01/05/14
El Salvador	07/07/97	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14
Estonia	11/09/97	09/04/08-01/05/14	09/04/08-01/05/14	09/04/08-01/05/14	09/04/08-01/05/14
Finland	19/10/77	18/03/08-01/05/14	18/03/08-01/05/14	18/03/08-01/05/14	18/03/08-01/05/14
France	23/01/79	21/12/07-01/05/14	21/12/07-01/05/14	21/12/07-01/05/14	18/07/08-01/05/14
Germany	09/02/86	21/12/07-01/05/14	21/12/07-01/05/14	21/12/07-01/05/14	18/07/08-01/05/14
Greece	19/07/90	14/12/07-08/03/12	14/12/07-08/03/12	14/12/07-08/03/12	14/12/07-08/03/12
Guatemala	08/07/97	16/01/08-01/05/14	16/01/08-01/05/14	16/01/08-01/05/14	16/01/08-01/05/14
Hong Kong	09/11/88	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14
Hungary	18/07/89	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14
Iceland	24/05/89	10/04/08-01/05/14	10/04/08-01/05/14	10/04/08-01/05/14	10/04/08-01/05/14
Indonesia	14/03/94	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14
Ireland	15/07/87	03/03/08-01/05/14	03/03/08-01/05/14	03/03/08-01/05/14	03/03/08-01/05/14
Israel	02/11/95	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14
Italy	10/10/86	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14	18/07/08-01/05/14
Jamaica	30/03/98	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14
Japan	01/10/81	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14
Kazakhstan	11/11/96	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14
Korea	18/11/86	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14
Latvia	17/12/97	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14
Lithuania	04/09/96	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14
Malaysia	18/01/86	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14	18/07/08-01/05/14
Malta	14/03/94	14/12/07-27/11/12	14/12/07-01/05/14	14/12/07-01/05/14	18/07/08-01/05/14
Morocco	22/07/99	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14
Netherlands	10/01/86	10/04/08-01/05/14	10/04/08-01/05/14	10/04/08-01/05/14	10/04/08-01/05/14
Norway	12/01/78	04/12/08-01/05/14	04/12/08-01/05/14	04/12/08-01/05/14	04/12/08-01/05/14
Pakistan	23/11/94	18/06/08-01/05/14	18/06/08-01/05/14	18/06/08-01/05/14	18/06/08-01/05/14
Panama	30/06/58	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14
Peru	05/02/96	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14
Philippines	01/07/93	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14
Poland	01/06/95	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14
Portugal	18/11/86	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14
Qatar	29/01/96	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14
Romania	06/03/96	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14
Russia	07/10/96	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14
Saudi Arabia	03/10/94	15/02/11-01/05/14	15/02/11-01/05/14	12/08/10-01/05/14	15/02/11-01/05/14
Serbia	14/07/13	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14
Singapore	20/09/89	31/10/08-01/05/14	31/10/08-01/05/14	31/10/08-01/05/14	31/10/08-01/05/14
Slovakia	15/05/95	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14
Slovenia	08/05/96	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14
Spain	03/02/88	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14
Sri Lanka	22/09/10	21/06/11-01/05/14	21/06/11-01/05/14	21/06/11-01/05/14	21/06/11-01/05/14
Sweden	10/11/77	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14	14/12/07-01/05/14
Switzerland	20/01/82	03/03/10-01/05/14	03/03/10-01/05/14	03/03/10-01/05/14	03/03/10-01/05/14
Taiwan	24/03/94	21/06/11-01/05/14	21/06/11-01/05/14	21/06/11-01/05/14	21/06/11-01/05/14
Thailand	01/08/89	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14
Turkey	05/05/92	08/10/08-01/05/14	08/10/08-01/05/14	08/10/08-01/05/14	08/10/08-01/05/14
Ukraine	06/02/98	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14	02/01/08-01/05/14
United Kingdom	29/01/96	24/04/08-01/05/14	24/04/08-01/05/14	24/04/08-01/05/14	24/04/08-01/05/14
USA	05/02/49	18/07/08-01/05/14	18/07/08-01/05/14	07/01/08-01/05/14	18/07/08-01/05/14
Uruguay	15/10/93	04/11/08-01/05/14	04/11/08-01/05/14	04/11/08-01/05/14	04/11/08-01/05/14
Venezuela	29/12/76	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14	29/02/08-01/05/14

Table 4: Dataset: Moody's ratings and CDS on government bonds (1/2)