Optimal Monetary Interventions in Credit Markets

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Abstract

In an environment based on Lagos and Wright (2005) but with two rounds of pairwise meetings, we introduce imperfect monitoring that resembles operations of unsecured loans. We characterize the set of implementable allocations satisfying individual rationality and pairwise core in bilateral meetings. We introduce a class of expansionary monetary policies that use the seignorage revenue to purchase privately issued debts that resemble unconventional monetary policies. We show that under the optimal trading mechanism, both money and debt circulate in the economy and the optimal inflation rate is positive, except for very high discount factors under which money alone achieves the first-best. Our model captures the view that unconventional monetary policy encourages lending while it may create inflation.

1 Introduction

The provision of adequate liquidity has become a prominent issue in monetary policy discussions. In particular, central banks now use “unconventional” monetary policies, such as the creation of lending facilities and the purchase of private debt, to directly provide liquidity to the private sector. Although the purpose of these policies is to stimulate lending in credit markets hampered by a lack of liquidity, their overall effect, particularly their potential inflationary impact, is still under debate.¹ From the empirical perspective, a new strand of literature has emerged to study both the inflationary and the output

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¹Economists and policymakers have expressed concerns about the inflationary impacts of unconventional monetary policies. For example, John Taylor (2007), in his testimony before the Congress, argued that “This large expansion of reserve balances creates risks. If it is not undone, then the bank reserves will eventually pour out into the economy, causing inflation.” In turn, Spencer Dale, chief economist of the Bank of England has said that “with little slack in the economy, businesses would put up prices if extra quantitative easing (QE) found its way into consumers’ pockets.” http://www.theguardian.com/business/2013/mar/15/bank-england-economist-quantitative-easing
implications of unconventional monetary policies. Preliminary results suggest that the impacts on both dimensions are significant (see, for example, Joyce et al. (2012) for a survey). From the theoretical perspective, however, there is no systematic study of the optimality of monetary interventions in credit markets. In this paper, we propose a monetary model that can account for the both effects of unconventional monetary policies, and characterize optimal arrangement of such policies.

There are at least two strands of literature that emphasize the role of liquidity in the real economy. Both strands are based on the limited commitment friction, and show that the inability of individuals to commit to honor their future obligations is the key friction that renders liquidity, or their lack thereof, a relevant issue. In one strand, DSGE models with financial frictions (see, e.g., Gertler and Kiyotaki (2010)) demonstrate that borrowing constraints can amplify business cycle fluctuations. However, in their study of unconventional monetary policy, Gertler and Karadi (2011) assumes a cashless economy and the potential cost of the such policies is not coming from their inflationary impacts but from an exogenous cost of government intermediation. The other strand, in contrast, provides an explicit description of economic environments under which either liquid assets or credit arrangements are necessary to conduct transactions (see, e.g., Wallace (2010) and Lagos, Rocheteau, and Wright (2014)). Besides the limited commitment friction, lack of record-keeping is typically assumed in these models to give money a role for transactions. Such assumptions make credit arrangement difficult to sustain, and hence, these models rarely allow for credit arrangement.\(^2\)

We contribute to the literature in two aspects. First, based on monetary models in the aforementioned second strand, we provide an environment where both money and debt circulate as means-of-payments under the optimal trading arrangements, and features an endogenous debt limit subject to the limited commitment friction. Second, we study optimality of a new class of monetary policies, labeled as expansionary monetary policies, which directly purchase privately issued debt and finance such purchases with newly printed money. Expansionary monetary policies resemble the unconventional monetary policies recently implemented by many central banks, and, while being inflationary, these policies encourage lending in the credit market by relaxing the borrowing constraint. In particular, both their implementation and effect differ drastically from the lump-sum transfer scheme to implement inflation typically considered in monetary models.

Our model is based on the Lagos-Wright (2005) environment (LW henceforth) to keep tractability, but we introduce three key modifications. First, we allow for the use of debt by assuming an imperfect monitoring technology which records promises-to-pay from buyers and through which sellers can access past records in a fraction of meetings. Second, we adopt a mechanism-design approach that endogenously determines the terms of trade and the means-of-payments to be used, depending on the characteristics of the meeting, and subject to the availability of the monitoring technology and incentive compatibility. Third, we have three stages in each period: the first two stages correspond to the decentralized market (DM) in LW, and the last stage corresponds to the centralized market (CM).

\(^2\)See Gu, Mattesini, and Wright (2013) for a more detailed discussions.
Although introduction of imperfect monitoring into this type of models is not new, our formulation is novel in that it only requires minimum information for credit arrangement. As such, it ensures the liquidity role for money and resembles the typical operations of unsecured loans, including credit cards and commercial papers. Our monitoring technology only records the identities of the agents involved in the transaction and the debt incurred by the buyer. In particular, it does not keep track of transfers of real balances. The recorded histories of a buyer are updated periodically into credit records (e.g., his FICO score or credit rating) which may be accessed through the monitoring technology only by his future partners. The monitoring technology is imperfect in that only a fraction of sellers have access to it, and buyers can only issue debt when the technology is available. To determine which meetings to use the technology, we take a mechanism design approach that respects the overall constraint on its availability.

The mechanism design approach has been used to study a pure currency economy in the LW environment with one DM round. Hu, Kennan, and Wallace (2009) (HKW henceforth) show that a constant money supply can achieve any allocation achievable under perfect monitoring (but under limited commitment). Gu, Mattesini, and Wright (2013) demonstrate a similar point by showing that either money or credit is sufficient under a class of trading mechanisms. As a result, in a model with one round of DM, money and debt cannot circulate simultaneously in a meaningful way. In contrast, our model with two rounds of DM, together with limited availability of the monitoring technology, features a relevant role for both money and debt in the optimal trading mechanism.

Because of the limited availability of the monitoring technology, the mechanism design approach dictates that the means of payments, either money or debt or both, to be used in a specific meeting are determined endogenously to maximize the social welfare. Feasibility requires that buyers can issue debt in a meeting only if the mechanism specifies that the corresponding seller has the monitoring technology. The proposed trades are also set optimally in each meeting, subject to incentive compatibility constraints. The amount of debt buyers can issue is hence limited so that their repayments are self-enforcing subject to the limited commitment. Moreover, the terms of trade are set to respect both the individual-rationality constraint and the pairwise-core requirement. We obtain a full characterization of implementable allocations, and how different monetary policies affect implementability.

Our main result shows that the optimal expansionary monetary policy generically involves a strictly positive amount of debt purchases in our environment. Because it uses monetary expansions in the CM to purchase private debt issued by buyers in monitored DM meetings, this also implies that the optimal inflation rate is positive. This class of policies generally has the following two implications. First, they alleviate the borrowing constraints faced by the buyers in monitored meetings and hence increase lending in those meetings, which may increase the welfare. Second, they create inflation which tightens liquidity in non-monitored meetings and hence may decrease the quantity traded in those meetings.

\[\text{3}\] Some papers obtain coexistence by introducing additional frictions. For instance, Sanches and Williamson (2010) introduce theft that reduces the benefit of money, and Williamson (2012) considers a cost of using currency.
meetings. The optimal mechanism trades off these implications but generically finds positive inflation optimal.

To resolve this trade-off, the crucial determinant in choosing which meetings to be monitored is the tightness of the meeting in terms of liquidity needs, that is, how restrictive the endogenous liquidity constraint is. To efficiently redistribute liquidity across meetings, it is optimal to monitor meetings that are tight in liquidity, and to use money in meetings with relatively abundant liquidity. The optimal mechanism then taxes all buyers through inflation and subsidizes only trades in monitored meetings. As a result, generically the efficient policy has a positive inflation rate and newly created money is used to purchase private debt. We also obtain a full characterization of the optimal policy with respect to the economic fundamentals.

Our characterization result shows that both the optimal inflation rate and the amount of debt purchases depend on the fundamentals in a nontrivial way. In particular, the optimal policy depends not only on the overall (technology or preference) shock to the entire economy (such as the TFP), but also on how the shock affects different sectors of the economy. We provide numerical examples where beneficial shocks to the credit sector would increase the optimal inflation rate while beneficial shocks to the money sector would decrease it.

We not only characterize the optimal expansionary monetary policy, but also show that it is generically optimal among all monetary interventions which respect voluntary participation and incentive compatibility, and are constrained by the information released by the monitoring technology. Note that these constraints imply that enforced lump-sum taxes necessary to implement the Friedman rule is not feasible. Moreover, all taxation or subsidies can only happen in monitored meetings, contingent on recorded information. In particular, we show that both inflation through lump-sum transfers and deflation through taxing monitored trades are both suboptimal.

These results suggest that unconventional monetary policies can be welfare improving whenever both money and credit play essential liquidity roles, if implemented optimally. Moreover, in contrast to most other papers that focus on unconventional monetary policies as short-run responses, we find beneficial impacts of such policies in a stationary environment. However, our results also suggest that the precise amount of debt purchases and the corresponding inflation rate depend on the details of the economy; in particular, the distribution of shocks to different sectors may play a crucial role. This also implies that a rigid target of inflation rate may be suboptimal, even in the long run.

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4 This also implies that all taxation or subsidies can only happen in monitored meetings, contingent on recorded information. Therefore, the policy labeled “interest-bearing money” proposed by Andolfatto (2009) and the similar scheme in Wallace (2014) are excluded as well. A similar restriction is also imposed by Gomis-Porqueras and Sanches (2013).
1.1 Related Literature

The use of mechanism design to study optimal monetary policy under both limited commitment and under imperfect monitoring goes back to Cavalcanti and Wallace (1999) and Cavalcanti, Erosa, and Temzelides (1999). Those papers assume indivisible asset holdings and focus on circulation of insider money, while inflation is not emphasized. To allow for divisible asset holdings, we build on later papers that adopt the mechanism design approach to the LW environment. Our construction of optimal mechanisms extend the ones proposed in Hu, Kennan, and Wallace (2009) to an environment with two DM rounds and with credit arrangements. We also borrow the debt-limit construction in Bethune, Hu, and Rocheteau (2014), who study a pure credit economy and relax the “not-too-tight” solvency constraint in Alvarez and Jermann (2001), and extend it to our economy with both money and credit.

A few other papers based on LW also analyze imperfect monitoring and endogenous borrowing constraints in a monetary economy, but with one round of DM. Gu, Mattesini, and Wright (2013) show that money and credit cannot be coessential. Lotz and Zhang (2013) obtain coexistence of money and credit by limiting credit access to a fraction of meetings. However, their result crucially relies on the particular trading mechanism adopted, but would not survive under the optimal trading mechanism, as shown in HKW. In a similar model, Gomis-Porqueras and Sanches (2013) study monetary policies similar to those proposed in Andolfatto (2009), and show the optimality of positive inflation. However, as shown in HKW, under the optimal mechanism, zero inflation rate is optimal in that environment. Berentsen, Camera, and Waller (2007) introduce financial intermediaries in the LW model that allow buyers to make deposits or to take a (cash) loan before entering the DM, while money is still the only means-of-payment. They model inflation with lump-sum transfers and find a positive inflation rate optimal. Although we also find positive inflation optimal, in contrast to those papers, our expansionary policy differs significantly from those papers.

There are also other papers with two rounds of DM in the LW framework. Berentsen, Camera, and Waller (2005) study the short-run neutrality of money in a pure currency economy in such an environment. Guerrieri and Lorenzoni (2009) study the amplification mechanism in a similar model, but introduce credit in one of the DM rounds, with perfect enforcement. Telyukova and Wright (2008) explain the credit card debt puzzle in a model where buyers can use credit in one round of DM but have to use money in the other. Different from our focus, they assume perfect enforcement.

Finally, Deviatov and Wallace (2009) study optimal monetary policy in an environment with debt and money and two rounds of interactions between agents in every period, a DM round and a CM round. In contrast to the LW setup, money is indivisible and the CM round is only used to implement monetary policy. They construct a numerical example where the optimal monetary policy involves loans to monitored agents which is used to fund their purchases in the goods market. These loans bear some resemblance to the...
optimal policy we obtain in our model. They lack, however, a clear mapping from the primitives of the environment to changes in liquidity and the implied optimal monetary policy.

The paper proceeds as follows. In the next section, we present the environment, define trading mechanisms, strategies and equilibrium. We also present results in the case where the monitoring technology is accessible in all meetings and the case where it is never accessible and the supply of money is constant. In section 3 we introduce expansionary monetary policies and characterize the set of implementable allocations under such policies. We consider optimality and relate optimality to the coessentiality of money and debt. We also consider alternative monetary policies. Section 4 presents extensions and section 5 concludes. All the proofs are in the Appendix.

2 Model

This section begins with the description of the environment. We then define trading mechanisms, strategies, and equilibrium.

2.1 Environment

Time is discrete and the horizon is infinite. The economy is populated by two types of agents, labeled as buyers and sellers. The set of buyers is denoted $\mathbb{B}$ and the set of sellers is partitioned into two subsets, $S_1$ and $S_2$, both with measure one. Each period is divided into three stages. Buyers randomly meet sellers in $S_i$ in stage $i \in \{1, 2\}$, and the probability of a successful meeting is $\sigma_i$. There are three goods, one for each stage. At stage $i = 1, 2$, a seller from $S_i$ can produce $x_i$ units of round 1 good for a buyer at cost $c_i(x_i)$ and the buyer’s utility is $u_i(x_i)$. Let $x_i^*$ be the solution to $u'_i(x) = c'_i(x)$, the quantity that maximizes the surplus. In the last stage, agents meet in a centralized market. In this market, they can all consume and produce, and the utility is linear, represented by $z$ (negative values are interpreted as disutility for production). Agents maximize their life-time expected utility with discount factor $\delta$. We let $\rho = \frac{1-\delta}{\delta}$. We call the first two stages DM rounds and the last stage CM round.

There exists a technology which keeps track of buyers’ trading histories in some meetings. We call a meeting a monitored meeting if the technology is accessible, and a non-monitored meeting otherwise. This technology works as follows. For each buyer $b \in \mathbb{B}$, a recorded history at period $t$ is a triple, $h = (h_1, h_2, h_3) \in H$, such that for $i = 1, 2$, $h_i = (b, s_i, z_{i,c})$ keeps track of the buyer’s round $i$ DM promise to the seller, where $b$ is the identity of the buyer, $s_i$ is the identity of the seller, and $z_{i,c}$ is the promise-to-pay.

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6The meeting pattern that buyers always meet sellers from $S_i$ at stage $i$ is special. In the Appendix we show that our results are robust to more general meeting patterns.
in terms of CM good (debt), and \( h_3 \in \mathbb{R} \) keeps track of the total repayment.\(^7\) Here we assume that the repayment is first used to repay the seller from \( S_1 \), if any, before used to repay the seller from \( S_2 \).\(^8\) If the buyer does not meet a seller in round-\( i \), or if the buyer meets a seller but there is no trade, \( h_i \) is empty. The recorded history \( h_i \) is also empty in non-monitored meetings. There also exists a technology, comprised of a set of records \( R \) and a function, \( \omega : R \times H \to R \), which updates the record of the buyer based on his recorded history. This technology is only accessible to sellers in monitored meetings and allows the seller to observe the record \( r \in R \) of the buyer.

The number of monitored DM rounds is given by \( \ell \in \{0, 1, 2\} \). Henceforth, we say that monitoring is unlimited if \( \ell = 2 \), monitoring is limited if \( \ell = 1 \), and there is no monitoring if \( \ell = 0 \). We are mainly interested in the limited monitoring case, which is meant to capture the idea that monitoring is costly and the society can only afford monitoring a proportion of transactions.\(^9\)

Our monitoring technology resembles the typical operations of unsecured loans, such as commercial papers. It only records the identities of the agents involved in the transaction and the amount of debt. In particular, it does not record agents’ money holdings or refusal to trade. In that sense, even unlimited monitoring differs from the usual perfect monitoring assumption in repeated games. The debt is unsecured due to the limited commitment friction. The credit records correspond to credit scores (FICO scores, for example) or agency ratings.

Lastly, there is an intrinsically useless, divisible, and storable object, called money. The money supply at the end of period \( t \) is denoted by \( M_t \).

### 2.2 Trading mechanisms

Instead of imposing a particular trading mechanism, we allow arbitrary trading mechanisms that are incentive compatible subject to the frictions in the environment. Taking a mechanism-design approach, we consider a proposal consisting of the following objects:

(P1) A subset \( C \subseteq \{1, 2\} \) of monitored DM rounds.

(P2) A sequence of debt limits, \( \{D_t\}_{t=0}^{\infty} \), two records, \( G \) (Good) and \( B \) (Bad); and an updating function \( \omega \) such that:

(i) \( \omega(r, \emptyset) = r \) for \( r \in \{G, B\} \);

(ii) \( \omega(B, h) = B \) for all \( h \in H \);

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\(^7\)We are assuming that buyers must settle all debt in the same period. Since the utility function in the CM is linear, this assumption is without loss of generality.

\(^8\)We could allow the buyer to choose whom to repay to but this would only complicate the notation without adding any insight.

\(^9\)We restrict the values of \( \ell \) to be in the set \( \{0, 1, 2\} \) for simplicity, but our results do not depend on this restriction. See also discussions in Section 4.1. Note also that even when monitoring is unlimited, our technology is much weaker than the notion of memory put forth by Kocherlakota (1998), which includes all actions of all direct and indirect partners of an agent.
(ii) \( \omega(G, h) = G \) iff \( h_2 \geq \min\{D_t, z_{1,c} + z_{2,c}\} \).

We also assume that, if \( C = \{2\} \), the seller observes the buyer’s available debt limit, \( D_t - z_{1,c} \), in the second DM round. Intuitively, \( D_t \) sets the maximum amount of debt a buyer can credibly incur in a given period.

(P3) The proposed trades are given by a function \( o^i \) defined as follows: if \( i \in C \), then

\[
o^i_t(m, r, d) = (x, z_{i,c}, z_{i,m}),\]

where \( m \) is the buyer’s announcement of real balances holdings, \( r \) is his record, \( d \) is his available debt limit, and \( (x, z_{i,c}, z_{i,m}) \) is the proposed trade—\( x \) is the quantity to be produced by the seller, \( z_{i,c} \) is the promise the buyer makes to the seller, and \( z_{i,m} \) is the transfer of real balances from the buyer to the seller; if \( i \notin C \), then

\[
o^i(m) = (x, z_{i,m}),\]

where \( m \) is the buyer’s announcement of real balance holdings and \( (x, z_{i,m}) \) is the trade.

(P4) The price for money \( \phi_t \) in the CM, which implies aggregate real balances \( Z_t = \phi_t M_t \), and an initial distribution of money holdings \( \mu \).

In the proposal, (P1) chooses the meetings to be monitored, which will be the ones with access to credit, only subject to the overall constraint on the number of DM rounds to be monitored given by \( \ell \). Thus, in contrast to the previous literature on money and credit, here access to credit is endogenously determined.

In (P2), we implicitly assume that \( |R| = 2 \) and a particular updating rule that uses debt limits, even though our technology allows for an arbitrary finite set of credit records and an arbitrary updating rule. As argued in Bethune, Hu, and Rocheteau (2014), this is without loss of generality.

The functions \( o^i \) in (P3) map the credit records (if applicable) and the announced money holdings of the buyer to a proposed trade. The announcement is necessary because money holdings are private information. To implement this proposed trade in a decentralized manner, we use the following trading protocol in meetings in the DM. The buyer first announces his real balances, and then both the buyer and the seller respond with yes or no to the corresponding proposed trade. If both respond with yes then they move to the next stage; otherwise, the meeting is autarkic. If they move to the next stage, the buyer makes a take-it-or-leave-it offer, which is implemented if the seller responds with yes while the originally proposed trade by the mechanism is carried out otherwise. In turn, the trading mechanism in the CM stage is as follows. Each buyer chooses how much (if any) to repay of his promises, and agents trade competitively against \( \phi_t \) to rebalance their money holdings.

Our trading protocol is in the spirit of a direct mechanism. In particular, we allow arbitrary ways to split the trading surpluses only subject to individual rationality and coalition-proofness. This trading mechanism generalizes the trading protocols considered in Zhu (2008) and Hu, Kennan, and Wallace (2009) to our setting with monitored meetings. As in those papers, the first stage ensures that the mechanism satisfies individual
rationality, and the second stage ensures that it satisfies the pairwise core requirement and hence is coalition-proof.\footnote{Note that, since we allow the buyer to make a take it or leave it offer which may differ from the trade proposed by the mechanism, the restriction to an updating function \( \omega \) which only gives a bad record to a buyer who does not repay the promise (up to the debt limit) is with loss of generality. In particular, one could prevent a buyer from making a different offer by conferring a bad record in case he does so. We do not allow such punishments for two reasons. First, punishing a buyer from making a different promise, even if he repays the promise, seems implausible. Second, our results on constrained efficient allocations do not depend on this restriction, and it renders the analysis more tractable.}

In what follows, we focus only on stationary proposals, which can be written as

\[ P = [C, D, (o_1, o_2), (Z, \mu)]. \]  

(1)

\subsection*{2.3 Strategies and equilibrium}

We denote by \( s_b \) the strategy of a buyer \( b \in B \). In each DM round, \( s_b \) maps the buyer’s real balance holdings, his record, and the available debt limit, to the buyer’s announcement, \( m \), and to his response \( \{yes, no\} \). Obviously, \( s_b \) may also differ for monitored meetings and non-monitored meetings. In turn, conditional on both the buyer and the seller responding with \( yes \), \( s_b \) gives the buyer offer to the seller. In the CM round \( s_b \) maps the buyer’s recorded history in the first and second DM rounds to his repayment decisions and to his final money holdings after the CM closes.\footnote{We are assuming that the buyer’s strategy does not depend on his private history, meant as the part of his history which will never be observed by any seller. This assumption is in the same spirit as the public perfect equilibrium in the repeated-games literature, and, as far as constrained-efficient allocations are concerned, is without loss of generality.}

We denote by \( s_i \) the strategy of a seller \( i \in S_i \), where \( i \in \{1, 2\} \). In the DM round, the strategy \( s_i \) maps the buyer’s announcement of real balances and his record (if observable by the seller) to the seller’s response \( \{yes, no\} \), and, conditional on both responding \( yes \), another function that maps the buyer’s offer to \( \{yes, no\} \). We assume that sellers do not carry money across periods.

We restrict attention to symmetric and stationary strategies, and hence a strategy profile may be denoted \((s_0, s_1, s_2)\), where \( s_0 \) is the buyer strategy for all buyers \( b \), and \( s_i \) is the strategy for all sellers from \( S_i \). We define an equilibrium, consisting of a proposal \( P \) and a strategy profile \( s \) as follows.

\textbf{Definition 2.1.} An equilibrium is a list

\[ E = \langle (s_0, s_1, s_2), [C, D, (o_1, o_2), (Z, \mu)] \rangle, \]

such that, given the price of money, each strategy is sequentially rational conditional on other players’ strategies; and the centralized market for money clears at every date.

Throughout the paper we restrict attention to equilibria with the following characteristics: (1) buyers always truthfully announce their real balance holdings, (2) buyers and
sellers respond with yes in all DM meetings and buyers always offer the trades proposed by the mechanism; (3) the initial distribution of money across buyers is degenerate - all buyers hold $M_0$ units of money; (4) buyers in state $G$ always repay their debt. We call such equilibria simple equilibria.

In what follows, to simplify notations and to convey our main insights, we restrict attention to the case where $\sigma_1 = 1$. Our results are robust to the case where $\sigma_1 < 1$ and we give a more detailed discussion in Section 4.2. Under the assumption that $\sigma_1 = 1$, an allocation associated with a simple equilibrium can be denoted by

$$L = [(x_1, x_2), (z_1, z_2)],$$

where $x_i$ denotes a buyer's consumption in round-$i$ DM and $z_i$ denotes CM consumption of a round-$i$ seller. Moreover, we restrict our attention to allocations that satisfy $z_1 \leq u_1(x_1) \leq u_1(x_1^*)$ and $z_2 \leq u_2(x_2) \leq u_2(x_2^*)$. This restriction is without loss of generality as far as optimal allocations are concerned, but it avoids many uninteresting cases. Lastly, note that, without taking implementability into account, the optimal allocation is given by $x_1 = x_1^*$ and $x_2 = x_2^*$. It does not depend on $z_1$ and $z_2$ as the utility in the CM is linear.

3 Implementation without monetary intervention

In this section, we characterize the set of implementable allocations with a constant money supply. We first consider two benchmark cases: a pure credit economy with unlimited monitoring, and a pure currency economy with no monitoring. We then consider limited monitoring with money.

3.1 Pure credit economy with unlimited monitoring

Given an allocation, $L = [(x_1, x_2), (z_1, z_2)]$, the buyer's ex ante expected discounted lifetime payoff is given by

$$\sum_{t=0}^{\infty} \delta^t \{[u_1(x_1) - z_1] + \sigma_2[u_2(x_2) - z_2]\} = \frac{1 + \rho}{\rho} \{[u_1(x_1) - z_1] + \sigma_2[u_2(x_2) - z_2]\}.$$

For $L$ to be implementable, it is then necessary that repaying the maximum equilibrium debt, which would amount to $z_1 + z_2$, and continuing with the equilibrium future payoffs, is preferred to repaying nothing and receiving no trade in all future periods, that is,

$$\rho(z_1 + z_2) \leq [u_1(x_1) - z_1] + \sigma_2[u_2(x_2) - z_2].$$

For a seller from $S_i$ to be willing to produce in the DM, his production cost must be covered by his payoffs in the CM, that is,

$$z_1 \geq c_1(x_1) \text{ and } z_2 \geq c_2(x_2).$$
Finally, to ensure that the pairwise core requirement is satisfied, the proposed pairwise round-1 surplus plus the buyer’s round-2 surplus should be higher than the pairwise round-1 surplus for the output level \( \hat{x}_1 \) given by

\[
\hat{x}_1 = \min\{x_1^*, c_1^{-1}(z_1 + z_2)\}.
\]

Note that the buyer has enough liquidity to induce the seller to produce \( \hat{x}_1 \) because \( c_1(\hat{x}_1) \leq z_1 + z_2 \). Formally, the condition is given by

\[
u_1(x_1) - c_1(x_1) + \sigma_2[u_2(x_2) - z_2] \geq u_1(\hat{x}_1) - c_1(\hat{x}_1).
\]

Indeed, if (4) does not hold, then the buyer would deviate to offering \((\hat{x}_1, \hat{z}_1)\) for some promise \( \hat{z}_1 \) to make both agents better off than the proposed trade.

The following theorem shows that these three necessary conditions are also sufficient.

**Theorem 3.1 (Implementability under Unlimited Monitoring).** Let \( \ell = 2 \) and assume that money has no value. An allocation \( L = [(x_1, x_2), (z_1, z_2)] \) is implementable if and only if it satisfies (2), (3), and (4).

To prove sufficiency, we take \( D = z_1 + z_2 \) to be the debt limit, and the buyer can keep a good record as long as he either repays in full or at least \( D \) from obligations made in the two stages. Conditions (2) and (3) ensure that buyers are willing to repay \( D \) in the CM and sellers are willing to produce. To ensure that the buyer has no profitable deviating offers other than the one involving \( \hat{x}_1 \), we use the Hu, Kennan, and Wallace (2009) mechanism to implement the round-2 allocation so that the buyer can receive a positive round-2 surplus only if his available debt limit when entering round-2 DM is at least \( z_2 \). Then, we show that (4) is sufficient to ensure that the deviating offer with \( \hat{x}_1 \) is not profitable.

### 3.2 Pure currency economy with no monitoring

In the absence of monitoring, money is necessary to implement any positive production. First we remark that conditions (2) and (3) are still necessary for individual rationality: without (2) buyers are better off not participating in any trades; without (3) sellers will not produce. Similarly, (4) is still necessary for the pairwise core requirement. Indeed, if it does not hold, the buyer can deviate and offer \( \hat{x}_1 \) as output and some monetary payment to make both agents better off.

However, with money alone, one more condition is necessary, because the buyer can hold real balances that are only sufficient for round-2 DM trade but skip round 1. Without the monitoring technology such deviation is not detectable. This leads to the following condition:

\[
-\rho z_1 + [u_1(x_1) - z_1] \geq 0.
\]

According to condition (5), the surplus for the buyer in round-1 DM, \( u_1(x_1) - z_1 \), has to be sufficiently large to compensate for his cost of holding \( z_1 \) units of real balances across periods.
The following theorem shows that these necessary conditions are also sufficient.\textsuperscript{12}

\textbf{Theorem 3.2} (Implementability under No Monitoring). Let \( \ell = 0 \) and assume that the money supply is constant. An allocation \( L = [(x_1, x_2), (z_1, z_2)] \) is implementable if and only if (2), (3), (4), and (5) are satisfied.

Compared to Theorem 3.1, Theorem 3.2 requires an additional condition, (5). As a result of this additional constraint, implementation in a pure currency economy is more restrictive than a pure credit economy.

\section*{3.3 Constant money supply with limited monitoring}

In the presence of limited monitoring, money is necessary to sustain positive production in non-monitored meetings.\textsuperscript{13} It turns out that, if monitoring is limited, credit trades cannot meaningfully expand the set of implementable outcomes than money alone. In particular, (5) is still necessary in the presence of limited monitoring, irrespective of whether \( C = \{1\} \) or \( C = \{2\} \). When \( C = \{1\} \), (5) is necessary to ensure that the buyers are willing to repay their debts. Indeed, in the CM the minimum repayment is \( z_1 \) and the future surpluses from monitored trades is

\[
\sum_{t=1}^{\infty} \delta^t [u_1(x_1) - z_1] = \frac{1}{\rho} [u_1(x_1) - z_1],
\]

and this implies that (5) is necessary for repayment to be individually rational. Similarly, if \( C = \{2\} \), then money is necessary to finance the trades in round-1 DM meetings. Hence, (5) is necessary for otherwise the buyer would prefer to skip round-1 meetings. We have the following lemma.

\textbf{Lemma 3.1}. Let \( \ell = 1 \) and assume that the money supply is constant. An allocation \( L = [(x_1, x_2), (z_1, z_2)] \) is implementable under a constant money supply only if it satisfies (2), (3), and (5).

Under \( C = \{1\} \), (4) is also necessary because in round-1 DM, the buyer can use both money and debt, and hence the previous logic applies. Under \( C = \{2\} \), however, this condition may not be necessary because the buyer cannot transfer debt from round-2 DM to round-1 DM.\textsuperscript{14} Nevertheless, as shown below, (4) is not binding when we look for constrained efficient allocations and hence relaxing it does not help.

\textsuperscript{12}We extend the HKW mechanism to show the sufficiency. However, while in HKW the implementation can be achieved by a mechanism that punishes the buyer with zero surpluses unless he brings at least the right amount of real balances, here in round-2 trades we use a continuous mechanism to ensure a continuous continuation value at the round-1 DM. This is useful because continuity guarantees existence of the proposed trades as the solution to a maximization problem.

\textsuperscript{13}In the Appendix we show this necessity formally.

\textsuperscript{14}It may be surprising that (4) is necessary under unlimited monitoring but not under limited monitoring. This has to do with our restriction to updating rules which only give a bad record to a buyer who does not repay his debt. Thus, if debt is accepted in both rounds, a buyer can deviate by moving debt from round-2 DM to round-1 DM. He cannot do that if monitoring is limited and \( C = \{2\} \) because debt is not accepted in round-1 DM.
Lemma 3.1, compared against Theorem 3.2, shows that under limited monitoring, the use of debt does not expand the set of implementable outcomes. However, it turns that this result is overturned when we consider monetary interventions, particularly the ones that look like unconventional monetary policies.

4 Monetary intervention under limited monitoring

In this section, we introduce a class of monetary interventions which increase the set of implementable allocations under limited monitoring. This class of interventions is purely monetary in the sense that no taxation other than inflation is allowed. We first characterize the set of implementable allocations and then look at the optimal monetary policy within this class. In the last subsection, we discuss alternative monetary interventions, for which fiscal actions are allowed, as long as they are consistent with our monitoring technology, and show that they are suboptimal.

4.1 Expansionary monetary policies (EMP)

We consider interventions that use the seigniorage revenue from money creation to purchase privately issued debt. We label these interventions expansionary monetary policies (henceforth EMP). In our stationary environment, we are mainly concerned with the long-run implications of these policies. In particular, the EMP is fully anticipated and the policymaker has full commitment power.

Consider an environment with limited monitoring and a proposal with $C = \{i\}$, that is, round-$i$ DM has monitored meetings. The EMP sets a maximum amount $k$ of private debt that the mechanism will redeem. That is, for any recorded promise at period $t$, $(b, s_i, z_{i,c}^b)$, the mechanism will print new money and use it to purchase $\min\{k, z_{i,c}^b\}$ of the debt from the seller $s_i$. Feasibility then implies that the inflation rate $\pi$ must satisfy

$$\int_{b \in \mathbb{B}} \min\{z_{i,c}^b, k\} db = \pi \phi_t M_{t-1},$$

(6)

where $M_{t-1}$ is the amount of money in the economy in period $t$, right before the monetary intervention. Thus, we require that, for every $t$, the amount of private debt outstanding is consistent with the inflation rate and with the amount of this debt which is purchased by the EMP. Because all debts are matured within one period, the purchased debts are retired immediately and hence represent a subsidy to the debt issuers.\(^{15}\)

Our EMP resembles unconventional monetary policies implemented by central banks after the 2008 financial crisis. In particular, the left-hand side of (6) captures the direct purchase of private debts, such as commercial papers, and the right-hand side corresponds

\(^{15}\)As mentioned earlier, all debts considered here are of one period because of quasi-linearity and hence, even though we could introduce long-term debts, they would not change the results. Here the EMP captures the implicit subsidies present in the unconventional policies implemented after the crisis.
to its inflationary implications. Being fully anticipated by the agents, the purchase by 
the EMP is an implicit subsidy to the credit sector, funded by an inflation tax in the 
monetary sector.\footnote{The unconventional monetary policy recently implemented also includes other measures, such as 
buying mortgage-backed securities that are typically used for collateralized borrowing. Our EMP captures 
such measures in so far as such lendings have an unsecured element (in the sense that it is without 
default risk) and such purchases imply some implicit subsidies to the credit market. See also discussions 
in Section 5.3.}

Formally, a proposal now includes

$$\mathcal{P} = [C, D, (o_1, o_2), (Z, \mu)],$$

and an EMP \((k, \pi)\). We say that an EMP is active if \(k > 0\). An allocation, \(\mathcal{L} = 
[(x_1, x_2), (z_1, z_2)]\), is implementable with EMP if it is implementable under a proposal \(\mathcal{P}\) 
and an EMP \((k, \pi)\).\footnote{Alternatively, we can formulate the expansionary monetary policy as a proportional subsidy. More 
precisely, the policy sets a fraction \(\kappa\) for the monitored stage \(i\), and the mechanism commits to purchase 
\(\kappa\) fraction of the debts issued by buyers to stage-\(i\) sellers. To avoid buyers from overissuing their debts one 
can choose the debt limit appropriately and it can be shown that, in terms of implementability, this 
scheme is equivalent to the scheme considered above.}

Theorem 4.1 provides a full characterization of implementable allocations with EMP 
under limited monitoring. We distinguish two cases: the first uses \(C = \{1\}\) while the 
second uses \(C = \{2\}\).

**Theorem 4.1 (Expansionary Monetary Policies).** Assume limited monitoring.

(i) An allocation, \(\mathcal{L} = [(x_1, x_2), (z_1, z_2)]\), is implementable with EMP and \(C = \{1\}\) if and 
only if (2), (3), and (4).

(ii) An allocation, \(\mathcal{L} = [(x_1, x_2), (z_1, z_2)]\), is implementable with EMP and \(C = \{2\}\) if and 
only if (3), (5), and

$$\{-\rho z_1 + [u_1(x) - z_1]\} + \frac{(\rho + 1)\sigma_2}{\rho + \sigma_2}\{-\rho z_2 + \sigma_2[u_2(x_2) - z_2]\} \geq 0. \tag{7}$$

Theorem 4.1 (i) shows that, when \(C = \{1\}\), the pure credit economy with unlimited 
monitoring and the EMP with limited monitoring implement exactly the same set of 
allocations. If (5) holds, the EMP is inactive and, as shown in Theorem 3.2, a constant 
money supply suffices even if debt is not used. However, if (5) does not hold, an active 
EMP is necessary to implement the allocations achieved with unlimited monitoring. Under 
EMP, the buyer has incentive to repay his debt if and only if

$$-\rho(z_1 - k) + [u_1(x_1) - z_1 + k] \geq 0.$$ 

To satisfy the above inequality, the minimal amount of debt to be purchased by the EMP 
and the corresponding inflation rate are given by

$$k_1 = z_1 - \frac{u_1(x_1)}{1 + \rho} \text{ and } \pi_1 = \frac{k_1}{z_2}. \tag{8}$$
It turns out that, by (2), this inflation rate is consistent with participation in the round-2 monetary trade.

Theorem 4.1 (ii) gives different conditions from (i) for \( C = \{2\} \). Because there is no monitoring in round-1 DM, (5) is necessary to ensure that buyers are willing to hold money and participate round-1 trades. Now, if \(-\rho z_2 + \sigma_2 [u_2(x_2) - z_2] \geq 0\), then a constant money supply suffices and debt it not needed. When that inequality fails, however, EMP is necessary, and under the EMP, the buyer has incentive to repay his debt incurred in round-2 trades if and only if

\[-\rho(z_2 - k) + \sigma_2 [u_2(x_2) - z_2 + k] \geq 0.\]

To satisfy the above inequality, the minimal amount of debt to be purchased by the EMP and the corresponding inflation rate are given by

\[k_2 = z_2 - \frac{\sigma_2 u_2(x_2)}{\sigma_2 + \rho} \quad \text{and} \quad \pi_2 = \frac{\sigma_2 k_2}{z_1}. \quad (9)\]

Different from the case with \( C = \{1\} \), however, when \( C = \{2\} \) an active EMP can implement allocations which cannot be implemented with unlimited monitoring. Indeed, there are allocations which do not satisfy (2) but satisfy (3), (5), and (7). As we shall see later, these may include constrained-efficient allocations. The intuition for this result runs as follows. Under unlimited monitoring but without the EMP, a buyer who participates in the two DM rounds incurs a cost \( z_1 + z_2 \) in the CM round in order to redeem the debts issued in exchange for the DM goods. Under limited monitoring but with EMP and \( C = \{2\} \), the cost associated with obtaining the same amount of DM goods is given by \((1 + \pi_2)z_1 + z_2 - k_2\), i.e., the direct cost of redeeming part of the debts issued by the buyer himself, and the indirect cost of holding \( z_1 \) real balances to participate in the first DM round when the inflation rate is given by \( \pi_2 \). Feasibility of the EMP implies \( \pi_2 z_1 = \sigma_2 k_2 \), and we can rewrite the difference in the costs with and without EMP, \(-k_2 + \pi_2 z_1\), as \(- (1 - \sigma_2) k_2\), which is negative whenever \( \sigma_2 < 1 \).

### 4.2 Optimal monetary policy

We now characterize the set of optimal allocations. Our main focus is on the necessity of the EMP to achieve such allocations. For a given allocation, \( \mathcal{L} = [(x_1, x_2), (z_1, z_2)] \), social welfare is given by

\[\mathcal{W}(\mathcal{L}) = \frac{1 + \rho}{\rho} \left\{ [u_1(x_1) - c_1(x_1)] + \sigma_2 [u_2(x_2) - c_2(x_2)] \right\}. \quad (10)\]

We say that an allocation is constrained efficient if it maximizes (10) among all implementable allocations.

To maximize social welfare, it is without loss of generality to have the constraint (3) binding, i.e., to consider only allocations of the form \( \mathcal{L} = [(x_1, x_2), (c_1(x_1), c_2(x_2))] \). We say that a pair \((x_1, x_2)\) is a constrained-efficient allocation if \([(x_1, x_2), (c_1(x_1), c_2(x_2))]\) is
a constrained-efficient allocation. Note that, although we are interested in the case of limited monitoring, this result applies irrespective of the degree of monitoring.

If the first-best allocation is implementable under no monitoring and a constant money supply, then debt is not essential in the sense that it is not needed to implement the constrained-efficient allocation. Now, by Theorem 3.2, to determine whether a first-best allocation, \((x_1^*, x_2^*)\), is implementable under no monitoring amounts to check whether the conditions (2) and (5) hold under that allocation, and we have the following corollary. Note that (4) is trivially satisfied for any first-best allocation.

**Corollary 4.1.** The first-best allocation, \((x_1^*, x_2^*)\), is implementable under no monitoring and with a constant money supply if and only if

\[
\rho \leq \rho^M \equiv \min \left\{ \frac{[u_1(x_1^*) - c_1(x_1^*)]}{c_1(x_1^*)} + \frac{\sigma_2 [u_2(x_2^*) - c_2(x_2^*)]}{c_1(x_1^*)} + \frac{u_1(x_1^*) - c_1(x_1^*)}{c_1(x_1^*)}, \frac{u_1(x_1^*) - c_1(x_1^*)}{c_1(x_1^*)} \right\}. \tag{11}
\]

The first term inside the min operator corresponds to (2), and the second term corresponds to (5). By Corollary 4.1, when \(\rho \leq \rho^M\), debt is not necessary to implement the first-best, and we are only interested in the case where \(\rho > \rho^M\).

Now we turn to constrained-efficient allocation with EMP. We consider two candidates according to \(C = \{1\}\) or \(C = \{2\}\). According to Theorem 4.1 (i) for the case \(C = \{1\}\), the relevant constraints include (2) and (4), and according to Theorem 4.1 (ii) for the case \(C = \{2\}\), (5) and (7). These two sets of constraints correspond to two candidates for the constrained efficient allocations. The first, denoted by \((x_1^{C1}, x_2^{C1})\), maximizes (10) subject to (2) with \((z_1, z_2) = (c_1(x_1^1), c_2(x_2^1))\). The lemma below shows that (4) is not binding. The second, denoted by \((x_1^{C2}, x_2^{C2})\), maximizes (10) subject to (5) and (7).

**Lemma 4.1.** Assume limited monitoring. Both \((x_1^{C1}, x_2^{C1})\) and \((x_1^{C2}, x_2^{C2})\) are implementable with EMP, and either one of them is the constrained-efficient allocation.

Because, by Lemma 3.1, both (2) and 5 are necessary for a constant money supply to implement an allocation, the constrained efficient allocation under limited monitoring without intervention cannot do better than \((x_1^{C1}, x_2^{C1})\). Thus, by Lemma 4.1, if (5) fails for \((x_1^{C1}, x_2^{C1})\), then an active EMP is necessary. Our next theorem shows that, at least generically, even when (5) holds for \((x_1^{C1}, x_2^{C1})\), EMP is still necessary to implement the constrained-efficient allocation whenever \(\rho > \rho^M\). The genericity condition first requires \(\sigma_2 < 1\). Second, it rules out the knife-edge case where the relevant three conditions, (2), (5), and (7) are all binding for the constrained efficient allocation. Formally, this amounts to ruling out the case where the constrained-efficient allocation is equal to \((\bar{x}_1, \bar{x}_2)\), defined as the unique positive solution to

\[
-\rho c_1(\bar{x}_1) + [u_1(\bar{x}_1) - c_1(\bar{x}_1)] = -\rho c_2(\bar{x}_2) + \sigma_2 [u_2(\bar{x}_2) - c_2(\bar{x}_2)] = 0. \tag{12}
\]

Generically, \((\bar{x}_1, \bar{x}_2) \neq (x_1^{C1}, x_2^{C1})\), as the latter has to satisfy the FOC’s implied by the maximization problem as well. We have the following theorem.
Theorem 4.2. Suppose that \( \rho > \rho^M, \sigma_2 < 1, \) and that \((\bar{x}_1, \bar{x}_2) \neq (x_1^{C1}, x_2^{C1})\). Then, the constrained efficient allocation can only be implemented with EMP but not with a constant money supply.

As noted earlier, if (5) fails for \((x_1^{C1}, x_2^{C1})\), then an active EMP is necessary. To prove Theorem 4.2, we show that when (5) holds for \((x_1^{C1}, x_2^{C1})\), we can construct another allocation that satisfies (5) and (7) and that gives a higher welfare than that of \((x_1^{C1}, x_2^{C1})\), and hence \((x_1^{C2}, x_2^{C2})\) is the constrained-efficient allocation and EMP is necessary. The crucial observation is that when \(\sigma_2 < 1\) and when (5) is slack, (7) allows for better allocations than (2).

Theorem 4.2 shows that, except for the case where money alone can implement the first-best, generically, an active EMP is necessary to achieve the constrained efficient allocation. Moreover, depending upon which of the two candidates is the constrained-efficient allocation, generically, an active EMP is necessary to achieve the constrained efficient allocation that satisfies (5) and (7) and that gives a higher welfare than that of (2).

We illustrate these results by some numerical examples, using a family of fairly standard functional forms: for \(i = 1, 2\), \(u_i(x_i) = x_i^a\), \(a \in (0, 1)\), and \(c_i(x_i) = x_i\). Here we set \(a_1 = 0.99\) and \(a_2 = 0.97\). When \(\sigma_1 = 1\), and \(\sigma_2 = 0.95\), \(\rho^M = 1.01\%\), and the first-best is implementable with EMP if and only if \(\rho \leq 1.97\%\). Moreover, the constrained-efficient allocation is \((x_1^{C1}, x_2^{C1})\) for \(\rho \in (1.01\%, 15\%)\), and the optimal EMP is depicted in Figures 1 and 2. For such \(\rho\)'s, the formula (8) is then relevant.

Note that the optimal intervention is not monotonic in the discount factor: both the optimal \(k\) (as a fraction of total output, \(c_1(x_1^{C1}) + c_2(x_2^{C1})\)) and \(\pi\) are increasing in \(\rho\) when the first-best is implementable but both are decreasing for larger \(\rho\)'s. When the first-best is implementable \(x_1^{C1} = x_1\) and hence, by (8), both optimal \(k\) and \(\pi\) decrease with \(\rho\). For larger \(\rho\)'s, however, \(x_1^{C1}\) decreases with \(\rho\) and hence we have two opposing effects, and this explains the non-monotonicity.

We also remark that, for any \(\rho > \rho^M\), the constrained-efficient allocation is given by \((x_1^{C2}, x_2^{C2})\) if \(\sigma_2\) is sufficiently small. In the above example, for \(\rho \in (1.01\%, 2\%)\), the constrained-efficient allocation is given by \((x_1^{C2}, x_2^{C2})\) if \(\sigma_2 < 0.29\). Thus, the fundamentals also matter for the choice of which sector should be endowed with the monitoring technology under the optimal trading mechanism.
Figure 1: Optimal EMP—$k$ as a fraction of total output

Figure 2: Optimal EMP—optimal inflation rate (%)
Here we give some examples to illustrate how the optimal EMP may respond to productivity shocks. We only focus on comparative statics across steady states, but our model can be extended to allow for persistent shocks and the results there would be similar to findings reported here. We use the above functional forms, but introduce shocks to both stages: for $i = 1, 2$, $u_i(x_i) = \theta_i x_i^{a_i}$, $a_i \in (0, 1)$, and $c_i(x_i) = x_i$, and we set $a_1 = 0.99$, $a_2 = 0.97$, and $\rho = 2\%$. In this case, the optimal EMP not only depends on the magnitude of $\theta_1$ and $\theta_2$, but it also depends on the relative size of the two. Figure 3 shows the optimal inflation rates for $(\theta_1, \theta_2) \in [0.9.1.1]^2$. We remark that under this range of parameters, the optimal mechanism has $C = \{1\}$ and the optimal policies are given by (8). In Figure 3, the optimal inflation rate increases with $\theta_1$ but decreases with $\theta_2$. This implies that, to determine the optimal monetary policy, not only how the shock affects the overall economy matters, but how the shock affects the monitored sector relative to the non-monitored sector also matters. In particular, if the shock is more beneficial to the monitored sector, i.e, if $\theta_1$ increases more, then the optimal inflation rate is pro-cyclical. In contrast, if the shock is more beneficial to the non-monitored sector, i.e, if $\theta_2$ increases more, then the optimal inflation rate should be counter-cyclical.

To illustrate this point, we control $\theta_1$ and $\theta_2$ by a parameter $\eta$ as follows:

$$\theta_1 = 2 \times (\eta + 0.45) \quad \text{and} \quad \theta_2 = \frac{0.17}{0.22 - \eta}.$$ 

Under this parametrization, both $\theta_1$ and $\theta_2$ increase with $\eta$, but the relative increase depends on the value of $\eta$. The above findings then imply that the optimal inflation
rate should increase with $\eta$ when $\theta_1$ increases more than $\theta_2$, that is, when $\eta$ is relatively small, while it should decrease with $\eta$ when $\theta_1$ increases more than $\theta_2$, that is, when $\eta$ is relatively large. Figure 4 illustrates this result: the optimal inflation rate, $\pi$, measured in the vertical axis, first increases and then decreases with $\eta$, measured in the horizontal axis.

These results demonstrate that liquidity needs are endogenously determined by the fundamentals. Moreover, the optimal policy response in terms of liquidity provision requires a detailed knowledge about how shocks to the fundamentals affect different sectors, especially the distribution between the monitored and non-monitored sectors in the economy.\footnote{Although this is not an issue in this example, in general under aggregate shocks one may want to introduce costs to switch the monitored sectors. However, the point that the distribution of shocks is crucial for the determination of optimal monetary policies does not seem to be affected by such costs.}

4.3 Alternative monetary policies

Here we consider alternative monetary policies and show that the EMP dominates in terms of social welfare. Given the limited commitment friction, we only need to consider policies that are consistent with our monitoring technology. This has two implications. First, in terms taxation, inflation is the only possible tax for non-monitored meetings, and taxation in monitored meetings is possible but has to be voluntary in the sense that the only punishment is to give a (presumably bad) record that is observable only in monitored meetings. In terms of subsidies, the mechanism can subsidize recorded trades, as done in EMP, or it can subsidize interest on money through deflation.\footnote{In particular, as there is no record of agents’ money holdings, the mechanism cannot pay interest on money conditional on agents’ money holdings. Examples of such schemes include the interest-bearing money in Andolfatto (2010) and the progressive and regressive schemes in Wallace (2013). The mechanism specifies the amounts of money transferred to an agent as a function of the agent’s money-holdings. As pointed out in Sanches and Gomis-Porqueras (2013), such schemes require some monitoring. Indeed, the mechanism has to keep track of each agent’s transfer to avoid double withdraw. Moreover, as pointed out}
we consider only two types of alternative policies: inflation through lump-sum transfers and deflationary policies that use taxes from monitored meetings.

**Lump-sum transfers of money**

The most commonly discussed monetary policy in the literature is the lump-sum transfer of money. Here we assume that, as typically in the literature, that newly created money is given to all buyers with equal amount in a lump-sum fashion before the CM opens.\(^{20}\)

Let \( \pi \) be the net money growth rate and let \( M_t \) be the average money holdings at the beginning of period \( t \). Then, the policy gives each buyer \( \pi M_t \) units of money at the beginning of period-\( t \) CM in a lump-sum fashion. The following theorem shows that such policy shrinks the set of implementable outcomes even against a constant money supply.

**Theorem 4.3** *(Implementability under lump-sum transfers)*. Assume imperfect monitoring and let the net money growth rate be \( \pi \) with lump-sum transfers. Let \( \zeta = (1 + \pi - \delta)/\delta \geq \rho \). Then, an allocation, \( L = [(x_1, x_2), (z_1, z_2)] \), is implementable under \( \pi > 0 \) only if it satisfies (3), (5), and

\[
-\rho z_1 - \zeta z_2 + [u_1(x_1) - z_1] + \sigma_2 [u_2(x_2) - z_2] \geq 0, \tag{13}
\]

or

\[
-\zeta z_1 - \rho z_2 + [u_1(x_1) - z_1] + \sigma_2 [u_2(x_2) - z_2] \geq 0. \tag{14}
\]

Compared against Lemma 3.1, the conditions in Theorem 4.3 are more restrictive: (13) and (14) are more restrictive than (2) while (3) and (5) are the same. As in Lemma 3.1, (4) is necessary under \( C = \{1\} \) but may not be necessary under \( C = \{2\} \). Because (4) is never binding for constrained-efficient allocations subject to constant money supply, these results imply that such inflation is never optimal even against a constant money supply.

**Deflationary monetary policy**

Now we turn to interventions that use taxes or fees. To be consistent with our environment, we assume that the mechanism may tax the agent only if the agent is in a monitored meeting and decides to engage in a monitored trade. Such taxes can be thought of as a fee to use the monitoring technology. In particular, this implies that lump-sum taxes are not feasible. Because the mechanism can only punish agents conditional on records, it is without loss of generality to assume that the only punishment for not paying the fees is

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\(^{20}\)Because of the lump-sum nature, the assumption that only buyers receive the transfers and the assumption that every buyer receive the same amount are not crucial. For example, the result would be exactly the same if the money transfer is given randomly in a lump-sum fashion. In fact, this random transfer would be more consistent with our environment because it does not require the money to be sent to each agent, which may require some monitoring on agents’ money holdings.

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to give the individual a bad credit record. Using these fees to subsidise credit trades will be either neutral or harmful. Because monetary trades are anonymous, to only way to subsidise that sector is to use the fees to buy back money and hence provide interest on money holdings. We call such interventions deflationary monetary policy (DMP).

Consider a mechanism where round-DM has monitored meetings. Then buyers may issue debt that is recorded for meetings where the technology is available. The DMP sets a fee (in terms of the CM good), \( \eta \), on the use of the monitoring technology and then buy back money with those revenues. Therefore, if a buyer \( b \) chooses to accept a monitored trade at period \( t \), the buyer has to pay extra \( \eta \) to keep his good record. Let \( \tau \) be the net money contraction rate. Thus, for each \( t \), \( M_{t+1} = (1 - \tau)M_t \) and we focus only on proposals with constant real balances, that is, \( \phi_{t+1} = \phi_t/(1 - \tau) \). Then, if \( \beta \) measure of buyers use the monitored trades, feasibility requires a corresponding deflation rate \( \tau \) such that

\[
\beta \eta = \tau \phi_t M_{t-1}. \tag{15}
\]

An allocation is implementable with \( (\eta, \tau) \) that satisfies (15) if there exists some proposal \( \langle \mathcal{P}, (\eta, \tau) \rangle \) such that the allocation is consistent with the simple equilibrium outcome under such proposal. Note that if an outcome is implementable under a constant money supply, it is implementable with DMP. We have the following theorem.

**Theorem 4.4** (Deflationary Monetary Policy). Assume imperfect monitoring. An allocation, \( \mathcal{L} = [(x_1, x_2), (z_1, z_2)] \), is implementable with DMP only if either

(i) it satisfies (2), (3), and (5), or

(ii) it satisfies (3) and

\[
\frac{\sigma_2 + \rho}{(1 + \rho)\sigma_2} \{-\rho z_1 + [u_1(x_1) - z_1]\} + \{-\rho z_2 + \sigma_2[u_2(x_2) - z_2]\} \geq 0, \tag{16}
\]

\[
-\rho z_2 + \sigma_2[u_2(x_2) - z_2] \geq 0. \tag{17}
\]

The necessary conditions in Theorem 4.4 are almost sufficient. What is missing is the pairwise core requirement, which may be required in some cases. In particular, to implement the proposed allocation under \( C = \{1\} \) always requires (4), but not so under \( C = \{2\} \). Because the DMP can use \( C = \{2\} \) to implement an allocation that does not satisfy (5) while EMP can only use \( C = \{1\} \) for such allocations, there may be allocations that are not implementable with EMP due to (4) but are implementable with DMP. However, (4) never binds for constrained-efficient allocations.\(^{21}\) Ignoring the pairwise core requirement, (4), Theorem 4.4 shows that implementation with DMP is more restrictive than with EMP. Indeed, any allocation satisfies (4) and conditions in Theorem 4.4 (i) can also be implemented with constant money supply, while any allocation that satisfies (4) and conditions in Theorem 4.4 (ii), according to Theorem 4.1 (i), can also be implemented with EMP under \( C = \{1\} \). Notice that the conditions (16) and (17) are more restrictive

\(^{21}\)We can only show this result for sufficiently high \( \sigma_1 \)'s, and hence, theoretically speaking, DMP may be useful for low \( \sigma_1 \)'s. However, in all our numerical examples, the PC requirement never binds, even for fairly small \( \sigma_1 \)'s.
than (2). As a result, one can show that unless the DMP can implement the first-best, EMP strictly dominates DMP generically.

The crucial factor that leads EMP to dominate DMP is that EMP, by using inflation, taxes all agents while it only subsidise a subset of agents who engage credit trades. In contrast, DMP can only tax a subset of agents (those who engage in credit trades), but it has to subsidise all agents as monetary trades are anonymous. Our result demonstrates that unconventional policies are efficient in reallocating liquidities.

5 Concluding Remarks

Here we discuss the robustness of our results to some assumptions and some possible extensions.

5.1 Alternative meeting patterns

General limited monitoring

In the main text we only consider three cases: $\ell = 0$, $\ell = 1$, and $\ell = 2$. Our methodology and results, however, can be generalized to cover other cases as well. In particular, because we can always design the trading mechanism so that the terms of trades do not depend on buyers’ credit records, whatever is implementable under $\ell = 1$ is also implementable under $\ell > 1$. Moreover, as long as $\ell < 2$, money is still necessary to allow trades to happen in some meetings, the essentiality of money and credit and the essentiality of EMP remains. Moreover, for any discount factor such that EMP is essential under $\ell = 1$, by continuity, it remains essential for $\ell$ sufficiently close to one. We also remark here that in a model with only one DM stage (i.e., with $\sigma_2 = 0$), credit is not essential even with $\ell < 1$; see the Supplemental Appendix for a precise proof.

5.1.1 Meeting probabilities with $\sigma_1 < 1$

Most of our results extend to the case where $\sigma_1 < 1$. First note that the first-best allocation is independent of $\sigma_1$. Hence, using the same logic of Theorem 4.2, we can show that for any $\sigma_1 \leq 1$, generically, there exists a $\rho^0 > \rho^M$ such that for all $\rho \in (\rho^M, \rho^0]$, the first-best is implementable only with EMP and hence the optimal inflation rate is positive. Moreover, by continuity, we can show that the optimality of EMP extends to lower $\rho$’s as well: there exists $\rho^1 > \rho^0$ such that for all $\rho \in (\rho^0, \rho^1]$, the first-best is not implementable but the constrained-efficient allocation is only implementable with EMP and hence the optimal inflation rate is positive. As a result, for any $\sigma_1$, unless money alone implements the first-best, the EMP is generically optimal as long as the discount factor is not too low. Moreover, for any $\rho$, the EMP is generically essential for sufficiently high $\sigma_1$’s. See the Supplemental Appendix for precise statements and proofs. Under the functional form
\[ u_i(x_i) = x_i^\alpha, \quad a \in (0, 1), \] and \[ c_i(x_i) = x_i, \] setting \( a_1 = 0.99, \ a_2 = 0.97, \) and \( \sigma_2 = 0.95, \) we can show that for any \( \sigma_1 \geq 0.95, \) the corresponding \( \rho^1 \) is at least 30\%, for any \( \sigma_1 \geq 0.9, \) the corresponding \( \rho^1 \) is at least 14\%, and for any \( \sigma_1 \geq 0.8, \) the corresponding \( \rho^1 \) is at least 9\%.

**Meeting both seller types at both rounds**

Our results are also robust to other meeting patterns as well. In particular, one special feature in our model is that buyers can only meet sellers from \( S_i \) at stage \( i. \) However, our results are robust to other arrangements as well. In particular, we can accommodate the following setting. In round-1 DM, a buyer may meet a seller from \( S_1 \) or \( S_2 \) or none. The probability of a successful meeting is \( 2\sigma_1 \leq 1 \) and the probability of meeting a seller from \( S_j \) is \( \sigma_j \) for both \( j = 1, 2. \) For simplicity we assume that only buyers with a successful meeting at round-1 has a chance to meet a seller from a different sector at round-2, which happens with probability \( \sigma_2/\sigma_1 \leq 1. \) We also assume that a seller may meet at most one buyer at each period. Note that the ex ante probability of a buyer to meet a sector-\( j \) seller is \( \sigma = \sigma_1 + \sigma_2 \) for both \( j = 1, 2. \) We can show that under this setting the main result, Theorem 4.2 still hold when \( \sigma_1 = \sigma_2; \) the assymetric case is similar to the case where \( \sigma_1 < 1 \) in our main model. See the Supplemental Appendix for full details.

### 5.2 Multi-stage DM’s

We can also introduce many rounds of DM’s, say, \( T \) rounds. It is then straightforward to derive conditions for implementation of the first-best allocations using our techniques. In particular, for \( T \geq 3, \) it can be the case that both money and credit are necessary to circulate in any optimal mechanism when \( \ell < T, \) and the EMP may not be necessary for some parameter values. Nevertheless, EMP with positive inflation will still be optimal for a large set of parameters (among all feasible policies).

### 5.3 Other assets

In our model we only allow unsecured credit arrangements. In reality many credit arrangements involve both collateralized and unsecured elements. To take such arrangements seriously, it is necessary to introduce other assets, such as capital or nominal bonds, that would coexist with money as means-of-payments. This coexistence issue has been difficult to deal with. One possible route is provided by Hu and Rocheteau (2013, 2014), who show that coexistence of money and assets with higher rate-of-returns can coexist under the optimal monetary policy. We conjecture that, as long as credit or collateralized trades

\[22\] This assumption allows us to focus on symmetric allocations across different meeting patterns for different buyers, and it plays a very similar role as \( \sigma_1 = 1 \) in previous sections. Results in this new setting are robust to this assumption in the same way as results in previous sections are robust to the assumption \( \sigma_1 = 1. \)
still include some unsecured elements and hence face endogenous borrowing constraints, policy analogous to our EMP may still be beneficial. A related issue is that all debts are of short maturities in our model due to quasi-linearity. To have a model where long term debts are useful it seems that one has to relax the quasi-linearity assumption to give some room for the risk-sharing role of long-term debts, and that is beyond the scope of the current paper.

6 Appendix: Proofs

Proof of Theorem 3.1

(⇒) First we prove necessity. We have proved the necessity of (2) and (3) in the text. Now consider (4). Suppose that it does not hold and hence

\[ u_1(\hat{x}_1) - c_1(\hat{x}_1) > [u_1(x_1) - z_1] + \sigma_2[u_2(x_2) - z_2] + [z_1 - c_1(x_1)]. \]  

(18)

Note that the first two terms of the right side of (18) is the expected surpluses for the buyer (from the 2 DM’s) and the last term is the surplus for the round-1 seller. By (18) there exists \( \hat{z}_1 \in (0, z_1 + z_2] \) such that

\[ u_1(\hat{x}_1) - \hat{z}_1 > [u_1(x_1) - z_1] + \sigma_2[u_2(x_2) - z_2] \]  

and hence the buyer has a profitable deviation to offering \((\hat{x}_1, \hat{z}_1)\). Note that this deviation does not affect the buyer’s credit record as long as he repays \( \hat{z}_1 \) in the CM.

(⇐) Here we prove sufficiency. First we formulate the proposed trades on the equilibrium path. The debt limit is given by \( D = z_1 + z_2 \). Because money has no value, a trade \((x, z)\) only consists of output \( x \) and promise \( z \) and the proposed trade has no real balances as an argument; moreover, in state \( B \) the debt limit is 0 and is omitted. In state \( B \), the buyer always gets no trade: \( o_i(B) = (0, 0) \) for \( i = 1, 2 \). In state \( G \), \( o_1(G, D) = (x_1, z_1) \) and \( o_2(G, D - z_1) = (x_2, z_2) \). Because \( u_i(x_i) \geq z_i \geq c_i(x_i) \), both agents are willing to accept the proposed trade against no trade for both rounds. However, it remains to show that the buyer has no profitable deviating offers at both DM rounds, and we need to specify \( o_2(G, d) \) for any \( d \in [0, D] \).

Let \( \xi(d) = u_2(x_2) - z_2 \) if \( d \geq D - z_1 = z_2 \) and let \( \xi(d) = 0 \) if \( d < z_2 \). The value \( \xi(d) \) will be the buyer’s surplus in round-2 DM if his available debt limit is \( d \) when entering that round. Then, \( o_2(G, d) \) solves

\[
\max_{(x,y) \in \mathbb{R}_+ \times [0,d]} -c_2(x) + y \\
\text{s.t. } u_2(x) - y \geq \xi(d).
\]  

(19)

The solution to (19) exists for all \( d \in [0, D] \) and is unique. We first show that \( o_2(G, z_2) = (x_2, z_2) \) and then we show that there is no profitable deviating offer for the buyer at round 1. For the first claim, suppose, by contradiction, that \((x, y) \neq (x_2, z_2)\) is the solution to
However, because $z$ in the text. Consider (2). Note that in equilibrium the buyer has to hold at least from CM to the next CM is at most $(2)$. First we prove necessity. The necessity of (3) is clear. We proved the necessity for (5) in the text. Consider (2). Note that in equilibrium the buyer has to hold at least $c_2(x) + y > -c_2(x_2) + z_2$ and $u_2(x) - y \geq u_2(x_2) - z_2$

and hence

$$u_2(x) - c_2(x) > u_2(x_2) - c_2(x_2).$$

Because $u_2(x^*_2) \geq u_2(x_2) \geq c_2(x_2)$, it follows that $x > x_2$ and hence $y > z_2$, a contradiction. Thus, $o_2(G, z_2) = (x_2, z_2)$. This also implies that the buyer has no profitable deviating offer at round-2 DM. Finally, we show that the buyer has no profitable deviating offer at round-1 DM. Suppose, by contradiction, there exists such a profitable deviating offer, $(x, y)$, at round-1 DM. Then,

$$u_1(x) - y + \sigma_2 \xi(D - y) > u_1(x_1) - z_1 + \sigma_2 \xi(z_2) \text{ and } y - c_1(x) \geq z_1 - c_1(x_1),$$

and hence

$$u_1(x) - c_1(x) + \sigma_2 \xi(D - y) > u_1(x_1) - c_1(x_1) + \sigma_2 [u_2(x_2) - z_2].$$

(20)

Consider two cases.

(a) $y \leq z_1$. Then, $c_1(x) - c_1(x_1) \leq y - z_1 \leq 0$ and hence $x \leq x_1 \leq x_1^*$. Thus,

$$u_1(x) - c_1(x) + \sigma_2 \xi(D - y) \leq u_1(x_1) - c_1(x_1) + \sigma_2 \xi(D - z_1).$$

Note that because $y \leq z_1$, $\xi(D - y) = \xi(z_2)$, and this leads to a contradiction to (20).

(b) $y > z_1$. Then, $\xi(D - y) = 0$ and hence, by (20),

$$u_1(x) - c_1(x) > u_1(x_1) - c_1(x_1) + \sigma_2 [u_2(x_2) - c_2(x_2)].$$

However, because $y \leq D$, it follows that $u_1(x) - c_1(x) \leq u_1(\hat{x}_1) - c_1(\hat{x}_1)$, and this leads to contradiction to (4).

**Proof of Theorem 3.2**

$\Rightarrow$ First we prove necessity. The necessity of (3) is clear. We proved the necessity for (5) in the text. Consider (2). Note that in equilibrium the buyer has to hold at least $z_1 + z_2$ units of real balances. Following the equilibrium path, the buyer’s total payoff from CM to the next CM is at most

$$-(z_1 + z_2) + \delta \{u_1(x_1) + \sigma_2 u_2(x_2) + (1 - \sigma_2) z_2\} = \delta \{-\rho(z_1 + z_2) + [u_1(x_1) - z_1] + \sigma_2 [u_2(x_2) - z_2]\},$$

while deviating to holding zero balances in the CM the buyer can guarantees himself a zero payoff. Hence, (2) is necessary. The necessity of (4) follows exactly the same argument as in the proof of Theorem 3.1.

$\Leftarrow$ Here we prove sufficiency. First we formulate the proposed trades on the equilibrium path. The real balance buyers hold at the end of the CM is given by $z_1 + z_2$. Because there
is no credit meetings, a trade \((x, z)\) only consists of output \(x\) and money transfer \(z\) and the proposed trade has no debt limit or credit record as an argument. On equilibrium path, \(o_1(z_1 + z_2) = (x_1, z_1)\) and \(o_2(z_2) = (x_2, z_2)\). By standard arguments in the Lagos-Wright model, the CM value function is linear, that is, \(W(m) = m + W(0)\), where \(W(m)\) is the continuation value for a buyer entering the CM with \(m\) units of real balances. Because \(u_i(x_i) \geq z_i \geq c_i(x_i)\), both agents are willing to accept the proposed trade against no trade for both rounds. However, it remains to show that the buyer has no profitable deviating offers at both DM rounds and the buyer is willing to hold \(z_1 + z_2\) real balances to leave the CM, and we need to specify \(o_1(m)\) and \(o_2(m)\) for all \(m\).

Let \(\epsilon \in (0, z_2)\) be so small that
\[
\epsilon < \frac{1}{2} \min \left\{ \frac{c_1'(x_1)\sigma_2[u_2(x_2) - z_2]}{u_1'(x_1) - c_1'(x_1)}, \frac{\sigma_2[u_2(x_2) - z_2]}{\rho} \right\}. \tag{21}
\]

Let
\[
\xi(m) = \begin{cases} 
  u_2(x_2) - z_2 & \text{if } m \geq z_2; \\
  0 & \text{if } z \leq z_2 - \epsilon; \\
  \left[1 - \frac{z - m}{\epsilon}\right] [u_2(x_2) - z_2] & \text{if } m \in (z_2 - \epsilon, z_2).
\end{cases}
\]

Note that \(\xi\) is a piecewise linear continuous function. Then, \(o_2(m)\) solves
\[
\max_{(x, y) \in \mathbb{R}_+ \times [0, m]} -c_2(x) + y \quad \text{s.t. } u_2(x) - y \geq \xi(m). \tag{22}
\]

The solution to (22) exists for all \(m\) and is unique with the constraint binding. Moreover, following exactly the same arguments as in the proof of Theorem 3.1, \(o_2(z_2) = (x_2, z_2)\). This also implies that the buyer has no profitable deviating offer at round-2 DM.

Now we formulate \(o_1(m)\). Let \(\eta(m) = u_1(x_1) - z_1 + \sigma_2 \xi(z_2)\) if \(m \geq z_1 + z_2\) and let \(\eta(m) = \sigma_2 \xi(m)\) otherwise. For each \(m \in \mathbb{R}_+\), let \(o_1(m)\) be a solution to
\[
\max_{(x, y) \in \mathbb{R}_+ \times [0, m]} -c_1(x) + y \quad \text{s.t. } u_1(x) - y + \sigma_2 \xi(m - y) \geq \eta(m). \tag{23}
\]

Because \(\xi\) is continuous, a solution to (23) exists.

For any solution to the problem, the constraint is binding. If not, then either \(y < m\) and we can increase \(y\) slightly to increase the seller surplus without violating the constraint, or \(y = m\) and hence \(x > 0\) and we can decrease \(x\) slightly to increase the seller surplus without violating the constraint. Thus, although there may be multiple solutions, we can pick any one of them and we know that under \(o_1\), the expected buyer surpluses from the two DM rounds for a buyer leaving the CM with \(m\) units of real balances is \(\eta(m)\).

Now we show that when \(m = z_1 + z_2\), \((x_1, z_1)\) is a solution. Suppose, by contradiction, \((x, y)\) gives seller a higher surplus without violating the constraint. Hence,
\[
u_1(x) - y + \sigma_2 \xi(z_1 + z_2 - y) \geq u_1(x_1) - z_1 + \sigma_2 \xi(z_2) \quad \text{and} \quad y - c_1(x) > z_1 - c_1(x_1),
\]
and hence
\[ u_1(x) - c_1(x) + \sigma_2 \xi(z_1 + z_2 - y) > u_1(x_1) - c_1(x_1) + \sigma_2[u_2(x_2) - z_2]. \] (24)

Consider three cases.

(a) \( y \leq z_1 \). Then, the argument is exactly the same as in the proof of Theorem 3.1.

(b) \( y \geq z_1 + \epsilon \). Then, \( \xi(z_1 + z_2 - y) = 0 \) and we can obtain a contradiction to (4) as in the proof of Theorem 3.1.

(c) \( y \in (z_1, z_1 + \epsilon) \) and let \( \epsilon' = y - z_1 \). Then,
\[ \xi(z_1 + z_2 - y) = \left(1 - \frac{\epsilon'}{\epsilon}\right)[u_2(x_2) - z_2]. \]

However, because \( y - c_1(x) > z_1 - c_1(x_1) \),
\[ c'_1(x_1)[x - x_1] \leq c_1(x) - c_1(x_1) < y - z_1 = \epsilon'. \]

From the above two conditions and the definition of \( \epsilon \), (21),
\[ \frac{[u_1(x) - c_1(x)] - [u_1(x_1) - c_1(x_1)]}{[u'_1(x_1) - c'_1(x_1)](x - x_1)} < \frac{\epsilon'}{\epsilon} \sigma_2[u_2(x_2) - z_2] = \sigma_2[\xi(z_2) - \xi(z_1 + z_2 - y)], \]
a contradiction to (24). This also implies that the buyer has no profitable deviating offers at round-1 DM.

Finally, we show that at the CM it is optimal for the buyer to leave with \( z_1 + z_2 \) units of real balances. Now, a buyer who leaves with \( m \) units of real balances has the expected payoff
\[ -m + \delta [\eta(m) + m + W(0)] = \delta \{-\rho m + \eta(m) + W(0)\}. \]
Recall that \( \eta(m) = u_1(x_1) - z_1 + \sigma_2 \xi(z_2) \) if \( m \geq z_1 + z_2 \) and \( \eta(m) = \sigma_2 \xi(m) \) otherwise. We distinguish three cases.

(a) \( m \geq z_1 + z_2 \). Because \( \eta(m) \) is constant for all \( m \geq z_1 + z_2 \) but the cost of holding money increases with \( m \), any \( m > z_1 + z_2 \) is strictly dominated by \( m = z_1 + z_2 \).

(b) \( m \in (z_2, z_1 + z_2] \). Because \( \xi(m) \) is constant for \( m \geq z_2 \), any \( m \in (z_2, z_1 + z_2] \) is strictly dominated by \( z_2 \).

(c) \( m < z_2 \). Here we show that for any \( \epsilon' \in (0, \epsilon] \), \( z_2 - \epsilon' \) is strictly dominated by \( z_2 \). This will be the case if
\[ -\rho(z_2 - \epsilon') + \sigma_2 \xi(z_2 - \epsilon') < -\rho z_2 + \sigma_2 \xi(z_2), \]
which is equivalent to
\[ \rho \epsilon' < \sigma_2[\xi(z_2) - \xi(z_2 - \epsilon')] = \sigma_2 \frac{\epsilon'}{\epsilon}[u_2(x_2) - z_2], \]
which holds by (21). Moreover, for any \( m \leq z_2 - \epsilon \), it is strictly dominated by zero as \( \xi(m) \) is constant below \( z_2 - \epsilon \).

Given the above discussion, to show that holding \( z_1 + z_2 \) is optimal, it is sufficient to show that it is better than \( z_2 \) and 0; the first follows from (5) and the second follows from (2).

**Proof of Lemma 3.1**

We show the necessity of (5). Suppose first that \( C = \{1\} \). Let \( z_{1,m} \) and \( z_{1,c} \) be the transfer of real balances and promise to pay in round-1 DM. Let \( z \) be the real balances the buyer has to hold when leaving CM. Then, the buyer may deviate to repaying nothing and holding only \( z - z_{1,m} \) units of real balances but still accept equilibrium trades at round-2 DM’s in all future periods. This deviation is profitable if

\[
-(z - z_{1,m}) + \frac{1}{\rho} \sigma_2[u_2(x_2) - z_2] > -(z + z_{1,c}) + \frac{1}{\rho} \{[u_1(x_2) - z_1] + \sigma_2[u_2(x_2) - z_2]\},
\]

that is, if

\[
-\rho(z_{1,c} + z_{1,m}) + [u_1(x_2) - z_1] < 0.
\]

Hence, (5) is necessary to prevent this profitable deviation. Now suppose that \( C = \{2\} \). Then, the round-1 DM trade has to be financed by transfer of real balances. (5) is then necessary for buyers not to skip round-1 DM trades.

**Proof of Theorem 4.1**

**Proof of (i) \( \Rightarrow \)** We now prove necessity. The necessity of (3) and (5) follows previous arguments. To prove the necessity of (2), consider an arbitrary EM policy, \( (k, \pi) \) that satisfies (6). This implies (2).

\( \Leftrightarrow \) Here we prove sufficiency. First we formulate the EM policy. If \(-\rho z_1 + [u_1(x_1) - z_1] \geq 0\), then we set \( k = \pi = 0 \). Otherwise, the EM policy is such that

\[
-\rho(z_1 - k) + [u_1(x_1) - (z_1 - k)] = 0,
\]

that is,

\[
k = z_1 - \frac{u_1(x_1)}{1 + \rho} \in (0, z_1). \tag{25}
\]

We will set buyers to hold \( z_2 \) real balances. Feasibility then implies \( k = \pi z_2 \), or \( \pi = k/z_2 \). The proposal is given by: \( C = \{1\}, \phi_t M_{t-1} = z_2 \) for all \( t \), and the debt limit is given by \( D = z_1 - k \). It remains to specify the proposed trades.

We start with \( o_2 \). Because the second round is a non-credit meeting, \( o_2 \) only depends on the buyer’s announcement of real balances, \( m \). Let \( \epsilon \in (0, z_2) \) be so small that

\[
\epsilon < \frac{1}{2} \min \left\{ \frac{c'_1(x_1) \sigma_2[u_2(x_2) - z_2]}{u'_1(x_1) - c'_1(x_1)}, \frac{\sigma_2[u_2(x_2) - z_2]}{\rho + (1 + \rho)\pi} \right\}. \tag{26}
\]
Note that condition (26) is exactly the same as (21) except the term $\rho$, which is replaced by $[\rho + (1 + \rho)\pi]$, reflecting the difference in the cost of holding money. Let

$$
\xi(m) = \begin{cases} 
    u_2(x_2) - z_2 & \text{if } m \geq z_2; \\
    0 & \text{if } m \leq z_2 - \epsilon; \\
\frac{1 - \frac{z_2 - m}{\epsilon}}{1 - \frac{z_2 - m}{\epsilon}} [u_2(x_2) - z_2] & \text{if } m \in (z_2 - \epsilon, z_2). 
\end{cases}
$$

Note that $\xi$ is a piecewise linear continuous function. Then, $o_2(m)$ solves

$$
\max_{(x,y)\in \mathbb{R}_+ \times [0,m]} -c_2(x) + y \\
\text{s.t. } u_2(x) - y \geq \xi(m).
$$

The solution to (27) exists for all $m$ and is unique with the constraint binding. Moreover, following exactly the same arguments as in the proof of Theorem 3.1, $o_2(z_2) = (x_2, z_2)$. This also implies that the buyer has no profitable deviating offer at round-2 DM.

Now we formulate $o_1(m, r, d)$. Note that $d$ can only take two values: $d = D$ when $r = G$ and $d = 0$ when $r = B$. Let $\eta(m, G) = u_1(x_1) - (z_1 - k) + \sigma_2 \xi(m)$ and let $\eta(m, B) = \sigma_2 \xi(m)$. $\eta$ is continuous as $\xi$ is. When $r = G$, for each $m \in \mathbb{R}_+$, let $o_1(m, G, D)$ be a solution to

$$
\max_{(x,y,z)\in \mathbb{R}_+ \times [0,D+k] \times [0,m]} -c_1(x) + y_c + y_m \\
\text{s.t. } u_1(x) - \max(y_c - k, 0) - y_m + \sigma_2 \xi(m - y_m) \geq \eta(m, G).
$$

When $r = B$, for each $m \in \mathbb{R}_+$, let $o_1(m, B, 0)$ be a solution to

$$
\max_{(x,y)\in \mathbb{R}_+ \times [0,m]} -c_1(x) + y_m \\
\text{s.t. } u_1(x) - y_m + \sigma_2 \xi(m - y_m) \geq \eta(m, B).
$$

Because $\xi$ is continuous, a solution to (28) and (29) exists. For any solution to the problem, the constraint is binding, following exactly the same arguments as in the proof of Theorem 3.1. Thus, although there may be multiple solutions, we can pick any one of them and we know that under $o_1$, the expected buyer surpluses from the two DM rounds for a buyer leaving the CM with $m$ units of real balances is $\eta(m, r)$ for both $r = G, B$. Note that, under $r = B$, the EM does not purchase any IOU from the buyer.

Now we show that when $m = z_2$, $(x_1, z_1, 0)$ is a solution to (28). Suppose, by contradiction, $(x, y_c, y_m)$ gives seller a higher surplus without violating the constraint. We may assume that $y_c \geq k$, for otherwise we can increase $y_c$ to give the seller even a higher surplus without changing the buyer’s. Hence,

$$
u_1(x) - (y_c - k) - y_m + \sigma_2 \xi(z_2 - y_m) \geq u_1(x_1) - (z_1 - k) + \sigma_2 \xi(z_2), \quad y_m + y_c - c_1(x) > z_1 - c_1(x_1),$$

and hence

$$u_1(x) - c_1(x) + \sigma_2 \xi(z_2 - y_m) > u_1(x_1) - c_1(x_1) + \sigma_2 [u_2(x_2) - z_2].$$  

(30)
Consider two cases.

(a) $y_m \geq \epsilon$. Then, $\xi(z_2 - y) = 0$ and we can obtain a contradiction to (4) as in the proof of Theorem 3.1.

(b) $y_m \in (0, \epsilon)$. Then,

$$\xi(z_2 - y_m) = \left(1 - \frac{y_m}{\epsilon}\right) [u_2(x_2) - z_2].$$

However, because $y_m + y_c - c_1(x) > z_1 - c_1(x_1)$ and because $y_c \leq z_1$,

$$c_1'(x_1)[x - x_1] \leq c_1(x) - c_1(x_1) < y_m.$$

From the above two conditions and the definition of $\epsilon$, (26),

$$\left[u_1(x) - c_1(x)\right] - \left[u_1(x_1) - c_1(x_1)\right] \leq \left[u_1'(x_1) - c_1'(x_1)\right](x - x_1) < \frac{\left[u_1'(x_1) - c_1'(x_1)\right]y_m}{c_1'(x_1)}$$

$$< \frac{y_m}{\epsilon} [u_2(x_2) - z_2] = \sigma_2 [\xi(z_2) - \xi(z_2 - y_m)],$$

a contradiction to (30). This also implies that the buyer has no profitable deviating offers at round-1 DM.

Now we show that the following strategies form a simple equilibrium. All agents respond with yes to the proposed trades and buyers offer the proposed trades, on both equilibrium and off-equilibrium paths. Buyers under state $G$ always repay their IOU’s up to $D$, and buyers under state $B$ never repay anything. All buyers leave the CM with $z_2$ units of real balances.

By construction, the outcome functions $o_1(m, r, d)$ and $o_2(m)$ ensure that buyers and sellers always prefer to say yes, buyers are willing to offer the proposed trade, and buyers always announce their money holdings truthfully. Moreover, because $\eta(m, G) - \eta(m, B) = u_1(x_1) - (z_1 - k)$ is independent of $m$, we can write the continuation values as follows. Let $V_B^c = 0$,

$$V_G^c = \frac{1}{1 - \delta} [u_1(x_1) - (z_1 - k)],$$

$$V(m) = \sigma_2 \xi(m) + W(0),$$

and

$$W(0) = -(1 + \pi)z_2 + \frac{\delta}{1 - \delta} \left\{ \sigma_2 u_2(x_2) + (1 - \sigma_2)z_2 - (1 + \pi)z_2 \right\}.$$

Then, the continuation value for a buyer entering DM with credit record $r$ and real balances $m$ is $V_r^c + V(m)$. This implies that the choice of real balances in the CM and the repayment decision for the debts are independent from each other.

Here we show that buyers are willing to leave the CM with $z_2$ units of real balances. Now, a buyer who leaves with $m$ units of real balances has the expected payoff (regardless of the amount of repayment to his debts)

$$-(1 + \pi)m + \delta [\eta(m, r) + m + W(0)] = \delta \left\{ -\rho + (1 + \rho)\pi \right\} m + \eta(m, r) + W(0).$$
Recall that
\[ \eta(m, r) = \begin{cases} 1_{r \in G}[u_1(x_1) - z_1 + k] + \sigma_2 \xi(z_2) & \text{if } m \geq z_2, \\ 1_{r \in G}[u_1(x_1) - z_1 + k] + \sigma_2 \xi(m) & \text{otherwise.} \end{cases} \]
Because \( \eta(m, r) \) is constant for all \( m \geq z_2 \) but the cost of holding money increases with \( m \), any \( m > z_2 \) is strictly dominated by \( m = z_2 \).

Here we show that for any \( \epsilon' \in (0, \epsilon] \), \( z_2 - \epsilon' \) is strictly dominated by \( z_2 \). This will be the case if
\[ -[\rho + (1 + \rho) \pi](z_2 - \epsilon') + \sigma_2 \xi(z_2 - \epsilon') < -[\rho + (1 + \rho) \pi]z_2 + \sigma_2 \xi(z_2), \]
which is equivalent to
\[ [\rho + (1 + \rho) \pi]\epsilon' < \sigma_2 [\xi(z_2) - \xi(z_2 - \epsilon')] = \sigma_2 \frac{\epsilon'}{\epsilon} [u_2(x_2) - z_2], \]
which holds by (26). Moreover, for any \( m \leq z_2 - \epsilon \), it is strictly dominated by zero as \( \xi(m) \) is constant below \( z_2 - \epsilon \). Thus, to show that holding \( z_2 \) is optimal, it is sufficient to show that it is better than 0, and this will be the case if and only if
\[ -[\rho + (1 + \rho) \pi]z_2 + \sigma_2 [u_2(x_2) - z_2] \geq 0. \]
Using \( \pi z_2 = k = z_1 - \frac{u_1(x_1)}{1 + \rho} \), we can rewrite this inequality as
\[ -\rho(z_1 + z_2) + [u_1(x_1) - z_1] + \sigma_2 [u_2(z_2) - z_2] \geq 0, \]
which corresponds to (2).

Finally, we show that a buyer under state \( G \) has incentive to repay \( D = z_1 - k \) whenever his IOU is at least \( z_1 \) (where the EM pays \( k \) for him). Because the buyer’s payoff are affected by his record only through \( \eta \) and the buyer holds \( z_2 \) units of real balances when leaving the CM regardless of his records, he has incentive to repay \( D \) if and only if
\[ -(z_1 - k) + \frac{\delta}{1 - \delta} \eta(z_2, G) \geq \frac{\delta}{1 - \delta} \eta(z_2, B), \]
which is equivalent to
\[ -\rho(z_1 - k) + u_1(x_1) - (z_1 - k) \geq 0. \]
This holds by (25).

**Proof of (ii) \((\Leftarrow)\)** We start sufficiency. First we formulate the EM policy. If \( -\rho z_2 + [u_2(x_2) - z_2] \geq 0 \), then set \( k = \pi = 0 \). Otherwise, let
\[ k = z_2 - \frac{\sigma_2}{\sigma_2 + \rho} u_2(x_2) \in (0, z_2). \tag{31} \]
We will buyers to hold \( z_1 \) real balances. Feasibility then implies \( \sigma_2 k = \pi z_1 \), or \( \pi = \sigma_2 k / z_1 \). The proposal is given by: \( C = \{ 2 \} \), \( \phi_t M_{t-1} = z_1 \) for all \( t \). The debt limit is given by \( D = z_2 - k \).
Now we formulate the proposed trades. We start with $o_2$. Since the second round is a credit meeting, $o_2$ depends on the buyer’s announcement of real balances $m$ and on his record $r$. Note that the available credit limit can only take two values: $d = D$ when $r = G$ and $d = 0$ when $r = B$. Let $o_2(m, G, D)$ be a solution to

$$\max_{(x, y_c, y_m) \in \mathbb{R}_+ \times [0, D+k] \times [0, m]} -c_2(x) + y_c + y_m$$

s.t. $u_2(x) - \max(y_c - k, 0) - y_m \geq u_2(x_2) - (z_2 - k)$.  \hspace{1cm} (32)

Let $o_2(m, B, 0)$ be a solution to

$$\max_{(x, y_m) \in \mathbb{R}_+ \times [0, m]} -c_2(x) + y$$

s.t. $u_2(x) - y_m \geq 0$.  \hspace{1cm} (33)

The solutions to (32) and (33) exist and are unique. Moreover, the constraints are always binding. The fact that $o_2(0, G, D) = (x_2, z_2)$ follows from $x_2 \leq x_2^*$.  

We now move to $o_1$. Because the first round is a non-credit meeting, $o_1$ only depends on the buyer’s announcement of real balances. Let $\eta(m) = u_1(x_1) - z_1$ if $m \geq z_1$ and let $\eta(m) = 0$ otherwise. Let $o_1(m)$ be a solution to

$$\max_{(x, y) \in \mathbb{R}_+ \times [0, m]} -c_1(x) + y$$

s.t. $u_1(x) - y \geq \eta(m)$.  \hspace{1cm} (34)

The solutions to (34) exist and are unique. Moreover, the constraint on the reservation utility of the buyer is always binding. The fact that $o_1(z_1) = (x_1, z_1)$ follows from $x_1 \leq x_1^*$.  

Now we show that the following strategies form a simple equilibrium. All agents respond with yes to the proposed trades and buyers offer the proposed trades, on both equilibrium and off-equilibrium paths. Buyers under state $G$ always repay their IOU’s up to $D$, and buyers under state $B$ never repay anything. All buyers leave the CM with $z_1$ units of real balances.

The proposed trades $o_1(m)$ and $o_2(r, m, d)$ ensure that buyers and sellers always prefer to say yes, buyers are willing to offer the prosed trades, and buyers always announce their money holdings truthfully. Moreover, we can write the continuation values as follows. Let $V_B^c = 0$,

$$V_G^c = \frac{1}{1-\delta} \sigma_2 [u_2(x_2) - (z_2 - k)]$$

and $V_B^c = 0$,

$$V(m) = \eta(m) + W(0),$$

and

$$W(0) = -(1 + \pi)z_1 + \frac{\delta}{1-\delta} \left\{ u_1(x_1) - (1 + \pi)z_1 \right\}.$$  

Then, the continuation value for a buyer entering DM with credit record $r$ and real balances $m$ is $V_r^c + V(m)$. This implies that the choice of real balances in the CM and the repayment decision for the IOU’s are independent from each other.
Now, consider the incentive of a buyer to repay his debt up to $D$ under state $G$. For a buyer in the CM under state $G$, he has incentive to repay $D$ if and only if

$$-(z_2 - k) + \delta V^c_G \geq \delta V^c_B,$$

that is,

$$-\rho(z_2 - k) + \sigma_2[u_2(x_2) - (z_2 - k)] \geq 0,$$

which holds with equality by (31).

We now show that the buyer has incentive to carry $z_1$ real balances. A buyer who leaves CM with $m$ units of real balances has the expected payoff (regardless of his records or the amount of repayment to his debts)

$$-(1 + \pi)m + \delta [\eta(m) + m + W(0)] = \delta \{ -[\rho + (1 + \rho)\pi]m + \eta(m) + W(0) \}.$$

Note that $\eta(m)$ is constant for all $m \geq z_2$ and is constant for all $m \in [0, z_2)$. Thus, we only need to show that bringing $z_2$ is better than zero, which will be the case if and only if

$$-[\rho + (1 + \rho)\pi]z_1 + u_1(x_1) - z_1 \geq 0.$$

Using $\pi z_1 = \sigma_2 k = \sigma_2 z_2 - \sigma_2 \frac{\sigma_2}{\sigma_2 + \rho}u_2(x_2)$, we can rewrite this inequality as

$$\{ -\rho z_1 + u_1(x_1) - z_1 \} + \frac{(1 + \rho)\sigma_2}{\sigma_2 + \rho} \{ -\rho z_2 + \sigma_2 [u_2(x_2) - z_2] \} \geq 0,$$

which corresponds to (7).

($\Rightarrow$) We now prove necessity. As in the case, of $C = \{1\}$, buyers are indifferent between repaying and not repaying the debt when $k$ satisfies (31). Thus, we cannot implement $L$ for lower values of $k$, as buyers would have no incentive to keep a record $G$. In turn, for higher values of $k$, we would relax the incentives of buyers to keep a record $G$ but we reduce the incentives of buyers to participate in the first DM round. This implies that there is no EM policy which can do better than the chosen one. Since the chosen EM policy requires (7), this condition is necessary. In turn, (3) is necessary, otherwise sellers would not be willing to participate. Finally, (5) is necessary otherwise the buyer would never carry $z_1$ real balances.$\square$

### 6.0.1 Proof of Lemma 4.1

By definition, the allocation $(x_1^{C_2}, x_2^{C_2})$ is implementable with EM under $C = \{2\}$. Next, we show that the allocation $(x_1^{C_1}, x_2^{C_1})$ is implementable with EM under $C = \{1\}$. It suffices to show that it satisfies (4). Suppose, by contradiction, that for $\hat{x}_i = \min \{ x_1^*, c_i^{-1}(z_1 + z_2) \}$ with $z_i = c_i(\hat{x}_i^{C_1})$, $i = 1, 2$, we have

$$u_1(x_1^{C_1}) - c_1(x_1^{C_1}) + \sigma_2[u_2(x_2^{C_1}) - z_2] < u_1(\hat{x}_1) - c_1(\hat{x}_1).$$

(35)

Because $(x_1^{C_1}, x_2^{C_1})$ satisfies (2), (35) implies that the pair $(\hat{x}_1, 0)$ satisfies (2) as well. Note that $c_1(\hat{x}_1) \leq z_1 + z_2$. But (35) implies that it generates a higher welfare than $(x_1^{C_1}, x_2^{C_1})$, which is a contradiction. Therefore, $(x_1^{C_1}, x_2^{C_1})$ is implementable with EM under $C = \{1\}$. $\square$
a contradiction. Thus, \((x_1^{C_1}, x_2^{C_1})\) satisfies (4) and hence is implementable with \(C = \{1\}\) and with EM.

Finally, because, by Lemma 3.1 and Theorem 4.1, the set of implementable allocations with EM includes all allocations implementable under constant money supply, the constrained-efficient allocation is either \((x_1^{C_1}, x_2^{C_1})\) or \((x_1^{C_2}, x_2^{C_2})\).

**Proof of Theorem 4.2**

We consider two cases.

(i) Suppose that \((x_1^{C_1}, x_2^{C_1})\) does not satisfy (5), and hence is not implementable with constant money supply. If \((x_1^{C_1}, x_2^{C_1})\) is a constrained efficient allocation, then EMP is necessary to implement it. Otherwise, \((x_1^{C_2}, x_2^{C_2})\) achieves a better welfare than \((x_1^{C_1}, x_2^{C_1})\), while a constant money supply can only never achieve a welfare higher than that of \((x_1^{C_1}, x_2^{C_1})\). Then, the constrained efficient allocation can only be implemented with EM but not with constant money supply.

(ii) Suppose that \((x_1^{C_1}, x_2^{C_1})\) is implementable with constant money supply and has the maximum welfare among all allocations that can be implemented by a constant money supply. Then, (5) holds, i.e., \(-\rho c_1(x_1^{C_1}) + [u_1(x_1^{C_1}) - c_1(x_1^{C_1})] \geq 0\). Also, \(\rho > \rho_M\) implies \((x_1^{C_1}, x_2^{C_1}) \neq (x_1^*, x_2^*)\). This also implies that constraint (2) is binding at \((x_1^{C_1}, x_2^{C_1})\). Thus, if \(-\rho c_1(x_1^{C_1}) + [u_1(x_1^{C_1}) - c_1(x_1^{C_1})] = 0\), then \(-\rho c_2(x_2^{C_1}) + \sigma_2[u_2(x_2^{C_1}) - c_2(x_2^{C_1})] = 0\). That is, \((x_1^{C_1}, x_2^{C_1}) = (x_1^*, x_2^*)\), a contradiction. Therefore,

\[
u_1(x_1^{C_1}) - (1 + \rho)c_1(x_1^{C_1}) > 0 \]

This inequality, together with \(\sigma_2 < 1\) and (2) for \((x_1^{C_1}, x_2^{C_1})\), implies

\[
[u_1(x_1^{C_1}) - (1 + \rho)c_1(x_1^{C_1})] + \frac{(\rho + 1)\sigma_2}{\rho + \sigma_2}[\sigma_2 u_2(x_2^{C_1}) - (\sigma_2 + \rho)c_2(x_2^{C_1})] > 0.
\]

Because \(x_1^{C_1} < x_1^*\), there exists \(\epsilon > 0\) such that \(x_1' = x_1^{C_1} + \epsilon < x_1^*\) and

\[
u_1(x_1') - (1 + \rho)c_1(x_1') > 0,
\]

\[
[u_1(x_1') - (1 + \rho)c_1(x_1')] + \frac{(\rho + 1)\sigma_2}{\rho + \sigma_2}[\sigma_2 u_2(x_2^{C_1}) - (\sigma_2 + \rho)c_2(x_2^{C_1})] > 0.
\]

By Theorem 4.1 (ii), \((x_1', x_2^{C_1})\) is implementable with EM, but it has a strictly higher welfare than \((x_1^{C_1}, x_2^{C_1})\). Thus, the constrained efficient allocation is \((x_1^{C_2}, x_2^{C_2})\), and it is not implementable without EMP. \(\Box\)

**Proof of Theorem 4.3**

Suppose that \(L = [(x_1, x_2), (z_1, z_2)]\) is implementable under \(\pi > 0\). We consider two cases.

(i) \(C = \{1\}\). Because the buyer can always skip round-1 DM, the necessity of (5) follows exactly the same proof as in Lemma 3.1. Because only money can be used to finance
trades in round-2 DM and hence buyers have to hold at least $z_2$ units of real balances in equilibrium and repay $z_1$, (13) is necessary to avoid buyers from not participating the whole scheme. Note that buyers can obtain the lump-sum transfer independent of his behavior.

(ii) $C = \{2\}$. Because the buyer can always skip round-1 DM and because only money can be used to finance the round-1 consumption, (5) with $r$ replaced by $\zeta > r$ is necessary. Because buyers have to hold at least $z_1$ units of real balances in equilibrium and repay $z_2$, (14) is necessary to avoid buyers from not participating the whole scheme. Note that buyers can obtain the lump-sum transfer independent of his behavior.

Proof of Theorem 4.4

Suppose that $\mathcal{L}$ is implementable with DMP. We consider two cases.

(i) $C = \{1\}$. Because the buyer can always skip the round-1 DM, (5) is necessary. Let $(z_{1,c}, z_{1,m})$ be the composition of payments by buyers in round-1 DM with $z_1 = z_{1,c} + z_{1,m}$. In order to avoid the buyer staying at the autarky, the equilibrium continuation payoff for a buyer at the beginning of the CM has to be at least zero, that is,

$$-z_{1,c} - \eta - (1 - \tau)(z_{1,m} + z_2) + \frac{1}{\rho} \{u_1(x_1) - z_{1,c} - (1 - \tau)z_1 + \sigma_2[u_2(x_2) - z_2] + \tau z_2\} \geq 0.$$  

By (15), $\tau(z_{1,m} + z_2) = \eta$, and the above inequality reduces to (2). Finally, (4) is still necessary.

(ii) $C = \{2\}$. Let $(z_{2,c}, z_{2,m})$ be the composition of payments by buyers in round-1 DM with $z_2 = z_{2,c} + z_{2,m}$. In order to avoid the buyer staying at the autarky, the equilibrium continuation payoff for a buyer at the beginning of the CM has to be at least zero, that is,

$$-z_{2,c} - \eta - (1 - \tau)(z_{2,m} + z_1) + \frac{1}{\rho} \{u_1(x_1) - z_{1,c} + \sigma_2[u_2(x_2) - z_2 - \eta] + \tau(z_{2,m} + z_1)\} \geq 0.$$  

By (15), $\tau(z_{2,m} + z_1) = \sigma_2 \eta$, and the above inequality reduces to

$$-z_2 - z_1 - (1 - \sigma_2) \eta + \frac{1}{\rho} \{u_1(x_1) - z_1 + \sigma_2[u_2(x_2) - z_2]\} \geq 0.$$  

Because $\eta \geq 0$, this implies (2). If (5) holds, then we have case (i) in the theorem. Suppose that (5) does not hold. Then, by (36), (17) holds with strict inequality. Moreover, buyers do not skip round-1 meetings only if

$$-\rho z_1 + (1 + \rho)\tau z_1 + [u_1(x_1) - z_1] \geq 0,$$

that is, only if

$$\tau z_1 \geq \frac{1}{1 + \rho} \{\rho z_1 - [u_1(x_1) - z_1]\}.$$  

This gives a lower bound on $\tau$ and hence on $\eta$, and plug in this lower bound into (36) we obtain (16). $\square$
References


