Coordinating Business Cycles*

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Abstract

We develop a quantitative theory of business cycles with coordination failures. Because of a standard aggregate demand externality, firms want to coordinate production. The presence of a non-convex capacity decision generates multiple equilibria under complete information. We use a global game approach to show that, under incomplete information, the multiplicity of equilibria disappears to give rise to a unique equilibrium with two stable steady states. The economy exhibits coordination traps: after a negative shock of sufficient size or duration, coordination on the good steady state is harder to achieve, leading to quasi-permanent recessions. In our calibration, the coordination channel improves on the neoclassical growth model in terms of business cycle asymmetries and skewness. The model also accounts for features of the 2007-2009 recession and its aftermath. Government spending is harmful in general as the coordination problem magnifies the crowding out. It can, however, increase welfare — without nominal rigidities — when the economy is about to transition to the bad steady state. Simple subsidies implement the efficient allocation.

JEL Classifications: E32, D83, E62

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1 Introduction

Economies are complex systems of interactions between agents in which coordination plays a central role. In typical neoclassical models, perfect coordination is achieved by the means of a well-functioning price system. An enduring tradition in macroeconomics has, however, questioned this aspect of neoclassical theory by proposing the view that the price mechanism may not always suffice and that the economy may be subject to coordination failures.

In this paper, we revisit this idea and propose a quantitative general equilibrium theory of coordination failures without substantial departure from standard macroeconomic models. Using a global game approach to resolve the multiplicity of equilibria that naturally arises from imperfect coordination, our theory offers a framework to analyze the impact of coordination on business cycle fluctuations and its implications for economic policy. Two main insights emerge from the theory. First, it provides a foundation for Keynesian-type demand-deficient equilibria. As firms may fail to coordinate on a higher output level, the economy may become stuck in long-lasting recessions. Second, government spending is in general detrimental to coordination but may sometimes improve welfare, even in the absence of nominal rigidities, when the economy is about to enter a recession.

Our model builds on the standard neoclassical growth model with monopolistic competition. In this environment, firms are subject to an externality as they take into account the level of aggregate demand when making production and pricing decisions. This externality links production decisions across firms and provides them with a motive to coordinate their actions. Consistent with empirical evidence, we enrich the model with a non-convex capacity utilization decision. This additional margin creates a sufficiently strong feedback from income to production to sustain multiple rational expectation equilibria. In each period, the economy admits a high-output and a low-output equilibrium. In the high-output equilibrium, firms operate at high capacity and aggregate employment and investment are high. On the opposite, in the low-output equilibrium, firms produce at low capacity and the economy is depressed.

How firms coordinate depends on their expectations of aggregate demand and on the economy's fundamentals. The role of capital is critical for the dynamics of the economy as it creates complementarities across periods. An abundance of capital fuels the growth of firms and favors the high-output equilibrium. Consequently, investment is high, facilitating coordination on the high-output equilibrium in the future. On the opposite, failure to coordinate on high production in one period may lead to a long-lasting recession. Coordination is therefore persistent.

While the multiplicity of equilibria generates interesting sunspot dynamics, the equilibrium indeterminacy is problematic for policy and quantitative applications. We therefore use a global game approach to resolve the equilibrium multiplicity. For that purpose, we introduce a small amount of incomplete information and endow firms with private signals about the state of nature. We show that a unique recursive equilibrium exists in this economy.

The unique equilibrium of our incomplete information environment preserves the rich dynamics of the model with multiple equilibria. In particular, the economy can feature *coordination traps*: the dynamics of the capital stock may have two stable steady states. The high steady state exhibits high levels of output, employment and capacity utilization while these values are depressed in the low steady state. After a bad shock of sufficient size and duration, as the capital stock declines, the economy may be too depressed for firms to coordinate on the high output and the economy runs the risk of falling into a chronic state of depression. Only large positive shocks to productivity or policy interventions can bring the economy back to the high steady state.

We calibrate the model on the United States economy and show that it performs similarly to a real business cycle model in terms of standard deviation of major aggregates and their correlation with output. It, however, outperforms the standard model in explaining business cycles asymmetries as it generates a substantial negative skewness more in line with the data.

We calibrate productivity shocks to replicate the US experience during the 2007-2009 recession and compare the time series simulated by the model with the actual data. In 2009, TFP had fallen by 5% before stabilizing to 2% below trend in the recovery period. Output, consumption and employment dropped to 5% below trend and have remained at that level, seemingly settling in a new steady state. The behavior of these times series, including investment and capacity utilization, is broadly consistent with the series generated by our model when the economy falls in the lower steady state.

Coordination failures are often used to motivate government intervention, including government spending policies. Because of the monopolistic distortions and the aggregate demand externality, the competitive equilibrium is inefficient, and government intervention is potentially required. Our findings suggest that government spending is detrimental to welfare in most of the state space, as the coordination problem magnifies the dynamic welfare losses due to the crowding out of private spending. However, perhaps surprisingly, government spending may sometimes increase welfare when the economy is about to enter a recession. The intuition can be stated as follows. When preferences allow for a wealth effect on the labor supply, an increase in government spending puts downward pressure on wages. As a result, the costs of production decline and firms can coordinate more easily on high output. Through this mechanism, government spending helps coordination.

Government spending is, however, suboptimal. We thus consider the problem of a social planner in the economy. Two simple subsidies are enough to implement the efficient allocation. First, an input subsidies corrects for the inefficient firm size that results from the monopoly distortions. Second, a profit subsidy is able to make firms internalize the aggregate demand externality on their capacity decision.

The existing literature on coordination failures often requires implausibly large increasing returns to generate multiple equilibria. We rely instead on non-convexities as a source of multiplicity. Micro-level data shows indeed that firms use various margins to adjust production over the business cycle, many of which involve non-convexities. In particular, a large empirical literature has documented the presence of large fixed costs in adjusting factors of production. For instance, Doms and Dunne (1998) and Cooper and Haltiwanger (2006) focus on capital adjustment while Caballero et al. (1997) consider costs to adjusting labor. Ramey (1991) estimates cost functions for six manufacturing industries and finds support for non-convexities. Bresnahan and Ramey

(1994) document important lumpiness in hours, overtime hours and plant shutdowns using weekly production data from the automobile industry. Also using data from the automobile industry, Hall (2000) provides evidence of non-convexities in capital utilization as firms adjust discretely the number of production shifts and operating plants over the cycle. We capture these various margins of adjustment in the simplest possible way through a discrete choice over capacity utilization.

Our paper belongs to a long tradition in macroeconomics that views recessions as episodes of coordination failures. A seminal paper in this literature, Diamond (1982) proposes a search model of the goods market subject to a thick market externality. The model features multiple rational expectation equilibria that can be viewed as a potential source of coordination failures. Kiyotaki (1988) builds a 2-period monopolistic competition model with increasing returns to generate equilibrium multiplicity. Cooper and John (1988) offer a unifying game-theoretic approach to show how complementarities in payoffs can generally give rise to multiple equilibria in static models. Jones and Manuelli (1992) propose a static framework to examine the key modeling features required to generate coordination failures. In contrast to these papers, we propose a full dynamic theory of coordination failures in an otherwise standard business cycle model. More closely related to our approach, Benhabib and Farmer (1994) and Farmer and Guo (1994) introduce increasing returns in a real business cycle model. They show that the economy admits multiple equilibrium path after linearizing around the unique steady state of the economy. Our contribution emphasizes the steady-state multiplicity that coordination problems can give rise to and their non-linear impact on the dynamics of the economy.

More importantly, a distinguishing feature of our paper is the use of a global game approach to discipline the equilibrium selection. The previous literature relied instead on sunspots to select equilibria. Even though they can produce rich dynamic patterns, including regime switches or endogenous cycles, sunspots usually raise a number of methodological issues. A first problem is the potential lack of discipline behind the equilibrium selection or, at least, the absence of a general consensus on the way to evaluate these models quantitatively. Second, the impact of various policies on the economy depends crucially on how the equilibrium in selected. As sunspots usually select equilibria in a completely exogenous way, they are subject to the Lucas critique: ignoring the impact of policies on equilibrium selection may lead to false policy recommendations. In response to these challenges, we let the model pick the equilibrium endogenously through a global game environment and explore the quantitative and policy implications of this approach.

Our paper also relates to the global game literature from which it borrows a number of insights and techniques. The key result, that departing from common knowledge may restore uniqueness in coordination games, stem from the seminal articles of Carlsson and Van Damme (1993) and Morris and Shin (1998). Our paper further relates to the dynamic global game literature as in Morris and Shin (1999), Steiner (2008) and Mathevet and Steiner (2013). In comparison to these paper, we consider a macroeconomic application to business cycles. Angeletos et al. (2007) proposes a dynamic global game in which agents learn about a fixed environment and uniqueness does not obtain. In our model, the fundamentals of the economy evolves stochastically, thereby preventing excessive accumulation of public knowledge and preserving the uniqueness result. The welfare criterion that we use to evaluate policies originates from Angeletos and Pavan (2007).

Related to the dynamic global game literature are the works of Burdzy et al. (2001) and Frankel and Pauzner (2000) who resolve the equilibrium indeterminacy in dynamic coordination games by introducing time-varying payoffs and a sufficient amount of frictions to prevent agents to take action in every period. Most closely related to our paper in this tradition is the work of Guimaraes and Machado (2014) which examines the impact of investment subsidies in an extension of the Frankel and Pauzner (2000) model to monopolistic competition and staggered technology choice. In their model, firms receive exogenous opportunities to change their technology according to a Calvo-type Poisson process. In line with our optimal policy result, their findings suggest subsidizing the adoption of technology upon switching to make firms internalize the aggregate demand externality.

Our approach is also reminiscent of the sentiment-driven business cycle literature as in the recent contributions of Angeletos and La'O (2013) and Benhabib et al. (2014). In these papers, in contrast to ours, the introduction of incomplete information leads to multiplicity of equilibria by allowing for correlation between information sets. As a result, the economy is subject to non-fundamental fluctuations. In our paper, we begin with a multiple equilibrium model and use a global game refinement to suppress all non-fundamentalness in the equilibrium. Hence, changes in fundamentals may trigger changes in coordination, but the economy is exempt from "animal spirits" or sentiment-driven fluctuations.

Finally, our paper touches upon various themes familiar to the poverty trap literature in growth theory. Murphy et al. (1989) propose a formal model of the Big Push idea that an economy can escape a no-industrialization trap if various sectors are simultaneously industrialized. Their theory relies on the interaction of demand spillovers and a choice between a constant and an increasing returns to scale technologies to generate multiplicity of equilibria. Similar forces are present in our model, but we use a global game approach to discipline the equilibrium selection and embed the mechanism in a business cycle model. In terms of the dynamics generated by the model, our paper is more closely related to Azariadis and Drazen (1990) who introduce threshold externalities in the neoclassical growth model to allow for multiplicity of locally stable steady states. Our paper relies on a demand-driven coordination problem to achieve similar transition stages in the dynamics of the economy and studies their implications for business cycles.

The paper is structured as follows. Section 2 introduces the environment and presents our baseline model under complete information. Section 3 describes the incomplete information version of the model and establishes our main uniqueness result. In section 4, we calibrate the model and show that the economy may replicate salient features of the recovery from the 2007-2009 recession. Section 5 analyzes the policy implications of the model and concludes with our findings on government spending. Proofs can be found in the appendix.

2 Complete Information

We begin by introducing the physical environment of our model, which remains the same throughout the paper, under the assumption of complete information. This simple framework allows us to build intuition about the source of equilibrium multiplicity and the role of coordination in this economy. We characterize the equilibria and discuss their dynamic implications.

2.1 Environment

Time is discrete and goes on forever. The economy consists of a representative household, a final good sector and an intermediate good sector. The final good is used for both consumption and investment. The intermediate goods consist of a continuum of varieties solely used for the production of the final good.

Households and preferences

The preferences of the representative household are given by

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}U\left(C_{t},L_{t}\right), \tag{1}$$

where $0<\beta<1$ is the discount factor, $C_t \geqslant 0$ is consumption of the final good and $L_t \geqslant 0$ is labor. We adopt the period utility function of Greenwood et al. (1988) (GHH hereafter):[1]

$$U\left(C_{t},L_{t}\right)=\frac{1}{1-\gamma}\left(C_{t}-\frac{L_{t}^{1+\nu}}{1+\nu}\right)^{1-\gamma},\gamma>0,\nu>0.$$

The representative household takes prices as given. It supplies capital K_t and labor L_t on perfectly competitive markets and owns the firms. It faces the sequence of budget constraints

$$P_{t}\left(C_{t}+K_{t+1}-\left(1-\delta\right)K_{t}\right)\leqslant W_{t}L_{t}+R_{t}K_{t}+\Pi_{t}, \tag{2}$$

where P_t is the price of the final good, W_t the wage rate, R_t the rental rate of capital and Π_t the total profits. Capital depreciates at rate $0<\delta<1$.

Final good producers

The final good is produced by a perfectly competitive, representative firm that combines a continuum of differentiated intermediate goods, indexed by $j \in (0,1)$, using the CES production function

[1] The preferences of Greenwood et al. (1988) allow us to derive analytical expressions for many equilibrium quantities, but are not critical for most of our results. We relax this assumption in our policy discussion as the preference specification matters for the effect of fiscal policy.

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}, \tag{3}$$

where $\sigma > 1$ is the elasticity of substitution between varieties, Y_t is the total output of the final good and Y_{jt} denotes the input of intermediate good j. Profit maximization, taking output price P_t and input prices P_{jt} as given, imply the usual factor demand curves and the price of the final good,

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\sigma} Y_t \text{ and } P_t = \left(\int_0^1 P_{jt}^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}. \tag{4}$$

Intermediate good producers

Intermediate good j is produced by a monopolist that uses a constant returns to scale production function with capital K_{jt} and labor L_{jt},

$$Y_{jt} = Ae^{\theta_t} u_{jt} K_{jt}^{\alpha} L_{jt}^{1-\alpha}, \tag{5}$$

where $0 < \alpha < 1$ is the capital intensity and u_{jt} is capacity utilization. Total factor productivity Ae^{θ} depends on a fundamental θ that follows an AR(1) process,

$$\theta_t = \rho \theta_{t-1} + \epsilon_{\theta t},$$

where $\epsilon_{\theta t} \sim \text{iid } \mathcal{N}\left(0, \gamma_\theta^{-1}\right)$.

Capacity u_{jt} can either take a low value, normalized to $u_l = 1$, or a high value $u_h = \omega > 1$. Producing at high capacity incurs a fixed cost $f > 0$ in terms of the final good. We denote by $A_h(\theta) \equiv \omega A e^{\theta}$ and $A_l(\theta) \equiv A e^{\theta}$ the effective TFP of firms with high and low capacity.

This capacity decision captures in a simple and tractable way different margins of adjustment, such as plants shutdowns and restarts or changes in the number of shifts and production lines, that firms use to adjust production over the cycle.[2] Importantly, this binary decision breaks the convexity of the cost function of the firms.[3] As a result, firms are able to expand their production swiftly in response to changes in aggregate condition, which is crucial to sustain multiple equilibria in this economy.

Intermediate producers take the rental rate of capital R_t and the wage W_t as given. For each capacity utilization u_i, $i \in \{h, l\}$, they solve the following static problem:

$$\Pi_{it} = \max_{Y_{it}, P_{it}, K_{it}, L_{it}} P_{it} Y_{it} - R_t K_{it} - W_t L_{it}, \tag{6}$$

subject to their demand curve (4) and production technology (5). Intermediate producer j then

[2] Because it acts as a TFP shifter, a broader interpretation of our capacity decision could include R&D, trade, etc. We focus on capacity utilization for its static nature and relevance in our quantitative exercise.

[3] We restrict capacity to take only two values for simplicity and tight theoretical results. It is straightforward to include additional capacity levels or model capacity choice as a continuous decision.

picks the capacity u_{jt} that maximizes its profits

$$u_{jt} = \underset{u_i \in \{u_h, u_l\}}{\operatorname{argmax}} \left\{ \Pi_{ht} - P_t f, \Pi_{lt} \right\}.$$

2.2 Equilibrium Definition

We are now ready to define a competitive equilibrium for this economy. Denote the complete history of shocks $\theta^t = (\theta_t, \theta_{t-1}, \ldots)$.

Definition 1. *A competitive equilibrium is a sequence of policies for the representative household* $\left\{C_t\left(\theta^t\right), K_{t+1}\left(\theta^t\right), L_t\left(\theta^t\right)\right\}_{t=0}^{\infty}$*, policies for firms* $\left\{Y_{it}\left(\theta^t\right), K_{it}\left(\theta^t\right), L_{it}\left(\theta^t\right)\right\}_{t=0}^{\infty}$*,* $i \in \{h, l\}$*, a measure* $m_t\left(\theta^t\right) \in [0,1]$ *of firms operating at high capacity and prices* $\left\{P_t\left(\theta^t\right), R_t\left(\theta^t\right), W_t\left(\theta^t\right)\right\}_{t=0}^{\infty}$ *such that i) the household maximizes utility (1) subject to (2); ii) intermediate producers solve their problem (6); iii) prices clear all markets; and iv) the measure of firms* $m_t\left(\theta^t\right)$ *satisfies*

$$m_t\left(\theta^t\right) = \begin{cases} 1 & \text{if } \Pi_{ht} - P_t f > \Pi_{lt}, \\ \in (0,1) & \text{if } \Pi_{ht} - P_t f = \Pi_{lt}, \\ 0 & \text{if } \Pi_{ht} - P_t f < \Pi_{lt}. \end{cases} \tag{7}$$

Our equilibrium concept is fully standard. Notice that the definition introduces the equilibrium measure $m_t\left(\theta^t\right)$ of firms with high capacity, which must be consistent with individual capacity decisions (7).

2.3 Characterization

Two features of our environment simplify the characterization of the equilibria: *i)* under GHH preferences, the amount of labor supplied by the household is independent of its consumption-savings decision, *ii)* the problems of the final and intermediate goods producers are static. We can therefore characterize the equilibrium in two stages: we first solve for the *static* equilibrium in every period, which determines production and capacity, and we then turn to the *dynamic* equilibrium, which uses the first stage as an input, to characterize the optimal consumption-savings decision and dynamics of the economy.

Partial equilibrium

We first characterize the decision of intermediate producers in partial equilibrium to highlight the role of aggregate demand and factor prices in their capacity choice. Substituting in the demand curve (4) in the expression for profits (6), the first-order conditions in terms of capital and labor yield

$$R_t K_{it} = \alpha \frac{\sigma - 1}{\sigma} P_{it} Y_{it} \text{ and } W_t L_{it} = (1 - \alpha) \frac{\sigma - 1}{\sigma} P_{it} Y_{it}. \tag{8}$$

Total factor expenses is therefore equal to a fraction $\frac{\sigma-1}{\sigma}$ of total sales, so that

$$\Pi_{it} = \frac{1}{\sigma} P_{it} Y_{it} = \frac{1}{\sigma} \left(\frac{P_t}{P_{it}} \right)^{\sigma-1} P_t Y_t,$$

where we have substituted the demand curve (4). In a monopolistic setup, production decisions are linked across firms as the total income generated by the private sector affects the level of demand faced by each individual producers. As a result, profits, gross of the fixed cost, depend on the firm's *relative* price and on aggregate demand Y_t. In particular, when aggregate demand is high, firms have stronger incentives to expand. This demand linkage is the main source of strategic complementarity in our model.

We can now simplify the capacity decision to

$$u_{it} = \underset{u_i \in \{u_h, u_l\}}{\operatorname{argmax}} \left\{ \frac{1}{\sigma} \left(\frac{P_t}{P_{ht}} \right)^{\sigma-1} Y_t - f, \frac{1}{\sigma} \left(\frac{P_t}{P_{lt}} \right)^{\sigma-1} Y_t \right\}, \tag{9}$$

where the individual prices are optimally set at a constant markup over marginal cost, $P_{it} = \frac{\sigma}{\sigma-1} \frac{1}{A_i(\theta_t)} \left(\frac{R_t}{\alpha} \right)^{\alpha} \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha}$ for $i \in \{h, l\}$.[4]

Expression (9) highlights the key forces that determine the choice of capacity in our environment. Firms with high capacity enjoy lower marginal costs of production and therefore sell their products at lower prices. Equation (9) tells us that, when choosing between the two capacity levels, firms compare two affine functions of aggregate demand—the one associated with high capacity having a higher slope but a lower intercept than that associated with the low capacity. As a result, firms pick the high capacity when aggregate demand is high. Intuitively, when demand is high, firms face high variable costs in capital and labor and have strong incentives to pay the fixed amount f in order to exploit economies of scale and save on these costs. On the other hand, firms have little interest to pay the fixed cost when demand is low and total variable costs relatively small.

General equilibrium

We now turn to the general equilibrium characterization. Under GHH preferences, we can derive analytical expressions for aggregate quantities as a function of the measure m_t of firms with high capacity.

Proposition 1. *For a given measure m_t of firms with high capacity the equilibrium output of the final good is given by*

$$Y_t = \overline{A} \left(\theta_t, m_t \right) K_t^{\alpha} L_t^{1-\alpha}, \tag{10}$$

[4] See Appendix (A) for a full derivation.

where $\overline{A}(\theta_t, m_t) = \left(m_t A_h(\theta_t)^{\sigma-1} + (1-m_t) A_l(\theta_t)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}$ *and aggregate labor is*

$$L_t = \left[(1-\alpha)\frac{\sigma-1}{\sigma}\overline{A}(\theta_t, m_t) K_t^{\alpha}\right]^{\frac{1}{\alpha+\nu}}. \tag{11}$$

The corresponding production and profit levels of intermediate firms are, for $i \in \{h, l\}$,

$$Y_{it} = \left(\frac{A_i(\theta_t)}{\overline{A}(\theta_t, m_t)}\right)^{\sigma} Y_t \text{ and } \Pi_{it} = \frac{1}{\sigma}\left(\frac{A_i(\theta_t)}{\overline{A}(\theta_t, m_t)}\right)^{\sigma-1} P_t Y_t. \tag{12}$$

Proof. All proofs are in the Appendix. □

Proposition 1 establishes a number of important results. We see from equation (10) that the economy aggregates into a Cobb-Douglas production function with TFP $\overline{A}(\theta_t, m_t)$. Importantly, aggregate TFP is an endogenous object that consists of some average of intermediate firms' effective productivities. As a result, aggregate output increases with the measure of firms m_t, as high capacity firms operate a more productive technology.

The last result is important as it completes our exposition about the nature of the complementarities in our environment: higher aggregate demand encourages firms to choose the high capacity; more firms choosing the high capacity, in turn, implies higher aggregate demand. Multiple equilibria arise in our environment when this two-way feedback between demand and capacity is sufficiently strong. This picture remains incomplete, however, if one ignores the role of relative prices P_{it}/P_t in capacity decisions. These relative prices depend on factor prices, which are affected by the measure of high capacity firms m_t in equilibrium, since firms compete on factor markets. Whether there is strategic complementarity in capacity decisions in our setup ultimately depends on which of these two forces dominate: *complementarity* through aggregate demand linkages or *substitutability* through factor market competition.

Equilibrium multiplicity

Using our analytical results on equilibrium production and profits, we now proceed to characterize the static equilibrium capacity decision for some given stock of capital K and productivity fundamental θ.

Proposition 2 (Multiplicity). *Consider the following condition on parameters:*

$$\frac{1+\nu}{\alpha+\nu} > \sigma - 1. \tag{13}$$

Under condition (13), there exist thresholds $B_H < B_L$ *such that:*
i) if $Ae^{\theta}K^{\alpha} < B_H$*, the static equilibrium is unique and such that all firms choose low capacity,* $m = 0$*;*
ii) if $Ae^{\theta}K^{\alpha} > B_L$*, the static equilibrium is unique and such that all firms choose high capacity,*

$m = 1$;
iii) if $B_H \leqslant Ae^{\theta}K^{\alpha} \leqslant B_L$, there are three static equilibria: two in pure strategies, $m = 1$ and $m = 0$, and one in mixed strategies, $m \in (0, 1)$.
If condition (13) is not satisfied, the static equilibrium is always unique.

Multiple equilibria arise under condition (13). In regions of the state space where capital is abundant and productivity θ is high, such that $Ae^{\theta}K^{\alpha} \geqslant B_H$, a *high equilibrium* exists in which all firms choose the high capacity, $m = 1$. In these regions, renting capital is inexpensive and technology is productive, so firms operate at a large scale. As a result, total income and aggregate demand are high, which further encourages firms to expand and adopt the high capacity. On the opposite, in regions of the state space where capital is scarce and productivity low, such that $Ae^{\theta}K^{\alpha} \leqslant B_L$, only a *low equilibrium* exists with $m = 0$: firms operate at a small scale and do not find it worthwhile to pay the fixed cost f to expand their production. For the intermediate region $B_H \leqslant Ae^{\theta}K^{\alpha} \leqslant B_L$ the two equilibria coexist. The economy is then subject to self-fulfilling prophecies: depending on firms' expectations, it may end up in either the high or the low equilibrium. Figure 1 depicts the situation described in the proposition.

The condition for multiplicity (13) characterizes the conflict between the strategic substitutability from competition in the factor markets, on the left-hand side, and the demand-side complementarity, captured by σ. This condition is satisfied when the intermediate good varieties are strong complements, if σ is low, or when the left-hand side is large. The latter term, $\frac{1+\nu}{\alpha+\nu}$, is the elasticity of aggregate production with respect to changes in TFP: it captures the scalability of the economy to changes in average capacity. Multiple equilibria are thus more likely to arise when the scalability is high, which happens when the labor supply is elastic (ν small) and when production is intensive in the flexible factor, labor (α small). This scalability term captures, in particular, the idea that multiple equilibria can only be sustained if factor prices react moderately to changes in m. We assume that condition (13) is satisfied from now on.

Efficiency

At this stage, it is natural to wonder whether a planner should intervene to improve the outcome of the coordination game. We consider the following planning problem:

$$\max_{K_{t+1}, L_t, m_t} \mathbb{E} \sum_{t=0}^{\infty} \beta^t U \left(\left(m_t Y_{ht}^{\frac{\sigma-1}{\sigma}} + (1 - m_t) Y_{lt}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} + (1 - \delta) K_t - m_t f - K_{t+1}, L_t \right),$$

subject to the production function (5) and the market clearing conditions on capital and labor markets. Proposition (3) describes the efficient allocation.

Proposition 3. *If $\frac{1+\nu}{\nu+\alpha} > \sigma - 1$, there exists a threshold B_{SP}, with $B_{SP} \leq B_L$, such that the planner makes all firms use the high capacity, $m_t = 1$, if $Ae^{\theta_t}K_t^{\alpha} \geqslant B_{SP}$ and firms use the low capacity, $m_t = 0$, if $Ae^{\theta_t}K_t^{\alpha} \leqslant B_{SP}$. The threshold B_{SP} is lower than B_H for σ small.*

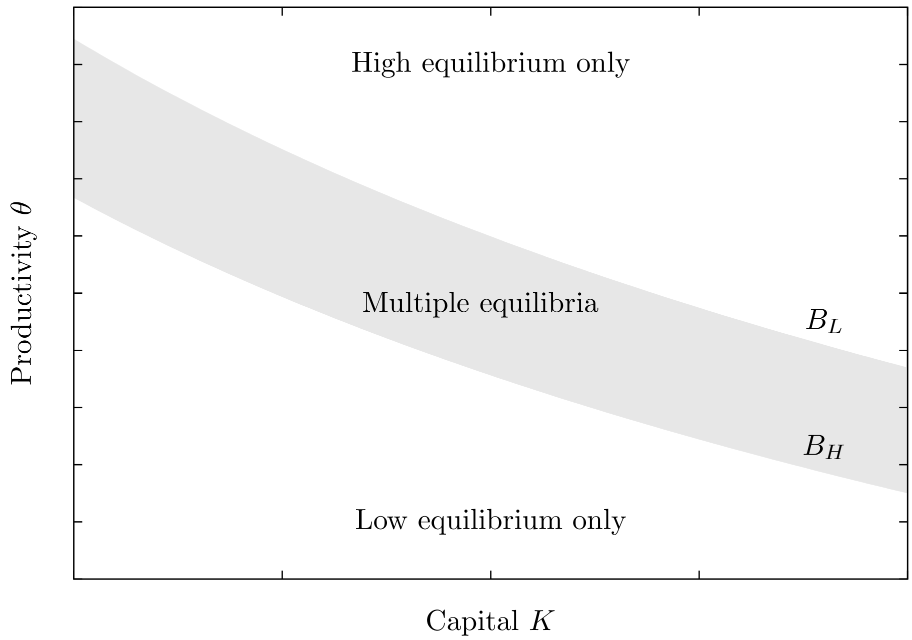


Figure 1: Multiplicity in the static game as a function of the state space

Under the condition for multiple equilibria (13) in the competitive economy, the planning problem is convex in m_t and the planner always chooses a corner solution, either $m_t = 0$ or 1. Since coordinating on the high capacity is costly, the planning solution is non-trivial and there exists a threshold B_{SP} such that all firms adopt the high capacity if and only if $Ae^{\theta_t}K_t^{\alpha} \geqslant B_{SP}$. When the productivity level and the capital stock are low, using the high capacity is too expensive and it is efficient to coordinate firms on the low equilibrium.

Because of the demand externality, the efficient allocation differs in important ways from the competitive outcome. Figure 2 shows the social planner's (SP) threshold, B_{SP}, together with the thresholds of the competitive economy (CE), B_L and B_H. Proposition 3 shows that B_{SP} always lies below B_L which indicates that the planner is more prone to pick the high capacity. This result is a direct consequence of the demand externality: firms do not internalize that by choosing the high capacity, they would generate more income to be spent on other firms' products, while the planner does. The competitive equilibrium therefore suffers from *coordination failures*: in the area limited by the dashed lines, between B_L and B_{SP}, the planner always picks the high equilibrium while firms in the competitive economy may coordinate on a different equilibrium.

Figure 2 depicts a particular case in which B_{SP} lies below B_H, which happens when the degree of complementarity is strong (σ low). When σ is large, the planner's threshold B_{SP} lies between B_H and B_L and cases may arise in which the planner would pick the low equilibrium when the competitive economy could coordinate on the high equilibrium.

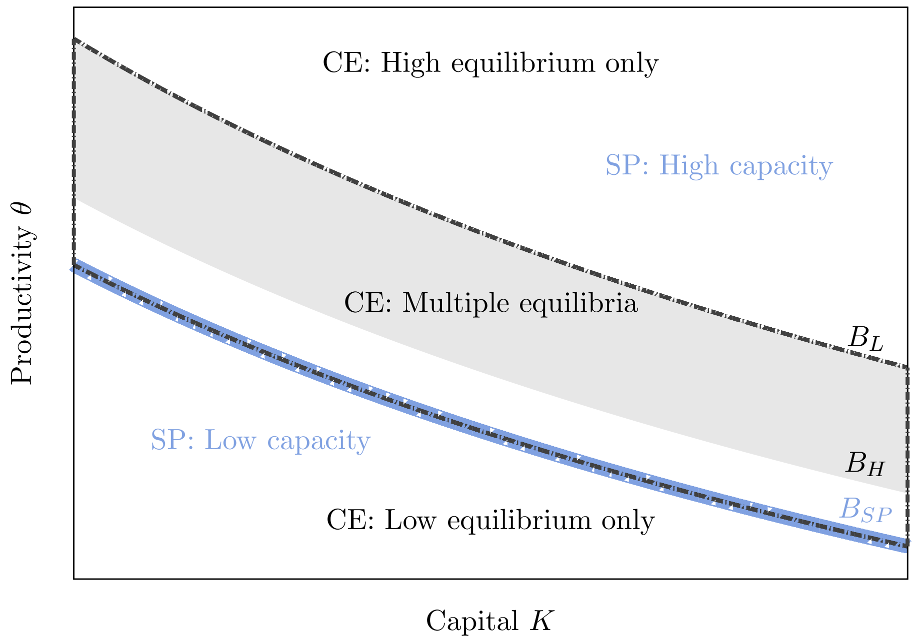


Figure 2: Planner's decision versus the outcome in the competitive economy.

2.4 Dynamics

Having characterized some properties of the static equilibria, we now study how coordination affects the dynamics of the economy. The multiplicity of equilibria in the static coordination game generates a multiplicity of equilibria in the dynamic economy. In other words, multiple equilibrium paths can be sustained with appropriate sunspots. To illustrate the rich dynamic implications of the model, we discuss one simple type of sunspot equilibrium and postpone to the next section a full description of the dynamics under our global game equilibrium selection device.

Consider the following naive sunspot equilibrium of regime switches in which firms resolve the multiplicity by repeating the equilibrium previously played. As a result, the economy only switches between equilibria when it visits a region of the state space in which the equilibrium previously selected no longer exists. Figure 3 illustrates the dynamics of this economy. In the upper right region, above B_L, only the high equilibrium is played. Similarly, below B_H, only the low equilibrium is selected. In the intermediate region, if the economy enters the shaded region from above B_L the high equilibrium is played (high regime), while the firms coordinate on the low equilibria (low regime) if the economy enters the shaded area from below B_H.

In both high and low regimes, the economy behaves as in a standard neoclassical growth model with aggregate TFP given by $\omega A e^{\theta}$ or $A e^{\theta}$. In both cases, the economy moves towards the solid lines, $\overline{K}_h(\theta)$ or $\overline{K}_l(\theta)$, represented by the light blue and black lines on figure 3. These lines represents the stationary locus points, where $\dot{K}_i(\theta) = 0$ for $i \in \{h, l\}$, to which the economy would converge if θ remained constant for a sufficiently long time. The arrows represent the evolution of the economy in these regions. The dynamics in the shaded region depends on the past history of

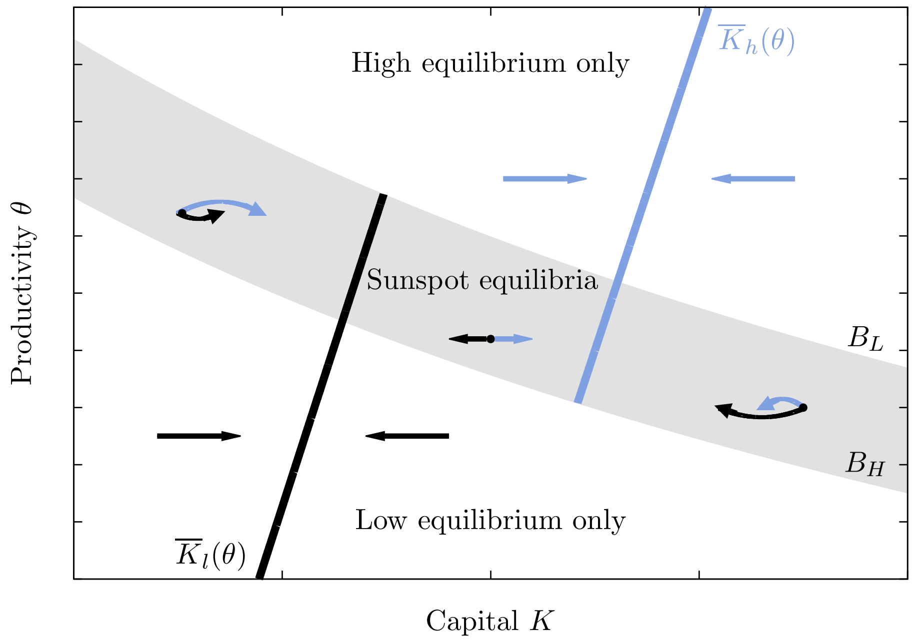


Figure 3: Phase diagram of the sunspot equilibrium

the economy. In this area, whenever the high equilibrium is played, the economy moves towards $\overline{K}_h(\theta)$, as indicated by the blue arrows. The black arrows indicate the dynamics when the low equilibrium is played.

In this example, switches between $m = 0$ and $m = 1$ occur only when the economy visits one of the two extreme regions with a unique static equilibrium. As a result, the economy tends to remain along one of the two steady states for a long time, as it takes large shocks in productivity θ to push the economy towards the other regime. The equilibrium dynamics therefore displays path-dependence and persistence. This persistence is further reinforced by the interaction between coordination and capital accumulation. For instance, whenever the economy is in the low regime, the productivity shock required to push the economy to the high regime is larger the lower is the capital stock.

This equilibrium is only one example of many potential sunspot equilibria, but it is useful to illustrate the dynamics in the complete information case. It provides a good intuition for the types of dynamics that arise in the incomplete information environment, to which we now turn.

3 Incomplete Information

The equilibrium multiplicity that arises in the model with complete information makes it unsuitable for policy and quantitative analysis. This multiplicity is, however, fragile and survives only under restrictive assumptions on the information structure. In this section, we adopt a global game approach by introducing incomplete information in our model and show that uniqueness of the full dynamic general equilibrium model obtains for a small departure from common knowledge.

3.1 Environment

The production structure of the economy, along with the capital and labor markets, remains the same as in part 2. However, we slightly modify the timing of events and the information structure so that firms are uncertain, but have private information, about productivity θ when they choose their capacity utilization

Information and timing

Each period t is split in two stages: 1) a first stage in which information is incomplete and intermediate producers choose their capacity, 2) a second stage in which information is complete, production decisions take place and all markets clear.

In the first stage, agents start with the information set $\mathcal{I}_t = (\theta_{t-1}, \theta_{t-2}, \dots)$ which contains all the past fundamentals of the economy. The new productivity level θ_t is drawn from to same process as in part 2 but remains unobserved. From Bayes' rule, ex-ante beliefs are given by

$$\theta_t \mid \mathcal{I}_t \sim \mathcal{N}\left(\rho\theta_{t-1}, \gamma_\theta^{-1}\right).$$

In comparison to the model with complete information in which agents could observe the fundamental θ_t immediately, we assume here that each intermediate producer j only receives a noisy signal v_j defined as

$$v_{jt} = \theta_t + \varepsilon_{jt}^v,$$

where the noise $\varepsilon_{jt}^v \sim \mathcal{N}\left(0, \gamma_v^{-1}\right)$ is iid across agents and time. Firms update their beliefs using Bayes' rule so that

$$\theta_t | \mathcal{I}_t, v_{jt} \sim \mathcal{N}\left(\frac{\gamma_\theta \rho\theta_{t-1} + \gamma_v v_{jt}}{\gamma_\theta + \gamma_v}, \frac{1}{\gamma_\theta + \gamma_v}\right). \tag{14}$$

Intermediate producers make their capacity decisions based on their individual beliefs in the first stage.

In the second stage, consumption-savings decisions are made, production takes place and all markets clear. The observation of production and aggregate prices reveals the aggregate productivity θ_t. Since the input choices and production take place simultaneously, these decisions are made under complete information. As a result, the equilibrium expressions derived in proposition 1 are still valid, with the exception that m_t is now the solution to the first stage coordination game under incomplete information.

Capacity decision

Intermediate producers choose the high capacity when the expected surplus from using the high technology,

$$\Delta\Pi\left(K_t,\theta_{t-1},m_t,v_{jt}\right)\equiv\mathbb{E}_\theta\left[U_c\left(C_t,L_t\right)\left(\Pi_h\left(K_t,\theta_t,m_t\right)-f-\Pi_l\left(K_t,\theta_t,m_t\right)\right)\mid\theta_{t-1},v_{jt}\right],\qquad(15)$$

is positive. An agent with private signal v_j therefore chooses to operate at high capacity if and only if $\Delta\Pi\left(K_t,\theta_{t-1},m_{\mathrm{t}},v_{jt}\right)\geqslant 0$, taking the potentially random measure of high capacity firms m_t as given. Three important features of expression (15) are worth emphasizing. First, in contrast to the complete information case, agents must compute the expectation of profits with respect to their own individual beliefs, given by (14). Second, not only is there uncertainty about the fundamental θ_t, but there is also strategic uncertainty: since other agents base their decisions on their own noisy private signals, the measure of firms with high capacity m_t is itself uncertain. Third, because of the uncertainty within the period, between stage 1 and 2, intermediate producers also take into account the fact that the household might not value consumption equally in all states of the world. As a result, firms use the representative household's stochastic discount factor $U_c\left(C,L\right)$ to evaluate profits.

Equilibrium definition

Because of the Markovian structure of the economy, it is convenient to define a recursive competitive equilibrium for our economy. We use θ_{-1} to denote the productivity of the previous period.

Definition 2. A recursive competitive equilibrium consists of i) a value function for the household $V\left(k;K,\theta,m\right)$ and decision rules $\left\{c\left(k;K,\theta,m\right),l\left(k;K,\theta,m\right),k'\left(k;K,\theta,m\right)\right\}$; ii) decision rules for individual intermediate producers $\left\{Y_i\left(K,\theta,m\right),K_i\left(K,\theta,m\right),L_i\left(K,\theta,m\right),\Pi_i\left(K,\theta,m\right)\right\}$ for $i\in\{h,l\}$; iii) aggregates $\left\{Y\left(K,\theta,m\right),L\left(K,\theta,m\right),\Pi\left(K,\theta,m\right)\right\}$; iv) price schedules $\left\{R\left(K,\theta,m\right),W\left(K,\theta,m\right)\right\}$; v) a law of motion for aggregate capital $H\left(K,\theta,m\right)$; and vi) iv) a measure $m\left(K,\theta_{-1},\theta\right)$ of firms with high capacity such that:

1. The household solves the problem

$$V\left(k;K,\theta,m\right)=\max_{c,l,k'}U\left(c,l\right)+\beta\mathbb{E}\left[V\left(k';H\left(K,\theta,m\right),\theta',m'\right)|\theta\right]$$

subject to $c+k'-\left(1-\delta\right)k\leqslant R\left(K,\theta,m\right)k+W\left(K,\theta,m\right)l+\Pi\left(K,\theta,m\right)$;

2. Intermediate producers of type $i\in\{h,l\}$ solve the problem

$$\Pi_i\left(K,\theta,m\right)=\max_{P_i,Y_i,K_i,L_i}P_iY_i-R\left(K,\theta,m\right)K_i-W\left(K,\theta,m\right)L_i,$$

subject to $Y_i=P_i^{-\sigma}Y\left(K,\theta,m\right)$ and $Y_i=A_i\left(\theta\right)K_i^{\alpha}L_i^{1-\alpha}$;

3. Aggregates are given by

$$Y(K,\theta,m) = \left(mY_h(K,\theta,m)^{\frac{\sigma-1}{\sigma}} + (1-m)Y_l(K,\theta,m)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$
$$\Pi(K,\theta,m) = m(\Pi_h(K,\theta,m) - f) + (1-m)\Pi_l(K,\theta,m);$$

4. Capital and labor markets clear

$$K = mK_h(K,\theta,m) + (1-m)K_l(K,\theta,m),$$
$$l(K;K,\theta,m) = mL_h(K,\theta,m) + (1-m)L_l(K,\theta,m);$$

5. Consistency of individual and aggregate capital decisions

$$H(K,\theta,m) = k'(K;K,\theta,m);$$

6. The aggregate resource constraint is satisfied

$$c(K;K,\theta,m) + H(K,\theta,m) = Y(K,\theta,m) + (1-\delta)K - mf;$$

7. For all K, θ_{-1} and θ, the measure of firms with high capacity $m(K,\theta_{-1},\theta)$ solves the fixed point problem

$$m(K,\theta_{-1},\theta) = \int \mathbb{1}\left[\Delta\Pi(K,\theta,m,v_j) \geqslant 0\right]\sqrt{\gamma_v}\phi\left(\sqrt{\gamma_v}(v_j-\theta)\right)dv_j, \tag{16}$$

   where ϕ is the probability density function of a unit normal.

Our definition of a recursive equilibrium is standard except for condition (16) which corresponds to the global game played by the firms: the measure m must solve a fixed point problem which defines it as the aggregation of individual capacity decisions when agents have the correct expectation of its equilibrium distribution.

3.2 Existence and Uniqueness

When they choose their capacity utilization, firms play a global game as in Carlsson and Van Damme (1993) and Morris and Shin (1998). We build on the insight from the global game literature that the multiplicity of equilibria in some coordination games is a consequence of an unrealistic information structure. In particular, full knowledge about the strategy of other players allows agents to coordinate perfectly in a way that leads to multiplicity. The introduction of a small amount of strategic uncertainty, however, can eliminate this multiplicity. We extend their results to our dynamic general equilibrium environment.

Applications of global games to market economies are sometimes problematic as prices may reveal enough information to restore common knowledge and multiplicity. In our setup, prices do reveal the true value of the fundamental, but since they are only determined at the production stage, after the capacity decisions are taken, we retain the uniqueness result.[5]

Another contribution is to show how uniqueness of the static capacity decision game extends to the rest of the dynamic environment. This result is not a straightforward application of standard global game techniques for several reasons. First, there is a complex two-way feedback between the capacity decision and the dynamic consumption-savings choice. Second, firms' capacity decisions sums up at the aggregate level into a non-concave production function with endogenous TFP. Third, our economy is subject to many distortions due to the coordination game and monopolistic power. All these reasons prevent us from using standard techniques.

We now state our main existence and uniqueness result.

Proposition 4. *For γ_v large and f sufficiently small such that, in particular, the following condition holds,*

$$\frac{\sqrt{\gamma_v}}{\gamma_\theta} > \frac{1}{\sqrt{2\pi}} \frac{\omega^{\sigma-1} - 1}{\sigma - 1}, \tag{17}$$

there exists a unique dynamic equilibrium. The equilibrium capacity decision takes the form of a continuous cutoff $\hat{v}(K, \theta_{-1})$ such that firm j invests if and only if $v_j \geqslant \hat{v}(K, \theta_{-1})$. Furthermore, the cutoff is a decreasing function of its arguments.

The proof of proposition 4 is structured according to the natural separation that arises in our model between the static capacity-production stage and the dynamic consumption-savings decision. In a first part, we focus on the global game, taking some equilibrium stochastic discount factor as given, and provide sufficient conditions for the uniqueness of the static capacity decision game. In a second part, using the endogenous TFP that results from the equilibrium capacity decisions, we focus on showing the uniqueness and existence of consumption and investment paths in the neoclassical growth model.

Part 1 proceeds in two steps. First, we show that when private signals are precise, such that γ_v is large, we can ignore the stochastic discount factor in expression (15) as risk vanishes. This step is particularly useful as it allows us to solve the global game, using iterated deletion of dominated strategies, independently from the consumption-savings decision of the household.

Going through the initial steps of the iterated deletion procedure is useful to understand the role of strategic uncertainty in eliminating the multiplicity. Notice first that it is a dominant strategy for a firm to choose the high capacity for a high enough signal: even in the event that all other firms choose to operate at low capacity, expectations of productivity are high enough to justify paying the fixed cost f. As a result, we can define a threshold $\hat{v}^0$ such that high capacity is dominant when $m = 0$ if and only if $v_j \geqslant \hat{v}^0$. Because of uncertainty and heterogeneity in beliefs, a firm that gets a signal equal to that threshold understands that a non-zero mass of other firms with signals better than its own would also choose the high capacity. In that case, m must be greater than zero and the

[5] The uniqueness result still obtains with simultaneous capacity and production decisions as long as prices and aggregates are not fully revealing, for instance because of noise traders, decentralized trading, etc. Uniqueness is still ensured for small enough private noise (see Angeletos and Werning (2006)).

threshold $\hat{v}^1$ at which high capacity becomes dominant when other firms play strategy $\hat{v}^0$ must be lower: $\hat{v}^1 < \hat{v}^0$. Iterating on this process, one obtains a decreasing sequence of thresholds $(\hat{v}^n)_{n\geqslant 0}$ which eventually converges to some limit. Repeating the process for regions where low capacity is dominant, a large number of strategies can be eliminated at every step when uncertainty is large enough, thereby reducing the equilibrium multiplicity. Condition (17) is a sufficient condition that guarantees that the deletion process converges to a unique equilibrium, which takes the form of a cutoff strategy. It states in particular that the fundamental θ must be sufficiently uncertain (γ_θ small) and, perhaps surprisingly, that private signals must be sufficiently precise (γ_v large). This last condition is required to generate enough strategic uncertainty: since firms put more weight on their heterogenous signals when they are precise, γ_v must be sufficiently large to generate enough dispersion in beliefs, and therefore in strategies.

Part 2 of the proof deals with the consumption-savings problem of the household. Once the capacity decision has been made, the model reduces to an almost standard neoclassical growth model with monopolistic distortions and an endogenous TFP. These two differences prevent the equilibrium from being cast into the problem of a fictitious social planner. Instead, we show directly that the Euler equation admits a unique positive fixed point by exploiting its monotonicity. Our proof builds on the lattice-theoretic work of Coleman and John (2000) and Datta et al. (2002), and uses a version of Tarski's fixed-point theorem on lattices, which states that monotone operators on lattices have a non-empty set of fixed points.[6] The proof proceeds by showing that the Euler equation is a well-defined monotone operator on a suitable space of consumption functions. Monotonicity requires f to be sufficiently small so that aggregate production, net of fixed costs, is always increasing in K. Uniqueness then obtains by pseudo-concavity of the Euler equation. Our proof extends earlier work to our setup and, in particular, to endogenous TFP and GHH preferences.

According to proposition 4, the optimal capacity decision takes the form of a decreasing cutoff $\hat{v}(K, \theta_{-1})$ such that only firms with private signals $v_j \geqslant \hat{v}(K, \theta_{-1})$ produce at high capacity. As a result, the equilibrium measure of firms with high capacity $m(K, \theta_{-1}, \theta)$ increases with K. Figure 4 compares the equilibrium aggregate output $Y(K, \theta, m)$ under incomplete information to the three possible equilibria of the complete information model ($m = 1$, $m = 0$ and the mixed equilibrium). As the figure illustrates, the global game tends to "select" the low equilibrium when the stock of capital is low and the high equilibrium for high values of K, with a gradual transition in the shaded region where multiplicity prevailed under complete information. This gradual transition is due to the progressive adoption of high capacity by firms with dispersed beliefs. While the outcomes are similar in the non-multiplicity regions, the equilibrium that the global game selects in the shaded region differs quite substantially from its complete information counterparts. In particular, the global game leads to a capacity level that increases in the economy's fundamentals, in contrast to the mixed equilibrium.

[6] See Topkis (1998) for additional references on lattice theory.

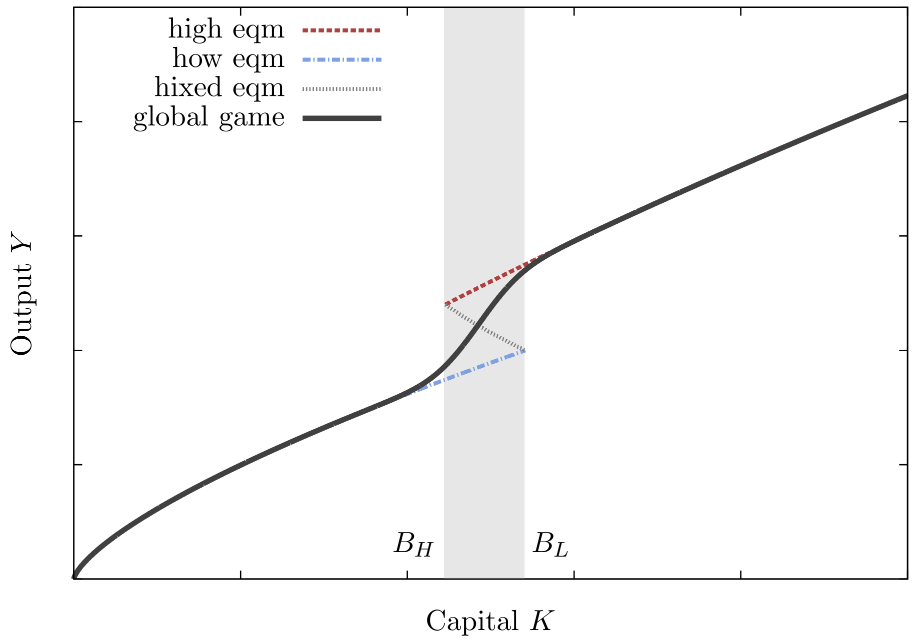


Figure 4: Dynamics of K for a given productivity θ

3.3 Dynamics

We now explore the dynamical properties of the economy. As was mentioned before, the incomplete information model aggregates into a neoclassical growth model with an endogenous TFP that breaks the convexity of the production set. Because of this non-convexity, aggregate output Y is an S-shaped function of capital K, as shown in figure 4. Intuitively, when capital is scarce firms prefer to have a small size and, therefore, to produce at low capacity. As capital becomes more abundant, the low rental rate increases the individual incentives to use the high capacity, which are further magnified by the adoption of the high capacity by other firms through the demand externality. The steep part of the S-shaped curve corresponds to the transition between the low and the high capacity.

Aggregate quantities such as consumption, employment and, importantly for the dynamics, investment inherit this S-shaped relationship to capital. The left panel of figure 5 displays the laws of motion of capital for various values of productivity θ. As the figure illustrates, for a high θ, the law of motion of K intersects the 45°-line once, at a high capital level. Similarly, when productivity is low, the only intersection occurs at a low level of capital. However, for intermediate values of productivity, the law of motion features three stationary points: a high and a low stationary point, both stable, and an unstable one in the middle region.[7]

The right panel of figure 5 is a phase diagram that summarizes the dynamics of the economy

[7] In a previous paper,Fajgelbaum et al. (2014), we described a dynamic system with similar properties in a distinct setup with social learning and irreversibilities in investment.

over the whole state space.[8] The two black lines represent the stationary points in the dynamics of capital ($\Delta K_t = 0$). We label the upper right locus as the *high regime* which corresponds to steady states where output, employment and capital are high, and where most firms operate at high capacity; the lower left locus is the *low regime* where aggregates are depressed and firms operate at low capacity. Both regimes are stable and the corresponding basins of attraction are indicated on the figure by the shaded area for the low regime and the white area for the high regime.

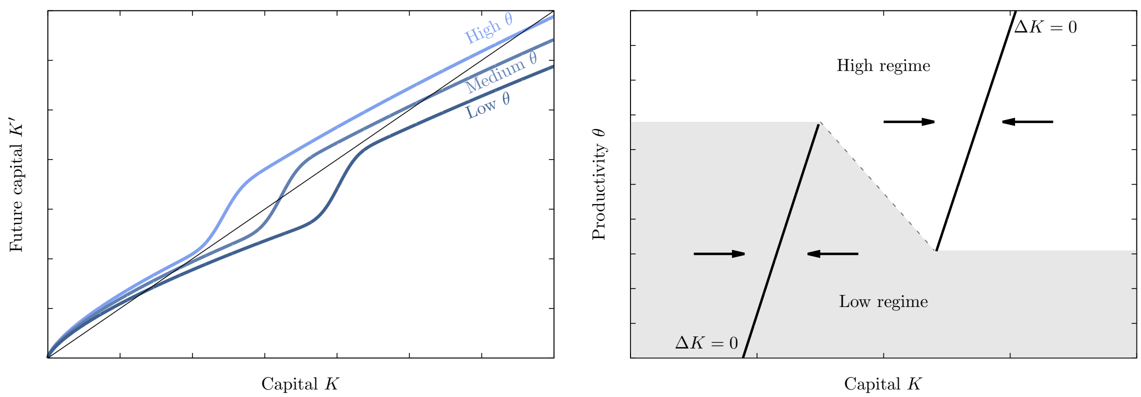


Figure 5: Multiple steady states and basins of attraction

In the absence of aggregate productivity shocks, the economy converges towards the steady state which corresponds to the basin of attraction it belongs to. Exogenous shocks to productivity θ can however push the economy from one basin of attraction to the other. When this happens, the economy starts converging towards its new steady state and the average capacity utilization adjusts accordingly.

A few features of the dynamics are worth mentioning. First, small shocks to θ can have a large impact on output when firms change their capacity utilization. Second, temporary shocks to θ can push the economy towards the other regime and therefore have a quasi-permanent impact on output. In this sense, the economy exhibits fragility. Third, shocks that destroy capital, as in Shimer (2012), would also have a long-lasting impact on the economy.

As we have seen, this type of dynamic system, with multiple steady states, displays strong non-linearities, path dependence and large persistence. We show in the next section that these characteristics have important implications for the dynamics of our calibrated economy.

4 Calibration

We now calibrate the model to the United States economy to evaluate its quantitative implications.

[8] The full state space is (K, θ_{-1}, θ). However, we omit θ_{-1} in our phase diagram because it becomes irrelevant when γ_v is large.

4.1 Parametrization

We calibrate the model at a quarterly frequency. The capital share α, the discount rate β and the depreciation rate δ are set to standard values. The literature uses a wide range of values for the elasticity of substitution. We follow (Hsieh and Klenow, 2014) and pick $\sigma = 3$.[9] For the preferences of the household, we use log utility, so that $\gamma = 1$, and follow (Jaimovich and Rebelo, 2009) in setting $\nu = 0.4$, implying a Frisch elasticity of 2.5 in line with macro-level estimates. The fundamental productivity process θ is parametrized to have a persistence of 0.95, as in (Cooley and Prescott, 1985), and a long-run standard deviation of 0.017% so as to fit the standard deviation of output.[10][11]

We are left with three parameters f, ω and γ_v to calibrate. As long as it is high enough to satisfy our uniqueness result, the precision of the private signals γ_v does not matter much for the simulations and is set to an arbitrary large number. The US time series display significant negative skewness, especially when the 2007-2009 recession is included in the dataset, which typical real business cycle models cannot explain. By varying the cost of using high capacity f, we can adjust the frequency at which the economy visits either one of the two steady states and explain a fraction of the observed skewness. We pick the fixed cost f to maximize the negative skewness of output. For the productivity gain from using the high capacity ω, we use data from the Federal Reserve Board and compare the average utilization rate between 2005 and 2008 to the average rate between 2010 to 2014.[12] The difference is of 1.7% and we therefore set $\omega = 1.017$. Table 1 summarizes the parameters.

4.2 Computations

The calibrated economy features multiple steady states in the dynamics of capital for average values of θ, as was explained in section 3.3. To illustrate the unusual dynamic properties that result from the coordination problem, we simulate the model for one million periods and plot the ergodic distributions of productivity, output, investment, employment, capacity utilization and TFP in log deviations on figure 6. While productivity θ is normally distributed, the other aggregates are negatively skewed, a sign that the economy spends a substantial amount of time in a depressed state, near the low steady state. Note in addition that the ergodic distributions of several variables are bimodal a feature that we observe for output in the data as shown on figure 11 in Appendix I.

[9] For robustness, we simulate the economy with higher values of the elasticity of substitution, such as $\sigma = 5$, and find that the economy still exhibits multiple steady states with roughly similar quantitative results.

[10] Because of the amplification provided by coordination, a relatively small standard deviation for shocks to θ is enough to fit the standard deviation of output.

[11] Appendix II provides the details of the data sources

[12] Between 2008 and 2010, capacity utilization dropped substantially but quickly rebounded to stabilize at about 2% below trend. Since we are interested in understanding propagation and persistence, we set ω to match the average capacity during the recovery period 2010-2014.

Table 1: Parameters

Parameter	Value	Source/Target
Time period	one quarter	
Total factor productivity	$A = 1$	Normalization
Capital share	$\alpha = 0.3$	Labor share 0.7
Discount factor	$\beta = 0.95^{1/4}$	0.95 annual
Depreciation rate	$\delta = 1 - 0.9^{1/4}$	10% annual
Elasticity of substitution	$\sigma = 3$	(Hsieh and Klenow, 2014)
Risk aversion	$\gamma = 1$	log utility
Elasticity of labor supply	$\nu = 0.4$	(Jaimovich and Rebelo, 2009)
Persistence θ process	$\rho_\theta = 0.95$	(Cooley and Prescott, 1985)
Standard deviation of ϵ_θ	$\gamma_\theta = 300,000$	Standard deviation of log output
Fixed cost	$f = 0.2$	See text
TFP gain from high capacity	$\omega = 1.017$	See text
Precision of private signal	$\gamma_v = 100,000$	See text

Business cycles moments

To evaluate the fit of the model we compute various moments from simulated time series and compare them to their empirical counterparts.[13] The results are shown in table 2 together with moments generated from a real business cycles version of our model in which the coordination mechanism is turned off: $f = 0$ and $\sigma \to \infty$.[14] The differences between the Full model and the RBC model highlight the influence of the coordination channel on the dynamics of the economy. In terms of the standard deviation of aggregates and their correlation with output both models offer a similar performance in matching the data with the Full model providing a better fit for the behavior of investment. The Full model, however, clearly outperforms the RBC model in terms of skewness.[15] Indeed, because of the two steady states, the economy of the Full model spends a substantial amount of time in the depressed state.[16]

Impulse response functions

To illustrate the non-linear properties of the model, we now look at the response of various aggregates to productivity shocks. In all cases, starting in the high steady state, we hit the economy with three shocks of different sizes and durations represented in panel (a) of figure 7. For the solid

[13] To compare the models appropriately with the data, we simulate 100,000 time series of 190 quarters, starting in the high steady state, and compute the average of the moments across simulations. To be consistent with the way the data is treated, a linear trend is also removed from the log of the simulated data.

[14] The volatility of the productivity process is recalibrated to $\gamma_\theta = 125,000$ in the RBC model to fit the standard deviation of log output. The fundamental must be much more volatile since coordination does not amplify shocks in this case.

[15] Both $f > 0$ *and* $\sigma < \infty$ are necessary to generate non-zero skewness.

[16] If we only select simulated time series in which the economy reaches the low steady state, a situation we could associate with the Great Recession, the ergodic distributions of aggregates become even more negatively skewed.

Table 2: Dynamic properties of the data, full model and RBC model.

Correlation with output	Output	Investment	Hours	Consumption
Data	1.00	0.87	0.86	0.94
Full model	1.00	0.89	1.00	0.99
RBC model	1.00	0.96	1.00	0.99

(a) Correlations with output

Std. Dev. Relative to Output	Output	Investment	Hours	Consumption
Data	1.00	3.27	1.46	0.94
Full model	1.00	2.06	0.72	0.88
RBC model	1.00	1.72	0.71	0.84

(b) Standard deviations relative to output

Skewness	Output	Investment	Hours	Consumption
Data	-0.59	-0.31	-0.35	-0.44
Full model	-0.16	-0.14	-0.16	-0.14
RBC model	0.00	0.00	0.00	0.00

(c) Skewness

curve, the innovations in θ are set to minus three standard deviations for 4 quarters, for the dashed curve the innovations in θ drop to minus 2.7 standard deviations for 4 quarters. Finally, for the remaining dotted curve, the innovations in θ drop to minus 2 standard deviations for 2 quarters. These specific shocks were chosen to properly illustrates the various mechanisms at work in the economy.

After the small shock, firms reduce their size only slightly and keep using the high capacity utilization. As as result, the economy recovers fairly quickly to the high steady state. As firms keep coordinating on the high utilization throughout the duration of the shock, the response of the economy is essentially the same as what we would observe in a standard RBC model. The situation is different when the economy faces the shock of intermediate size represented by the dashed curve. In this case, firms reduce their production substantially, partly because of lower productivity and partly because of lower aggregate demand. Because of this failure of firms to coordinate, the economy takes more time to recover to the high steady state. Finally, after the large shock, capacity utilization drops massively and stays low for a long time. As a result, the household saves less and the capital stock declines, making coordination on the high capacity difficult. The economy converges to the low steady state and remains stuck there even after productivity θ has fully recovered. Once in the low regime, only a sufficiently large positive shock can move the economy into the basin of attraction of the high steady state.

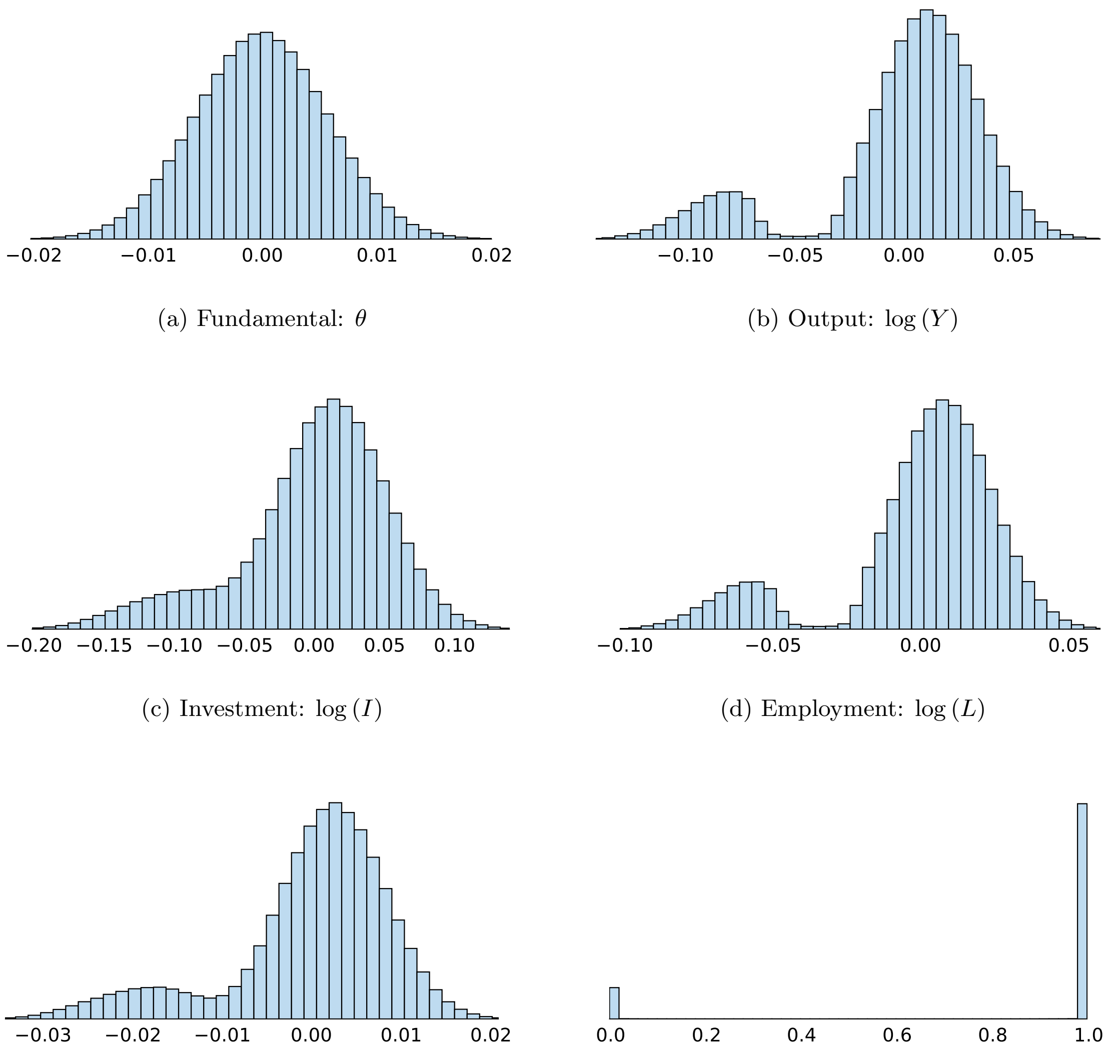
The aftermath of the 2007-2009 recession

We now consider whether the non-linear dynamics of the model could explain the aftermath of the 2007-2009 recession. Panel (a) of figure 8 shows the behavior of output, employment, investment, consumption and TFP[17] from 2005 to the second quarter of 2014. All series are normalized to 0 at the beginning of the recession. After the recession hit, consumption, output and employment slowly declined and stabilized at about 5% below their pre-recession levels. Similarly, investment initially dropped by about 20% before recovering to 10% below its pre-recession level.

From the perspective of our theory, the behavior of the economy in the aftermath of the Great Recession is consistent with a transition to the low steady state. To evaluate whether our model can quantitatively fit the data, we consider a transitory productivity shocks that pushes TFP to its level at the end of the recession, in the first second of 2009.[18] The response of various aggregates is shown in panel (b) of figure 8. As we can see, the fit of consumption, employment and TFP is quite good while output and investment drop too much in the model economy. Notice that the initial drop in endogenous TFP is due to the direct impact of the productivity shock together with the transition from high to low capacity by the firms. Its long-run behavior, however, is solely driven by the low capacity, as exogenous productivity θ has completely recovered by then.

[17]TFP is measured as the Solow residual and is analogous to the endogenous TFP $\overline{A}(\theta, m)$ from the model. See Appendix II for details.

[18]Specifically, we set the innovation of θ to minus 2.45 standard deviations for 6 quarters and 0 afterwards. Since the coordination problem essentially provides us with a propagation channel, the actual shock that hits the economy is unimportant for the long-run consequences of the recession in the model as long as the economy falls in the lower steady states. Our coordination mechanism would provide equally strong propagation to other theories of the recession based on financial shocks, uncertainty and others.



(a) Fundamental: θ

(b) Output: $\log(Y)$

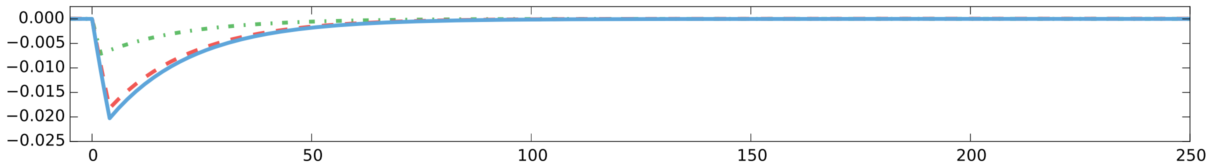
(c) Investment: $\log(I)$

(d) Employment: $\log(L)$

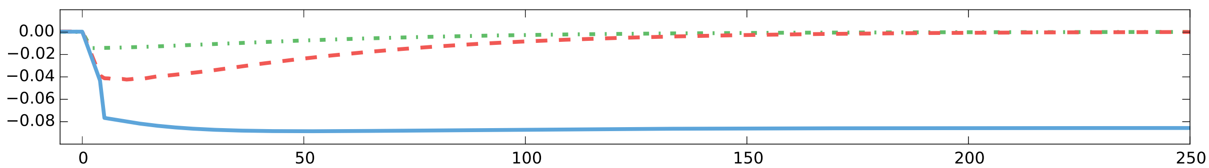
(e) Endogenous TFP: $\log\left(\bar{A}(\theta, m)\right)$

(f) Capacity utilization: m

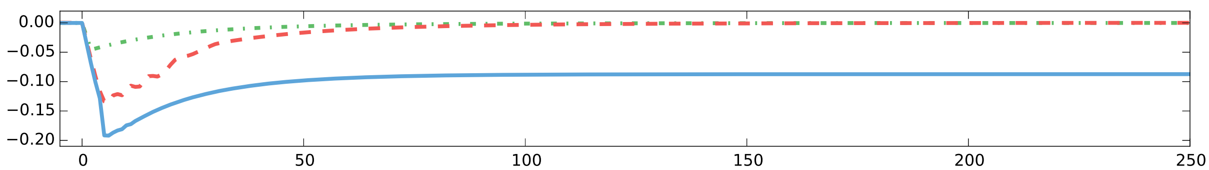
Figure 6: Ergodic distributions of aggregates



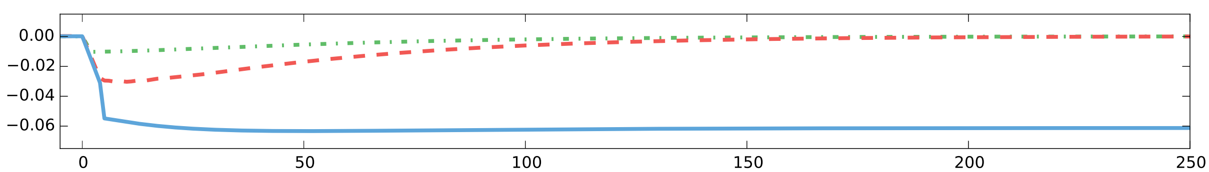
(a) Shocks to fundamental θ



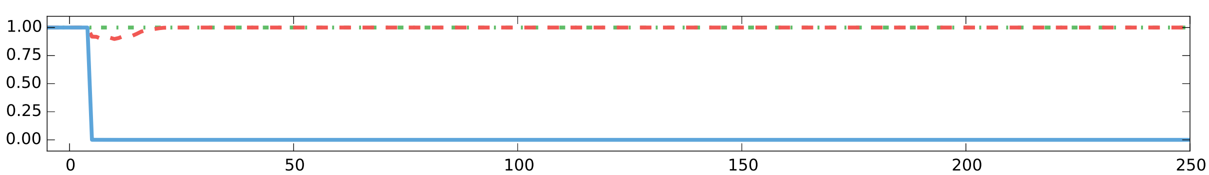
(b) Output $\log(Y)$



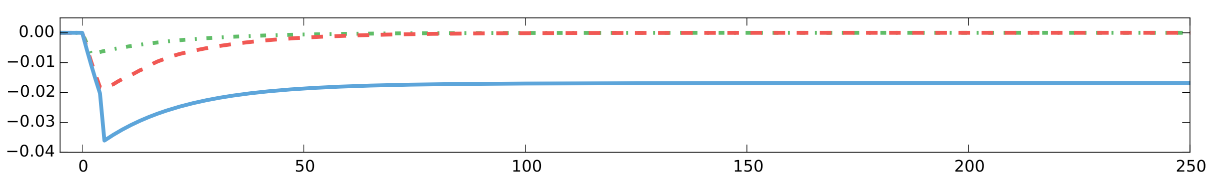
(c) Investment $\log(I)$



(d) Employment $\log(L)$

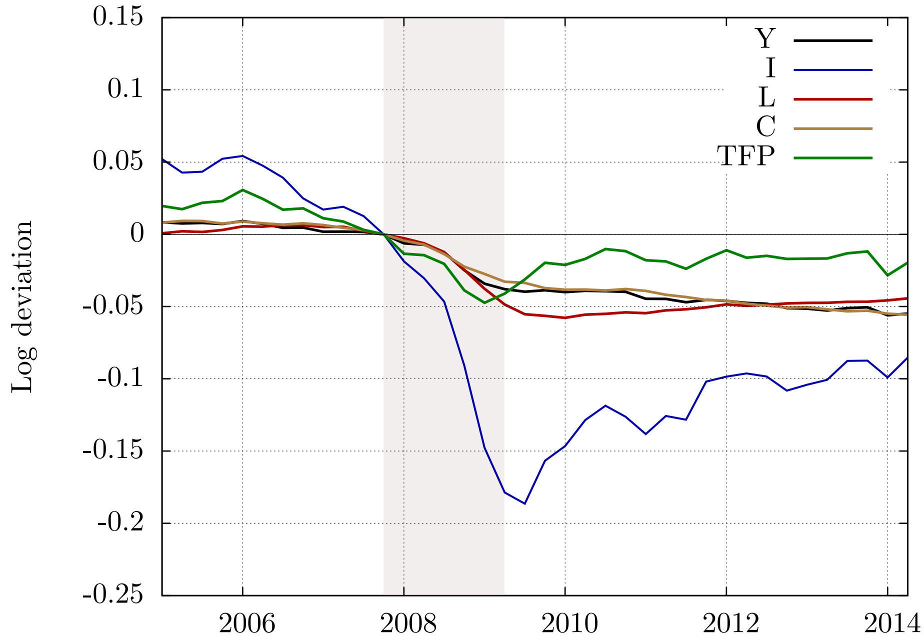


(e) Capacity utilization m

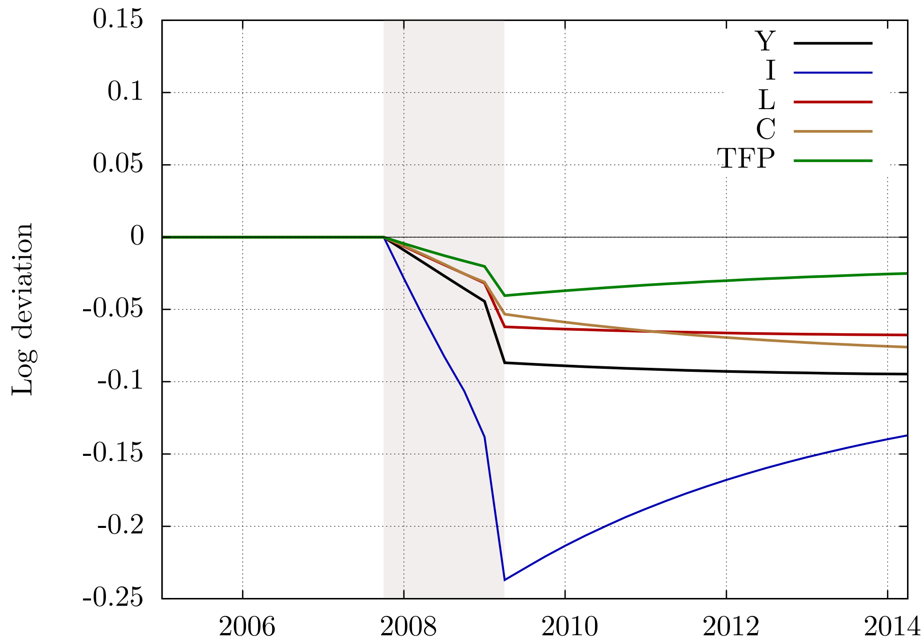


(f) Endogenous TFP $\log\left(\overline{A}(\theta, m)\right)$

Figure 7: Impulse response functions



(a) Data: US series centered on 2007:Q4



(b) Model

Figure 8: The 2007-2009 recession

5 Policy

The prospect of coordination failures is often used in policy debates to justify large government interventions, including in particular expansionary fiscal policies. In this section, we use our model to analyze the appropriate policy response when the economy is hindered by a coordination problem and discuss to what extent policies such as government spending may be beneficial, if at all desirable. We first solve for the efficient allocation and describe how it can be implemented using various subsidies. We then consider whether an increase in government spending can be welfare improving when the efficient subsidies are not available.

5.1 Efficient allocation

Our model economy suffers from two inefficiencies. The first inefficiency arises as firms use their monopoly power to price their products at a markup over their marginal cost. As a result, firms produce and sell too little. The second inefficiency is due to the aggregate demand externality which affects the capacity decision. Firms do not internalize that producing at high capacity positively impacts the demand that other producers face and therefore fail to coordinate on the efficient capacity level.

To shed light on these inefficiencies, we solve the problem of a constrained social planner that does not have access to more signals than the private agents and cannot aggregate their information as in Angeletos and Pavan (2007). He can, however, instruct each firm to use the high capacity with some probability $z(v) \in [0,1]$ as a function of its private signal v. With this policy instrument, the planner's problem is

$$
\begin{aligned}
V_{SP}(K,\theta_{-1}) = \max_{0\leqslant z(\cdot)\leqslant 1} \mathbb{E}\Bigg[& \max_{K',L} U\left(\bar{A}\left(\theta,m\right)K^{\alpha}L^{1-\alpha} - m\left(\theta,z\right)f - K' + (1-\delta)K, L\right) \\
& + \beta V_{SP}(K',\theta)\Bigg|\theta_{-1}\Bigg]
\end{aligned}
$$

where

$$
m\left(\theta,z\right) = \int \sqrt{\gamma_v}\phi\left(\sqrt{\gamma_v}\left(v-\theta\right)\right)z(v)dv.
$$

Notice that we use the result that the economy admits aggregation[19] and directly write the planner's problem using the aggregate production function.

We characterize the constrained efficient allocation and its implementation in the following proposition.

Proposition 5. *The competitive equilibrium with incomplete information is inefficient, but the constrained efficient allocation can be implemented with a lump-sum tax on the household, an input*

[19] See proof in Appendix G.

subsidy s_{kl} *and a profit subsidy* s_π *to intermediate goods producers such that* $1 - s_{kl} = \frac{\sigma-1}{\sigma}$ *and* $1 + s_\pi = \frac{\sigma}{\sigma-1}$.

Proposition 5 shows that the constrained efficient allocation can be implemented in the competitive economy using simple instruments that correct the two inefficiencies at their origins.[20]. To offset the distortions induced by the monopoly power, the planner uses an input subsidy s_{kl}, standard in the New-Keynesian literature,[21] to encourage firms to expand to the optimal scale of operation. Despite this input subsidy, firms still operate at a suboptimal capacity level because of the aggregate demand externality and the planner needs an additional instrument to induce the right capacity choice. Perhaps surprisingly, a simple linear profit subsidy s_π is enough to correct this margin in the global game. By increasing profits, this subsidy makes firms internalize the impact of their capacity choice on aggregate demand and incentivizes the adoption of the high capacity. Finally, a non-distortionary lump-sum tax on the household ensures that the government budget constraint balances every period. Note that the implementation is not unique, and we show, in the Appendix, that another implementation based on a single sales subsidy can correct both margins at the same time because of the specific structure implied by the Dixit-Stiglitz model of monopolistic competition.

5.2 Government spending

The optimal implementation result involves the use of input and profit subsidies, which are rarely called for in practice. In the event that such instruments are unavailable to policymakers, perhaps for political economy reasons, we consider the impact of government spending on the economy. Since firms operate at an inefficiently low capacity level in equilibrium, an increase in aggregate demand caused by government spending may, in principle, have a positive impact on welfare by raising the incentives to adopt the high capacity. We investigate this claim in the context of our model.

We find that, in general, government spending is detrimental to welfare because of a crowding out effect that hurts coordination in subsequent periods. Government spending thus creates dynamic welfare losses. We show, in fact, that government spending unambiguously reduces welfare under GHH preferences. However, we find that government spending can be welfare improving in a small region of the state space if the preferences of the household allow for a wealth effect on the labor supply.

We now describe these two channels in our model. To do so, we assume that government spending is pure government consumption that is not valued by the household, and that it is financed through a lump-sum tax on the household.[22]

[20] The problem of the firm with subsidies is written in Appendix D

[21] See for instance Christiano et al. (2011).

[22] As Ricardian equivalence holds in our environment the timing of taxes is irrelevant for the allocation.

Crowding out of private spending

As in the neoclassical growth model, an increase in government spending leads to a reduction in the wealth of the household who, as a result, consumes and invests less in physical capital. Consequently, the amount of capital available in the following periods is reduced and the measure of firms adopting the high capacity decreases, as we showed in proposition 4. Intuitively, a decline in the capital stock raises the rental rate of capital and causes firms to reduce their sizes. Incentives to adopt the high capacity are thus reduced, in contrast to what efficiency would require. In this sense, perhaps in contradiction with the general intuition, the coordination problem magnifies the crowding out effect of government spending.

We can precisely establish this point in our benchmark framework with GHH preferences. Under these preferences, there is no wealth effects on the labor supply and the crowding out effects associated to government spending unambiguously lead to welfare losses.

Proposition 6. *Under GHH preferences, for γ_v large, an unforeseen one-time increase in government spending financed by lump-sum taxes reduces welfare .*

The intuition behind this result is as follows. Under GHH preferences, the equilibrium output and employment only depend on current capital K, productivity θ and the measure of firms with high capacity m. When γ_v is large risk at the time of the capacity choice is irrelevant and the stochastic discount factor is negligible in the surplus expression (15). As a result, in the limit as $\gamma_v \to \infty$, government spending has no impact on the outcome of the coordination game. The measure m remains unaffected and only the crowding out effect remain. Government spending is thus a pure waste of resources.

Wealth effect on the labor supply

When the assumption of GHH preferences is relaxed, the labor supply curve of the household is affected by an increase in government spending. As the household gets poorer, the labor supply goes up, thereby putting downward pressure on wages. With cheaper inputs, firms expand and are more tempted to use the high capacity, which alleviates the coordination problem and may result in welfare gains.

Figure 9 explains the mechanism. The red and the blue curves represent the high and the low equilibria from the model with complete information. The black curves represent the unique equilibrium from the model with incomplete information with and without government spending G. As government spending increases, firms are more tempted to use the high capacity and the shaded zone with multiple equilibria shifts to the left. As a result, the low equilibrium ceases to exist for the range of K in the light grey zone. In the environment with incomplete information, the global game selects an equilibrium between the two equilibria of the complete information setup. The unique equilibrium curve therefore also shifts to the left, from the dashed curve to the solid one. Notice that for values of K in the light grey zone, the resulting increase in the mass of firms using the high capacity increases the endogenous TFP $\bar{A}$ which increases output, consumption and investment.

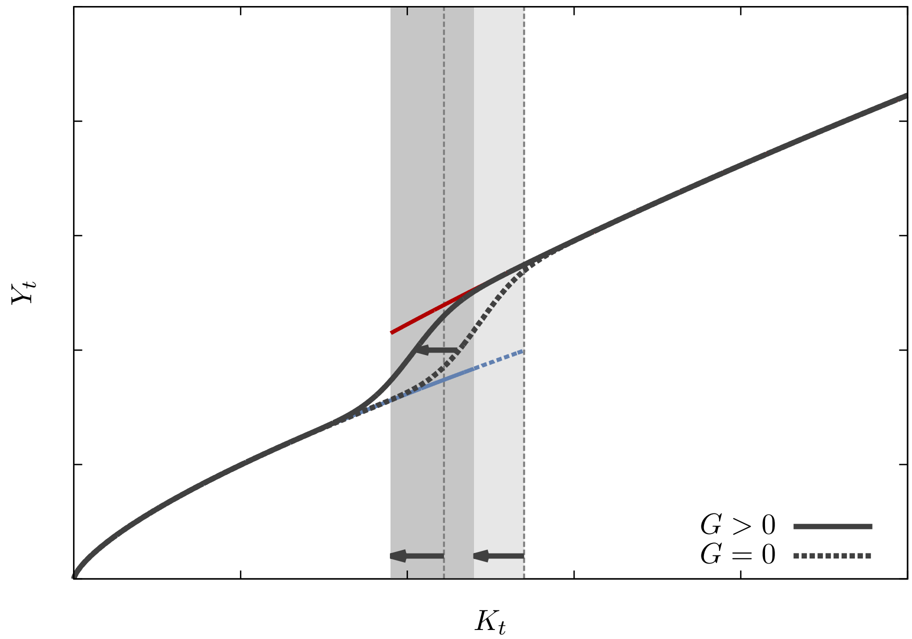


Figure 9: Impact of an increase in government spending on coordination

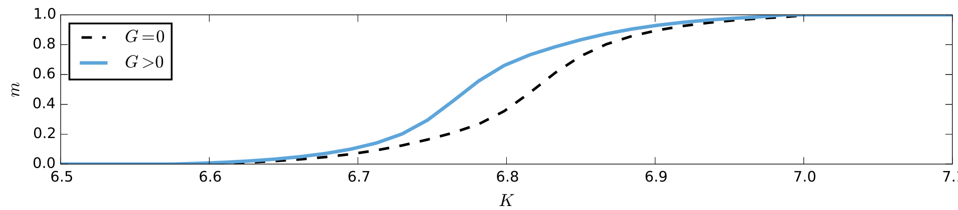
Numerical simulations

To investigate the overall impact of government spending on the economy, we proceed to a series of simulations. To allow for wealth effects on the labor supply, we relax our assumption of GHH preferences and use instead the separable preferences $U(C, L) = \log C - (1+\nu)^{-1} L^{1+\nu}$.[23] This new model is calibrated as the model in section 4. The parameters and the details of the calibration are included in Appendix III. We consider an economy in which government spending G_t is high $G_t = G > 0$ with probability 1/2 and low $G_t = 0$ with probability 1/2. The draws are independent across time. We calibrate G to be equal to 0.5% of the steady-state level of output and we assume that the value of G is revealed to all agents at the beginning of the period.

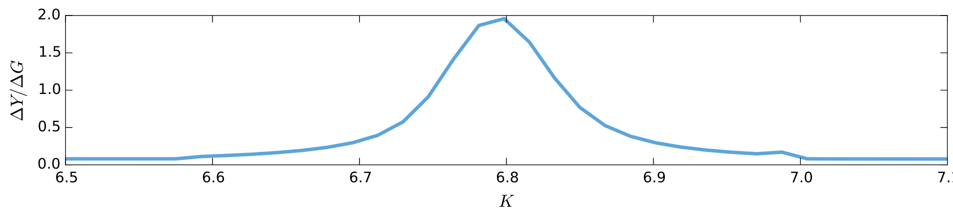
Figure 10 shows the outcome of these simulations. In the top panel, we see that an increase in government spending G helps firms coordinate on the high capacity in some region of the state space. Interestingly, this effect is only present for values of K in which the economy is close to the transition in m between the low and the high regime. Elsewhere, G has little to no impact on coordination. On panel 10b, we see that the interaction of coordination and government spending can give rise to large contemporaneous multipliers for output. When the gains from coordination are large enough

[23]We can no longer derive all our theoretical results under these new preferences, but the model can be solved numerically. We make sure, in particular, that uniqueness still obtains for the global game in our numerical simulations.

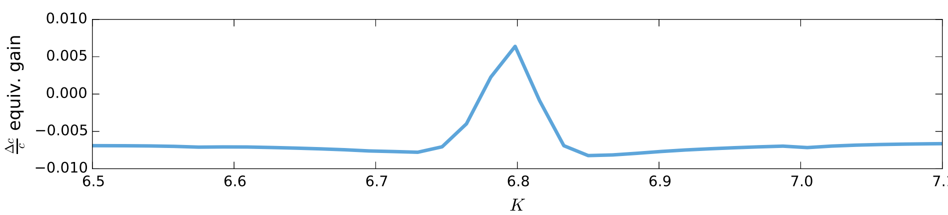
to offset its negative impact on capital accumulation, government spending may improve welfare, as expressed in consumption equivalent terms on panel 10c. Notice that government spending is generally detrimental to welfare, as the dynamic welfare losses coming from the crowding out effect dominate in most of the state space. Only in the region where the economy is close to a transition from the low to the high regime does government spending help coordination sufficiently to improve welfare.



(a) Impact of G on coordination



(b) Fiscal multiplier



(c) Impact of G on welfare

Figure 10: Impact of an increase in government spending

Consistent with our theory, in a recent empirical paper, Auerbach and Gorodnichenko (2012) find that the fiscal multiplier for total government spending in the United States is much larger during recessions. Christiano (2004) and Eggertson (2004) show that New-Keynesian models with a binding zero lower bound on the nominal interest rate can also generate large multipliers in recession. Our mechanism provides a rationale for large multipliers even when the economy is far from the zero lower bound, consistent with the findings of Auerbach and Gorodnichenko.

6 Conclusion

We develop a dynamic stochastic general equilibrium model of business cycles with coordination failures. The model provides an alternative foundation for Keynesian-type demand-deficient equilibria as the economy may fall into long-lasting recessions due to the failure of firms to coordinate on a higher output equilibrium. Government spending is potentially welfare-improving, without relying on nominal rigidities.

Non-convexities in the firm's problem are an essential part of our mechanism. In this paper, we have focused on a simple binary capacity utilization choice, but we believe that the central mechanism of the paper applies to a larger class of non-convexities. For instance, it would be interesting to extend the model to include fixed adjustment costs in capital or labor, which are have been widely documented.

More broadly, we believe that the interaction of non-convexities and complementarities can generate interesting mechanisms in other contexts. For instance, the possibility of falling in the low regime may have interesting asset pricing implications, as we can interpret our model as providing a theory of endogenous rare disasters. Another likely important factor influencing coordination is social learning. In another paper, we consider an environment in which people learn from the actions of others in a coordination game. The interaction of complementaries and social learning give rise to exuberant periods of economic activity followed by brutal crashes.

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Part I

Additional Graphs

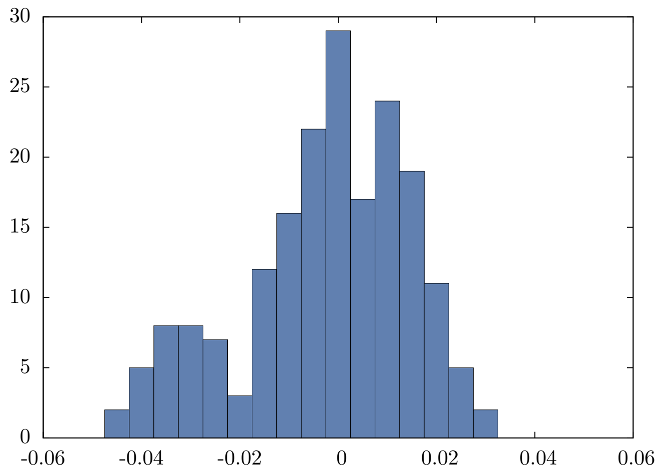

Figure 11: Empirical distribution of log output over 1967-2014 (linearly detrended)

Part II

Data

Table 3 details the sources of the data and how it is treated before analysis. All time series are quarterly from 1967Q1 to 2014Q2. All time series except capacity utilization are seasonally adjusted. For all time series except capacity utilization, we take the log of the data and remove a linear trend.

Table 3: Data sources

Variable	Source
Output	BEA - Real Gross Domestic Product
Investment	BEA - Real Gross Domestic Investment
Hours	BLS - Nonfarm Business sector: Hours of all persons
Consumption	BEA - Real Personal Consumption Expenditures
Capacity utilization	FRB - Capacity Utilization: Total Industry
Total Factor Productivity	Fernald (2014): Business Sector TFP

Part III

Simulation of Government Spending

Table 4 details the parameters of the model used to evaluate the impact of shocks to government spending. Most paper are identical to the calibrated model. The only difference is f which we pick so as to have multiple steady states in K for both values of G. The positive welfare impact of G is robust to reasonable changes to f.

Table 4: Parameters

Parameter	Value	Source/Target
Time period	one quarter	
Total factor productivity	$A = 1$	Normalization
Capital share	$\alpha = 0.3$	Labor share 0.7
Discount factor	$\beta = 0.95^{1/4}$	0.95 annual
Depreciation rate	$\delta = 1 - 0.9^{1/4}$	10% annual
Elasticity of substitution	$\sigma = 3$	Hsieh and Klenow (2014)
Risk aversion	$\gamma = 1$	log utility
Elasticity of labor supply	$\nu = 0.4$	Jaimovich and Rebelo (2009)
Persistence θ process	$\rho_\theta = 0.95$	Cooley and Prescott (1985)
Standard deviation of ϵ_θ	$\gamma_\theta = 300,000$	Standard deviation of log output
Fixed cost	$f = 0.01485$	See text
TFP gain from high capacity	$\omega = 1.017$	See text
Precision of private signal	$\gamma_v = 100,000$	See text
Size of government spending	$G = 0.00665$	0.5% of steady-state output

Part IV

Complete Information: Proofs

A Equilibrium characterization

Proposition 1. *For a given measure m_t of firms with high capacity the equilibrium output of the final good is given by*

$$Y_t = \overline{A}\left(\theta_t, m_t\right) K_t^{\alpha} L_t^{1-\alpha},$$

where $\overline{A}\left(\theta_t, m_t\right) = \left(m_t A_h\left(\theta_t\right)^{\sigma-1} + \left(1 - m_t\right) A_l\left(\theta_t\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}$ *and aggregate labor is*

$$L_t = \left[\left(1-\alpha\right)\frac{\sigma-1}{\sigma}\overline{A}\left(\theta_t, m_t\right) K_t^{\alpha}\right]^{\frac{1}{\alpha+\nu}}.$$

The corresponding production and profit levels of intermediate firms are, for $i \in \{h, l\}$,

$$Y_{it} = \left(\frac{A_i(\theta_t)}{\overline{A}(\theta_t, m_t)} \right)^{\sigma} Y_t \text{ and } \Pi_{it} = \frac{1}{\sigma} \left(\frac{A_i(\theta_t)}{\overline{A}(\theta_t, m_t)} \right)^{\sigma - 1} Y_t.$$

Proof. The household's problem delivers the two standard conditions

$$U_c(C_t, L_t) = \beta \mathbb{E}\left[(R_{t+1} + 1 - \delta) U_c(C_{t+1}, L_{t+1}) \right] \text{ and } L_t^{\nu} = \frac{W_t}{P_t}. \tag{18}$$

The first order conditions for an individual firm of type $i \in \{h, l\}$ in terms of capital and labor are

$$\alpha \frac{\sigma - 1}{\sigma} \frac{P_t Y_t^{\frac{1}{\sigma}} Y_{it}^{1 - \frac{1}{\sigma}}}{K_{it}} = R_t \text{ and } (1 - \alpha) \frac{\sigma - 1}{\sigma} \frac{P_t Y_t^{\frac{1}{\sigma}} Y_{it}^{1 - \frac{1}{\sigma}}}{L_{it}} = W_t. \tag{19}$$

Combining both equations, obtain the expression

$$\frac{\sigma - 1}{\sigma} Y_t^{\frac{1}{\sigma}} Y_{it}^{-\frac{1}{\sigma}} = \frac{1}{A_i(\theta_t)} \left(\frac{R_t}{\alpha} \right)^{\alpha} \left(\frac{W_t}{1 - \alpha} \right)^{1 - \alpha}.$$

Since $\frac{P_{it}}{P_t} = \left(\frac{Y_{it}}{Y_t} \right)^{-\frac{1}{\sigma}}$, we recognize in this expression the optimal strategy for firms to price their products at a constant markup $\frac{\sigma}{\sigma - 1}$ over marginal cost,

$$P_{it} = \frac{\sigma}{\sigma - 1} \frac{1}{A_i(\theta_t)} \left(\frac{R_t}{\alpha} \right)^{\alpha} \left(\frac{W_t}{1 - \alpha} \right)^{1 - \alpha}. \tag{20}$$

The price of the final good is

$$P_t = \left(m_t P_{ht}^{1 - \sigma} + (1 - m_t) P_{lt}^{1 - \sigma} \right)^{\frac{1}{1 - \sigma}} = \frac{\sigma}{\sigma - 1} \frac{1}{\overline{A}(\theta_t, m_t)} \left(\frac{R_t}{\alpha} \right)^{\alpha} \left(\frac{W_t}{1 - \alpha} \right)^{1 - \alpha}. \tag{21}$$

We may then express individual production

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\sigma} Y_t = \left(\frac{A_i(\theta_t)}{\overline{A}(\theta_t, m_t)} \right)^{\sigma} Y_t, \tag{22}$$

and aggregate output of the final good

$$Y_t = \left(m_t Y_{ht}^{\frac{\sigma - 1}{\sigma}} + (1 - m_t) Y_{lt}^{\frac{\sigma - 1}{\sigma}} \right)^{\frac{\sigma}{\sigma - 1}} = \overline{A}(\theta_t, m_t) K_t^{\alpha} L_t^{1 - \alpha}. \tag{23}$$

Derive the factor demands from the first order conditions,

$$K_{it} = \alpha \frac{\sigma-1}{\sigma} \frac{P_t Y_t^{\frac{1}{\sigma}} Y_{it}^{1-\frac{1}{\sigma}}}{R_t} = \alpha \frac{\sigma-1}{\sigma} \left(\frac{A_i(\theta_t)}{\overline{A}(\theta_t, m_t)} \right)^{\sigma-1} \frac{P_t Y_t}{R_t}$$
$$L_{it} = (1-\alpha) \frac{\sigma-1}{\sigma} \left(\frac{A_i(\theta_t)}{\overline{A}(\theta_t, m_t)} \right)^{\sigma-1} \frac{P_t Y_t}{W_t}.$$

Market clearing on the factor markets imply

$$K_t = m_t K_{ht} + (1-m_t) K_{lt} = \alpha \frac{\sigma-1}{\sigma} \frac{P_t Y_t}{R_t}$$
$$L_t = m_t L_{ht} + (1-m_t) L_{lt} = (1-\alpha) \frac{\sigma-1}{\sigma} \frac{P_t Y_t}{W_t}.$$

The equilibrium level of labor as a function of m_t can be obtained by combining the household's labor supply equation to the aggregate labor demand:

$$L_t = \left(\frac{W_t}{P_t} \right)^{\frac{1}{\nu}} = \left((1-\alpha) \frac{\sigma-1}{\sigma} \frac{Y_t}{L_t} \right)^{\frac{1}{\nu}},$$

which delivers the equilibrium labor and output levels

$$L_t = \left[(1-\alpha) \frac{\sigma-1}{\sigma} \overline{A}(\theta_t, m_t) K_t^{\alpha} \right]^{\frac{1}{\alpha+\nu}} \tag{24}$$
$$Y_t = \left((1-\alpha) \frac{\sigma-1}{\sigma} \right)^{\frac{1-\alpha}{\alpha+\nu}} \left(\overline{A}(\theta_t, m_t) K_t^{\alpha} \right)^{\frac{1+\nu}{\alpha+\nu}}. \tag{25}$$

Finally, we may now derive expressions for individual profits:

$$\Pi_{it} = P_{it} Y_{it} - R_t K_{it} - W_t L_{it} = \frac{1}{\sigma} P_{it} Y_{it} = \frac{1}{\sigma} \left(\frac{A_i(\theta_t)}{\overline{A}(\theta_t, m_t)} \right)^{\sigma-1} P_t Y_t. \tag{26}$$

□

B Multiplicity of equilibria

Proposition 2. *Consider the following condition on parameters:*

$$\frac{1+\nu}{\alpha+\nu} > \sigma - 1. \tag{13}$$

Under condition (13), there exist thresholds $B_H < B_L$ such that:
i) if $Ae^{\theta} K^{\alpha} < B_H$, the static equilibrium is unique and such that all firms choose low capacity, $m = 0$;
ii) if $Ae^{\theta} K^{\alpha} > B_L$, the static equilibrium is unique and such that all firms choose high capacity,

$m = 1$;

iii) if $B_H \leqslant Ae^{\theta}K^{\alpha} \leqslant B_L$, there are three static equilibria: two in pure strategies, $m = 1$ and $m = 0$, and one in mixed strategies, $m \in (0,1)$.

If condition (13) is not satisfied, the static equilibrium is always unique.

Proof. Substituting in the equilibrium profit functions derived in proposition 1, the capacity decision problem becomes

$$u_{it} = \underset{u_i \in \{u_h, u_l\}}{\operatorname{argmax}} \left\{ \frac{1}{\sigma} \left(\frac{A_h(\theta_t)}{\overline{A}(\theta_t, m_t)} \right)^{\sigma-1} Y_t - f, \frac{1}{\sigma} \left(\frac{A_l(\theta_t)}{\overline{A}(\theta_t, m_t)} \right)^{\sigma-1} Y_t \right\}.$$

The capacity decision is governed by the sign of the surplus from choosing high capacity which we define as

$$\Delta\Pi(K, \theta, m) \equiv \Pi_h - \Pi_l - f = \frac{1}{\sigma} \frac{A_h(\theta)^{\sigma-1} - A_l(\theta)^{\sigma-1}}{\overline{A}(\theta, m)^{\sigma-1}} \left((1-\alpha) \frac{\sigma-1}{\sigma} \right)^{\frac{1-\alpha}{\alpha+\nu}} \left(\overline{A}(\theta, m) K^{\alpha} \right)^{\frac{1+\nu}{\alpha+\nu}} - f.$$

The economy admits a *pure strategy equilibrium with high capacity* if and only if $\Delta\Pi(K, \theta, 1) \geqslant 0$, which is equivalent to

$$\frac{1}{\sigma} \frac{A_h(\theta)^{\sigma-1} - A_l(\theta)^{\sigma-1}}{A_h(\theta)^{\sigma-1}} \left((1-\alpha) \frac{\sigma-1}{\sigma} \right)^{\frac{1-\alpha}{\alpha+\nu}} \left(A_h(\theta) K^{\alpha} \right)^{\frac{1+\nu}{\alpha+\nu}} - f \geqslant 0.$$

A high equilibrium exists if and only if the following condition is satisfied:

$$Ae^{\theta}K^{\alpha} \geqslant \frac{1}{\omega} \frac{\sigma}{\sigma-1} \left(\frac{(\sigma-1)f}{1-\omega^{1-\sigma}} \right)^{\frac{\alpha+\nu}{1+\nu}} \equiv B_H.$$

Similarly, there exists a *pure strategy equilibrium with low capacity utilization* if and only if $\Delta\Pi(K, \theta, 0) \leqslant 0$, which is equivalent to

$$\frac{1}{\sigma} \frac{A_h(\theta)^{\sigma-1} - A_l(\theta)^{\sigma-1}}{A_l(\theta)^{\sigma-1}} \left((1-\alpha) \frac{\sigma-1}{\sigma} \right)^{\frac{1-\alpha}{\alpha+\nu}} \left(A_l(\theta) K^{\alpha} \right)^{\frac{1+\nu}{\alpha+\nu}} - f \leqslant 0.$$

A low equilibrium exists if and only if the following condition is satisfied:

$$A(\theta) K^{\alpha} \leqslant \frac{\sigma}{\sigma-1} \left(\frac{(\sigma-1)f}{\omega^{\sigma-1}-1} \right)^{\frac{\alpha+\nu}{1+\nu}} (1-\alpha)^{-\frac{1-\alpha}{1+\nu}} \equiv B_L.$$

The thresholds are such that $B_L > B_H$ if and only $\frac{1+\nu}{\alpha+\nu} > \sigma - 1$.

Next, let us consider the mixed strategy equilibrium. Firms are indifferent between both capacities if

$$\frac{1}{\sigma}\frac{A_h\left(\theta\right)^{\sigma-1}-A_l\left(\theta\right)^{\sigma-1}}{\overline{A}\left(\theta,m\right)^{\sigma-1}}\left(\left(1-\alpha\right)\frac{\sigma-1}{\sigma}\right)^{\frac{1-\alpha}{\alpha+\nu}}\left(\overline{A}\left(\theta,m\right)K^{\alpha}\right)^{\frac{1+\nu}{\alpha+\nu}}-f=0.$$

There is a mixed strategy equilibrium if there is a solution to this equation with $m\in(0,1)$. We can rewrite this equation as

$$Ae^{\theta}K^{\alpha}=\frac{\sigma}{\sigma-1}\left(\frac{\left(\sigma-1\right)f}{\omega^{\sigma-1}-1}\right)^{\frac{\alpha+\nu}{1+\nu}}\left(1-\alpha\right)^{-\frac{1-\alpha}{1+\nu}}\left(m\left(\omega^{\sigma-1}-1\right)+1\right)^{\frac{\alpha+\nu}{1+\nu}-\frac{1}{\sigma-1}}. \tag{27}$$

If $\frac{1+\nu}{\alpha+\nu}>\sigma-1$, the right-hand side is strictly decreasing in m and equals B_L for $m=0$ and B_H for $m=1$. Therefore, as long as $B_H<Ae^{\theta}K^{\alpha}<B_L$ there is a mixed strategy equilibrium in addition to the two others. Notice that the equilibrium m is decreasing in $Ae^{\theta}K^{\alpha}$.
If $\frac{1+\nu}{\alpha+\nu}\leqslant\sigma-1$, then $B_L\leqslant B_H$ and the right-hand side of (27) is increasing in m from B_L to B_H. There is therefore a unique static equilibrium for all $Ae^{\theta}K^{\alpha}$. □

C Efficiency

Proposition 3. *If $\frac{1+\nu}{\nu+\alpha}>\sigma-1$, there exists a threshold B_{SP}, with $B_{SP}\leq B_L$, such that the planner makes all firms use the high capacity, $m_t=1$, if $Ae^{\theta_t}K_t^{\alpha}\geqslant B_{SP}$ and firms use the low capacity, $m_t=0$, if $Ae^{\theta_t}K_t^{\alpha}\leqslant B_{SP}$. The threshold B_{SP} is lower than B_H for σ small.*

Proof. Consider the planning problem:

$$\max_{K_{t+1},L_t,m_t}\mathbb{E}\sum_{t=0}^{\infty}\beta^t U\left(\left(\int_0^1 m_t Y_{ht}^{\frac{\sigma-1}{\sigma}}+\left(1-m_t\right)Y_{lt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}-\left(1-\delta\right)K_t-m_t f-K_{t+1},L_t\right)$$

subject to

$$\begin{aligned}Y_{it}&=A_i\left(\theta_t\right)K_{it}^{\alpha}L_{it}^{1-\alpha},i\in\{h,l\}\\K_t&=m_tK_{ht}+\left(1-m_t\right)K_{lt}\\L_t&=m_tL_{ht}+\left(1-m_t\right)L_{lt}.\end{aligned}$$

In the optimal allocation, the marginal products are equalized across firms:

$$\alpha\frac{Y_t^{\frac{1}{\sigma}}Y_{ht}^{1-\frac{1}{\sigma}}}{K_{ht}}=\alpha\frac{Y_t^{\frac{1}{\sigma}}Y_{lt}^{1-\frac{1}{\sigma}}}{K_{lt}}\text{ and }\left(1-\alpha\right)\frac{Y_t^{\frac{1}{\sigma}}Y_{ht}^{1-\frac{1}{\sigma}}}{L_{ht}}=\left(1-\alpha\right)\frac{Y_t^{\frac{1}{\sigma}}Y_{lt}^{1-\frac{1}{\sigma}}}{L_{lt}}.$$

Combining these two equations, we obtain the same aggregation result that we derived in proposition (1), i.e., $Y_{it}=\left(\frac{A_i(\theta_t)}{\overline{A}(\theta_t,m_t)}\right)^{\sigma}Y_t$ and $Y_t=\overline{A}\left(\theta_t,m_t\right)K_t^{\alpha}L_t^{1-\alpha}$. The planner's problem then reduces to

$$\max_{K_{t+1},L_t,m_t} \mathbb{E}\sum_{t=0}^{\infty}\beta^t U\left(\overline{A}\left(\theta_t,m_t\right)K_t^{\alpha}L_t^{1-\alpha}-\left(1-\delta\right)K_t-m_tf-K_{t+1},L_t\right).$$

The optimality for labor is

$$\left(1-\alpha\right)\overline{A}\left(\theta_t,m_t\right)K_t^{\alpha}L_t^{-\alpha}=L_t^{\nu}.$$

In particular, the planner's problem inherits the structure of the competitive economy, which can be solved in two stages thanks to the GHH preferences. Solving for L_t, one obtains

$$L_t=\left[\left(1-\alpha\right)\overline{A}\left(\theta_t,m_t\right)K_t^{\alpha}\right]^{\frac{1}{\alpha+\nu}},$$

a similar expression as (11) for the competitive equilibrium, except that the monopoly distortions, captured by $\frac{\sigma}{\sigma-1}$ do not distort the factor demand. Plugging this expression back in the objective function and noticing that $Y_t-\frac{L_t^{1+\nu}}{1+\nu}=\frac{\alpha+\nu}{1+\nu}Y_t$, the planner's problem can be rewritten as

$$\max_{K_{t+1},m_t}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\frac{1}{1-\gamma}\left(\frac{\alpha+\nu}{1+\nu}\left(1-\alpha\right)^{\frac{1-\alpha}{\alpha+\nu}}\left(\overline{A}\left(\theta_t,m_t\right)K_t^{\alpha}\right)^{\frac{1+\nu}{\alpha+\nu}}-m_tf-\left(1-\delta\right)K_t-K_{t+1}\right)^{1-\gamma},$$

so that the maximization over m_t boils down to maximizing production net of disutility of labor,

$$\frac{\alpha+\nu}{1+\nu}\left(1-\alpha\right)^{\frac{1-\alpha}{\alpha+\nu}}\left(\overline{A}\left(\theta_t,m_t\right)K_t^{\alpha}\right)^{\frac{1+\nu}{\alpha+\nu}}-m_tf.$$

This problem is strictly convex in m when $\frac{1+\nu}{\alpha+\nu}>\sigma-1$, so that the planner always picks a corner solution $m_t=0$ or $m_t=1$. Comparing both values, the planner uses high capacity if and only if

$$\frac{\alpha+\nu}{1+\nu}(1-\alpha)^{\frac{1-\alpha}{\alpha+\nu}}\left(A_h\left(\theta_t\right)K_t^{\alpha}\right)^{\frac{1+\nu}{\alpha+\nu}}-f\geqslant\frac{\alpha+\nu}{1+\nu}(1-\alpha)^{\frac{1-\alpha}{\alpha+\nu}}\left(A_l\left(\theta_t\right)K_t^{\alpha}\right)^{\frac{1+\nu}{\alpha+\nu}},$$

which is equivalent to the condition

$$Ae^{\theta_t}K_t^{\alpha}\geqslant\left(\frac{1}{(1-\alpha)^{\frac{1-\alpha}{\alpha+\nu}}}\frac{1+\nu}{\alpha+\nu}\frac{f}{\omega^{\frac{1+\nu}{\alpha+\nu}}-1}\right)^{\frac{\alpha+\nu}{1+\nu}}\equiv B_{SP},$$

where B_{SP} is a threshold such that the planner picks the high capacity if and only if $Ae^{\theta_t}K_t^{\alpha}\geq B_{SP}$. First, let us show that $B_{SP}\leq B_L$:

$$B_{SP}\quad\leq\quad B_L\Leftrightarrow\frac{1+\nu}{\alpha+\nu}\frac{1}{\omega^{\frac{1+\nu}{\alpha+\nu}}-1}\leq\left(\frac{\sigma}{\sigma-1}\right)^{\frac{1+\nu}{\alpha+\nu}}\frac{\sigma-1}{\omega^{\sigma-1}-1}$$

which is satisfied if

$$\frac{1+\nu}{\alpha+\nu}\frac{1}{\omega^{\frac{1+\nu}{\alpha+\nu}}-1}\leq\frac{\sigma-1}{\omega^{\sigma-1}-1}.$$

Function $f(x) = \frac{1}{x}(\omega^x - 1)$ being increasing, we then conclude that $B_{SP} < B_L$ under the condition that $\frac{1+\nu}{\alpha+\nu} > \sigma - 1$.

Let us now compare B_{SP} and B_H:

$$B_{SP} \quad \leq \quad B_H \Leftrightarrow \frac{1+\nu}{\alpha+\nu} \frac{1}{\omega^{\frac{1+\nu}{\alpha+\nu}} - 1} \leq \left(\frac{\sigma}{\sigma-1}\right)^{\frac{1+\nu}{\nu+\alpha}} \frac{1}{\omega^{\frac{1+\nu}{\alpha+\nu} - \sigma + 1}} \frac{\sigma - 1}{\omega^{\sigma-1} - 1}.$$

The left-hand side is independent of σ. Since $\lim_{\sigma \to 1} \frac{\omega^{\sigma-1}-1}{\sigma-1} = \log \omega$, the right-hand side goes to ∞ as $\sigma \to 1$ from above and $B_{SP} \leq B_H$ for σ small enough. □

Part V

Incomplete Information: Proofs

D Notation and Definitions

This section introduces some useful notation and restates various equilibrium results established in the paper when the economy is subject to an input subsidy s_{kl}, a sales subsidy s_y, a profit subsidy s_π and a lump-sum tax on the household to finance the subsidies. Under these subsidies the problem of the firm becomes

$$\Pi_{it} = \max_{Y_{it}, P_{it}, K_{it}, L_{it}} (1 + s_y) P_{it} Y_{it} - (1 - s_{kl}) (R_t K_{it} + W_t L_{it})$$

subject to 4 and 5 and where the capacity choice is such that

$$u_j = u_h \Longleftrightarrow \mathbb{E}_\theta \left[U_c (C_t, L_t) ((1 + s_\pi) (\Pi_h (K_t, \theta_t, m_t) - \Pi_l (K_t, \theta_t, m_t)) - f) \mid \theta_{t-1}, v_{jt}\right] \geqslant 0.$$

D.1 Notation

As introduced in the main text, we denote by $\overline{A}$ the endogenous aggregate TFP of the economy, with

$$\overline{A}(\theta, m) \equiv A e^\theta \Omega(m),$$

where $\Omega(m) \equiv \left(m\left(\omega^{\sigma-1} - 1\right) + 1\right)^{\frac{1}{\sigma-1}}$ is an average of capacity across firms. The equilibrium level of output and labor as a function of K, θ and m is

$$Y(K, \theta, m) \equiv \left[(1 - \alpha) \frac{\sigma - 1}{\sigma} \frac{1 + s_y}{1 - s_{kl}}\right]^{\frac{1-\alpha}{\alpha+\nu}} \overline{A}(\theta, m)^{\frac{1+\nu}{\alpha+\nu}} K^{\alpha \frac{1+\nu}{\alpha+\nu}},$$

and labor

$$L(K, \theta, m) \equiv \left[(1 - \alpha) \frac{\sigma - 1}{\sigma} \frac{1 + s_y}{1 - s_{kl}}\right]^{\frac{1}{\alpha+\nu}} \overline{A}(\theta, m)^{\frac{1}{\alpha+\nu}} K^{\frac{\alpha}{\alpha+\nu}}.$$

The corresponding rental rate of capital is

$$R(K,\theta,m) \equiv \alpha \frac{\sigma-1}{\sigma} \frac{1+s_y}{1-s_{kl}} \frac{Y(K,\theta,m)}{K}.$$

To lighten notation, it is also useful to introduce the gross output level net of fixed costs and depreciation

$$y(K,\theta,m) \equiv Y(K,\theta,m) + (1-\delta) K - mf,$$

and the corresponding net interest rate

$$r(K,\theta,m) \equiv R(K,\theta,m) + 1 - \delta.$$

Finally, we denote the equilibrium output level for firms of type h and l by

$$Y_h(K,\theta,m) \equiv \frac{\omega^\sigma}{\Omega(m)^\sigma} Y(K,\theta,m) \text{ and } Y_l(K,\theta,m) \equiv \frac{1}{\Omega(m)^\sigma} Y(K,\theta,m),$$

and profit rates

$$\Pi_h(K,\theta,m) \equiv \frac{1}{\sigma} \frac{1+s_y}{1-s_{kl}} \frac{\omega^{\sigma-1}}{\Omega(m)^{\sigma-1}} Y(K,\theta,m) \text{ and } \Pi_l(K,\theta,m) \equiv \frac{1}{\sigma} \frac{1+s_y}{1-s_{kl}} \frac{1}{\Omega(m)^{\sigma-1}} Y(K,\theta,m).$$

We sometimes abuse notation in part F of the proofs, once conditions for existence and uniqueness of a solution to the global game have been established, by writing $Y(K,\theta_{-1},\theta) = Y(K,\theta,m(K,\theta_{-1},\theta))$, $R(K,\theta_{-1},\theta) = R(K,\theta,m(K,\theta_{-1},\theta))$, and so on. Furthermore, we use the vector notation $\boldsymbol{\theta} = (\theta_{-1},\theta)'$ in several parts of the proofs to avoid spelling out the entire state space.

D.2 Assumptions and Definitions

Our existence and uniqueness proofs require our value and policy functions to be bounded. We thus restrict our fundamental to remain between two bounds $[\underline{\theta},\overline{\theta}]$, chosen large enough that they contain the most part of the ergodic distribution of θ.[24]

Definition 3. Let $\Theta = [\underline{\theta},\overline{\theta}]$. Fundamental θ follows the autoregressive process

$$\theta = \min\left(\max\left(\rho\theta_{-1} + e_t^\theta, \underline{\theta}\right), \overline{\theta}\right),$$

and we denote its transition density $\pi(\theta, d\theta') = Pr\{\theta_{t+1} \in [\theta', \theta' + d\theta'] \mid \theta_t = \theta\}$.

Definition 4. Let $\mathbb{K} = [0, \overline{K}]$ where $\overline{K}$ is implificitly defined by

$$Y\left(\overline{K},\overline{\theta},1\right) + (1-\delta)\overline{K} - \frac{L\left(\overline{K},\overline{\theta},1\right)^{1+\nu}}{1+\nu} = \overline{K}.$$

[24] For arbitrarily large bounds, this restriction has no bearing on our quantitative results and the Bayesian updating rules for untruncated normals, that we use to update private beliefs in the static global game, provide an arbitrarily good approximation to the true beliefs with truncated normals for the relevant part of the ergodic distribution of θ.

Definition 4 defines the set in which the stock of capital lies and $\overline{K}$ which corresponds to the maximal output ever achievable is an upper bound on capital. The upper bound $\overline{K}$ exists and is unique under assumption 1 below.

Assumption 1. *The parameters satisfy*

$$\frac{\sigma-1}{\sigma}\frac{1-\alpha}{1+\nu}\frac{1+s_y}{1-s_{kl}}\leqslant 1.$$

Assumption 1 is a feasibility condition required by the GHH preferences. It guarantees that total production, $y(K,\boldsymbol{\theta})$, net of minimum consumption, $\frac{L(K,\boldsymbol{\theta})^{1+\nu}}{1+\nu}$, is positive, so that there exists a solution to the static equiibrium in production.

Assumption 2. *The lower bound $\underline{\theta}$ is chosen sufficiently small that there exists $K^->0$ such that $y(K^-,\underline{\theta},\underline{\theta})-\frac{L(K^-,\underline{\theta},\underline{\theta})^{1+\nu}}{1+\nu}>K^-$ and $\beta\mathbb{E}\left[r(K^-,\underline{\theta},\theta')\mid\theta=\underline{\theta}\right]\leqslant 1$.*

Assumption 2 is due to Coleman (1991) and is necessary to show the existence of a non-zero equilibrium. Note that there always exists a K^- that satisfies the first part of the definition given our choice for the production function and $m\simeq 0$. The key requirement comes from the second part and can be achieved by assuming that $\underline{\theta}$ is sufficiently low.

D.3 Main proposition

Proposition 4. *For γ_v large and f sufficiently small such that, in particular, the following condition holds,*

$$\frac{\sqrt{\gamma_v}}{\gamma_\theta}>\frac{1}{\sqrt{2\pi}}\frac{\omega^{\sigma-1}-1}{\sigma-1},\qquad(17)$$

there exists a unique dynamic equilibrium. The equilibrium capacity decision takes the form of a continuous cutoff $\hat{v}(K,\theta_{-1})$ such that firm j invests if and only if $v_j\geqslant\hat{v}(K,\theta_{-1})$. Furthermore, the cutoff is a decreasing function of its arguments.

Proof. We prove this proposition in several steps. In a first step, we show in section E below that, for a small departure from complete information, i.e., when γ_v is large, risk becomes irrelevant for firms and the stochastic discount factor drops from their capacity decision (lemma A1). In that case, we can solve the global game independently from the rest of the dynamic equilibrium. In proposition A1, we show that there exists a unique equilibrium to the global game under condition 17 and that the equilibrium capacity decision takes the form of a cutoff strategy $\hat{v}(K,\theta_{-1})$ such that firms choose high capacity if and only if they receive a signal v_j above that threshold.

Using the resulting $\hat{v}$ from the global game, we show the existence and uniqueness of the dynamic equilibrium in section F. Proposition A2 establishes the existence under several additional assumptions. First, assumption 1 ensures that the firm's decision is well defined and bounded by putting an upper bound on the subsidies that they receive. It is trivially satisfied for a competitive economy without government subsidies, but the proposition shows how the result extends to economies with input, sales and profit subsidies. Second, the condition that f is small ensures that aggregate production net of fixed costs is always increasing in K. This condition is sufficient but not necessary. It is required by our proof strategy which exploits the monotonicity of the Euler equation. Finally, the proposition uses assumption 2 which is relatively mild as it only requires us to choose a sufficiently low bound $\underline{\theta}$. Notice also that the proposition provides the existence of a strictly positive equilibrium, in the sense that consumption is non-zero whenever $K > 0$. Finally, proposition A3 establishes uniqueness of the (strictly positive) equilibrium under the same conditions on the parameters. □

E Global game

E.1 Description

This section describes the solution of the static game played every period between the intermediate goods producers.

The decision of intermediate producer j to operate at high capacity over low capacity is determined by the sign of the surplus,

$$\Delta\Pi\left(K,\theta_{-1},v_j,m\right)=\mathbb{E}_\theta\left[U_C\left(C,L\right)\left(\left(1+s_\pi\right)\left(\Pi_h\left(K,\theta,m\right)-\Pi_l\left(K,\theta,m\right)\right)-f\right)\mid\theta_{-1},v_j\right],$$

such that producer j chooses high capacity if and only if $\Delta\Pi \geqslant 0$. In equilibrium, C and L are functions of the aggregate state space (K,θ_{-1},θ), which we sometimes write $(K,\boldsymbol{\theta})$ with vector $\boldsymbol{\theta}=(\theta_{-1},\theta)'$. Anticipating on the rest of the proof, let us denote the inverse marginal utility of consumption $P\left(K,\boldsymbol{\theta}\right)=\left[U_C\left(C\left(K,\boldsymbol{\theta}\right),L\left(K,\boldsymbol{\theta}\right)\right)\right]^{-1}$. Substituting with the equilibrium value of profits, the expected surplus from operating at high vs. low capacity for firm j with a perceived mass of entrants $m\left(K,\theta,\theta_{-1}\right)$ is

$$\Delta\Pi\left(K,\theta_{-1},v_j,m\right)=\mathbb{E}_\theta\left[\frac{1}{P\left(K,\boldsymbol{\theta}\right)}\left(\frac{1}{\sigma}\frac{\left(1+s_\pi\right)\left(1+s_y\right)}{1-s_{kl}}\frac{\omega^{\sigma-1}-1}{\Omega\left(m\right)^{\sigma-1}}Y\left(K,\theta,m\right)-f\right)\mid\theta_{-1},v_j\right]. \tag{28}$$

The presence of the stochastic discount factor in the problem of the firm introduces an additional complication in comparison to standard global games without general equilibrium effects. Fortunately, under the assumption that γ_v is large, i.e., for a small deviation from common knowledge, the stochastic discount factor drops out of the equation and is, thus, asymptotically irrelevant. The outcome of the global game may thus be approximated by a simpler problem, which we describe below.

E.2 Approximation

Lemma A1. *Let $P : (K, \boldsymbol{\theta}) \in \mathbb{K} \times \Theta^2 \to \mathbb{R}$ bounded, continuous, positive and bounded away from 0 over $\left[\underline{K}, \overline{K}\right] \times \Theta^2$ for all $\underline{K} > 0$. Then for all $\hat{v} \in \mathbb{R}$,*

$$\left|\mathbb{E}_\theta\left[\frac{(1+s_\pi)\left(\Pi_h(K,\theta,m)-\Pi_l(K,\theta,m)\right)-f}{P(K,\theta_{-1},\theta)} \mid \theta_{-1}, v_j\right] - \mathbb{E}_\theta\left[\frac{(1+s_\pi)\left(\Pi_h(K,\theta,m)-\Pi_l(K,\theta,m)\right)-f}{P\left(K,\theta_{-1},\frac{\gamma_\theta\rho\theta_{-1}+\gamma_v v_j}{\gamma_\theta+\gamma_v}\right)} \mid \theta_{-1}, v_j\right]\right| \underset{\gamma_v\to\infty}{\longrightarrow} 0.$$

Furthermore, the convergence is uniform over $\left[\underline{K}, \overline{K}\right] \times \Theta^2$.

Proof. To lighten notation, denote

$$\Delta Y(K,\theta,m) \equiv (1+s_\pi)\left(\Pi_h(K,\theta,m)-\Pi_l(K,\theta,m)\right)-f = \frac{1}{\sigma}\frac{(1+s_\pi)(1+s_y)}{1-s_{kl}}\frac{\omega^{\sigma-1}-1}{\Omega(m)^{\sigma-1}}Y(K,\theta,m)-f.$$

Since $\theta \mid \theta_{-1}, v_j \sim \mathcal{N}\left(\frac{\gamma_\theta\rho\theta_{-1}+\gamma_v v_j}{\gamma_\theta+\gamma_v}, \frac{1}{\gamma_\theta+\gamma_v}\right)$, we can control the above expression as follows:

$$\left|\mathbb{E}_\theta\left[\frac{\Delta Y(K,\theta,m)}{P(K,\theta_{-1},\theta)} \mid \theta_{-1}, v_j\right] - \mathbb{E}_\theta\left[\frac{\Delta Y(K,\theta,m)}{P\left(K,\theta_{-1},\frac{\gamma_\theta\rho\theta_{-1}+\gamma_v v_j}{\gamma_\theta+\gamma_v}\right)} \mid \theta_{-1}, v_j\right]\right|$$
$$\leqslant \overline{\Delta Y}\frac{\overline{P}_\theta}{\inf_\theta P(K,\theta_{-1},\theta)^2}\mathbb{E}_\theta\left[\left|\theta - \frac{\gamma_\theta\rho\theta_{-1}+\gamma_v v_j}{\gamma_\theta+\gamma_v}\right| \mid \theta_{-1}, v_j\right] \leqslant \overline{\Delta Y}\frac{\overline{P}_\theta}{\inf_\theta P(K,\theta_{-1},\theta)^2}\frac{1}{\gamma_\theta+\gamma_v}\sqrt{\frac{2}{\pi}},$$

where $\overline{P}_\theta$ is the modulus of uniform continuity of P along θ. Therefore, we have pointwise convergence and uniform convergence on all segments $\left[\underline{K}, \overline{K}\right]$ with $\underline{K} > 0$ with the uniform bound $\overline{\Delta Y}\frac{\overline{P}_\theta}{\inf_{[\underline{K},\overline{K}]\times\Theta^2} P(\underline{K},\theta_{-1},\theta)^2}\frac{1}{\gamma_\theta+\gamma_v}\sqrt{\frac{2}{\pi}}$. □

Choosing the bound $\underline{K}$ low enough that $\left[\underline{K}, \overline{K}\right]$ contains all the stocks of capital ever visited along the equilibrium path, the above lemma tells us that, in the limit as $\gamma_v \to \infty$, we can approximate the surplus from choosing high capacity by the simpler expression:

$$\Delta\tilde{\Pi}(K,\theta_{-1},v_j,m) \equiv \frac{1}{P\left(K,\theta_{-1},\frac{\gamma_\theta\rho\theta_{-1}+\gamma_v v_j}{\gamma_\theta+\gamma_v}\right)}\mathbb{E}_\theta\left[\Delta Y(K,\theta,m) \mid \theta_{-1}, v_j\right].$$

The intuition behind this expression is that, as $\gamma_v \to \infty$, consumption risk vanishes and becomes irrelevant to firms. This does not mean, however, that uncertainty is unimportant: the firms' decision is then entirely driven by strategic concerns, captured by $E_\theta\left[\Delta Y \mid \theta_{-1}, v_j\right]$, in which uncertainty plays a crucial role.

We focus, from now on, on the cases where γ_v is high and the above approximation holds. Under that assumption, the decision of firm j is

$$u_j = u_h \Leftrightarrow \mathbb{E}_\theta\left[\Delta Y(K,\theta,m) \mid \theta_{-1}, v_j\right] \geqslant 0. \tag{29}$$

It is worth noting here that our numerical results suggest so far that the approximation is very accurate: under our benchmark calibration, the solutions of the global game using expression (28) and (29) are virtually indistinguishable.

E.3 Existence and Uniqueness

Proposition A1. *For γ_v large enough that approximation (29) holds and*

$$\frac{\sqrt{\gamma_v}}{\gamma_\theta} > \frac{1}{\sqrt{2\pi}} \frac{\omega^{\sigma-1} - 1}{\sigma - 1}, \tag{17}$$

then the optimal capacity decision takes the form of a unique cutoff strategy $\hat{v}(K, \theta_{-1})$ such that firm j chooses high capacity if and only if $v_j \geqslant \hat{v}$.

Proof. Fix $K \in \mathbb{K}$ and $\theta_{-1} \in \Theta$. Under the hypothesis that γ_v is large enough that the approximation (29) holds, firm j chooses high capacity if and only if

$$\Delta\tilde{\Pi}(K, \theta_{-1}, v_j, m) \geqslant 0 \Leftrightarrow \mathbb{E}_\theta \left[\frac{1}{\sigma} \frac{(1+s_\pi)(1+s_y)}{1-s_{kl}} \frac{\omega^{\sigma-1}-1}{\Omega(m)^{\sigma-1}} Y(K, \theta, m) - f \mid \theta_{-1}, v_j \right] \geqslant 0.$$

The proof proceeds in two steps. In a first step, we start solving the game by iterated deletion of dominated strategies. In a second step, we provide conditions under which this procedure converges towards a unique equilibrium.

■ Case $\frac{1+\nu}{\alpha+\nu} > \sigma - 1$

Step 1. To lighten notation, denote

$$\begin{aligned}\Delta Y(K, \theta, m) &\equiv \frac{1}{\sigma} \frac{(1+s_\pi)(1+s_y)}{1-s_{kl}} \frac{\omega^{\sigma-1}-1}{\Omega(m)^{\sigma-1}} Y(K, \theta, m) - f \\ &= \frac{1}{\sigma} \frac{(1+s_\pi)(1+s_y)}{1-s_{kl}} \left(\omega^{\sigma-1} - 1\right) \left[(1-\alpha) \frac{\sigma-1}{\sigma} \frac{1+s_y}{1-s_{kl}}\right]^{\frac{1-\alpha}{\alpha+\nu}} \left(Ae^\theta K^\alpha\right)^{\frac{1+\nu}{\alpha+\nu}} \Omega(m)^{\frac{1+\nu}{\alpha+\nu} - \sigma + 1} - f.\end{aligned}$$

When $\frac{1+\nu}{\alpha+\nu} > \sigma - 1$, ΔY is increasing in all its arguments. We proceed by iterated deletion of dominated strategies. Initialize the recursion by defining $\hat{v}^0 = \infty$ and $\hat{v}_0 = -\infty$, such that it is dominant to choose the high capacity for $v_j \geqslant \hat{v}^0$ and dominant to choose low capacity for $v_j \leqslant \hat{v}_0$. We now define $\hat{v}^1(K, \theta_{-1})$ such that

$$\mathbb{E}_\theta \left[\Delta Y(K, \theta, 0) \mid \theta_{-1}, \hat{v}^1\right] = \mathbb{E}_\theta \left[\Delta Y\left(K, \theta, \Phi\left(\sqrt{\gamma_v}\left(\theta - \hat{v}^0\right)\right)\right) \mid \theta_{-1}, \hat{v}^1\right] = 0,$$

which means that it is dominant to choose high capacity for all firms j such that $v_j \geqslant \hat{v}^1$, even if no one else did. Symmetrically, we can define $\hat{v}_1(K, \theta_{-1})$ such that

$$\mathbb{E}_\theta \left[\Delta Y(K, \theta, 1) \mid \theta_{-1}, \hat{v}_1\right] = \mathbb{E}_\theta \left[\Delta Y\left(K, \theta, \Phi\left(\sqrt{\gamma_v}\left(\theta - \hat{v}_0\right)\right)\right) \mid \theta_{-1}, \hat{v}_1\right] = 0,$$

such that it is dominant to choose low capacity for all firms j such that $v_j \leqslant \hat{v}_1$ even if all other firms choose the high capacity. By the properties of ΔY, we must have $\hat{v}_0 < \hat{v}_1 \leqslant \hat{v}^1 < \hat{v}^0$. This establishes the first iteration of our procedure. By induction, let $n \geqslant 2$ and assume that $\hat{v}_0 < \ldots < \hat{v}_{n-1} \leqslant \hat{v}^{n-1} < \ldots < \hat{v}^0$ such that it is dominant to choose high capacity if $v_j \geqslant \hat{v}^{n-1}$ and dominant to choose low for $v_j \leqslant \hat{v}_{n-1}$. Define $\hat{v}^n$ and $\hat{v}_n$ such that

$$\mathbb{E}_\theta \left[\Delta Y \left(K, \theta, \Phi \left(\sqrt{\gamma_v} \left(\theta - \hat{v}^{n-1}\right)\right)\right) \mid \theta_{-1}, \hat{v}^n\right] = 0,$$
$$\mathbb{E}_\theta \left[\Delta Y \left(K, \theta, \Phi \left(\sqrt{\gamma_v} \left(\theta - \hat{v}_{n-1}\right)\right)\right) \mid \theta_{-1}, \hat{v}_n\right] = 0.$$

By induction, $\hat{v}^{n-1} \geqslant \hat{v}_{n-1}$, so that $\Phi\left(\sqrt{\gamma_v}\left(\theta - \hat{v}^{n-1}\right)\right) \leqslant \Phi\left(\sqrt{\gamma_v}\left(\theta - \hat{v}_{n-1}\right)\right)$ and $\hat{v}^n \geqslant \hat{v}_n$. Also, since $\hat{v}^{n-1} < \hat{v}^{n-2}$, then $\Phi\left(\sqrt{\gamma_v}\left(\theta - \hat{v}^{n-1}\right)\right) > \Phi\left(\sqrt{\gamma_v}\left(\theta - \hat{v}^{n-2}\right)\right)$ and $\hat{v}^n < \hat{v}^{n-1}$. Symmetrically, we have $\hat{v}_n > \hat{v}_{n-1}$. This establishes the recursion.

Sequence $(\hat{v}_n)_{n \geqslant 0}$ is a strictly increasing bounded sequence, therefore it converges. Denote $\hat{v}_\infty$ its limit: $\hat{v}_n \underset{n\to\infty}{\to} \hat{v}_\infty$. Symmetrically, establish that $(\hat{v}^n)_{n\geqslant 0}$ converges towards some limit $\hat{v}^\infty \geqslant \hat{v}_\infty$. By continuity of ΔY, we have

$$\mathbb{E}_\theta \left[\Delta Y \left(K, \theta, \Phi \left(\sqrt{\gamma_v} \left(\theta - \hat{v}^{\infty}\right)\right)\right) \mid \theta_{-1}, \hat{v}^\infty\right] = 0 \text{ and } \mathbb{E}_\theta \left[\Delta Y \left(K, \theta, \Phi \left(\sqrt{\gamma_v} \left(\theta - \hat{v}_{\infty}\right)\right)\right) \mid \theta_{-1}, \hat{v}_\infty\right] = 0.$$

Step 2. Define $H(K, \theta_{-1}, \hat{v}) \equiv \mathbb{E}_\theta \left[\Delta Y \left(K, \theta, \Phi \left(\sqrt{\gamma_v} \left(\theta - \hat{v}\right)\right)\right) \mid \theta_{-1}, \hat{v}\right]$. We now provide sufficient conditions such that the implicit equation in $\hat{v}$,

$$H(K, \theta_{-1}, \hat{v}) = 0,$$

has a unique solution $\hat{v}(K, \theta_{-1})$. In particular, this condition is satisfied if H is strictly increasing in $\hat{v}$. Since $\theta \mid \theta_{-1}, v_j = \hat{v} \sim \mathcal{N}\left(\frac{\gamma_\theta \rho \theta_{-1} + \gamma_v \hat{v}}{\gamma_\theta + \gamma_v}, \frac{1}{\gamma_\theta + \gamma_v}\right)$,

$$\begin{aligned} H(K, \theta_{-1}, \hat{v}) &= \mathbb{E}_\theta \left[\Delta Y \left(K, \theta, \Phi \left(\sqrt{\gamma_v} \left(\theta - \hat{v}\right)\right)\right) \mid \theta_{-1}, \hat{v}\right] \\ &= \mathbb{E}_\varepsilon \left[c_0 \left(A e^{\frac{\gamma_\theta \rho \theta_{-1} + \gamma_v \hat{v}}{\gamma_\theta + \gamma_v} + \varepsilon} K^\alpha\right)^{\frac{1+\nu}{\alpha+\nu}} \Omega \left(\Phi \left(\sqrt{\gamma_v} \left(\frac{\gamma_\theta \rho \theta_{-1} + \gamma_v \hat{v}}{\gamma_\theta + \gamma_v} + \varepsilon - \hat{v}\right)\right)\right)^{\frac{1+\nu}{\alpha+\nu} - \sigma + 1} - f\right] \end{aligned}$$

where $c_0 = \frac{1}{\sigma} \frac{(1+s_\pi)(1+s_y)}{1-s_{kl}} \left(\omega^{\sigma-1} - 1\right) \left[(1-\alpha) \frac{\sigma-1}{\sigma} \frac{1+s_y}{1-s_{kl}}\right]^{\frac{1-\alpha}{\alpha+\nu}}$ and $\varepsilon = \theta - \frac{\gamma_\theta \rho \theta_{-1} + \gamma_v \hat{v}}{\gamma_\theta + \gamma_v} \sim \mathcal{N}\left(0, \frac{1}{\gamma_\theta + \gamma_v}\right)$. Compute the derivative:

$$\frac{\partial \log H}{\partial \hat{v}} = \frac{1+\nu}{\alpha+\nu} \frac{\gamma_v}{\gamma_\theta + \gamma_v} + \frac{\partial}{\partial \hat{v}} \log H_2(\theta_{-1}, \hat{v})$$

where $H_2(\theta_{-1},\hat{v})=\mathbb{E}_\varepsilon\left[e^{\frac{1+\nu}{\alpha+\nu}\varepsilon}\Omega\left(\Phi\left(\sqrt{\gamma_v}\left(\frac{\gamma_\theta(\rho\theta_{-1}-\hat{v})}{\gamma_\theta+\gamma_v}+\varepsilon\right)\right)\right)^{\frac{1+\nu}{\alpha+\nu}-\sigma+1}\right]$. Compute the last term,

$$\frac{\partial H_2}{\partial \hat{v}}=-\frac{\sqrt{\gamma_v}\gamma_\theta}{\gamma_\theta+\gamma_v}\left(\frac{1+\nu}{\alpha+\nu}-\sigma+1\right)\mathbb{E}_\varepsilon\left[\phi\left(\sqrt{\gamma_v}\left(\frac{\gamma_\theta(\rho\theta_{-1}-\hat{v})}{\gamma_\theta+\gamma_v}+\varepsilon\right)\right)\frac{\Omega'\left(\Phi\left(\sqrt{\gamma_v}\left(\frac{\gamma_\theta(\rho\theta_{-1}-\hat{v})}{\gamma_\theta+\gamma_v}+\varepsilon\right)\right)\right)}{\Omega\left(\Phi\left(\sqrt{\gamma_v}\left(\frac{\gamma_\theta(\rho\theta_{-1}-\hat{v})}{\gamma_\theta+\gamma_v}+\varepsilon\right)\right)\right)}\right.$$
$$\left.\times\, e^{\frac{1+\nu}{\alpha+\nu}\varepsilon}\Omega\left(\Phi\left(\sqrt{\gamma_v}\left(\frac{\gamma_\theta(\rho\theta_{-1}-\hat{v})}{\gamma_\theta+\gamma_v}+\varepsilon\right)\right)\right)^{\frac{1+\nu}{\alpha+\nu}-\sigma+1}\right]$$

so that $\left|\frac{\partial H_2}{\partial \hat{v}}\right|\leqslant\frac{\sqrt{\gamma_v}\gamma_\theta}{\gamma_\theta+\gamma_v}\left(\frac{1+\nu}{\alpha+\nu}-\sigma+1\right)\frac{1}{\sqrt{2\pi}}\overline{\frac{\Omega'}{\Omega}}H_2(\theta_{-1},\hat{v})$. Since $\left|\frac{\Omega'(m)}{\Omega(m)}\right|\leqslant\frac{\omega^{\sigma-1}-1}{\sigma-1}$, we have

$$\left|\frac{\partial\log H_2}{\partial\hat{v}}\right|\leqslant\frac{\sqrt{\gamma_v}\gamma_\theta}{\gamma_\theta+\gamma_v}\left(\frac{1+\nu}{\alpha+\nu}-\sigma+1\right)\frac{1}{\sqrt{2\pi}}\frac{\omega^{\sigma-1}-1}{\sigma-1}.$$

We may now conclude that

$$\frac{\partial\log H}{\partial\hat{v}}\geqslant\frac{1+\nu}{\alpha+\nu}\frac{\gamma_v}{\gamma_\theta+\gamma_v}-\frac{\sqrt{\gamma_v}\gamma_\theta}{\gamma_\theta+\gamma_v}\frac{1}{\sqrt{2\pi}}\left(\frac{1+\nu}{\alpha+\nu}-\sigma+1\right)\frac{\omega^{\sigma-1}-1}{\sigma-1}.$$

Therefore, a sufficient condition that guarantees that H is strictly increasing in $\hat{v}$ is

$$\frac{\sqrt{\gamma_v}}{\gamma_\theta}>\frac{1}{\sqrt{2\pi}}\frac{\omega^{\sigma-1}-1}{\sigma-1}.$$

Note in addition that $H\underset{\hat{v}\to\infty}{\longrightarrow}\infty$ and $H\underset{\hat{v}\to-\infty}{\longrightarrow}-\infty$, therefore there exists a unique solution $\hat{v}(K,\theta_{-1})$ to the equation $H(K,\theta_{-1},\hat{v}(K,\theta_{-1}))=0$.

Conclusion. Under the condition (17), there exists a unique solution to the equation

$$\mathbb{E}_\theta\left[\Delta Y\left(K,\theta,\Phi\left(\sqrt{\gamma_v}(\theta-\hat{v})\right)\right)\mid\theta_{-1},\hat{v}\right]=0,$$

which is satisfied by both $\hat{v}^\infty$ and $\hat{v}_\infty$. Therefore, $\hat{v}^\infty=\hat{v}_\infty=\hat{v}(K,\theta_{-1})$ and the solution to the global game is the unique cutoff strategy $\hat{v}(K,\theta_{-1})$ such that firm j chooses high capacity if and only if $v_j\geqslant\hat{v}(K,\theta_{-1})$.

■ Case $\frac{1+\nu}{\alpha+\nu}\leqslant\sigma-1$

In the case that the condition for multiplicity is not satisfied, the proof is similar but easier since there is strategic substitutability between firms. By iterated deletion of dominant strategies, define the monotone sequences $(\hat{v}_n)_{n\geqslant0}$ and $(\hat{v}^n)_{n\geqslant0}$ by

$$\mathbb{E}_\theta\left[\Delta Y\left(K,\theta,\Phi\left(\sqrt{\gamma_v}(\theta-\hat{v}_{n-1})\right)\right)\mid\theta_{-1},\hat{v}^n\right]=0,$$
$$\mathbb{E}_\theta\left[\Delta Y\left(K,\theta,\Phi\left(\sqrt{\gamma_v}\left(\theta-\hat{v}^{n-1}\right)\right)\right)\mid\theta_{-1},\hat{v}_n\right]=0.$$

Then, function $\mathbb{E}_\theta\left[\Delta Y\left(K,\theta,\Phi\left(\sqrt{\gamma_v}(\theta-\hat{v})\right)\right)\mid\theta_{-1},\hat{v}\right]$ is strictly increasing in $\hat{v}$ without additional restrictions on the parameter. Conclude as in the previous case. □

E.4 Regularity

In this section, we establish a number of regularity conditions and properties of $\hat{v}$, m, $\overline{A}$ and Y.

Lemma A2. *Under the conditions of proposition A1, (i) $\hat{v}(K,\theta_{-1})$ is continuous and weakly decreasing in K and θ_{-1}, (ii) $m(K,\theta_{-1},\theta)$ and $\overline{A}(K,\theta_{-1},\theta)$ are bounded, continuous and weakly increasing, (iii) $y(K,\theta_{-1},\theta)$ is bounded, continuous and, for f sufficiently small, increasing in K, (iv) if in addition assumption 1 is verified, $y(K,\theta_{-1},\theta)-\frac{L(K,\theta_{-1},\theta)^{1+\nu}}{1+\nu}$ is increasing in K.*

Proof. (i) *Continuity and monotonicity of $\hat{v}(K,\theta_{-1})$.* Cutoff $\hat{v}(K,\theta_{-1})$ is implicitly defined by the function $\Delta\tilde{\Pi}(K,\theta_{-1},\hat{v}(K,\theta_{-1}),\hat{v}(K,\theta_{-1}))=0$ which is a continosly differentiable function of K, θ_{-1} and $\hat{v}$. Under the conditions of proposition A1, $\frac{\partial}{\partial\hat{v}}\Delta\tilde{\Pi}>0$, so the implicit function theorem tells us that $\hat{v}$ is continuous in a neighborhood of (K,θ_{-1}). Since $\Delta\Pi$ is strictly increasing in K and $\hat{v}$, this implies that $\hat{v}$ is decreasing in K. Similarly, since $\Delta\Pi$ is increasing in θ_{-1}, $\hat{v}$ is also decreasing in θ_{-1}.
(ii) *Continuity and monotonicity of m and A.* The continuity of m and A is inherited from that of $\hat{v}$ since

$$m(K,\boldsymbol{\theta})=\Phi\left(\sqrt{\gamma_v}\left(\theta-\hat{v}(K,\theta_{-1})\right)\right) \text{ and } \overline{A}(K,\boldsymbol{\theta})=Ae^{\theta}\left(m(K,\boldsymbol{\theta})\left(\omega^{\sigma-1}-1\right)+1\right)^{\frac{1}{\sigma-1}},$$

which are bounded on $\mathbb{K}\times\Theta^2$. The monotonicity of m and A is inherited from that of $\hat{v}$.
(iii) *Continuity and monotonicity of $y(K,\boldsymbol{\theta})$.* Recall the definition of y:

$$y(K,\boldsymbol{\theta})\equiv\left[(1-\alpha)\frac{\sigma-1}{\sigma}\frac{1+s_y}{1-s_{kl}}\right]^{\frac{1-\alpha}{\alpha+\nu}}\left(Ae^{\theta}\Omega\left(m(K,\boldsymbol{\theta})\right)\right)^{\frac{1+\nu}{\alpha+\nu}}K^{\alpha\frac{1+\nu}{\alpha+\nu}}+(1-\delta)K-m(K,\boldsymbol{\theta})f.$$

The implicit function theorem applied to the global game tells us that $\hat{v}$ and m are differentiable. Therefore, we have:

$$\begin{aligned}\frac{\partial y}{\partial K}&=\frac{\partial y\left(K,\theta,m(K,\theta_{-1},\theta)\right)}{\partial K}\\&=\alpha\frac{1+\nu}{\alpha+\nu}\left[(1-\alpha)\frac{\sigma-1}{\sigma}\frac{1+s_y}{1-s_{kl}}\right]^{\frac{1-\alpha}{\alpha+\nu}}\left(Ae^{\theta}\Omega\left(m(K,\boldsymbol{\theta})\right)\right)^{\frac{1+\nu}{\alpha+\nu}}K^{\alpha\frac{1+\nu}{\alpha+\nu}-1}+1-\delta+\frac{\partial y}{\partial m}\frac{\partial m}{\partial K}.\end{aligned}$$

The first term and $\frac{\partial m}{\partial K}$ are positive, so we must only compute the sign of $\frac{\partial y}{\partial m}$. Compute the following:

$$\frac{\partial y}{\partial m}=\frac{1+\nu}{\alpha+\nu}\frac{1}{\sigma-1}\left[(1-\alpha)\frac{\sigma-1}{\sigma}\frac{1+s_y}{1-s_{kl}}\right]^{\frac{1-\alpha}{\alpha+\nu}}\left(Ae^{\theta}K^{\alpha}\right)^{\frac{1+\nu}{\alpha+\nu}}\left(\omega^{\sigma-1}-1\right)\Omega\left(m(K,\boldsymbol{\theta})\right)^{\frac{1+\nu}{\alpha+\nu}-\sigma+1}-f.$$

Applying the implicit function theorem to the global game, one can obtain a uniform bound for $\left|\frac{\partial m}{\partial K}\right|$. For f small, as long as

$$\left|\frac{\partial m}{\partial K}\right|f\leqslant\alpha\frac{1+\nu}{\alpha+\nu}\left[(1-\alpha)\frac{\sigma-1}{\sigma}\frac{1+s_y}{1-s_{kl}}\right]^{\frac{1-\alpha}{\alpha+\nu}}\left(Ae^{\underline{\theta}}\right)^{\frac{1+\nu}{\alpha+\nu}}\overline{K}^{\alpha\frac{1+\nu}{\alpha+\nu}-1},$$

then $\frac{\partial y}{\partial K}$ is strictly positive. The signs of $\frac{\partial y}{\partial \theta}$ and $\frac{\partial y}{\partial \theta_{-1}}$ are in general ambiguous.
(iv) This property is used in several lemmas. The argument is the same as above:

$$y(K,\boldsymbol{\theta}) - \frac{L(K,\boldsymbol{\theta})^{1+\nu}}{1+\nu} = \left(1 - \frac{\sigma-1}{\sigma}\frac{1-\alpha}{1+\nu}\frac{1+s_y}{1-s_{kl}}\right) y(K,\boldsymbol{\theta}) + (1-\delta)K - m(K,\boldsymbol{\theta}) f.$$

Therefore, under all the previous assumptions and assumption 1, then we can always find f sufficiently small that $y(K,\boldsymbol{\theta}) - \frac{L(K,\boldsymbol{\theta})^{1+\nu}}{1+\nu}$ is increasing in K. □

F Existence and Uniqueness of the Dynamic Equilibrium

This proof builds on the monotone operator and lattice-theoretic techniques developed in Coleman (1991), Coleman and John (2000), Datta et al. (2002) or Morand and Reffett (2003) and extends it to the features present in our setup. The proof uses the following version of Tarski's fixed point theorem (see Tarski et al. (1955)):

Theorem. ***[Tarski, 1955]*** *Suppose that $(X, \geqslant)$ is a nonempty complete lattice and $T : X \to X$ is an increasing mapping. Then, the set of fixed points of T is a nonempty complete lattice.*

F.1 Description and Definitions

The objective of this proof is to show the existence and uniqueness of a solution to the Euler equation in some particular space. For reasons that will appear clearer later, it is useful to represent the Euler equation in the space of *inverse marginal utility*, which we denote as p, instead of consumption functions directly.[25] That is to say, we will go back and forth between the spaces of inverse marginal values and consumption functions through the following mapping,

$$p(K,\boldsymbol{\theta}) = U_C\left(c(K,\boldsymbol{\theta}), L(K,\boldsymbol{\theta})\right)^{-1}.$$

Definition 5. Let $\mathscr{P}$ be the set

$$\begin{aligned}\mathscr{P} = \Big\{ & p(K,\boldsymbol{\theta}) \mid c : \mathbb{K}\times\Theta^2 \longrightarrow \mathbb{K} \text{ such that} \\ & (a)\, 0 \leqslant p(K,\boldsymbol{\theta}) \leqslant U_C\left(y(K,\boldsymbol{\theta}), L(K,\boldsymbol{\theta})\right)^{-1} \text{ for } (K,\boldsymbol{\theta}) \in \mathbb{K}\times\Theta^2; \\ & (b)\, p \text{ weakly increasing in } K \Big\}.\end{aligned}$$

Definition 5 describes the set in which the equilibrium inverse marginal utility p lies. We may now introduce the following definitions which sets up the environment and the Euler equation that we must solve:

[25] A similar existence proof can be written in the space of consumption functions as in Coleman (1991). The uniqueness is, however, problematic in that space since the operator corresponding to the Euler equation is not pseudo-concave without restrictive assumptions on the preferences. On the other hand, that same operator is naturally pseudo-concave in the space of inverse marginal utilities as noted by Coleman (2000) and Datta et al. (2002).

Definition 6. (i) The mapping from marginal consumption value to consumption is

$$C : \mathbb{R}^{+} \times \mathbb{K} \times \Theta^{2} \longrightarrow \mathbb{R}$$
$$(p, K, \boldsymbol{\theta}) \mapsto U_{C}^{-1}\left(p, L\left(K, \boldsymbol{\theta}\right)\right) = p^{\frac{1}{\gamma}} + \frac{L\left(K, \boldsymbol{\theta}\right)^{1+\nu}}{1+\nu};$$

(ii) The mapping corresponding to the Euler equation is

$$Z : \mathbb{R}^{+} \times \mathscr{P} \times \mathbb{K} \times \Theta^{2} \longrightarrow \mathbb{R} \cup \{-\infty, \infty\}$$
$$(p, P, K, \boldsymbol{\theta}) \mapsto \begin{cases} 0 \text{ if } p = 0 \text{ and } \Big(K = 0 \text{ or } P\Big(y\left(K, \boldsymbol{\theta}\right) - C\left(0, K, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\Big) = 0\Big) \\ \frac{1}{p} - \beta \mathbb{E}\left[\frac{r\Big(y(K,\boldsymbol{\theta}) - C(p,K,\boldsymbol{\theta}), \boldsymbol{\theta}'\Big)}{P\Big(y(K,\boldsymbol{\theta}) - C(p,K,\boldsymbol{\theta}), \boldsymbol{\theta}'\Big)}\right], \text{ otherwise;} \end{cases}$$

(iii) The operator providing solutions to the Euler equation is

$$T\left(P\right) = \left\{p \in \mathscr{P} \mid Z\left(p\left(K, \boldsymbol{\theta}\right), P, K, \boldsymbol{\theta}\right) = 0 \text{ for } K \in \mathbb{K}, \boldsymbol{\theta} \in \Theta^{2}\right\}.$$

F.2 Existence

We endow the space $\mathscr{P}$ with the pointwise partial order $\leqslant$, such that $p \leqslant \hat{p}$ if $p\left(K, \boldsymbol{\theta}\right) \leqslant \hat{p}\left(K, \boldsymbol{\theta}\right)$ for all $\left(K, \boldsymbol{\theta}\right) \in \mathbb{K} \times \Theta^{2}$ and two binary operations that we refer to as the *meet* $\left(p \wedge \hat{p}\right)$ and the *join* $\left(p \vee \hat{p}\right)$ for any two points $p, \hat{p} \in \mathscr{P}$. The meet is the greatest lower bound of two elements, i.e.,

$$\left(p \wedge \hat{p}\right)\left(K, \boldsymbol{\theta}\right) = \min\ \left\{p\left(K, \boldsymbol{\theta}\right), \hat{p}\left(K, \boldsymbol{\theta}\right)\right\},$$

and the join is the least upper bound, defined as

$$\left(p \vee \hat{p}\right)\left(K, \boldsymbol{\theta}\right) = \max\ \left\{p\left(K, \boldsymbol{\theta}\right), \hat{p}\left(K, \boldsymbol{\theta}\right)\right\}.$$

Lemma A3. $(\mathscr{P}, \leqslant)$ *is a complete lattice.*

Proof. A lattice is complete if each subset has a supremum and an infimum. Consider a subset $X \subset \mathscr{P}$. Clearly, the join of all elements in X, $\sup_{p \in X} p$, satisfies $\sup_{p \in X} p \leqslant U_{C}\left(y\left(K, \boldsymbol{\theta}\right), L\left(K, \boldsymbol{\theta}\right)\right)^{-1}$ and $\sup_{p \in X} p$ is weakly increasing in K, so $\sup_{p \in X} p \in \mathscr{P}$. A symmetric argument tells us that the meet of all elements in X, $\inf_{p \in X} p$, belongs to $\mathscr{P}$. Therefore, $\mathscr{P}$ is a complete lattice. $\square$

We now show that mapping T, which associates the solution to the Euler equation for any future inverse marginal utility $P \in \mathscr{P}$ is a well-defined monotone mapping from $\mathscr{P}$ to $\mathscr{P}$.

Lemma A4. *Under the conditions of proposition A1, assumption 1 and f small enough, T is a well-defined self-map on $\mathscr{P}$.*

Proof. Notice, first, from the definition of Z and $\mathscr{P}$ that Z is strictly decreasing in p but strictly increasing in P.

Step 1: T is well defined. Fix $K > 0$, $\boldsymbol{\theta}$ and P. Note that as $p \to 0$, $Z(p, P, K, \boldsymbol{\theta}) \to \infty$ and as $p \to U_C(y(K, \boldsymbol{\theta}), L(K, \boldsymbol{\theta}))^{-1}$, $Z(p, P, K, \boldsymbol{\theta}) \to -\infty$. Since Z is continuous and strictly decreasing in p, there exists a unique $0 < p(K, \boldsymbol{\theta}) < Y(K, \boldsymbol{\theta})$ such that $Z(p(K, \boldsymbol{\theta}), P, K, \boldsymbol{\theta}) = 0$.

Step 2: T maps $\mathscr{P}$ onto itself. We must check properties (a)-(b) in the definition of $\mathscr{P}$:

(a) Already verified in step 1.

(b) Pick $0 < K \leqslant \hat{K}$. Denote $p = T(P)$. By definition $Z(p(K, \boldsymbol{\theta}), P, K, \boldsymbol{\theta}) = 0$. Evaluate Z at $p(K, \boldsymbol{\theta})$ for $\hat{K}$:

$$Z\left(p(K, \boldsymbol{\theta}), P, \hat{K}, \boldsymbol{\theta}\right) = \frac{1}{p(K, \boldsymbol{\theta})} - \beta \mathbb{E}\left[\frac{r\left(y\left(\hat{K}, \boldsymbol{\theta}\right) - C\left(p(K, \boldsymbol{\theta}), \hat{K}, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\right)}{P\left(y\left(\hat{K}, \boldsymbol{\theta}\right) - C\left(p(K, \boldsymbol{\theta}), \hat{K}, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\right)}\right].$$

Compute the following term:

$$\begin{aligned} y\left(\hat{K}, \boldsymbol{\theta}\right) - C\left(p(K, \boldsymbol{\theta}), \hat{K}, \boldsymbol{\theta}\right) &= y\left(\hat{K}, \boldsymbol{\theta}\right) - \frac{L\left(\hat{K}, \boldsymbol{\theta}\right)^{1+\nu}}{1+\nu} - p(K, \boldsymbol{\theta})^{\frac{1}{\gamma}} \\ &= \left(1 - \frac{\sigma - 1}{\sigma} \frac{1-\alpha}{1+\nu} \frac{1+s_y}{1-s_{kl}}\right) y\left(\hat{K}, \boldsymbol{\theta}\right) - m\left(\hat{K}, \boldsymbol{\theta}\right) f + (1-\delta)\hat{K} - p(K, \boldsymbol{\theta})^{\frac{1}{\gamma}} \\ &\geqslant y(K, \boldsymbol{\theta}) - C(p(K, \boldsymbol{\theta}), K, \boldsymbol{\theta}), \end{aligned}$$

where the inequality is due to the fact that y is increasing in K for ω close enough to 1 (lemma A2(iv)). Therefore,

$$Z\left(p(K, \boldsymbol{\theta}), P, \hat{K}, \boldsymbol{\theta}\right) \geqslant Z(p(K, \boldsymbol{\theta}), P, K, \boldsymbol{\theta}) = 0,$$

which implies that $p\left(\hat{K}, \boldsymbol{\theta}\right) \geqslant p(K, \boldsymbol{\theta})$ since Z is strictly decreasing in p. □

Lemma A5. *Under the conditions of proposition A1, assumption 1 and f sufficiently small, T is continuous and monotone.*

Proof. Step 1: Monotonicity.

Take $p \leqslant \hat{p}$ in the sense that $p(K, \boldsymbol{\theta}) \leqslant \hat{p}(K, \boldsymbol{\theta})$ for all $(K, \boldsymbol{\theta})$. Evaluate Z at

$$\begin{aligned} Z(Tp(K, \boldsymbol{\theta}), \hat{p}, K, \boldsymbol{\theta}) &= \frac{1}{Tp(K, \boldsymbol{\theta})} - \beta \mathbb{E}\left[\frac{r\left(y(K, \boldsymbol{\theta}) - C\left(Tp(K, \boldsymbol{\theta}), \hat{K}, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\right)}{\hat{p}\left(y(K, \boldsymbol{\theta}) - C\left(Tp(K, \boldsymbol{\theta}), \hat{K}, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\right)}\right] \\ &\geqslant Z(Tp(K, \boldsymbol{\theta}), p, K, \boldsymbol{\theta}) = 0, \end{aligned}$$

which implies that $T\hat{p}(K,\boldsymbol{\theta}) \geqslant Tp(K,\boldsymbol{\theta})$. Therefore, $Tp \leqslant T\hat{p}$.

Step 2: Continuity.

Fix $p \in \mathscr{P}$. Pick $\varepsilon > 0$ and some $\hat{p} \in \mathscr{P}$ such that $\|\hat{p} - p\| \leqslant \varepsilon$. Fix $K > 0, \boldsymbol{\theta} \in \Theta^2$. For all $\tilde{p} \in \mathbb{R}$,

$$\begin{aligned} Z(\tilde{p}, \hat{p}, K, \boldsymbol{\theta}) &= \frac{1}{\tilde{p}} - \beta \mathbb{E}\left[\frac{r\Big(y(K,\boldsymbol{\theta}) - C\left(\tilde{p}, \hat{K}, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\Big)}{\hat{p}\Big(y(K,\boldsymbol{\theta}) - C\left(\tilde{p}, \hat{K}, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\Big)}\right] \\ &\leqslant \frac{1}{\tilde{p}} - \beta \mathbb{E}\left[\frac{r\Big(y(K,\boldsymbol{\theta}) - C\left(\tilde{p}, \hat{K}, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\Big)}{p\Big(y(K,\boldsymbol{\theta}) - C\left(\tilde{p}, \hat{K}, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\Big) + \varepsilon}\right], \end{aligned}$$

which means that $T\hat{p} \leqslant T(p+\varepsilon)$. A similar argument yields $T\hat{p} \geqslant T(p-\varepsilon)$. By definition,

$$Z(T[p+\varepsilon](K,\boldsymbol{\theta}), p+\varepsilon, K, \boldsymbol{\theta}) = \frac{1}{T[p+\varepsilon](K,\boldsymbol{\theta})} - \beta\mathbb{E}\left[\frac{r\Big(y(K,\boldsymbol{\theta}) - C\left(T[p+\varepsilon](K,\boldsymbol{\theta}), \hat{K}, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\Big)}{p\Big(y(K,\boldsymbol{\theta}) - C\left(T[p+\varepsilon](K,\boldsymbol{\theta}), \hat{K}, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\Big) + \varepsilon}\right].$$

Using the fact that $T(p+\varepsilon) \geqslant Tp$ and r/p is decreasing in K, we obtain:

$$0 \leqslant \frac{1}{T[p+\varepsilon](K,\boldsymbol{\theta})} - \beta\mathbb{E}\left[\frac{r\Big(y(K,\boldsymbol{\theta}) - C\left(Tp(K,\boldsymbol{\theta}), \hat{K}, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\Big)}{p\Big(y(K,\boldsymbol{\theta}) - C\left(Tp(K,\boldsymbol{\theta}), \hat{K}, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\Big) + \varepsilon}\right].$$

Thus,

$$\begin{aligned} T[p+\varepsilon](K,\boldsymbol{\theta}) &\leqslant \beta^{-1}\mathbb{E}\left[\frac{r\Big(y(K,\boldsymbol{\theta}) - C\left(Tp(K,\boldsymbol{\theta}), \hat{K}, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\Big)}{p\Big(y(K,\boldsymbol{\theta}) - C\left(Tp(K,\boldsymbol{\theta}), \hat{K}, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\Big) + \varepsilon}\right]^{-1} \\ &\leqslant \beta^{-1}\mathbb{E}\left[\frac{p\Big(y(K,\boldsymbol{\theta}) - C\left(Tp(K,\boldsymbol{\theta}), \hat{K}, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\Big) + \varepsilon}{r\Big(y(K,\boldsymbol{\theta}) - C\left(Tp(K,\boldsymbol{\theta}), \hat{K}, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\Big)}\right] \quad \text{(Jensen)} \\ &\leqslant Tp(K,\boldsymbol{\theta}) + \beta^{-1}\varepsilon\mathbb{E}\left[r\Big(y(K,\boldsymbol{\theta}) - C\left(Tp(K,\boldsymbol{\theta}), \hat{K}, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\Big)^{-1}\right] \\ &\leqslant Tp(K,\boldsymbol{\theta}) + \beta^{-1} r\Big(y\left(\overline{K}, \overline{\boldsymbol{\theta}}\right), \overline{\boldsymbol{\theta}}\Big)^{-1}\varepsilon. \end{aligned}$$

The same argument applied to $p - \varepsilon$ yields $Tp \leqslant T(p-\varepsilon) + \beta^{-1} r\Big(y\left(\overline{K}, \overline{\boldsymbol{\theta}}\right), \overline{\boldsymbol{\theta}}\Big)^{-1}\varepsilon$. We can now conclude that T is a continuous mapping on $\mathscr{P}$, since $\|\hat{p} - p\| \leqslant \varepsilon$ implies

$$\begin{aligned} \|T\hat{p} - Tp\| &\leqslant \max\left(\|T(p+\varepsilon) - Tp\|, \|T(p-\varepsilon) - Tp\|\right) \\ &\leqslant \beta^{-1} r\Big(y\left(\overline{K}, \overline{\boldsymbol{\theta}}\right), \overline{\boldsymbol{\theta}}\Big)^{-1}\varepsilon. \end{aligned}$$

□

Proposition A2. *Under the conditions of proposition A1, assumptions 1-2 and f sufficiently small, there exists a strictly positive equilibrium function $p^* \in \mathscr{P}$.*

Proof. The existence of a fixed point is simply given by Tarski's fixed point theorem applied to the monotone self-map T on the complete lattice $(\mathscr{P}, \leqslant)$.

We now construct a strictly positive fixed point p^*. Note that we are abusing language when using the expression "strictly positive", since our setup is such that $p^*(0, \boldsymbol{\theta}) = 0$ for all $\boldsymbol{\theta}$. Thus, by "strictly positive", we mean that $p^*(K, \boldsymbol{\theta}) > 0$ for all $K > 0$. We proceed in three steps.

Step 1. Define the sequence $(p_n)_{n \geqslant 0}$ such that $p_0(K, \boldsymbol{\theta}) = U_C(y(K, \boldsymbol{\theta}), L(K, \boldsymbol{\theta}))^{-1}$ and $p_n = T^n p_0$. By construction, the first iteration is mapped downward ($p_1 \leqslant p_0$) and we obtain a decreasing sequence which converges pointwise towards a function p^*. Clearly, $p^* = \inf_{n \geqslant 0} p_n$ so $p^* \in \mathscr{P}$. Furthermore, since T is continuous, $p^* = Tp^*$, so p^* is a fixed point of T.

Step 2. We first show that p^* is not 0. From assumption 2, take K^-such that $y(K^-, \underline{\boldsymbol{\theta}}) - \frac{L(K^-, \underline{\boldsymbol{\theta}})^{1+\nu}}{1+\nu} > K^-$ and $\beta \mathbb{E}\left[r\left(K^-, \underline{\theta}, \theta'\right) \mid \underline{\theta}\right] \leqslant 1$. Pick an $\alpha > 0$ such that $C(\alpha, K^-, \underline{\boldsymbol{\theta}}) < y(K^-, \underline{\boldsymbol{\theta}}) - K^-$, i.e., such that $0 < \alpha^{\frac{1}{\gamma}} < y(K^-, \underline{\boldsymbol{\theta}}) - \frac{L(K^-, \underline{\boldsymbol{\theta}})^{1+\nu}}{1+\nu} - K^-$. Assume some $p \in \mathscr{P}$ is such that $p(K^-, \underline{\theta}, \theta') \geqslant \alpha$ for all θ', then we show that $Tp(K^-, \underline{\boldsymbol{\theta}}) \geqslant \alpha$ by simply evaluating $Z(\alpha, p, K^-, \underline{\boldsymbol{\theta}})$:

$$
\begin{aligned}
Z\left(\alpha, p, K^-, \underline{\boldsymbol{\theta}}\right) &= \frac{1}{\alpha} - \beta \mathbb{E}\left[\frac{r\Big(y(K^-, \underline{\boldsymbol{\theta}}) - C(\alpha, K^-, \underline{\boldsymbol{\theta}}), \underline{\theta}, \theta'\Big)}{p\Big(y(K^-, \underline{\boldsymbol{\theta}}) - C(\alpha, K^-, \underline{\boldsymbol{\theta}}), \underline{\theta}, \theta'\Big)}\right] \\
&\geqslant \frac{1}{\alpha} - \beta \mathbb{E}\left[\frac{r\Big(K^-, \underline{\theta}, \theta'\Big)}{p\Big(K^-, \underline{\theta}, \theta'\Big)}\right] \geqslant \frac{1}{\alpha} - \frac{\beta}{\alpha} \mathbb{E}\left[r\Big(K^-, \underline{\theta}, \theta'\Big)\right] \geqslant 0.
\end{aligned}
$$

This establishes that $Tp(K^-, \boldsymbol{\theta}) \geqslant \alpha$. Since we start our iterations with

$$
p_0\left(K^-, \boldsymbol{\theta}\right) = U_C\left(y\left(K^-, \boldsymbol{\theta}\right), L\left(K^-, \boldsymbol{\theta}\right)\right)^{-1},
$$

i.e., such that $C(p_0, K^-, \underline{\theta}, \theta') = y(K^-, \underline{\theta}, \theta')$ and therefore $p_0(K^-, \underline{\theta}, \theta') > \alpha$, we have $p^*(K^-, \underline{\theta}, \theta') \geqslant \alpha > 0$.

Step 3. We now want to show that p^* is strictly positive for all $K > 0$. Assume, by contradiction, that p^* is not strictly positive. This means, that there exists $(K_0, \boldsymbol{\theta}_0)$ such that $p^*(K_0, \boldsymbol{\theta}_0) = 0$. Since p^* is increasing in all its arguments, this means that $p^*(K, \underline{\boldsymbol{\theta}}) = 0$ for all $K \leqslant K_0$. Define

$$
\tilde{K} = \sup_{K \leqslant K^-} \left\{p^*(K, \underline{\boldsymbol{\theta}}) = 0\right\}.
$$

With the assumption that p^* is not strictly positive, $\tilde{K} > 0$. Since $\tilde{K} \leqslant K^-$, then we have that

$y\left(\tilde{K},\underline{\boldsymbol{\theta}}\right)-\frac{L\left(\tilde{K},\underline{\boldsymbol{\theta}}\right)^{1+\nu}}{1+\nu}>\tilde{K}$. The right hand side of the Euler equation evaluated at $\tilde{K}$ and $\underline{\boldsymbol{\theta}}$ gives

$$0\leqslant\beta\mathbb{E}\left[\frac{r\left(y\left(\tilde{K},\underline{\boldsymbol{\theta}}\right)-C\left(0,\tilde{K},\underline{\boldsymbol{\theta}}\right),\boldsymbol{\theta}'\right)}{p^{*}\left(y\left(\tilde{K},\underline{\boldsymbol{\theta}}\right)-C\left(0,\tilde{K},\underline{\boldsymbol{\theta}}\right),\boldsymbol{\theta}'\right)}\right]\leqslant\beta\mathbb{E}\left[\frac{r\left(\tilde{K},\boldsymbol{\theta}'\right)}{p^{*}\left(y\left(\tilde{K},\underline{\boldsymbol{\theta}}\right)-C\left(0,\tilde{K},\underline{\boldsymbol{\theta}}\right),\boldsymbol{\theta}'\right)}\right],$$

which is finite since $p^{*}\left(y\left(\tilde{K},\underline{\boldsymbol{\theta}}\right)-C\left(0,\tilde{K},\underline{\boldsymbol{\theta}}\right),\boldsymbol{\theta}'\right)>0$. Thus, we obtain a contradiction since $p^{*}\left(\tilde{K},\underline{\theta}\right)=0$ cannot be a solution. Therefore, $\tilde{K}$ must be 0 and p^{*} is strictly positive everywhere except at $K=0$. □

F.3 Uniqueness

The proof for uniqueness relies on showing that the operator T is pseudo-concave. The following definitions are useful for that purpose. First, define a pseudo-concave operator:

Definition 7. A monotone operator $T:\mathscr{P}\longrightarrow\mathscr{P}$ is pseudo-concave if for any strictly positive $p\in\mathscr{P}$ and $t\in(0,1)$, $T(tp)(K,\boldsymbol{\theta})>tTp(K,\boldsymbol{\theta})$ for all $K>0$, $\boldsymbol{\theta}\in\Theta^{2}$.

We now define the concept of K_0-monotonicity for an operator.

Definition 8. An operator $T:\mathscr{P}\longrightarrow\mathscr{P}$ is K_0-monotone if it is monotone and if, for any strictly positive fixed point p^{*}, there exists $K_0>0$ such that for any $0\leqslant K_1\leqslant K_0$ and any $p\in\mathscr{P}$ such that $p(K,\boldsymbol{\theta})\leqslant p^{*}(K,\boldsymbol{\theta}),\forall K\geqslant K_1,\boldsymbol{\theta}$, then

$$p^{*}(K,\boldsymbol{\theta})\geqslant Tp(K,\boldsymbol{\theta}),\forall K\geqslant K_1,\boldsymbol{\theta}.$$

We now proceed to show that T is K_0-monotone, which we will then use to prove its pseudo-concavity. In order to do so, the following preliminary result is useful:

Lemma A6. *Under the conditions of proposition A1, assumption 1 and f sufficiently small, suppose $P\in\mathscr{P}$ and let $p=T(P)$, then for all $\left(\theta_{-1},\hat{\theta}_{-1}\right)\in\Theta^{2}$,*

$$\left|C\left(p\left(K,\hat{\theta}_{-1},\theta\right),K,\hat{\theta}_{-1},\theta\right)-C\left(p\left(K,\theta_{-1},\theta\right),K,\theta_{-1},\theta\right)\right|\leqslant\left|y\left(K,\hat{\theta}_{-1},\theta\right)-y\left(K,\theta_{-1},\theta\right)\right|.$$

Proof. Pick $\left(\theta_{-1},\hat{\theta}_{-1}\right)\in\Theta^{2}$ and assume WLOG that $Y\left(K,\hat{\theta}_{-1},\theta\right)\geqslant Y\left(K,\theta_{-1},\theta\right)$ and that ω has been chosen close enough to 1 that

$$y\left(K,\hat{\theta}_{-1},\theta\right)-\frac{L\left(K,\hat{\theta}_{-1},\theta\right)^{1+\nu}}{1+\nu}\geqslant y\left(K,\theta_{-1},\theta\right)-\frac{L\left(K,\theta_{-1},\theta\right)^{1+\nu}}{1+\nu}.$$

Step 1. By definition $Z\left(p\left(K,\theta_{-1},\theta\right),P,K,\theta_{-1},\theta\right)=0$. Evaluate $Z\left(\tilde{p},P,K,\hat{\theta}_{-1},\theta\right)$ at $\tilde{p}$ such that

$$C\left(\tilde{p},P,K,\hat{\theta}_{-1},\theta\right)=C\left(p\left(K,\theta_{-1},\theta\right),K,\theta_{-1},\theta\right)+y\left(K,\hat{\theta}_{-1},\theta\right)-y\left(K,\theta_{-1},\theta\right),$$

in other words,

$$\tilde{p}^{\frac{1}{\gamma}} + \frac{L\left(K, \hat{\theta}_{-1}, \theta\right)^{1+\nu}}{1+\nu} = p^{\frac{1}{\gamma}} + \frac{L\left(K, \theta_{-1}, \theta\right)^{1+\nu}}{1+\nu} + y\left(K, \hat{\theta}_{-1}, \theta\right) - y\left(K, \theta_{-1}, \theta\right).$$

Assume WLOG that $\tilde{p} \geqslant p\left(K, \theta_{-1}, \theta\right)$. Then, we have

$$\begin{aligned} Z\left(\tilde{p}, P, K, \hat{\theta}_{-1}, \theta\right) &= \frac{1}{\tilde{p}} - \beta\mathbb{E}\left[\frac{r\Big(y\left(K, \hat{\theta}_{-1}, \theta\right) - C\left(\tilde{p}, K, \hat{\theta}_{-1}, \theta\right), \theta, \theta'\Big)}{P\Big(y\left(K, \hat{\theta}_{-1}, \theta\right) - C\left(\tilde{p}, K, \hat{\theta}_{-1}, \theta\right), \theta, \theta'\Big)}\right] \\ &= \frac{1}{\tilde{p}} - \beta\mathbb{E}\left[\frac{r\Big(y\left(K, \theta_{-1}, \theta\right) - C\left(\tilde{p}, K, \theta_{-1}, \theta\right), \theta, \theta'\Big)}{P\Big(y\left(K, \theta_{-1}, \theta\right) - C\left(\tilde{p}, K, \theta_{-1}, \theta\right), \theta, \theta'\Big)}\right] \\ &\leqslant 0, \end{aligned}$$

which tells us that $p\left(K, \hat{\theta}_{-1}, \theta\right) \leqslant \tilde{p}$. Thus, we have:

$$\begin{aligned} &C\left(p\left(K, \hat{\theta}_{-1}, \theta\right), K, \hat{\theta}_{-1}, \theta\right) - C\left(p\left(K, \theta_{-1}, \theta\right), K, \theta_{-1}, \theta\right) \\ &\leqslant C\left(\tilde{p}, K, \hat{\theta}_{-1}, \theta\right) - C\left(p\left(K, \theta_{-1}, \theta\right), K, \theta_{-1}, \theta\right) \\ &\leqslant y\left(K, \hat{\theta}_{-1}, \theta\right) - y\left(K, \theta_{-1}, \theta\right). \end{aligned}$$

Step 2. We now evaluate the other side of the inequality. Evaluate $Z\Big(p\left(K, \theta_{-1}, \theta\right), P, K, \hat{\theta}_{-1}, \theta\Big)$:

$$\begin{aligned} Z\Big(p\left(K, \theta_{-1}, \theta\right), P, K, \hat{\theta}_{-1}, \theta\Big) &= \frac{1}{p\left(K, \theta_{-1}, \theta\right)} - \beta\mathbb{E}\left[\frac{r\Big(y\left(K, \hat{\theta}_{-1}, \theta\right) - C\left(p\left(K, \theta_{-1}, \theta\right), K, \hat{\theta}_{-1}, \theta\right), \theta, \theta'\Big)}{P\Big(y\left(K, \hat{\theta}_{-1}, \theta\right) - C\left(p\left(K, \theta_{-1}, \theta\right), K, \hat{\theta}_{-1}, \theta\right), \theta, \theta'\Big)}\right] \\ &\geqslant \frac{1}{p\left(K, \theta_{-1}, \theta\right)} - \beta\mathbb{E}\left[\frac{r\Big(y\left(K, \theta_{-1}, \theta\right) - C\left(p\left(K, \theta_{-1}, \theta\right), K, \theta_{-1}, \theta\right), \theta, \theta'\Big)}{P\Big(y\left(K, \theta_{-1}, \theta\right) - C\left(p\left(K, \theta_{-1}, \theta\right), K, \theta_{-1}, \theta\right), \theta, \theta'\Big)}\right], \end{aligned}$$

which implies that $p\left(K, \hat{\theta}_{-1}, \theta\right) \geqslant p\left(K, \theta_{-1}, \theta\right)$. Therefore,

$$C\left(p\left(K, \hat{\theta}_{-1}, \theta\right), K, \hat{\theta}_{-1}, \theta\right) - C\left(p\left(K, \theta_{-1}, \theta\right), K, \theta_{-1}, \theta\right) \geqslant 0,$$

which establishes the desired result. $\square$

Lemma A7. *Under the conditions of proposition A1, assumptions 1-2 and f sufficiently small, T is K_0-monotone.*

Proof. The proof proceeds in two steps.

Step 1. Let us show that there exists $K_0 > 0$ such that $y(K, \boldsymbol{\theta}) - C(p^*(K, \boldsymbol{\theta}), K, \boldsymbol{\theta}) \geqslant K, \forall K \leqslant K_0, \forall \boldsymbol{\theta}$. Pick a strictly positive fixed point p^*. By contradiction, suppose that for all $K_0 > 0$, there exists a $K \leqslant K_0$ and a $\boldsymbol{\theta} = (\theta_{-1}, \theta)'$ such that $y(K, \boldsymbol{\theta}) - C(p^*(K, \boldsymbol{\theta}), K, \boldsymbol{\theta}) < K$. Suppose, by contradiction, that $y(K, \theta, \theta) - C(p^*(K, \theta, \theta), K, \theta, \theta) \geqslant K$, then we would have

$$C(p^*(K, \boldsymbol{\theta}), K, \boldsymbol{\theta}) - C(p^*(K, \theta, \theta), K, \theta, \theta) > y(K, \theta_{-1}, \theta) - y(K, \theta, \theta),$$

which cannot be true for $p^* \in \mathscr{P}$ according to lemma A6. Therefore, $y(K, \theta, \theta) - C(p^*(K, \theta, \theta), K, \theta, \theta) < K$. By the definition of p^*:

$$\begin{aligned} \frac{1}{p^*(K, \theta, \theta)} &= \beta \mathbb{E}\left[\frac{r(y(K, \theta, \theta) - C(p^*(K, \theta, \theta), K, \theta, \theta), \theta, \theta')}{p^*(y(K, \theta, \theta) - C(p^*(K, \theta, \theta), K, \theta, \theta), \theta, \theta')}\right] \\ &> \beta \mathbb{E}\left[\frac{r(K, \theta, \theta')}{p^*(K, \theta, \theta')}\right] = \beta \int \frac{r(K, \theta, \theta')}{p^*(K, \theta, \theta')} \pi\left(d\theta', \theta\right), \end{aligned}$$

where $\pi(d\theta', \theta)$ denotes the marginal density of θ' conditional on θ. Since $p^*(K, \theta, \theta')$ is weakly increasing in θ', we have

$$\begin{aligned} &\beta \int \frac{r(K, \theta, \theta')}{p^*(K, \theta, \theta')} \pi\left(d\theta', \theta\right) \\ &\geqslant \frac{\beta}{p^*(K, \theta, \theta)} \int_{\theta' \leqslant \theta} r\left(K, \theta, \theta'\right) \pi\left(d\theta', \theta\right) \\ &= \frac{\beta}{p^*(K, \theta, \theta)} r(K, \underline{\theta}, \underline{\theta}) \mathbb{P}\left(\theta' \leqslant \theta\right), \end{aligned}$$

where $r(K, \underline{\theta}, \underline{\theta}) > 0$. Given our specification of the stochastic process followed by θ, denote $\underline{\pi}$ the lower bound on the probability that θ' falls below current θ, i.e., $\underline{\pi} = \inf_{\theta \in \Theta} \mathbb{P}(\theta' \leqslant \theta \mid \theta)$. In our setting, $\underline{\pi}$ exists and is strictly positive. Since $r(K, \underline{\theta}, \underline{\theta}) \to \infty$ as $K \to 0$, we can choose K_0 small enough that $\beta r(K, \underline{\theta}, \underline{\theta}) \underline{\pi} > 1$, then

$$\begin{aligned} \frac{1}{p^*(K, \theta, \theta)} &> \frac{\beta}{p^*(K, \theta, \theta)} r(K, \underline{\theta}, \underline{\theta}) \underline{\pi}, \\ &> \frac{1}{p^*(K, \theta, \theta)}. \end{aligned}$$

Hence, we have a contradiction.

Step 2. Keeping the same K_0 given by step 1, pick a $K_1 \leqslant K_0$ with a p such that $p(K, \boldsymbol{\theta}) \leqslant p^*(K, \boldsymbol{\theta}), \forall K \geqslant K_1, \forall \boldsymbol{\theta}$. Since $y(K, \boldsymbol{\theta}) - C(p^*(K, \boldsymbol{\theta}), K, \boldsymbol{\theta}) \geqslant K_1$, then for all $K \geqslant K_1$ and $\boldsymbol{\theta}$,$\boldsymbol{\theta}$':

$$p\left(y(K, \boldsymbol{\theta}) - C(p^*(K, \boldsymbol{\theta}), K, \boldsymbol{\theta}), \boldsymbol{\theta}'\right) \leqslant p^*\left(y(K, \boldsymbol{\theta}) - C(p^*(K, \boldsymbol{\theta}), K, \boldsymbol{\theta}), \boldsymbol{\theta}'\right)$$

Therefore, $Z(p^*(K, \boldsymbol{\theta}), p^*(K, \boldsymbol{\theta}), K, \boldsymbol{\theta}) = 0 \geqslant Z(p^*(K, \boldsymbol{\theta}), p(K, \boldsymbol{\theta}), K, \boldsymbol{\theta})$, which implies that $Tp(K, \boldsymbol{\theta}) \leqslant p^*(K, \boldsymbol{\theta})$. T is K_0-monotone. □

Lemma A8. *Under the conditions of proposition A1, assumptions 1-2 and f sufficiently small, T is pseudo-concave.*

Proof. We want for $t \in (0,1)$ that $T[tp](K,\boldsymbol{\theta}) > tT[p](K,\boldsymbol{\theta})$ for $K > 0$. Since Z is strictly decreasing in p, it is equivalent to show that

$$0 = Z\left(T[tp](K,\boldsymbol{\theta}), tp, K, \boldsymbol{\theta}\right) < Z\left(tT[p](K,\boldsymbol{\theta}), tp, K, \boldsymbol{\theta}\right).$$

$$\begin{aligned} &Z\left(tT[p](K,\boldsymbol{\theta}), tp, K, \boldsymbol{\theta}\right) \\ &= \frac{1}{tT[p](K,\boldsymbol{\theta})} - \beta\mathbb{E}\left[\frac{r\left(y(K,\boldsymbol{\theta}) - C\left(tT[p](K,\boldsymbol{\theta}), K, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\right)}{tp\left(y(K,\boldsymbol{\theta}) - C\left(tT[p](K,\boldsymbol{\theta}), K, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\right)}\right] \\ &= \frac{1}{t}\left\{\frac{1}{T[p](K,\boldsymbol{\theta})} - \beta\mathbb{E}\left[\frac{r\left(y(K,\boldsymbol{\theta}) - C\left(tT[p](K,\boldsymbol{\theta}), K, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\right)}{p\left(y(K,\boldsymbol{\theta}) - C\left(tT[p](K,\boldsymbol{\theta}), K, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\right)}\right]\right\}, \end{aligned}$$

since C is strictly increasing in p, $C\left(tT[p](K,\boldsymbol{\theta}), K, \boldsymbol{\theta}\right) < C\left(T[p](K,\boldsymbol{\theta}), K, \boldsymbol{\theta}\right)$. Since $\frac{r}{p}$ is strictly decreasing in K, then

$$\begin{aligned} Z(tAv, tv, K, \boldsymbol{\theta}) &= \frac{1}{t}\left\{\frac{1}{T[p](K,\boldsymbol{\theta})} - \beta\mathbb{E}\left[\frac{r\left(y(K,\boldsymbol{\theta}) - C\left(tT[p](K,\boldsymbol{\theta}), K, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\right)}{p\left(y(K,\boldsymbol{\theta}) - C\left(tT[p](K,\boldsymbol{\theta}), K, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\right)}\right]\right\} \\ &> \frac{1}{T[p](K,\boldsymbol{\theta})} - \beta\mathbb{E}\left[\frac{r\left(y(K,\boldsymbol{\theta}) - C\left(T[p](K,\boldsymbol{\theta}), K, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\right)}{p\left(y(K,\boldsymbol{\theta}) - C\left(T[p](K,\boldsymbol{\theta}), K, \boldsymbol{\theta}\right), \boldsymbol{\theta}'\right)}\right] = 0, \end{aligned}$$

which shows that $T[tp](K,\boldsymbol{\theta}) > tT[p](K,\boldsymbol{\theta})$ for $K > 0$. Therefore, T is a pseudo-concave mapping. □

Proposition A3. *Under the conditions of proposition A1, assumptions 1-2 and f sufficiently small, there is a unique strictly positive equilibrium $p \in \mathscr{P}$.*

Proof. T being K_0-monotone, pseudo-concave has at most one strictly positive fixed point. Take two fixed points p_1^* and p_2^*. Suppose for some $K > 0, \boldsymbol{\theta} \in \Theta^2$, $p_1^*(K,\boldsymbol{\theta}) < p_2^*(K,\boldsymbol{\theta})$. Pick the K_0 from the K_0-monotonicity and choose $t \in (0,1)$ such that $p_1^*(K,\boldsymbol{\theta}) \geqslant tp_2^*(K,\boldsymbol{\theta})$, i.e., choose $t = \inf_{K \geqslant K_0} \frac{p_1^*(K,\boldsymbol{\theta})}{p_2^*(K,\boldsymbol{\theta})}$ which is finite, strictly positive (recall that the p's are increasing in K, strictly positive and bounded) and strictly less than 1 by assumption. Then, by K_0-monotonicity, for all $K \geqslant K_0$,

$$\begin{aligned} p_1^*(K,\boldsymbol{\theta}) &\geqslant T[tp_2^*](K,\boldsymbol{\theta}) \\ &> tT[p_2^*](K,\boldsymbol{\theta}) \\ &> tp_2^*(K,\boldsymbol{\theta}) \end{aligned}$$

which contradicts the fact that t was the infimum. Therefore, the equilibrium is unique. □

G Policy

Proposition 5. *The competitive equilibrium with incomplete information is inefficient, but the constrained efficient allocation can be implemented with a lump-sum tax on the household, an input subsidy* s_{kl} *and a profit subsidy* s_π *to intermediate goods producers such that* $1 - s_{kl} = \frac{\sigma-1}{\sigma}$ *and* $1 + s_\pi = \frac{\sigma}{\sigma-1}$.

Proof. We define the constrained planner problem as selecting a schedule $z(v_j)$ of probabilities to use high capacity as a function of an agent's private signal v_j and levels of production Y_h and Y_l for capacity levels. Define the planner's Bellman equation

$$\begin{aligned} V_{SP}(K, \theta_{-1}) = \max_{0 \leqslant z(\cdot) \leqslant 1} \mathbb{E}_\theta \Big[& \max_{K', L, K_i, L_i} U\left(Y - m(\theta, z) f - K' + (1-\delta)K, L\right) \\ & + \beta V_{SP}\left(K', \theta\right) \mid \theta_{-1} \Big] \end{aligned}$$

subject to

$$\begin{aligned} Y &= \left(\int_0^1 m(\theta, z) Y_h^{\frac{\sigma-1}{\sigma}} + (1 - m(\theta, z)) Y_l^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \\ Y_i &= A_i(\theta) K_i^\alpha L_i^{1-\alpha}, i \in \{h, l\} \\ K &= m(\theta, z) K_h + (1 - m(\theta, z)) K_l \\ L &= m(\theta, z) L_h + (1 - m(\theta, z)) L_l \\ m(\theta, z) &= \int \sqrt{\gamma_v} \phi\left(\sqrt{\gamma_v}(v - \theta)\right) z(v) dv. \end{aligned}$$

The first-order conditions with respect to K_i and L_i tell us that the marginal products are equalized across firms,

$$\begin{aligned} \alpha Y_h^{\frac{\sigma-1}{\sigma}-1} Y^{\frac{1}{\sigma}} \frac{Y_h}{K_h} &= \alpha Y_l^{\frac{\sigma-1}{\sigma}-1} Y^{\frac{1}{\sigma}} \frac{Y_l}{K_l} \\ (1-\alpha) Y_h^{\frac{\sigma-1}{\sigma}-1} Y^{\frac{1}{\sigma}} \frac{Y_h}{L_h} &= (1-\alpha) Y_l^{\frac{\sigma-1}{\sigma}-1} Y^{\frac{1}{\sigma}} \frac{Y_l}{L_l}, \end{aligned}$$

and we have equality between the marginal product of labor and the marginal rate of substitution,

$$(1-\alpha) Y_i^{\frac{\sigma-1}{\sigma}-1} Y^{\frac{1}{\sigma}} \frac{Y_i}{L_i} = \frac{U_L(C, L)}{U_C(C, L)} = L^\nu.$$

Solving this system of equation, we obtain the following efficient output level and labor,

$$Y_{SP}(K, \theta, m) = (1-\alpha)^{\frac{1-\alpha}{\alpha+\nu}} \left(\overline{A}(\theta, m) K^\alpha\right)^{\frac{1+\nu}{\alpha+\nu}} \text{ and } L_{SP}(K, \theta, m) = (1-\alpha)^{\frac{1}{\alpha+\nu}} \left(\overline{A}(\theta, m) K^\alpha\right)^{\frac{1}{\alpha+\nu}}. \tag{30}$$

The first order condition on $z(v)$ is

$$\mathbb{E}_\theta\left[\sqrt{\gamma_v}\phi\left(\sqrt{\gamma_v}(\theta-v)\right)U_C(C,L)\left(\frac{\overline{A}_m(\theta,m)}{\overline{A}(\theta,m)}Y_{SP}-f\right)\mid\theta_{-1}\right]\gtreqless 0,$$

with corresponding complementary slackness conditions. Substituting in the values of $\overline{A}$ and Y_{SP}, we get

$$\mathbb{E}_\theta\left[\sqrt{\gamma_v}\phi\left(\sqrt{\gamma_v}(\theta-v)\right)U_C(C,L)\left(\frac{1}{\sigma-1}\frac{\omega^{\sigma-1}-1}{\Omega(m)^{\sigma-1}}(1-\alpha)^{\frac{1-\alpha}{\alpha+\nu}}\left(\overline{A}(\theta,m)K^\alpha\right)^{\frac{1+\nu}{\alpha+\nu}}-f\right)\mid\theta_{-1}\right]\gtreqless 0.$$

It is now useful to notice that, since $\theta=\rho\theta_{-1}+\sqrt{\gamma_\theta}\varepsilon_\theta$ and $v_j=\theta+\sqrt{\gamma_v}\varepsilon_{vj}$, with $(\varepsilon_\theta,\varepsilon_{vj})$ unit normals, then for any arbitrary function $f(K,\theta,m)$, the following equality holds

$$\begin{aligned}\mathbb{E}_\theta\left[f(K,\theta,m)\mid\theta_{-1},v_j\right] &= \int f(K,\theta,m)\sqrt{\gamma_\theta}\phi\left(\sqrt{\gamma_\theta}(\theta-\rho\theta_{-1})\right)\sqrt{\gamma_v}\phi\left(\sqrt{\gamma_v}(v-\theta)\right)d\theta/\pi(v\mid\theta_{-1})\\ &= \mathbb{E}_\theta\left[f(K,\theta,m)\sqrt{\gamma_v}\phi\left(\sqrt{\gamma_v}(v-\theta)\right)\mid\theta_{-1}\right]/\pi(v\mid\theta_{-1}).\end{aligned}$$

Thus, going back to the planner's problem, a firm with signal v chooses high capacity with positive probability if and only if

$$\mathbb{E}_\theta\left[U_C(C,L)\left(\frac{1}{\sigma-1}\frac{\omega^{\sigma-1}-1}{\Omega(m)^{\sigma-1}}(1-\alpha)^{\frac{1-\alpha}{\alpha+\nu}}\left(\overline{A}(\theta,m)K^\alpha\right)^{\frac{1+\nu}{\alpha+\nu}}-f\right)\mid\theta_{-1},v_j\right]\geqslant 0.$$

This problem is familiar and we recognize a condition similar to the one that characterizes the solution of the global game. Using the same arguments as before, we know that the stochastic discount factor U_C drops from the equation when $\gamma_v\to\infty$, which simplifies the problem to

$$\mathbb{E}_\theta\left[\frac{1}{\sigma-1}\frac{\omega^{\sigma-1}-1}{\Omega(m)^{\sigma-1}}(1-\alpha)^{\frac{1-\alpha}{\alpha+\nu}}\left(\overline{A}(\theta,m)K^\alpha\right)^{\frac{1+\nu}{\alpha+\nu}}-f\mid\theta_{-1},v\right]\geqslant 0.$$

Then, under the same hypothesis as proposition A1, i.e., that $\frac{\sqrt{\gamma_v}}{\gamma_\theta}>\frac{1}{\sqrt{2\pi}}\frac{\omega^{\sigma-1}-1}{\sigma-1}$, we know that the only solution to the above equation is a cutoff $\hat{v}_{SP}(K,\theta_{-1})$ such that $z(v)=1$ for $v>\hat{v}_{SP}(K,\theta_{-1})$ and $z(v)=0$ for $v<\hat{v}_{SP}(K,\theta_{-1})$. The cutoff is such that

$$\mathbb{E}_\theta\left[\frac{1}{\sigma-1}\frac{\omega^{\sigma-1}-1}{\Omega(m)^{\sigma-1}}(1-\alpha)^{\frac{1-\alpha}{\alpha+\nu}}\left(\overline{A}\left(\theta,\Phi\left(\sqrt{\gamma_v}(\theta-\hat{v}_{SP})\right)\right)K^\alpha\right)^{\frac{1+\nu}{\alpha+\nu}}-f\mid\theta_{-1},\hat{v}_{SP}\right]=0. \tag{31}$$

Comparing the two conditions (30) and (31) to that of the competitive economy, we see that the conditions coincide either with the input subsidy $1-s_{kl}=\frac{\sigma-1}{\sigma}$ so as to offset the markup and the profit subsidy $1+s_\pi=\frac{\sigma}{\sigma-1}$ to induce the right entry; or more simply, just using a sales subsidy $1+s_y=\frac{\sigma}{\sigma-1}$. Under the conditions of proposition A3, we know that these two first order conditions uniquely determine the equilibrium. Therefore, the two economies coincide under this optimal sales subsidy and the economy without subsisdy is inefficient. □

Proposition 6. *Under GHH preferences, for γ_v large, an unforeseen one-time increase in government spending financed by lump-sum taxes reduces welfare.*

Proof. Consider the case of an unforeseen shock to government spending $G_0 > 0$ that lasts only one period, $G_t = 0$ for $t \geqslant 1$ financed by a lump-sum tax $T_0 = G_0$.[26] Notice that, under our assumption of GHH preferences, our expressions for equilibrium output $Y(K, \theta, m)$ and labor $L(K, \theta, m)$ from proposition 1 remain unaffected by government spending. The only channel by which spending may influence output is through the coordination game by affecting the measure of firms with high capacity m. As shown in lemma A1, as γ_v becomes large, the within-period uncertainty vanishes and the stochastic discount factor disappears from the surplus from choosing the high capacity which, in the absence of other subsidies, can be approximated by

$$\Delta\tilde{\Pi}(K, \theta_{-1}, v_j, m) = \mathbb{E}_\theta \left[\frac{1}{\sigma} \frac{\omega^{\sigma-1} - 1}{\Omega(m)^{\sigma-1}} \left[(1-\alpha) \frac{\sigma - 1}{\sigma} \frac{1 + s_y}{1 - s_{kl}} \right]^{\frac{1-\alpha}{\alpha+\nu}} \overline{A}(\theta, m)^{\frac{1+\nu}{\alpha+\nu}} K^{\alpha \frac{1+\nu}{\alpha+\nu}} - f \mid \theta_{-1}, v_j \right].$$

As a result, when γ_v is large, equilibrium consumption C drops from the equation and the solution $\hat{v}(K, \theta_{-1})$ to the global game is independent from government spending G. The equilibrium production Y is thus unaffected.

Consider now the equilibrium allocation $\{C_t(\theta^t), K_{t+1}(\theta^t)\}$ in the economy hit by the government spending shock. Because equilibrium production $Y_t(\theta^t)$ and labor $L_t(\theta^t)$ are unaffected by government spending, hence prices as well, the same allocation is feasible in an economy without government spending: it satisfies both the household's budget constraint and the aggregate resource constraint with some extra resources left from the unused government consumption. By increasing consumption in period 0 by G_0 exactly, the household can choose an allocation that remains feasible and strictly increases its welfare. As a conclusion, welfare in the economy without spending is strictly greater than in the economy with government spending shock. □

[26] Ricardian equivalence obtains in our experiment and the actual timing of taxes is irrelevant.