On the limits of macroprudential policy

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Abstract

This paper considers a canonical New Keynesian macrofinancial model to analyze how macroprudential policy tools can help the monetary authority in reaching a selection of dual stabilization objectives. We show that using the loan-to-value ratio as an additional policy instrument does not allow to resolve the standard inflation-output volatility tradeoff. Simultaneous stabilization of inflation and either credit or house prices with monetary and macroprudential policy is possible only if the role of credit in the economy is very small. Overall, our results suggest that macroprudential policy has important limits as a complement to monetary policy.

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1 Introduction

It is well understood that not all business cycle fluctuations can be considered efficient and hence an appropriately concocted set of policy measures that are aimed at limiting volatility of selected macrocategories can improve social welfare. Consistently with this view, most of contemporary macroeconomic models that are used to inform policymakers assign an important role to stabilization policy. At the same time, it is well understood that such a policy has some limits. One of them is related to the number of available instruments that can be used in a timely manner being usually smaller than the number of targets that one might want to hit. This violates the famous Tinbergen rule that postulates the need for at least as many independent instruments as the number of policy goals. In consequence, policymakers have to resolve a number of tradeoffs between various stabilization objectives.\textsuperscript{1}

This type of dilemmas also applies to monetary policy, which is widely considered the preferred policy to stabilize business cycles, also because of its not being subject to big implementation lags. In particular, it is in general not possible to perfectly stabilize both inflation and economic activity using just the short-term interest rate (Gertler et al., 1999; Woodford, 2003), a result that is often referred to as the inflation-output stabilization trade-off. Naturally, the policy dilemmas become even more difficult to resolve in the presence of financial frictions as these make additional stabilization objectives relevant, or if the central bank is explicitly responsible for maintaining financial stability.\textsuperscript{2}

Following the recent financial crisis, a relatively new type of policy, dubbed macroprudential, has been proposed as a promising way of handling imbalances arising from financial market imperfections. This policy uses a set of tools typically used by the financial supervisory authorities at a microeconomic level, and applies them to limit various vulnerabilities of the whole financial system.\textsuperscript{3} Various macroprudential policy instruments have been proposed by the standard-setting or policy making institutions like the Bank for International Settlements, International Monetary Fund, or used by central banks in various countries (IMF, 2011). Many of these tools are aimed to operate at business cycle frequencies. In particular, countercyclical use of capital buffers was advocated by the Basel Committee on Banking Supervision in its Basel III recommendation to smooth the credit cycle, i.e. prevent excessive growth of credit during booms and ensure its availability during periods of financial stress (BCBS, 2010). As evidenced by the experience of some emerging market economies including Hong Kong and South Korea, adjustments in the loan to value (LTV) ratio can

\textsuperscript{1}There is a long list of papers discussing how these tradeoffs should be resolved. In the context of a standard monetary-fiscal policy mix, see e.g. Dixit and Lambertini (2003), Schmitt-Grohe and Uribe (2004), Siu (2004) or Correia et al. (2008).

\textsuperscript{2}There is extensive literature on whether and how monetary policy should respond to asset prices to reduce the cost of a possible financial crisis. See e.g. Bernanke and Gertler (2001), Carlstrom and Fuerst (2007) or Faia and Monacelli (2007).

\textsuperscript{3}See Clement (2010) for the origins and evolution of macroprudential policy.
serve a similar purpose.

A natural question that emerges in this context is: How efficient can these new policy tools be in meeting particular stabilization objectives, and how do they interact with monetary policy? A number of papers have looked into this issue and the literature is growing fast. The findings are usually moderately positive as they suggest that macroprudential policy instruments are quite efficient in stabilizing the financial sector and hence can be considered a useful complement to monetary policy, see Galati and Moessner (2013) for a survey. However, there may be important interactions between the two policies that justify the need to coordinate them (De Paoli and Paustian, 2013; Cecchetti and Kohler, 2014). This literature also includes papers that use dynamic stochastic general equilibrium (DSGE) models with sticky prices and financial frictions, in which the central bank fully controls the short-term interest rate while the macroprudential authority sets the LTV or bank capital adequacy ratio. They offer conclusions that are based on stochastic simulations for various specifications of feedback rules describing the decisions made by the monetary and macroprudential authorities.

This paper seeks to add to our understanding of how macroprudential policy can contribute to macroeconomic stability by taking a more analytical perspective. To this end, we consider a simple New Keynesian framework augmented with housing and collateral constraints as in Iacoviello (2005), and define the LTV ratio as an additional policy instrument. This simple framework is then used to state and prove several important qualitative statements about the ability of monetary and macroprudential policy to jointly meet a selection of relevant stabilization objectives. More specifically, we ask the following questions: Does macroprudential policy help to resolve the standard inflation-output volatility tradeoff? To what extent can this policy stabilize the financial sector without jeopardizing the price stability objective pursued by the monetary authority? While answering these questions we compare the outcomes obtainable with the LTV ratio to those that can be achieved with simple government spending policy.

As regards the methodology, we use a linear approximation to the model equilibrium conditions. This yields the system of equations that is tractable enough to allow us to prove the key propositions analytically. Our analytical approach can be contrasted with the previous DSGE literature that is based on simulations with larger models, for which qualitative results cannot be derived and hence one might be concerned whether the reported findings go through for alternative and plausible calibration choices. The local flavor of the framework used in this paper also makes it different from the recent research that employs global methods to study macroprudential policy in the presence of occasionally binding

\footnote{See e.g. Christensen et al. (2011), Darracq-Pariés et al. (2011), Angeloni and Faia (2013) or Lambertini et al. (2013). Quint and Rabanal (2014) and Brzoza-Brzezina et al. (2015) look at this issue in the context of a heterogeneous monetary union. Brzoza-Brzezina et al. (2014) show how macroprudential policy may be limited by the presence of multi-period loans and occasionally binding collateral constraints.}
constraints (e.g. Jeanne and Korinek, 2010; Benigno et al., 2013; Brunnermeier and Sannikov, 2014).

The main findings obtained with our benchmark New Keynesian macro-financial model can be summarized as follows. First, there does not exist a stable equilibrium in which fluctuations in inflation and output are eliminated by an appropriate mix of monetary and macroprudential actions. In other words, having the LTV ratio as an additional instrument to the short-term interest rate controlled by the central bank does not resolve the standard inflation-output stabilization tradeoff. In this respect, macroprudential policy is less efficient than simple government spending policy, which, if combined with monetary policy, can achieve this dual objective, at least for standard calibration. Second, using monetary and macroprudential policy to simultaneously stabilize inflation and credit is possible only if the role of the latter in the economy is very small. This time, however, government spending policy cannot be considered as a more attractive alternative to macroprudential policy as applying it together with monetary policy such that inflation and credit are stable always leads to indeterminacy and hence cannot be considered implementable in the sense of Schmitt-Grohe and Uribe (2007). Third, similar conclusions hold for the ability of monetary and macroprudential (or fiscal) policy to perfectly stabilize inflation and house prices.

Overall, these results suggest that macroprudential policy is not as powerful complement to conventional monetary policy as one might hope. It does not help to deal with the standard inflation-output volatility tradeoff. Even more disappointingly, it does not necessarily allow to achieve one additional financial target (credit or house price stabilization) in addition to price stability delivered by monetary policy. In other words, meeting the Tinbergen rule in the context of simultaneous use of monetary and macroprudential policy is insufficient if one aims at resolving the standard macroeconomic or macrofinancial tradeoffs.

The rest of this paper is organized as follows. Section two presents the benchmark New Keynesian macrofinancial model and its log-linearized version. Calibration choices underlying the illustrative impulse responses are explained in section three. Section four establishes and discusses our main results. Section five concludes. A full list of model equations, as well as proofs and more details of all propositions stated in the text are presented in the Appendix.

## 2 Benchmark model

Our benchmark model is a standard New Keynesian (NK) setup, extended to include housing and financially constrained agents that can borrow only against housing collateral as in Iacoviello (2005). There are three authorities that fully control three policy instruments: the central bank sets the short-term interest rate, the government determines its purchase of final goods, and the macroprudential authority controls the loan-to-value (LTV) ratio.
The rest of this section briefly describes the problems facing the agents and a log-linearized version of the model. A full list of equations making up the original model is given in the Appendix.

2.1 Theoretical framework

2.1.1 Households

There are two types of households whose preferences differ in the degree to which they discount the future utility flows. This makes impatient households natural borrowers, and the patient ones natural lenders. We denote these two types by \( I \) and \( P \), respectively, and the size of the former by \( \omega \), with the total measure of households normalized to unity. Within each group \( i = \{I, P\} \), a representative agent maximizes

\[
E_0 \left\{ \sum_{t=0}^{\infty} \beta_t^i \left[ \log c_{i,t} + A_\chi \nu_i(\chi_{i,t}) - \frac{n_{i,t}^{1+\varphi}}{1+\varphi} \right] \right\}
\]

where \( A_\chi > 0 \), \( 0 < \beta_I < \beta_P < 1 \), \( c_t \) is consumption, \( n_t \) is labor supply, and \( \nu_i \) is a function describing how housing stock \( \chi_t \) affects the utility.

Patient households’ maximization is subject to a standard budget constraint

\[
P_t c_{P,t} + P_\chi t(\chi_{P,t} - \chi_{P,t-1}) + D_t + T_t \leq W_t n_{P,t} + R_{t-1} D_{t-1} + \Xi_t
\]

where \( P_t \) is the price of final goods, \( P_\chi t \) denotes the price of housing, \( W_t \) is nominal wage, \( \Xi_t \) denotes profits from monopolistically competitive firms, \( T_t \) is lump-sum taxes, while \( D_t \) stands for deposits paying risk-free rate \( R_t \).

Impatient households’ budget constraint is similar, except that these agents do not own firms nor pay taxes, and the interest charged on their loans \( L_t \) is also given by \( R_t \)

\[
P_t c_{I,t} + P_\chi t(\chi_{I,t} - \chi_{I,t-1}) + R_{t-1} L_{t-1} \leq L_t + W_t n_{I,t}
\]

Additionally, impatient households’ optimization is subject to the collateral constraint

\[
R_t L_t \leq m_t P_\chi t \chi_{I,t}
\]

where \( m_t \) denotes the LTV ratio, the steady-state of which is assumed to equal unity.
2.1.2 Firms

Final output $y_t$ is produced by perfectly competitive firms that aggregate intermediate goods indexed by $\nu$ according to

$$y_t = \left[ \int_0^1 y_t(\nu)^{\frac{1}{\mu}} d\nu \right]^\mu$$

where $\mu > 1$.

Intermediate goods producing firms operate in a monopolistically competitive environment and use a production function

$$y_t(\nu) = \varepsilon_z t [\omega n_{I,t}(\nu) + (1 - \omega) n_{P,t}(\nu)]$$

where $\varepsilon_z t$ is exogenous productivity. These firms set their prices according to the Calvo scheme, with the probability of not receiving the price change signal given by $\theta$.

2.1.3 Market clearing

We impose a standard set of market clearing conditions. In particular, we assume that the stock of housing is fixed at $\chi$ and hence the housing market clearing condition can be written as

$$\chi = \omega \chi_{I,t} + (1 - \omega) \chi_{P,t}$$

(7)

Equilibrium in the labor market implies

$$n_{i,t} = \int_0^1 n_{i,t}(\nu) d\nu$$

(8)

Finally, the aggregate resource constraint is

$$y_t = \omega c_{I,t} + (1 - \omega) c_{P,t} + g_t$$

(9)

where $g_t$ is government spending that is fully financed by lump sum taxes levied on patient households so that $P_t g_t = T_t$.

2.1.4 Functional forms

The functional form for impatient agents’ housing utility is chosen to be $\nu t(\chi_{I,t}) \equiv \log \chi_{I,t}$. As regards that for patient households, we follow Justiniano et al. (2014) and assume that their preferences imply a rigid demand for housing at some fixed value $\chi_{P}$. Such an asymmetric modeling of utility implies that there is no reallocation of houses across the two types of agents and that they are effectively priced by leveraged households, which is consistent with
Geanakoplos (2010). This simplifying assumption also makes the analytical derivations of our results more tractable. However, as a robustness check, we will also check how our main findings change if we assume symmetric preferences, i.e. \( \nu_P(\chi_{P,t}) \equiv \log \chi_{P,t} \).

### 2.2 Log-linearized model

A log-linearized version of our benchmark model is given by ten equations that we list below. In what follows, variables without time subscripts denote their steady state values while a hat over a variable indicates its log-deviation from the non-stochastic steady state.

The standard consumption Euler equation for patient households is given by

\[
\hat{c}_{P,t} = \mathbb{E}_t \{ \hat{c}_{P,t+1} \} - \hat{R}_t + \mathbb{E}_t \{ \hat{\pi}_{t+1} \}
\]  

while that describing intertemporal consumption choices of impatient households

\[
\hat{c}_{I,t} = \frac{\beta_I}{\beta_P} \mathbb{E}_t \{ \hat{c}_{I,t+1} \} - \hat{R}_t + \frac{\beta_I}{\beta_P} \mathbb{E}_t \{ \hat{\pi}_{t+1} \} - \left( 1 - \frac{\beta_I}{\beta_P} \right) \hat{\Theta}_t
\]

where \( \pi_t \equiv P_t/P_{t-1} \) is the (gross) inflation rate, assumed equal to unity in the steady state, and \( \Theta_t \) is the Lagrange multiplier on the collateral constraint (4). As can be seen, the difference in the consumption Euler equations across both types of agents results from their different discounting factors.

The log-linearized version of impatient households’ budget constraint (3) can be written as

\[
\frac{c_I}{l} \hat{c}_{I,t} + \frac{1}{\beta_P} (\hat{R}_{t-1} - \hat{\pi}_t + \hat{l}_{t-1}) = \frac{wn_I}{l} (\hat{w}_t + \hat{n}_{I,t}) + \hat{l}_t
\]

while the collateral constraint becomes\(^5\)

\[
\hat{R}_t + \hat{l}_t = \hat{m}_t + \hat{p}_{\chi,t} + \hat{\chi}_{t,t}
\]

House prices are determined by the following housing Euler equation

\[
(1 - \beta_P + \beta_I) \hat{p}_{\chi,t} = \beta_I \mathbb{E}_t \{ \hat{p}_{\chi,t+1} \} + \hat{c}_{I,t} - \beta_I \mathbb{E}_t \{ \hat{c}_{I,t+1} \} + (\beta_P - \beta_I) (\hat{\Theta}_t + \hat{m}_t)
\]

and hence depend positively on the tightness of the collateral constraint.

Optimal labor supply schedules (for \( i = \{I, P\} \)) are given by

\[
\hat{w}_t - \hat{c}_{i,t} = \varphi \hat{n}_{i,t}
\]

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\(^5\)Since \( \beta_I < \beta_P \), the collateral constraint is binding in (and sufficiently close to) the non-stochastic steady state, which allows us to write it as an equality.
and postulate equalization of the real wage with the marginal rate of substitution between consumption and leisure.

Solving intermediate good firms’ problem gives the standard Phillips curve

\[ \hat{\pi}_t = \beta P \mathbb{E}_t \{ \hat{\pi}_{t+1} \} + \frac{(1 - \theta)(1 - \beta P \theta)}{\theta} (\hat{w}_t - \hat{\varepsilon}_{z,t}) \tag{16} \]

and the log-linearized version of the aggregate production function can be written as

\[ \hat{y}_t = \hat{\varepsilon}_{z,t} + \omega_n \hat{h}_{I,t} + (1 - \omega_n) \hat{h}_{P,t} \tag{17} \]

where \( \omega_n \equiv \omega n_I / n \).

Log-linearizing the goods market clearing condition (9) yields

\[ \hat{y}_t = \omega_c \hat{c}_{I,t} + (1 - \omega_c - \omega_g) \hat{c}_{P,t} + \omega_g \hat{g}_t \tag{18} \]

where \( \omega_c \equiv \omega c_I / y \) and \( \omega_g \equiv g / y \).

Finally, rather than writing explicit rules for the three instruments controlled by the government, i.e. the interest rate \( \hat{R}_t \), the LTV ratio \( \hat{m}_t \) and government spending \( \hat{g}_t \), we describe them in terms of the stabilization goals that they are aimed to hit.

The model economy is driven by stochastic shocks to productivity \( \hat{\varepsilon}_{z,t} \). It has to be noted that our findings for the ability of various policy mixes to achieve certain stabilization objectives depend on dynamic properties of the system and not on the sources of aggregate fluctuations. Hence, restricting our attention to productivity shocks and using them to provide intuition for our main results should not be seen as restrictive.

### 3 Calibration

Wherever possible, our main results are derived analytically for all admissible parameter values. However, to illustrate and provide intuition for some of the findings, we also offer impulse response analysis. In this section we discuss a benchmark calibration that underlies this part of the discussion and is based on the US macroeconomic data.

We set the discount factor of patient households \( \beta_P \) to 0.995 to match the average annual real interest rate of 2%. Following Campbell and Hercowitz (2009), the relative impatience of borrowers is calibrated at 0.5%. The inverse of the Frisch elasticity of labor supply \( \varphi \) is set to a conventional value of 2. The relative share of impatient households \( \omega \) and housing weight in utility \( A_x \) are fixed at 0.25 and 0.045, respectively, to match the credit-to-GDP ratio of 0.45 and housing-to-GDP ratio of 1.2. We also assume a standard value for the price markup \( (\mu = 1.2) \) and the Calvo probability \( (\theta = 0.75) \). The share of government spending in output is set at its long-run average of 16.5%. Finally, technology shocks are assumed to
follow a simple first-order autoregressive process with the autoregression coefficient $\rho = 0.95$ and standard deviation of innovation $\sigma = 0.01$.

4 Results

We use the benchmark macro-financial NK model laid down in the previous sections to check if macroprudential policy, by using the LTV ratio as its instrument, can complement monetary policy in resolving the standard inflation-output volatility tradeoff or stabilize the financial sector without jeopardizing the central bank’s price stability objective. To this end, we assume that the monetary authority always sets the short-term interest rate to keep inflation at zero at all times, and analyze if there exist a unique and stable equilibrium in which appropriate adjustments in the LTV ratio allow to perfectly stabilize either output, credit or house prices. While discussing the outcomes under such a policy mix, we contrast it with a more traditional one, in which monetary policy actions are complemented with adjustments in fiscal policy that uses government spending as its instrument.

4.1 Inflation-output volatility tradeoff

We start with the classical macroeconomic policy tradeoff, which concerns simultaneous stabilization of prices and economic activity. In general, and in line with the Tinbergen rule, the central bank cannot achieve both of these objectives using only one single instrument which is the short-term interest rate. However, one could hope that adding the LTV ratio as an additional instrument will allow to hit both targets at the same time. Unfortunately, this turns out not to be the case. Formally, we state this finding in form of the following proposition.

Proposition 1. In the benchmark macro-financial NK model, there does not exist a stable equilibrium in which monetary and macroprudential policy achieve full stabilization of inflation and output.

More specifically, as we show in the Appendix, a coordinated attempt of the central bank and macroprudential authority to perfectly stabilize both inflation and output necessarily leads to explosive behavior in credit.

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6 It has to be noted that while price stickiness makes stabilization of inflation one of the relevant targets to a benevolent policy maker, it is less clear that she should always try to limit fluctuations in the other three variables. Hence, our stabilization experiments are more motivated by observation on how policy targets are set in practice than implied by the underlying economic structure. Deriving a welfare-based loss measure in the spirit of Benigno and Woodford (2012) is beyond the scope of this paper. Needless to say, it would be also quite complicated and problematic because of heterogeneity between borrowers and lenders.
To gain some intuition for this result, it is instructive to assume for a moment that the macroprudential policy follows a simple feedback rule of the following form

\[
\hat{m}_t = -\psi_y \hat{y}_t
\]  

(19)

and investigate the effects of increasing the response parameter. As before, the central bank is assumed to set the short-term interest rate such that inflation is constant at all times.

Figure 1 shows the results of this experiment, using the impulse responses to a positive technology shock as an example. If macroprudential policy does not step in \((\psi_y = 0)\), output expands in response to a positive technology shock. A downward pressure on marginal cost means that the central bank must design a monetary expansion to keep inflation constant.\(^7\) This leads to an increase in credit that makes the boom even stronger. Allowing for countercyclical adjustments of the LTV ratio does not help to stabilize output. The responses of this variable for different values of \(\psi_y\) are actually indistinguishable from each other, despite big differences in the paths for the LTV ratio and hence credit. The reason for it can be understood by looking at the responses of the interest rate, which imply more expansionary monetary policy in reaction to more aggressive tightening of the lending standards. Whenever macroprudential policy tries to fight an increase in economic activity with a decrease in the LTV ratio, keeping inflation stable requires such monetary expansion on the part of the central bank that output actually expands at the rate observed under no macroprudential policy interventions.

These findings suggest that defining monetary and macroprudential policy objectives as stabilization of inflation and output is incompatible with what these two policies can jointly achieve. Interestingly, this conflict is not present if we assign the same two objectives to be fulfilled by monetary and fiscal policy, the latter controlling the government spending. Figure 2 shows how perfect stabilization of inflation and output can be achieved by these two policies, using again a positive technology shock as a disturbance. If the government sufficiently cuts its expenditures, expansion in economic activity can be completely neutralized (at least for our baseline calibration), even though this intervention sparks more monetary expansion and hence stronger boom in credit. Overall, fiscal policy emerges as a better complement of monetary policy if the goal is to stabilize prices and output.

### 4.2 Credit stabilization

We have seen that macroprudential policy is not helpful in resolving the classical inflation-output volatility tradeoff. However, this policy is usually thought of as designed to stabilize

\(^7\)As can be seen in the figure, the short-term interest rate actually increases on impact before falling below its steady-state value. However, the sum of the current and future deviations of the interest rate is negative so the policy can be described as expansionary.
the financial sector rather than the real economy. We start by checking how this policy can be combined with monetary policy to simultaneously eliminate fluctuations in the price level and credit. Our finding can be summarized in the following proposition.

**Proposition 2.** In the benchmark macro-financial NK model, a stable equilibrium in which monetary and macroprudential policy achieve full stabilization of inflation and credit exists only if the role of the latter in the economy is sufficiently small.

What this proposition tells us that, under some model parametrization, perfect stabilization of inflation and credit is possible without leading to explosive behavior. In the proof of the proposition documented in the Appendix, we show that this parametrization must be such that the following restriction holds

$$\frac{\varphi \omega_c + \omega_n}{\varphi(1 - \omega_c - \omega_g) + 1 - \omega_n} \leq \frac{1 - \beta_p}{2\varphi} \left(1 + \varphi + \frac{1}{A\chi}\right)$$  \hspace{1cm} (20)

Holding other parameters fixed at our baseline calibration values, this inequality is satisfied for the share of impatient households $\omega$ at most 0.077 or, alternatively, housing weight in utility $A\chi$ no more than 0.011. These threshold values imply (annualized) credit-to-GDP ratio of around 0.14 and 0.12, respectively, which is well below what one can see in the data. Hence, perfect stabilization of inflation and credit using monetary and macroprudential policy is possible only in a low credit environment.

As before, we illustrate these findings using the impulse response analysis. Figure 3 plots the reactions to a positive technology shock if credit is sufficiently low, which we achieve by setting $\omega = 0.054$, implying the credit-to-GDP ratio of 0.1 (annualized). An appropriate decrease in the LTV ratio eliminates the credit boom. Monetary policy needs to be more expansionary in the short-term to offset the deflationary effects of macroprudential policy tightening so that the response of output is indistinguishable from the constant LTV case.

A different picture emerges for an economy in which credit is not that small. Figure 4 plots the responses for several degrees of macroprudential policy aggressiveness, assuming again that the LTV ratio is set according to a simple feedback rule, but this time responding to credit

$$\hat{m}_t = -\psi_l \hat{l}_t$$  \hspace{1cm} (21)

If $\psi_l = 10$, macroprudential policy manages to dampen much of the credit boom. However, to withstand the resulting deflationary pressure and keep inflation stable, the central bank must provide substantial monetary easing. Moreover, the interest rate path is not smooth as its sharp decrease one quarter after the shock fuels credit demand and hence inflationary pressure, which next needs to be counteracted. If macroprudential policy attempts to stabilize credit more by adjusting the LTV ratio more aggressively to credit developments, the oscillations in the interest rate, and hence in credit and the LTV ratio, become more pro-

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nounced. As a result, loans fluctuate even more than under moderate $\psi_t$ and full stabilization of both loans and inflation is not possible.

As previously, we compare these outcomes to those that can be achieved using fiscal rather than macroprudential policy as a complement to monetary policy. Our finding can be summarized by the following proposition.

Proposition 3. In the benchmark macro-financial NK model, using monetary and fiscal policy to fully stabilize inflation and credit leads to indeterminacy.

More precisely, as we show in the Appendix, appropriate adjustments in government spending and the short-term interest rate can ensure perfect stabilization of both inflation and credit without making the economy explode, but this comes at a cost, i.e. the equilibrium is not unique as sunspot equilibria arise. Overall, one can conclude that while macroprudential policy has clear limits in ensuring stable credit without jeopardizing the standard price stability objective pursued by the central bank, use of fiscal policy in this context is also not recommended.

4.3 House price stabilization

Finally, we investigate the ability of macroprudential policy to house prices. We find by simulating the model for various parameter values that all three outcomes, i.e. unique, multiple or no stable equilibria, are possible. Again, the key parameters are those affecting leverage in the economy. Figure 5 illustrates how the type of equilibrium depends on the relative size of impatient households $\omega$ and weight of housing in utility $A_{x}$. A unique stable equilibrium exists only if either of the two parameters is very small, which means that the role of credit in the economy is tiny. For parametrizations implying realistic ratios of credit and housing to GDP, perfect stabilization of inflation and house prices results in indeterminacy (our benchmark calibration) or explosive behavior (higher leverage than in benchmark calibration).

As regards fiscal policy, we can formulate our conclusions in the following proposition.

Proposition 4. In the benchmark macro-financial NK model, using monetary and fiscal policy to fully stabilize inflation and house prices leads to indeterminacy.

Hence, independent of the model parametrization, using the short-term interest rate and government spending to eliminate fluctuations in inflation and house prices results in sunspot equilibria. Overall, one can conclude that neither macroprudential nor fiscal policy are efficient in complementing monetary policy when the primary stabilization objectives are defined as inflation and house price stability.
4.4 Extensions

All results discussed in this section are based on the model variant in which patient households’ housing demand is assumed to be rigid. This assumption combined with fixed total housing stock effectively shut down any trade in housing between the two types of agents, which renders the model more tractable and the propositions relatively easy to prove. In this subsection we briefly discuss how flexible housing demand changes our main findings regarding macroprudential policy. To this end, we assume that housing utility of patient agents is of the same log form as the one applying to impatient households.\(^8\)

Starting with the inflation-output volatility tradeoff, we find our conclusions unaffected. It is not possible to use the short-term interest rate and the LTV ratio to fully stabilize inflation and output such that the model economy does not explode. As regards the ability of macroprudential policy to stabilize credit if the central bank fully stabilizes inflation, our findings are qualitatively similar to those derived for the benchmark model, i.e. there exist a unique stable equilibrium if the role of credit is sufficiently small. This time, however, the threshold leverage in the economy is much higher. In particular, our benchmark calibration is consistent with a unique stable equilibrium in which monetary and macroprudential policy meet their stabilization objectives defined as price and credit stability. Finally, allowing for flexible housing demand of lenders makes the conclusions regarding house price stabilization stronger as this time an attempt to achieve it using the LTV ratio always leads to instability as long as the central bank keeps inflation constant.

Summing up, our general findings on the limited power of macroprudential policy in complementing monetary policy in achieving standard macroeconomic or additional financial stabilization objectives are robust to dropping the assumption on rigid housing demand of patient agents.

4.5 Non-zero stabilization targets

We have already noted that the qualitative results discussed above do not depend on the type of aggregate uncertainty. This is because, as we show in the Appendix, these results are derived from the properties of the dynamic systems where no distinction needs to be made between various types of shocks. Similar considerations lead to a conclusion that our findings can be generalized to non-zero stabilization targets as long as they can be expressed as linear combinations of shocks hitting the economy. This is a relevant case as policy makers might want to perfectly stabilize some macrovariables following one type of shocks and let them respond to other sources of disturbances.

\(^8\)Analytical derivations and simulations underlying the findings discussed below are available from the author upon request.
5 Conclusions

In this paper we have analyzed how macroprudential policy that uses the LTV ratio as its instrument can contribute to macroeconomic stability by helping the central bank in achieving a selection of alternative stabilization objectives. According to our results, what such defined policy can achieve has important limits. In particular, macroprudential policy does not help to resolve the inflation-output volatility tradeoff nor allows to achieve one additional financial target (credit or house price stabilization) without conflicting with the price stability objective.

Naturally, these results do not mean that macroprudential policy cannot be useful, especially in preventing financial crises. However, one should not expect that implementing it at a business cycle frequency will dramatically improve the key macroeconomic or macrofinancial stabilization tradeoffs.

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Tables and figures

Table 1: Calibration

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<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_P$</td>
<td>0.995</td>
<td>Discount factor, patient HHs</td>
</tr>
<tr>
<td>$\beta_I$</td>
<td>0.99</td>
<td>Discount factor, impatient HHs</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.25</td>
<td>Share of impatient HHs in population</td>
</tr>
<tr>
<td>$A_\chi$</td>
<td>0.045</td>
<td>Weight on housing in utility</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>2</td>
<td>Inverse of Frisch elasticity of labor supply</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.2</td>
<td>Steady state product markup</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.75</td>
<td>Calvo probability for prices</td>
</tr>
<tr>
<td>$g_y$</td>
<td>0.165</td>
<td>Share of government spending in output</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.95</td>
<td>Inertia of productivity shocks</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.01</td>
<td>Standard deviation of productivity shocks</td>
</tr>
</tbody>
</table>

Figure 1: Inability of monetary and macroprudential policy to fully stabilize inflation and output

![Figure 1](image_url)

Note: The plots present the responses to a 1% productivity shock for different strength of macroprudential policy response to output gap, assuming that the central bank fully stabilizes inflation.
Figure 2: Full stabilization of inflation and output with monetary and fiscal policy

Note: The plots present the responses to a 1% productivity shock under constant government spending or its adjustments making output constant. In both cases, the central bank fully stabilizes inflation.

Figure 3: Full stabilization of inflation and credit with monetary and macroprudential policy - low credit environment

Note: The plots present the responses to a 1% productivity shock under constant LTV ratio or its adjustments making output constant. In both cases, the central bank fully stabilizes inflation and the share of impatient households (and hence credit-to-GDP ratio) is small.
Figure 4: Inability of monetary and macroprudential policy to fully stabilize inflation and credit - high credit environment

![Output vs. Time](image1)

![Credit vs. Time](image2)

![Interest rate vs. Time](image3)

![LTV vs. Time](image4)

Note: The plots present the responses to a 1% productivity shock for different strength of macroprudential policy response to credit, assuming that the central bank fully stabilizes inflation.

Figure 5: Equilibrium stability regions if monetary and macroprudential policy fully stabilize inflation and house prices

![Equilibrium Stability Regions](image5)

Note: The plot present the equilibrium stability regions as a function of two key parameters, both presented on the log scale. The light gray spot indicates our baseline calibration.
Appendix

A.1 Model equations

In this section of the Appendix we present a full list of equations making up the benchmark NK macrofinancial model. The variables without time subscripts denote their steady state values.

Households

Euler equation for patient households

\[ c^{-1}_{P,t} = \beta_P \mathbb{E}_t \{ c^{-1}_{P,t+1} \pi_t^{-1} \} R_t \]  \hspace{1cm} (A.1)

Impatient households’ budget constraint

\[ c_{I,t} + p_{\chi,t}(\chi_{I,t} - \chi_{I,t-1}) + R_{t-1} l_{t-1} \pi_t^{-1} = l_t + w_t n_{I,t} \]  \hspace{1cm} (A.2)

Collateral constraints

\[ R_t l_t = m_t p_{\chi,t} \chi_{I,t} \]  \hspace{1cm} (A.3)

Euler equations for impatient households

\[ c_{I,t}^{-1} = R_t \{ \beta_I \mathbb{E}_t \{ c_{I,t+1}^{-1} \pi_t^{-1} \} + \Theta_t \} \]  \hspace{1cm} (A.4)

Rigid housing demand of patient households

\[ \chi_{P,t} = \chi_P \]  \hspace{1cm} (A.5)

Housing Euler equation for impatient households

\[ c_{I,t}^{-1} p_{\chi,t} = A_{\chi} \chi_{I,t}^{-\sigma_{\chi}} + \beta_I \mathbb{E}_t \{ c_{I,t+1}^{-1} p_{\chi,t+1} \} + \Theta_t m_t p_{\chi,t} \]  \hspace{1cm} (A.6)

Labor supply (for \( i = \{I, P\} \))

\[ w_t c_{i,t}^{-1} = A_n n_{i,t}^{\gamma} \]  \hspace{1cm} (A.7)

Labor aggregate

\[ n_t = \omega n_{I,t} + (1 - \omega) n_{P,t} \]  \hspace{1cm} (A.8)
Firms

Marginal cost

\[ mc_t = \frac{w_t}{\varepsilon_{z,t}} \] (A.9)

Optimal price set by reoptimizing firms

\[ \tilde{p}_t = \mu \frac{\Omega_t}{\Upsilon_t} \] (A.10)

Auxiliary functions for optimal price

\[ \Omega_t = c_{P_t}^-mc_t y_t + \beta P_t \theta E_t \left\{ \frac{\mu}{\pi_t^{\frac{1}{1-\mu}}} \Omega_{t+1} \right\} \] (A.11)

\[ \Upsilon_t = c_{P_t}^-y_t + \beta P_t \theta E_t \left\{ \frac{1}{\pi_t^{\frac{1}{1-\mu}}} \Upsilon_{t+1} \right\} \] (A.12)

Price indexes

\[ 1 = \theta \left( \frac{\pi}{\pi_t} \right) \frac{1}{1-\mu} + (1 - \theta) \tilde{p}_t^{\frac{1}{1-\mu}} \] (A.13)

Market clearing

Goods market

\[ y_t = \omega c_{I,t} + (1 - \omega)c_{P,t} + g_t \] (A.14)

Aggregate output

\[ y_t \Delta_t = \varepsilon_{z,t} \eta_t \] (A.15)

Price dispersion index

\[ \Delta_t = \theta \Delta_{t-1} \pi_t^{\frac{\mu}{1-\mu}} + (1 - \theta) \tilde{p}_t^{\frac{\mu}{1-\mu}} \] (A.16)

Housing market

\[ \chi = \omega_I \chi_{I,t} + (1 - \omega_I) \chi_{P,t} \] (A.17)

A.2 Proof of Proposition 1 (inflation and output stabilization using monetary and macroprudential policy)

We prove Proposition 1 by showing that perfect stabilization of both inflation and output using the short-term interest rate and LTV ratio are inconsistent with the existence of a stable rational expectations equilibrium.

First note that the aggregate production function (17) combined with optimal labor
schedules (15) and market clearing condition (18) imply
\[ \omega_c \hat{c}_{I,t} + (1 - \omega_c - \omega_g) \hat{c}_{P,t} + \omega_g \hat{g}_t = \hat{\varepsilon}_{z,t} + \varphi^{-1} \hat{w}_t - \varphi^{-1}[\omega_n \hat{c}_{I,t} + (1 - \omega_n) \hat{c}_{P,t}] \] (A.18)

If output is constant at all times, the market clearing condition (18) implies
\[ \omega_c \hat{c}_{I,t} + (1 - \omega_c - \omega_g) \hat{c}_{P,t} + \omega_g \hat{g}_t = 0 \] (A.19)

If additionally inflation is zero at all times, the Phillips curve implies \( \hat{w}_t = \hat{\varepsilon}_{z,t} \) and hence equation (A.18) can be rewritten as
\[ \omega_n \hat{c}_{I,t} + (1 - \omega_n) \hat{c}_{P,t} - (1 + \varphi) \hat{\varepsilon}_{z,t} = 0 \] (A.20)

With inactive fiscal policy (\( \hat{g}_t = 0 \)), these two equations which can be solved for \( \hat{c}_{I,t} \) and \( \hat{c}_{P,t} \) as functions of \( \hat{\varepsilon}_{z,t} \) only or, more generally, any exogenous shocks hitting the economy at time \( t \), including fiscal policy shocks.

Further, the Euler equation for patient households (10) implies that \( \hat{R}_t \) can be written as a function of exogenous shocks only, which together with the Euler equation for impatient households (11) leads to the same result for \( \hat{\Theta}_t \).

Let us denote any linear function of exogenous variables at time \( t \) as \( exo_t \), and any vector of such functions as \( Exo_t \). Then, after using the collateral constraint (13) to eliminate \( \hat{m}_t \), our benchmark NK macrofinancial model can be reduced to the following three equations

\[ \hat{R}_t = exo_t \] (A.21)

\[ \frac{1}{\beta_P} (\hat{R}_{t-1} + \hat{l}_{t-1}) - \hat{l}_t = exo_t \] (A.22)

\[ \hat{p}_{\chi,t} - \beta_I \mathbb{E}_t \{ \hat{p}_{\chi,t+1} \} + (\beta_I - \beta_P) \hat{l}_t = exo_t \] (A.23)

It is clear by combining equations (A.21) and (A.22) that credit is explosive in response to shocks. More formally, system (A.21)-(A.23) can be cast in matrix notation

\[ \Gamma_0 \begin{bmatrix} \hat{R}_t \\ \hat{l}_t \\ E_t \hat{p}_{\chi,t+1} \end{bmatrix} = \Gamma_1 \begin{bmatrix} \hat{R}_{t-1} \\ \hat{l}_{t-1} \\ \hat{p}_{\chi,t} \end{bmatrix} + Exo_t \] (A.24)

where

\[ \Gamma_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \beta_P - \beta_I & \beta_I \end{bmatrix} \]

\[ \Gamma_1 = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{\beta_P} & \frac{1}{\beta_P} & 0 \\ 0 & 0 & 1 \end{bmatrix} \]
The determinant of $\Gamma_0$ is equal to $\beta_I > 0$, which allows us to write

\[
\begin{bmatrix}
\hat{R}_t \\
\hat{i}_t \\
E_t \hat{p}_{x,t+1}
\end{bmatrix} = \Gamma
\begin{bmatrix}
\hat{R}_{t-1} \\
\hat{i}_{t-1} \\
\hat{p}_{x,t}
\end{bmatrix} + E x o_t
\] (A.25)

The eigenvalues of $\Gamma \equiv \Gamma_0^{-1} \Gamma_1$ are $[ \beta_I^{-1} \beta_P^{-1} 0 ]$. Hence, there are two eigenvalues outside the unit circle and only one forward-looking variable in the associated system (A.24), which means that no stable equilibrium exists, see Blanchard and Kahn (1980).

### A.3 Proof of Proposition 2 (inflation and credit stabilization using monetary and macroprudential policy)

We prove Proposition 2 by showing that perfect stabilization of both inflation and credit using the short-term interest rate and LTV ratio is consistent with the existence of a stable rational expectations equilibrium only if the model parameters satisfy a certain restriction implying relatively minor role of credit in the economy.

If inflation is zero at all times, equation (A.18) and the Phillips curve imply

\[
\phi (\omega_c \hat{c}_{I,t} + (1 - \omega_c - \omega_g) \hat{c}_{P,t} + \omega_g \hat{g}_t) + \omega_n \hat{c}_{I,t} + (1 - \omega_n) \hat{c}_{P,t} - (1 + \phi) \hat{\varepsilon}_{z,t} = 0
\] (A.26)

which allows us to express patient households’ consumption as function of that of impatient ones and exogenous variables

\[
\hat{c}_{P,t} = -a \hat{c}_{I,t} + E x o_t
\] (A.27)

where $a \equiv \frac{\omega_c \phi + \omega_n}{\phi (1 - \omega_c - \omega_g) + 1 - \omega_n} > 0$ and $E x o_t$ denotes any linear function of exogenous variables at time $t$, including the ones not considered in our benchmark model.

Plugging this condition into the model equations and restricting credit to be constant at all times leads to the following system of equations

\[-a \hat{c}_{I,t} + a \hat{E}_t \{ \hat{c}_{I,t+1} \} + \hat{R}_t = E x o_t\] (A.28)

\[s_c \hat{c}_{I,t} + \frac{1}{\beta_P} \hat{R}_{t-1} = E x o_t\] (A.29)

\[\hat{c}_{I,t} - \hat{p}_{x,t} + \beta_I \hat{E}_t \{ -\hat{c}_{I,t+1} + \hat{p}_{x,t+1} \} + (\beta_P - \beta_I) (\hat{\Theta}_t + \hat{m}_t + \hat{p}_{x,t}) = E x o_t\] (A.30)

\[\beta_P (\hat{c}_{I,t} + \hat{R}_t) - \beta_I \hat{E}_t \{ \hat{c}_{I,t+1} \} + (\beta_P - \beta_I) \hat{\Theta}_t = E x o_t\] (A.31)

\[\hat{R}_t = \hat{m}_t + \hat{p}_{x,t} + E x o_t\] (A.32)

where $s_c \equiv \frac{\phi}{1 + \frac{\omega_I}{\omega}} > 0$.

If we use equation (A.29) to eliminate $\hat{c}_{I,t}$, equation (A.31) to eliminate $\hat{\Theta}_t$, and equation
(A.32) to eliminate \( \hat{m}_t \), the system reduces to two equations in two endogenous variables

\[
\frac{a}{s_c \beta_p} \hat{R}_{t-1} + (1 - \frac{a}{s_c \beta_p}) \hat{R}_t = exo_t
\]

(A.33)

\[-\frac{1 - \beta P}{s_c \beta_p} \hat{R}_{t-1} - \hat{p}_{\chi,t} - \beta_I \hat{R}_t + \beta_I \mathbb{E}_t \{ \hat{p}_{\chi,t+1} \} = exo_t
\]

(A.34)

Casting it in matrix notation yields

\[
\begin{bmatrix}
R_t \\
E_t \hat{p}_{\chi,t+1}
\end{bmatrix} =
\Gamma_0
\begin{bmatrix}
R_{t-1} \\
p_{\chi,t}
\end{bmatrix} + Exo_t
\]

(A.35)

where

\[
\Gamma_0 =
\begin{bmatrix}
1 - \frac{a}{s_c \beta_p} & 0 \\
-\beta_I & \beta_I
\end{bmatrix}
\Gamma_1 =
\begin{bmatrix}
-\frac{a}{s_c \beta_p} & 0 \\
\frac{1 - \beta P}{s_c \beta_p} & 1
\end{bmatrix}
\]

Note that the determinant of \( \Gamma_0 \) is \( (1 - \frac{a}{s_c \beta_p}) \beta_I \), which is non-zero except for very special parametrization, so we can write

\[
\begin{bmatrix}
R_t \\
E_t \hat{p}_{\chi,t+1}
\end{bmatrix} = \Gamma
\begin{bmatrix}
R_{t-1} \\
p_{\chi,t}
\end{bmatrix} + Exo_t
\]

(A.36)

The two eigenvalues of \( \Gamma \) are \( 1/\beta_I \) and \( a/(a - \beta_P s_c) \). The first one clearly lies outside the unit circle. Since there is one forward-looking variable in system (A.35), and both \( a \) and \( s_c \) are strictly positive, for the stable equilibrium to exist we must have \( 2a \leq \beta_P s_c \).

Let us now work more on the formula for \( s_c \). First note that the Euler equation (A.1) evaluated in the steady state implies

\[
R = \beta_P^{-1}
\]

(A.37)

This together with the Euler equation for impatient agents (A.4) yields

\[
\Theta = c_I^{-1}(\beta_P - \beta_I)
\]

(A.38)

This formula can be used to substitute for \( \Theta \) in the housing Euler equation (A.6), which gives

\[
1 - \beta_P = \frac{A_{\chi} c_I}{\hat{p}_{\chi,I}}
\]

(A.39)

After using the collateral constraint (A.3) to substitute for \( p_{\chi,I} \) we obtain

\[
\frac{c_I}{I} = \frac{1 - \beta_P}{A_{\chi} \beta_P}
\]

(A.40)

Next note that the budget constraint of impatient households (A.2) evaluated at the steady
state implies
\[ \frac{wn_I}{l} = c_I + \beta_P^{-1} - 1 \]  
(A.41)

Now we are ready to derive the formula for \( s_c \) as a function of deep model parameters
\[ s_c = \frac{c_I}{l} + \varphi^{-1} \frac{wn_I}{l} = (1 + \varphi^{-1}) \frac{c_I}{l} + \varphi^{-1} (\beta_P^{-1} - 1) = \frac{1 - \beta_P}{\varphi \beta_P} \left( 1 + \frac{\varphi + 1}{A_\chi} \right) \]  
(A.42)

Plugging this equation into condition \( 2a \leq \beta_P s_c \) gives
\[ \frac{\varphi \omega_c + \omega_n}{\varphi (1 - \omega_c - \omega_g) + 1 - \omega_n} \leq \frac{1 - \beta_P}{2\varphi} \left( 1 + \frac{\varphi + 1}{A_\chi} \right) \]
which is restriction (20) in the main text.

This condition is sufficient for the existence of a stable and unique equilibrium since the rank condition (see Blanchard and Kahn 1980 or Klein, 2000) holds for any parameter values. To see it, consider the Jordan decomposition of \( \Gamma \)
\[ \Gamma = P \Lambda P^{-1} \]

It can be easily shown that \( P^{-1} \) is of the form
\[ P^{-1} = \begin{bmatrix} p_{11} & 0 \\ p_{21} & 1 \end{bmatrix} \]
and hence entry \((2,2)\) of \( P^{-1} \) which is associated with the forward-looking variable is non-zero.

### A.4 Proof of Proposition 3 (inflation and credit stabilization using monetary and government spending policy)

Proposition 3 is proved by demonstrating that perfect stabilization of both inflation and credit using the short-term interest rate and government spending results in non-uniqueness or non-existence of a stable equilibrium.

First note that, if inflation and credit are both constant at all times and macroprudential policy is inactive (i.e. not responding to endogenous variables), the model can be reduced to the following system of equations
\[ \hat{c}_{P,t} - \mathbb{E}_t \{ \hat{c}_{P,t+1} \} + \hat{R}_t = exo_t \]  
(A.43)
\[ s_c \hat{c}_{I,t} + \frac{1}{\beta_P} \hat{R}_{t-1} = exo_t \]  
(A.44)
\[
\begin{align*}
\dot{c}_{I,t} - \hat{p}_{X,t} + \beta_I \mathbb{E}_t \{-\dot{c}_{I,t+1} + \hat{p}_{X,t+1}\} + (\beta_P - \beta_I)\hat{\Theta}_t = & \text{exo}_t \quad (A.45) \\
\beta_P(\dot{c}_{I,t} + \hat{R}_t) - \beta_I \mathbb{E}_t \{\dot{c}_{I,t+1}\} + (\beta_P - \beta_I)\hat{\Theta}_t = & \text{exo}_t \quad (A.46) \\
\hat{R}_t = & \hat{p}_{X,t} + \text{exo}_t \quad (A.47)
\end{align*}
\]

where \(s_c \equiv \frac{c_t}{t} + \frac{w_{n,t}}{\varphi t} > 0\) and exo\(_t\) denotes any linear function of any exogenous variables showing up in the model at time \(t\).

Eliminating \(\hat{\Theta}_t, \hat{R}_t\) and \(\dot{c}_{I,t}\) allows us to write

\[
\begin{align*}
\dot{c}_{P,t} - \mathbb{E}_t \{\dot{c}_{P,t+1}\} + \hat{p}_{X,t} = & \text{exo}_t \quad (A.48) \\
\frac{1 - \beta_P}{s_c \beta_P} \hat{p}_{X,t-1} + (1 + \beta_I)\hat{p}_{X,t} - \beta_I \mathbb{E}_t \{\hat{p}_{X,t+1}\} = & \text{exo}_t \quad (A.49)
\end{align*}
\]

After defining \(\hat{p}_{X,t} \equiv \hat{p}_{X,t-1}\), the system can be cast in matrix notation as follows

\[
\Gamma_0 \begin{bmatrix} \hat{p}_{X,t+1} \\ E_t \hat{c}_{P,t+1} \\ E_t \hat{p}_{X,t+1} \end{bmatrix} = \Gamma_1 \begin{bmatrix} \hat{p}_{X,t} \\ \hat{c}_{P,t} \\ \hat{p}_{X,t} \end{bmatrix} + \text{exo}_t \quad (A.50)
\]

where

\[
\Gamma_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \beta_I \end{bmatrix}, \quad \Gamma_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ \frac{1 - \beta_P}{s_c \beta_P} & 0 & 1 + \beta_I \end{bmatrix}
\]

Note that the determinant of \(\Gamma_0\) is \(\beta_I > 0\), hence we can write

\[
\begin{bmatrix} \hat{p}_{X,t+1} \\ E_t \hat{c}_{P,t+1} \\ E_t \hat{p}_{X,t+1} \end{bmatrix} = \Gamma \begin{bmatrix} \hat{p}_{X,t} \\ \hat{c}_{P,t} \\ \hat{p}_{X,t} \end{bmatrix} + \text{exo}_t \quad (A.51)
\]

It is easy to verify that one of the eigenvalues of \(\Gamma = \Gamma_0^{-1}\Gamma_1\) is 1, while the other two are given by the following formula

\[
\frac{\beta_I + 1 \pm \sqrt{(\beta_I + 1)^2 + \frac{4\beta_I(1 - \beta_P)}{\beta_P s_c}}}{2\beta_I}
\]

Of these two eigenvalues, one is clearly larger than 1. There are two forward-looking variables in the reduced system (A.50). Hence, for there to exist a unique stable equilibrium, the other eigenvalue must be smaller than −1. For this to be the case, the following condition must hold

\[
s_c < \frac{1 - \beta_P}{(2\beta_I + 1)\beta_P} \quad (A.52)
\]
If it does not, there exist multiple stable solutions (indeterminacy).

Note that, for reasonable values of the discount factors $\beta_P$ and $\beta_I$, the right-hand side of (A.52) is small and hence this condition will not be met. In what follows we will show that even if inequality (A.52) is satisfied, and hence the Blanchard-Kahn condition is met, the so-called rank condition fails. To see it, consider the Jordan decomposition of $\Gamma$

$$\Gamma = PA P^{-1}$$

and order $\Lambda$ such that the unit eigenvalue of $\Gamma$ enters as element $[1,1]$. It can be easily verified that $P^{-1}$ is of the form

$$P^{-1} = \begin{bmatrix} p_{11} & 1 & p_{13} \\ p_{21} & 0 & p_{23} \\ -p_{21} & 0 & -p_{23} \end{bmatrix}$$

Clearly, the part of $P^{-1}$ associated with the two forward-looking variables is rank deficient and the last two rows of this matrix are perfectly collinear. This allows us to conclude that the model has multiple stable solutions, see Blanchard and Kahn (1980) and Klein (2000).

### A.5 Proof of Proposition 4 (inflation and house price stabilization using monetary and government spending policy)

As in the case of Proposition 3, Proposition 4 is proved by demonstrating that perfect stabilization of both inflation and credit using the short-term interest rate and government spending results in non-uniqueness of the equilibrium.

First use the assumption on constant inflation and house prices together with inactive macroprudential policy to reduce the model to the following system of equations

\begin{align*}
\hat{c}_{P,t} - \mathbb{E}_t \{\hat{c}_{P,t+1}\} + \hat{R}_t &= \text{exo}_t \quad \text{(A.53)} \\
sc \hat{c}_{I,t} + \frac{1}{\beta_P} (\hat{R}_{t-1} + \hat{l}_{t-1}) - \hat{l}_t &= \text{exo}_t \quad \text{(A.54)} \\
\hat{c}_{I,t} - \beta_I \mathbb{E}_t \{\hat{c}_{I,t+1}\} + (\beta_P - \beta_I) \hat{\Theta}_t &= \text{exo}_t \quad \text{(A.55)} \\
\beta_P (\hat{c}_{I,t} + \hat{R}_t) - \beta_I \mathbb{E}_t \{\hat{c}_{I,t+1}\} + (\beta_P - \beta_I) \hat{\Theta}_t &= \text{exo}_t \quad \text{(A.56)} \\
\hat{R}_t + \hat{l}_t &= \text{exo}_t \quad \text{(A.57)}
\end{align*}

where, as previously, $s_c \equiv \frac{c_I}{\ell} + \frac{w_I}{\ell} > 0$ and $\text{exo}_t$ denotes any linear function of exogenous variables at time $t$.

Now eliminate $\hat{\Theta}_t$ and $\hat{l}_t$ to obtain

\begin{align*}
\hat{c}_{P,t} - \mathbb{E}_t \{\hat{c}_{P,t+1}\} + \hat{R}_t &= \text{exo}_t \quad \text{(A.58)}
\end{align*}
\[ s_c \hat{c}_{I,t} + \hat{R}_t = exo_t \]  
(A.59)

\[ (1 - \beta p)\hat{c}_{I,t} - \beta p \hat{R}_t = exo_t \]  
(A.60)

Since equations (A.59) and (A.60) can be solved for \( \hat{c}_{I,t} \) and \( \hat{R}_t \) as functions of exogenous variables only, we can write equation (A.58) as

\[ \hat{c}_{P,t} - \mathbb{E}_t \{ \hat{c}_{P,t+1} \} = exo_t \]  
(A.61)

There is a unit eigenvalue associated with this equation and it includes one forward-looking variable, hence the equilibrium is indeterminate, see Blanchard and Kahn (1980).