

Optimal Automatic Stabilizers

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Abstract

This paper studies the design of fiscal policies that serve as automatic stabilizers in an incomplete markets economy affected by inefficient business cycle fluctuations. We make three contributions. First, we provide a model that combines nominal rigidities, idiosyncratic income shocks and incomplete markets, but which is sufficiently simple that we can analyze it with an AS-AD diagram to show how sticky prices and incomplete markets interact to determine the effect of and desirability of the automatic stabilizers. Second, we characterize social welfare and show that it depends on the variance of an output gap and inflation as well as on a measure of time-varying inequality. The interaction of nominal rigidities and incomplete markets raises the costs of business cycles making room for stabilization policy to achieve large gains. Third, we calibrate the model to match the main facts about inequality in order to solve for the optimal set of automatic stabilizers. We show that stabilization concerns make the income tax more progressive, unemployment benefits and income support policies more generous.

1 Introduction

How generous should the unemployment insurance system be? How progressive should the tax system be? These questions have been studied extensively and there are well-known trade-offs between social insurance and incentives. Typically these issues are explored in the context of a stationary economy. These policies, however, also serve as automatic stabilizers that alter the dynamics of the business cycle. The purpose of this paper is to ask how and when aggregate stabilization objectives call for, say, more generous unemployment benefits or a more progressive tax system than would be desirable in a stationary economy.

Our model has two main ingredients: nominal rigidities and uninsurable idiosyncratic risk. While markets are incomplete, the model nonetheless aggregates in a no-trade equilibrium along the lines of Sterk and Ravn (2013). This tractability allows us to analyze the dynamics of the economy through an aggregate-supply-aggregate-demand (AS-AD) diagram. The model's tractability also allows us to express a utilitarian social welfare function in terms of aggregate variables. The model also includes nominal rigidities which make aggregate demand relevant for the business cycle. This is important in that automatic stabilizers mainly operate through aggregate demand channels.

We consider two classic automatic stabilizers: unemployment benefits and progressive taxation. Both of these policies have roles in redistributing income and in providing social insurance. Redistribution affects aggregate demand in our model because households differ in their marginal propensities to consume. Social insurance affects aggregate demand through precautionary savings decisions because markets are incomplete. In addition to unemployment insurance and progressive taxation, we also consider a fiscal rule that makes government spending respond automatically to the state of the economy.

Our focus is on the manner in which the optimal fiscal structure of the economy is altered by aggregate stabilization concerns. Increasing the scope of the automatic stabilizers can lead to welfare gains if they raise equilibrium output when it would otherwise be inefficiently low and vice versa. Therefore, it is not stabilization per se that is the objective but rather eliminating inefficient fluctuations. An important aspect of the model specifica-

tion is therefore the extent of inefficient business cycle fluctuations. Our model generates inefficient fluctuations because prices are sticky and monetary policy cannot fully eliminate the distortions. We show that in a reasonable calibration, more generous unemployment benefits and more progressive taxation are helpful in reducing these inefficiencies. Simply put, if unemployment is high when there is a negative output gap, a larger unemployment benefit will stimulate aggregate demand when it is inefficiently low thereby raising welfare. Similarly, if idiosyncratic risk is high when there is a negative output gap,¹ providing social insurance through more progressive taxation will also increase welfare.

Our work is related to several strands of literature. Baily (1978) derives a condition for optimal unemployment insurance that balances insurance with moral hazard. Chetty (2006) shows that this condition applies in a wide range of settings. Of particular relevance for our work, Landais, Michaillat, and Saez (2010) show that the Baily-Chetty formula needs to be adjusted in general equilibrium to incorporate the effect of unemployment benefits on labor market tightness, which might differ from the socially optimal level. In their analysis, the level of unemployment benefits affects labor market tightness through the worker's choice of search intensity and the firms' hiring decisions. Our analysis also adjusts the incentives-insurance trade-off to account for general equilibrium effects. However, we focus on the impact of unemployment benefits and other policies on aggregate demand. Our analysis differs from that of Landais, Michaillat, and Saez (2010), in three ways: first, the key inefficiency in our model is nominal rigidity as opposed to search frictions; second, we consider other policies that affect aggregate demand in addition to unemployment insurance; and third, we focus on fixed fiscal rules (e.g. a constant unemployment benefit) as opposed to rules that respond to the cycle (e.g. counter-cyclical benefits).

The optimal progressivity of the tax system has been studied by, among others, Mirrlees (1971), Varian (1980), Conesa and Krueger (2006), Conesa, Kitao, and Krueger (2009), Heathcote, Storesletten, and Violante (2014), and Krueger and Ludwig (2014). We know of no existing work that considers the optimal progressivity of the tax code in presence of

¹Guvenen, Ozkan, and Song (2014) show that the distribution of earnings growth rates becomes negatively skewed in a recession.

business cycles.

Farhi and Werning (2013) and Korinek and Simsek (2014) have considered how the distribution of resources across households can interact with aggregate inefficiencies that result from nominal rigidities. Those authors investigate how macro-prudential regulation in the form of credit market interventions can alter the distribution of wealth in ways that raise welfare. In short, an episode in which a negative output gap results from household deleveraging can be avoided by limiting household debt ex ante. Our work does not consider credit market interventions, but instead redistribution and social insurance through the tax-and-transfer system.

In our previous work, McKay and Reis (2014), we have considered how the automatic stabilizers implemented in the US alter the dynamics of the business cycle. Here we are concerned with the optimal fiscal system as opposed to the observed one. In addition, the model we present here includes stronger links between the business cycle and inequality that both make aggregate fluctuations more costly for welfare and also create a larger role for automatic stabilizers in altering aggregate dynamics.

2 The environment

2.1 Population, preferences and endowments

There is a unit continuum of households and each household i is endowed with individual labor productivity $\alpha_{i,t}$ at time t . Households may be employed or unemployed. Households survive from one period to the next with probability $1 - \delta$. Households derive utility from consumption, c and publicly provided goods, G , and derive disutility from working for pay, h , and searching for work, q , as specified by

$$\mathbb{E}_0 \sum_t \hat{\beta}^t \left[\log(c_t) - \frac{h_t^{1+\gamma}}{1+\gamma} - \frac{q_t^{1+\kappa}}{1+\kappa} + \chi \log(G_t) \right]$$

We use $\beta \equiv \hat{\beta}(1 - \delta)$ to denote the joint effect of pure time preference and discounting due to mortality risk.

When a household dies, it is replaced by a household with no assets and $\alpha_{i,t}$ normalized to 1. The mortality risk allows for a stationary cross-sectional distribution of productivity along with permanent shocks. Mortality is independent of household characteristics.

The productivity of surviving households evolves as

$$\alpha_{i,t+1} = \alpha_{i,t}\epsilon_{i,t+1}.$$

$\log \epsilon_{i,t+1}$ is distributed normally with variance $\sigma_{\epsilon,t+1}^2$ and mean $-\sigma_{\epsilon,t+1}^2/2$. This implies that the average labor productivity in the population is constant and equal to one. We assume that $\sigma_{\epsilon,t+1}^2$ is related to aggregate conditions according to

$$\sigma_{\epsilon,t+1}^2 = \bar{\sigma}_{\epsilon}^2 \left(\frac{Y_t}{\bar{Y}} \right)^{\xi^{\sigma}} \eta_t^{\epsilon}, \quad (1)$$

where η_t^{ϵ} is an exogenous shock to the level of risk in the economy.

Households transition between employment and unemployment. We assume that unemployment is distributed i.i.d. across households. Given the high (quarterly) job-finding rates in the US, the assumption of i.i.d. unemployment spells is not too poor an approximation. At the start of the period, a fraction v of households loses employment and must search to regain employment. Search effort q_t leads to employment with probability $M_t q_t$. M_t plays the role of labor market tightness in our model and we build in an exogenous relationship between the level of output and the labor market tightness to reflect Okun's Law:

$$M_t = \bar{M} \left(\frac{Y_t}{\bar{Y}} \right)^{\xi^M} \eta_t^M, \quad (2)$$

where η_t^M is an exogenous shock to the labor market. As we will show, all searching households will select the same search effort. It then follows that the unemployment rate is then

given by

$$u_t = v(1 - q_t M_t). \quad (3)$$

As we will establish, all employed households will choose the same level of work effort, h_t . It then follows that aggregate labor supply is $h_t \int \alpha_{i,t} n_{i,t} di$, where $n_{i,t}$ indicates whether household i is employed. Using the facts that the cross-sectional expectation of $\alpha_{i,t}$ is equal to one and $n_{i,t}$ is independent of $\alpha_{i,t}$, it follows that aggregate labor supply is $(1 - u_t)h_t$.

2.2 Technology and market structure

Households consume a final good that is a Dixit-Stiglitz aggregate of intermediate inputs

$$Y_t = \left(\int_0^1 y_t(j)^{1/\mu} dj \right)^\mu$$

leading to price index

$$p_t = \left(\int_0^1 p_t(j)^{1/(1-\mu)} dj \right)^{1-\mu}.$$

As usual the demand for variety j is

$$y_t(j) = \left(\frac{p_t(j)}{p_t} \right)^{\mu/(1-\mu)} Y_t. \quad (4)$$

Intermediate goods are produced from labor alone with $y_t(j) = A_t n_t(j)$ where $n_t(j)$ is employment of effective units of labor and A_t is the aggregate productivity, which follows

$$\log A_t = \rho^A \log A_{t-1} + \eta_t^A. \quad (5)$$

Intermediate goods firms face nominal rigidities in the style of Calvo (1983). Each firm has a probability θ of updating its price each period.

The profits of intermediate goods firms are distributed among the employed workers in proportion to their skill. The profits received by a household with productivity α can be

written as αd_t .

The labor market is a Walrasian spot market with a wage w_t per efficiency unit of labor. Markets are incomplete in that there are no insurance markets for idiosyncratic productivity and employment shocks. Households can only save in an annuity that pays

$$R_t = \tilde{R}_t / (1 - \delta)$$

where \tilde{R}_t is the return on a risk-free bond. The underlying bond is in zero net supply and as a result the annuity is also in zero net supply.

Total income in this economy is the sum of labor earnings and dividend income:

$$Y_t = \int \alpha_{i,t} (w_t h_t + d_t) n_{i,t} di.$$

As $\alpha_{i,t}$ integrates to one, this implies

$$Y_t = (1 - u_t)(w_t h_t + d_t). \tag{6}$$

Integrating (4) across firms and using labor market clearing results in the aggregate production function

$$S_t Y_t = A_t h_t (1 - u_t), \tag{7}$$

where $S_t \equiv \int (p_t(j)/p_t)^{\mu/(1-\mu)} dj$ reflects the efficiency loss due to price dispersion. The dynamics of S_t are described by

$$S_t = (1 - \theta) S_{t-1} \pi_t^{-\mu/(1-\mu)} + \theta \left(\frac{p_t^*}{p_t} \right)^{\mu/(1-\mu)}, \tag{8}$$

where π_t is the (gross) inflation rate from $t - 1$ to t and p_t^* is the price chosen by firms that

update at date t . The inflation rate and the optimal price are related according to

$$\pi_t = \left(\frac{1 - \theta}{1 - \theta \left(\frac{p_t^*}{p_t} \right)^{1/(1-\mu)}} \right)^{1-\mu}. \quad (9)$$

Finally, the aggregate resource constraint is

$$Y_t = C_t + G_t. \quad (10)$$

2.3 Government

The government raises tax revenue to finance government purchases and an unemployment benefit system. Households receive four types of income in the model: labor earnings, dividends, UI payments and interest payments. We assume that a progressive tax system applies to the first three types of income. Interest income is untaxed for simplicity.

If $x_{i,t}$ is taxable income, then after-tax income is given by $\lambda_t x^{1-\tau}$. λ_t determines the overall level of taxes and τ determines the progressivity of the tax system.

Unemployment benefits are paid in proportion to what the unemployed worker would earn if she were employed. Specifically the benefit is $b\alpha_{i,t}(w_t h_t + d_t)$, where b is a constant that determines the replacement rate and h_t is the level of hours chosen by all employed households.

Government purchases are given by the rule

$$G_t = \bar{G} \left(\frac{Y_{t+1}}{\bar{Y}} \right)^\phi \eta_t^G, \quad (11)$$

where η_t^G is an exogenous government spending shock.

Budget balance then requires

$$\int x_{i,t} - \lambda_t x_{i,t}^{1-\tau} di = G_t + b(w_t h_t + d_t) \int (1 - n_{i,t}) \alpha_{i,t} di,$$

which implies

$$\begin{aligned} \frac{Y_t}{1-u_t}(1-u_t+u_t b) - \lambda_t \left(\frac{Y_t}{1-u_t}\right)^{1-\tau} (1-u_t+u_t b^{1-\tau}) \mathbb{E}_i [\alpha_{i,t}^{1-\tau}] &= G_t + b \frac{Y_t}{1-u_t} u_t \\ Y_t - G_t &= \lambda_t \left(\frac{Y_t}{1-u_t}\right)^{1-\tau} (1-u_t+u_t b^{1-\tau}) \mathbb{E}_i [\alpha_{i,t}^{1-\tau}] \end{aligned}$$

The cross-sectional moment $\mathbb{E}_i [\alpha_{i,t}^{1-\tau}]$ evolves according to

$$\begin{aligned} \mathbb{E}_i [\alpha_{i,t}^{1-\tau}] &= (1-\delta) \mathbb{E}_i [\alpha_{i,t-1}^{1-\tau}] \mathbb{E}_i [\epsilon_{i,t}^{1-\tau}] + \delta \\ \mathbb{E}_i [\alpha_{i,t}^{1-\tau}] &= (1-\delta) \mathbb{E}_i [\alpha_{i,t-1}^{1-\tau}] e^{\frac{\sigma_{\epsilon,t}^2}{2}(\tau^2-\tau)} + \delta. \end{aligned} \tag{12}$$

Note that with $\tau \in (0, 1)$, $\mathbb{E}_i [\alpha_{i,t}^{1-\tau}]$ will be decreasing in $\sigma_{\epsilon,t}^2$.

Monetary policy follows a Taylor rule that sets the nominal interest rate as a function of current inflation

$$I_t = \bar{I} \pi_t^\omega. \tag{13}$$

The real interest rate is then determined by the Fisher equation

$$R_t = I_t / \mathbb{E}_t [\pi_{t+1}]. \tag{14}$$

2.4 Decision problems

2.4.1 Household problem

Here we analyze the household's problem in a no-trade equilibrium. There is no trade in equilibrium because the borrowing constraint is maximally tight. If no household can borrow there are no assets in equilibrium and no household can save. This approach is inspired by Krusell, Mukoyama, and Smith Jr (2011) and Sterk and Ravn (2013).

Let \mathcal{S} be the collection of aggregate states, with law of motion $\mathcal{S}' = H(\mathcal{S}, \eta')$, where η' is a vector of aggregate shocks. The decision problem of an employed household is captured

by

$$V(a, \alpha, n, \mathcal{S}) = \max_{a', c, h} \left\{ \log c - \frac{h^{1+\gamma}}{1+\gamma} + \beta \mathbb{E} [(1-v)V(a', \alpha', 1, \mathcal{S}') + vV^s(a', \alpha', \mathcal{S}')] \right\}$$

subject to

$$a' + c = Ra + \lambda [(n + (1-n)b)\alpha(wh + d)]^{1-\tau}$$

and $a' \geq 0$, where a represents annuity holdings and n is an indicator for employment status.

For a household searching for a job, the decision problem is captured by

$$V^s(a, \alpha, \mathcal{S}) = \max_q \left\{ MqV(a, \alpha, 1, \mathcal{S}) + (1-Mq)V(a, \alpha, 0, \mathcal{S}) - \frac{q^{1+\kappa}}{1+\kappa} \right\}.$$

We will analyze the household's problem in three stages: first we will show that all employed households choose the same level of hours regardless of skill. Second, we will show that all searching households select the same level search effort regardless of skill. Finally, we will show that all employed households have the same Euler equation and all unemployed households have the same Euler equation regardless of skill.

The intra-temporal labor supply condition for an employed household is

$$h_{i,t}^\gamma = \frac{1}{c_{i,t}} \lambda_t \alpha_{i,t}^{1-\tau} (1-\tau) (w_t h_{i,t} + d_t)^{-\tau} w_t \quad (15)$$

In the no-trade equilibrium, the employed household will consume $\lambda_t (w_t h_{i,t} + d_t)^{1-\tau} \alpha_{i,t}^{1-\tau}$ so (15) becomes

$$h_{i,t}^\gamma = \frac{(1-\tau)w_t}{w_t h_{i,t} + d_t}. \quad (16)$$

Notice that this equation pins down $h_{i,t}$ for all employed households regardless of their skill. Therefore all households select the same level of hours. Using (6) and (7), equation (16)

becomes

$$h_t^\gamma = \frac{(1 - \tau)w_t S_t}{A_t h_t}. \quad (17)$$

And finally we have

$$h_t = \left[\frac{(1 - \tau)w_t S_t}{A_t} \right]^{1/(1+\gamma)}, \quad (18)$$

which is the aggregate labor supply curve.

Turn now to the searching household's choice of search effort. The first order condition for search effort is

$$V(a, \alpha, 1, \mathcal{S}) - V(a, \alpha, 0, \mathcal{S}) = \frac{1}{M} q^\kappa. \quad (19)$$

We will now establish that in the no-trade equilibrium ($a = 0$), the difference on the left-hand side is independent of α , which implies that the value of q that solves (19) is independent of α . This all follows straightforwardly from the following result.

Claim 1. *In the no-trade equilibrium, $V(0, \alpha, n, \mathcal{S})$ can be written as $x \log(\alpha) + \bar{V}(n, \mathcal{S})$ for some constant x and function \bar{V} .*

Proof. Suppose that the value function is of this form. We will establish that the Bellman equation maps functions in this class into itself, which implies that the fixed point of the Bellman equation is in this class by the contraction mapping theorem. V^s will inherit the same form as V . To see this, note that

$$V^s(0, \alpha, \mathcal{S}) = x \log(\alpha) + Mq\bar{V}(1, \mathcal{S}) + (1 - Mq)\bar{V}(0, \mathcal{S}) - \frac{q^{1+\kappa}}{1 + \kappa}$$

and the choice of q is independent of α by the argument above. Moreover this implies

$$(1-v)V(0, \alpha, 1, \mathcal{S}) + vV^s(0, \alpha, \mathcal{S}) = x \log(\alpha) + [1 - v(1 - Mq)]\bar{V}(1, \mathcal{S}) + v \left\{ (1 - Mq)\bar{V}(0, \mathcal{S}) - \frac{q^{1+\kappa}}{1 + \kappa} \right\}.$$

So the continuation value inside the expectation in the Bellman equation is of the form $x \log(\alpha') + g(\mathcal{S}')$ where $g(\mathcal{S})$ is defined by the second and third terms in the above expression.

Turning now to the Bellman equation, in the no-trade equilibrium we have

$$\begin{aligned} V(a = 0, \alpha, n, \mathcal{S}) &= \log \left[\lambda ((n + (1 - n)b)\alpha(wh + d))^{1-\tau} \right] - \frac{h^{1+\gamma}}{1 + \gamma} + \beta \mathbb{E} [x \log(\alpha') + g(\mathcal{S}')] \\ &= (1 - \tau + \beta x) \log(\alpha) + \log \left[\lambda ((n + (1 - n)b)(wh + d))^{1-\tau} \right] \\ &\quad - \frac{h^{1+\gamma}}{1 + \gamma} + \beta \mathbb{E} \left[-\frac{x}{2} \sigma_\epsilon^{2'} + g(\mathcal{S}') \right]. \end{aligned}$$

Choosing $x = (1 - \tau)/(1 - \beta)$ delivers the desired functional form. \square

To calculate the optimal search effort, we must know the difference between the value of employment and the value of unemployment.

Claim 2. *In the no-trade equilibrium, $V(0, \alpha, 1, \mathcal{S}) - V(0, \alpha, 0, \mathcal{S}) = -\frac{h^{1+\gamma}}{1+\gamma} - (1 - \tau) \log b$.*

Proof. In the no-trade equilibrium, the continuation value in the Bellman equation

$$\beta \mathbb{E} [(1 - v)V(0, \alpha', 1, \mathcal{S}') + vV^s(0, \alpha', \mathcal{S}')]]$$

is independent of n . Therefore the difference in value functions comes from the payoffs within the period. Substituting in for consumption using the budget constraint yields the result. \square

Using Claim 2.4.1, we can rewrite (19) as

$$-\frac{h^{1+\gamma}}{1 + \gamma} - (1 - \tau) \log b = \frac{1}{M} q^\kappa. \quad (20)$$

We turn now to the Euler equation, which is

$$\frac{1}{c_{i,t}} \geq \beta R_t \mathbb{E} \left[\frac{1}{c_{i,t+1}} \right].$$

For an employed household this becomes:

$$\frac{1}{\lambda_t \alpha_{i,t}^{1-\tau} (w_t h_t + d_t)^{1-\tau}} \geq \beta R_t \mathbb{E} \left\{ \left[(\lambda_{t+1} \alpha_{i,t+1}^{1-\tau} (w_{t+1} h_{t+1} + d_{t+1})^{1-\tau})^{-1} \right] [(1 - u_{t+1}) + u_{t+1} b^{-1+\tau}] \right\}.$$

For an unemployed household the Euler equation is:

$$\frac{1}{b^{1-\tau} \lambda_t \alpha_{i,t}^{1-\tau} (w_t h_t + d_t)^{1-\tau}} \geq \beta R_t \mathbb{E} \left\{ \left[(\lambda_{t+1} \alpha_{i,t+1}^{1-\tau} (w_{t+1} h_{t+1} + d_{t+1})^{1-\tau})^{-1} \right] [(1 - u_{t+1}) + u_{t+1} b^{-1+\tau}] \right\}.$$

We can rewrite this as

$$\frac{1}{\lambda_t (w_t h_t + d_t)^{1-\tau}} \geq \beta R_t \mathbb{E} \left\{ \left[(\lambda_{t+1} (w_{t+1} h_{t+1} + d_{t+1})^{1-\tau})^{-1} \right] [(1 - u_{t+1}) + u_{t+1} b^{-1+\tau}] \right\} \mathbb{E} \left[\frac{\alpha_{i,t+1}^{1-\tau}}{\alpha_{i,t}^{1-\tau}} \right] D_{i,t}.$$

Where $D_{i,t} = 1$ for an employed household and $D_{i,t} = b^{1-\tau}$ for an unemployed household. Note that $\mathbb{E} \left[\frac{\alpha_{i,t}^{1-\tau}}{\alpha_{i,t+1}^{1-\tau}} \right] = \mathbb{E} [\epsilon_{i,t+1}^{\tau-1}] = e^{\frac{\sigma_{\epsilon,t+1}^2}{2} [\tau^2 - 3\tau + 2]}$. So in the end, the Euler equation of any household can be written as

$$\frac{1}{\lambda_t (w_t h_t + d_t)^{1-\tau}} \geq \beta R_t \mathbb{E} \left\{ \left[(\lambda_{t+1} (w_{t+1} h_{t+1} + d_{t+1})^{1-\tau})^{-1} \right] [(1 - u_{t+1}) + u_{t+1} b^{-1+\tau}] \right\} e^{\frac{\sigma_{\epsilon,t+1}^2}{2} [\tau^2 - 3\tau + 2]} D_{i,t}.$$

This only differs across households due to the term $D_{i,t}$. Assuming the unemployment benefit replacement rate is less than one, $D_{i,t}$ will be larger for employed than unemployed so in equilibrium all unemployed will be constrained and the Euler equation will hold with equality for all employed.

2.4.2 Consumption shares and aggregate Euler equation

For the analysis that will follow, it is useful to derive the consumption share of an individual with states $\alpha_{i,t}$ and $n_{i,t}$. Aggregate consumption is $C_t = Y_t - G_t$. The consumption of an individual is $c_{i,t} = \lambda_t \alpha_{i,t}^{1-\tau} (w_t h_t + d_t)^{1-\tau} (n_{i,t} + (1 - n_{i,t}) b^{1-\tau})$. Using the government budget

constraint we have

$$C_t = \lambda_t (w_t h_t + d_t)^{1-\tau} (1 - u_t + u_t b^{1-\tau}) \mathbb{E}_i [\alpha_{i,t}^{1-\tau}]. \quad (21)$$

Dividing $c_{i,t}$ by C_t we arrive at

$$s_{i,t} \equiv \frac{\alpha_{i,t}^{1-\tau} (n_{i,t} + (1 - n_{i,t}) b^{1-\tau})}{(1 - u_t + u_t b^{1-\tau}) \mathbb{E}_i [\alpha_{i,t}^{1-\tau}]},$$

where $s_{i,t}$ is the consumption share of individual i .

The consumption of individual i is $s_{i,t} C_t$ so we have

$$c_{i,t} = \alpha_{i,t}^{1-\tau} (n_{i,t} + (1 - n_{i,t}) b^{1-\tau}) \frac{C_t}{(1 - u_t + u_t b^{1-\tau}) \mathbb{E}_i [\alpha_{i,t}^{1-\tau}]}. \quad (22)$$

It is useful to interpret the terms in the denominator of the right-hand side of this equation. These terms represent the cross-sectional expectation of the consumption shares. Suppose C_t is fixed. Then conditional on $\alpha_{i,t}$ and $n_{i,t}$, individual i will get more consumption if others get less. For $b < 1$ an increase in unemployment raises $c_{i,t}$. Similarly, the share of an individual is higher if others pay high average tax rates because then the overall level of taxes can be lower. In a progressive tax system, a mean preserving spread of income will generate more revenue. This effect of the income distribution on the consumption share of individual i is captured by $\mathbb{E}_i [\alpha_{i,t}^{1-\tau}]$.

We can also use equation (22) to understand the determination of C_t given the consumption of an individual with certain characteristics. Suppose agent i is employed with $\alpha_{i,t} = 1$. Then we have

$$C_t = c_{i,t} (1 - u_t + u_t b^{1-\tau}) \mathbb{E}_i [\alpha_{i,t}^{1-\tau}].$$

For a given $c_{i,t}$, aggregate consumption is lower if unemployment rises as the unemployed consume less, but less so if b is high. Similarly, a mean-preserving spread of aggregate income lowers aggregate consumption because consumption is a concave function of pre-tax income.²

²Here it is important that we are considering consumption as a function of individual income and fixing

Define

$$Q_{t+1} \equiv \frac{(1 - u_{t+1} + u_{t+1}b^{1-\tau})\mathbb{E}_i [\alpha_{i,t+1}^{1-\tau}]}{(1 - u_t + u_t b^{1-\tau})\mathbb{E}_i [\alpha_{i,t}^{1-\tau}]} = \underbrace{\frac{(1 - u_{t+1} + u_{t+1}b^{1-\tau})}{(1 - u_t + u_t b^{1-\tau})}}_{\equiv Q_{t+1}^u} \underbrace{\frac{\mathbb{E}_i [\alpha_{i,t+1}^{1-\tau}]}{\mathbb{E}_i [\alpha_{i,t}^{1-\tau}]}}_{\equiv Q_{t+1}^\alpha}. \quad (23)$$

Then the Euler equation of the employed can be written as

$$C_t^{-1} = \beta R_t \mathbb{E} \left\{ Q_{t+1} C_{t+1}^{-1} [(1 - u_{t+1}) + u_{t+1}b^{-1+\tau}] \right\} e^{\frac{\sigma_{\epsilon,t+1}^2}{2} [\tau^2 - 3\tau + 2]}. \quad (24)$$

This Euler equation differs from the standard $C_t^{-1} = \beta R_t \mathbb{E}[C_{t+1}^{-1}]$ for three reasons. First it reflects the anticipated change in consumption shares due to changes in taxes, Q_{t+1} . Second, it reflects the risk of becoming unemployed in which case marginal utility is higher by a factor of $b^{-1+\tau}$. And finally, it reflects the risk of changes in α due to shocks to ϵ .

2.4.3 Price-setting problem

In this incomplete markets problem it is not immediately clear how to discount the future profits of the intermediate goods firm. We assume the firm discounts future profits using the real interest rate. The intermediate goods producer's problem is

$$\max_{p_t^*, \{y_s(j), N_s(j)\}_{s=t}^\infty} \mathbb{E}_t \sum_{s=t}^\infty R_{t,s} (1 - \theta)^{s-t} \left(\frac{p_t^*}{p_s} y_s(j) - w_s n_s(j) \right)$$

subject to

$$y_s(j) = \left(\frac{p_t^*}{p_s} \right)^{\mu/(1-\mu)} Y_s,$$

$$y_s(j) = A_s n_s(j),$$

the level of $c_{i,t}$, which pins down the level of the consumption function. In the aggregate we have $C_t = Y_t - G_t$ so a mean-preserving spread of income does not change aggregate consumption.

where $R_{t,s} \equiv \prod_{s'=t+1}^s R_{s'}$. The solution to this problem satisfies

$$\frac{p_t^*}{p_t} = \frac{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s} (1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{\mu/(1-\mu)} Y_s \mu w_s / A_s}{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s} (1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{1/(1-\mu)} Y_s}. \quad (25)$$

2.5 Equilibrium

The dynamics of the economy can be calculated from a system of 17 equations in 17 variables.

The variables are

$$C_t, u_t, \mathbb{E}_i [\alpha_{i,t}^{1-\tau}], Q_t, R_t, \sigma_{\epsilon,t}, I_t, \pi_t, Y_t, G_t, h_t, w_t, S_t, A_t, \frac{p_t^*}{p_t}, M_t, q_t.$$

And the equations are: (1), (2), (3), (5), (7), (8), (9), (10), (11), (12), (13), (14), (18), (20), (23), (24), (25).

3 Welfare function

A household with states $\alpha_{i,t}$ and $n_{i,t}$ receives a share of aggregate consumption equal to

$$s_{i,t} \equiv \frac{\alpha_{i,t}^{1-\tau} (n_{i,t} + (1 - n_{i,t})b^{1-\tau})}{\mathbb{E}_i [\alpha_{i,t}^{1-\tau} (n_{i,t} + (1 - n_{i,t})b^{1-\tau})]} = \frac{\alpha_{i,t}^{1-\tau} (n_{i,t} + (1 - n_{i,t})b^{1-\tau})}{\mathbb{E}_i [\alpha_{i,t}^{1-\tau}] (1 - u_t + u_t b^{1-\tau})}.$$

The utilitarian welfare function is then

$$\begin{aligned} & \mathbb{E}_i \mathbb{E}_0 \sum_{t=0}^{\infty} \hat{\beta}^t \left[\log(s_{i,t}) + \log(C_t) - \frac{h_{i,t}^{1+\gamma}}{1+\gamma} - \frac{q_{i,t}^{1+\kappa}}{1+\kappa} + \chi \log(G_t) \right] \\ &= \mathbb{E}_0 \sum_{t=0}^{\infty} \hat{\beta}^t \mathbb{E}_i \log(s_{i,t}) \\ &+ \mathbb{E}_0 \sum_{t=0}^{\infty} \hat{\beta}^t \left[\log(C_t) - (1 - u_t) \frac{h_t^{1+\gamma}}{1+\gamma} - v \frac{q_t^{1+\kappa}}{1+\kappa} + \chi \log(G_t) \right]. \end{aligned}$$

First observation: if the shares $s_{i,t}$ are exogenous (e.g. $\xi^\sigma = \xi^u = 0$) then inequality just creates a term that is independent of policy.

$\mathbb{E}_i \log(s_{i,t})$ can be decomposed into two terms that reflect the effects of dispersion in $\alpha_{i,t}$ and employment status

$$\begin{aligned}\mathbb{E}_i \log(s_{i,t}) &= \mathbb{E}_i \log \left(\frac{\alpha_{i,t}^{1-\tau}}{\mathbb{E}_i [\alpha_{i,t}^{1-\tau}]} \right) + \mathbb{E}_i \log \left(\frac{n_{i,t} + (1 - n_{i,t})b^{1-\tau}}{1 - u_t + u_t b^{1-\tau}} \right) \\ &= \mathbb{E}_i \log (\alpha_{i,t}^{1-\tau}) - \log (\mathbb{E}_i [\alpha_{i,t}^{1-\tau}]) + u_t \log (b^{1-\tau}) - \log (1 - u_t + u_t b^{1-\tau}).\end{aligned}$$

Calculations for numerical evaluation $\mathbb{E}_i \log (\alpha_{i,t})$ evolves according to

$$\begin{aligned}\mathbb{E}_i \log (\alpha_{i,t}) &= (1 - \delta) [\mathbb{E}_i \log (\alpha_{i,t-1}) + \mathbb{E}_i \log (\epsilon_{i,t})] \\ \mathbb{E}_i \log (\alpha_{i,t}) &= (1 - \delta) \left[\mathbb{E}_i \log (\alpha_{i,t-1}) - \frac{\sigma_{\epsilon,t}^2}{2} \right] \\ \mathbb{E}_i \log (\alpha_{i,t}) &= -(1 - \delta) \left[\frac{\sigma_{\epsilon,t}^2}{2} + (1 - \delta) \frac{\sigma_{\epsilon,t-1}^2}{2} + (1 - \delta)^2 \frac{\sigma_{\epsilon,t-2}^2}{2} + \dots + (1 - \delta)^{t-1} \frac{\sigma_{\epsilon,1}^2}{2} \right] + (1 - \delta)^t \mathbb{E}_i \log (\alpha_{i,0})\end{aligned}$$

3.1 Log-normal approximation to $\alpha_{i,t}$

$\log \alpha_{i,t}$ is distributed according to a mixture of normals. Here we analyze welfare under an approximation in which $\log \alpha_{i,t}$ is normally distributed with the same mean and variance. If $\alpha_{i,t}$ is log-normal then the term $\mathbb{E}_i \log (\alpha_{i,t}^{1-\tau}) - \log (\mathbb{E}_i [\alpha_{i,t}^{1-\tau}])$ simplifies to $-\frac{\sigma_{\alpha,t}^2(1-\tau)^2}{2}$, where $\sigma_{\alpha,t}^2$ is the variance of $\log(\alpha_{i,t})$. It follows that welfare is decreasing in $\sigma_{\alpha,t}^2$.

$\sigma_{\alpha,t}^2$ evolves as follows:

$$\begin{aligned}\sigma_{\alpha,t}^2 &= \mathbb{E}_i [(\log \alpha_{i,t})^2] - \mathbb{E}_i [\log \alpha_{i,t}]^2 \\ &= (1 - \delta) \{ \mathbb{E}_i [(\log \alpha_{i,t-1} + \log \epsilon_{i,t})^2] - \mathbb{E}_i [\log \alpha_{i,t-1} + \log \epsilon_{i,t}]^2 \} \\ &= (1 - \delta) [\sigma_{\alpha,t-1}^2 + \sigma_{\epsilon,t}^2].\end{aligned}$$

Therefore, an increase in $\sigma_{\epsilon,t}^2$ reduces welfare in period t but also in all future periods. The welfare impact of an increase in $\sigma_{\epsilon,t}^2$ fades over time for two reasons. First, there is the usual discounting of future payoffs and second there is the fact that the dispersion in α fades with

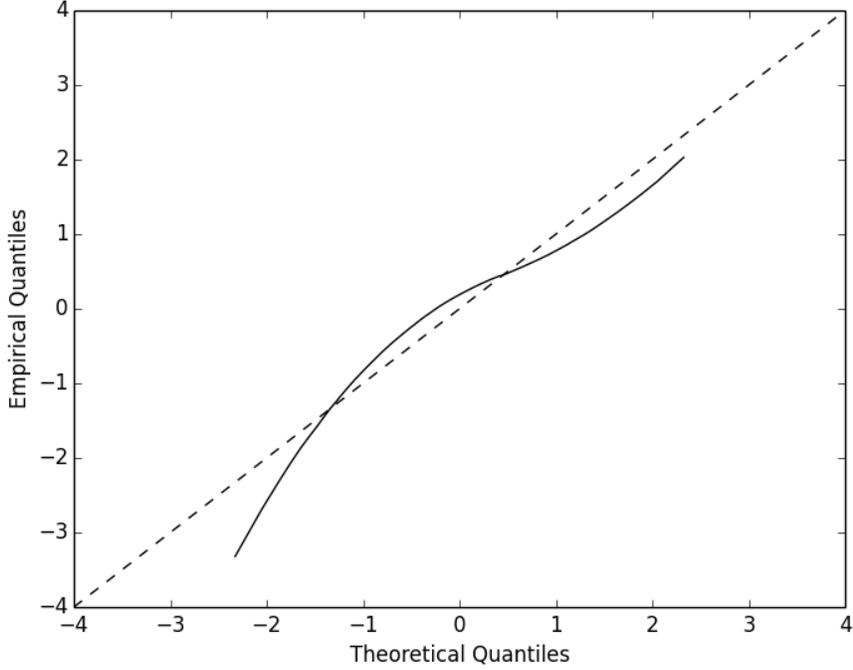


Figure 1: QQ plot of simulated distribution of $\log \alpha$ versus normal distribution.

mortality. Combining these two forces, the total welfare impact of $\sigma_{\epsilon,t}^2$ is

$$-\frac{(1-\tau)^2}{2} \frac{(1-\delta)\sigma_{\epsilon,t}^2}{1-\hat{\beta}(1-\delta)}.$$

As $\sigma_{\epsilon,t}^2$ is a decreasing function of Y_t , this is a source of welfare loss associated with low levels of Y_t .

Is this log-normal approximation at all reasonable? I did a simple calculation to check it. I simulated a population of households with $\sigma_{\alpha,t}^2 = 0.005$ and $\delta = 0.005$. I then standardized the simulated distribution of $\log \alpha$ by the sample mean and standard deviation and plotted the empirical quantiles versus the quantiles of the standard normal. Figure 1 shows the fit is decent but not great. For numerical solutions we do not need to make this approximation.

3.2 Welfare losses from inequality due to unemployment

Claim 3. For $b \in (0, 1)$ and $u \in [0, 0.5]$, the term $u_t \log(b^{1-\tau}) - \log(1 - u_t + u_t b^{1-\tau})$ is decreasing in u .

Proof. For the purposes of this proof, define

$$f(u, b) \equiv u \log(b^{1-\tau}) - \log(1 - u + ub^{1-\tau}).$$

The claim is then that $\frac{\partial f}{\partial u} < 0$ for $u \in [0, 0.5]$ and $b \in (0, 1)$. We have

$$\frac{\partial f}{\partial u} = \log(b) + \frac{1 - b}{1 - (1 - b)u}.$$

$u = 0.5$ maximizes $\frac{\partial f}{\partial u}$ on the domain $u \in [0, 0.5]$ for all $b \in (0, 1)$ therefore it is sufficient to check that $\frac{\partial f}{\partial u}|_{u=0.5} < 0$ for all $b \in (0, 1)$. This can be established by continuity and observing that $\frac{\partial f}{\partial u}|_{u=0.5, b=1} = 0$ and $\frac{\partial^2 f}{\partial u \partial b}|_{u=0.5} > 0$ for all $b \in (0, 1)$.

The cross partial is

$$\begin{aligned} \frac{\partial^2 f}{\partial u \partial b}|_{u=0.5} &= \frac{1}{b} + \frac{-(1 - (1 - b)/2) - (1 - b)/2}{(1 - (1 - b)/2)^2} > 0 \\ &\frac{1}{b} > \frac{1}{(1 - (1 - b)/2)^2} \\ &b < (1 - (1 - b)/2)^2 \\ &b < 1 - (1 - b) + (1 - b)^2/4 \\ &b < b + (1 - b)^2/4 \\ &0 < (1 - b)^2/4. \end{aligned}$$

□

If the unemployment rate is decreasing in Y , then the term $u_t \log(b^{1-\tau}) - \log(1 - u_t + u_t b^{1-\tau})$ is an increasing function of Y_t .

3.3 Approximate welfare function

Under the log-normal approximation described above, we can write the welfare function as

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \hat{\beta}^t \left[\log(C_t) - (1 - u_t) \frac{h_t^{1+\gamma}}{1 + \gamma} - v \frac{q_t^{1+\kappa}}{1 + \kappa} + \chi \log(G_t) + F(Y_t) \right],$$

where $F(Y_t)$ is increasing in Y_t and captures the welfare losses that result from the link between Y_t and inequality. $F(\cdot)$ will depend on the elasticity of inequality with respect to output (the ξ terms), how long-lived the idiosyncratic shocks are (captured by δ), and the extent to which social insurance shares this risk (τ and b).

4 AS-AD analysis

To provide intuition for how aggregate dynamics are affected by the automatic stabilizers we derive two equations that relate the price level to the level of output for a simplified version of our model. One equation captures the demand side of the economy and we call it the aggregate demand (curve) and the other equation captures aggregate supply (AS).

In this section, we use several modifications to simplify the model to make the analysis tractable. We assume that the economy begins at date $t = 0$ with known paths for the exogenous shocks. We assume A_t , η_t^G , η_t^u , and η_t^σ are constant and equal to their steady state values for $t \geq 1$. We assume that a fraction θ of firms cannot adjust their price in period 0, but all firms can adjust their prices in every period for $t \geq 1$. Let p_0^e denote the price of firms that cannot adjust in period 0. We assume that household search effort is exogenous and constant. Finally, we wish to eliminate the variable $\mathbb{E}_i [\alpha_{i,t}^{1-\tau}]$ as an endogenous state. One way to do this is to assume that monetary policy fully offsets the impact of Q_{t+1}^α in the Euler equation. Q_{t+1}^α plays a role of a demand shock which can be completely neutralized by monetary policy. If we assume that such a monetary policy is in place, it is not necessary to track $\mathbb{E}_i [\alpha_{i,t}^{1-\tau}]$ as a state because this variable only enters the model dynamics through

Q_{t+1}^α . Specifically, we will assume that monetary policy is such that the real interest rate is

$$R_t = \bar{R}(p_t) \frac{1}{Q_{t+1}^\alpha}.$$

4.1 The AD curve

Under our simplifying assumptions the Euler equation becomes

$$\frac{(1 - u_0 + u_0 b^{1-\tau})}{C_0} = \beta \bar{R}(p_0) \mathbb{E} \left\{ \frac{(1 - u_1 + u_1 b^{1-\tau})(1 - u_1 + u_1 b^{-1+\tau})}{C_1} \right\} e^{\frac{\sigma_{\epsilon,1}^2}{2}[\tau^2 - 3\tau + 2]}.$$

For dates $t \geq 1$ the economy will be in steady state because there are no endogenous states in this version of the model and we have assumed the exogenous variables return to steady state for $t \geq 1$. The expectation term in the Euler equation then is a known constant, call it E . While this constant may depend on policy parameters, it will not depend on the shocks that occur in period 0 or the values of Y_0 or p_0 . Therefore we do not need to analyze E to understand how policies affect the slope of the AD curve or how much it shifts in response to demand shocks.

We can then write the Euler equation as

$$(Y_0 - G_0) \frac{1}{1 - u_0 + u_0 b^{1-\tau}} e^{\frac{\sigma_{\epsilon,1}^2}{2}[\tau^2 - 3\tau + 2]} = \frac{1}{\beta \bar{R}(p_0) E}. \quad (26)$$

Taking logs of both sides of (26), using the approximation $\log(1 - u + ub^{1-\tau}) = -(1 - b^{1-\tau})u$, and arranging we arrive at

$$\log(Y_0) = -\log\left(1_0 - \frac{G_0}{Y_0}\right) - (1 - b^{1-\tau})u_0 - \frac{\sigma_{\epsilon,1}^2}{2}(\tau^2 - 3\tau + 2) - \log(\beta \bar{R}(p_0) E). \quad (27)$$

We call this relationship between $\log(Y_0)$ and the price level the AD curve. Note that u_0 and $\sigma_{\epsilon,1}$ are functions of Y_0 . The first three terms on the right-hand side of equation (27) correspond to the three automatic stabilizers in the model. We will consider each in turn analyzing how the fiscal structure alters both aggregate demand shocks and the slope of the

AD curve.

We begin with $-\log\left(1 - \frac{G_0}{Y_0}\right)$. Notice that an increase in G/Y raises aggregate demand. Therefore, exogenous changes in government spending are themselves aggregate demand shocks. In addition, if G/Y is negatively related to changes in Y there is an endogenous feedback from increases in $\log Y$ to decreases in G/Y that serve to stabilize output. This makes the AD curve less elastic.

Next we consider $-(1 - b^{1-\tau})u_0$. This term reflects the fact that increases in the unemployment rate reduce aggregate demand because the unemployed consume less than the employed. This effect is tempered by the unemployment benefit. Higher benefits therefore reduce the aggregate demand consequences of shocks to unemployment. If u_0 is itself a decreasing function of Y_0 , the aggregate demand curve becomes more elastic as a result because any change in Y_0 is further reinforced through changes in the unemployment rate. Again, this effect is tempered by unemployment benefits. So unemployment benefits make the aggregate demand curve less elastic and less sensitive to unemployment shocks.

Finally we consider the term $-\frac{\sigma_{\epsilon,1}^2}{2}(\tau^2 - 3\tau + 2)$. Note that $(\tau^2 - 3\tau + 2)$ is positive and decreasing in τ on the interval $[0, 1]$. An increase in σ_ϵ creates a precautionary savings motive that reduces aggregate demand. This precautionary savings motive is tempered by progressive taxation that provides social insurance. If there is negative relationship σ_ϵ and Y then the aggregate demand curve again becomes more elastic because any increase in output is further reinforced by a reduction in risk and a reduction in precautionary savings motives. This feedback is also tempered by social insurance. So progressive taxation makes the aggregate demand curve less elastic and less sensitive to changes in idiosyncratic risk.

Finally we turn to the last term on the right hand side of (27). The term E depends on the policy variables, but by assumption it is independent of Y_0 and u_0 . So the effects of the policies on E reflect the fact that these policies shift the aggregate demand curve through wealth effects and through the provision of social insurance against future unemployment spells. While in this analysis we have treated u_1 as a known constant, in a more general setting there could be volatility in the expectation of u_1 that would induce an additional

source of variable in precautionary savings motives. The unemployment benefit reduces the aggregate demand consequences of these unemployment fears.

4.2 AS curve

First, we can derive a relationship between the price level, p_0 , and the wage, w_0 . Any firm that can update its price in period 0 will choose to set it to the optimal markup over current marginal costs because all prices will be re-optimized in the next period. Therefore $p_0^* = \mu w_0 / A_0$. Using the aggregate price index we arrive at

$$p_0 = \left(\theta \left(\frac{\mu w_0}{A_0} \right)^{1/(1-\mu)} + (1-\theta) p_0^e \right)^{1-\mu}.$$

This is an increasing relationship between w_0 and p_0 . If we invert this we can consider w_0 as an increasing function of p_0 .

We use the labor supply condition and the production function to arrive at

$$Y_0 = \left(\frac{A_0}{S_0} \right)^{\gamma/(1+\gamma)} (1-u_0) [(1-\tau)w_0]^{1/(1+\gamma)}.$$

Taking logs of both sides

$$\log Y_0 = \frac{\gamma}{1+\gamma} [\log(A_0) - \log(S_0)] - u_0 + \frac{1}{1+\gamma} [\log(1-\tau) + \log(w_0)].$$

$\log S_0$ increases as p_0 deviates from p_0^e in either direction, but is zero to a first-order approximation. Therefore we have an upward sloping aggregate supply curve in the neighborhood of $p_0 = p_0^e$.

Aggregate supply is decreasing in the unemployment rate. Any negative feedback from Y_0 to u_0 will make aggregate supply more elastic as an increase in output is reinforced by a decline in the unemployment rate. Neither shocks to the unemployment rate nor the slope of the aggregate supply curve are affected by policy. In fact, the only place that the stabilizers appear is that the disincentive effect of progressive taxation serves to shift the aggregate

supply curve in.

5 Optimal Automatic Stabilizers

To be written.

6 Conclusion

To be written.

References

- BAILY, M. N. (1978): “Some aspects of optimal unemployment insurance,” *Journal of Public Economics*, 10(3), 379–402.
- CALVO, G. A. (1983): “Staggered Prices in a Utility-Maximizing Framework,” *Journal of Monetary Economics*, 12(3), 383 – 398.
- CHETTY, R. (2006): “A general formula for the optimal level of social insurance,” *Journal of Public Economics*, 90(10), 1879–1901.
- CONESA, J. C., S. KITAO, AND D. KRUEGER (2009): “Taxing Capital? Not a Bad Idea After All!,” *American Economic Review*, 99(1), 25–48.
- CONESA, J. C., AND D. KRUEGER (2006): “On the optimal progressivity of the income tax code,” *Journal of Monetary Economics*, 53(7), 1425–1450.
- FARHI, E., AND I. WERNING (2013): “A Theory of Macroprudential Policies in the Presence of Nominal Rigidities,” MIT working paper.
- GUVENEN, F., S. OZKAN, AND J. SONG (2014): “The Nature of Countercyclical Income Risk,” *Journal of Political Economy*, 122(3), pp. 621–660.
- HEATHCOTE, J., K. STORESLETTEN, AND G. L. VIOLANTE (2014): “Optimal Tax Progressivity: An Analytical Framework,” Discussion paper, National Bureau of Economic Research.
- KORINEK, A., AND A. SIMSEK (2014): “Liquidity Trap and Excessive Leverage,” MIT working paper.
- KRUEGER, D., AND A. LUDWIG (2014): “Optimal Capital and Progressive Labor Income Taxation with Endogenous Schooling Decisions and Intergenerational Transfers,” University of Pennsylvania working paper.
- KRUSELL, P., T. MUKOYAMA, AND A. A. SMITH JR (2011): “Asset prices in a Huggett economy,” *Journal of Economic Theory*, 146(3), 812–844.
- LANDAIS, C., P. MICHAILLAT, AND E. SAEZ (2010): “Optimal unemployment insurance over the business cycle,” Discussion paper, National Bureau of Economic Research.
- MCKAY, A., AND R. REIS (2014): “The Role of Automatic Stabilizers in the U.S. Business Cycle,” Boston University Working Paper.
- MIRRLEES, J. A. (1971): “An exploration in the theory of optimum income taxation,” *The review of economic studies*, pp. 175–208.
- STERK, V., AND M. RAVN (2013): “Job Uncertainty and Deep Recessions,” UCL Working Paper.

VARIAN, H. R. (1980): "Redistributive taxation as social insurance," *Journal of Public Economics*, 14(1), 49–68.